

The Costs of Investment

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Abstract: This paper describes a method of structural estimation of firm-level investment costs, based on a dynamic investment model that incorporates fix and irreversibility costs of investment. These cost components are consistent with the stylized facts documented by the empirical literature over the past decade. Our approach is novel in two respects: (1) the model makes distinction between cheap (cost-free) replacement investment and costly new investment, and we also make such a distinction at the empirical level; (2) we estimate the structural cost parameters of the model with a modified version of the indirect inference method. To estimate the model, we use an unbalanced panel of US manufacturing firms between 1959-87. Our results indicate that fixed and irreversibility costs are indeed significant. In particular, we find strong evidence of partial irreversibility, which is of much higher extent than estimated previously.

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I - Introduction

The importance of understanding investment cannot be overstated. For example, Hungarian data about investment (Gross Asset Formation) and GDP shows that investment is responsible for about 20-25% of total (nominal) GDP (*Table 1*). The GDP-share of investment is of similar magnitude in more developed countries; in the United States, for example, it was between 16.91% (1992) and 21.98% (1979) during the 1975-2004 period.¹

Year	Gross Asset Formation (bn HUF)	Gross Domestic Product (bn HUF)	Gross Asset Formation as % of GDP
1995	1125.389	5700.278	19.74%
1996	1475.538	6900.262	21.38%
1997	1898.917	8550.109	22.21%
1998	2384.615	10031.925	23.77%
1999	2724.532	11198.808	24.33%
2000	3099.131	12834.343	24.15%
2001	3492.990	14694.638	23.77%
2002	3916.892	16657.534	23.51%

Table 1. Investment (gross asset formation) and GDP in Hungary, 1995-2002. Source: Central Statistical Office of Hungary (KSH)

Investment, however, not only constitutes a large proportion of the GDP, but it is also the most volatile part of it, and it is also one of the main determinants of medium- and long-term growth. These are just a few reasons why it has been one of the more pervasive questions in economics to understand the determinants of investment.

Early investment models (known as accelerator models, see for example *Koyck (1954)*) relate investment to sales and output. Though these models performed relatively well to explain aggregate investment activity, they did not provide an underlying theory of why exactly these variables should be included into investment regressions. In search for an underlying theory, *Jorgenson (1963)* set up a model in which he assumed that firms could instantaneously and costlessly adjust their stock of capital. Under these assumptions he showed that firms always equate the marginal productivity of capital to the user cost of capital. Since there are no frictions in Jorgenson's model, the firms' decision about their capital stock (and hence investment) is a simple static problem.

Later models departed from the unrealistic assumption of cost-free capital adjustment. If there are frictions to adjust capital stock, however, the investment decision becomes a dynamic problem in which firms have to consider future conditions when deciding about their current investment. The first models of this type considered the convex costs of investment as such a friction. Among others, *Abel (1983)* showed that in a dynamic model with convex adjustment costs investment is an increasing function of the marginal value of capital (known as Tobin's marginal *Q*), connecting this way *Tobin (1969)*'s *Q*-theory to the neoclassical model with adjustment costs.

¹ This share was calculated as the ratio of gross fixed capital formation and nominal GDP. Source: *International Financial Statistics online*, <http://ifs.apdi.net/imf/ifsBrowser.aspx>.

Despite their theoretical appeal, however, these traditional models did not get much empirical support. As demonstrated by several surveys (see, for example, *Caballero (1999)*), in empirical specifications investment was found to have low or no responsiveness to investment fundamentals.

Empirical work over the past decade has shown that there are at least two important factors missing from earlier models. First, by investigating the investment pattern of a panel of US manufacturing firms over 17 years, *Doms and Dunne (1998)* showed that firm-level investment is lumpy: a typical firm has huge investment bursts followed by periods of inactivity. This shows that the possibility of continuous adjustment of capital stock (a consequence of convex adjustment costs) is not realistic, and indicates the existence of other types of costs of capital adjustment.

A second important factor missing from traditional models was documented in another influential paper by *Ramey and Shapiro (2001)*, who show that capital sales can entail irreversibility costs: the sales price of capital can be significantly lower than the purchase price (or replacement value) of capital. Irreversibility is a cost of investment because it makes capital more expensive: if firms could sell their capital at the same price as they purchased them, then after a negative shock they would be able to get back the original price of investment, so the initial *decision to invest* would not entail any sunk cost. On the other hand, if the sales price of used capital is smaller than the purchase price, that is, if we have at least partial irreversibility, the *decision to invest* entails sunk costs.

New investment models (for example *Abel and Eberly (1994)*, *Bertola and Caballero (1994)*) incorporate fixed and irreversibility costs of investment. This paper presents a structural estimation of fixed, convex and irreversibility investment costs in a model that is similar to the model of *Abel and Eberly*. Our approach, however, has some novel elements. First, in our investment model we make distinction between the relatively cheap (cost-free) replacement investment and costly new investment. Second, the estimation technique is a somewhat modified version of indirect inference (for a description, see *Gourieroux and Monfort (1996)*), which was previously used in a similar framework by several studies (see for example *Bayraktar, Sakellaris and Vermuelen (2005)*, though we use an unbalanced panel for the estimation). In the current paper we modify the indirect inference method in such a way that it leads to a better identification for all of the parameters.

The estimation of the different investment cost parameters is important for at least two reasons. First, the estimate of the irreversibility parameter is interesting on its own right, because it is directly related to disinvestment; the ease of which is one of the major determinants of economic flexibility and speed of adjustment to shocks. Further, as data is generally available only for gross investment, negative investment is hidden behind the generally bigger positive investment; therefore disinvestment can be directly observed only on exceptional occasions. Currently we know about two direct estimations of the extent of irreversibility. The first is the study of *Ramey and Shapiro (2001)* that we have already referred to: based on the asset sales of a closing US aerospace plant, they report that the average ratio of the sales price and calculated replacement value of capital assets is only 28%. In other words, capital sales can be done by a discount as high as 72% on average, which is quite substantial. Besides this,

Reiff (2004) has similar findings on a data set about the asset auction of a discontinuing Hungarian manufacturing plant.²

The second reason why the estimation of the different investment cost parameters is important is because with the estimated cost parameters we can gain better insights into micro-level investment behavior, and based on this we can also investigate the aggregate implications of the micro-based investment models. Using simulation techniques, we can examine the responsiveness of *aggregate* investment to *aggregate* shocks, which can be different from the corresponding micro-level responsiveness (see Caballero (1992) about the “fallacy of composition”). Better insights into this aggregate responsiveness can give us better understanding of investment related policies.

The paper is organized as follows. In *section II* we set up the basic investment model under our focus, and investigate the theoretical implications of fixed, irreversibility and convex costs to the responsiveness of investment to shocks (or, in the terminology of this paper, to the investment-shock relationships). The line of argument in doing so may seem straightforward, but this is still important as later we identify the different cost components based on this investment-shock relationships. *Section III* gives an overview of the general strategy of the estimation, highlighting the new elements of our estimation technique. In *section IV* we discuss the first stage of estimation: after describing the data, we present the estimation results. *Section V* discusses the second stage of estimation: it describes the steps of the simulation exercise, presents the results and discusses the estimated cost parameters. A short analysis of aggregate implications is also provided. *Section VI* is a summary of the main the results.

II - The Model

Let us consider a general investment model, in which firms maximize the present value of their future profits, net of future investment costs:³

$$V(A_0, K_0) = \max_{\{I_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} b^t [\Pi(A_t, K_t) - C(I_t, K_t)] \right\}, \quad (1)$$

where profit at time t is given by $\Pi(A_t, K_t)$, with A_t and K_t denoting profitability shock and capital stock at time t , respectively,⁴ the cost of investment I_t is $C(I_t, K_t)$, and b is a discount factor. The capital stock depreciates at a rate of $d > 0$, and the profitability shock is assumed to be a first-order Markov-process, so the transition equations are

² Of course, while these estimates on closing plants cannot be directly compared to other estimates of the extent of irreversibility that are based on panels of continuously operating firms, they clearly indicate that the extent of irreversibility can be significant.

³ The general structure of this model is similar to the models presented by Abel and Eberly (1994) and Stokey (2001).

⁴ The “profitability shock” and the $\Pi(A_t, K_t)$ profit function will be defined explicitly in *section 4*.

$$K_{t+1} = (1-d)K_t + I_t, \quad (2)$$

$$A_{t+1}|A_t \text{ is a random variable with known distribution.} \quad (3)$$

Firms then maximize (1) with constraints (2) and (3).⁵ Omitting the time indices, and denoting future values of the variables by primes, the solution entails solving the following maximization problem in each time period:

$$\max_I \{-C(I, K) + bE_{A'|A} V(A', K' = (1-d)K + I|A)\}, \quad (4)^6$$

and the solution is

$$(q =) bE_{A'|A} \frac{\partial V(A', K')}{\partial K} = bE_{A'|A} V_K(A', K') = C_I(I, K).^7 \quad (5)$$

This is a well-known optimum condition, stating that the (expected) discounted marginal value of capital for the firms (left-hand side) must be equal to the marginal cost of capital (right-hand side).⁸

Obviously, this solution depends crucially on the exact formulation of the cost function. In this paper we use a general formulation of the investment cost function $C(I_t, K_t)$, and we assume that it has three components. In the following, along the lines of *Stokey (2001)*, we examine these components one by one, with a special emphasis on their effect on investment decisions.

The first component of the investment cost function is the fixed cost F , which has to be paid whenever investment is non-zero:⁹

⁵ Thus we have a dynamic optimization problem with state variables (A, K) and control I .

⁶ More precisely, the resulting value function is given by the Bellman equation

$$V(A, K) = \max_I \{\Pi(A, K) - C(I, K) + E_{A'|A} V(A', K'|A)\}$$

⁷ V_K is the marginal value of capital for the firm, often denoted by q (Tobin's marginal q).

⁸ The timing of the model is the following: firms have an initial capital stock K , and then they learn the value of the profitability shock A . This influences the expected discounted marginal value of capital (left-hand side of (5)). Finally, firms choose I to make the marginal cost (right-hand side) equal to the marginal value of capital (taking into account that the choice of I also influences the marginal value of capital through K'), and enter the next period with their new capital stock K' . Effectively, we describe this sequence of events with the "investment-shock relationship": in each time period, firms respond to profitability shock A with an optimal investment rate I or $I^*(A)$.

⁹ This fixed cost is assumed to be independent from I , but not necessarily from K . In fact it is a common assumption in the literature that the investment cost function is homogenous of degree 1 in (I, K) , and therefore the fixed cost is assumed to be proportional to K . To ease exposition, for the time being we simply use F instead of FK .

$$C(I, K) = \begin{cases} F + \Gamma(I, K), & I \neq 0. \\ 0, & I = 0. \end{cases} \quad (6)$$

where $\Gamma(I, K)$ is the cost of investment other than fixed costs (time indices are dropped once again to ease exposition).

