The Welfare Losses of Price Rigidities*

José Dorich†

November 2007

Abstract

Conventional wisdom in macroeconomics suggests that price stability is a sufficient condition to avoid important welfare losses arising from price stickiness. This idea is supported by sticky price models in which firms only react to aggregate shocks. However, it is well documented that firms also face idiosyncratic productivity shocks. In this paper, I investigate how the introduction of these shocks affects the welfare implications of price rigidities. I develop an analytical framework to measure the welfare losses when firms face idiosyncratic productivity shocks. Then, I compute these losses by using two alternative price settings and different calibration exercises that match the data on individual price changes. Several interesting results emerge. First, even when the aggregate price level is stable, an economy can incur quantitatively important welfare losses. In particular, these losses can add up to 4.4 percent of steady state consumption with time dependent pricing; while they can reach up to 2.3 percent of steady state consumption with state dependent pricing. Second, welfare losses are always significantly higher with time dependent pricing. Third, the variance of the idiosyncratic productivity shock and the frequency of price adjustments are the most important factors in determining the size of the welfare losses.

Keywords: Idiosyncratic Shocks, Price Rigidities, Welfare Analysis
JELclassification: D40, D60, E31

*I thank Jordi Galí for his valuable guidance, as well as Fernando Broner, Fabio Canova, Davide Debortoli, Francisco Grippa, Albert Marcet, Alberto Martin, Ricardo Nunes, Michael Reiter and Thijs Van Rens for helpful discussions. I also thank seminar participants in the Macroeconomics Workshop and Macroeconomics Break at Universitat Pompeu Fabra. Any remaining errors are mine.

†Universitat Pompeu Fabra. Contact author: joseantonio.dorich@upf.edu

1 Introduction

The existence of nominal price rigidity seems uncontroversial. The fact that individual goods prices adjust sluggishly has been well documented by different studies for the United States and the Euro Area[1]. This fact naturally raises the following question: What are the welfare consequences of this rigidity in the economy? Theoretically, in the face of exogenous shocks, price stickiness can cause welfare losses by creating relative price distortions that lead to an inefficient sectoral allocation of resources. The general belief in macroeconomics is that these losses would be negligible if monetary policy were to fully stabilize the aggregate price level. This idea is supported by models with price rigidities in which firms face only aggregate shocks[2]. The story behind all these models is that by attaining zero inflation, relative price distortions are eliminated and the economy reaches the flexible price allocation.

Empirical evidence suggests that firms are also hit by idiosyncratic productivity shocks[3]. In this paper, I consider these shocks in the analysis of the welfare losses of price rigidities. In a simple model, in which firms face both aggregate and idiosyncratic productivity shocks, I develop a general framework that allows for the measurement of the welfare losses of price rigidities. These losses are defined as the difference between the households' utility under sticky prices and the one under flexible prices. I then derive a second order approximation of the utility function and obtain the analytical expression for the welfare losses. I show that these losses depend on two different elements, independently of the way the price setting is modeled. The first is the aggregate output gap, which measures the deviation of total output from the natural output[4]. The second component is the dispersion of output gaps across goods. This component indicates how inefficient the sectoral allocation of goods is, given the aggregate output. Moreover, I show that a direct relationship exists between the dispersion of output gaps across goods and the dispersion of price gaps across goods[5]. The latter measures how distorted

[1] Among these studies, Bils and Klenow (2004) point out that the average duration of a price spell is 7 months for US; whereas Dhyne et al.(2006) find that this duration is 13 months for the Euro Area. Both studies used the monthly price records underlying the computation of the CPI.

[2] See Goodfriend and King (1997), King and Wolman (1999), Chari et al.(2000), Galí (2003), Woodford (2003) among others.

[3] See Blundell and Bond(2000), Cooper et al.(2004) among others.

[4] The natural output is defined as the equilibrium level of output that would prevail if prices were flexible.

[5] The price gap is defined as the diference between the actual price and the one that would be set if prices were flexible.

relative prices are. Therefore, I confirm the intuition that inefficient output composition is associated with relative price distortions.

Once I find the analytical expression for the welfare losses, I need to assume a price setting structure in order to compute these. Given the lack of consensus about how price stickiness should be modeled, I use two alternative price settings to evaluate the magnitude of the welfare losses. The first one is the time-dependent pricing and the second one is the state-dependent pricing. The main difference between these two approaches is that the timing of price changes is exogenous in the time-dependent framework, while it is endogenous in the state-dependent one. In the latter case, the timing depends basically on how far the price of a firm is from its optimal price.

The introduction of idiosyncratic shocks has important consequences regarding the welfare losses associated with price rigidities. Accounting for all the uncertainty that exists on the structure of the economy, I find that these losses are between 0.5 and 4.4 percent of steady state consumption when the time dependent pricing is considered, while they are between 0.1 and 2.3 percent of steady state consumption when the state-dependent pricing is used. In both cases, these losses arise even if price stability is followed. These results suggest that price rigidities are relevant from a welfare point of view; and consequently, that it is important to think more carefully about their determinants in order to investigate if there exist alternative policies that can help to reduce the welfare losses arising from price stickiness. Moreover, the results show that the size of the welfare losses is very sensitive to the price setting, to the variance of the idiosyncratic productivity shock and to the frequency of price adjustments. Regarding the sensitivity to the price setting, I show that price rigidities in the form of pricing policies that are state-dependent are always significantly less harmful than those based on time-dependent rules. The intuition of this result is related to the existence of the selection effect identified by Golosov and Lucas (2007) in the case of the state-dependent pricing. In the latter case, firms that are further away from their optimal price are more likely to change their price, diminishing the distortions that price rigidities can cause.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 derives an analytical expression for the welfare losses and shows some important analytical results. Section 4 introduces the standard Calvo price setting in order to compute the welfare losses when the pricing decisions are time-dependent. In this case, I show analytically that the dispersion of output gaps across goods depends, in the long run, on aggregate inflation and on the variance of the idiosyncratic productivity shock. The part of the dispersion that is due to idiosyncratic shocks is independent of aggregate macroeconomic variables, and consequently, independent of monetary policy. Therefore, it is concluded that there does not exist any monetary policy that can reach the flexible price allocation when some firms cannot adjust prices to their idiosyncratic shocks. The welfare losses are computed under different plausible calibration exercises and assuming that price stability is followed. Section 5 presents a modified version of the Generalized Ss model developed by Caballero and Engel (2007). This model is used in order to compute the welfare losses when the pricing decisions are state-dependent. Section 6 concludes.

2 The Model

Most of the structure of the model developed in this section is taken from the one developed in Galí (2007). The main difference is that firms are hit also by idiosyncratic productivity shocks.

2.1 Households

The representative household seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \tag{1}$$

where $0 < \beta < 1$ is the discount factor, C_t is an index of consumption goods and H_t is the number of hours worked in period t. The household purchases differentiated goods and combines them into a composite good using a Dixit-Stiglitz aggregator:

$$C_t = \left(\int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)} \tag{2}$$

where $C_t(i)$ is the differentiated good of type i and $\epsilon > 1$ is the constant elasticity of substitution among goods. The households maximize the index C_t, given the total cost of all differentiated goods and their nominal prices $\{P_t(i)\}$. Then, the demand for each good is given by:

$$C_t(i) \quad = \quad \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \tag{3}$$

where P_t is the aggregate price level and is defined as follows:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)} \tag{4}$$

The maximization of the expected utility is subject to an intertemporal budget constraint of the form:

$$\sum_{t=0}^{\infty} E_0 Q_{0,t} P_t C_t \leq B_0 + \sum_{t=0}^{\infty} E_0 Q_{0,t} \left[(1+\eta) W_t H_t - T_t \right] \tag{5}$$

where B_0 is the initial level of wealth, W_t is the nominal wage per hour worked, T_t represents a lump sum tax and η denotes a constant rate of employment subsidy that is funded by the lump sum tax. This subsidy is introduced in the model in order to offset the distortion associated with imperfect competition in goods markets. Moreover, $Q_{0,t}$ is a stochastic discount factor that satisfies $Q_{0,0} = 1$ and $E_0 Q_{0,t} = \prod_{s=0}^{t-1} (1+i_s)^{-1}$ where i_t denotes the interest rate at period t. The labor market is perfectly competitive and wages are flexible.

The household's optimization problem is then to choose processes C_t and H_t for all dates t satisfying (5), given its initial wealth B_0, the goods prices, the nominal wage and the stochastic discount factors that it expects to face, so as to maximize (1).

For the purpose of this paper, the intratemporal first order condition (associated with labor supply) is the only one to be presented. This condition is:

$$\frac{-U_h}{U_c} = (1+\eta) \frac{W_t}{P_t} \tag{6}$$

2.2 Firms

Each firm i has a production function of the form:

$$Y_t(i) = \widetilde{A_t} A_t(i) H_t(i)^{\alpha} \tag{7}$$

where $Y_t(i)$ is the level of output at period t of firm i, $\widetilde{A_t}$ is the aggregate level of productivity in period t, $A_t(i)$ is the firm i's idiosyncratic productivity level at period t and $H_t(i)$ is the total hours hired by firm i in period t. The idiosyncratic productivity level is assumed to follow an AR(1) process of the form:

$$\log A_t(i) = \rho \log A_{t-1}(i) + \varepsilon_t(i) \tag{8}$$

where $\varepsilon_t(i)$ follows an i.i.d process with zero mean and constant variance σ_ε^2. Firms face a rigidity in changing their price. Two ways of modeling this rigidity are explored in the paper: the Calvo pricing (1983) and a modified version of the Generalized Ss model developed by Caballero and Engel (2007). The details on the price settings proposed in these models are left for Sections 4 and 5 respectively.

2.3 Equilibrium

Market clearing in the goods market requires that $C_t(i) = Y_t(i)$ for all i and at all times. This implies that the index of aggregate consumption C_t must at all times equal the index of aggregate output $Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)}$.

