Optimal Monetary Policy with Credit Augmented Liquidity Cycles

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Abstract

The optimal response of monetary policy to financial instability is a long standing question whose policy relevance is now emphasized by the increase in available liquidity and in firms' financial exposure. Bernanke, Gertler and Gilchrist (1998) build a model in which credit frictions occur on the demand for capital investment and induce *demand driven fluctuations* which exacerbate shock transmission. In this context the policy maker does not face a trade-off as output stabilization is achieved through inflation targeting. I build a sticky price DSGE model in which the demand for working capital is affected both by a cost channel and an external finance premium. In this context the policy instrument affects the cost of collateralizable loans which in turn affects firms' marginal cost and inflation dynamics (*supply side driven fluctuations)*. The optimal monetary policy design is based upon both constrained and global Ramsey policies. Results show that: a) the optimal inflation level lies between zero and the one prescribed by the Friedman rule, b) the optimal dynamic path features deviations from price stability, c) the optimal rule features asset price targeting.

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1 Introduction

The optimal response of monetary policy to financial instability, mostly in presence of lending distortions, is a long standing question whose policy relevance has increased nowadays. The recent global financial turmoil has occurred, despite the ample availability of liquidity, while several central banks had declared and defended a mandate of strict price stability or inflation targeting, with little or not attention to financial market indicators. The current functioning of the financial market is very complex due to the securitization phenomenon and disentangling its interactions with the monetary transmission mechanism is beyond the scope of this paper. The focus of this paper will be on showing that even in presence of a simple channel in which firms' liquidity for working capital is affected by moral hazard problems, monetary policy might find optimal to target financial indicators, thereby deviating from strict inflation stabilization. The reason for that arises from a trade-off faced by the monetary policy maker whose actions on the Federal Funds Rate Target, by affecting firms' lending rate, pass-through into firms' marginal cost and inflation.

It is well-known in the literature that credit frictions have a significant impact on the transmission of shocks. A prominent role has been played by the *financial accelerator* mechanism of Bernanke, Gertler and Gilchrist 1998 - BGG 1998 hereafter - which assumes the presence of credit frictions, in the form of an external finance premium, on the demand for capital investment. Those frictions generate *demand driven fluctuations which* do not induce policy trade-offs as the policy maker can achieve output stabilization by pursuing inflation targeting[1] as it would do in the standard new Keynesian model. The presence of an external finance premium which depends on firms' collateralizable wealth tends to exacerbate business cycle fluctuations without altering the main policy trade-offs.

Different is the case in which credit frictions apply to working capital. In this case the policy instrument, the Federal Fund Rate, by affecting firms' lending rate, has an impact on firms' marginal costs and inflation. In this context and in response to any type of shocks, the monetary authority is unable to achieve the flexible price allocation as any movements in the nominal interest rate would act as endogenous cost push shocks and produce second-round fluctuations in marginal cost and inflation. Those fluctuations are exacerbated by the presence of a credit channel.

To study those issues I augment a DSGE model with a standard liquidity channel (see cost channel a' la Barth and Ramey 2001, Christiano, Eichembaum and Evans 2001, Ravenna and Walsh 2006) by introducing a credit channel on firms' working capital. Specifically I assume that firms

[1] See also Bernanke and Gertler 1998, Faia and Monacelli 2007, Faia 2007.

must borrow one period in advance to finance their working capital and that those findings are subject to moral hazard and asymmetric information problems which are rationalized through a costly state verification contract between the firm and the bank. In this context the firm must pay an external finance premium which directly affects its marginal costs. This model is characterized by *supply side driven fluctuations* which are exacerbated by the dependence of the external finance premium on firms' leverage.

In this economy I study the optimal design of monetary policy by solving both *constrained* and *global Ramsey policies*. The first case (constrained Ramsey policy) allows us to solve the optimal targeting problem as the allocation is constrained by a well-specified set of operational policy rules. The second case (global solution to the Ramsey policy) allows us to characterize the optimal path of all variables.

Three results stand out. First, the optimal long-run level of inflation lies between zero and the one associated with the Friedman rule. Second, an optimal policy rule features asset price targeting alongside with inflation targeting. Overall welfare tends to increase when the monetary authority targets either asset prices or the leverage ratio. Third, the optimal dynamic path features deviations from price stability and more muted response of nominal interest rates. Overall the activist role of monetary policy is revived.

Importantly in our context the optimal policy design follows the classical Ramsey approach (Ramsey 1927, Atkinson and Stiglitz 1980, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1992) in which a social planner maximizes household's welfare subject to a resource constraint and to the constraints describing the equilibrium in the private sector economy. Such approach allows to study the design of optimal policy in presence of wedges that affect both the long run steady state of the economy and the dynamic path of variables[2]. In the context of the present model the cost channel induces an efficiency wedge both in the labour market equilibrium condition and in the optimal allocation of capital, while the external finance premium tends to amplify fluctuations driven by the abovementioned efficiency distortions.

The rest of the paper is divided as follows. Section 2 presents the model economy. Section 3 presents the dynamic properties of the model under standard Taylor rules. Section 4 shows the implementation of the optimal monetary policy design and the results concerning the optimal targeting rule. Section 5 solves for the long run policy. Section 6 presents the dynamic properties of the optimal policy and section 7 concludes.

[2] See Adao, Correia and Teles 2003, Schmitt-Grohe and Uribe 2003, Khan, King and Wolman 2003, Faia 2007a and Faia and Monacelli 2007a for further applications of this approach to new keynesian models.

2 The Model Economy

The laboratory economy is populated by representative agents who consume, supply labour and capital to firms and invest in money and deposits. Monopolistically competitive firms produce different varieties merging capital and labour through a Cobb-Douglas technology. Firms' face both adjustment costs a' la Rotemberg 1982 on their pricing decisions and costs for working capital as factors of production must be paid before the proceeds from the sale of output are received. In addition the external funds raised to pay for the factors of production are provided by a risk neutral intermediary that is unable to observe the ex-post realization of firms' revenues. Due to both an asymmetric information and a moral hazard problem, raising external funds requires the payment of an external finance premium whose behavior is characterized via a costly state verification debt contract a' la Gale and Hellwig 1983.

2.1 Households

In this economy there is continuum of households, each indexed by $i \in (0, 1)$. They consume the final good, c_t,invest in safe bank deposits, d_t, real money balances, m_t, and capital, k_t, supply labor, n_t, and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative worker chooses the set of processes $\{c_t,\ n_t\}_{t=0}^{\infty}$ and $\{d_t, k_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{p_t, w_t,\ (1 + r_t^n), q_t\}_{t=0}^{\infty}$and the initial condition d_0, k_0 to maximize[3]:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right\} \tag{1}$$

subject to the sequence of budget constraints:

$$p_t c_t + p_t d_{t+1} + m_{t+1} \leq (1 + r_t^n) p_t d_t + m_t + w_t n_t + \Theta_t + t_t + q_t k_t - p_t i_t \tag{2}$$

w_t is the nominal wage while p_t is the CPI price index, r_t^d is the nominal interest rate paid on deposits, Θ_t are the real profits that households receive from running production in the monopolistic sector, t_t are lump sum taxes/transfers from the fiscal authority and i_t is investment in capital that evolves according to the following law of motion:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{3}$$

[3] In this context I follow the convention of considering real money balances as numeraire.

and to the cash in advance constraint:

$$p_t(c_t + i_t) = m_t + w_t n_t - d_t \tag{4}$$

which states that consumers can finance consumption using resources left over from the previous period. Let's define λ_t as the lagrange multiplier on constraint (2) and η_t as the lagrange multiplier on constraint (4). The first order conditions of the above problem read as follows:

$$u_{c,t} = \lambda_t + \eta_t \tag{5}$$

$$u_{c,t} = \beta(1 + r_t^d)E_t\left\{u_{c,t+1}\frac{p_t}{p_{t+1}}\right\} \tag{6}$$

$$u_{c,t}\frac{w_t}{p_t} = -u_{n,t} \tag{7}$$

$$u_{c,t} = \beta E_t\left\{u_{c,t+1}[(1-\delta)p_{t+1} + q_{t+1}]\right\} \tag{8}$$

Equation (5) defines the marginal utility on consumption. Equation (6) gives the optimal intertemporal allocation of consumption. Equation (7) gives the optimal allocation of labour supply and equation (8) gives the optimal allocation of capital. To achieve optimality this set of first order conditions must be satisfied along with a No- Ponzi condition on total asset allocation and with the cash in advance constraint, (4). Let's define the real interest rate as:

$$(1 + r_t) = (1 + r_t^d)E_t\left\{\frac{p_t}{p_{t+1}}\right\} \tag{9}$$

2.2 Final Good Sector

The aggregate final good y is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by i, via the aggregate production function:

$$y_t \equiv \left(\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{10}$$

where ε is the demand elasticity for variety i. Maximization of profits yields typical demand functions:

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon} y_t \tag{11}$$

for all i, where $p_t \equiv \left(\int_0^1 p_t(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

2.3 Firms

Firms choose input demands knowing that they are liquidity constrained and choose optimal prices by facing a cost of adjustment a' la Rotemberg 1982.

