Optimal Policy with Occasionally Binding Credit Constraints: How Much Precautionary Policy in Emerging Markets?

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Abstract

We study optimal policy in a small-open economy in which a foreign borrowing constraint binds occasionally. The objective of the paper is to develop an optimal policy rule for both crisis periods when the borrowing constraints binds, and for periods of relative tranquility away from the crisis. Our approach to optimal policy allows us to determins the extent to which the optimal policy has a precautionary component. That is, should the government intervene before the constraint actually binds? In the model, the policy instrument is a distortionary tax on consumption of non-tradable goods. The tax affects the relative price of nontradable goods and is interpreted as an intervention to sustain the real exchange rate. We find that the optimal policy is highly nonlinear. If the constraint is not binding, the optimal tax rate is zero, as in an economy without a credit constraint, and hence there is no exchange rate intervention. Therefore there is no precautionary component to policy. If the constraint is binding, the optimal policy is to intervene aggressively by subsidizing the consumption of non-tradable goods. The welfare gains associated with such a policy are significant, and come about by both inducing lower self-insurance on the part of the private sector and alleviating the constraint when it does bind.

1　Introduction

Emerging market countries have experienced periodic crises that cause significant economic dislocation. These episodes, labeled "sudden stops" (Calvo, 1998), are characterized by a sharp reversal in private capital flows, large drops in output and consumption, coupled with large declines in asset prices and the real exchange rate. Progress has been made in understanding optimal policy responses in models in which the economy *is* in a sudden stop.[1] In this paper we address the complementary issue of optimal policy for an economy that *might* face a sudden stop. In doing so we solve for optimal policy both in and away from the crisis. Our approach allows us to address a key issue related to the design of policy in emerging markets: How much of a precautionary component should there be in the optimal policy? That is, at what point before a possible sudden stop should the government intervene? Should the government wait until the sudden stop strikes, or should it intervene as the conditional probability of the crisis rises?

We model sudden stops as a situation in which an international borrowing constraint becomes binding. The constraint in our model binds endogenously, depending on the history of production and savings decisions, as well as an exogenous technology shock. When the constraint does not bind the model economy exhibits normal business cycle fluctuations. The presence of the borrowing constraint, though, leaves the economy vulnerable to the possibility that a sequence of bad technology shocks can push the economy into the borrowing constraint, for certain level of foreign indebtedness. When this happens, the economy enters a crisis period and suffers the economic dislocation typically associated with a sudden stop episode.

To solve for optimal policy in this model we develop a global solution method. That is, we solve simultaneously for a policy rule across all states of the world–both when the constraint binds and when it does not. Such an approach enforces that the rule away from the crisis periods is designed with full knowledge of what the rule will be when we enter the

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[1]See Braggion, Christiano, Roldos (2007), Caballero and Panageas (2007), Christiano, Gust, and Roldos (2004), Cúrdia (2007), Caballero and Krishnamurthy (2005) on the optimal (monetary) policy response to these crisis periods.

sudden stop. This is true for both the policy maker and the agents in the economy. This solution method, while computationally costly, is critical for understanding the interaction between precautionary behavior on the part of the private sector with precautionary behavior on the part of the policy maker. The technical challenge in solving such a model is that the constraint binds only occasionally and changes location in the state space of the model.

To our knowledge, there are no contributions on the analysis of optimal policy in an environment where a constraint both binds occasionally and is endogenous to the decisions in the model. The most closely related work to ours includes Durdu and Mendoza (2005) who analyze broad alternative policy rules in a similar environment, but do not characterize optimal policy. Adams and Billi (2006a and b) study optimal monetary policy in a closed economy new Keynesian model in which there is zero lower bound on interest rates. Their zero-bound constraint is fixed and does not evolve endogenously. Bordo and Jeanne (2002) and Devereaux and Poon (2004) investigate precautionary components of optimal monetary policy responses to asset prices and sudden stops, respectively, but not in the context of a fully specified DSGE models.

Our endogenous borrowing credit constraint is embeded in a standard two-sector (tradable and non-tradable good) small open economy in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002). The asset menu is restricted to a one period risk-free bond paying off the exogenously given foreign interest rate. In addition, we assume that access to foreign financing is constrained to a fraction of households' total income. Foreign borrowing is denominated in units of the tradable good, while total income serves as collateral. This specification of the borrowing constraint captures "liability dollarization,"one of the key features of emerging market capital structure. In this class of models, agents self insure against the low-probability but high cost possibility of a sudden stop generated by the occasionally binding credit constraint. This is accomplished through precautionary saving by accumulation of net foreign assets.

Our policy instrument is a tax on non-traded consumption, which can be interpreted as an intervention on the real exchange rate. When the tax instrument is used, the government balances its budget period-by-period with lump sum taxation. The borrowing constraint

is on the individual level, so the agent knows that his work effort affects his income (and hence collateral for borrowing). However, in representative agent models, the individual takes prices (i.e. the relative price of the nontradeables and wages) as given. The worker then does not internalize the impact of his labor supply decisions on the relative price of nontradeables. In a sudden stop, when the relative price of the nontradedable is falling, the worker does not internalize the fact that by working less he is further lowering this price. Optimal policy then intervenes to counteract this externality by supporting the price of the nontraded good.

We find that the optimal policy is highly non-linear. If the credit constraint is not binding, optimal policy would mimic the one that would arise in an economy without a credit constraint (a zero tax rate in our model). Therefore, there is no precautionary component to the optimal policy. If instead the credit constraint is binding, the optimal policy is to intervene aggressively to subsidize the price of non-tradable consumption (i.e. defend the real exchange rate). This subsidy increases demand for nontradable goods. The worker then receives a higher wage, which increases labor supply and by extension the supply of nontraded goods. The increase in income, which serves as collateral for the debt, is just enough to keep the borrowing constraint from binding.

The commitment to implement the optimal policy has significant welfare implications. When comparing the solution of the model with and without the optimal policy agents in the model with the optimal policy accumulate 3.4 percent more foreign liabilities than in the economy without the optimal policy. This additional foreign borrowing allows agents to increase consumption by saving less (i.e. accumulating more foreign debt). In essence, agents can forgo less consumption to self insurance. In welfare terms, the gain from committing to the optimal policy is non-trivial. The amount one would pay in consumption equivalents to move from a world without the policy to one with the policy is about 0.2 percent of consumption at every date and stare of the world.[2] Relative to the size of the welfare gains reported in the business cycle literature, this is a significant number. Additionally, the

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[2]By comparison, the gain from moving from the world with the borrowing constraint to a world with no constraint is 0.5 percent of consumption.

welfare gain is larger for those economies with greater relative debt positions.

This paper is related to two other literatures. The first focuses on financial frictions that may help replicate the main features of the business cycle in emerging market economies (for example, Mendoza 1991, 2002, 2006, Neumeyer and Perri 2005 and Oviedo 2006). The second focuses on the analysis of optimal fiscal and monetary policy in dynamic general equilibrium models (for example, Chari and Kehoe, 1999; Schmitt-Grohé and Uribe, 2004). While studies of emerging market business cycles can provide a realistic description of the economic environment in which these economies operate, the question of how policy should be set in such environments, particularly outside of the crisis period, is very much open. The optimal fiscal and monetary policies developed in standard open economy models may not be appropriate to provide insight on how policy should be set in the environment faced by emerging market economies.

The rest of the paper is organized as follows. Section 2 presents analytic results from a simple two-period model to highight the intuition for our main numerical results. Section 3 describes a fully speficied DSGE model. Section 4 discusses its calibration and solution. Section 5 contains the main results of the paper, characterizing the optimal policy, discussing its working, and showing the welfare impact of the opitmal policy. Section 6 reports on the sensitivity of the results to key parameters. Section 7 concludes. Technical details, including the numerical algorithm we use to solve the model, are in appendix.

2   Lessons from a deterministic, two-period economy

Before turning to the full model we first study a simple two-period deterministic economy that can be solved analytically. We consider two cases: a one good economy and a two-good economy. The objective is to illustrate the core of our optimal policy problem by comparing the solution of the competitive economy with the planner's solution in both cases. Doing so provides intuition for the difference between the problem that the planner is solving and that of the competitive equilibrium. It will also show when there will be a role for opitmal policy to move us from the competitive equilibrium to the planners solution.

2.1 The one-sector case

Consider a two-period, deterministic small open economy. Production takes place only in the first period and the economy is subject to an endogenous borrowing constraint. The household maximizes the two-period utility flow:

$$\log c_1 - \frac{h_1^d}{d} + \beta \log c_2 \tag{1}$$

where c_1 and c_2 are consumption in period 1 and 2, and h_1 is period 1 labor. The household is subject to the following period-specific budget constraints:

$$w_1 h_1 + \pi + b_1 - T = (1 - \tau)c_1 + b_2$$

$$c_2 = b_2(1 + r) + Y_2$$

The wage rate in period 1 is w_1, b_i is the net foreign asset position in period i, π is the total profit from owning the firm, T are lump-sum taxes, r is the net real interest rate, τ is the distortionary tax on period 1 consumption, and Y_2 is the period 2 endowment of the tradeable good. Borrowing is resticted and cannot exceed the following constraint:

$$b_2 \geq -\frac{1 - \varphi}{\varphi}\left(w_1 h_1 + \pi\right),$$

where φ is a parameter that governs the tightness of the borrowing constraint. The household solves the usual lifetime utility maximization problem subject to the two budget budget constraints and the borrowing constraint.

The household owns the firm, which produces the traded good with the following production function:

$$Y_1 = z l_1^\alpha$$

where z is a scale factor and l_1 is the labor input. The firms' profit maximization is static. The firm maximizes its profits, π, by choosing the amount of labor input:

$$\max \pi = z l_1^\alpha - w_1 l_1.$$

Government policy in this model is the use of a distortionary tax on consumption. The tax can be used to subsidize the price of the consumption good. We assume that the government

runs a balanced budget rule so that the distortionary tax (subsidy) on consumption is offset with lump sum taxes (i.e. $T=\tau c_1$).