Moreover, labor supply must equal labor demand, which means:

$$H_t = \int_0^1 H_t(i) di \tag{9}$$

By using the market clearing condition in the goods markets, the demand for goods and the production function of the firm, the market clearing condition in the labor market implies:

$$H_t = \left(\frac{Y_t}{\widetilde{A}_t}\right)^{1/\alpha} \int_0^1 \left(\frac{Y_t(i)/Y_t}{A_t(i)}\right)^{1/\alpha} di \tag{10}$$

Taking logs in (10), I get:

$$\alpha h_t = y_t - \widetilde{a}_t + d_t \tag{11}$$

where the lower case letters are used to denote the logs of original variables and $d_t = \alpha \log \int_0^1 \left(\frac{Y_t(i)/Y_t}{A_t(i)}\right)^{1/\alpha}$ is a measure of output dispersion across goods adjusted by the presence of idiosyncratic shocks. Throughout this paper, I will use the term adjusted output dispersion when I refer to d_t. This term captures how the composition of output between firms affects total output. Alternatively, d_t can be written as: $d_t = \alpha \log \int_0^1 \left(\frac{1}{A_t(i)}\right)^{1/\alpha} \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon/\alpha} di$.

3 The Welfare Losses of Price Rigidities

The welfare losses of price rigidities are given by the difference between the households' utility under sticky prices and the one under flexible prices. In this section I derive a second order approximation of the utility function around a zero inflation steady state. I then evaluate this approximation under sticky prices and flexible prices. Finally, I obtain the analytical expression for the welfare losses.

3.1 A Second Order Approximation to Utility

The second order Taylor expansion of U_t around a steady state (C,N) with zero inflation yields:

$$U_t - U \simeq U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) + U_h H \left(\widehat{h}_t + \frac{1+\chi}{2} \widehat{h}_t^2 \right) \tag{12}$$

where hat variables represent log deviations from steady state, $\sigma = -\frac{U_{cc}}{U_c} C$ and $\chi = \frac{U_{hh}}{U_h} H$. Moreover, I have made use of the market clearing condition $\widehat{y}_t = \widehat{c}_t$. Next, it is convenient to rewrite $\widehat{h}_t$ in terms of $\widehat{y}_t$ by using (11) and the fact that d_t is a term of second order around a zero inflation steady state. Then, we have:

$$U_t - U \simeq U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) + \frac{U_h H}{\alpha} \left(\widehat{y}_t - \widetilde{a}_t + d_t \right) + U_h H \frac{1+\chi}{2\alpha^2} \left(\widehat{y}_t - \widetilde{a}_t \right)^2 \tag{13}$$

Efficiency in the zero inflation steady state, which is guaranteed by the government subsidy to labor, implies that $\frac{-U_h}{U_c} = \alpha \frac{Y}{H}$. Therefore, period t utility function can be written as:

$$\frac{U_t - U}{U_c C} \simeq \frac{1-\sigma}{2} \widehat{y}_t^2 + \widetilde{a}_t - d_t - \frac{1+\chi}{2\alpha} \left(\widehat{y}_t - \widetilde{a}_t \right)^2 \tag{14}$$

The latter expression measures the deviation of period utility from its steady state. It is expressed as a fraction of steady state consumption.

3.2 An Analytical Expression for the Welfare Losses

The welfare losses of price stickiness, expressed as a fraction of steady state consumption, can be defined as follows:

$$L_t = \frac{U_t - U_t^F}{U_c C} \tag{15}$$

where U_t and U_t^F are the utilities under sticky prices and flexible prices respectively. Therefore, in order to compute the welfare losses, it is necessary to obtain utilities under both scenarios. The deviation of utility from the steady state under flexible prices can be expressed as:

$$\frac{U_t^F - U}{U_c C} \simeq \frac{1-\sigma}{2}\widehat{y^n}_t^2 + \widetilde{a}_t - d_t^n - \frac{1+\chi}{2\alpha}\left(\widehat{y}_t^n - \widetilde{a}_t\right)^2 \tag{16}$$

where $\widehat{y}_t^n$ and d_t^n denote the natural output and the adjusted output dispersion without price rigidities respectively. By using (14), I can define the deviation of utility from the steady state under sticky prices. Then, by taking into account that $\widehat{y}_t^n = \frac{1+\chi}{\alpha\sigma+1-\alpha+\chi}\widetilde{a}_t$ and substracting (16) from (14), I get the following expression for the welfare losses of price rigidities:

$$L_t = -\left[\frac{\alpha\sigma + 1 - \alpha + \chi}{2\alpha}\right]\left(\widehat{y}_t - \widehat{y}_t^n\right)^2 - \left(d_t - d_t^n\right) \tag{17}$$

It can be seen that these losses depend on two different components. The first one, known in the literature as the output gap, measures how close total output is from the natural output. The second element has two possible interpretations. One is that it captures how distorted relative prices are. The other one is that it reflects how inefficient the sectoral allocation of goods is.

In order to illustrate the two possible interpretations of the difference between d_t and d_t^n, it is helpful to define two concepts. The first one is the dispersion of price gaps across goods. It is defined as the variance across goods of the difference between actual prices and the ones that these goods would have if prices were flexible. In Appendix B, it is shown that the dispersion of the price gaps across goods is related to the second component in (17) in the following way:

$$d_t - d_t^n = \frac{\epsilon}{2\Theta} Var_i\left\{p_t(i) - p_t^f(i)\right\} \tag{18}$$

where $\Theta = \frac{\alpha}{\alpha+(1-\alpha)\epsilon}$, $p_t(i)$ is the logarithm of the actual price of good i and $p_t^f(i)$ is the logarithm of the price that a good i would have if price rigidities were permanently removed. The magnitude of the variance in (18) measures how distorted relative prices are[6]. From expression (18), it is clear that higher relative price distortions due to price stickiness imply more welfare losses. Moreover, by using (18), it is obvious that the second element in (17)

[6]Notice that $Var_i\left\{p_t(i) - p_t^f(i)\right\} = Var_i\left\{p_t(i) - p_t - (p_t^f(i) - p_t^f)\right\}$ where p_t and p_t^f are the price levels under sticky and flexible prices respectively.

is always non-negative. This implies that there always exist welfare losses in this model, unless the flexible price allocation is reached.

The second concept that is useful to develop is the dispersion of output gaps across goods. This is equal to the variance across goods of the difference between actual output of good i and the natural output of good i. The size of this variance measures how inefficient the sectoral composition of output is, given the aggregate output. By using the structure of the demand for good i, it is straightforward to see that the dispersion of price gaps across goods and the dispersion of output gaps across goods are related in the following way:

$$Var_i \left\{ p_t(i) - p_t^f(i) \right\} = \frac{1}{\epsilon^2} Var_i \left\{ y_t(i) - y_t^n(i) \right\} \tag{19}$$

where $y_t(i)$ is the logarithm of the actual output of good i and $y_t^n(i)$ is the logarithm of the natural output of good i. Expression (19) confirms the intuition that there is a direct relationship between relative price distortions and the inefficiency in sectoral allocation of real resources. Moreover, by using (18) and (19), it can be concluded that the second interpretation of the difference between d_t and d_t^n is also right. Given this interpretation, throughout the rest of the paper, I will refer to the gap $d_t - d_t^n$ as the dispersion of output gaps across goods.

To conclude this section, it is convenient to show the particular form of the welfare losses when there are no idiosyncratic shocks. In this case, the frictionless price is the same for every firm i. Therefore, the dispersion of price gaps across goods can be expressed only as a function of the cross sectional variance of actual prices. This means that expression (17) can be written as the standard welfare losses in the literature on optimal monetary policy with $d_t - d_t^n = -\frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) \right\}$.

4 The Welfare Losses with Calvo Pricing

In this section, first, I present the model with Calvo price setting. Then, I show how to use it in order to compute the welfare losses when this price setting takes place. Finally, I present estimates of the welfare losses under different plausible calibrations of the parameters of the model.

4.1 Calvo Price Setting

Firms set prices as in the sticky price model of Calvo (1983). In this model, during each period, a randomly chosen fraction of firms $(1 - \theta)$ is allowed to change the prices; whereas the other fraction θ do not change. Those firms resetting prices will choose an optimal price $P_t^*(i)$. Notice that in this case, given the idiosyncratic productivity shock, the optimal price for each firm would not be the same among those firms that change.

4.1.1 Optimal Price Setting

A firm reoptimizing in period t will choose a price $P_t^*(i)$ that maximizes the current market value of the profits generated while that price remains effective. This means solving the following problem:

$$\max_{P_t^*(i)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} (P_t^*(i) Y_{t+k}(i) - W_{t+k} H_{t+k}(i)) \right\} \tag{20}$$

subject to the sequence of demand constraints and production functions.

The first order condition associated with this problem, up to a first order approximation around the zero inflation steady state, is:

$$p_t^*(i) = \Theta \left\{ \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t x_{t+k} \right\} - \frac{(1-\beta\theta)}{\alpha} \Theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t a_{t+k}(i) \tag{21}$$

where the lower case letters are used to denote the logs of original variables, $\mu = \log \frac{\epsilon}{\epsilon - 1}$ and x_t is given by the following expression:

$$x_t = -\log \alpha + w_t - \frac{1}{\alpha} \widetilde{a}_t + \frac{1-\alpha}{\alpha} (\epsilon p_t + y_t) \tag{22}$$

Notice from (21) that the optimal price has two components: the first one is a macro component (common across firms) and the second one is a firm specific component. Then, it is convenient to express this condition as:

$$p_t^*(i) = p_t^C - \frac{(1-\beta\theta)}{\alpha} \Theta \sum_{k=0}^{\infty} (\beta\theta)^k E_t a_{t+k}(i) \tag{23}$$

where $p_t^C = \Theta \left\{ \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t x_{t+k} \right\}$. Finally, by using the fact that the idiosyncratic shock follows an AR(1) process, (23) can be written as:

$$p_t^*(i) = p_t^C - \frac{(1-\beta\theta)\Theta}{\alpha(1-\beta\theta\rho)} a_t(i) \tag{24}$$

4.1.2 Aggregate Price Level Dynamics

Using the definition of the aggregate price level, the log of the price level can be written as:

$$p_t = \int_0^1 p_t(i)di \tag{25}$$

Then, by using the Calvo pricing, this relation can be written as:

$$p_t = \theta \int_0^1 p_{t-1}(i)di + (1-\theta) \int_0^1 p_t^*(i)di \tag{26}$$

Finally, by combining (24) and (26), I get:

$$p_t^C - p_t = \frac{\theta}{1-\theta}\pi_t \tag{27}$$

where π_t is the inflation rate between periods $t-1$ and t.