2.3.1 Optimal input demands

Each firm borrows external funds to finance input demand up to an amount:

$$l_t \leq \frac{w_t}{p_t} n_t + q_t k_t \tag{12}$$

External funds are available from a risk neutral intermediary at a real gross rate $(1 + r_t^l)$. Each firm assembles labor and capital to operate a constant return to scale production function for the variety i of the intermediate good:

$$y_t(i) = a_t \omega_t n_t(i)^{1-\alpha} k_t(i)^{\alpha} \tag{13}$$

where a_t is a productivity shifter common to all entrepreneurs and ω_t is an idiosyncratic productivity shifter that follows a uniform distribution, $f(\omega_t)$over the interval $[a, b]$[4]. For notational convenience I am assuming that the idiosyncratic shock does not have a firm index: it will be shown later that the index can be dropped due to the linear nature of the monitoring technology characterizing the optimal contract. The firm can observe the idiosyncratic shock ex-ante, while the intermediary can do so only ex-post and by paying a monitoring cost. As all firms in equilibrium will charge the same price and produce the same output we can now skip the index i. However only the firm which are able to repay the external funds will be operating next period: the mass of firms which remain operative is given by the probability that the idiosyncratic shock is above the threshold value, with the latter being defined by the following condition:

$$\mathcal{G}(l_t^j) \varpi_t^j \equiv (1 + r_t^l) l_t^j \tag{14}$$

where $\mathcal{G}(l_t^j)$ are firms' marginal revenues and will be defined later on, while $(1 + r_t^l)$ is the gross lending rate paid by non-defaulting firms.

Cost minimization is an important building block of the transmission mechanism in our model as it provide the link between the lending rate and the firms' marginal cost. Firm choose labour and capital inputs by minimizing the following cost function:

$$Min \frac{w_t}{p_t} n_t + q_t k_t + (1 + r_t^l)(\frac{w_t}{p_t} n_t + q_t k_t) \tag{15}$$

[4]This assumption is needed to guarantee uniqueness of the solution to the debt contract.

subject to equation (13) and (12). Let's define mc_t, the lagrange multiplier on (13), as the real marginal cost. First order conditions for this problem give:

- n_t :

$$\frac{w_t}{p_t}(1 + (1 + r_t^l)) = mc_t a_t \omega_t (1 - \alpha) n_t^{-\alpha} k_t^{\alpha} \tag{16}$$

- k_t :

$$q_t(1 + (1 + r_t^l)) = mc_t a_t \omega_t \alpha n_t^{1-\alpha} k_t^{\alpha-1} \tag{17}$$

By substituting (16) into (17) we obtain the following expression for the marginal cost:

$$\frac{w_t}{p_t}\frac{(1 + (1 + r_t^l))}{a_t \omega_t (1 - \alpha)}(\frac{n_t}{k_t})^{\alpha} = mc_t \tag{18}$$

The above expression manifest the mechanism through which movements in the nominal interest rate by affecting the cost of loans have an impact on firms' marginal costs. A few considerations are worth at this point. First, notice that higher lending rates tend to raise firms' marginal costs. Second, the higher is the default threshold (the lower the default probability), the lower are the aggregate marginal costs. An increase in the mass of operative firms tend to reduce unitary costs per output produced. Third, the higher the volatility of the firm idiosyncratic productivity the higher is the volatility of the marginal costs, which implies that the latter tracks the riskiness of the business sector.

An important role in the lending relation between the bank and the firm is played by firms' marginal revenues. It will prove convenient to express firm's marginal revenues as function of firms' borrowing. To this purpose we use the factor input ratio together with the liquidity constraint, (12), to obtain the following optimal input demands:

$$k_t = \alpha \frac{l_t}{q_t} \tag{19}$$

$$n_t = (1 - \alpha) \frac{l_t}{\frac{w_t}{p_t}} \tag{20}$$

Substituting optimal input demands into the production functions we obtain optimal firms revenues as function of loan demand:

$$\mathcal{G}(l_t) = p_t y_t = p_t a_t \omega_t \left(\frac{(1 - \alpha)}{\frac{w_t}{p_t}} \right)^{(1-\alpha)} \left(\frac{\alpha}{q_t} \right)^{\alpha} l_t \tag{21}$$

2.3.2 Pricing decisions

In our model each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to $\frac{\theta_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2$, where the parameter θ_p measures the degree of nominal price rigidity. The problem of each domestic monopolistic firm is the one of choosing the sequence $\{p_t(i)\}_{t=0}^{\infty}$, given optimal input demands, in order to maximize expected discounted real profits:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u_{c,t}\Theta_t(i)\right\} \tag{22}$$

where:

$$\Theta_t(i) \equiv y_t(i) - \left(\frac{w_t}{p_t}n_t(i) + q_t k_t(i)(1+(1+r_t^l))\right) - \frac{\theta_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2 \tag{23}$$

subject to demand constraint, (11) and the production constraint (13). Let's denote by mc_t, the lagrange multiplier on (13), as the real marginal cost, by $\widetilde{p}_t \equiv \frac{P_t(i)}{P_t}$ the relative price of variety i and $\pi_t = \frac{p_t}{p_{t-1}}$ as the inflation rate. The first order condition of the above problem reads as follows:

$$\begin{aligned} 0 \;=\; & y_t\widetilde{p}_t^{\,-\varepsilon}\left((1-\varepsilon)+\varepsilon mc_t\right) - \theta_p\left(\pi_t\frac{\widetilde{p}_t}{\widetilde{p}_{t-1}}-1\right)\frac{\pi_t}{\widetilde{p}_{t-1}} \\ & +\beta\theta_p\left(\pi_{t+1}\frac{\widetilde{p}_{t+1}}{\widetilde{p}_t}-1\right)\pi_{t+1}\frac{\widetilde{p}_{t+1}}{\widetilde{p}_t^{\,2}} \end{aligned} \tag{24}$$

where I have suppressed the superscript i since all firms employ an identical capital/labor ratio in equilibrium. After imposing symmetry the above condition can be written as follows:

$$0 = y_t\left((1-\varepsilon)+\varepsilon mc_t\right) - \theta_p\left(\pi_t-1\right)\pi_t + \beta\theta_p\left(\pi_{t+1}-1\right)\pi_{t+1} \tag{25}$$

Equation (25) gives a standard Phillips curve relating current inflation to expected one and the real marginal cost. By substituting the marginal cost equation, 18, into the Phillips curve relation, 25, it becomes evident the link between the lending rate and the inflation process. Any monetary policy action, which affects the lending rate through the Federal Fund Rate, will also affect inflation. This clearly manifest the monetary policy trade-offs as for instance an increase in the Federal Fund Rate, undertaken with the goal of reducing the inflation gap, will increase inflation in a second round because of its impact on firms' marginal costs. The higher is the volatility of the idiosyncratic shocks, which proxies the riskiness of the business sector, the steeper is the policy trade-off.