The competitive equilibrium in this economy is characterized by the first order conditions of the household and the firm and the government budget constraint. The equilibrium conditions for this model are straightforward to derive. (See the Technical Appendix for a full derivation of the solution). The key equation governing the competitive equilibrium allocations is:

$$h_1^{d-1} = \left[\frac{1}{c_1} - \frac{1-\varphi}{\varphi} \left(\frac{1}{c_1(1-\tau)} - \frac{1}{c_2}\beta(1+r) \right) \right] z\alpha h_1^{\alpha-1} \tag{2}$$

We next turn to the problem that the planner would solve so that we can compare the solutions. The planner maximizes the household's utility (1) subject to the aggregate resource constraint and the borrowing constraint. The period budget constraints become:

$$zh_1^\alpha + b_1 = c_1 + b_2,$$

$$c_2 = b_2(1+r),$$

while the borrowing constraint can be expressed as:

$$b_2 \geq -\frac{1-\varphi}{\varphi} z h_1^\alpha.$$

The key difference between the competitive equilibrium and the planners problem is that the planner chooses quantities directly, rather than quantities being driven by price movements as in the competitive equilibrium. Also, the planner's problem, while respecting the borrowing constraint, does not include a choice of taxes. An optimal policy then will be a tax rule that equates the competitive equilibrium allocation to the allocation from the planners problem. The solution of the planners problem (see Technical Appendix for full derivation) is given by the following equilibrium condition:

$$h_1^{d-1} = \left[\frac{1}{c_1} - \frac{1-\varphi}{\varphi} \left(\frac{1}{c_1} - \frac{1}{c_2}\beta(1+r) \right) \right] z\alpha h_1^{\alpha-1} \tag{3}$$

By comparing (2) with (3) it is evident that the two allocations will be equivalent if and only if $\tau = 0$, regardless of whether the borrowing constraint binds or not. That is, optimal policy replicates the planners solution by choosing a tax rate of zero regardless of the state of the world. In this case, the distortion imposed by the borrowing constraint cannot be undone as the planner is unable to relax the borrowing constraint with a fiscal distortion. Thus, the competitive allocation is constrained-efficient in this case (i.e., constrained by the existence of the borrowing constraint). In this one-good economy there is no role for policy. We will see that the case with two goods is different. This result motivates both our use of a multisector economy and our specification in the full model that the collateral constraint be defined over total income, rather than just income from the traded goods sector.

2.2 The two-sector case

Consider a two-sector small open economy. To keep the analysis simple, we assume that non-tradeable goods are produced and consumed only in the first period, while tradeable goods are endowed and consumed in both periods. Thus, the household maximizes the following utility flow:

$$\gamma \log c_1^T + (1 - \gamma) \log c_1^N - \frac{h_1^d}{d} + \frac{1}{2}\beta \log c_2^T \tag{4}$$

where c_1^T and c_1^N are the consumption of the tradeable and non-tradeable goods in period 1, while c_2^T is period 2 consumption of tradeable goods. The household is subject to the following period budget constraints:

$$w_1 h_1 + \pi + b_1 - T = (1 - \tau)p_1^N c_1^N + c_1^T + b_2$$

$$c_2^T = b_2(1 + r) + Y_2,$$

in which p_1^N denotes the relative price of non-tradeable in terms of tradeable, Y_1 is the endowment of tradeable goods in period 1, and distortionary taxation applies to non-tradeable consumption. The borrowing constraint is similar to before as agents can borrow only a fraction of the current income flow:

$$b_2 \geq -\frac{1 - \phi}{\phi}(w_1 h_1 + \pi).$$

The first order conditions of the household becomes:

$$\frac{\gamma}{c_1^T} = \lambda_1,$$

$$\frac{1-\gamma}{c_1^N} = \lambda_1 \lambda_1 (1-\tau) p_1^N,$$

$$\frac{1}{2} \frac{\beta}{c_2^T} = \lambda_2$$

$$-\lambda_1 + \lambda_2 \beta (1+r) + \mu = 0,$$

$$h_1^{d-1} = \left(\lambda_1 + \frac{1-\phi}{\phi} \mu \right) w_1$$

The firm's problem is also similar to the one sector case, the only difference being that now production choices are related to the non-tradeable sector: the firm maximizes profits,

$$\pi = Y_1 + p_1^N z h_1^\alpha - w_1 h_1,$$

by choosing only the labour input, and its first order condition is

$$w_1 = p_1^N z \alpha h_1^{\alpha-1}.$$

As before, the government budget constraint is

$$T = \tau p_1^N c_1^N.$$

The competitive allocation for this case is then characterized by the first order conditions of the household and the firm, the government budget constraint, and the equilibrium conditions in the tradeable and non-tradeable goods and labor markets.

Consider the corresponding planner problem. The planner maximizes the household's utility (4) subject to the aggregate resource constraint, the borrowing constraint, and the first order conditions of the competitive equilibrium by choosing τ :

$$c_1^T + b_2 = Y_1 + b_1$$

$$c_1^N = z h_1^\alpha,$$

$$c_2^T = Y_2 + b_2(1 + r).$$

The borrowing constraint from the planner's perspective can be written as:

$$b_2 \geq -\frac{1 - \varphi}{\varphi} \left(Y_1 + p_1^N z \left(l_1^N \right)^\alpha \right),$$

with $\frac{(1-\gamma)}{\gamma} \left(\frac{c^T}{c^N} \right) \frac{1}{(1-\tau)} = p_1^N$, so that we can rewrite it as:

$$b_2 \geq -\frac{1 - \varphi}{\varphi} \left(Y_1 + \frac{(1 - \gamma)}{\gamma} \frac{c_1^T}{(1 - \tau)} \right)$$

Thus, first order conditions for the competitive equilibrium become:

$$h_1^{d-1} = z \alpha h_1^{\alpha-1} \frac{1 - \gamma}{\gamma} \frac{c_1^T}{c_1^N} \frac{1}{\phi(1 - \tau)} \left[\gamma \left(\frac{c_1^N}{c_1^T} \right)^{1-\gamma} u_{C_1} - (1 - \phi) u_{C_2}(1 + r)\beta \right],$$

$$\frac{\gamma}{c_1^T} = \frac{(1 + r)\beta}{2 c_1^T}, \qquad \text{if } b_2 > -\frac{1 - \phi}{\phi} \left(Y_1 + \frac{1 - \gamma}{\gamma} \frac{c_1^T}{1 - \tau} \right),$$

$$\frac{\gamma}{c_1^T} \geq \frac{(1 + r)\beta}{2 c_1^T}, \qquad \text{if } b_2 = -\frac{1 - \phi}{\phi} \left(Y_1 + \frac{1 - \gamma}{\gamma} \frac{c_1^T}{1 - \tau} \right).$$

Unlike in the one-sector economy, the planners problem differs from the competitive equilibrium because the planner internalizes the effect of varying τ on the relative price p_1^N, while agents take p_1^N as given in the competitive equilibrium.

It is easy to see that if the economy is at a competitive equilibrium in which the collateral constraint is not binding and $\tau = 0$, then $\tau = 0$ is optimal from the planner's perspective as well.

We now focus on the case in which the constraint is binding and analyze the properties of the optimal policy problem. Without loss of generality, assume that $r = 0$ and $\beta = 1$ and, $\phi = 1/2$, and normalize $z = 1$. Then note first that, under this parametrization, the credit constraint is slack (i.e, $c_1^T = 2\gamma c_2^T$) at the competitive equilibrium under $\tau = 0$ if and only if[3]

$$b_1 \geq -\frac{4Y_1 + (2 - 4\gamma)Y_2}{3 - 2\gamma} \equiv b_{1*}. \tag{5}$$

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[3]Suppose first the credit constraint is slack (i.e, $c_1^T = 2\gamma c_2^T$), we have

$$b_2 = \frac{Y_1 + Y_2 + b_1}{1 + 2\gamma} - Y_2.$$

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Note in addition that, if condition (5) is violated (i.e., the credit constraint is not slack), there is always a $\tau_* > 0$ such that for $\tau \in [\tau_*, 1)$ there is one solution of the problem, not necessarily the optimal solution, such that the credit constraint holds with slack and $c_1^T = 2\gamma c_2^T$. In fact, if condition (5) is violated, from the credit constraint we have:

$$b_2 + Y_1 + \frac{1-\gamma}{\gamma} \frac{c_1^T}{1-\tau} \geq 0.$$

It is thus evident that for sufficiently large τ this last inequality always holds and there is always a value of the subsidy that can make the constraint slack so that $c_1^T = 2\gamma c_2^T$. The smallest value of τ (i.e., the smallest subsidy) for which the credit constraint holds with equality is

$$\tau_* = 1 - \frac{2(1-\gamma)(Y_1 + Y_2 + b_1)}{2\gamma(Y_2 - Y_1) - (2Y_1 + b_1)}.$$

In addition, utility is decreasing in τ, as τ gets further away from 0.

Consider now the case in which the credit constraint is binding and the allocation satisfies $2\gamma c_2^T > c_1^T$. It is possible to show that the boundary value of τ for which the condition $2\gamma c_2^T \geq c_1^T$ holds is the same as τ_* above. Notice however that, in the case in which the constraint is not slack, the feasible set of τ may include the negative region (taxation instead of subsidy) for some values for the endowments $Y_1 + Y_2 + b_1$. Specifically, there are two subcases for the feasible region of τ, depending on the value of b_1 relative to the endowments and the value of γ :

1. If $-(Y_1 + Y_2) \leq b_1 \leq -2Y_1$ and $\gamma \in (1/2, 1)$ or $-(Y_1 + Y_2) \leq b_1 \leq b_{1*}$ and $\gamma \in (0, 1/2]$, there is a solution that satisfies $c_1^T < 2\gamma c_2^T$ if and only if $\tau \in (\tau_*, 1)$.

2. If $-2Y_1 \leq b_1 \leq b_{1*}$ and $\gamma \in (1/2, 1)$, a solution satisfies $c_1^T < 2\gamma c_2^T$ if and only if $\tau < \tau_*$.

Substituting this back into the credit constraint and setting $\tau = 0$, we get (5). Suppose now condition (5) is violated (at $\tau = 0$ and with $Y_1 + b_1 + Y_2 \geq 0$). The credit constraint can be rewritten as

$$\frac{Y_1 + Y_2 + b_1}{1 + 2\gamma} - Y_2 + Y_1 + \frac{1-\gamma}{\gamma} \frac{2\gamma}{1+2\gamma}(Y_1 + Y_2 + b_1) \geq 0.$$

It is straightforward to see that the left hand side is no less than 0 if and only if $b_1 \geq b_*$.

From the first subcase we have two allocations for a given $\tau \in (\tau_*, 1)$: one with $c_1^T = 2\gamma c_2^T$ from the discussion above, and the other with $c_1^T < 2\gamma c_2^T$ from the first subcase 1. However, in the solution with $c_1^T < 2\gamma c_2^T$ the credit constraint binds, and thus it is dominated by the solution with $c_1^T = 2\gamma c_2^T$. In addition, utility decreases for $\tau \geq \tau_* > 0$. It follows that τ_* achieves the global maximum.

In subcase 2, however, we only have one allocation for every value of $\tau < 1$. That is, the solution satisfies $c_1^T = 2\gamma c_2^T$ when $\tau \geq \tau_*$ and satisfies $c_1^T < 2\gamma c_2^T$ when $\tau < \tau_*$. It follows that τ_* does not achieves the global maximum. For instance, if $Y_1 = 1$, $Y_2 = 3$, $b_1 = 1.2$, $\gamma = 0.95$, $\alpha = 0.5$, $d = 1.5$, then $\tau_* = 0.133$, but optimal $\tau = 0.0467$. Therefore, the optimal τ is not necessarily equal to τ_* in the second subcase.