4.1.3 The New Keynesian Phillips Curve with Idiosyncratic Shocks

The first step to derive the aggregate supply curve with idiosyncratic shocks consists in defining the economy´s real average marginal cost (mc_t) as the difference between the real wage and the economy´s average product of labor. Then, this definition implies:

$$mc_t = w_t - p_t - \frac{1}{\alpha}\widetilde{a}_t + \frac{1-\alpha}{\alpha}y_t - \log\alpha \tag{28}$$

By combining the previous definition with the one of x_t, I get:

$$x_t = mc_t + \frac{1}{\Theta}p_t \tag{29}$$

Plugging the latter relationship into the definition of p_t^C and rearranging some terms, I obtain:

$$p_t^C = (1-\beta\theta)\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t\widehat{mc}_{t+k} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t p_{t+k} \tag{30}$$

Substracting p_{t-1} from both sides, I get:

$$p_t^C - p_{t-1} = (1-\beta\theta)\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t\widehat{mc}_{t+k} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\pi_{t+k} \quad (31)$$

Notice that the previous expression can be rewritten more compactly as a difference equation in the following way:

$$p_t^C - p_{t-1} = \beta\theta(p_{t+1}^C - p_t) + (1-\beta\theta)\Theta\widehat{mc}_{t+k} + \pi_t \quad (32)$$

Finally, by using the fact that $p_t^C - p_{t-1} = \frac{\pi_t}{1-\theta}$, which is derived from equation (27), equation (32) yields the following inflation equation:

$$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta\widehat{mc}_{t+k} \quad (33)$$

It has been shown that the existence of idiosyncratic shocks does not affect the first order approximation of the standard relationship between inflation and real marginal costs. This is because the mean of the idiosyncratic productivity shocks is zero. Now, for the welfare analysis, it is convenient to obtain a relationship between inflation and the output gap. Galí (2007) shows that the following relationship between the economy´s real average marginal cost and the output gap holds in the model developed in Section 2[7]:

$$\widehat{mc}_{t+k} = \left[\sigma + \frac{\chi+1-\alpha}{\alpha}\right](\widehat{y}_t - \widehat{y}_t^n) \quad (34)$$

To conclude the derivation of the relationship between inflation and the output gap, I combine (33) and (34) to obtain:

$$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left[\sigma + \frac{\chi+1-\alpha}{\alpha}\right]\Theta(\widehat{y}_t - \widehat{y}_t^n) \quad (35)$$

4.2 Measuring Welfare Losses

In this case, it is convenient to write the welfare losses as in equation (17):

$$L_t = -\left[\frac{\alpha\sigma+1-\alpha+\chi}{2\alpha}\right](\widehat{y}_t - \widehat{y}_t^n)^2 - (d_t - d_t^n)$$

Now, it is necessary to find an expression for the dispersion of output gaps across goods that depends on aggregate inflation and on the variance of the

[7]Notice that the existence of idiosyncratic shocks does not affect Galí´s result on this relationship because the mean of firm-specific productivity shocks is zero. The latter means that these shocks do not have any impact on the average of aggregate variables.

idiosyncratic component of productivity. This expression will be useful in order to decompose the welfare losses of price rigidities in two parts: one that is dependent of monetary policy and another one that is not. By using the lemmas developed in Appendix C, it can be shown that the dispersion of output gaps across goods, as $t \rightarrow \infty$, is given by:

$$d_t - d_t^n = \frac{\epsilon}{2\Theta} \frac{\theta}{(1-\theta)} \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 + \left\{ \frac{\epsilon}{2\Theta} \phi^2 + \frac{1+(\epsilon-1)\Theta}{2\alpha} - \frac{\epsilon(1-\theta)}{\alpha(1-\theta\rho)} \phi \right\} \sigma_a^2 \tag{36}$$

where $\phi = \frac{(1-\beta\theta)\Theta}{(1-\beta\theta\rho)\alpha}$ and $\sigma_a^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2}$.

Some comments about the last expression are useful. First, in the long run, the dispersion of output gaps across goods depends on aggregate inflation and on the variance of the idiosyncratic productivity shock. Second, the first component in (36) measures the dispersion that is generated due to the fact that some firms cannot adjust prices to aggregate shocks; whereas the second component in (36) measures the dispersion that is created because the same firms cannot adjust prices to their idiosyncratic shocks. Under sticky prices ($0 < \theta < 1$), both components are always non negative. Third, when $\theta = 0$, it can be shown that the dispersion is zero, which implies $d_t = d_t^n$. Fourth, the part of the dispersion that is due to idiosyncratic shocks is independent of aggregate macroeconomic variables, and consequently, independent of monetary policy. Therefore, it can be concluded that no monetary policy exists that can reach the flexible price composition of output among goods when some firms cannot adjust their prices to their idiosyncratic shocks. Fifth, the dispersion is increasing in the elasticity of substitution among goods, in the degree of price rigidity and in the variance of the idiosyncratic productivity shock.

By using (36), it is clear that we can decompose the welfare losses of price rigidities in two parts: one that is dependent of monetary policy and another one that is not. The losses that depend on monetary policy are given by the following expression:

$$L_t^P = -\frac{\epsilon}{2\Theta} \frac{\theta}{(1-\theta)} \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 - \left[\frac{\alpha\sigma + 1 - \alpha + \chi}{2\alpha} \right] (\widehat{y}_t - \widehat{y}_t^n)^2 \tag{37}$$

whereas the ones that are independent are given by:

$$L_t^{IP} = -\left\{ \frac{\epsilon}{2\Theta} \phi^2 + \frac{1+(\epsilon-1)\Theta}{2\alpha} - \frac{\epsilon(1-\theta)}{\alpha(1-\theta\rho)} \phi \right\} \frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{38}$$

Then, the natural question is: how big are these welfare losses? Clearly, L_t^P will depend on the monetary policy that is followed. For simplicity, I assume a policy that fully stabilizes the price level. This implies that the output gap is also zero up to a first order approximation, according to the Phillips Curve presented in (35). Consequently, under zero inflation, L_t^P is zero up to a second order approximation[8]. Therefore, the only source of welfare losses is L_t^{IP}, which can be measured in the model without resorting to the monetary policy. The next subsection seeks to quantify that term.

4.3 Quantifying L_t^{IP}

In order to measure L_t^{IP}, it is necessary to calibrate the parameters of the model. The frequency chosen to perform this exercise is monthly. The baseline calibration is shown in Table 1. Before discussing this calibration, it is worth mentioning that four out of six of the structural parameters are calibrated by using information from the Dominick´s database and some relationships derived from the model[9]. These parameters are ϵ, α, θ and σ_ε^2. The main advantage of calibrating the majority of parameters by using the same database is that it provides consistency between the different choices of parameters.

$Table 1$

β	ϵ	α	θ	ρ	σ_ε^2
0.997	3.00	0.990	0.80	0.95	0.0036

It is assumed that $\beta = 0.997$, implying a steady state real return of financial assets of about four percent in annual terms. I set $\epsilon = 3$, based on the evidence provided by Chevalier, Kayshap and Rossi (2003). They estimate price elasticities using the quantity and price data from Dominick´s database. Most of their elasticity estimates range between 2 and 4. I set α so that it equals the average labor income share (0.66 in this calibration) times

[8] When firms face idiosyncratic productivity shocks, a zero inflation policy cannot attain the natural level of output. In fact, the second order approximation of the standard New Keynesian Phillips curve is different from the one derived when firms are hit by idiosyncratic shocks. In the latter, there is a constant term than depends on the variance of the idiosyncratic shocks. Therefore, zero inflation cannot lead to a zero output gap, up to a second or higher order approximation. However, the impact of non zero output gap on the welfare function is of third or higher order with price stability.

[9] The Dominick´s database contains nine years (from 1989 to 1997) of weekly store level data on the prices and quantities of more than 4500 products for 86 stores in the Chicago area. For more details on this database, see Midrigan (2006).

the markup implied by the choice of ϵ[10]. On price stickiness, it is assumed $\theta = 0.8$ such that the model matches the average price duration of five months estimated by Midrigan (2006) using the Dominick´s database[11]. This price duration is also close to those found in the studies performed by Bils and Klenow (2004) and Altig et al.(2004). The persistence of the idiosyncratic component of productivity is assumed to be very high by setting $\rho = 0.95$. This is the preferred point estimate of ρ in Blundell and Bond (2000)[12]. They estimate an AR(1) process for the firm´s idiosyncratic productivity by using a panel data covering 509 U. S. manufacturing companies observed for 8 years. Finally, the calibration of σ_ε^2 is performed such that I match the observed variance of individual price changes. This is done by using the following expression derived from the model presented above[13]:

$$\sigma_\varepsilon^2 = \frac{(1-\theta\rho)(1-\rho^2)}{2(1-\rho)(1-\theta)\phi^2}\left\{Var_i\left\{\pi(i)\right\} - \frac{2\theta}{1-\theta}\pi^2\right\} \tag{39}$$

where $Var_i\left\{\pi(i)\right\}$ is the variance of monthly individual price changes across goods and π is the monthly inflation. Now, it is straightforward how σ_ε^2 is computed. Given equation (39), the values set above for $\beta, \epsilon, \alpha, \theta$ and ρ, a constant monthly inflation of 0.03/12 and a variance of monthly individual price changes across goods equal to 0.002116 (consistent with the observed standard deviation of monthly individual price changes of 4.6 percent found in the Dominick´s database), it yields $\sigma_\varepsilon^2 = 0.0036$. The latter value is slightly lower than the one set by Golosov and Lucas (2007)[14]. Under this calibration, the welfare losses of price rigidities are equivalent to 1.7 percent of steady state consumption.

[10] Firms' profits maximization in the steady state implies that $1 = \frac{\epsilon}{\epsilon-1}mc_t$ where mc_t denotes the real marginal cost. Moreover, the assumption about technology implies that the real marginal cost is equal to the labor share (ls) divided by α.Therefore, $\alpha = \frac{\epsilon}{\epsilon-1}ls$.

[11] The average price duration is computed by considering regular prices only (no sales). See Midrigan (2006) for the details on this calculation.

[12] They provide an estimate of ρ equal to 0.565 in annual frequency. In order to translate this estimate into the monthly frequency, I use $\rho_a = \rho_m^{12}$. This approximation assumes that productivity is end of period sampled and interprets it as a stock variable. I use "a" to denote annual frequency and "m"to denote monthly frequency.

[13] See the Appendix E for the derivation of this expression.