2.4 The Financial Contract between Firms and Intermediaries

The financial contract between firms and intermediaries assumes the form of an optimal debt contract à la Gale and Hellwig 1983. When the idiosyncratic shock to production is above the cut-off value which determines the default states the entrepreneurs repay an amount $(1+r_t^l)l_t^j$[5]. On the contrary, in the default states, the bank monitors the production activity (and pays a monitoring cost μ) and repossesses the firms' revenues. Default occurs when the revenues from production $\omega_t^j \mathcal{G}(l_t^j)$ falls short of the amount that needs to be repaid $(1+r_t^l)l_t^j$[6]. Hence the *default space* is implicitly defined as the range for ω such that :

$$\omega_t^j < \varpi_t^j \equiv \frac{(1+r_t^l)l_t^j}{\mathcal{G}(l_t^j)} \tag{26}$$

where ϖ_t^j is a cutoff value for the idiosyncratic productivity shock. Let's define by $\Gamma(\varpi^j) \equiv \int_0^{\varpi_t^j} \omega_t^j f(\omega)d\omega + \varpi_t^j \int_{\varpi_t}^{\infty} f(\omega)d\omega$ and $1-\Gamma(\varpi_t^j)$ the fractions of net capital output received by the intermediary and the firm respectively. Expected bankruptcy costs are defined as $\mu M(\varpi_t^j) \equiv \mu \int_0^{\varpi_t^j} \omega_t^j f(\omega)d\omega$ with the *net share* accruing to the intermediary being $\Gamma(\varpi_t^j) - \mu M(\varpi_{t+1}^j)$. The real return paid on deposits is given by the safe rate, $(1+r_t^n)$, which as such corresponds, for the intermediary, to the opportunity cost of financing capital. The intermediary must hold a fraction of deposits in the form of reserves required by the central bank for insurance purposes. Let's define ξ as the fraction of required reserves. Therefore in equilibrium:

$$l_t = d_t(1-\xi)$$

The *participation constraint* for the intermediary states that the expected return from the lending activity should not fall short of the opportunity cost of finance:

$$\mathcal{G}(l_t^j)(\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)) \geq (1+r_t^n)(1-\xi)l_t \tag{27}$$

The contract specifies a pair $\left\{\varpi_t^j, l_t^j\right\}$ which solves the following maximization problem:

$$Max\ (1-\Gamma(\varpi_t^j))\mathcal{G}(l_t^j) \tag{28}$$

subject to the participation constraint (27). Since unitary monitoring costs are constant this will give raise to linear relations that allow aggregation, hence we can skip the index j from now

[5] In every period t this amount must be independent from the idiosyncratic shock in order to satisfy incentive compatibility conditions.

[6] Notice that this contract starts and ends in a single period as it finances working capital which is used for production in the same period. This differ from BGG 1998 that have a intra-period contract as loans are required to finance capital which pays returns in future periods.

on. Using the first order conditions with respect $\{\varpi_t, l_t\}$ and aggregating yields a wedge between marginal revenues and the safe return paid on deposits:

$$\rho(\varpi_t) = \left[\frac{(1-\Gamma(\varpi_t))(\Gamma'(\varpi_t) - \mu M'(\varpi_t))}{\Gamma'(\varpi_t)} + (\Gamma(\varpi_t) - \mu M(\varpi_t)) \right]^{-1}$$

which is positively related to the default threshold. Let's define $(1 + r_t^p) = \mathcal{G}'(l_t)$, the marginal revenues from the production activity, as the return from working capital investment. We can now define the *external finance premium* as:

$$efp_t \equiv \frac{(1 + r_t^p)}{(1 + r_t^n)(1 - \xi)} = \rho(\varpi_t) \tag{29}$$

By combining the above expression with (27) and with the expression for $\rho(\varpi_t)$ it is possible to write a relation between the external finance premium, efp_t, and firms leverage, $\frac{l_t}{\mathcal{G}(l_t^j)}$[7]:

$$efp_t = \rho(\frac{l_t}{\mathcal{G}(l_t)}) \tag{30}$$

A decrease in the leverage ratio reduces the optimal cut-off value, as shown by equation (26). A fall in the cut-off value, by reducing the size of the default space, implies a fall in the size of the bankruptcy costs and of the external finance premium.

2.5 The Credit Augmented Cost Channel

Before turning to the analysis of optimal policy it is useful to disentangle the effects that characterize the monetary transmission mechanism in this model. First, monetary policy has a supply-side effect arising from the pure cost channel. To see this let's for the time being shut-off the external finance premium and let's work with the relations describing the flexible price equilibrium. Particularly the latter assumption implies the marginal cost is equal to the inverse of the mark-up, $mc = \frac{\varepsilon - 1}{\varepsilon}$. In this case the labour demand schedule is given by:

$$\frac{w_t}{p_t} = \frac{\frac{\varepsilon-1}{\varepsilon} a_t (1-\alpha) n_t^{-\alpha} k_t^{\alpha}}{(1 + (1 + r_t^n))} \tag{31}$$

This schedule can be depicted as a downward-sloping demand schedule in the real wage-employment space. Other things being equal a decrease in the nominal interest rate shifts the labour demand to the right. The same line of reasoning holds for the demand of capital:

$$q_t = \frac{\frac{\varepsilon-1}{\varepsilon} a_t \alpha n_t^{1-\alpha} k_t^{\alpha-1}}{(1 + (1 + r_t^n))} \tag{32}$$

[7]Notice that I abstract from the presence of internal funding. This implies that the external finance premium represents a wedge between investment in working capital or in alternative risk free activities.

The equilibrium response of employment to changes in the nominal interest rate can be recovered by looking at the equilibrium condition in the labour market which is obtained by merging (18) with (7):

$$mc_t = -\frac{u_{n,t}}{u_{c,t}} \frac{(1 + (1 + r_t^n))}{a_t(1 - \alpha)} (\frac{n_t}{k_t})^\alpha \tag{33}$$

The last equation also provides an expression for the marginal cost in this economy. As marginal costs are given by the inverse of the mark-up and since the latter must be kept constant in an equilibrium with flexible prices, it follows that a decrease in the nominal interest rate must be accompanied by an increase in employment.

Let's now consider the implications of the sticky price assumption. For the time being we are still assuming that there are no informational frictions on the lending activity. The presence of sticky prices links marginal costs to inflation dynamic through endogenous mark-up movements. To see this we can substitute the above expression for the marginal cost, (33), into the Phillips curve relation to obtain:

$$0 = a_t n_t^{1-\alpha} k_t^\alpha \left((1 - \varepsilon) + \varepsilon \left[-\frac{u_{n,t}}{u_{c,t}} \frac{(1 + (1 + r_t^n))}{a_t(1 - \alpha)} (\frac{n_t}{k_t})^\alpha \right] \right) - \theta_p (\pi_t - 1) \pi_t + \beta \theta_p (\pi_{t+1} - 1) \pi_{t+1} \tag{34}$$

This expression manifests the link between movements in the nominal interest rate and inflation. An increase in the nominal interest, implemented for instance to close the output and the inflation gaps, works as a cost-push shock as by raising marginal cost also raises inflation.

Let's now reintroduce the credit channel. Two observations arise. First, our model features an endogenous time-varying mark-up:

$$\mu_t = \frac{1}{mc_t} = -\frac{u_{c,t}}{u_{n,t}} \frac{a_t \omega_t (1 - \alpha)}{(1 + (1 + r_t^l))} (\frac{k_t}{n_t})^\alpha \tag{35}$$

whose dynamic properties (volatility and persistence) are affected by the lending rate and by firms' idiosyncratic productivity. Secondly, the reduced form expression for the Phillips curve will now read as follows:

$$0 = a_t \omega_t n_t^{1-\alpha} k_t^\alpha \left((1 - \varepsilon) + \varepsilon \left[-\frac{u_{n,t}}{u_{c,t}} \frac{(1 + (1 + r_t^l))}{a_t \omega_t (1 - \alpha)} (\frac{n_t}{k_t})^\alpha \right] \right) - \theta_p (\pi_t - 1) \pi_t + \beta \theta_p (\pi_{t+1} - 1) \pi_{t+1} \tag{36}$$

As it stand clear the role of the credit frictions is that of reinforcing the effects of the cost channel by linking the evolution of marginal costs to lending conditions and firm dynamic. Overall the transmission mechanism in this model is characterized by *supply side driven fluctuations* which are exacerbated by the credit channel.

2.6 Definition of finance premia and interest rate differentials

As emphasized in Goodfriend and McCallum 2006 when setting its policy the monetary authority should take into account the effects of changes in the policy instrument on the full array of asset returns that are involved in the monetary transmission mechanism. In our model changes in the nominal interest rate have an impact on the return of real money balances, on the cost of working capital and on the costs of servicing collateralizable loans. Because of those links the direction, the extent and the effectiveness of monetary policy actions depend on the impact that movements in the Federal Fund Rate have on each of those interest rates. Hence the monetary transmission mechanism can be disentangled in various blocks corresponding to the movements in the various external finance premia defined as differences between all the interest rates characterizing our economy. In the context of the present model we can define at least three different spreads between asset returns.

The first is a *liquidity term premium* which is given by the differential between the interest rate on deposits, $(1+r_t^d)$, and the cost of real money balances. This differential is simply given by the fraction of held reserve deposits:

$$\frac{(1+r_t^d)}{(1+r_t^n)} = (1-\xi) \tag{37}$$

As the fraction, ξ, is assumed constant in our case such spread should not have a significant quantitative impact on the monetary policy transmission.