More genrally, one should expect that, if the allocation is such that the collateral constraint is not satisfied with either equality or slack (i.e., an allocation that, for given τ, would violate the collateral constraint), for any given τ, under certain values of the exogenous variables and the model parameters, only one of the two conditions above hold for given values values of τ (that is to say that the set of feasible allocations for given τ contains only one element and there is a one to one mapping between tau and allocations). For other values of the exogenous variables, instead, both situations may be valid (that is to say that the set of feasible allocations for given τ contains two elements). However, in the latter case, since last condition imposes the credit constraint onto the planner's problem, the solution with the first dominates the solution with the second, and therefore we can rank them.

To summarize, the planner allocation is achieved at $\tau = 0$ when (5) is satisfied. Otherwise, in the first subcase, τ_* achieves the global maximum. In the second subcase, the optimal value of τ may be strictly smaller than τ_*. Note that in the latter case we have a strictly positive multiplier attached to the credit constraint at the planner solution.

3   Model

This section simplifies the two-good, small open, production economy, with an occasionally binding credit constraint, originally proposed by Mendoza (2002). Compared to that model,

we consider only one source of disturbance, to aggregate productivity in the tradable sector of the economy. The specification of endogenous discounting is also simplified by assuming that the agents' discount rate depends on aggregate consumption as opposed to the individual one, as in Schmitt-Grohé and Uribe (2003).[4]

3.1 Households

There is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \left\{ \sum_{t=0}^{\infty} \exp(-\theta_t) \, u \left(C_t^j - z \left(H_t^j \right) \right) \right\}, \tag{6}$$

with C^j denoting the individual consumption basket and H^j the individual supply of labor. We assume that the expected utility includes an endogenous discount factor as:

$$\theta_t = \theta_{t-1} + \beta \ln \left(1 + C \left(C_t^T, C_t^N \right) - z \left(H_t \right) \right)$$

$$\theta_0 = 1,$$

with C denoting aggregate per capita consumption that the individual household takes as given.[5]

The functional form of the period utility function is,

$$u \left(C \left(C_t^T, C_t^N \right) - z \left(H_t \right) \right) \equiv \frac{1}{1 - \rho} \left(C_t - \frac{H_t^{\delta}}{\delta} \right)^{1 - \rho}, \tag{7}$$

omitting for simplicity the superscript j, and where δ is the elasticity of labor supply with respect to the real wage, and ρ is the coefficient of relative risk aversion. The consumption basket C is a composite of tradable and non-tradables goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{\frac{\kappa - 1}{\kappa}} + (1 - \omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{\frac{\kappa - 1}{\kappa}} \right]^{\frac{\kappa}{\kappa - 1}}. \tag{8}$$

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[4]So, our formulation corresponds to what Schmitt-Grohé and Uribe (2003) call "endogenous discount factor without internalization. Due to precautionary savings it may not be necessary in the stochastic model.

[5]Endogenous discounting pins down a well defined net foreign asset position in the deterministic steady state of the model.

The parameter κ denotes the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω represents a weighting factor. The corresponding aggregate price index is given by

$$P_t = \left[\omega + (1 - \omega) \left(P_t^N \right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}};$$

where the price of tradables is normalized to 1.

Households maximize utility subject to the following period budget constraint expressed in units of tradable consumption (where again for simplicity we omit the superscript j):

$$C_t^T + \left(1 + \tau_t^N \right) P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} - (1 + i) B_t - T_t^T - P_t^N T^N, \qquad (9)$$

where W_t is the real wage, B_{t+1} denotes the amount of bonds issued with gross real return $1 + i$, τ_t^N is a distortionary taxes on non-tradables consumption, and T^T and T^N are lump sum taxes in units of tradables and non-tradables, respectively. π_t represents per capita firm profits and $W_t H_t$ represents the household labor income.

International financial markets are incomplete and access to them is also imperfect. International borrowing cannot be made state-contingent, because the asset menu includes only a one period bond denominated in units of tradable consumption, paying off the exogenously given foreign interest rate. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geqslant -\frac{1 - \phi}{\phi} \left[\pi_t + W_t H_t \right]. \qquad (10)$$

This constraint (10) depends endogenously on the current realization of profits and wage income. One important feature of (10), is that it captures the effects of liability dollarization since foreign borrowing is denominated in units of tradables while part of the income that serves as a collateral comes from the nontradables sector. Roughly speaking, this constraint assumes that only a fraction of current income can be effectively claimed in the event of default, so lenders are unwilling to permit borrowing beyond that limit.

As emphasized in Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction

between a risk-averse borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981). It is also consistent with anecdotal evidence on lending criteria and guidelines used in mortgage and consumer financing. However, it is not derived as the outcome of an optimal credit contract.

Households maximize (6) subject to (9) and (10) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first order conditions of this problem are the following:[6]

$$\frac{C_{C_t^N}}{C_{C_t^T}} = \left(1 + \tau_t^N\right) P_t^N, \tag{11}$$

$$u_{C_t} C_{C_t^T} = \mu_t, \tag{12}$$

$$\mu_t + \lambda_t = \exp\left(-\beta \ln\left(1 + C\left(C_t^T, C_t^N\right) - z\left(H_t\right)\right)\right)(1+i) E_t\left[\mu_{t+1}\right], \tag{13}$$

and

$$z_H(H_t) = C_{C_t^T} W_t \left[1 + \frac{\lambda_t}{\mu_t}\frac{1-\phi}{\phi}\right]. \tag{14}$$

μ_t and λ_t are the multipliers on the budget and liquidity constraint, respectively. As usual, the relevant transversality conditions are assumed to be satisfied. Equation (11) determines the optimal allocation of consumption across tradable and nontradable goods by equating the marginal rate of substitution between C_t^N and C_t^T with the relative price of non-tradable and distortionary taxation in the nontradable sector. Equation (12) determines the multiplier μ_t in terms of the marginal utility of tradable consumption. Equation (13) determines the optimal choices of foreign bonds and thus saving. Note that when the the credit constraint is binding, and hence the multiplier $\lambda_t > 0$, the standard Euler equation incorporates effects that can be interpreted as arising from a country-specific risk premium on external financing, influenced by the degree of risk aversion of the households. Finally (14) determines the optimal supply of labor as a function of the relevant real wage and the multipliers. Again,

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[6]We denote with $C_{C_t^N}$ the partial derivative of the consumption index C with respect to non-tradable consumption. u_C denotes the partial derivative of the period utility function with respect to consumption and z_H denotes the derivative of labor disutility with respect to labor.

it is important to note that a binding international credit constraint increases the marginal benefit of supplying one unit of labor since this improves households' borrowing capacity. Consumption, saving, equilibrium labor effort, and output are therefore distorted by the presence of a binding credit constraint in a manner that depends on the interaction between the two effects highlighted above.

3.2 Firms

Our small open economy is endowed with a stochastic stream of tradable goods, $\exp(\varepsilon_t^T)Y^T$, where ε_t^T is a random Markov disturbance, and produces non-tradable goods, Y^N. Unlike Mendoza (2002), we assume that ε^T follows a standard autoregressive process of the first order, AR(1). However, we abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables, the world interest rate, and the tax rate for simplicity.

Firms produce non-tradables goods Y_t^N based on the following Cobb-Douglas technology

$$Y_t^N = AK^\alpha H_t^{1-\alpha},$$

where K is a constant level of capital stock, and A is a scaling factor. Since capital stock is given, the firm's problem is static and current-period profits (π_t) are:

$$\pi_t = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N AK^\alpha H_t^{1-\alpha} - W_t H_t.$$

The first order condition for labor demand, in equilibirum , gives:

$$W_t = (1-\alpha) P_t^N AK^\alpha H_t^{-\alpha}, \tag{15}$$

so the real wage (W_t) is equal to the value of the marginal product of labor.

3.3 Government

The government runs a balanced budget in each period, so that the consolidated government budget constraint is given by

$$\exp(G_t^T) + P_t^N \exp\left(G_t^N\right) = \tau_t^N P_t^N C_t^N + T_t^T + P_t^N T_t^N.$$

Stabilization policy is implemented by means of a distortionary tax rate τ_t^N on private domestic non-tradables consumption. Mendoza and Uribe (2000) emphasize how movements in this tax rate can approximate some of the effects induced by currency depreciation in monetary models of exchange rate determination. As such, it captures one important aspect of monetary policy in emerging markets, which is distinct from the more conventional role of monetary policy in the presence of nominal rigidities.

Movements in the primary fiscal balance are offset via lump-sum rebates or taxes. Specifically, we assume that the government keeps a constant level of non-tradable expenditure financed by a constant lump-sum tax (i.e. $\exp\left(G^N\right) = T^N$). Thus, changes in the policy variable τ^N are financed by a combination of changes in the lump-sum transfer on tradables, T_t^T, and the endogenous response of the relative prices, for given public expenditures on tradable and non-tradables. This simplifying assumption implies that we abstract from the important practical issue of how to finance changes in the tax rate in the case in which they are negative (i.e., $\tau^N < 0$ is a subsidy). This allow us to focus on the implications of the occasionally binding constraint for the design of the tax policy abstracting from other optimal tax policy considerations.

3.4 Aggregation and equilibrium

We now consider the aggregate equilibrium conditions. Combining the household budget constraint, government budget constraint, and the firm profits we have that the aggregate constraint for the small open economy can be rewritten as

$$C_t^T + P_t^N C_t^N + B_{t+1} = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y_t^N + (1+i) B_t - \exp\left(G_t^T\right) - P_t^N \exp\left(G_t^N\right),$$

and the equilibrium condition in the non-tradeable good sector is

$$C_t^N + \exp\left(G_t^N\right) = Y_t^N = A K^\alpha H_t^{1-\alpha}. \tag{16}$$

Combining these two equations we have

$$C_t^T = Y_t^T - \exp(G_t^T) - B_{t+1} + (1+i) B_t, \tag{17}$$

that represents the evolution of the net foreign asset position as if there were no international borrowing constraint. In this model though, using the definitions of firm profit and wages, the liquidity constraint implies that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi} \left[\exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y^N \right]. \tag{18}$$

4   Calibration and solution

In this section we discuss the calibration of model parameters and describe the solution method. We then solve for the competitive equilibrium in the model and show how a sudden stop episode occurs in our model.

4.1   Calibration

The calibration of the model is reported in Table 1 and follows the work of Mendoza (2002, 2006) and Kim and Ruhl (2005).[7] We follow Mendoza (2002) in setting the world interest rate to $i = 0.0159$, which yields an annual real rate of interest of 6.5 percent.[8] The elasticity of intratemporal substitution between tradables and nontradables follows from Ostry and Reinhart (1992) who estimates a value of $\kappa = 0.760$ for developing countries.[9] The elasticity of labor supply for the non-tradable sector is unitary, implying that $\delta = 2$ while the elasticity of intertemporal substitution is set to $\rho = 2$. These values are consistent with many studies in the real business cycle literature.[10] The value of the liquidity parameter determines the tightness of the constraint, and hence the probability of a sudden stop. We choose this parameter so that the probability of a sudden stop is 2 percent, which is consistent with

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[7]We will investigate the sensistivity of our results to this calibration. Our sensitivity analysis will be aimed at matching features of other emergings markets, such as a higher probability of a sudden stop than we observe in the Mexican data.