The second component of the investment cost function is a linear term, which represents the buying (P) and selling (p) price of capital ($P \geq p \geq 0$). Thus $\Gamma(I, K)$ can be further divided as

$$\Gamma(I, K) = \begin{cases} PI + g(I, K), & I \geq 0. \\ pI + g(I, K), & I < 0. \end{cases} \quad (7)$$

Finally, the third component of the investment cost function is $g(I, K)$, which is the usual convex adjustment cost; we make the general assumption that $g(I, K)$ is a parabola-like function, with a minimum value of 0, and also a possible kink at $I = 0$.¹⁰ Therefore the partial derivative of this function with respect to I is non-decreasing, with negative values for $I < 0$ and positive values for $I > 0$, and this derivative may be discontinuous at $I = 0$ if and only if there is a kink in the $g(I, K)$ function there.

More specifically, in this paper we define the fixed component of the investment cost function as FK , and the convex component as $g(I, K) = \frac{g}{2} \left(\frac{I}{K} \right)^2 K$, so that the investment cost function is linearly homogenous in (I, K) . We normalize the model to the buying cost of capital, and assume that $P = 1$, from which it follows that for the selling cost of capital $0 \leq p \leq 1$ must hold.¹¹

Additionally, we will also distinguish between replacement investment and new investment. There are several reasons why replacement investment is not as costly as new investment: (1) when undertaking replacement investment, firms often have their tools and machines checked, certain parts exchanged or upgraded, and this entails contacting well-known suppliers at much lower costs; (2) learning costs are also likely to be much lower in this case. Though replacement investment may also entail adjustment costs, it seems to be a reasonable approximation to treat replacement investment cost-free, as opposed to costly new investment. Specifically, we assume that investments up to the size of dK (the depreciated part of capital) have no convex or fixed costs, and firms have to pay adjustment costs after that part of investment that

¹⁰ Specifically, $g(I, K) = 0$ is assumed to be twice continuously differentiable except possibly at $I = 0$, weakly convex, non-decreasing in $|I|$, with $g(0, K) = 0$.

¹¹ So if $p = 1$, then there is no irreversibility. Complete irreversibility will be characterized by $p = 0$.

exceeds this amount. (Of course, when undertaking replacement investment, firms still have to pay the unit purchase price of investment goods.)

Thus the final specification of the investment cost function is the following:

$$\frac{C(I, K)}{K} = \begin{cases} \frac{I}{K}, & 0 \leq \frac{I}{K} \leq d, \\ F + \frac{I}{K} + \frac{g}{2} \left(\frac{I - dK}{K} \right)^2, & \frac{I}{K} > d, \\ F + p \frac{I}{K} + \frac{g}{2} \left(\frac{I}{K} \right)^2, & \frac{I}{K} < 0. \end{cases} \quad (8)$$

For the remaining of this section we return to the general specification of the investment cost function (see expression (6)), and also neglect the distinction between replacement and new investment, with the purpose of investigating the effects of each cost components. A careful investigation of the effects of the various cost components is important because a full understanding of the role of the model's key parameters will ease their empirical identification.

Specifically, we examine the shape of $C_I(I, K)$ on the right-hand side of (5); so let us make a graph about the partial derivatives of each cost components. First, denote the limits of the partial derivative of $g(I, K)$ (with respect to I) as $q_a = \lim_{I \rightarrow 0-0} g_I(I, K)$ and $q_A = \lim_{I \rightarrow 0+0} g_I(I, K)$ (as in Figure 1); our assumptions ensure that $q_a \leq 0 \leq q_A$.

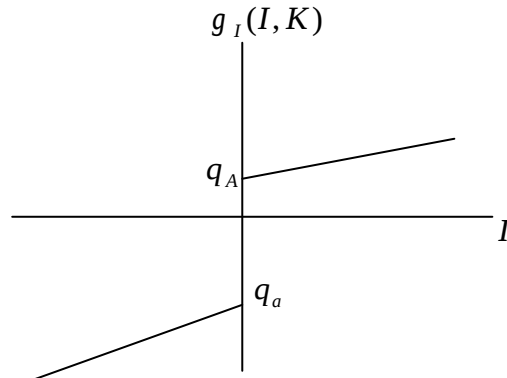


Figure 1. The general shape of the function $g_I(I, K)$.

Moreover, the partial derivative of $\Gamma(I, K)$ with respect to I is $P + g_I(I, K)$ for $I > 0$, and $p + g_I(I, K)$ for $I < 0$, so the shape of this function is as in Figure 2.¹²

¹² Here $q_1 \geq q_2$ follows from $P \geq p$ and $q_A \geq q_a$.

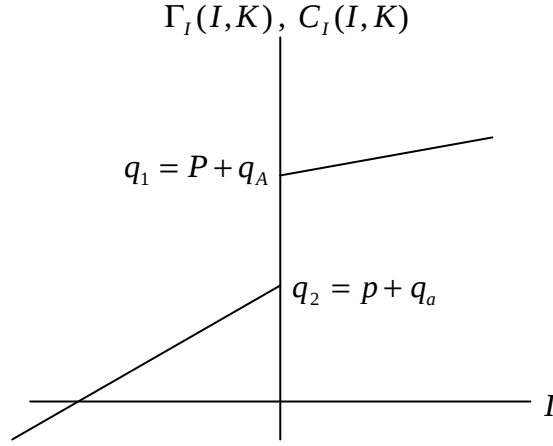


Figure 2. The general shape of the function $\Gamma_I(I, K)$ and $C_I(I, K)$.

Finally, we have $C_I(I, K) = \Gamma_I(I, K)$ by the definition of the cost function.

Going back to the first order condition in (5), without fixed costs we have the following solution:

- if for the current capital stock K and profit shock A we have $(q =) E_{A|A} V_K(A', (1-d)K) > q_1$,¹³ then the optimal level of investment will be positive;
- if $(q =) E_{A|A} V_K(A', (1-d)K) < q_2$ then the optimal investment will be negative;
- and if $q_1 \geq (q =) E_{A|A} V_K(A', (1-d)K) \geq q_2$, then the optimal investment will be zero.

In this case we have an inaction region as long as $q_1 > q_2$ (i.e., either if there is a kink at $I = 0$ in the adjustment cost function ($q_A > q_a$) or if there is no perfect reversibility ($P > p$)), so the investment function $I^*(q)$ (i.e. investment as a function of the underlying fundamental, the Tobin's q) is flat at $I^* = 0$ for q -s in a certain region (if $q \in [q_1; q_2]$), but it is continuous: even small investment episodes will occur.

¹³ $E_{A|A} V_K(A', (1-d)K)$ is the expected marginal value of capital when investment is zero; its value depends on the current value of the controls.

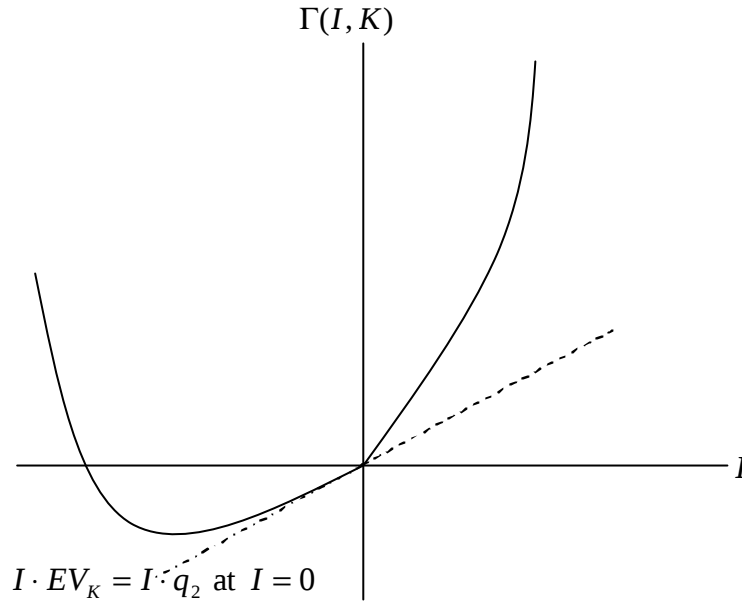


Figure 3. The firms' investment problem without fixed costs—graphically.

If there are fixed costs, there is a slight difference. To see this, consider the $\Gamma(I, K)$ function, which is the sum of a convex and a linear function, with $\Gamma(0, K) = 0$, and a possible kink at $I = 0$. Moreover, its derivative should be $\Gamma_I(I, K)$ as shown on Figure 2. Then Figure 3 illustrates the possible shape of $\Gamma(I, K)$, with the additional assumption that currently $E_{A|A} V_K(A', (1-d)K) = q_2$ (that is, if there is no new investment, the marginal value of depreciating capital for the firm is exactly q_2).

If we have a situation like on Figure 3, then optimal investment is zero, but $EV_K = E_{A|A} V_K(A', (1-d)K)$ has the smallest possible value (q_2) (or the slope of the dashed line is the smallest) so that in the absence of fixed costs investment is non-negative. If we decrease $E_{A|A} V_K(A', (1-d)K) = EV_K$ marginally, then, still assuming that fixed costs are 0, the optimal decision will be a marginal disinvestment, as the IV_K line, which represents the benefit from investment in terms of future profits, will be locally above the $\Gamma(I, K)$ function, the function representing the costs of investment. If there are positive fixed costs, however, then after a marginal decrease of $E_{A|A} V_K(A', (1-d)K) = EV_K$, it will be the zero investment that would be still optimal: the expected marginal net benefit ($I \cdot EV_K - \Gamma(I, K)$) would be so small that it would not compensate for the fixed costs that would have to be paid.

Indeed, if $F > 0$, then in case of the situation illustrated on Figure 3 we will only see a disinvestment if we decrease the value of $E_{A|A} V_K(A', (1-d)K) = EV_K$ substantially enough below q_2 ; so that the net benefit from disinvestment (the highest difference between $I \cdot EV_K$ and $\Gamma(I, K)$ in the graph) should be at least F (see Figure 4).

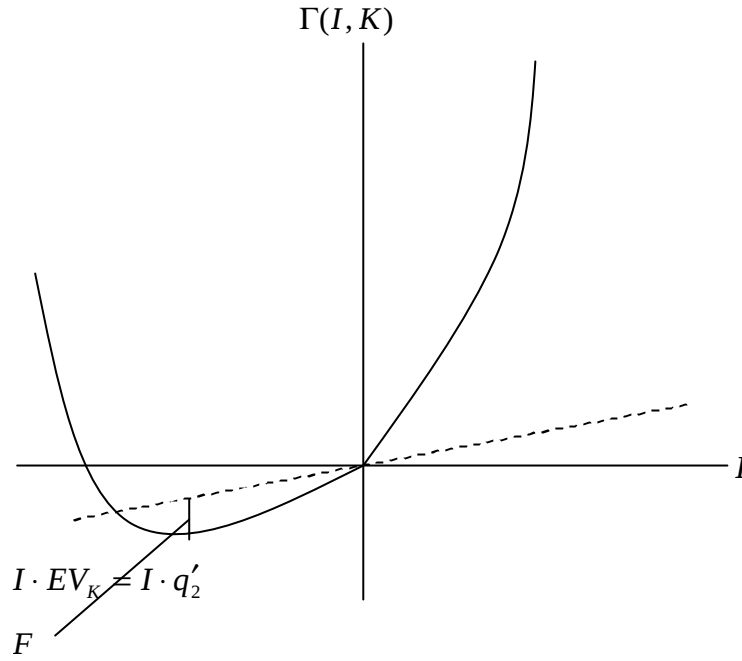


Figure 4. The firms' investment problem with fixed costs—graphically.