[14] They choose a variance equal to 0.011 in their baseline calibration in quarterly frequency. Then, in order to translate my estimate into quarterly frequency and compare it with the one of Golosov and Lucas (2007), I apply the following relation: $\sigma_{\varepsilon q}^2 = (1+\rho_m^2+\rho_m^4)\sigma_{\varepsilon m}^2$. My monthly estimate is equivalent to a quarterly estimate of 0.010. I use "q" to denote quarterly frequency and "m"to denote monthly frequency.

4.3.1 Robustness Exercise

Four important sources of uncertainty can affect the baseline estimate. First, even assuming that the Dominick´s database is a representative sample of the economy, there exists uncertainty about the persistence of the idiosyncratic component of productivity and the elasticity of substitution[15]. Second, there is uncertainty about the determinants of the observed heterogeneity in the size of individual price changes. In the baseline calibration, it has been assumed that the variance of the idiosyncratic productivity shock can account for almost all the variance of individual price changes. However, it is possible that there exist ex-ante heterogeneity, like different frequencies of price adjustment, that can help to explain this variance. Third, the estimates of ϵ and θ, obtained by using the Dominick´s database, are significantly lower than others presented in alternative studies. Therefore, there is uncertainty about how well the economy is represented by the information contained in the Dominick´s database. Moreover, given the way I calibrate the variance of the firm specific productivity shock, this third source of uncertainty introduces a fourth one on σ_ε^2 and its relation with ϵ and θ. In this subsection, I analyze and discuss how the baseline estimate changes when we consider all these sources of uncertainty separately.

In order to show how much the first source of uncertainty may matter, Table 2 presents the welfare losses by allowing the parameters ρ and ϵ to vary between reasonable values[16]. In all these cases, α and σ_ε^2 are also changed appropriately such that the procedure followed to obtain the baseline estimate is the same, except in the choice of ρ and ϵ. From this table, it can be seen that the welfare losses are very sensitive to the elasticity of substitution. This sensitivity is not significantly affected by the values of ρ. The degree of autocorrelation of the firm´s productivity is less important in order to determine the welfare losses for low values of ϵ.

[15] Notice that the uncertainty in ρ and ϵ leads to uncertainty in σ_ε^2. Given that the latter is pinned down from all the other parameters and from the standard deviation of price changes, it is not considered that σ_ε^2 induce uncertainty by itself.

[16] The two standard error confidence interval for ρ, implied by Blundell and Bond´s estimation, is [0.93,0.97]. I also consider 0.99 in order to see what happens when the idiosyncratic productivity is very close to a unit root process. In the case of ϵ, the range [2,4] has been chosen based on the evidence provided by Chevalier et al (2003) using the Dominick´s database.

$Table2 : Welfare\ Losses (in\ \%)$

	ρ			
ϵ	0.93	0.95	0.97	0.99
2	0.62	0.56	0.50	0.45
3	1.85	1.68	1.51	1.35
4	3.70	3.36	3.02	2.71

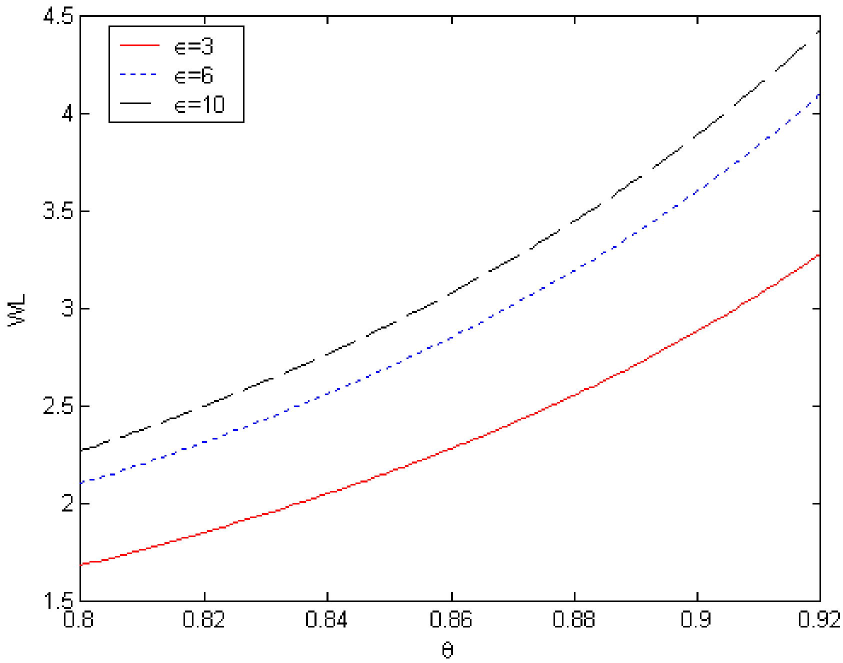
The second source of uncertainty is explored by analyzing how the baseline estimate changes when only the variance of the idiosyncratic productivity shock varies. Table 3 presents this sensitivity analysis. I consider five different values for σ_ε^2 in the table. The first column corresponds to the baseline estimate. The second row in the table indicates the fraction of the observed variance of individual price changes that is explained by the model. Clearly, in the baseline calibration, this fraction is 1, which means that basically all the observed heterogeneity is due to idiosyncratic shocks. However, it could be argued that there exists some ex-ante heterogeneity that can also account for the variance of the individual price changes. Midrigan (2006) performs an analysis by using the Dominick´s database and concludes that only 20 percent of the variance of price changes could be explained by ex-ante heterogeneity. This case corresponds to the calibration in the second column. Given that ex-ante heterogeneity is not incorporated in the model, I calibrate the variance of the idiosyncratic productivity shock such that only 80 percent of the variance of individual price changes is explained by the model. It can be seen that the estimate of the welfare losses diminishes to 1.34 percent of steady state consumption in this case. This result is not so different from the one obtained with the baseline calibration.

$Table3 : Sensitivity\ Analysis\ for\ \sigma_\varepsilon^2$

	σ_ε^2				
	0.0036	0.0029	0.0025	0.0022	0.0018
$\frac{Var_i^M\{\pi(i)\}}{Var_i^O\{\pi(i)\}}$	1.00	0.80	0.70	0.60	0.50
$Welfare\ Losses\ (in\ \%)$	1.68	1.34	1.17	1.00	0.83

The third source of uncertainty is related with the convenience of employing the Dominick´s database to calibrate some parameters. There exist some evidence that can cast doubt on the usefulness of this database. In particular, this evidence suggests that the degree of price rigidity and the elasticity of substitution among goods are much higher than the ones estimated with the Dominick´s database. Nakamura and Steinsson (2007) report that the average price duration is between 11.6 and 13 months; while Klenow and Kristow (2007) find that the average price duration is 8.6 months. Golosov

Figure 1: Sensitivity Analysis for θ and ϵ

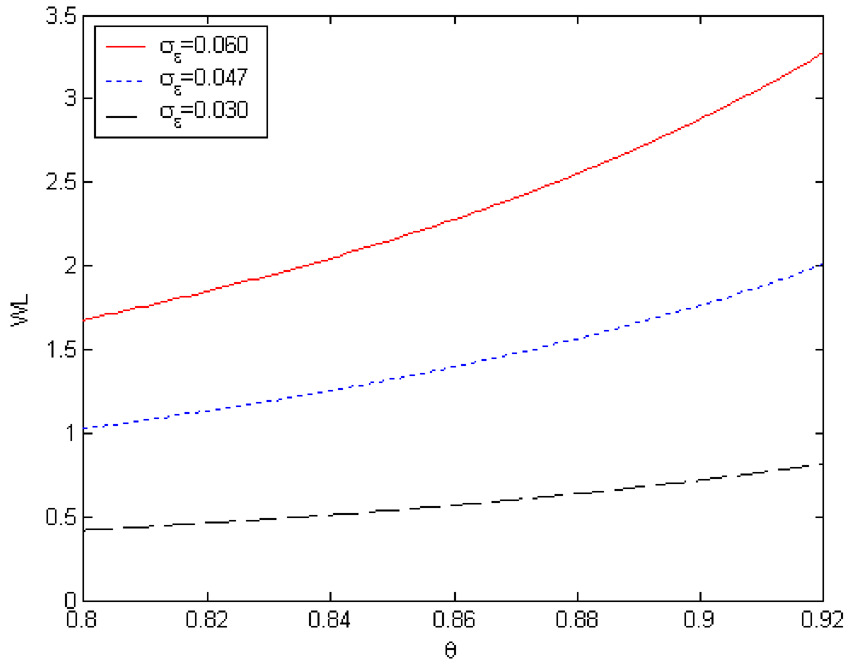


and Lucas (2007) mention that ϵ typically falls in the range between 6 and 10. This implies different values from for α as well[17]. Therefore, given the conflicting evidence for θ and ϵ, it is necessary to perform a sensitivity analysis to the baseline calibration by changing only these parameters and α accordingly. Notice that σ_{ε}^2 ,in this analysis, corresponds to the one used in the baseline estimate, given that the information on the variance of individual price changes is not available in the alternative studies. I evaluate later how welfare losses change when σ_{ε}^2 varies for different values of θ and ϵ.

Figure 1 shows the results of this exercise. On the vertical axis, the welfare losses are measured as percentage of steady state consumption. The parameter θ is allowed to vary between 0.8 and 0.92, which implies that average price duration is between 5 and 13 months. The lines in the graph describe how welfare losses change with the degree of price rigidity for three different levels of the elasticity of substitution among goods. Two interesting results arise from this picture. First, for any degree of price rigidity, the estimation of the welfare losses is very sensitive to variations in ϵ in the range between 3 and 6; while it is not severely affected when ϵ moves between 6

[17]Notice that the model establishes a relationship between ϵ and α.