The second spread is a *finance premium* linking the interest rate on deposits and the one on loans. This is obtained by merging equations (27) and (26):

$$\frac{(1+r_t^l)}{(1+r_t^d)} = \frac{\varpi_t^j}{\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)} \tag{38}$$

This spread depends on the ratio between the optimal default threshold and the fraction of surplus accruing to the intermediary as, for given interest rate on deposits, the cost of servicing a collateralizable loan increases when the default probability raises or when the expected net surplus for the intermediary falls. The ratio in (38) manifests the countercyclical nature of this spread which decreases in response to an aggregate productivity that by increasing revenues tends to decrease the default threshold (see equation (26)).

The third spread is given by a pure *external finance premium* obtained as differential between the rate on collateralizable assets and the interest rate on working capital investment:

$$\frac{(1+r_t^p)}{(1+r_t^l)} = \rho(\varpi_t)\frac{\varpi_t^j}{\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)} \tag{39}$$

Both the components, $\rho(\varpi_t)$ and $\frac{\varpi_t^j}{\Gamma(\varpi_t^j)-\mu M(\varpi_t^j)}$, characterizing this spread behave *counter-cyclically*, as both of them are positively related to the default threshold.

Countercyclical spreads in general tend to exacerbate business cycle fluctuations in response to shocks. This is so since productivity shocks have both a direct and an indirect effect on factor demands. To understand this mechanism, let's rely on the following intuitive argument. Let's assume that a technology improvement occurs in a flexible price equilibrium. In this context an increase in technology has two effects. First, it raises input demand on impact as by raising productivity it shifts to the right the input demand schedules. Second, by shrinking the *external finance premia* it allows to protract and amplify the positive boost on input demands.

2.6.1 Market Clearing Conditions and Competitive Equilibrium

Equilibrium in the final good market requires that the production of the final good be allocated to private consumption by households, investment and to resource costs that originate from the adjustment of prices as well as from the lender's monitoring of the investment activity. Under standard aggregation assumptions the resource constraint reads as follows:

$$y_t = c_t + i_t + g_t + \frac{\theta_p}{2}(\pi_t - 1)^2 + \mu M(\varpi_t)\mathcal{G}(l_t^j) \tag{40}$$

where g_t is an exogenous government expenditure process. Fiscal balance is achieved by financing government expenditure with lump sum taxation.

Definition 1. *A distorted competitive equilibrium for this economy is a sequence of allocation and prices* $\{c_t, n_t, m_t, k_{t+1}, i_t, l_t, \varpi_t, mc_t, \pi_t, q_t, w_t, r_t^n, r_t^p\}_{t=0}^{\infty}$ *which, for given initial* d_0,k_0, m_0, *satisfies equations*

(6), (7),(8), (4), (12), (13),(16),(17),(25),(26),(27),(29),

(40).

3 Dynamics Responses of the Competitive Equilibrium Allocation

Before turning to the optimal monetary policy design it is instructive to consider how the economy behaves when confronted with simple operational Taylor rules of the following type:

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) \tag{41}$$

where $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. The analysis of the dynamic responses under simple rules allows us to disentangle the effects induced by the credit augmented liquidity cycle.

3.1 Computation and Calibration

To solve for the dynamic responses of the competitive economy I compute *second order approximations* of the equilibrium conditions. The use of those methods is particularly useful in economies with credit frictions as they allow us to account for the effects of volatility on mean levels of variables therefore providing more accurate solution (see Schmitt-Grohe and Uribe 2001 for details). Parameters' calibration is as follows.

Preferences. I set the discount factor $\beta = 0.99$, so that the annual interest rate is equal to 4 percent. Utility is chosen separable in consumption and labor:

$$U(c_t, n_t) = \log(c_t) - \gamma \log(1 - n_t)$$

The parameter γ is set equal to 3 as this guarantees that in steady state one third of time is spent working. Sensitivity analysis is done to assess the robustness of the results.

Technology. I set the share of capital in the production functions equal to $\alpha = 0.35$ as in Christiano 1988, the quarterly depreciation rate $\delta = 0.021$ as in Christiano 1991, the steady state mark-up value to $\frac{\varepsilon}{\varepsilon - 1} = 1.2$ which corresponds to a value for the elasticity of demand, $\varepsilon = 11$ as in Khan, King and Wolman 2003. The benchmark value for the adjustment cost parameter, θ_p, is 100. This parameter is varied in the simulations to test robustness.

Financial frictions parameters: The financial frictions parameters are obtained by solving the steady state version of the competitive economy under the optimal contracting problem. Some primitive parameters are set so as to match values for industrialized countries. The idiosyncratic shock is assumed to follow a uniform distribution within the range $[1 - A, 1 + A]$: the value for A is chosen to as to generate a external finance premium of 187 basis points. The monitoring cost is varied between 0.03 and 0.4, as those values are compatible with data for bankruptcy cost for industrialized countries.

Shocks. I simulate the model under productivity shocks which follow $AR(1)$ processes. Persistence and volatility are calibrated as in the RBC literature. government consumption evolves according to the following exogenous process, $\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g$, where the steady-state share of government consumption, g, is set so that $\frac{g}{y} = 0.25$ and ε_t^g is an i.i.d. shock with standard deviation σ_g. Empirical evidence for the US in Perotti (2004) suggests $\sigma_g = 0.008$ and $\rho_g = 0.9$.

3.2 Dynamic Properties of the Model

In figure 1 I show dynamic responses of selected variables to a one percent productivity shock under two different parametrization of the monitoring cost (which vary, within an admissible empirical range, from 0.1 to 0.4). In response to an increase in productivity consumption, output and employment raise, while inflation falls. The default threshold decreases reducing the mass of firms which defaults. The decrease in the default threshold tends to increase the marginal costs but at the same time reduces the lending rate as the project riskiness decreases. The second effect prevails on the first, which implies that marginal costs are reduced. Two considerations are worth. First marginal cost sensitivity to shocks is amplified by the endogenous response of monetary policy as the Federal Fund Rate feeds into marginal costs through the lending rate. Second, the higher the monitoring costs the higher is the acceleration effect featured in this economy. This is so since as the default threshold moves pro-cyclically (the higher the aggregate productivity the lower is firms' default probability) the external finance premium must move counter-cyclally thereby amplifying the effects of shocks.

Figure 2 shows the response to productivity shocks under different values foe the adjustment costs parameter which is now moved from 100 to 200, while keeping the monitoring cost fixed at 0.3. Impulse responses are more amplified when the cost of adjusting prices is higher. The reason for this is as follows. With higher cost of adjusting prices, firms tend to change mark-ups more relatively to demand. As mark-ups behave countercyclically an increase in their sensitivity tends to amplify the economy's response to shocks. Notice also that as marginal costs are given by the inverse of the mark-up the higher sensitivity of the latter is translated to the inflation and the price path.

Figure 3 shows impulse response functions of selected variables under government expenditure shocks and assuming different values for the monitoring costs. An increase in government consumptions tends to offset private consumption. Overall however the increase in aggregate demand tends to increase output, employment and prices. In this case the impact of credit frictions, associated with different values of the monitoring cost, is more muted as the sensitivity of the marginal shock is lower in response to demand shocks.

4 The Optimal Policy Problem

To analyze the optimal policy problem I will rely on two different aspects of the monetary policy design. In general the optimal policy problem is solved by following the Ramsey approach in which the planner maximizes agents' utility subject to the constraints of the competitive equilibrium. As I first step I analyze the optimal targeting problem by solving a Ramsey plan in which the allocation is constrained also by a well specified set of operational monetary policy rules. As a second step I study the optimal dynamic path of variables by solving a full-fledged Ramsey plan. Alternatively one could see the first step as the implementation, through well specified operational policy rules, of the optimal policy plan as designed in step two.

4.1 Constrained Ramsey Plan

As specified above the optimal policy problem in this context is solved by assuming that the monetary authority maximizes households welfare subject to the competitive equilibrium conditions and the class of monetary policy rules represented by:

$$\begin{aligned} \ln\left(\frac{1+r_t^n}{1+r^n}\right) &= (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_q \ln\left(\frac{q_t}{q}\right)\right) \\ &\quad +\phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) \end{aligned} \tag{42}$$

where ϕ_q represents a response to asset price. I also consider the alternative specification given by:

$$\begin{aligned} \ln\left(\frac{1+r_t^n}{1+r^n}\right) &= (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_{lk} \ln\left(\frac{l_t/k_t}{(l/k)}\right)\right) \\ &\quad +\phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) \end{aligned} \tag{43}$$

where ϕ_{lk} represents the response to the leverage ratio. I therefore search for parametrization of interest rate rules that satisfy the following 3 conditions: a) they are simple since they involve only observable variables, b) they guarantee uniqueness of the rational expectation equilibrium, c) they maximize the expected life-time utility of the representative agent.

Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. Indeed in an economy with a distorted steady state stochastic volatility affects both first and second moments of those variables that are critical for welfare. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions[8]. Additionally one needs to focus on the *conditional* expected discounted utility of the representative agent. This allows to account for the transitional

[8] See Kim and Kim (2003) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.

effects from the deterministic to the different stochastic steady states respectively implied by each alternative policy rule[9]. Define Ω as the fraction of household's consumption that would be needed to equate conditional welfare $\mathcal{W}_0$ under a generic interest rate policy to the level of welfare $\widetilde{\mathcal{W}_0}$ implied by the optimal rule. Hence Ω should satisfy the following equation:

$$\mathcal{W}_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1+\Omega)c_t, n_t) \right\} = \widetilde{\mathcal{W}_0}$$

Under a given specification of utility one can solve for Ω and obtain:

$$\Omega = \exp \left\{ \left(\widetilde{\mathcal{W}_0} - \mathcal{W}_0 \right) (1-\beta) \right\} - 1$$

Given this welfare metric I simulate the model economy under the two sources of aggregate uncertainty, productivity and government consumption shocks. I then conduct two experiments. First, I compute welfare under different (ad hoc) specifications of the monetary policy rule. The rules are the following:

(i) *Simple Taylor rule*, with $\phi_\pi = 1.5$, $\phi_y = 0.5/4, \phi_q = \phi_r = 0$;

(ii) *Simple Taylor rule with smoothing*, with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4, \phi_q = 0, \phi_r = 0.8$;

(iii) *Strict inflation targeting,* $\phi_\pi = 2, \phi_y = 0.5/4, \phi_q = \phi_r = 0$;

(iv) *Price stability,* $\phi_\pi = 2, \phi_y = 0.5/4, \phi_q = \phi_r = 0$;

(v) *Response to asset prices*, with $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, $\phi_q = 0.5/4, \phi_r = 0$.

(vi) *Response to asset prices and interest rate smoothing*, with $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, $\phi_q = 0.5/4, \phi_r = 0$.

(vii) *Strong response to inflation and to asset prices*, with $\phi_\pi = 3$, $\phi_q = 0.5/4$, $\phi_y = \phi_r = 0$.

Secondly, I search in the grid of parameters $\left\{\phi_\pi, \phi_y, \phi_q, \phi_r\right\}$ for the rule which delivers the highest level of welfare, which is defined as the optimal policy rule[10] and I compare welfare under optimal policy and simple rules.

The choice of including asset prices or leverage ratios as independent targets is motivated by the consideration that the monetary authority might want to de-amplify fluctuations in marginal

[9] See Kim and Levin (2004) for a detailed analysis on this point.

[10] The search is made over the following ranges: $[1.5, 4]$ for ϕ_π, $[0, 2]$ for ϕ_q, $[0, 2]$ for ϕ_y. Notice that the parameters ϕ_y and ϕ_q are divided by four given the standard assumption on the length of a period (quarterly) and given that inflation in Taylor type rules is expressed at annual rates. I also compare rules with interest rate smoothing ($\phi_r = 0.9$) to rules without smoothing ($\phi_r = 0$). It is judged as admissible a combination of policy parameters that delivered a unique rational expectations equilibrium.

costs which ultimately depend on the cost of working capital. As both the price of working capital and the leverage ratio are proxies of the cost for working capital it seems reasonable to include them as independent targets. The fact that optimality might require assigning a positive weight to any of those additional targets alongside with inflation implies that strict inflation targeting or price stability policies are suboptimal.

Table 1 summarizes the findings in terms of the welfare loss Ω (relative to the optimal policy) of alternative simple rules.

Results are as follows. First, responding to asset prices along with inflation is the optimal rule. More specifically the optimal rule features the following coefficients: $\phi_\pi = 3$, $\phi_q = 0.5/4$, $\phi_y = 0, \phi_r = 0$. This is so since the policy maker faces a trade-off between stabilizing inflation, as this would allow to close the gap between the flexible and the sticky price allocation, and easing up liquidity, which by reducing the cost of working capital would stabilize fluctuations in marginal cost. The optimal management of this policy trade-off is resolved by targeting the price of working capital on top and above inflation as this guarantees a certain degree of marginal cost stabilization.

Secondly, responding to output along with inflation and asset prices is slightly welfare detrimental. This result is consistent with the one obtained by Schmitt-Grohe and Uribe 2003, Faia and Monacelli 2007, Faia 2007 by using alternative specification of the new Keynesian economy. The intuition for this result in the context of the present paper lies in the fact that the policy maker aims at stabilizing only variables or gaps which proxy more closely the inefficiency. Asset prices are a better proxies for the cost of working capital than output is. Third, interest smoothing is slightly welfare improving as this helps to smooth fluctuations in lending rates and marginal costs.

Finally figures 4 and 5 the welfare gains relative to the optimal rule of targeting asset prices and the leverage ratio respectively. the gains are clearly increasing for any level of inflation targeting.

4.2 Globally Optimal Ramsey Plan Under Commitment

I now turn to the specification of a general set-up for the optimal policy conduct. The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983. There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe 2002 to analyze a problem of joint determination of optimal monetary and fiscal policy.

The optimality conditions for the consumer are summarized by equations, (6), (7),(8). Next, firms' optimizing conditions can be summarized by the following equations:

$$w_t^r(1+\varpi_t\frac{\mathcal{G}(l_t)}{l_t}) = mc_t a_t \omega_t (1-\alpha) n_t^{-\alpha} k_t^{\alpha} \tag{44}$$

$$q_t(1+\varpi_t\frac{\mathcal{G}(l_t)}{l_t}) = mc_t a_t \omega_t \alpha n_t^{1-\alpha} k_t^{\alpha-1} \tag{45}$$

$$0 = a_t \omega_t n_t^{1-\alpha} k_t^{\alpha}\left((1-\varepsilon)+\varepsilon mc_t\right) - \theta_p\left(\pi_t-1\right)\pi_t + \beta\theta_p\left(\pi_{t+1}-1\right)\pi_{t+1} \tag{46}$$

where $w_t^r = \frac{w_t}{p_t}, l_t = \frac{w_t}{p_t}n_t + q_t k_t$. The conditions summarizing the relations for the optimal contract are:

$$\frac{\mathcal{G}(l_t^j)}{l_t} \geq \frac{(1+r_t^n)(1-\xi)}{(\Gamma(\varpi_t^j)-\mu M(\varpi_t^j))} \tag{47}$$

$$\frac{\mathcal{G}'(l_t^j)}{(1+r_t^n)(1-\xi)} = \rho(\varpi_t) \tag{48}$$

Finally we need to consider the resource constraint:

$$a_t \omega_t n_t^{1-\alpha} k_t^{\alpha} = c_t + k_{t+1} - (1-\delta)k_t + \frac{\theta_p}{2}\left(\pi_t-1\right)^2 + \mu M(\varpi_t)\mathcal{G}(l_t^j) \tag{49}$$

Two considerations are worth. First, we do not need to include the government resource constraints among the equilibrium conditions as it is assumed the absence of distortionary taxation. Second, we do not need to include the cash in advance constraint as the conditions relevant to the Ramsey planner are the given by the minimum set of conditions needed to achieve a determinate real equilibrium.

Definition 2. *Let $\Lambda_t^n = \{\lambda_{1,t},\lambda_{2,t},\lambda_{3,t},\lambda_{4,t},\lambda_{5,t},\lambda_{6,t},\lambda_{7,t},\lambda_{8,t},\lambda_{9,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (6), (7),(8), (44), (45), (46),(47),(48),*

(49) respectively. Let k_0, be given. Then for given stochastic process $\{a_t,\omega_t\}_{t=0}^{\infty}$, plans for the control variables $\Xi_t^n \equiv \{c_t, n_t, w_t^r, q_t, k_{t+1}, mc_t, \pi_t, r_t^n, \varpi_t\}_{t=0}^{\infty}$ and for the co-state variables

$\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}} \; Max_{\{\Xi_t^n\}_{t=0}^{\infty}} \; E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right\} \tag{50}$$

subject to (6), (7),(8), (44), (45), (46), (47),(48),
(49).