In Figure 4 we can see that the highest q'_2 for which the optimal investment is negative, is smaller than q_2 in Figure 3, because of the fixed cost F . With similar reasoning it is easy to see that the lowest q for which the optimal decision is to have positive investment (i.e., the upper bound of the inaction region) is $q'_1 > q_1$.

It should be obvious from Figure 4 that the presence of fixed costs generates discontinuities in the investment function: the threshold between the inaction region and the disinvestment region is q'_2 , but for q -s slightly below this, the optimal disinvestment is not marginal.

To conclude this section, it may be useful to summarize the role of the different cost parameters in the theoretical model. We have seen that:

- **irreversibility** generates an inaction region (i.e., if the marginal value of capital is inside a certain band, firms will neither invest nor disinvest), but leaves the investment function continuous;
- **fixed costs** also generate an inaction region (or in the presence of irreversibility costs they further widen it), and create discontinuities in the investment function (i.e., there will be no small investments undertaken).

To further illustrate this point, we solved numerically the model for some simple cost structures.¹⁴ Appendix A contains the investment functions (investment as

¹⁴ For the numerical solution, we assumed that $b = 0.95$, $d = 0.07$, two common assumptions in the literature dealing with US data. We solved the dynamic optimization problem with parametric value function iteration as described by Judd (1998), with a bi-variate cubic specification for the value function. (We also solved the problem with the more accurate value function iteration for appropriately discretised state space, and found that the cubic approximation of the value function was quite close to this more accurate solution.) We assume that the profitability shock behaves as estimated from real data (see section 4.).

a function of $\log(\text{profitability shock})$ in various cases: when investment is cost-free, when there are only convex costs of investment, when there are only fixed costs of investment, and when there is irreversibility. The effects of the various cost components can also be observed on these figures.

III - Estimation of Cost Parameters: General Strategy

In this paper, we use *indirect inference* (as described by *Gourieroux and Monfort (1996)*) to estimate structural investment cost parameters. The general idea behind the identification of the structural parameters is to match the investment-shock relationship obtained from the theoretical model to the observed investment-shock relationship. The “philosophy” of this approach is that we believe that we are able to observe the “true” investment-shock relationship fairly precisely; and then we choose those cost parameters in the theoretical model, for which the theoretical investment-shock relationship is very close to the observed one.

This identification strategy is an indirect one when compared to more conventional methods, which (for example) start out from the first-order conditions, and identify structural parameters on the basis that these conditions should be met empirically. This conventional method (or its variations), however, cannot be used directly in the context of our model, since the existence of an inaction region means that the conventional first-order conditions are not always equalities: for observations when there is inaction, we have only an inequality, stating that the current marginal value of capital, q is somewhere between the left-hand side derivative of the investment cost function, q_1 and the right-hand side derivative of the investment cost function, q_2 . So we use an indirect method because the direct, first-order condition based approach cannot be used in the usual manner.¹⁵

However, our identification strategy is not straightforward either, as there is no closed-form solution of our model, and we can not derive analytically the theoretical investment-shock relationship as a function of structural cost parameters. As discussed in the previous section and also in *Appendix A*, the theoretical investment-shock relationship is non-linear, and in case of positive fixed costs it is not even continuous. In recent literature using indirect inference (see for example *Bayraktar, Sakellaris, Vermeulen (2005)* and the initial version of *Cooper-Haltiwanger (2005)*), it has been very popular to identify the cost parameters based on a quadratic shock-investment relationship that captures non-linearity, but fails to capture discontinuity and inaction. In the following we argue that though this method can be useful in identifying the convexity and irreversibility parameters, it is not sufficient to identify the fixed cost parameter.¹⁶

¹⁵ *Cooper, Haltiwanger and Willis (2005)* investigate the possibility of modifying the usual first-order condition based approach so that it remains applicable in this context. Their modification solves the problem of inequality-type first-order conditions for inactive observations by using the data of the active firms only, together with the lengths of inactive spell of inactive firms. They also correct for the endogenous selection, which arises because of the exclusion of the inactive observations.

¹⁶ This is not surprising: the fixed cost parameter is the one that creates discontinuity and inaction, none of which is present in the quadratic equation.

In recent literature, the usual quadratic reduced form regression (the “shock-investment relationship”, based on which the parameters are identified) applied when using indirect inference is the following:

$$\tilde{i}_{it} = f_0 + f_1 \tilde{a}_{it} + f_2 \tilde{a}_{it}^2 + f_3 \tilde{a}_{it-1} + m_t + u_{it}, \quad (9)$$

where i denotes the investment rate, a denotes the profitability shock, m_t is a time-dummy, u is a well-behaving error term, and the variables with tildes denote deviations from plant-specific means. In this specification the parameter f_2 is meant to capture the non-linearity of the investment-shock relationship (as higher profitability shocks are assumed to lead to proportionally higher investment activity in absolute value). According to the usual arguments, parameter f_3 represents the lumpiness of investment: because of inaction, shocks sometimes lead to lagged effects – following a positive profitability shock, for example, the investment threshold may be passed only in later periods. (Or, alternatively, current shocks may trigger immediate investment, and then inaction for many periods.) The parameter f_3 therefore is included to account for the possible inaction region, and captures both the effects of irreversibility and fixed costs.

To investigate the effect of structural cost parameters to these regression parameters, we estimated the reduced regression parameters in certain simple cases. We have already referred to *Appendix A* to illustrate the investment-shock relationship under basic cost structures; now we examine the estimated reduced regression parameters for the same cases.

- In the **cost-free case** ($F = 0, g = 0, p = 1$), we have $\hat{y}_1 = 2.5233$, $\hat{y}_2 = 0.4384$, $\hat{y}_3 = -2.5151$. Here the significantly positive $\hat{y}_2$ represents the slight convexity of this relationship due to diminishing returns to capital (a further discussion of this is provided in *Appendix A*), and we can also see that investment is highly responsive for shocks: the absolute values of the estimated parameters are relatively high.
- When there are **only irreversibility costs** ($F = 0, g = 0, p = 0.95$), we estimate $\hat{y}_1 = 0.8520$, $\hat{y}_2 = 0.3928$, $\hat{y}_3 = -0.5564$. Because of the inaction region, the estimated shock-investment relationship became more convex, which is apparent from the increase in the relative magnitude of $\hat{y}_2$. On the other hand, investment is much less responsive to shocks, also because of the inaction region; this is obvious from the smaller absolute values of the estimated reduced regression parameters.
- If we have **only convex costs** of adjustment ($F = 0, g = 0.2, p = 1$) we see the estimated reduced regression parameters as $\hat{y}_1 = 0.4657$, $\hat{y}_2 = 0.0672$, $\hat{y}_3 = -0.2557$. As can be seen in *Figure A/3*, the only difference between this case and the cost-free case is that the investment-shock relationship became flatter, and this is apparent from the proportional decrease of the estimated reduced regression parameters.

- When there are **only fixed costs** of investment ($F = 0.001, g = 0, p = 1$), the shock-investment relationship is basically the same as in the frictionless case, with its middle part (when the absolute value of shocks is small) missing; see *Figure A/4*. It is not surprising, therefore, that the estimated regression coefficients are quite similar to the estimated regression coefficients in the cost-free case: now they are $\hat{y}_1 = 2.4475$, $\hat{y}_2 = 0.4423$, $\hat{y}_3 = -2.4165$.¹⁷ This indicates that *changes in F do not lead to changes in the estimated reduced regression parameters*.

Thus the general responsiveness of investment to shocks (in other words, the absolute value of the estimated reduced regression parameters) identifies the convex component of the investment cost function (g). Also, the relative magnitude of $\hat{y}_2$ identifies the irreversibility costs (p). However, *these regression parameters do not contain any information* based on which one could identify F , the fixed cost of investment.

Therefore, while reduced form regression (9) is useful to estimate g and p , we should look for a different type of information if we also want to identify F . To do this, it seems to be obvious to use some property of the investment-shock relationship that is exclusively due to the presence of fixed costs.

In *section 2* we saw that in the theoretical model, *increasing fixed costs* lead to wider inaction region *and* larger discontinuity in the investment-shock relationship, while *increasing irreversibility* leads to higher inaction *without* affecting the continuity of the investment-shock relationship. This makes us think that matching theoretical discontinuity with empirically observed discontinuity in the investment-shock relationship could easily identify fixed costs. But given that this discontinuity is very hard to observe empirically (see observed gross and new investment rate distributions in *section 4*, both of which are continuous), this method does not work.¹⁸

Because of this, we chose a somewhat more indirect method to identify the fixed cost parameter: we try to match theoretical and observed inaction rates. (As discussed later, inaction is easily observable when we distinguish between new and replacement investment.) The general idea behind this is the following: inaction can emerge both because of fixed costs and irreversibility. But given that reduced regression (9) identifies irreversibility (and also irreversibility-induced inaction), from the observed inaction, together with the irreversibility-induced inaction, we can infer fixed cost-induced inaction and fixed costs themselves.

An alternative way to identify irreversibility separately from fixed costs is to investigate the asymmetry of the investment rate distribution. It seems to be obvious that irreversibility creates asymmetric behavior on the positive and negative ends at the micro level, while fixed costs do not lead to such asymmetry. So when identifying the structural cost parameters, we will also control for the asymmetry of the theoretical investment distribution, hoping that the direct identification of the irreversibility parameter indirectly identifies fixed costs (through matching the inaction rates). Thus we also match the theoretical and observed asymmetry to each other.

¹⁷ In the cost-free case they were $\hat{y}_1 = 2.5233$, $\hat{y}_2 = 0.4384$, $\hat{y}_3 = -2.5151$.

¹⁸ *Cooper and Haltiwanger (2005)* also report continuous investment distribution, based on a different establishment-level data set.

To conclude this section, let us summarize our estimation strategy. To identify the investment cost function's structural parameters, we will *match the estimated regression parameters of equation (9), along with the inaction rate and investment rate distribution skewness in the theoretical model to similar parameters observed in real data.*

IV – Data and empirical results

Data set and main variables

To estimate the structural parameters of the investment cost function, we use a panel data set about balance sheet and income statement data of publicly traded US manufacturing firms between 1959-1987, a part of the COMPUSTAT database.¹⁹ This is a well-known data set, and for a detailed documentation we simply refer to *Hall (1990)*. Here we give only a brief description of the main features of the data.