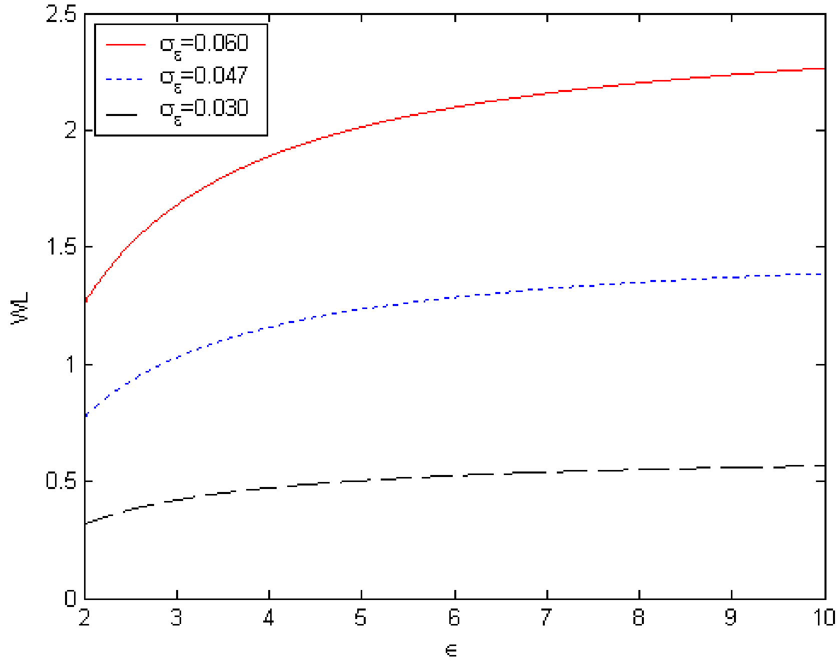
Figure 2: Sensitivity Analysis for θ and σ_ε



and 10. Second, the whole picture reveals that the uncertainty in θ and ϵ is translated in a huge uncertainty about the welfare losses, which vary from 1.7 percent $(\theta = 0.8, \epsilon = 3)$ to 4.4 percent of steady state consumption $(\theta = 0.92, \epsilon = 10)$.

To conclude the robustness exercises, I quantify how movements in σ_ε^2 affect the estimates of welfare losses for different degrees of θ and ϵ. Figures 2 and 3 present the results of these exercises. Figure 2 shows how welfare losses change with the degree of price rigidity for three different levels of σ_ε. It can be seen that the degree of price rigidity does not significantly affect the losses for low levels of volatility of the shock ($\sigma_\varepsilon = 0.03$ or less). The picture considers $\epsilon = 3$, but this result also holds if $\epsilon = 10$. Moreover, the uncertainty in θ and σ_ε also implies an enormous uncertainty about the welfare losses, which vary from 0.4 percent to 3.3 percent of steady state consumption. Figure 3 presents how the losses vary with the elasticity of substitution for the same levels of σ_ε. This picture shows that the uncertainty in the elasticity of substitution does not matter much for low levels of σ_ε. Besides, the impact of the uncertainty in ϵ is lower than the one of θ. Notice that the size of the range for the welfare losses is lower in figure 3 than in figure 2.

Figure 3: Sensitivity Analysis for ϵ and σ_ε



5 Welfare Losses with State-Dependent Pricing

In the previous section, the Calvo price setting was used in order to estimate the welfare losses resulting from price stickiness. One weakness of this approach is that it does not incorporate the fact that it is more likely that those firms that have their prices further away from their target prices have a higher probability of changing their prices[18]. In this section, I use a modified version of the Generalized Ss model proposed by Caballero and Engel (2007) in order to let the probability of changing prices be an increasing function of the difference between the actual price and the target price. The section is divided into three parts. First, I present the model. Then, I show how to use it in order to compute the welfare losses. Finally, I present the baseline calibration of the model and some robustness exercises.

[18]The cost of deviating from the target price is increasing with respect to the distance from this price. Therefore, adjustment is more likely when this distance is larger.

5.1 The Model

Consider a firm $i \in [0,1]$ at time t that sets its price at $P_t(i)$ but would choose its price at $P_t^*(i)$ if price rigidities were momentarily removed. Let the difference between these two prices (the actual and the target prices respectively) be defined, in logarithms, as follows:

$$x_t(i) = p_t(i) - p_t^*(i) \tag{40}$$

For simplicity, in this section I assume that there exists idiosyncratic productivity shocks only, which are independent across firms and across time. All these shocks have zero mean and variance σ_ε^2. Moreover, under the assumption that increments in productivity are approximately independent (over time for each i), I can approximate $p_t^*(i)$ by the following expression[19]:

$$p_t^*(i) \simeq \Omega + p_t^f(i) \tag{41}$$

where $p_t^f(i)$ is the log of the frictionless price (the price that a firm would choose if price rigidities were permanently removed) and Ω is an uninteresting constant. In general, the target price would be a weighted average of current and expected future frictionless prices. When productivity is very persistent (in the limit it is a unit root), it can be shown that the expectation of the future frictionless prices is approximately the current price[20]. Therefore, it holds that the target price is approximately given by the frictionless price[21].

From (41), it holds that:

$$\Delta p_t^*(i) \simeq \Delta p_t^f(i) \tag{42}$$

Notice from Section 2 that the frictionless price is given by:

$$p_t^f(i) = \Theta \left[-\log \alpha + w_t + \frac{1-\alpha}{\alpha}(\epsilon p_t + c_t) - \frac{1}{\alpha} a_t(i) \right] \tag{43}$$

Considering that there are no aggregate shocks and that inflation is equal to zero, it can be concluded that $w_t = \overline{w}$, $p_t = \overline{p}$ and $c_t = \overline{c}$. The latter implies that the target price follows the process:

$$\Delta p_t^*(i) \simeq \Delta p_t^f(i) = -\frac{\Theta}{\alpha} \Delta a_t(i) \tag{44}$$

[19] This assumption has been used in other applications by Caballero and Engel (1993a,1993b). It seems very plausible according to the empirical evidence provided by Blundell and Bond (2000). Notice that, in other words, this assumption means that the idiosyncratic productivity should be very persistent. In the limiting case, when $\rho = 1$, increments in productivity are independent.

[20] Notice that, in the limiting case, when the productivity is a unit root, the frictionless price is a unit root.

[21] See Caballero and Engel (1993b) for more details on this issue.

Given that ρ is very close to 1, $\Delta a_t(i) \simeq \varepsilon_t(i)$. Therefore:

$$\Delta p_t^*(i) \simeq -\frac{\Theta}{\alpha}\varepsilon_t(i) \tag{45}$$

The existence of idiosyncratic productivity shocks every period implies that the target price changes every period; and, consequently the price imbalance x also varies. To complete the model, I need to specify how firms would adjust their prices after being hit by the idiosyncratic shock. I assume that the probability that a firm i changes its price is equal to $\Lambda(x_t(i))$ where $\Lambda(x)$ represents the adjustment hazard. In this way, I capture the most distinguishing feature of state-dependent models: the fact that the disequilibrium variable $x_t(i)$ influences how likely it is that a firm adjusts its price in a given time period[22]. In principle, a hazard function could take any shape. Reasonable hazard functions should be increasing with respect to the absolute value of x, given that it seems unlikely that firms tolerate large deviations as much as they tolerate the small ones[23]. This feature is known in the literature as the increasing hazard property (Caballero and Engel 1993a).

The timing convention of the model is as follows. At the beginning of period t, firm i has a price imbalance of $x_{t-1}(i)$. Then, an idiosyncratic productivity shock hits the firm. This implies that x moves from $x_{t-1}(i)$ to $x_{t-1}(i) + \Delta p_t^*(i)$. Finally, the adjustment hazard is applied on the price deviation after the idiosyncratic shock. With probability $\Lambda(x_{t-1}(i) + \Delta p_t^*(i))$ the firm changes its price and eliminates the price imbalance[24] ($x_t(i) = 0$) and with probability $1 - \Lambda(x_{t-1}(i) + \Delta p_t^*(i))$ the firm does not change its price and keeps its price deviation in $x_{t-1}(i) + \Delta p_t^*(i)$. Therefore, for each firm i, the following process for $x_t(i)$ holds:

$$x_t(i) = I_t(i)\left[x_{t-1}(i) - \frac{\Theta}{\alpha}\varepsilon_t(i)\right] \tag{46}$$

where:

$$\begin{aligned} I_t(i) &= 1 \text{ with Probability } 1 - \Lambda(x_{t-1}(i) + \Delta p_t^*(i)) \\ &= 0 \text{ with Probability } \Lambda(x_{t-1}(i) + \Delta p_t^*(i)) \end{aligned}$$

[22]The adjustment hazard framework has been used by Caballero and Engel (1993a, 1993b, 2006, 2007). In their 2006 paper, they claim that almost any Ss model can be approximated by using the adjustment hazard framework.

[23]In fact, menu costs models are consistent with increasing hazard functions.

[24]When the price imbalance is positive (negative), eliminating it implies that the firm has decreased (increased) its price.

5.2 Measuring Welfare Losses

In this case, it is convenient to combine (17) with (18) in order to write the welfare losses as:

$$L_t = -\frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) - p_t^f(i) \right\} - \left[\frac{\alpha\sigma + 1 - \alpha + \chi}{2\alpha} \right] (\widehat{y}_t - \widehat{y}_t^n)^2 \tag{47}$$

Again, these losses have two parts: one that depends on policy and one that does not. Equation (45) is consistent with zero inflation, which is assumed. Moreover, I assume that the standard New Keynesian Phillips curve is still a good approximation to relate output gap and inflation[25]. Under this assumption, a zero inflation policy leads to a zero output gap, up to a first order approximation. Consequently, the welfare losses are given by:

$$L_t = -\frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) - p_t^f(i) \right\} \tag{48}$$

This implies that the only source of welfare losses is the dispersion of price gaps across goods. Given that the model is defined in terms of the price deviation from the desired price (or target price), it is convenient to rewrite the welfare losses as a function of the cross sectional variance of x_t. By using (41) in (48), the welfare losses can be expressed as:

$$L_t = -\frac{\epsilon}{2\Theta} Var_i \left\{ x_t(i) \right\} \tag{49}$$

5.3 Quantifying $Var_i \left\{ x_t(i) \right\}$

The cross sectional variance is estimated by finding the variance of the ergodic distribution of the state variable x for a given firm i. In order to simulate the process x I need to assume a functional form for $\Lambda(x)$. Following Caballero and Engel (2006), I assume the simplest quadratic hazard they present in their paper, which is given by the following expression:

$$\begin{aligned} \Lambda(x) &= \delta_p x^2 \text{ , } x \leq 0 \\ &= \delta_n x^2 \text{ , } x \geq 0 \end{aligned} \tag{50}$$

The parameters ϵ and α are the same as those in the baseline calibration in Section 4. The remaining parameters δ_p, δ_n and σ_ε^2 are calibrated in two

[25] Gertler and Leahy (2006) show that the standard New Keynesian Phillips curve is consistent with state-dependent pricing. The main difference with respect to the time dependent pricing is the sensitivity of the output gap to movements in inflation. In the latter case, the sensitivity is much higher.

slightly different ways. In the first one, I impose $\delta_p = \delta_n$ and calibrate the parameters such that I match the fraction of price adjustments and the standard deviation of individual price changes observed in the Dominick's database[26]. In the second one, I remove the restriction $\delta_p = \delta_n$, such that I can match additionally the fraction of positive price changes[27].