4.2.1 Non-recursivity and Initial Conditions

As a result of constraints (6), (8) and (46) exhibiting future expectations of control variables, the maximization problem as spelled out in (50) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, and then developed by Marcet and Marimon 1999, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that I denote $\chi_{1,t}$, $\chi_{2,t}$ and $\chi_{3,t}$ for (6), (8) and (46) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

$$\begin{aligned} \chi_{1,t+1} &= \lambda_{1,t} \\ \chi_{2,t+1} &= \lambda_{2,t} \\ \chi_{3,t+1} &= \lambda_{3,t} \end{aligned} \tag{51}$$

Using the new co-state variable so far described I amplify the state space of the Ramsey allocation to be $\{a_t, \chi_{1,t}, \chi_{2,t}, \chi_{3,t}\}_{t=0}^{\infty}$ and I define a new saddle point problem which is recursive in the new state space. Consistently with a *timeless perspective* I set the values of the three co-state variables at time zero equal to their solution in the steady state. I will return on this point in the next subsection.

4.3 Monetary Policy Trade-offs

Before turning to the solution of the optimal plan it is instructive to analyze the constraints faced by the monetary authority to highlight the relevant policy trade-offs. In the standard new Keynesian framework with sticky prices but in absence of the liquidity effects associated with the cost channel and in absence of credit frictions the policy maker achieves optimality by replicating the flexible price allocation. The relevant costs in this case are represented by the resource wasted in adjusting prices, therefore implying that optimality requires closing the *gaps* between the flexible price and the sticky price allocation. To close the gaps the monetary authority must simply set $\frac{\theta_p}{2}\left(\pi_t - 1\right)^2 = 0$ which implies following price stability rules[11]. In our model however the flexible price allocation with constant mark-ups can never be achieved. In fact by imposing zero inflation on the reduced form Phillips curve (52) we obtain the following relation:

$$mc_t = \frac{1-\varepsilon_t}{\varepsilon_t} = \left[-\frac{u_{n,t}}{u_{c,t}} \frac{n_t{}^{\alpha}}{a_t \omega_t (1-\alpha) k_t^{\alpha}} (1 + (1 + r_t^l)) \right]^{-1} \tag{52}$$

which shows that marginal costs are time-varying as lending rates and default threshold move in response to any type of shock.

5 Long Run Optimal Policy

Before turning to the optimal stabilization policy in response to shocks we need to characterize the log-run optimal policy, which is the one to which the policy maker would like to converge. To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the *modified golden rule* steady state. To determine the long-run inflation rate associated to the optimal policy problem above, one needs to solve the steady-state version of the set of efficiency conditions. Notice in particular that the first order condition with respect to inflation reads as follows:

$$\chi_{1,t}\left(\frac{u_{c,t}}{\pi_t^2}\right) + (\lambda_{2,t} - \chi_{2,t})\theta_p(2\pi_t - 1) - \lambda_{3,t}\theta_p\left(\pi_t - 1\right) = 0 \tag{53}$$

For the whole set of optimality conditions of the Ramsey plan to be satisfied in the steady state a necessary condition is that equation (53) is satisfied in the steady state. In that steady-state, we have $\lambda_{2,t} = \lambda_{2,t-1} = \chi_{2,t}$. Hence condition (53) immediately implies:

$$\chi_1\left(\frac{u_c}{\pi^2}\right) + \lambda_3\theta_p\left(\pi - 1\right) = 0 \tag{54}$$

The above expression shows clearly that the zero (net) inflation policy is not a solution to the above condition. The Ramsey planner faces a tension between closing the gap with the flexible price allocation and setting the nominal interest rate to zero in order to reduce the transaction

[11] This is certainly true in presence of productivity shocks (see Clarida, Gali' and Gertler (1998)). A caveat must be made for government expenditure shocks as by allowing for fluctuations in the ratio of demand to output they endow the policy maker with a leverage represented by countercyclical mark-up (see King and Wolman (1998), Adao, Correia and Teles (2003), Khan, King and Wolman (2003) and Ravenna and Walsh (2006)).

cost of real money balances. In particular positive levels of the nominal interest rate (which imply deflation) by increasing the cost of funds for working capital tend to increase firms' marginal cost and inflation. This second channel acquires stronger relevance when considering that firms' are subject to additional costs of external finance due to informational asymmetries. Overall the optimal level of (net) inflation stems between zero and the one satisfying the Friedman rule.

6 Dynamic Properties of the Optimal Plan

Here I evaluate the dynamic properties of the optimal plan based on impulse response functions, optimal volatilities and welfare costs. A quantitative assessment is indeed necessary in order to evaluate a series of things among which the importance of deviations from price stability.

6.1 Dynamic Responses to Shocks of the Optimal Plan

To solve for the optimal stabilization policy I compute second order approximations[12] of the first order conditions of the Lagrangian problem described in definition 3. Technically I compute the stationary allocation that characterize the deterministic steady state of the first order conditions to the Ramsey plan. I then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady already for a long period of time (under timeless perspective).

Figure 6 shows impulse responses to productivity shocks under the Ramsey plan. First, the Ramsey plan deviates from strict price stability as in fact this is not fully implementable. Second, as the Ramsey plan tries to trade-offs between the cost of adjusting prices and the cost of available funds to run production inflation lies between the one under the price stability policy and the one under Taylor rule. Finally the default threshold falls by more under this policy (than under Taylor rules) as the planner tries to take full advantage of the beneficial effects of aggregate productivity shocks.

7 Conclusions

This paper studies optimal monetary policy in a model that combines a cost channel with a credit channel. The presence of a cost channel implies that fluctuations in nominal interest rate have an

[12]Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2004a,b) among others.

impact on firms marginal costs and inflation dynamics therefore inducing a policy trade-off. The addition of credit frictions, generated by informational asymmetries and moral hazard, tends to amplify those types of supply side driven fluctuations.

Results show that the monetary authority should deviate from the price stability prescription both in the long run and in the short run. An optimal monetary policy rule should feature asset price targeting alongside with inflation targeting.

The current analysis focused on the role of liquidity and credit frictions for the firm problem. One of the feature of the recent crises on the sub-prime lending market is that much of the lending activity has involved households investment in durable goods. One promising avenue of research comes from exploring the role of those type of frictions for the households problem.

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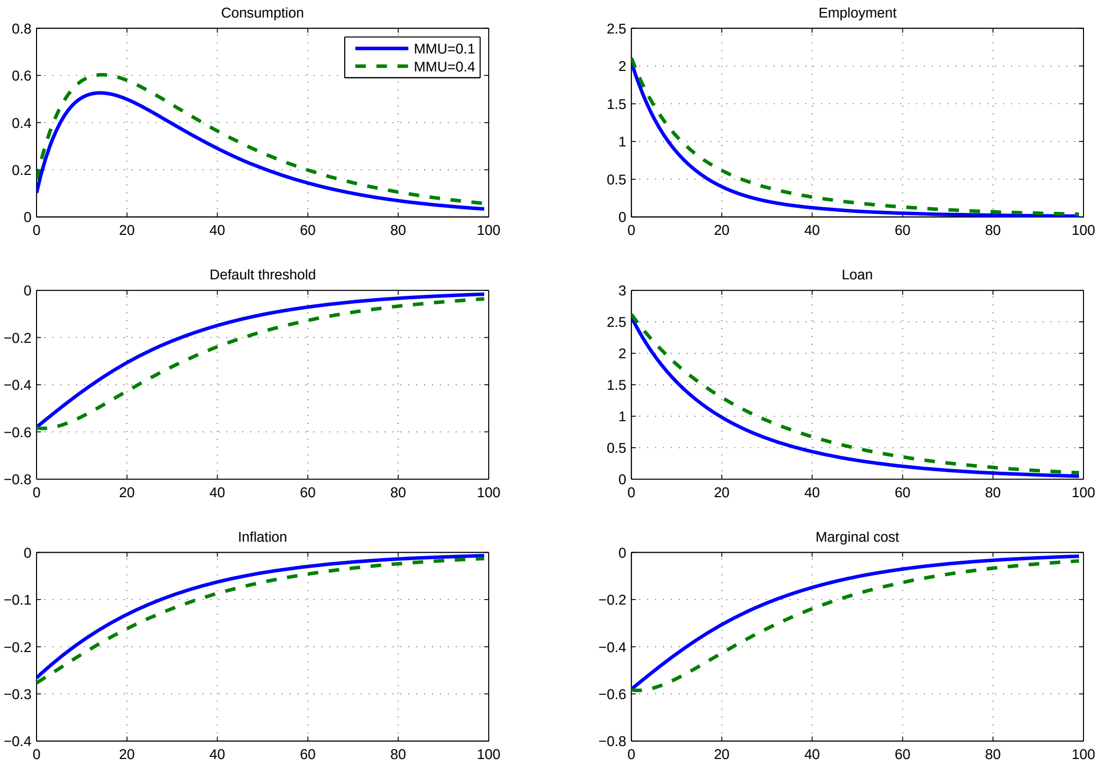