[8]This interest rate is greater than the 5 percent real rate in Kehoe and Ruhl (2005), and less than 8.6 real rate in Mendoza (2006).

[9]There is considerable debate about the value of this parameter (see Ruhl (2004)). The Ostry and Reinhart estimate we use is consistent with Kehoe and Ruhl (2005) who set this parameter to $\kappa = 0.5$.

[10]Since estimates for the elasticity of labor supply in the non-tradable sectore are not available for Mexico, Mendoza (2002) sets it to unitary value.

Mexican data. We also include a second calibration following Jeanne and Ranciere (2008) who use a probability of 10 percent.

The labor share of production in the non tradable output sector is $\alpha = 0.636$. This value is the value that Kehoe and Ruhl (2005) estimate using an input-output matrix for Mexico for the year 1989. We then set β, ω, and AK^α to obtain a steady state foreign borrowing to GDP ratio of 43 percent[11], a steady state ratio of tradable to non-tradables output of 64.8 percent, and the steady state relative price of non-tradables equal to one. The implied discount factor is $\beta = 0.0177$, which is slightly lower than in Mendoza (2002) because of the different specification of endogenous discounting. The tax rate on nontradable consumption in the steady state is fixed at $\tau = 0$. Government spending and lump sum taxes are also set to zero in the steady state. We normalize output by setting $Y_T = 1$ in the steady state.

In our analysis we focus on the behavior of the economy following a stochastic shock to tradeable output, which we model as an AR(1) process. Specifically, the shock process is ε_t,

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \sigma_v v_t, \tag{19}$$

where v_t is an iid N(0,1) innovation, and σ_v is a scaling factor. The parameters of the AR(1) process are chosen to match the standard deviation and serial correlation of tradeable output in Mexico of 3.36 percent and 0.553, respectively.

4.2   Solution

The algorithm for computing the competitive equilibrium is adapted from Kydland (1989) and Coleman (1991). The method involves solving a seqeuence of equilibria to construct a recursive competitive equilibrium. The algorithm involves first solving a static model, followed by solving a two-period model using the solution to the static model as the continuation utility. The algorithm then continues in this backward fashion to construct the entire equilibrium. Key ingredients of the procedure include the solution to a set of functions governing the equilibrium prices and quantities, as well as the maximization of the value function. A complete description of our procedures, including the functional equations

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[11]This corresponds to the estimates obtained by Lane and Milesi Ferretti (2006) for the period 1990-2006.

we solve and optimization techniques employed, is in the Appendix. It is important to note that our optimization step does respect the occasionally binding borrowing constraint.

4.3 Competitive Equilibrium

In Figure 1 and 2 we compare the equilibrium decision rule (or policy function) for our constrained model economy and an unconstrained economy (i.e. the real business cycle small open economy case). Figure 1 plots the policy functions for foreign borrowing, $B_{t+1} = g\left(B_t, \varepsilon_t^T\right)$,[12] conditional on a specific value of the tradable shock (i.e., the negative of one standard deviation of its marginal distribution). This intersects the $45°$ line at the boundary of the constrained region; that is, if the economy perpetually received this realization of the shock, it would converge to a level of external debt for which the credit constraint is just binding. If the economy happened to find itself in the interior of the constrained region, it would diverge to $B = -\infty$, violating the implicit trasversality condition that requires long-run solvency. Therefore the decision rules must be truncated at the boundary. This divergence would occur in any state in which there exists a positive probability of entering the interior of the constrained region, and this probability is always positive with an AR(1) process.

Figure 2 compares the equilibrium decision rules, with and without the credit constraint, for the real wage (W_t), the relative price of non tradable (P_t^N), aggregate consumption (C_t), employment (H_t). The first point to observe is that both employment and consumption are lower in the presence of the credit constraint, reflecting precautionary savings driven by the possibility of hitting the credit constraint. Consumption is lower because households save a higher fraction of their income to accumulate foreign assets. Even if the constraint is not binding, the reduction in consumption drives down the relative price of non-tradables because demand of non-tradables falls more than the supply of non tradables since agents save more for precautionary reasons. Supply of non-tradables falls as the negative effect of a reduction in labor demand dominates the positive effect on labor supply of a decline in the relative price of nontradables.

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[12] As there is only one asset, gross and net foreign liabilities or assets (NFA) coincide.

Equilibrium real wages, relative price and labor fall sharply as the economy approaches the region in which the constraint becomes binding since, as NFA deteriorates, the precautionary motive determines a bigger drop in consumption and the possibility of hitting the constraint amplifies the equilibrium response.

Precautionary saving induced by the occasionally binding credit constraint is quantitatively significant in the model. For instance, the average NFA position in the ergodic distribution of the economy with no collateral constraint is $B = -3.0$ (this corresponds to about -30 percent of annual average GDP), while in the economy with the constraint $B = -2.37$ (or -22 percent of annual average GDP).[13]

This difference is large, considering the small shocks that hit this economy and the relatively low degree of risk aversion. In contrast, Aiyagari (1994) finds that measured uninsurable idiosyncratic earnings risk, which is an order of magnitude larger than the shocks considered here, generates only a 3 percent increase in the aggregate capital stock. The main reason why precautionary savings is larger here is that the return on saving increases as the price of non-tradables falls, since gross real interest rate in terms of consumption good increases. So additional saving does not reduce its return, a mechanism that tends to weaken precautionary balances in a closed economy setting, such as the one studied by Aiyagari (1994).

5 Optimal stabilization policy

Once we have examined the implications of endogenous borrowing constraint for the small open economy, it is natural to ask how stabilization policy should be designed in this environment. Does optimal policy exhibit any precautionary motive? And, how does optimal policy affect private agents' decisions?

To address these questions and solve the optimal tax problem, the only change needed to the solution method discussed in the previous section is to introduce a second control into the problem to (??) —the tax rate on non-tradable consumption, τ_t^N—yielding the functional

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[13]See below on the welfare implications of this precautionary saving.

equation

$$\widehat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}, \tau_t^N} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t))) E\left[V^n\left(B_{t+1}, \varepsilon_{t+1}^T\right)\right] \right\}. \tag{20}$$

The decision rule for τ_t^N, as well as the implied lump sum transfer over GDP ($P_t^N T_t^T / Y$), which adjust in order to satisfy the balanced budget rule by the government, are plotted in Figure 3. Figure 4 plots the same decision rules reported in Figure 2, with and without optimal policy for τ_t^N. Figure 5 plots the policy function for the net foreign asset position with and without the optimal policy. Figure 6 reports the ergodic distribution of net foreign asset position for the economy without borrowing constraint, the one with borrowing constraint, and the one in which policy expressed in terms of τ_t^N is optimally set.

Optimal policy has two possible roles in the model.[14] The first role is related to the existence of the occasionally binding constraint, and there are two goals for policy due to the presence of the credit constraint (though not exclusive)—to reduce the probability of reaching the region in which the constraint is binding and to minimize the effects when it binds by increasing the value of the collateral. The second possible role, as in other incomplete market models (such as Aiyagari 1995), is to increase welfare by choosing policies that reduce agents precautionary savings. This role for policy is independent of the presence of the credit constraint and relies only on the general inefficiency of incomplete market models. As we will see, there is no scope for this second role for policy with this formulation of the policy problem.

As we can see from Figure 3, the optimal policy schedule, the equilibrium decision rule for the policy instrument τ^N is highly non-linear: in states of the world in which the constraint is not binding (in normal or tranquil times) the optimal policy is "no policy action", i.e., $\tau^N = 0$, while in states of the world in which the constraint is binding (in the "sudden stop" region), the optimal policy is a subsidy to non-tradable consumption, i.e. $\tau^N < 0$.

This result shows that there is no precautionary motive for the optimal policy either

[14]Our model also features an externality—the endogenous discount factor depends on aggregate consumption, and therefore agents do not internalize the effect of current consumption and labor supply on discounting. But this effect should be minor since the discount factor is nearly inelastic.

related the presence of the borrowing constraint or market incompleteness. When the constraint is not binding, policy is set as to minimize the distortion associated with the use of the policy instrument so that $\tau^N = 0$, like in the unconstrained economy.[15] In our relatively simple model, the financing of the optimal policy response to the sudden stop is cost free. Hence, a large subsidy can be applied right when the constraint binds. If the budget were to be balanced by a distortionary source of financing (e.g., a tax on labor or capital income rather then a lump-sum transfer), the government would find it optimal to start using the consumption subsidy before hitting the credit constraint.[16]

When the constraint is not binding, there is a trade-off between efficiency (i.e., to minimize marginal distortions by setting $\tau^N = 0$), and the need to mitigate the effects of the credit constraint. The planner does so by subsidizing non tradable consumption, which increases the value of the collateral in the sudden stop region, "lifting" the decision rules for any level of foreign borrowing, thus relaxing the borrowing constraint. The worse is the state of the world in terms of negative net foreign asset position, the bigger is the subsidy required to rise the value of the collateral.

Specifically, with such a subsidy, demand and to a lesser extent supply, of non-tradable goods increases, as a result the relative price of non-tradable goods rises, so that the value of the collateral also increases. Demand of non-tradables raises relative to demand of tradables to match the tax-adjusted relative price of non-tradable goods. Supply of non-tradable goods also increases, as the post-tax real wage increases and stimulates labor supply, but by less than the increase in non-tradable demand. In equilibrium, a higher non-tradable relative price stimulates labor demand by increasing the value of the marginal product of labor, but also offsets in part the expansionary effect of a lower tax on labor supply because it decreases the pre-tax real wage relevant for households decisions.

Optimal policy affects private agents' behavior even when the constraint is not binding. As we can see from Figure 4, in the model with optimal policy, agents consume more, and the equilibrium real wage, the relative price of non-tradable, and labor are higher than in the

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[15]In the model without credit constraint, there is no policy trade off, and setting $\tau^N = 0$ is always optimal, because incompleteness of the international asset market is imposed as a constraint on the planner problem.

[16]Extending the analysis in this direction is work in progress.

model without optimal policy for all levels of foreign borrowing. Private agents anticipate the policy response in the binding region and they reduce their precautionary saving by increasing consumption, so that P_N is higher compared to the model without optimal policy. A higher relative price of non-tradable stimulates labor demand, but tends to dampen labor supply. As a result, in equilibrium, both labor and the wages are higher.