Table 3 : Calibration of the Hazard Models

	Data	Models		
		1	2	3
STATISTICS (In %)				
Fraction of Price Adjustments	20	20	20	20
Standard Deviation of Price Changes	4.6	4.6	4.6	4.6
Fraction of Positive Price Changes	13	10	13	10
|Mean of Price Adjustments|	7.7	9.8	9.0	7.6
Mean of Price Increase		9.8	6.8	7.6
Mean of Price Decrease		9.8	13.4	7.6
PARAMETERS				
δ_p		50	205	-
δ_n		50	15	-
σ_ε		0.047	0.047	0.047
WELFARE LOSSES (In %)		**0.28**	**0.37**	**1.33**

Table 3 summarizes the results of the simulations. Model 1 reports the symmetric quadratic hazard model. The model fails to match the mean of the absolute value of individual price changes (it overestimates it). This is consistent with the failure of menu costs models to generate many small price changes.[28] By using this model, the welfare losses are 0.3 percent of steady state consumption. Model 2 reports the asymmetric quadratic hazard model. Notice that δ_p is much higher than δ_n in order to capture the fact that price increases occur more frequently than price reductions. Moreover, the asymmetric hazard allows for matching the fraction of price increases, which is higher than the one of price decreases. Like Model 1, it predicts an absolute value of price changes that is much higher than the one observed in the data. With this model, the welfare losses are 0.4 percent of steady state consumption. Model 3 reports the constant-hazard model (Calvo 1983). This model has been calibrated so that $\Lambda(x) = 1 - \theta = 0.2$. In contrast to the

[26] Notice that the fraction of price adjustments (f) is approximately related to the average price duration (d) by the following expression: $f \approx d^{-1}$

[27] Of course, there exists other dimensions of the data that could be matched. It would be interesting to see how they affect our understanding of the welfare losses of price rigidities.

[28] As an example, see the menu cost model developed in Golosov and Lucas (2007).

previous two models, it matches fairly well the mean of the absolute value of price adjustments. However, it does not capture (by construction) the higher probability of a price increase. By using this model, the welfare losses are much higher (1.3 percent of steady state consumption). Finally, notice that the welfare losses estimated by using model 3 are a very good approximation to the ones estimated by using the complete structure of the Calvo model under the assumption that the idiosyncratic productivity is highly persistent. In fact, when using the adjustment hazard approach, the estimated losses are 1.33 percent; whereas when using the model of section 4 with $\rho = 0.99$, these losses are 1.35 percent.

5.3.1 Robustness Exercise

In the previous calibration exercises, there exist two important sources of uncertainty. Conditional on the representativity of the Dominick´s database, the first source is the estimation of the elasticity of substitution. In fact, estimates of this parameter based on the use of the Dominick´s database are in the range 2-4. This implies, after following the same type of procedure performed in table 3, that the welfare losses are between 0.1 and 0.6 percent of the steady state consumption if model 1, with $\delta_p = \delta_n$, is used. When model 2 is considered to perform this robustness analysis, the range for the welfare losses is 0.1-0.7 percent of the steady state consumption.

The second source of uncertainty is related with the convenience of using the Dominick´s database. As mentioned before, other studies present estimates of the elasticity of substitution among goods and the average price duration that are much higher than those obtained by using this database. For this reason, I also perform some additional calibration exercises of the welfare losses that consider: a) lower frequency of price adjustments (average price duration equal to 13 months instead of 5 months) b) three different estimates for the elasticity of substitution among goods and c) two different values for σ_ε. All these exercises are performed by calibrating model 2 (with $\delta_p \neq \delta_n$) such that the fraction of price adjustments and the fraction of positive price changes are the same as in the data on individual price changes due to Nakamura and Steinsson (2007). Results are presented in Table 4.

Several interesting results emerge from this robustness exercise. First, the impact of the degree of price rigidity on the welfare losses is crucially affected by the size of the standard deviation of the idiosyncratic productivity shock. When $\sigma_\varepsilon = 0.03$, these losses are between 0.4 and 0.6 percent of the steady state consumption; while they are between 1.8 and 2.3 percent when the standard deviation of the idiosyncratic productivity shock is doubled. In both cases, the degree of price rigidity is the same. Second, based on the ability of the different calibrations of the model to match the data on the size of individual price changes, it is difficult to take a position on the amount of the welfare losses. In particular, calibrations 1 and 5 do a great job in matching the absolute value of the median of price adjustments and the median of price increases but yield completely different welfare losses (0.4 versus 2.3 percent). Independent evidence on the variance of the idiosyncratic productivity shocks is necessary in order to obtain a more precise estimate of the welfare losses. Third, given σ_ε, the impact of varying ϵ on welfare is not very important. This result holds because in order to match the fraction of price changes and the fraction of positive adjustments, an increase in ϵ implies a reduction in the variance of the price imbalance x_t. Fourth, this exercise shows clearly that a lower frequency of price adjustments would not necessarily imply significantly more welfare losses. If we compare the result obtained by using calibration 1 with the baseline estimate of 0.37 percent, we see that the difference between the two is small. This is because it is plausible that economies with lower frequency of price adjustments are economies with smaller idiosyncratic productivity shocks. Fifth, the model does not fit the disaggregated data on prices when $\epsilon = 10$. A higher variance of the idiosyncratic productivity shock will solve this problem. In general, when choosing any value for the elasticity of demand higher than 6, the model would require a higher σ_ε in order to match adequately the data on individual price changes.

Table 4 : Robustness Exercise

	Data	Alternative Calibrations for Model 2					
		1	2	3	4	5	6
STATISTICS (In %)							
Frac. of Price Adj.	8	8	8	8	8	8	8
St. Dev. of Price Ch.	n.a	2.9	1.4	0.9	5.9	3.0	1.7
Frac. of Pos. Adj.	5	5	5	5	5	5	5
|Median of Price Adj.|	8.5	8.7	4.2	2.4	17.0	8.6	5.1
Median of Price Inc	7.3	7.2	3.6	2.1	14.4	7.3	4.3
Median of Price Decr	10.5	11.7	5.8	3.7	23.6	11.9	7.1
PARAMETERS							
δ_p		64	250	785	15	60	167
δ_n		9	36	90	2	9	25
ϵ		3	6	10	3	6	10
σ_ε		0.03	0.03	0.03	0.06	0.06	0.06
WL (In %)		**0.43**	**0.55**	**0.60**	**1.79**	**2.27**	**2.33**

6 Concluding Remarks

I have presented a new perspective on the importance of the study of price rigidities. Traditionally, these rigidities have been analyzed in order to understand the real effects of monetary policy, inflation persistence or the design of optimal monetary policy. In this sense, price stickiness has been an important element in monetary policy analysis. Here I provide an additional motivation to pay attention to price rigidities. In particular, I emphasize that price stickiness is relevant because it can cause important welfare losses, even in economies with price stability. This conclusion has been obtained after considering idiosyncratic productivity shocks in the welfare analysis of price rigidities.

The results of this paper also allow for the identification of two aspects of price rigidities that are relevant from a welfare point of view. First, they highlight how crucial it is to understand why firms would decide in favor of state dependent behavior or time dependent behavior[29]. In fact, this study has shown that the welfare losses are significantly higher with time dependent pricing. Secondly, they emphasize the importance of investigating the determinants of the frequency of price adjustments. According to my results, this variable is a key factor in determining the size of welfare losses[30]. Research on these two aspects would also be helpful in order to see if there exist policies that can help to reduce the negative impact of price rigidities.

[29] Alvarez (2007) develops an econometric analysis in this line of research. He estimates a multinomial logit model with Spanish Survey data in order to explain the relationship between the use of time dependent pricing strategies and industry characteristics. He finds that time dependent behavior is associated with higher labor intensity in the production, lower degree of competition and large firms.

[30] The other factor is the variance of the idiosyncratic productivity shocks. Clearly, this factor is exogenous.

References

[1] Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Linde (2004): “Firm-Specific Capital, Nominal Rigidities and the Business Cycle”, mimeo, Northwestern University.

[2] Alvarez, Luis J. (2007): "What do Micro Price Data Tell Us on the Validity of the New Keynesian Phillips Curve?, Bank of Spain Working Paper N^o 0729.

[3] Bils, Mark and Peter J. Klenow (2004): "Some Evidence on the Importance of Sticky Prices", Journal of Political Economy 112: 947-985.

[4] Blundell, Richard and Stephen Bond (2000): "GMM Estimation with Persistent Panel Data: an Application to Production Functions", Econometric Reviews 19 (3) :321-340

[5] Caballero, R.J. and Engel, E.M.R.A. (1993a): “Microeconomic Adjustment Hazards”, Quarterly Journal of Economics, 108 (2), 359–383.

[6] Caballero, R.J. and Engel, E.M.R.A. (1993b): “Microeconomic Rigidities and Aggregate Price Dynamics”, European Economic Review, 37 (4), 697–711.

[7] Caballero, R.J. and Engel, E.M.R.A. (2006): “Price Stickiness in Ss models: Basic Properties”, European Economic Review, 37 (4), 697–711.

[8] Caballero, R.J. and Engel, E.M.R.A. (2007): "Price Stickiness in Ss models: New Interpretations for Old Results", Journal of Monetary Economics 54: 100-121.

[9] Calvo, Guillermo (1983): "Staggered Prices in a Utility Maximizing Framework", Journal of Monetary Economics 12: 383-398.

[10] Chari, V. V., Lawrence J. Christiano, and Ellen R. McGrattan (2000): "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", Econometrica 68: 1151-1179.

[11] Chevalier, Judith, Anil Kashyap and Peter Rossi (2003): “Why Don’t Prices Rise During Periods of Peak Demand? Evidence from Scanner Data”, American Economic Review 93(1): 15-37.