The “Manufacturing Sector Master File” is a data set about 2,726 large US manufacturing firms between 1959-87. The unique feature of this data set is that it contains information about the reasons of exits, indicating any domestic/foreign acquisitions, privatizations, leveraged buyouts, bankruptcies, liquidations, reorganizations, and name changes. This makes it possible (by dropping only those firms who were acquired, reorganized, bought out) to build a data set that contains companies with continuous operation together with companies that were either bankrupt or liquidated. Excluding also the bankrupt or liquidated firms could lead to selectivity bias by excluding many companies with presumably large negative profitability shocks and negative investment, which is a significant loss of information when (among others) we investigate irreversibility.

The raw data set contains 49,225 year-observations about 2,726 companies. As a first step, we excluded all merged, acquired, privatized firms from the data set (along with those companies for which the reason of exit is unknown), and obtained a data set containing the continuously operating, bankrupt or liquidated firms. This reduced the size of our data set to 31,297 year-observations about 1,664 companies. We had to narrow our sample further as there are some companies for which we do not have any information about their net value of capital; due to this fact the size of the panel is decreased to 29,548 year-observations about 1,617 companies. Finally, at later stages we will use sales revenues as a weighting variable; in some cases this is missing, or it is unreasonably small.²⁰ After deleting these observations the size of the data set decreases to 29,500 year-observations about 1,616 firms. *Table 2* contains information about the entry and exit dates of these 1,616 companies.

From now on, we will refer to these 29,500 year-observations about 1,616 companies as the “*full sample*”. But for comparison purposes we also created a

¹⁹ I am grateful to *Plutarchos Sakellaris* for giving me access to these data.

²⁰ This latter category includes newly created firms: we can observe in case of these that the sales revenue is virtually zero, while having huge losses. We assume that this phenomenon is due to initial investment, and therefore does not represent normal operation, so we deleted these few cases from our data set.

balanced sub-sample between 1972-87²¹ of the entire data set, labeled thereafter as “*balanced*”. This sub-sample contains 15,088 year-observations about 943 firms, and its composition is as in the shaded-striped area of *Table 2*.

		LAST OBSERVATION													SUM
		75	76	77	78	79	80	81	82	83	84	85	86	87	
F I R S T	59				1	1	2	4						278	286
	60	1	3	2	2	2	2	3	1	2	1	2		124	145
	61		1		1	1	1	1		2				24	31
	62		1					1		1		1		60	64
	63					1				1	3	2		42	49
	64			1	1			1		1		1		26	31
	65					1				2		1		33	37
	66				1						1			37	39
	67	1		2		1	1			1		1	1	46	54
	68		1		2	1	1	3	1	2	2	2	2	126	143
O B S E R V A T I O N	69		1			1	2	3			1	1	1	38	48
	70		1				1	2			1			21	25
	71			2	1	1	1	4	1					42	54
	72		1	2		1	1	2			2	1		46	53
	73								1		2	1		43	47
	74						1	1	1	2	1	3		86	95
	75							1						12	13
	76										1	1		14	16
	77						1	1						7	9
	78													24	24
	79						1		2	1				34	38
	80							2		1		1	1	43	48
	81												1	23	24
	82											1	1	65	67
	83												1	55	57
	84										1			37	37
	85													44	44
	86													38	38
SUM		2	9	7	9	11	14	29	7	16	16	19	9	1468	1616

Table 2. Entry and exit dates of the firms in our sample

To measure the capital stock of the firms, we use inflation-adjusted net plant value (NPLANT). This variable was calculated by “multiplying the book plant value by the ratio of the US GNP deflator for fixed nonresidential investment in the current year to the GNP deflator AA years ago”²², where AA stands for the average age of the plant and equipment for this particular firm. Thus this variable is a corrected book plant value of the firms, where the correction was made to express all previous capital purchases at current prices.

To measure gross investment, we used the difference between gross capital expenditures (UFCAP) and sales of property, plant and equipment (SFPPE), both reported from firms’ statements of changes. We preferred these variables to the main investment variable of the data set (INVEST) as this latter also includes the amount spent to acquisitions and other not strictly investment-related expenditures. (Note, however, that in the vast majority of the observations we have UFCAP = INVEST, so our results would not change dramatically if we used the other investment variable.)

To calculate an investment rate variable we first subtracted the capital sales (SFPPE) from gross capital expenditures (UFCAP), and then divided this by the previous year’s net plant value, and obtained the investment rate in year t :

$$INVRATE_t = \frac{UFCAP_t - SFPPE_t}{NPLANT_{t-1}}. \quad (10)$$

²¹ Cooper and Haltiwanger (2005) use a balanced sub-sample of the Longitudinal Research Database (LRD) between 1972-88.

²² Hall (1990), page 18.

This observed investment rate, however, contains both new and replacement investments, while we are mainly interested in the costly new investment rate. To be consistent with the assumptions of the theoretical model, we separated new investment and replacement investment based on the relationship between observed net capital expenditures ($UFCAP_t - SFPPE_t$) and depreciation ($ADJDEP_t$).²³

- If $UFCAP_t - SFPPE_t > ADJDEP_t$, then net capital expenditures exceeded depreciation, so net capital stock increased. We assume that in this “expansionary” case firms undertake as much replacement investment as possible (as this is relatively cheap), and only the increase in the value of net capital stock is the result of the costly new investment activity. So in this case, $NEWINVRATE_t = \frac{(UFCAP_t - SFPPE_t) - ADJDEP_t}{NPLANT_{t-1}}$.
- If $ADJDEP_t \geq UFCAP_t - SFPPE_t \geq 0$, then the firm’s net capital expenditures were positive, but since they did not entirely cover depreciation, the firm’s former capital stock depreciated to some extent. We assume in this case all capital expenditures were maintenance-type replacement expenditures, and therefore $NEWINVRATE_t = 0$.
- if $0 > UFCAP_t - SFPPE_t$, then the firm is obviously shrinking. It seems to be logical to assume in this case that no replacement investment was undertaken, as this could have been compensated for by costly capital sales. In this case $NEWINVRATE_t = INVRATE_t = \frac{UFCAP_t - SFPPE_t}{NPLANT_{t-1}}$.

The distributions of the calculated gross and new investment rates are depicted on Figures 5-6.

²³ ADJDEP is an adjusted measure of the depreciation, where (similarly to the correction of NPLANT) observed depreciation is deflated by an investment deflator AA (average age of capital) years ago, to get a measure of depreciation that is expressed in current prices (as opposed to historical purchase prices represented in the book value).