Another way to see the impact of the optimal policy on private sector behavior is to look at how the region in which the constraint is binding changes. Figure 5 illustrates the effect of optimal policy from this standpoint. The binding region of the constraint shrinks with optimal policy so that, for a given realization of the shock and initial net foreign asset position, the amount of foreign borrowing allowed before the constraint starts to bind is larger. So, in general equilibrium, optimal policy "creates room" for precautionary saving to fall and consumption and foreign borrowing to raise. Interestingly, in our model economy, optimal policy also lowers the likelihood of entering the binding region for each pair (B_t, ε_t^T), despite the fact that households reduce precautionary savings in the presence of optimal policy. In fact, the probability of hitting the constraint in the ergodic distribution decreases by about 15 percent, from 0.6 percent without optimal policy to 0.5 percent with it.

Consistent with the effects on the binding region and the probability to hit the constraint, the average NFA position in the ergodic distribution of the economy is not affected significantly by optimal policy. As Figure 6 shows, relative to the no-stabilization case (with the credit constraint), the average net foreign debt in terms of the deterministic steady state GDP increases only by 3.4 percent, to B=-2.45 under the optimal policy (or 22 percent of average GDP that is also higher with optimal policy). The welfare implications of optimal policy, however, are significant as we shall see below.

Figure 5 also shows that the optimal policy is such that the liquidity constraint becomes "just binding"; that is, the policy function for B_t is tangent to the binding region and the corresponding multiplier λ_t of the liquidity constraint remains 0. The goal of optimal policy is to distort the economy as little as possible, and any deviation of the shadow price of foreign borrowing from zero is costly. Therefore the planner relaxes the constraint just enough to make it non-binding. But the constraint is not relaxed beyond this, because that involves

additional distortions that are welfare-reducing.

The optimal policy is state-contingent, requiring knowledge of the unobservable shocks and net foreign asset position for its implementation. We therefore also explore the impact of simple, constant subsidy rules that are not state contingent, and can be easily financed (meaning relatively small). Figure 7 shows how the constrained regions change moving from the economy without policy to the ones with the non-state contingent subsidies that we consider ($\tau^N = 0$, $\tau^N = -0.01$, $\tau^N = -0.05$). This shows that the fixed tax rule moves the economy in the direction of the optimal policy.[17]

6   Sensitivity analysis

In this section we explore the robustness of the optimal policy results to alternative values of key structural parameters, as well as of the stochastic process for the tradable endowment. From the outset, it is important to mention that none of these changes affect the main result, namely the absence of a precautionary component in the optimal policy.[18] This suggests that the result is a robust qualitative feature of the model. The inner working of the model, as illustrated by the decision rules for the main endogenous variables, is also fairly robust to alternative parameterization.

As the optimal policy hinges on the labor effort behavior and the substitutability between tradable and non-tradable goods in consumption, it is important to consider alternative values for κ and δ. A second set of parameters potentially affecting the working of the model include the degree of risk aversion (ρ), the tightness of the credit constraint in the deterministic steady state (ϕ), and finally the parameters governing the stochastic process for the tradable endowment (ρ_ε and σ_n respectively).

We consider four alternative cases for κ and δ, two higher values and two lower values than assumed in the baseline, changing only one parameter at a time. Specifically, we consider the

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[17]Interestingly, this also suggests that a small overvalaution, which effectively subsidizes consumption of non-tradable goods, may be a desirable policy option.

[18]Adding endogenous capital accumulation without distortionary taxation also does not alter the main result of the analysis.

following alternative cases: $\kappa = .3$ or $\kappa = 0.9$ (less or more substitutability between tradable and non tradable goods in consumption than in the baseline) and $\delta = 1.2$ or $\delta = 5$ (higher and lower labor elasticity than in baseline); and four cases for $\rho = 5$ (more risk aversion), $\phi = 0.5$ (looser constraint in the deterministic steady state and less likely to be occasionally binding), $\rho_\varepsilon = 0.95$ and $\sigma_n = 0.05$ (more persistent or more volatile AR1 process).

The results are summarized in Figure 8. The results are robust, except in the case of a lower labor supply elasticity. When tradables and non-tradables goods become closer substitutes ($\kappa = 0.9$), optimal policy would cut taxes less aggressively compared to the baseline specification. The general principle of optimal policy is to relax the borrowing constraint by increasing the value of collateral when the constraint becomes binding (i.e. by raising $P_t^N Y_t^N$). When the intratemporal elasticity of substitution between tradables and non tradables is higher, it is more efficient to do so by increasing the relative price of non-tradables and decreasing non tradables production compared to the baseline parametrization. Indeed, for a given relative price of non tradables and a given subsidy, a higher substitutability between tradables and non tradables will push the demand for tradable goods higher. Since tradable output is exogenously given, demand needs to be decreased in order to clear the tradable goods market if the economy cannot borrow from abroad. For a relatively higher κ this could be achieved with a relatively lower subsidy. Non-tradables demand will rise relatively more than with a lower κ so that the relative price of non tradables is higher, real wages are lower, and non-tradables production is lower since agents will decrease their labor supply compared to the baseline case. The opposite logic applies in the case in which $\kappa = .3$.

When labor supply becomes more elastic ($\delta = 1.2$), optimal policy would cut taxes less aggressively compared to the baseline specification. Also in this case it is efficient to relax the borrowing constraint by decreasing non-tradables production and increasing the relative price of non-tradables compared to our baseline parametrization. The opposite logic applies in the case in which labor supply is less elastic ($\delta = 5$).

When the constraint is looser ($\phi = 0.5$), the probability that the constraint tends to zero and the economy tends to behave as the unconstrained one.

Higher risk aversion ($\rho = 5$) than the baseline parametrization doesn't make any signif-

icant difference in terms of the policy function. On the other hand higher persistence and volatility of the shock of the tradable shock would both imply a higher subsidy in the binding region and a lower level of debt beyond which the constraint starts to bind. Recall here that we are plotting the policy functions conditional on the value of the tradable shock that corresponds to the negative of the standard deviation of its marginal distribution. Increased volatility of the shock for the given state of net foreign asset position requires bigger subsidy since agents rise precautionary saving due to higher uncertainty. Similarly when the shock is more persistent, given a bad realization of the shock the economy is more likely to hit the borrowing limit. This increases precautionary saving and by reducing consumption requires a higher subsidy to relax the constraint in the binding region.

7   Welfare Gains of Optimal Policy

In order to quantify the welfare gains associated with the optimal stabilization policy we compute a "consumption equivalent" measure of welfare in the spirit of Lucas (1987). Specifically, we compute the percent change in the average lifetime consumption, at every date and state, that would leave the stand-in household indifferent between the economy with optimal policy and the benchmark economy.[19] We then compute an overall summary measure by weighting the welfare gain at each state by the probability of being in that state, using the ergodic distribution.

Note that there are two sources of potential welfare gain from the optimal policy in our model. The first is the efficiency gain or loss from altering the tax distortion. The second is welfare gain from mitigating the effects of the credit constraint and reducing the probability of its occurrence: as we shall see, the former is one order of magnitude larger than latter. To illustrate this we report results from three experiments in which we either remove the credit constraint, or the tax distortion, or both.

Table 2 reports the results of these welfare experiments. The gain from eliminating

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[19]To recall, the benchmark economy has a fixed tax rate of 7.93 percent and the occasionally binding credit constraint. Appendix B provides technical details on these calculations. Also note that the computed gain includes the costs and gains associated with the transition from one state to the other.

altogether the credit constraint while retaining the tax distortion is 0.5 percent in consumption equivalent terms, consistent with the welfare gain computed and reported by Mendoza (2002). The gain from moving to a zero tax rate regime while retaining the credit constraint is 3.41 percent in consumption equivalent terms. The gain from the joint removal of the tax distortion and the credit constraint, which is an upper bound for the welfare gain from optimal policy, is 3.84 percent.

The gains associated with the optimal stabilization policy are significant. Moving from the benchmark economy without optimal policy to the economy with the optimal policy, average lifetime consumption increases by 3.6 percent. Subtracting from this the gain from eliminating the tax distortion, which is 3.41 percent, we obtain a 0.2 percent gain from using the optimal policy to mitigate the impact of the credit constraint. This gain represents about 40 percent of the gain from the complete elimination of the constraint.

While Table 3 provides valuable measures of the welfare gains from various policies, it obscures the fact that the welfare gain also has a state contingent dimension. The amount the stand-in household would pay to change policy is in fact dependent on the current state of the world. In states near the binding constraint the gains can be much larger. Figure 9 plots the welfare gain in each state for the optimal policy rule, and the gains from moving to the optimal rule are much larger for debt levels that are close to the constrained region. One way to interpret this graph is that economies that spend more time near the constraint are going to have bigger gains from the optimal policy, since these states will get larger weights in the ergodic set. Alternatively, the gains from adopting the optimal policy are larger for those countries with higher debt levels, as the conditional probability of a crisis is higher.

It is important to note also that our welfare gain, which is large by the standards of the cost of business cycle literature, is accounting only part of the potential benefits from optimal policy. The true gain could be even bigger.In our model in fact there is no idiosyncratic risk, only aggregate risk, as markets are complete with respect to risk sharing across agents within the country. For instance, Chatterjee and Corbae (2007), who do account for idiosyncratic risk, show that the gains to eliminating the possibility of a crisis state can be as large as seven percent of annual consumption. In their model, households face idiosyncratic risk that

is correlated with the aggregate shock, as in İmrohoroğlu (1989). As idiosyncratic risk is important in emerging markets, we view the welfare numbers in Table 2 as a lower bound on the welfare gains.

8   Conclusions

In this paper we study the optimal stabilization problem for a small open economy subject to an occasionally binding borrowing constraint. In our benchmark economy, we characterize policy in terms of a distortionary tax on non-tradable consumption allowing for costless financing through lump sum transfers. There are two main results.

The first main result is that optimal policy is non linear: when the constraint is not binding, the tax rate is set to zero while in the binding region optimal policy subsidizes non -tradable consumption in order to relax the borrowing constraint. Thus, in our benchmark case, optimal policy does not exhibit any precautionary motive. An implication of this first result is that stabilization policy in an economy with an occasionally binding financial friction, when the friction is not binding, should be set as if the friction were not present. This first result is robust to the choice of alternative parameter values and the consideration of endogenous capital accumulation. We conjecture however that extending the model to include costly financing of the optimal policy would result in a small precautionary component in the optimal policy when the credit constraint is not binding.

A second result is that the commitment to optimal policy affects private agents' behavior even when the constraint is not binding: agents consume more and accumulate more debt. In welfare terms the gain from optimal policy are non- trivial and account up to 40 percent of the gain that would arise from eliminating the borrowing constraint.

A Appendix

This appendix reports the model steady state.