[12] Cooper, Russell W., John C. Haltiwanger and Jonathan Willis (2004): “Dynamics of Labor Demand: Evidence from Plant Level Observations and Aggregate Implications”, NBER Working Paper N^o 10297.

[13] Dhyne, E., Alvarez, L.J., Bhan, H.L., Veronese, G., Dias, D., Hoffmann, J., Jonker, N., Lunnemann, P., Rumler, F. and Vilmunen, J. (2006): "Price setting in the euro area and the United States: some facts from individual consumer price data", Journal of Economic Perspectives 20: 171–192.

[14] Gali, Jordi (2003): "New Perspectives on Monetary Policy, Inflation and the Business Cycle", in Mathias Dewatripoint; Lars Peter Hansen and Stephen Turnowsky, eds, Advances in Economics and Econometrics: Theory and Applications, Vol. 3. Cambridge: Cambridge University Press, pp 151-197.

[15] Gali, Jordi (2007): Monetary Policy, Inflation and the Business Cycle, unpublished monograph.

[16] Gertler, Mark and John Leahy (2006): “A Phillips Curve with an S-s Foundation," mimeo New York University.

[17] Golosov, Mikhail, and Robert E. Lucas Jr. (2007): "Menu Costs and Phillips Curves", Journal of Political Economy 115 (2): 171-199.

[18] Goodfriend Marvin and Robert King (1997): "The New Neoclassical Synthesis and the Role of Monetary Policy", in Ben Bernanke and J Rotemberg, NBER Macroeconomics Annual 1997, Vol. 12. Cambridge MA: MIT Press, pp 231-283.

[19] King, Robert G. and Alexander L. Wolman (1999): "What Should the Monetary Authority Do when Prices are Sticky?" in J.B. Taylor, ed., Monetary Policy Rules, Chicago: University of Chicago Press.

[20] Klenow, Peter J. and Oleksiy Krystov (2007): "State Dependent or Time Dependent Pricing: Does it matter for Recent U.S Inflation", mimeo Standford University.

[21] Midrigan, Virgiliu (2006): "Menu costs, Multi-Product Firms, and Aggregate Fluctuations" mimeo New York University.

[22] Nakamura, Emi and Jón Steinsson (2007): "Five Facts About Prices: A Reevaluation of the Menu Cost Models" manuscript, Harvard University.

[23] Woodford, Michael (2003): Interest and Prices, Princeton University Press: Princeton, New Jersey.

A The Adjusted Output Dispersion ($\mathbf{d}_t$)

The adjusted output dispersion, up to a second order approximation, can be expressed as:

$$d_t = \frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) \right\} + \frac{1}{2\alpha} Var_i \left\{ a_t(i) \right\} + \frac{\epsilon}{\alpha} Cov_i \left\{ p_t(i), a_t(i) \right\} \tag{51}$$

Proof: First, notice that the adjusted output dispersion (in logs) can be written as:

$$d_t = \alpha \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon/\alpha} \left(\frac{1}{A_t(i)} \right)^{1/\alpha} di \tag{52}$$

Then, a second order approximation of d_t around a zero inflation steady state is given by[31]:

$$d_t = \alpha \left[-\frac{\epsilon}{\alpha} \int_0^1 (p_t(i) - p_t) di + \frac{1}{2} \left(\frac{\epsilon}{\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di + \frac{1}{2\alpha^2} \int_0^1 a_t^2(i) di \right] \tag{53}$$

Now, by taking into account that $\int_0^1 (p_t(i) - p_t) di = -(1-\epsilon) \int_0^1 (p_t(i) - p_t)^2 di$ (from a second order approximation around a zero inflation of the identity of the price level), the following expression arises:

$$d_t = \frac{\epsilon}{2\Theta} \int_0^1 (p_t(i) - p_t)^2 di + \frac{1}{2\alpha} \int_0^1 a_t^2(i) di + \frac{\epsilon}{\alpha} \int_0^1 (p_t(i) - p_t)\, a_t(i) di \tag{54}$$

Finally, by noticing that p_t is the mean of $p_t(i)$ and that $a_t(i)$ has mean zero, then (54) can be written as (51).

[31] It has been taken into account that $a_t(i)$ has a zero mean, which means that $\int_0^1 a_t(i) di = 0$.

B The Relationship between $d_t - d_t^n$ and the Dispersion of Price Gaps Across Goods

The difference between d_t and d_t^n can be expressed as follows:

$$d_t - d_t^n = \frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) - p_t^f(i) \right\} \tag{55}$$

Proof: First, up to a second order approximation around a zero inflation, it holds that:

$$Var_i \left\{ p_t(i) - p_t^f(i) \right\} = Var_i \left\{ p_t(i) \right\} + Var_i \left\{ p_t^f(i) \right\} - 2Cov_i \left\{ p_t(i), p_t^f(i) \right\} \tag{56}$$

Now, notice that the frictionless price $p_t^f(i)$ is given by the following expression:

$$p_t^f(i) = \Theta \left[-\log \alpha + w_t - \frac{1}{\alpha}\widetilde{a}_t + \frac{1-\alpha}{\alpha}(\epsilon p_t + c_t) - \frac{1}{\alpha} a_t(i) \right] \tag{57}$$

Then, by using (57), $Cov_i \left\{ p_t(i), p_t^f(i) \right\} = -\frac{\Theta}{\alpha} Cov_i \left\{ p_t(i), a_t(i) \right\}$. This means that if both sides of (56) are multiplied by $\frac{\epsilon}{2\Theta}$ and terms are rearranged, the following expression holds:

$$\frac{\epsilon}{2\Theta} Var_i \left\{ p_t(i) \right\} + \frac{\epsilon}{\alpha} Cov_i \left\{ p_t(i), a_t(i) \right\} = \frac{\epsilon}{2\Theta} \left[Var_i \left\{ p_t(i) - p_t^f(i) \right\} - Var_i \left\{ p_t^f(i) \right\} \right] \tag{58}$$

The latter expression is useful to find an alternative expression of d_t that relates it with $Var_i \left\{ p_t(i) - p_t^f(i) \right\}$. In fact, by combining (51) and (58), I have:

$$d_t = \frac{\epsilon}{2\Theta} \left[Var_i \left\{ p_t(i) - p_t^f(i) \right\} - Var_i \left\{ p_t^f(i) \right\} \right] + \frac{1}{2\alpha} Var_i \left\{ a_t(i) \right\} \tag{59}$$

Using the latter, the adjusted output dispersion under flexible prices is given by:

$$d_t^n = -\frac{\epsilon}{2\Theta} Var_i \left\{ p_t^f(i) \right\} + \frac{1}{2\alpha} Var_i \left\{ a_t(i) \right\} \tag{60}$$

Finally, by substracting (60) from (59), (55) is obtained.

C Proofs of Lemmas

Lemma 1: The adjusted output dispersion d_t, up to a second order approximation, is given by the following expression:

$$d_t = \frac{\epsilon}{2\Theta} Var_i \{p_t(i)\} + \frac{1}{2\alpha} Var_i \{a_t(i)\} + \frac{\epsilon}{\alpha} Cov_i \{p_t(i), a_t(i)\} \tag{61}$$

Proof: See appendix A

Lemma 2: The adjusted output dispersion under flexible prices (d_t^n), up to a second order approximation, can be written as:

$$d_t^n = \frac{1-\epsilon}{2\alpha} \Theta Var_i \{a_t(i)\} \tag{62}$$

Proof: By using lemma 1, d_t^n can be expressed as:

$$d_t^n = \frac{\epsilon}{2\Theta} Var_i \left\{p_t^f(i)\right\} + \frac{1}{2\alpha} Var_i \{a_t(i)\} + \frac{\epsilon}{\alpha} Cov_i \left\{p_t^f(i), a_t(i)\right\} \tag{63}$$

Then, by considering (57) and the fact that the idiosyncratic shocks and the aggregate variables are uncorrelated, it is straightforward to find $Var_i \left\{p_t^f(i)\right\}$ and $Cov_i \left\{p_t^f(i), a_t(i)\right\}$ as a function of $Var_i \{a_t(i)\}$. More precisely:

$$Var_i \left\{p_t^f(i)\right\} = \left(\frac{\Theta}{\alpha}\right)^2 Var_i \{a_t(i)\} \tag{64}$$

$$Cov_i \left\{p_t^f(i), a_t(i)\right\} = \frac{-1}{\alpha + (1-\alpha)\epsilon} Var_i \{a_t(i)\} \tag{65}$$

Finally, by plugging (64) and (65) into (63) and adding all the resulting terms, lemma 2 is found.

Lemma 3: The variance of prices across goods, up to a second order approximation, is given by the following expression:

$$Var_i \{p_t(i)\} = \frac{\theta}{(1-\theta)} \sum_{j=0}^{\infty} \theta^j \pi_{t-j}^2 + (1-\theta)\phi^2 \sum_{j=0}^{\infty} \theta^j Var_i \{a_{t-j}(i)\} \tag{66}$$

where $\phi = \frac{(1-\beta\theta)\Theta}{(1-\beta\theta\rho)\alpha}$.

Proof: The proof can be divided into four steps.

Step 1: Define $E_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon/\alpha} di$. Then, up to a second order approximation around zero inflation, this variable (in logs) can be expressed as:

$$e_t = \frac{\epsilon}{2\Theta\alpha} Var_i \{p_t(i)\} \tag{67}$$

Step 2: Take into account the Calvo price setting. In this case, e_t can be written as:

$$e_t = \frac{\epsilon}{2\Theta\alpha} \left[\theta \int_0^1 (p_{t-1}(i) - p_t)^2 di + (1-\theta) \int_0^1 (p_t^*(i) - p_t)^2 di \right] \tag{68}$$

Now, by plugging $p_t = p_{t-1} + \pi_t$ and $p_t^*(i) = p_t^C - \phi a_t(i)$ into (68); and considering that $p_t^C - p_t = \frac{\theta}{1-\theta}\pi_t$ holds, then after some algebra I get:

$$e_t = \frac{\epsilon}{2\Theta\alpha} \left[\theta Var_i \{p_{t-1}(i)\} + \frac{\theta}{1-\theta}\pi_t^2 + (1-\theta)\phi^2 Var_i \{a_t(i)\} \right] \tag{69}$$

Step 3: Combine (67) and (69) in order to find:

$$Var_i \{p_t(i)\} = \theta Var_i \{p_{t-1}(i)\} + \frac{\theta}{1-\theta}\pi_t^2 + (1-\theta)\phi^2 Var_i \{a_t(i)\} \tag{70}$$

Step 4: Considering that $0 \leq \theta < 1$, (70) can be solved backward in order to obtain (66).