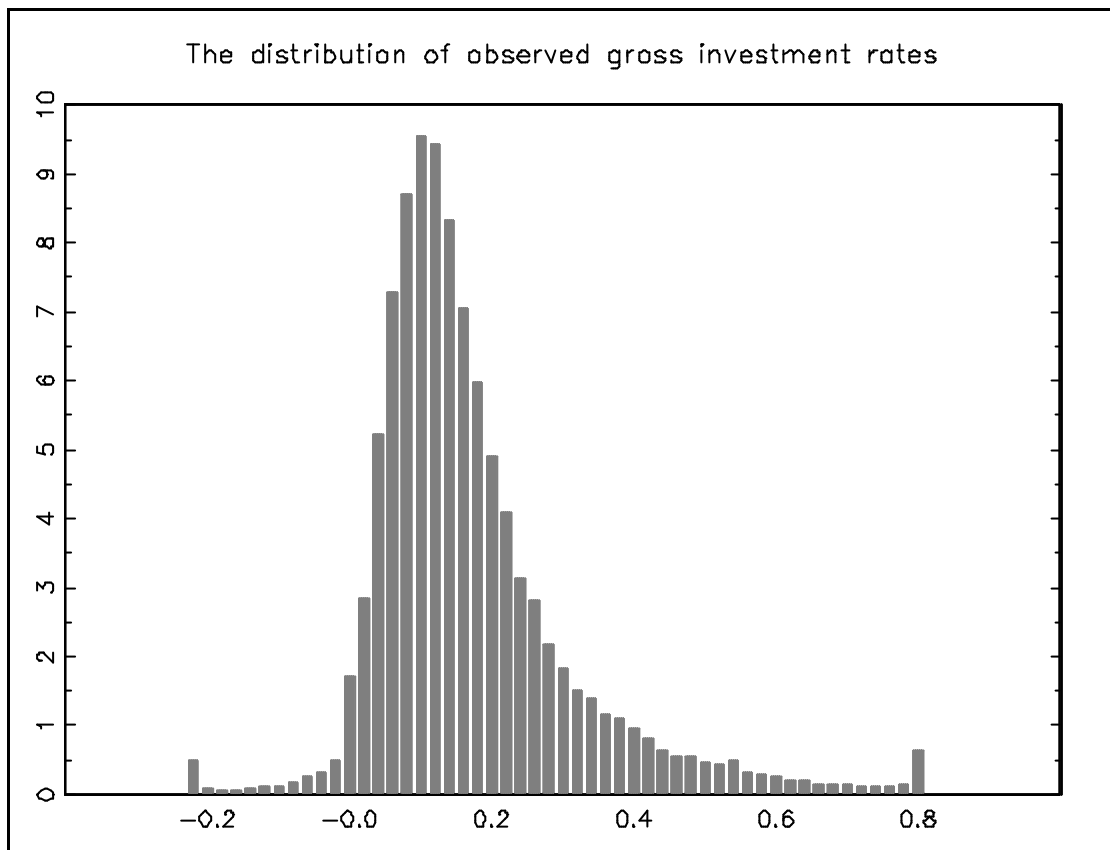


Figure 5. The distribution of observed gross investment rates, full sample

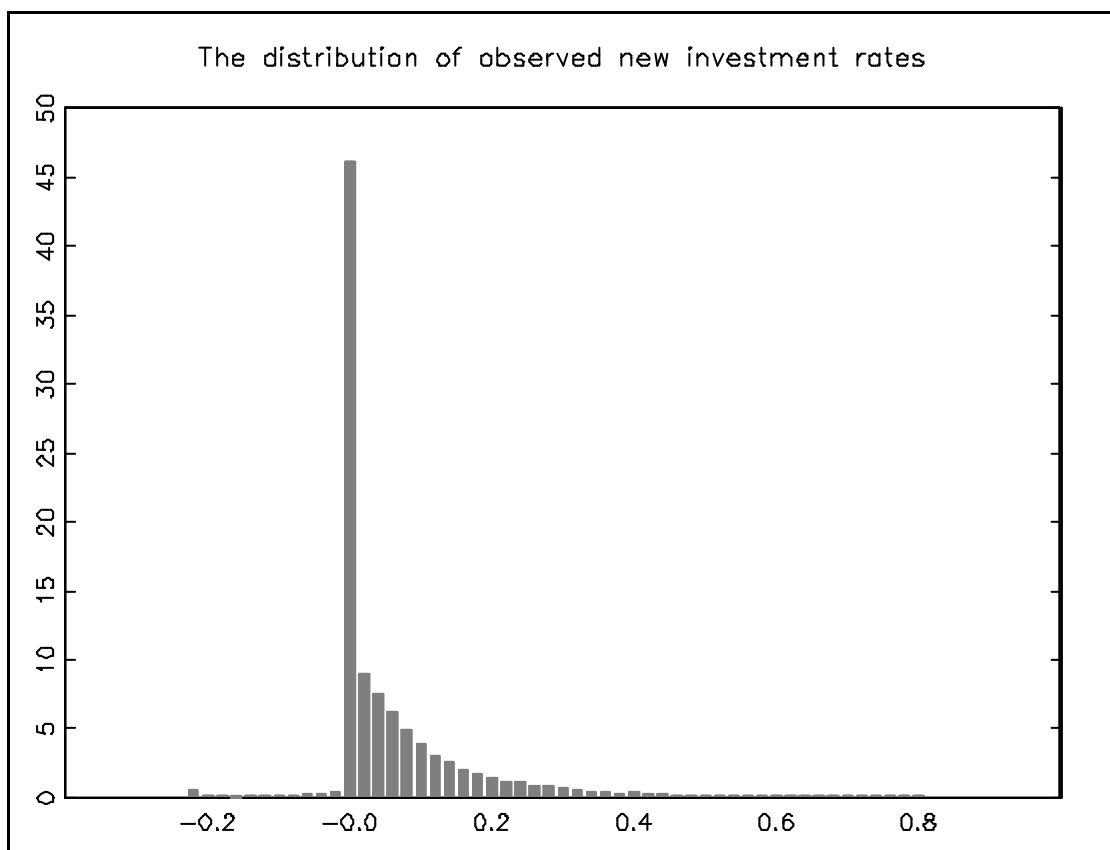


Figure 6. The distribution of observed new investment rates, full sample

The shape of the gross investment rate distribution is very similar to what is reported in *Cooper and Haltiwanger (2005)*, even though we use firm-level (as opposed to establishment-level) data. The mode of the distribution is at about 8% investment rate, probably reflecting usual replacement investment activity. In the new investment rate distribution we have a large peak at zero, reflecting the fact that we had many observations with net investment expenditure between 0 and observed depreciation. We also see the mode of the distribution at very low positive investment levels: further 12.51% of observed investment rates is in the [0;3%] range, while the proportion of rates in the [0;5%] range is 21.18%. We also see that the observed distributions are skewed to the left.

In certain steps of the analysis, we also use the following variables: operating income before depreciation (OPINC), sales revenue (SALES), and employment (EMPLY). *Appendix C* contains a full description of variable definitions.

Estimating the reduced regression parameters from data

As discussed in *section 3*, we estimate the investment cost parameters (F, g, p) by matching the observed reduced regression parameters, inaction rate and asymmetry of investment distribution with the same parameters calculated from theoretical investment models. Therefore, as a first step we estimate the reduced form regression parameters, calculate the inaction rate and asymmetry of investment distribution for our data set.

To estimate the reduced form regression (9) we first identify the yearly profitability shocks that hit the firms in our data. We do this by adopting the strategy of *Cooper and Haltiwanger (2005)*. First we assume that firms have identical, constant returns-to-scale Cobb-Douglas production functions:

$$Y_{it} = B_{it} L_{it}^{a_L} K_{it}^{1-a_L}, \quad (11)$$

where labor (L_{it}) can be adjusted in the short-run and can therefore be regarded in our yearly sample as being optimized, but capital (K_{it}) cannot be adjusted in the short-run. In this expression Y_{it} denotes production, B_{it} is production shock,²⁴ a_L is labor share. We also assume that firms face a constant elasticity (x) demand curve $D(p) = p^x$, so the inverse demand curve is $p(y) = y^{1/x}$. Therefore the firms' problem is:

$$\Pi_{it} = p_{it}(y_{it})y_{it} - wL_{it} = y_{it}^{\frac{1+x}{x}} - wL_{it} = B_{it}^{\frac{1+x}{x}} L_{it}^{a_L \frac{1+x}{x}} K_{it}^{(1-a_L) \frac{1+x}{x}} - wL_{it} \xrightarrow{L_{it}} \max, \quad (12)$$

²⁴ Note that this is not the profitability shock that we have in reduced regression (9).

where w denotes the wage rate (assumed to be constant). The first-order condition of this problem is $a_L \frac{1+x}{x} B_{it}^{\frac{1+x}{x}} K_{it}^{(1-a_L)\frac{1+x}{x}} L_{it}^{\frac{a_L(1+x)}{x}-1} = w$, from which the optimal labor usage is

$$L_{it}^* = \left(\frac{x}{a_L(1+x)} \right)^{\frac{x}{a_L(1+x)-x}} w^{\frac{x}{a_L(1+x)-x}} B_{it}^{\frac{1+x}{x-a_L(1+x)}} K_{it}^{\frac{(1+x)(1-a_L)}{x-a_L(1+x)}}. \quad (13)$$

Substituting this into the profit function (11), the optimal profit of the firm is

$$\begin{aligned} \Pi_{it}^* &= B_{it}^{\frac{1+x}{x}} (L_{it}^*)^{\frac{a_L(1+x)}{x}} K_{it}^{(1-a_L)\frac{1+x}{x}} - wL_{it}^* = \\ &= \left(\frac{x}{a_L(1+x)} \right)^{\frac{-x}{x-a_L(1+x)}} w^{\frac{-a_L(1+x)}{x-a_L(1+x)}} B_{it}^{\frac{1+x}{x-a_L(1+x)}} K_{it}^{\frac{(1+x)(1-a_L)}{x-a_L(1+x)}} \left[\frac{x}{a_L(1+x)} - 1 \right]. \end{aligned} \quad (14)$$

Hence if we write (14) as $\Pi_{it}^* = A_{it} K_{it}^q$, where A_{it} denotes the profitability shock²⁵ (as opposed to B_{it} , which was productivity shock), then $q = \frac{(1+x)(1-a_L)}{x-a_L(1+x)}$, a function of the demand elasticity and the labor share in the production function.

We identify firm-level profitability shocks simply by $A_{it} = \frac{\Pi_{it}^*}{K_{it}^q}$, which can be calculated from our data set if we have an estimate for q . We estimated $q = 0.6911$ in the full sample. (In the balanced sub-sample the estimated q is $\hat{q} = 0.4641$.)²⁶

We call the profitability shock calculated this way as type 1 shock. But as the resulting variance of the profitability shocks appears to be implausibly large (see *Tables 3a-b* later), replicating the strategy *Cooper and Haltiwanger (2005)*, we estimated the profitability shocks in an alternative way. A little algebra shows that the optimal profit in (14) can also be written as

²⁵ So the profitability shock consists of wages, demand elasticities, labor shares and productivity shocks. We could argue that wages are also changing over time, but as one can easily see, any aggregate time-series variation of wages is captured by the time dummies in the reduced form regression.

²⁶ We estimated q from a non-linear model using firm-level fixed effects: $\Pi_{it} = A_i K_{it}^q + e_{it}$. We estimated this model with non-linear least-squares, and treated the heteroscedasticity by weighting the observations with $1/Y_{it}$, where Y_{it} stands for the sales revenues. For a more detailed description of our estimation method see *Appendix B*.

$$\Pi_{it}^* = wL_{it}^* \left[\frac{x}{a_L(1+x)} - 1 \right] = wL_{it}^* \frac{1-a_L}{q a_L}, \quad (15)$$

therefore the A_{it} profitability shock can also be calculated as

$$A_{it} = \frac{\Pi_{it}^*}{K_{it}^q} = \frac{wL_{it}^*(x - a_L(1+x))}{K_{it}^q a_L(1+x)} = \frac{wL_{it}^*}{qK_{it}^q} \frac{1-a_L}{a_L}. \quad (16)$$

To compute the profitability shock this way, we should know the value of a_L . But as we only need the deviation of the (log) profit shocks from their plant-specific means (see reduced regression (9)), the parameter a_L becomes unimportant:

$\log(A_{it}) = \log\left(\frac{wL_{it}^*}{qK_{it}^q}\right) + \log\left(\frac{1-a_L}{a_L}\right)$, so when subtracting the plant-specific means, the

time-invariant term disappears. Therefore it is enough to calculate $A_{it} = \frac{wL_{it}^*}{qK_{it}^q}$, for which we have data. We will call the profitability shock calculated this way as type 2 shock.²⁷

With the calculated profitability shocks we are ready to estimate reduced regression (9). But later, when the theoretical model is solved numerically, we shall simulate the profitability shocks and draw random variables from the observed distribution of shocks, using the exact correlation structure. As this correlation structure is extremely rich (shocks are correlated both across individuals and through time), we decomposed the A_{it} profitability shocks (both types) into aggregate and idiosyncratic shocks.²⁸ Following the method of *Cooper and Haltiwanger (2005)*, we

define the aggregate shock simply as $A_t = \frac{\sum_{i=1}^{N_t} A_{it}}{N_t}$, where N_t stands for the number of observations in year t . The idiosyncratic shock is what remains: $w_{it} = \frac{A_{it}}{A_t}$.

Tables 3a-b contain the descriptive statistics of the identified type 1 and type 2 shocks in the full sample and balanced sub-sample.

²⁷ In fact we have data only on the size of labor force, not on labor costs. But similar considerations as in case of a_L lead us to conclude that the value of w is unimportant.

²⁸ Thus the common aggregate shock captures between-firm correlation of shocks; the autocorrelation of the aggregate and idiosyncratic shocks captures the within-firm correlation of shocks.

<i>Type 1 shocks</i>	<i>Full sample</i>	<i>Balanced panel</i>
Aggregate shock standard deviation	0.1359	0.1196
Aggregate shock autocorrelation	0.6979	0.5297
Idiosyncratic shock standard deviation	1.8355	2.1477
Idiosyncratic shock autocorrelation	0.4004	0.4975

Table 3a. Standard deviations and autocorrelations of type 1 shocks

<i>Type 2 shocks</i>	<i>Full sample</i>	<i>Balanced panel</i>
Aggregate shock standard deviation	0.0822	0.0407
Aggregate shock autocorrelation	0.9325	0.7449
Idiosyncratic shock standard deviation	0.2891	0.2482
Idiosyncratic shock autocorrelation	0.6410	0.7050

Table 3b. Standard deviations and autocorrelations of type 2 shocks

It is apparent from *Tables 3a-b* that type 2 shocks have much smaller variation than type 1 shocks, probably reflecting lower measurement error in the labor force variable than in the profit variable.²⁹ It is also intuitive that in case of the more reliable type 2 shocks if we identify the profitability shocks in the full sample, as opposed to the balanced sub-sample, we find the standard deviation of the shocks significantly (25-30%) higher. This difference could be attributed to the variance-increasing effect of the large negative shocks that probably hit the liquidated and bankrupt firms mostly excluded from the balanced panel.