A.1 Steady state

The deterministic steady state equilibrium conditions are given by the following set of equations. The first four correspond to the first order conditions for the household maximization problem,

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{C^T}{C^N}\right)^{\frac{1}{\kappa}} = (1+\tau^N)P^N,$$

$$\left(C - \frac{H^\delta}{\delta}\right) \omega^{\frac{1}{\kappa}} \left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} = \mu,$$

$$\left(1 + \frac{\lambda}{\mu}\right) = \exp\left(-\beta \ln\left(1 + C - \frac{H^\delta}{\delta}\right)\right)(1+i),$$

$$H^{\delta-1} = W\omega^{\frac{1}{\kappa}} \left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} \left[1 + \frac{\lambda}{\mu}\frac{1-\phi}{\phi}\right],$$

and the fifth is the definition of the consumption index:

$$C \equiv \left[\omega^{\frac{1}{\kappa}} \left(C^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C^N\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}.$$

The other equilibrium conditions are given by the liquidity constraint and the equilibrium condition in the tradable sector that determines the level of tradable consumption in the case in which the liquidity constraint is binding (i.e. $\lambda > 0$):

$$B \geqslant -\frac{1-\phi}{\phi}\left[Y^T + P^N Y^N\right],$$

$$C^T + B = Y^T + (1+i)B - G^T.$$

We then have the production function for the non-tradeable sector and the good market equilibrium for non tradeables:

$$Y^N = AK^\alpha H^{1-\alpha}$$

$$C^N + G^N = Y^N.$$

A.2 Detailed Model Solution

First, we guess at the value function $v^n (b, B, \varepsilon_T)$, the law of motion for the aggregate bond position $B' = G_B^n (B, \varepsilon_T)$, and two functions that map the aggregate state into aggregate labor input $N = G_N^n (B, \varepsilon_T)$ and the nontradeable price $P = G_P^n (B, \varepsilon_T)$. We update these guesses by solving the system of equations

$$
\begin{aligned}
N (B, \varepsilon_T) &= g_h (B, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T)) \\
AN (B, \varepsilon_T)^{1-\alpha} &= g_N (B, B, \varepsilon_T, P (B, \varepsilon_T), N (B, \varepsilon_T)) \\
G_B^{n+1} (B, \varepsilon_T) &= g_b (B, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T))
\end{aligned}
$$

for each (B, ε_T), where

$$
(g_T, g_N, g_h, g_b) (b, B, \varepsilon_T; P, N) = argmax_{c_T, c_N, h, b'} \left\{ \begin{array}{c} u (C (c_T, c_N), 1 - h) + \\ \beta (C (c_T, c_N), 1 - h) E [v^n (b', G_B^n (B, \varepsilon_T), \varepsilon_T') | \varepsilon_T] \end{array} \right\}
$$

subject to

$$
\begin{aligned}
c_T + Pc_N + b' &\leq (1 + r) b + P (1 - \alpha) AN^{-\alpha} h + \varepsilon_T + P\alpha AN^{1-\alpha} \\
b' &\geq -\frac{1 - \varphi}{\varphi} \left(P (1 - \alpha) AN^{-\alpha} h + \varepsilon_T + P\alpha AN^{1-\alpha} \right)
\end{aligned}
$$

and

$$
\begin{aligned}
v^{n+1} (b, B, \varepsilon_T) &= u (H (C, 1 - N)) + \\
&\quad \beta (H (C, 1 - N)) E [v^n (g_b (b, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T)), G_B^n (B, \varepsilon_T), \varepsilon_T') | \varepsilon_T]
\end{aligned}
$$

where

$$
\begin{aligned}
&H (C, 1 - N) = \\
&C (g_T (b, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T)), g_N (b, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T)), \\
&1 - g_h (b, B, \varepsilon_T; P (B, \varepsilon_T), N (B, \varepsilon_T)).
\end{aligned}
$$

The value function is approximated using a one-dimensional cubic spline in the b direction and a linear spline in the B direction. Due to numerical sensitivity with respect to the

derivatives, we solve the system of equations using a polytope method that minimizes the squared deviations. The optimization on the RHS of the Bellman equation is done using a feasible sequential quadratic programming algorithm (FSQP from AEM Design). Fortran programs to solve the problem are available upon request. A proof of convergence for similar models can be found in Coleman (1989,1997) and Greenwood and Huffman (1995).

For a competitive equilibrium with taxes, we have to add another function, the transfer $T = G_T^n (B, \varepsilon_T, \tau)$, and another equation, the government budget constraint:

$$\tau P (B, \varepsilon_T, \tau) g_N (B, B, \varepsilon_T; P (B, \varepsilon_T, \tau), N (B, \varepsilon_T, \tau), T (B, \varepsilon_T, \tau)) = T (B, \varepsilon_T, \tau).$$

Appropriate modifications to the optimization problem of the households and the updating of the value function are straightforward and are subsumed below into a discussion of the optimal policy problem.

For the optimal policy problem, we use the same procedure to obtain a competitive equilibrium for a given τ, by solving the system of equations:

$$
\begin{aligned}
N (B, \varepsilon_T) &= g_h (B, B, \varepsilon_T; P (B, \varepsilon_T, \tau), N (B, \varepsilon_T, \tau), T (B, \varepsilon_T, \tau), \tau) \\
AN (B, \varepsilon_T, \tau)^{1-\alpha} &= g_N (B, B, \varepsilon_T, P (B, \varepsilon_T, \tau), N (B, \varepsilon_T, \tau), T (B, \varepsilon_T, \tau), \tau) \\
G_B^{n+1} (B, \varepsilon_T, \tau) &= g_b (B, B, \varepsilon_T; P (B, \varepsilon_T, \tau), N (B, \varepsilon_T, \tau), T (B, \varepsilon_T, \tau), \tau) \\
T (B, \varepsilon_T, \tau) &= \tau P (B, \varepsilon_T, \tau) g_N (B, B, \varepsilon_T; P (B, \varepsilon_T, \tau), N (B, \varepsilon_T, \tau), T (B, \varepsilon_T, \tau))
\end{aligned}
$$

where

$$(g_T, g_N, g_h, g_b) (b, B, \varepsilon_T; P, N, T, \tau) =$$
$$argmax_{c_T, c_N, h, b'} \left\{ \begin{array}{c} u (C (c_T, c_N), 1 - h) + \\ \beta (C (c_T, c_N), 1 - h) E [v^n (b', G_B^n (B, \varepsilon_T, \tau), \varepsilon_T') | \varepsilon_T] \end{array} \right\}$$

subject to

$$
\begin{aligned}
c_T + (1 + \tau) P c_N + b' &\leq (1 + r) b + P (1 - \alpha) AN^{-\alpha} h + \varepsilon_T + P \alpha AN^{1-\alpha} + T \\
b' &\geq -\frac{1 - \varphi}{\varphi} \left(P (1 - \alpha) AN^{-\alpha} h + \varepsilon_T + P \alpha AN^{1-\alpha} \right).
\end{aligned}
$$

The value function is updated first to obtain welfare in the competitive equilibrium, conditional on tax policy:

$$V\left(B, \varepsilon_T, \tau\right) = u\left(H\left(C, 1-N\right)\right) + \beta\left(H\left(C, 1-N\right)\right) E\left[v^n\left(g_b\left(B, B,\right.\right.\right.$$

$$N\left(B, \varepsilon_T, \tau\right), T\left(B, \varepsilon_T, \tau\right), \tau, G_B^n\left(B, \varepsilon_T, \tau\right), \varepsilon_T'|\varepsilon_T\right]$$

where

$$H\left(C, 1-N\right) = C\left(B, B, \varepsilon_T; P\left(B, \varepsilon_T, \tau\right), N\left(B, \varepsilon_T, \tau\right), T\left(B, \varepsilon_T, \tau\right), \tau\right), 1-g_h\left(B, B, \varepsilon_T; P\left(B, \varepsilon_T, \tau\right), N\left(B,\right.\right.$$

Note that $V \neq v^{n+1}$, because τ is a function of the state and not a state itself and V is not a function of the individual state b. We then obtain the optimal tax policy by maximizing V with respect to τ:

$$\tau\left(B, \varepsilon_T\right) = argmax_\tau\left\{V\left(B, \varepsilon_T, \tau\right)\right\}.$$

This maximization is done by approximating V as a one-dimensional cubic spline with respect to τ and using Brent's method. Substituting this function into the individual level Bellman equation yields the updating equation for v:

$$\begin{aligned} v^{n+1}\left(b, B, \varepsilon_T\right) &= u\left(C\left(g_T, g_N\right), 1-g_h\right) + \beta\left(C\left(g_T, g_N\right), 1-g_h\right) E\left[v^n\left(g_b, G_B^{n+1}, \varepsilon_T'\right)|\varepsilon_T\right] \\ &= V\left(B, \varepsilon_T, \tau\left(B, \varepsilon_T\right)\right) \end{aligned}$$

where the arguments of the functions are omitted for clarity.

To accelerate convergence, we initially iterate on the Bellman equation without solving for and updating the competitive equilibrium function guesses. We also stop solving for those functions when they have converged and then iterate solely on the Bellman equation until v converges, which typically occurs much slower than the other functions.

A.3 Computing consumption equivalents

To compute welfare gains from optimal policy, we consider the functional equations

$$\begin{aligned} V_{PO}\left(B_t, \varepsilon_t^T\right) &= u(C_{PO}\left(B_t, \varepsilon_t^T\right) - z(H_{PO}\left(B_t, \varepsilon_t^T\right))) + \qquad\qquad (21) \\ &\quad \exp(-\beta\ln(C_{PO}\left(B_t, \varepsilon_t^T\right) - z(H_{PO}\left(B_t, \varepsilon_t^T\right))))E\left[V_{PO}\left(B_{PO}\left(B_t, \varepsilon_t^T\right), \varepsilon_{t+1}^T\right)\right] \end{aligned}$$

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and

$$V_{CE}\left(B_t, \varepsilon_t^T\right) = u(C_{CE}\left(B_t, \varepsilon_t^T\right) - z(H_{CE}\left(B_t, \varepsilon_t^T\right))) + \tag{22}$$
$$\exp(-\beta \ln(C_{CE}\left(B_t, \varepsilon_t^T\right) - z(H_{CE}\left(B_t, \varepsilon_t^T\right))))E\left[V_{CE}\left(B_{CE}\left(B_t, \varepsilon_t^T\right), \varepsilon_{t+1}^T\right)\right]$$

the first corresponds to the value function in the optimal allocation and the second to the value function in the economy without stabilization policy. We then inflate total consumption in (22) by a fraction χ, keeping the decision rules fixed, so that

$$V_{CE}\left(B_t, \varepsilon_t^T; \chi\right) = u((1+\chi)\,C_{CE}\left(B_t, \varepsilon_t^T\right) - z(H_{CE}\left(B_t, \varepsilon_t^T\right))) + \tag{23}$$
$$\exp(-\beta \ln((1+\chi)\,C_{CE}\left(B_t, \varepsilon_t^T\right) - z(H_{CE}\left(B_t, \varepsilon_t^T\right))))E\left[V_{CE}\left(B_{CE}\left(B_t, \varepsilon_t^T\right), \varepsilon_{t+1}^T\right)\right].$$

For each state $\left(B_t, \varepsilon_t^T\right)$, we set

$$V_{PO}\left(B_t, \varepsilon_t^T\right) = V_{CE}\left(B_t, \varepsilon_t^T; \chi\right)$$

and solve this nonlinear equation for χ, which yields the welfare gain from switching the optimal policy conditional on the current state. To obtain the average gain, we simulate using the decision rules from (22) and weight the states according to the ergodic distribution.