Lemma 4: The covariance between prices and the idiosyncratic productivity shocks can be expressed as:

$$Cov_i \{p_t(i), a_t(i)\} = -\phi(1-\theta) \sum_{j=0}^{\infty} \theta^j Cov_i \{a_t(i), a_{t-j}(i)\} \tag{71}$$

Proof: First, use the following definition:

$$Cov_i \{p_t(i), a_t(i)\} = \int_0^1 (p_t(i) - p_t)\, a_t(i) di \tag{72}$$

Second, considering the Calvo price setting, the previous expression can be written as:

$$Cov_i \{p_t(i), a_t(i)\} = \theta \int_0^1 (p_{t-1}(i) - p_t)\, a_t(i) di + (1-\theta) \int_0^1 (p_t^*(i) - p_t)\, a_t(i) di \tag{73}$$

Third, by plugging $p_t = p_{t-1} + \pi_t$ and $p_t^*(i) = p_t^C - \phi a_t(i)$ into the latter equation; and considering that the idiosyncratic and the aggregate variables are uncorrelated, I get after some algebra:

$$Cov_i \{p_t(i), a_t(i)\} = \theta Cov_i \{p_{t-1}(i), a_t(i)\} - (1-\theta)\,\phi Var_i \{a_t(i)\} \tag{74}$$

If the previous steps are repeated to find an expression for $Cov_i \{p_{t-1}(i), a_t(i)\}$, I get:

$$Cov_i \{p_t(i), a_t(i)\} = \theta^2 Cov_i \{p_{t-2}(i), a_t(i)\} - \phi(1-\theta) \sum_{j=0}^{1} \theta^j Cov_i \{a_t(i), a_{t-j}(i)\} \tag{75}$$

If this process is repeated infinitely many times, I get (71).

Lemma 5: As $t \to \infty$, $Var_i \{a_t(i)\} = \frac{\sigma_\varepsilon^2}{1-\rho^2} = \sigma_a^2$

Proof: Without loss of generality, assume that the initial level of productivity (in logs) for every firm is zero. Then, at time 0, it holds that $a_0(i) = \varepsilon_0(i)$. Then, $Var_i \{a_0(i)\} = \sigma_\varepsilon^2$. Now, at $t = 1$, $a_1(i) = \rho a_0(i) + \varepsilon_1(i)$, which implies that $Var_i \{a_1(i)\} = \sigma_\varepsilon^2 (1+\rho^2)$. In general, at $t = n$, $Var_i \{a_n(i)\} = \sigma_\varepsilon^2 (1 + \rho^2 + .. + \rho^{2n})$. Therefore, as $t \to \infty$, I get lemma 5.

Lemma 6: $Cov_i \{a_t(i), a_{t-j}(i)\} = \rho^j \sigma_a^2$

Proof: Notice that $Cov_i \{a_2(i), a_1(i)\} = \rho Var_i \{a_1(i)\}$; $Cov_i \{a_3(i), a_1(i)\} = \rho^2 Var_i \{a_1(i)\}$.

In general, $Cov_i \{a_t(i), a_{t-j}(i)\} = \rho^j \; Var_i \{a_{t-j}(i)\}$.

Finally, applying lemma 5 to the previous expression, I get lemma 6.

Lemma 7: $Cov_i \{p_t(i), p_{t-1}(i)\} = \theta Var_i \{p_{t-1}(i)\} - \phi(1-\theta) Cov_i \{a_t(i), p_{t-1}(i)\}$

Proof: Up to a second order approximation, the following identity holds:

$$Cov_i \{p_t(i), p_{t-1}(i)\} = \int_0^1 (p_t(i) - p_t)(p_{t-1}(i) - p_{t-1}) \tag{76}$$

Considering the Calvo price setting, the latter expression can be simplified until the lemma is finally proved in the following way:

$$\begin{aligned}
Cov_i\left\{p_t(i), p_{t-1}(i)\right\} &= \theta\int_0^1 \left(p_{t-1}(i) - p_t\right)\left(p_{t-1}(i) - p_{t-1}\right) di \\
&\quad + (1-\theta)\int_0^1 \left(p_t^*(i) - p_t\right)\left(p_{t-1}(i) - p_{t-1}\right) di \\
&= \theta\int_0^1 \left(p_{t-1}(i) - p_{t-1} + \pi_t\right)\left(p_{t-1}(i) - p_{t-1}\right) di \\
&\quad + (1-\theta)\int_0^1 \left(p_t^C - \phi a_t(i) - p_t\right)\left(p_{t-1}(i) - p_{t-1}\right) di \\
&= \theta Var_i\left\{p_{t-1}(i)\right\} - \phi(1-\theta)Cov_i\left\{a_t(i), p_{t-1}(i)\right\}
\end{aligned}$$

D Decomposing the Variance of Price Changes

The variance of price changes across goods, up to a second order approximation around zero inflation, is given by the following identity:

$$Var_i\left\{\pi_t(i)\right\} = Var_i\left\{p_t(i)\right\} - 2Cov_i\left\{p_t(i), p_{t-1}(i)\right\} + Var_i\left\{p_{t-1}(i)\right\} \quad (77)$$

By using lemma 7, the previous relationship can be expressed as:

$$Var_i\left\{\pi_t(i)\right\} = Var_i\left\{p_t(i)\right\} + \kappa Var_i\left\{p_{t-1}(i)\right\} + \lambda Cov_i\left\{a_t(i), p_{t-1}(i)\right\} \quad (78)$$

where $\kappa = (1-2\theta)$ and $\lambda = 2\phi(1-\theta)$.

From (78), $Var_i\left\{p_{t-1}(i)\right\}$can be expressed as:

$$Var_i\left\{p_{t-1}(i)\right\} = \frac{Var_i\left\{\pi_t(i)\right\} - Var_i\left\{p_t(i)\right\} - \lambda Cov_i\left\{a_t(i), p_{t-1}(i)\right\}}{\kappa} \quad (79)$$

Then, by plugging (79) into (70) and rearranging terms, I obtain:

$$\begin{aligned} Var_i\left\{p_t(i)\right\} &= \left(\frac{\theta}{1-\theta}\right)\left[Var_i\left\{\pi_t(i)\right\} + \frac{\kappa}{1-\theta}\pi_t^2\right] \\ &\quad -2\phi\theta Cov_i\left\{a_t(i), p_{t-1}(i)\right\} + \kappa\phi^2 Var_i a_t(i) \qquad (80) \end{aligned}$$

Combining the previous expression with (66), I get:

$$Var_i\left\{\pi_t(i)\right\} = \sum_{j=0}^{\infty}\theta^j \pi_{t-j}^2 - \left(\frac{1-2\theta}{1-\theta}\right)\pi_t^2 + idio \quad (81)$$

where *idio* is given by:

$$\begin{aligned} idio &= \frac{(1-\theta)^2}{\theta}\phi^2\sum_{j=0}^{\infty}\theta^j Var_i a_{t-j}(i) + \lambda Cov_i\left\{a_t(i), p_{t-1}(i)\right\} \\ &\quad -\frac{(1-\theta)\kappa\phi^2}{\theta}Var_i a_t(i) \qquad (82) \end{aligned}$$

Some comments about expression (81) are useful. First, notice that the variance of price changes across goods can be decomposed in two parts: one that is driven by aggregate shocks (summarized by current aggregate inflation and its lags) and another one that is driven by the idiosyncratic productivity shock (*idio*). Second, considering an annual inflation of 3 percent (0.03/12 in monthly frequency) an average price duration of 5 months ($\theta = 0.8$), and the values set in Section 4.3 for β, ϵ, α,and ρ, the model predicts that the variance of monthly individual price changes across goods is 0.7 percent. However, in the data, this variance is 4.6 percent[32]. The introduction of idiosyncratic productivity shocks helps the model to fit much better this dimension of the data.

[32] Midrigan (2006) reports that the standard deviation of price changes, conditional on price adjustment is 10.4 percent. Then, assuming that 80 percent of prices do not change (consistent with the average duration he found), it can be inferred that the standard deviation of all price changes (including zeros) is 4.6 percent.

E Calibration of the Variance of the Idiosyncratic Productivity Shock

In this appendix, I show the way how to derive (39) in order to calibrate the variance of the idiosyncratic productivity shock. First, it is convenient to find an expression for *idio* that depends only on the current and past cross sectional variances of the idiosyncratic productivity shock. By looking at (82), it is clear that I should obtain an alternative expression for $Cov_i\left\{a_t(i), p_{t-1}(i)\right\}$. The way to find it consists in plugging (71) into (74) to obtain, after rearranging some terms, the following expression:

$$Cov_i\left\{a_t(i), p_{t-1}(i)\right\} = -\frac{(1-\theta)\phi}{\theta}\sum_{j=1}^{\infty}\theta^j Cov_i\left\{a_t(i), a_{t-j}(i)\right\} \tag{83}$$

By replacing the previous expression into (82), and considering that a large enough period of time has passed until now (as $t \rightarrow \infty$), I can apply lemmas 5 and 6 in order to write *idio* as:

$$idio = \frac{2(1-\theta)(1-\rho)\phi^2}{(1-\theta\rho)}\frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{84}$$

As $t \rightarrow \infty$, I can also assume that $\pi_t = \pi$. Therefore, as $t \rightarrow \infty$, the cross sectional variance of price changes is given by:

$$Var_i\left\{\pi(i)\right\} = \frac{2\theta}{1-\theta}\pi^2 + \frac{2(1-\rho)(1-\theta)\phi^2}{(1-\theta\rho)}\frac{\sigma_\varepsilon^2}{1-\rho^2} \tag{85}$$

Finally, rearranging terms, I obtain the following expression for the variance of the idiosyncratic productivity shock:

$$\sigma_\varepsilon^2 = \frac{(1-\theta\rho)(1-\rho^2)}{2(1-\rho)(1-\theta)\phi^2}\left\{Var_i\left\{\pi(i)\right\} - \frac{2\theta}{1-\theta}\pi^2\right\} \tag{86}$$