With the identified profitability shocks, we now estimate the reduced form regression (9):

$$\tilde{i}_{it} = y_0 + y_1 \tilde{a}_{it} + y_2 (\tilde{a}_{it})^2 + y_3 \tilde{a}_{i,t-1} + m_t + u_{it}.^{30} \quad (9)$$

Table 4 reports the estimated parameters of this regression for the full sample and balanced sub-sample. The estimated reduced regression coefficients are quite similar to each other, the only difference is in the estimated y_2 parameter, which is significant only at the 10% level in the balanced sub-sample.³¹

²⁹ *Cooper and Haltiwanger (2005)* have similar findings on a different data set.

³⁰ Firms were quite heterogeneous with respect to the variation of shocks that hit them. To deal with this kind of firm-level heterogeneity, and to avoid larger influence of more volatile firms for the estimated parameters, we weighted each observation by one over the firm-level standard deviation of the identified shock. This makes our results more comparable to the simulation results (see next part), as in the simulation exercise we also assumed that the standard deviation of the shocks is the same for each firm.

³¹ This is probably because we have relatively few large shocks in the balanced sample, and the reduced regression can detect relatively modest non-linearity.

	<i>Full sample</i>	<i>Balanced sub-sample</i>
y_1	0.1150 (0.0085)	0.1032 (0.0099)
y_2	0.0822 (0.0147)	0.0310 (0.0289)
y_3	-0.0251 (0.0081)	-0.0368 (0.0092)
<i>R-squared</i>	0.0711	0.0528
<i>No. of firms</i>	1,554	941
<i>No. of obs.</i>	23,413	12,847

Table 4. Estimated reduced regression parameters. Standard errors are in parenthesis.

Estimating the inaction rate and asymmetry of investment distribution from data

We discussed in *section 3* that we also match theoretical inaction rate (the proportion of zero investments) to the observed inaction rate, because this way we can better identify the fixed cost parameter of the investment cost function. The observed inaction rate is 42.35% in the full sample, with a standard error of 0.29%.³² Similar inaction rate is reported for the balanced sub-sample in *Table 5*.

As usual, we use skewness to measure the asymmetry of the investment rate distribution. As discussed earlier, only the irreversibility parameter is likely to influence the asymmetry of this distribution, so this may lead to better identification of the irreversibility parameter. *Table 5* contains the estimated skewness values of the investment rate distribution: it is 1.2182 for the full sample, and 0.9866 for the balanced sub-sample. (The corresponding standard errors of the estimated skewness figures³³ are 0.0146 for the full sample, and 0.0200 for the balanced sub-sample.)

	<i>Full sample</i>	<i>Balanced sub-sample</i>
Skewness of investment rate distribution	1.2182 (0.0146)	0.9866 (0.0200)
Inaction rate	0.4235 (0.0029)	0.4625 (0.0041)

Table 5. Skewness of investment rates and observed inaction. Standard errors are in parenthesis.

³² The standard errors of the estimated proportions are $\sqrt{\frac{p(1-p)}{n}}$, with p denoting the estimated proportion, and n is the number of observations. We have apparently larger inaction rate in *Figure 6*, but that is actually the proportion of new investment rates between -1% and 1%.

From now on, we will work with the parameter estimates for the full sample as benchmark parameters. In the second step of the estimation procedure we will choose the structural parameters of the investment cost function in such a way, that the estimated reduced regression parameters from the theoretical model, along with the inaction rate and skewness of investment rate distribution, should be sufficiently close to these benchmark parameters.

Before finishing this section, it maybe useful to summarize those results that we will use to identify the theoretical cost parameters:

- *standard deviations and autocorrelations of the identified (type 2) aggregate and idiosyncratic shocks* (for the full sample, these are in column 2 in Table 3b). This information will be used to simulate aggregate and idiosyncratic shocks that are similar to observed shocks when we solve numerically the theoretical model and simulate investment paths;
- *estimated parameters of the reduced form regression* from the full sample (column 2 in Table 4), and the estimated variance-covariance matrix $(\hat{W})$ of the estimated parameters $y^{TRUE} = (\hat{y}_1, \hat{y}_2, \hat{y}_3)$;
- *estimated skewness* of the distribution of the investment rates in full sample, and the standard error of this (first entry of the 2nd column in Table 5);
- *observed inaction rate* in full sample, and the standard error of this (second entry of the 2nd column in Table 5).

V – Estimation Results

The estimation of the structural cost parameters involves three steps.

Step 1. We specify the investment cost function according to (8):

$$\frac{C(I, K)}{K} = \begin{cases} \frac{I}{K}, 0 \leq \frac{I}{K} \leq d, \\ F + \frac{I}{K} + \frac{g}{2} \left(\frac{I - dK}{K} \right)^2, \frac{I}{K} > d, \\ F + p \frac{I}{K} + \frac{g}{2} \left(\frac{I}{K} \right)^2, \frac{I}{K} < 0. \end{cases} \quad (8)$$

³³ The standard error of estimated skewness is calculated as $\sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}} \approx \sqrt{\frac{6}{n}}$, with n denoting the number of observations.

Then we solve numerically the theoretical investment model for any cost parameter vector (F, g, p) . When solving the model, we assume that $b = \frac{1}{1+r} = 0.95$, $d = 0.07$, common assumptions in the literature using US data. For the numerical solution we use parametric value function iteration (as described by Judd (1998)), with the value function assumed to be a bivariate cubic function of (A, K) .³⁴ During the solution we assume that the profitability shock A_t is the sum of aggregate and idiosyncratic shocks, where the aggregate shock is a 2-state Markov-process with standard deviation and autocorrelation estimated from real data (see Table 3b), and the idiosyncratic shock is an AR(1) process with normally distributed innovations (also matching the properties of idiosyncratic shocks reported in Table 3b).

Step 2 With the numerical solution of the theoretical model, we simulate capital and investment paths of hypothetical firms. To do this, first we simulate (aggregate and idiosyncratic) profitability shocks, using the descriptive statistics of the “true” (type-2) profitability shocks identified from the data (Table 3b). We also simulate the initial capital of the firms, then with the policy function obtained in Step1, along with the simulated shocks, we generate the capital and investment path of each firm.³⁵ Then we calculate the deviations from plant-specific means for both the investment rate and profitability shocks, and estimate the reduced form regression (9) on the simulated data set. We also calculate the skewness and the inaction rate in the simulated investment distribution. Let us denote the estimated set of parameters by $y(F, g, p)$, which expresses that these estimated parameters will depend on the structural cost parameters.³⁶

Step 3. For any cost parameter vector (F, g, p) , we calculate the “distance” between $y(F, g, p)$ and the parameter vector estimated from real data. The distance function is

$$D(F, g, p) = (y(F, g, p) - y^{TRUE})' \cdot \hat{W}^{-1} \cdot (y(F, g, p) - y^{TRUE}), \quad (18)$$

where $\hat{W}$ is the variance-covariance matrix of y^{TRUE} estimated from the data.³⁷ That is, for any (F, g, p) the distance is the weighted sum of squared deviation of the estimated parameters on simulated data from the “true” parameter set estimated on

³⁴ We solved the model initially with the more precise (but computationally more demanding) value function iteration, and chose the cubic functional form based on these results.

³⁵ To avoid problems arising from the misspecified distribution of the initial capital of the firms, we prepare the capital path of each firm for 129 periods (instead of 29), and only consider the data of the last 29 periods, which are not influenced by the initial level of the capital. The number of simulated firms is the same as the number of firms in our data, 1616.

³⁶ Note that here y is a vector containing 5 elements: (y_1, y_2, y_3) from the reduced form regression, and the skewness and the inaction rate in the investment rate distribution.

³⁷ $\hat{W}$ contains the estimated variance-covariance matrix of the reduced regression parameters, the estimated variance of the skewness of the investment distribution (0.0146^2) , and the estimated variance of the observed inaction rate (0.0029^2) . The pair-wise covariance between the estimated skewness, inaction rate and reduced regression parameters is assumed to be 0.

real data, with the weights being the estimated variance-covariance matrix of the “true” parameters.³⁸

We estimate the structural cost parameters (F, g, p) by minimizing the distance function (18). The results for the “full sample” are in *Tables 6-7*.

Table 6 contains the estimated structural cost parameters.³⁹ The magnitude of the fixed costs seems to be very small, but the estimated parameter is about 4.69% of the “regular” purchase price of capital when investment rate is 1%.⁴⁰ Moreover, the estimated irreversibility parameter indicates substantial irreversibility, a more than 30% average discount on capital sales. The estimated convex cost parameter is 0.4464.

estimated F	0.000469 (0.000151)
estimated g	0.4464 (0.0252)
estimated p	0.6962 (0.0923)
optimal LOSS	193.4723

Table 6. Estimated cost parameters from the full sample. Standard errors are in parenthesis

To further analyze the estimated cost parameters, suppose that we sell 1% of our existing capital stock. Then the fixed cost of this transaction is 0.000469, the convex cost is $(0.4464/2)*(-0.01)*(-0.01) = 0.00002232$, and the irreversibility cost is $0.3038*0.01 = 0.003038$ (the product of the discount at which we can sell capital, and the quantity sold). So the total adjustment cost to be paid is 0.003529, which is 35.29% of the price we would get for this capital sale in the absence of frictions (0.01). The relative importance of the different cost components is the following: 13.3% of total adjustment costs is fixed cost, 0.6% of total adjustment costs is convex cost, and the remaining 86.1% is irreversibility cost.⁴¹ The average positive investment rate in our data set is 6.34%; in this case the total adjustment costs are 2.15% of the purchase price, of which 34.3% are due to fixed costs, and 65.7% are due to convex costs. (Obviously, in case of positive investment rate there are no direct irreversibility costs.)

The estimated irreversibility parameter, $p = 0.6962$ (significantly smaller than 1) indicates that firms can sell their used capital at a 31% discount, or on average 31% of any dollar spent on investment is sunk. This is quite far from the estimate of *Ramey and Shapiro (2001)*, who find that at a discontinuing US plant the average discount on

³⁸ So parameters estimated with smaller standard errors have larger weights.

³⁹ Standard errors are calculated as described by *Gourieroux and Monfort (1996)*.

⁴⁰ Note that we normalized the model to the buying price of capital, and therefore when investment rate is 1%, the price of new capital is 0.01.