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Table 1. Calibrated parameters and steady state values for the model without capital

Structural parameters	Values
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 2$
Labor supply elasticity	$\delta = 2$
Credit constraint parameter	$\phi = 0.74$
Labor share in production	$\alpha = 0.636$
Relative weight of tradable and non-tradable goods	$\omega = 0.344$
Discount factor	$\beta = 0.0177$
Production factor	$AK^\alpha = 1.723$
Tax rate on non-tradable consumption	$\tau = 0.0793$

Endogenous variables	Steady state values
Per capita NFA	$B = -3.562$
Relative price of non-tradable	$P_N = 1$
World real interest rate	$i = 0.0159$
Tradable government consumption	$\exp(G_T) = 0.0170$
Nontradable government consumption	$\exp(G_N) = 0.218$
Per capita tradable consumption	$C_T = 0.607$
Per capita non-tradable consumption	$C_N = 1.093$
Per capita consumption	$C = 1.698$
Per capita tradable GDP	$Y_T = 1$
Per capita non-tradable GDP	$Y_N = 1.543$
Per capita GDP	$Y = 2.543$

Productivity process	
Persistence	$\rho_\varepsilon = 0.553$
Volatility	$\sigma_n = 0.028$

Table 2. Welfare Gains of Moving from Benchamerk Economy 1/

Gain 2/	Tax/Subsidy	Credit Constraint
0.50	0.079	NO
3.41	0.000	YES
3.84	0.000	NO
3.60	Optimal Policy	YES

1/ Benchmark is economy with fixed tax rate at 7.93 percent and credit constraint.
2/ Percent increase in average lifetime consumption.

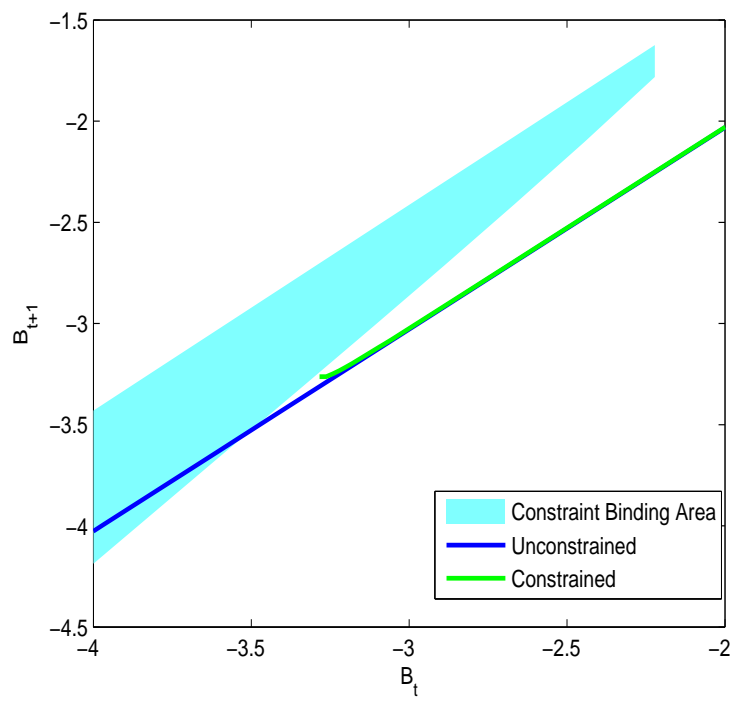Figure 1: NFA POLICY FUNCTION WITH AND WITHOUT CONSTRAINT

Figure 2: DECISION RULES FOR WAGES, PN, CONSUMPTION, HOUR (WITH AND WITHOUT CONSTRAINT)
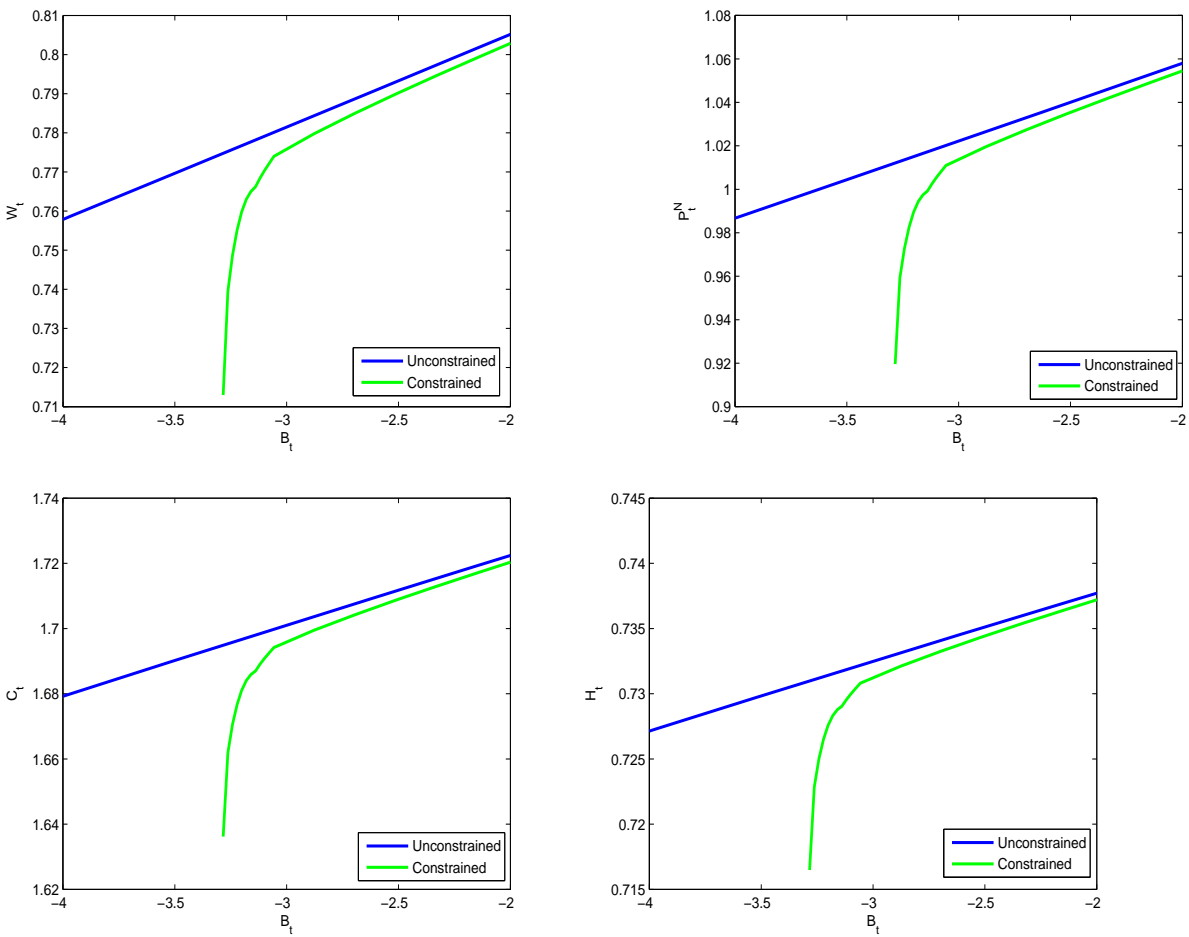
Figure 3: OPTIMAL TAU AND TNPN/Y (IS THIS THE GRAPH?)
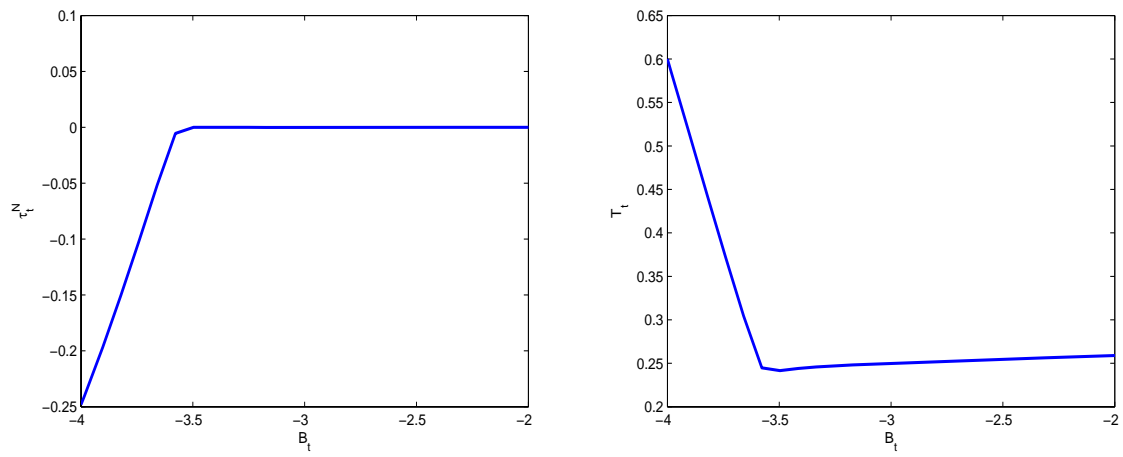
Figure 4: Decision Rules for wages, pn, consumption, hour (with and without optimal policy)
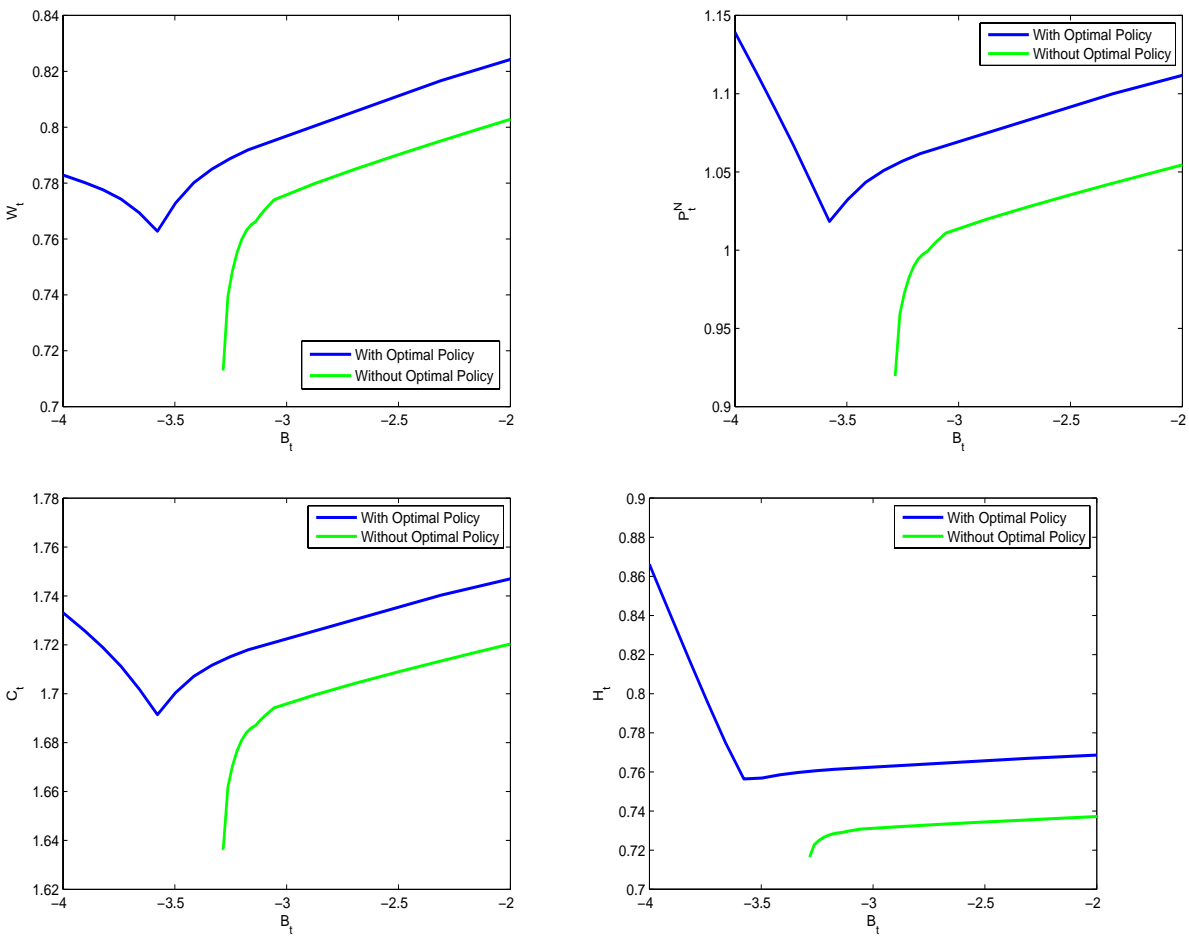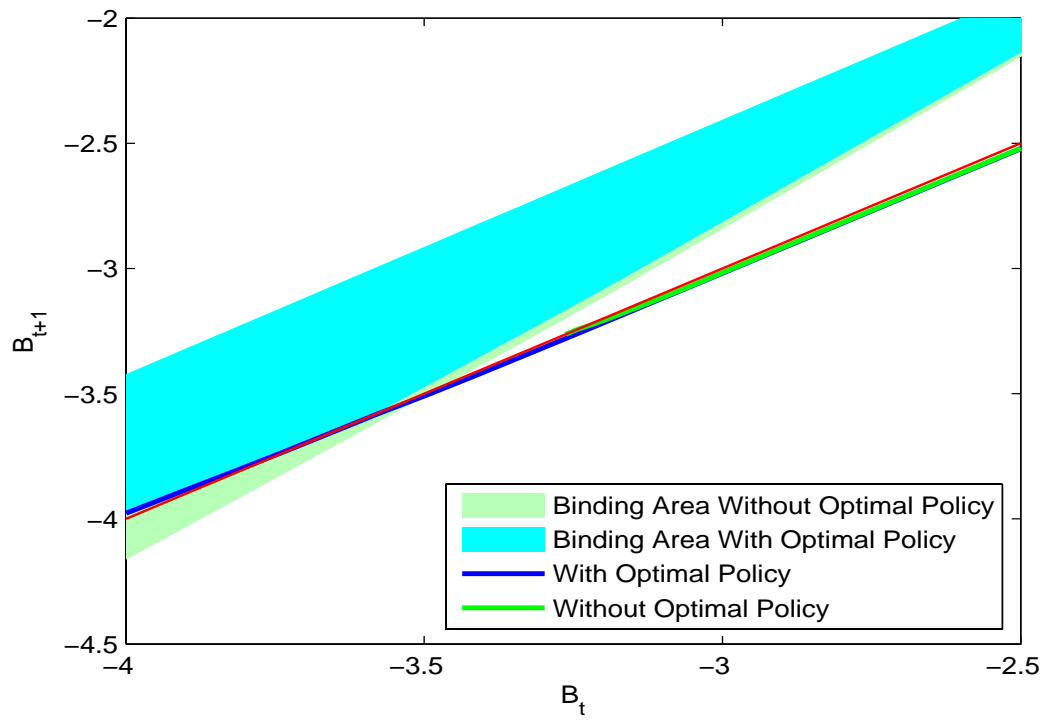
Figure 5: NFA WITH CONSTRAINT AND OPTIMAL POLICY