Figure 1: **Dynamic responses of selected variables to productivity shocks under two different values for the monitoring cost**

Table 1: **Welfare comparison of alternative monetary policy rules.**

Monetary Policy Rule	% Loss relative to optimal rule
	$\mu = 0.4$
$\phi_\pi = 1.5,\ \phi_y = 0.5/4, \phi_q = \phi_r = 0$	0.092
$\phi_\pi = 1.5,\ \phi_y = 0.5/4, \phi_q = 0, \phi_r = 0.8$	0.025
$\phi_\pi = 2,\ \phi_y = 0.5/4, \phi_q = \phi_r = 0$	0.0048
$\phi_\pi = 3,\ \phi_y = 0, \phi_q = \phi_r = 0$	0.8322
$\phi_\pi = 1.5,\ \phi_y = 0.5/4, \phi_q = 0.5/4, \phi_r = 0$	0.0092
$\phi_\pi = 1.5,\ \phi_y = 0.5/4, \phi_q = 0.5/4, \phi_r = 0.8$	0.0044
$\phi_\pi = 2,\ \phi_y = 0.5/4, \phi_q = 0.5/4\phi_r = 0$	0.0092
$\phi_\pi = 3,\ \phi_y = 0, \phi_q = 0.5/4, \phi_r = 0$	0

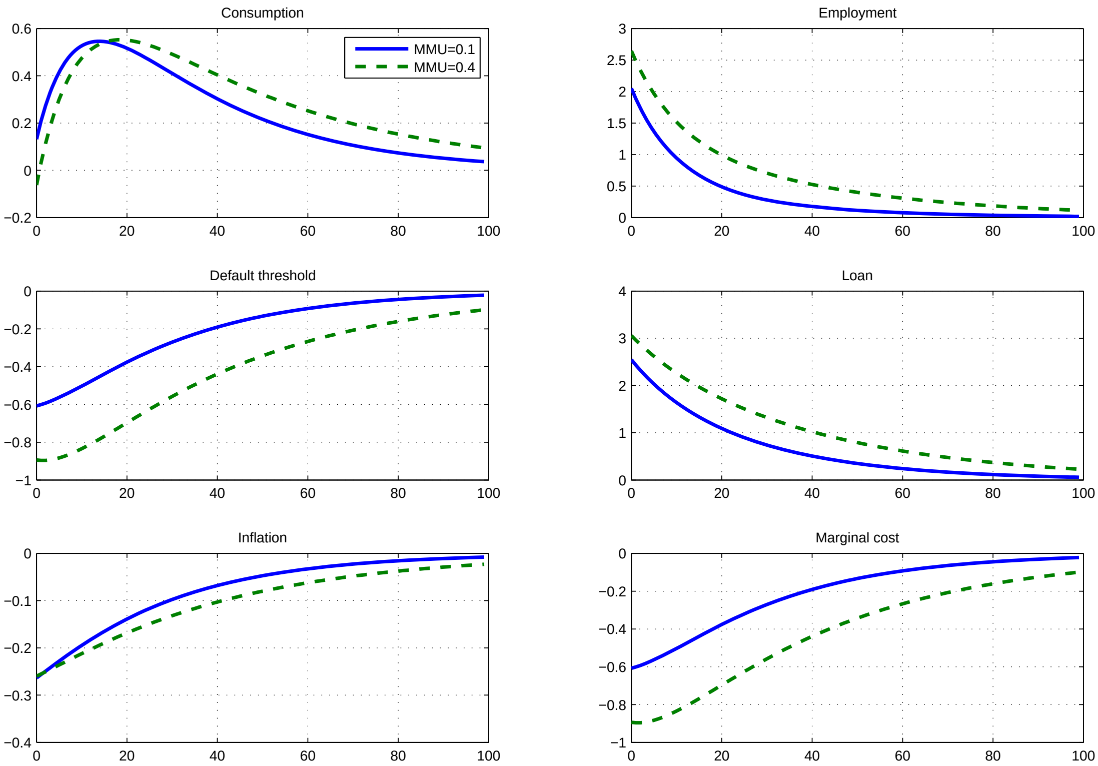

Figure 2: **Dynamic responses of selected variables to productivity shocks under two different values for the adjustment cost parameter**

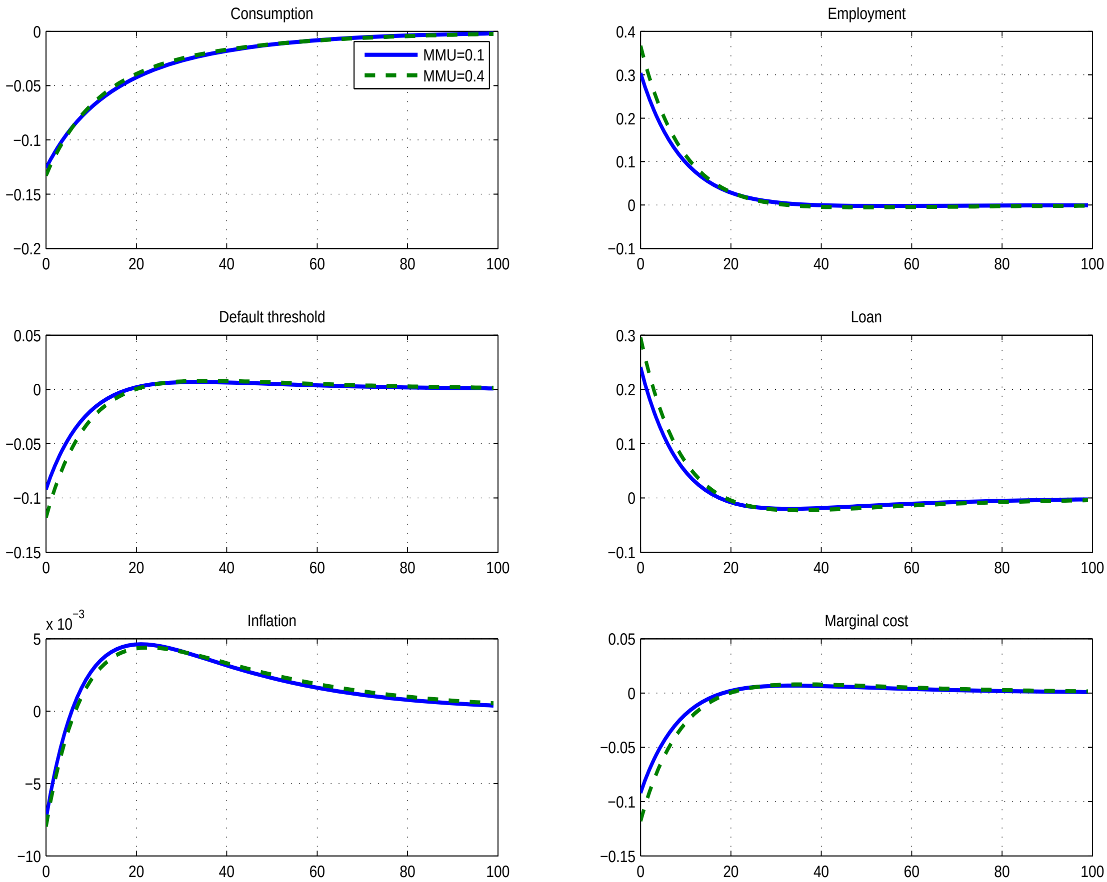

Figure 3: **Dynamic responses of selected variables to government expenditure shocks under two different values for the monitoring cost**