⁴¹ For different investment rates, these proportions change. Larger new investment activity generally increases the importance of adjustment costs, mainly because of relatively quickly increasing convex costs. Also, for larger investment projects convex costs will dominate fixed and irreversibility costs.

used capital is 72% of the replacement value. This value is also far from the findings of *Reiff (2004)*, where more than 50% discount is documented for a discontinuing Hungarian manufacturing plant. However, our estimate of $p = 0.69$ is based on continuously operating plants, as opposed to the total sell-out of assets at discontinuing plants, so these results cannot be directly compared. On the other hand, our parameter estimate is much smaller (and therefore indicates much higher irreversibility) than those results in the literature that use similar techniques to ours. Based on indirect inference, with reduced form regression (9) in a somewhat modified model, the initial version of *Bayraktar, Sakellaris and Vermuelen (2005)* estimate $p = 0.902$ for German manufacturing plants between 1992-2000. Further, for a balanced panel of US manufacturing plants *Cooper and Haltiwanger (2005)* estimate $p = 0.975$ with a simulated maximum likelihood method. These latter two results are estimated from a balanced panel, which may lead to significantly different results than estimation from an unbalanced panel. Moreover, our result of substantial irreversibility is primarily due to the control for the skewness of investment rate distribution, which is missing from other studies.

	<i>simulated</i>	<i>“true”</i>
reduced regression parameter y_1	0.1971	0.1150
reduced regression parameter y_2	0.0778	0.0822
reduced regression parameter y_3	-0.0767	-0.025
inaction rate	0.4179	0.4235
skewness of investment distribution	1.2087	1.2182
total LOSS	193.4723	

Table 7. Estimated reduced regression parameters, inaction rate and skewness

Table 7 reports the simulated reduced regression parameters, inaction rate and investment rate skewness, together with the observed values of the same parameters. We can see from this that our matching technique does quite well to match simulated regression parameters, inaction rate and skewness to their observed values.

Aggregate implications

With the estimated cost parameters one can investigate the aggregate implications of our results. To do this, we simulated a panel of firms that have the investment cost function as we estimated, and calculated the aggregate investment and

aggregate shock over the years in our simulated data set. *Table 8* contains the main descriptive statistics of the simulated aggregate variables with the corresponding descriptive statistics of the individual variables.

	<i>in plant-level data (real data)</i>	<i>in plant-level data (simulation)</i>	<i>in aggregate data (real data)</i>	<i>in aggregate data (simulation)</i>
$st.dev.(i_t)$	0.1258	0.0704	0.0354	0.0224
$corr(i_t, i_{t-1})$	0.2248	0.1143	0.3707	0.5695
$corr(i_t, a_t)$	0.0890	0.5179	0.5227	0.6416

Table 8. Descriptive statistics of aggregate and plant-level investment and shocks

It is obvious from *Table 8* that the standard deviation of the aggregate investment rate is naturally much smaller than that of the individual investment rate. Moreover, the autocorrelation of the investment rate is also much higher in the aggregate level, and the correlation between the investment rate and the profitability shock also increases. Aggregate investment behaves quite differently from individual investment.

We also estimated the investment-shock relationship on the aggregate level. Contrary to what we found on the plant-level, we could not detect any nonlinearity in this relationship. (The squared shock remained insignificant when we estimated a regression of investment on shocks.) We found that in the aggregate level there is a modest linear relationship between profitability shocks and investment – the estimated parameter of the profitability shock is 0.2392 in the real data, and 0.3394 in the simulated data (both of them are significant at the 5% level).

VI – Summary

The goal of this paper is to structurally estimate the most important costs of investment, based on a dynamic model. To estimate the cost components we use an unbalanced panel about US manufacturing plants between 1959-87. To do this, we estimate a reduced form regression that captures the effects (nonlinearity, lumpiness) of the above cost components, along with the inaction rate and skewness of investment distribution, for both real and simulated data.

Our results indicate that fixed costs may be an economically significant factor for the firms' investment activity, although their magnitude is relatively small if compared to the firms' capital stock. On the other hand, we find strong evidence of non-perfect reversibility: we estimate that firms in our data set (that are not necessarily closing firms, as in *Ramey and Shapiro (2001)* and *Reiff (2004)*) can sell their used capital at significantly lower prices than the purchase price. The estimated irreversibility parameter is somewhat smaller than in comparable studies (*Bayraktar, Sakellaris, Vermeulen (2003)*, and *Cooper and Haltiwanger (2005)*), indicating that the extent of irreversibility may be much higher than we thought earlier. Overall, our

parameter estimates support the generally accepted view about firm-level investment activity: there are investment peaks followed by periods of inactivity.

In line with the findings of *Caballero (1992)*, things are different at the aggregate level. Despite plant-level investment being non-linear (on terms of responses to shocks), no similar non-linearity can be detected at the aggregate level. We also find that aggregate investment is much more persistent than plant-level investment, and it is somewhat more responsive to aggregate profitability shocks.

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Appendix A

Investment functions with different simple cost structures

Figure A/1 illustrates the investment-shock relationship in the **cost-free case**. ($F = 0, g = 0, p = 1$). We see that investment is non-zero whenever the shock is non-zero, that is, we have instantaneous adjustment. We see that this function is slightly convex even in this case. This reflects the law of diminishing returns for the capital: when a large shock increases the marginal value of capital (q), we need a proportionally higher increase in the capital stock to restore the optimality condition of $q = 1/b$; therefore the shock-investment relationship is slightly convex: $\hat{y}_1 = 2.5233$, $\hat{y}_2 = 0.4384$, $\hat{y}_3 = -2.5151$. We also see that as investment is cost-free, investment rates are relatively high even for small shocks: a typical profitability shock (of one standard deviation, $\tilde{a} = 0.0822$) triggers a 21.04% ($2.5233*0.0822 + 0.4384*0.0822*0.0822$) investment rate.

Figure A/2: the case of **partially irreversible** investment ($F = 0, g = 0, p = 0.95$). Observe that irreversibility creates an inaction region, but the investment function remains continuous: small investments are still possible. Because of the inaction region, the shock-investment relationship became more convex, and as capital sales became more expensive, we need very large negative shocks ($< -60\%$) to induce negative investments. The estimated parameters of the usual reduced form regression $\hat{y}_1 = 0.8520$, $\hat{y}_2 = 0.3928$, $\hat{y}_3 = -0.5564$. Convexity is stronger (the relative size of $\hat{y}_2$ increased), and the absolute value of the parameters decreased, so effect of profitability shocks is much smaller (a 1 standard deviation profitability shock, $\tilde{a} = 0.0822$ leads to $7.27\% = 0.8520*0.0822 + 0.3928*0.0822*0.0822$ investment).

Figure A/3: we have **convex cost** of investment ($F = 0, g = 0.2, p = 1$). We see that investment is instantaneous (any shock leads to investment activity), but as the marginal cost increased, it is of smaller magnitude (the function became flatter). Estimated reduced regression parameters: $\hat{y}_1 = 0.4657$, $\hat{y}_2 = 0.0672$, $\hat{y}_3 = -0.2557$, so a 1 standard deviation profitability shock leads to an investment rate of 3.87% ($0.4657*0.0822 + 0.0672*0.0822*0.0822$), which is much smaller than in the frictionless case.

Figure A/4: investment function with **fixed costs** ($F = 0.001, g = 0, p = 1$). This is basically the same as in the frictionless case, but firms do not undertake small investments, when the net gain is smaller than fixed costs. So fixed costs create an inaction region, and also lead to discontinuity (as no small investment activity is observed). The estimated parameters of the reduced form regression (9) are: $\hat{y}_1 = 2.4475$, $\hat{y}_2 = 0.4423$, $\hat{y}_3 = -2.4165$, which is very similar to the frictionless case. This result is intuitive, as the graph of the investment function has not changed dramatically. A 1 standard deviation profitability shock leads to an investment rate of 20.42% ($2.4475*0.0822 + 0.4423*0.0822*0.0822$), which is also similar to the frictionless case.