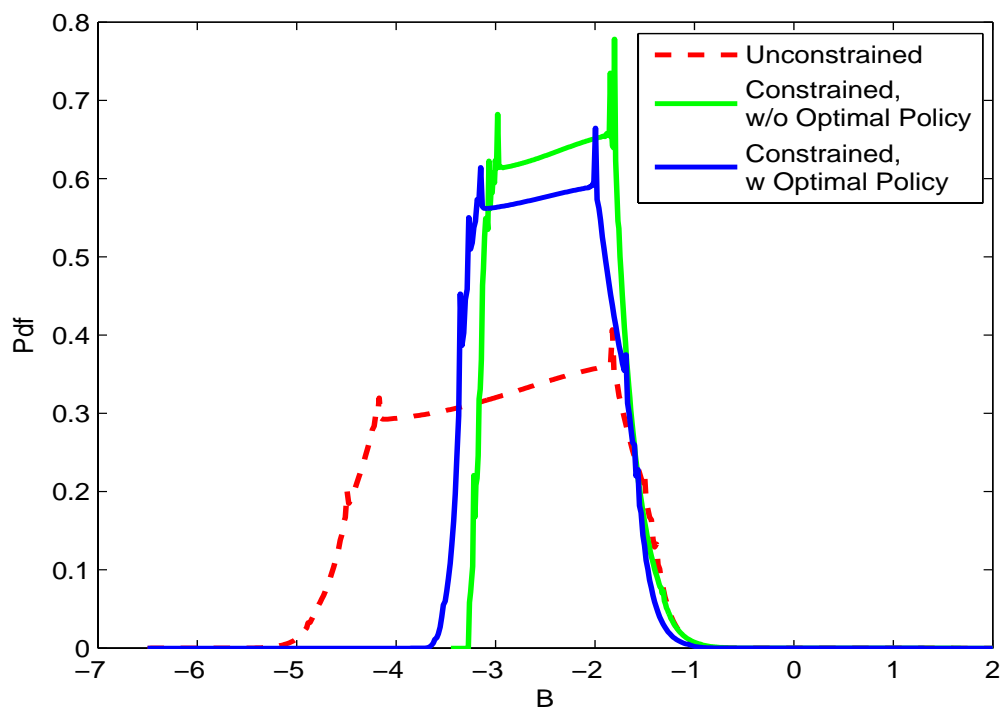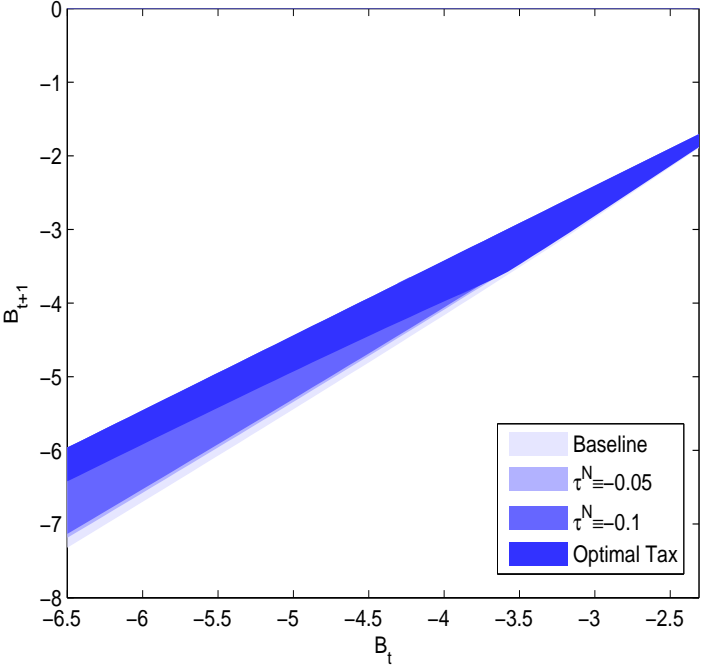Figure 6: ERGODIC DISTRIBUTION OF NFA

Figure 7: FIXED TAX RULES

Figure 8: SENSITIVITY ANALYSIS
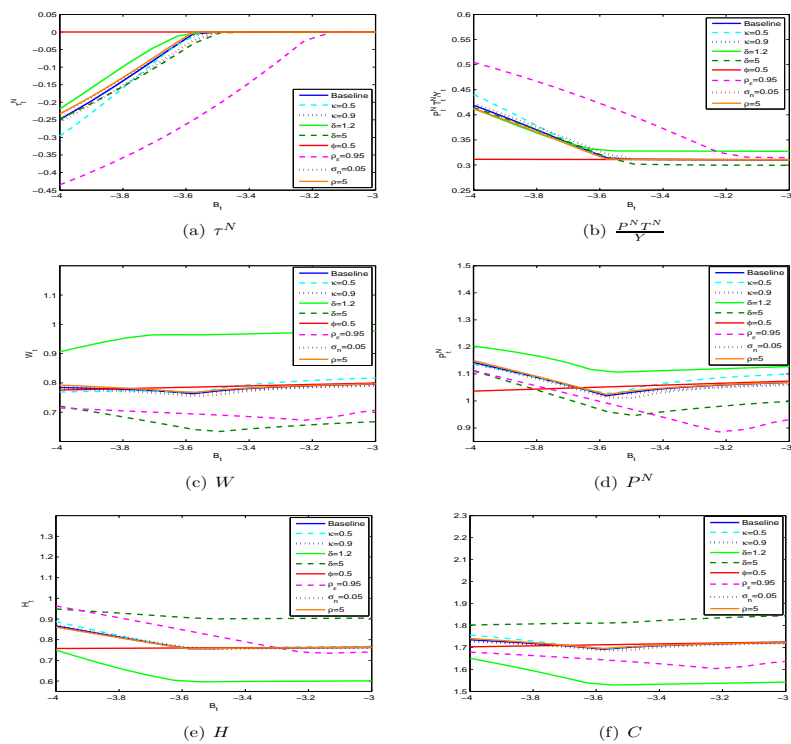
(a) τ^N

(b) $\frac{P^N T^N}{Y}$

(c) W

(d) P^N

(e) H

(f) C

Figure 9: WELFARE BY STATE