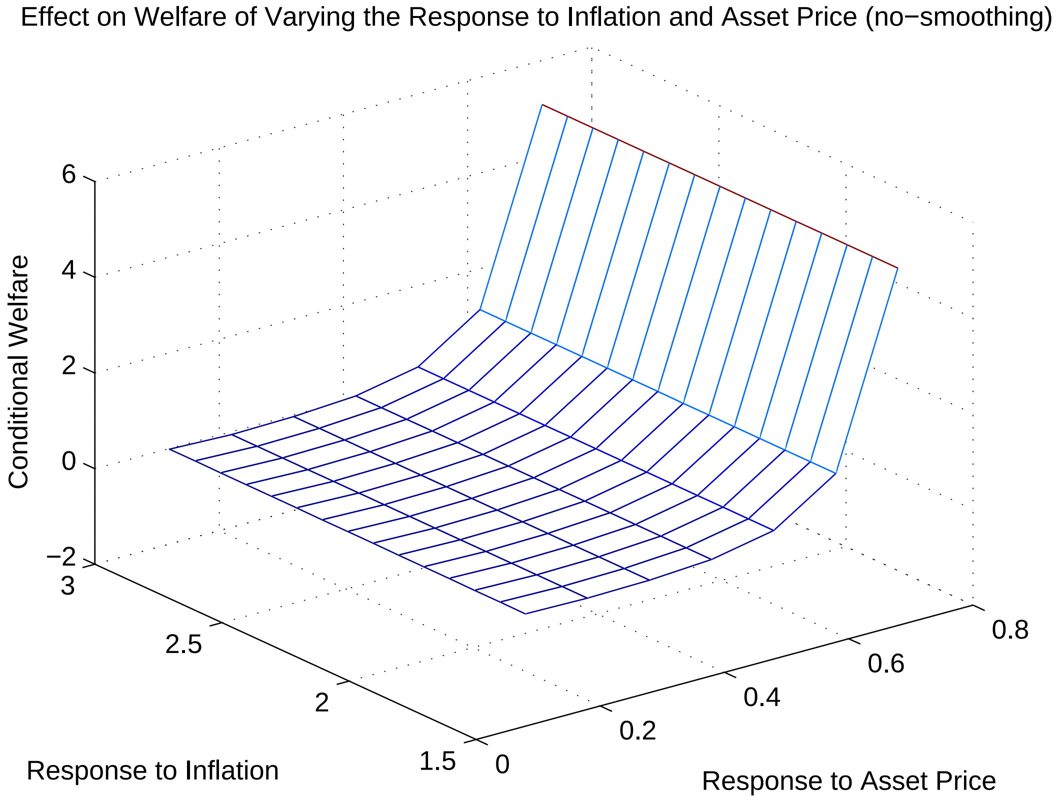
Effect on Welfare of Varying the Response to Inflation and Asset Price (no−smoothing)
Conditional Welfare
6
4
2
0
−2
3
2.5
2
1.5
Response to Inflation
0
0.2
0.4
0.6
0.8
Response to Asset Price

Figure 4:

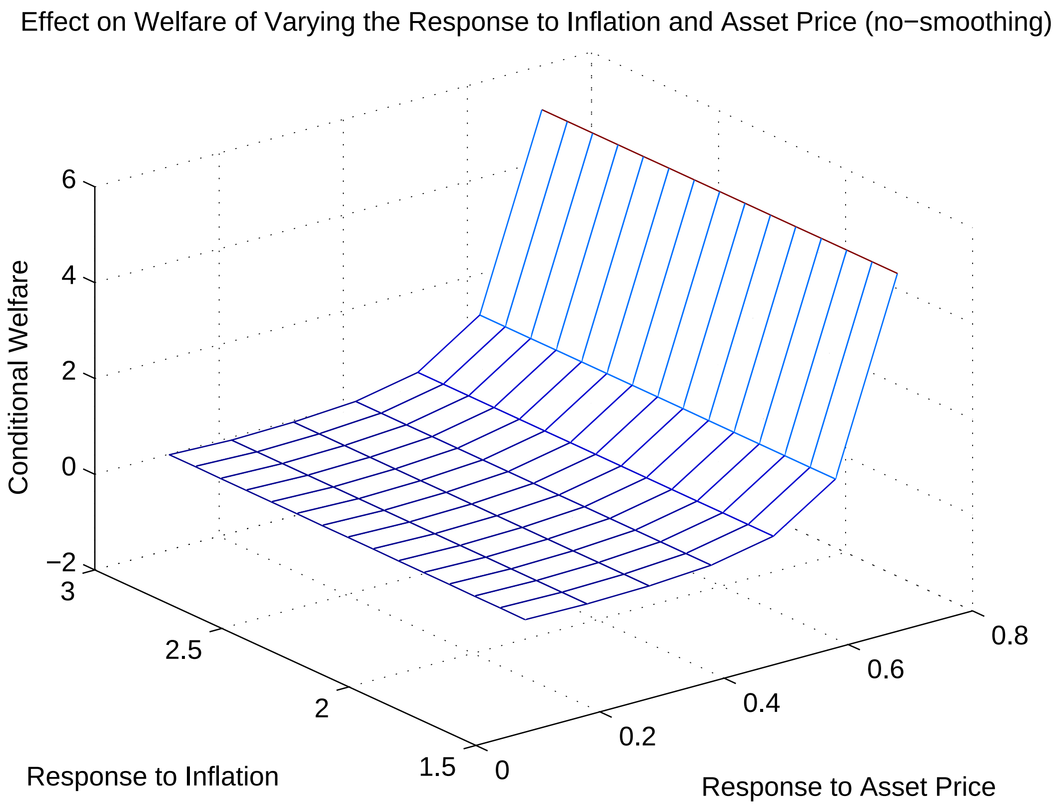
Effect on Welfare of Varying the Response to Inflation and Asset Price (no−smoothing)
Conditional Welfare
6
4
2
0
−2
3
2.5
2
1.5
Response to Inflation
0
0.2
0.4
0.6
0.8
Response to Asset Price

Figure 5:

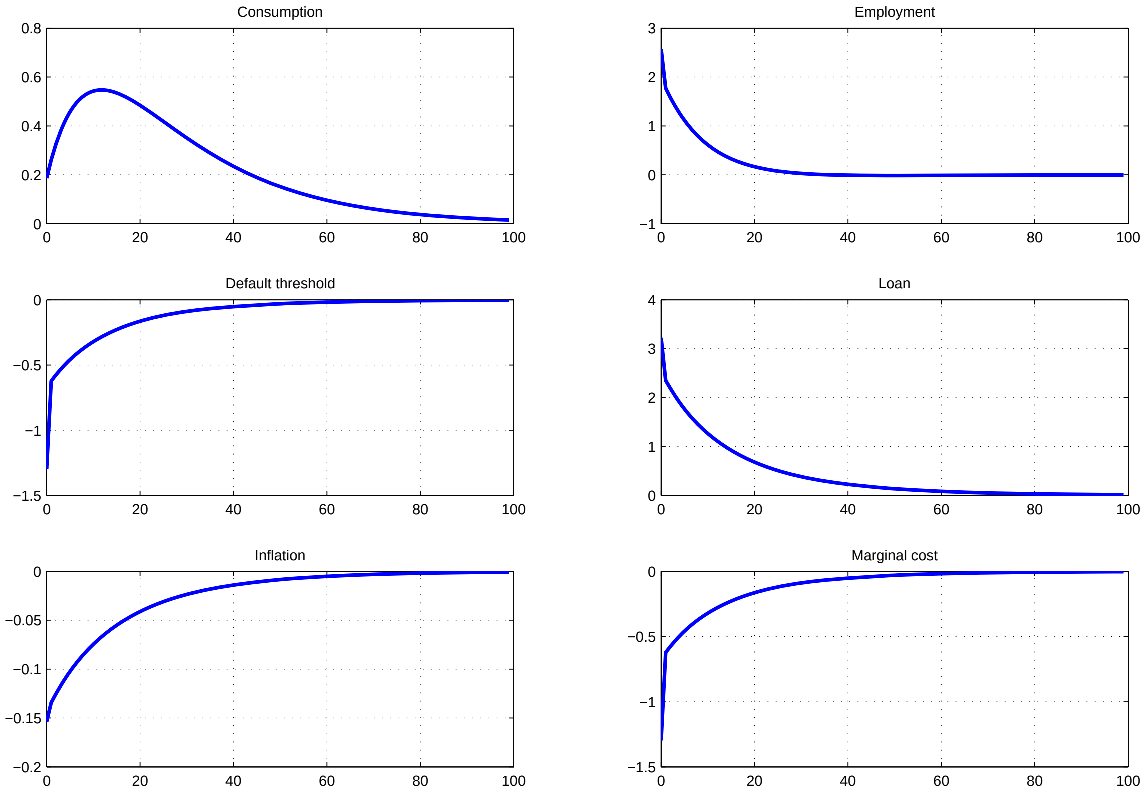

Figure 6: **Response to productivity shocks under Ramsey policy**