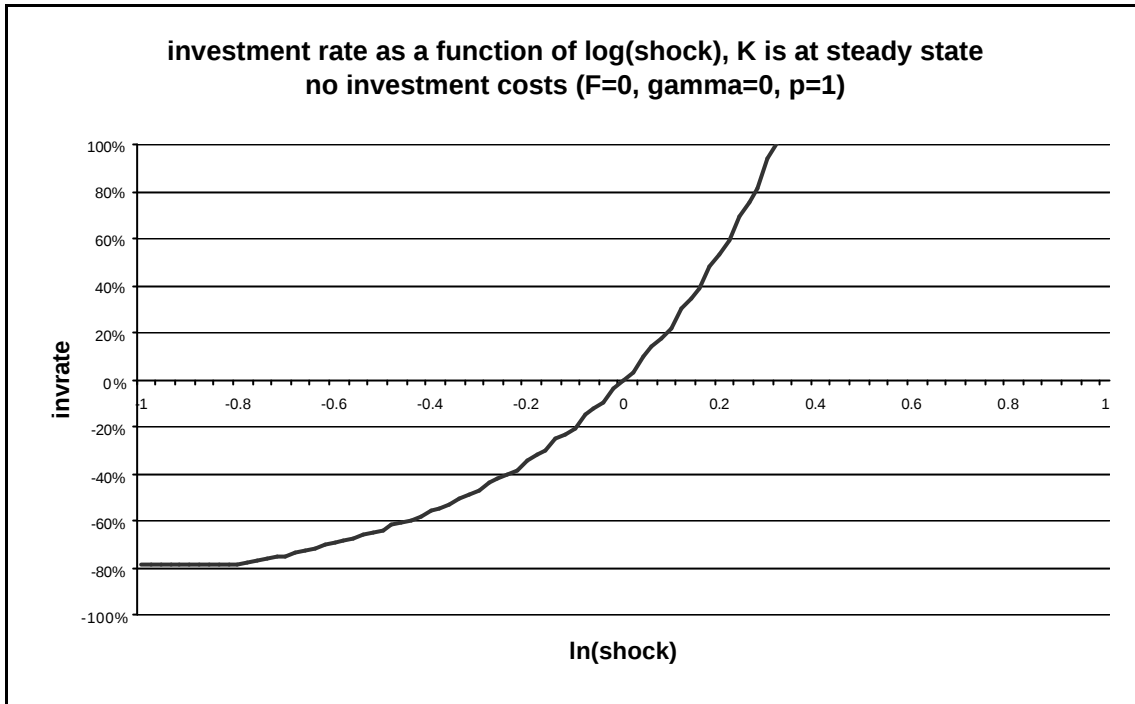


Figure A/1. The investment function in the costless case

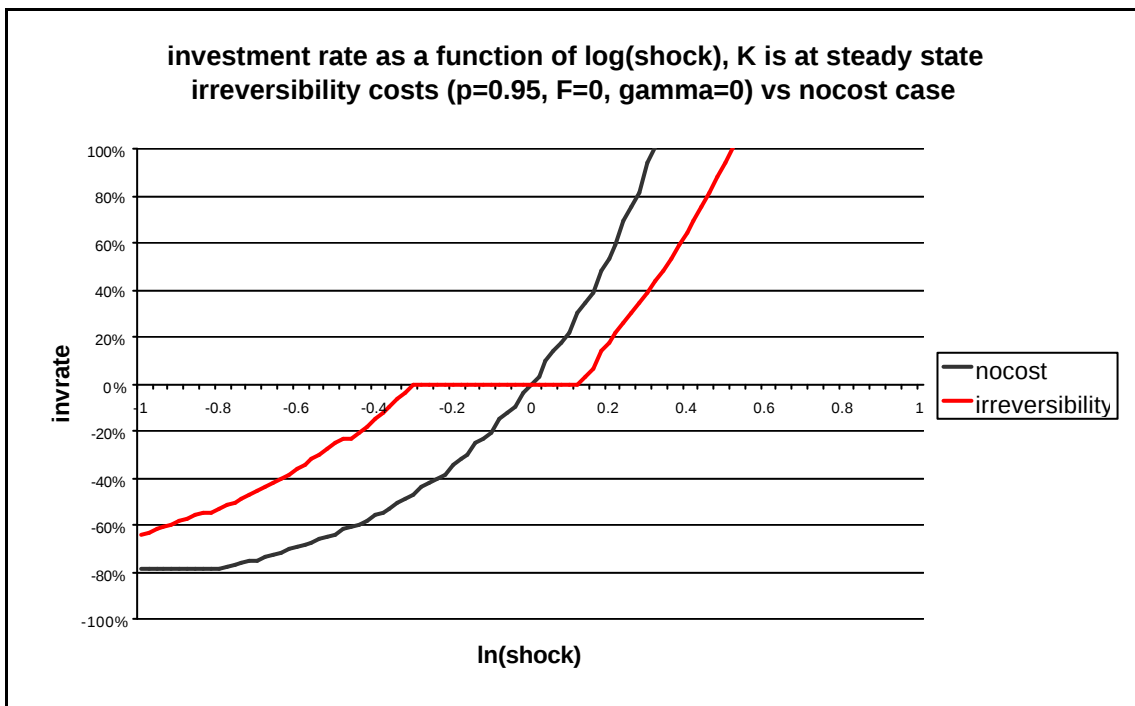


Figure A/2. The investment function if there is (partial) irreversibility

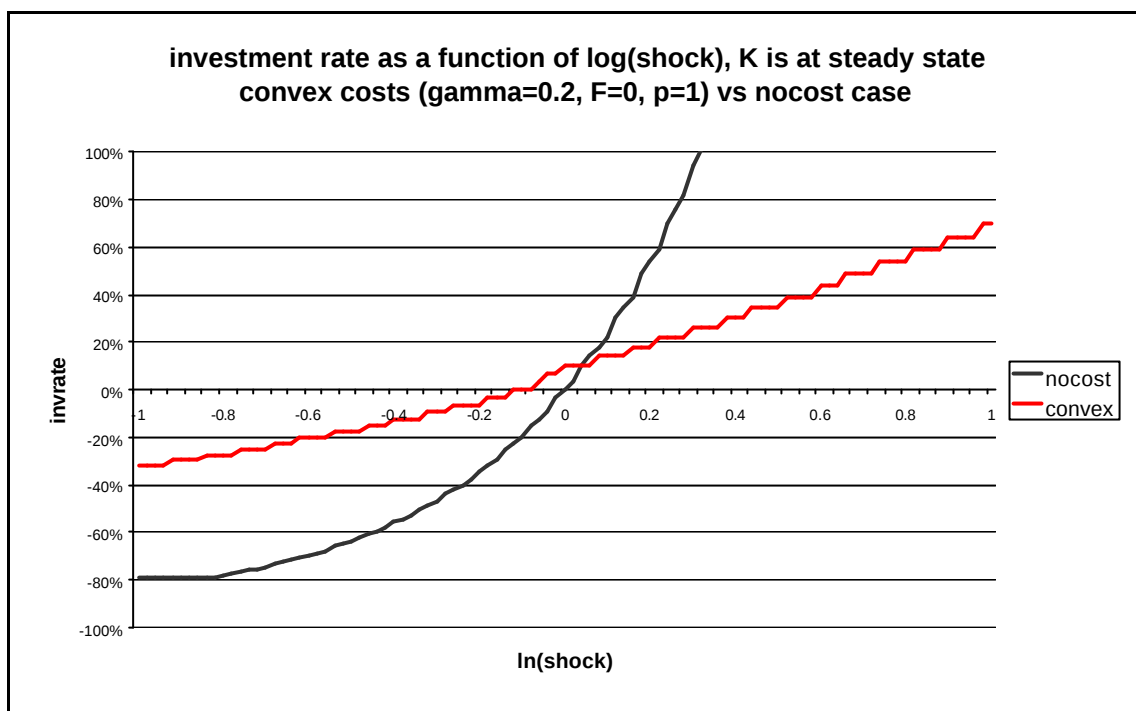


Figure A/3. The investment function if there is a convex cost of investment

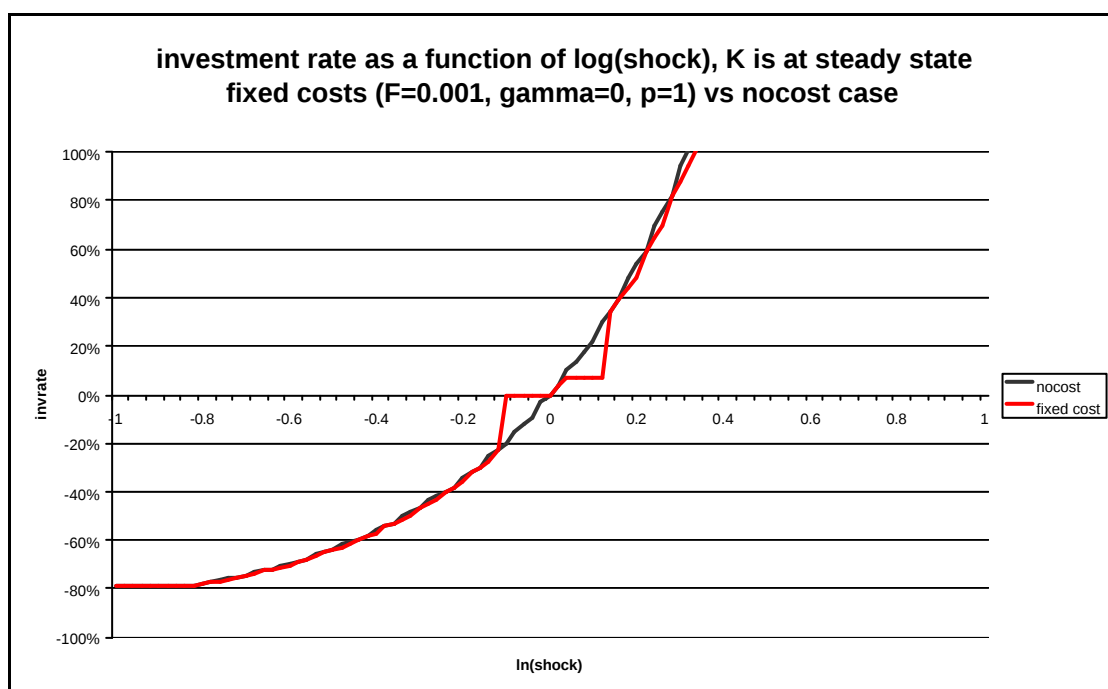


Figure A/4. The investment function if there is a fixed cost of investment

Appendix B

Methodology to estimate the curvature of the profit function

We want to estimate q in the profit function:

$$\Pi_{it} = A_i K_{it}^q + e_{it}, \quad (A1)$$

where Π_{it} and K_{it} are the profit and capital of firm i at year t , respectively, A_i is a firm-specific scaling parameter of the profit function, and e_{it} is a well-behaving error term. To estimate parameter q , we have to solve

$$\sum_i \sum_t (\Pi_{it} - A_i K_{it}^q)^2 \xrightarrow{q, A_i} \min. \quad (A2)$$

First-order condition with respect to q :

$$\sum_i \sum_t 2(\Pi_{it} - A_i K_{it}^q) (-A_i q K_{it}^{q-1}) = 0, \quad (A3)$$

that is,

$$\sum_i A_i \sum_t \Pi_{it} K_{it}^{q-1} = \sum_i A_i^2 \sum_t K_{it}^{2q-1}. \quad (A3')$$

First-order condition with respect to A_i :

$$\sum_{t=1}^{T_i} 2(\Pi_{it} - A_i K_{it}^q) (-K_{it}^q) = 0, \quad (A4)$$

or

$$\sum_t \Pi_{it} K_{it}^q = A_i \sum_t K_{it}^{2q}, \quad (A4')$$

therefore

$$A_i = \frac{\sum_t \Pi_{it} K_{it}^q}{\sum_t K_{it}^{2q}}. \quad (A5)$$

If we substitute (A5) back to (A3'), then we obtain an equation for $\hat{q}$.

Appendix C

Variable definitions

- NPLANT: capital stock. “The net value of the plant adjusted for inflation. This quantity is obtained by multiplying the book plant value by the ratio of the GNP deflator for fixed nonresidential investment in the current year to GNP deflator AA years ago. AA is the average age of the plant and equipment for this firm which is deduced in the following manner: an average age series is obtained as the ratio of accumulated depreciation (gross plant minus net plant) to depreciation this year. This assumes straight-line depreciation...”
- UFCAP: capital purchases. “Compustat data item #128, capital expenditures (from statement of changes).”
- SFPPE: capital sales. “Compustat data item #107, sale of plant, property and equipment (from statement of changes).”
- INVEST: alternative investment measure, not used because we want to exclude acquisitions. “Compustat data item #30, capital expenditures (gross investment). The amount spent for the construction and/or acquisition of property, plant and equipment, including that of purchased companies (acquisition).”
- ADJDEP: depreciation (to calculate new investment rate). “This year’s depreciation adjusted for the effects of inflation. This variable is DEPREC deflated by the ratio of the GNP deflator for fixed nonresidential investment AA (see NPLANT for a definition of AA, average age) years ago to the current GNP deflator.”
- OPINC: profit variable, before depreciation, which is consistent with expression (12). “Compustat data item #13, operating income before depreciation.”
- SALES: sales revenue, a weighting variable for NLLS-estimation of parameter q in *Appendix B* “Compustat data item #12, net sales. This is the amount of actual billings to customers for regular sales completed during the period, reduced by cash discounts, trade discounts, and returned sales for which credit is given to customers. Interest and equity income from unconsolidated subsidiaries, non-operating income, and income from discontinued operations are excluded.”
- EMPLY: number of employees. (Wage bill is unavailable.) “Compustat data item #29, number of employees. This is the number of company workers as reported to shareholders. It may be an average throughout the year or an end-of-year number; the latter is reported if both are given. It includes part-time employees and the employees of consolidated subsidiaries.”

♦ Variable definitions are quoted from *Hall (1990)*, pp. 13-22.