The Bank as Grim Reaper: Debt Composition and Recoveries on Defaulted Debt

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Abstract

We offer a model and evidence that private debtholders play a key role in setting the endogenous asset value threshold below which corporations declare bankruptcy. The model, in the spirit of Black and Cox (1976), implies that the recovery rate at emergence from bankruptcy on all of the firm's debt is related to the pre-bankruptcy share of private debt in all of the firm's debt. Empirical evidence supports this implication. Indeed, debt composition has a more economically material empirical influence on recovery than all other variables we try taken together. This special role of private debt in the capital structure has important implications for pricing models and risk management.

Keywords: credit risk, recovery rates, bankruptcy, debt default
JEL Codes: G12, G33, G32

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We offer a model and evidence showing that the composition of corporate debt strongly influences recovery rates on debt of bankrupt corporations. Our work is in the spirit of structural and strategic models of default, which appear to inform most researchers' intuition about recovery, but the locus of strategic behavior is different. In such models, the firm defaults when the value of its assets falls below a threshold. Implicitly or explicitly, debtholders recover the threshold value of assets, perhaps less a haircut for deadweight costs of bankruptcy. In early structural models of default, such as Merton (1974) or Longstaff and Schwartz (1995), the threshold is exogenous. In models of strategic default, such as Leland (1994) or Fan and Sundaresan (2000), equity holders choose the threshold endogenously to maximize the value of their claims. In our model, holders of private debt with strong covenants control the choice of threshold.

Our model is a generalization of the first-passage model of Black and Cox (1976) in which firm's private debtholders ("banks" for simplicity) endogenously choose the bankruptcy threshold value of assets. In our model, private debt has covenants that give banks the right to force a distressed firm into bankruptcy, even if the firm has made all debt payments. The firm's public debt is junior and has no material covenants. Because private debt is also senior, the bank has an incentive to foreclose only when the borrower's asset value drops to the neighborhood of the loan's face value, which can be well below the insolvency value of assets. Therefore, the lower the bank loan share in total debt, the lower the asset value of the borrower at bankruptcy and the lower the recovery to debt as a whole. Furthermore, our model has testable implications for the relationship between recovery rates and loan interest rate spreads and the volatility of the firm's assets.[1]

The locus of strategic behavior in structural models depends on the interpretation of the asset-sale restrictions that are invariably attached to debt contracts.[2] The branch of the literature in the spirit of Leland (1994) takes a strict view of these restrictions, which implies that coupon payments must be financed out-of-pocket by equity holders (or via new equity issues), and therefore that equityholders will default when the value of continuation of their call option on assets is below the required "new money" payment. A looser interpretation of asset-sale restrictions would constrain only attempts to divert assets. Even when (net) asset returns are negative, firms typically generate substantial (gross) cashflows. So long as enough cashflows can be diverted to keeping contractually required debt payments current, equityholders may be able to retain control well past the point of insolvency without having to make payments out-of-pocket. If asset-sale restrictions *never* bind on coupon payments, equityholders will never voluntarily default—all bankruptcies will be forced by banks. We follow the latter, extreme interpretation mainly for simplicity and to complement the well-developed literature that flows from the opposite view.

Debt composition has only recently been considered in the literature on structural models of default, as earlier models have assumed a single class of debt. Hackbarth et al. (forthcoming) examine optimal capital structure in a model in which firms can issue bank debt, public bonds, and equity. The special quality of bank debt in their model is the ability to renegotiate outside formal bankruptcy. Bank debt offers a better tradeoff between tax shields and bankruptcy costs,

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[1] Non-bankruptcy defaults and renegotiations of debt contract terms are a material source of credit losses. Indeed, a number of models of strategic default, such as Mella-Barral and Perraudin (1997) and Hackbarth et al. (forthcoming), focus on such events. We do not consider them because we believe understanding of bankruptcy payoffs is an important step in understanding non-bankruptcy defaults. Bargaining out of bankruptcy is likely to be influenced by expectations about bankruptcy timing and outcomes.

[2] Lando (2004, §2.13.2) discusses the fundamental role of assumptions on asset-sale restrictions in structural models of credit risk.

whereas non-renegotiable public debt offers higher debt capacity.[3] In our model, the special role of bank debt derives instead from the strong covenants that typically are attached to loans but not to public bonds. So far as we are aware, ours is the first model to explore the implications for bankruptcy thresholds and recovery rates of this ubiquitous feature of private debt.[4]

Our focus throughout this paper is on the endgame phase of the firm's life (bankruptcy and recovery). Upon the onset of severe financial distress, the costs of altering debt composition or raising new equity are likely to be high, and so it is reasonable to take the firm's capital structure as fixed. For firms with assets well above the insolvency value, however, debt composition should be a material endogenous decision for the firm's owners. For example, by choosing bank debt share, the firm can influence the states of the world in which deadweight costs of bankruptcy are incurred, just as the choice of leverage influences the incidence of such costs in the capital structure literature. As our model takes equityholders as passive, it is not well-suited to analysis of the firm's ex-ante choice of capital structure. An extension of the model to allow for active equityholders and explicit transfer of control rights, along the lines of Broadie et al. (forthcoming), is left for future work.

Empirically, we find a robust, economically and statistically significant relationship between recovery and bank debt share of total debt at default. A marginal one percentage point increase in bank debt share improves recovery at emergence from bankruptcy ("ultimate recovery") on all the firm's debt taken together ("firm-level" recovery) by about one-quarter percentage point. That is, an increase from a small amount of bank debt to all bank debt would be associated with an increase in recovery rate of 25 percentage points, other things equal, which is large relative to the sample mean recovery of about 50 percent. Moreover, we find evidence of the loan coupon interest rate effects implied by our model.

Our model also implies that bank debt should receive a high recovery rate on average and that a substantial proportion of bank loans should receive a full recovery. We find mean and median debt-instrument-level recoveries on loans of 79 and 97 percent, respectively, and that at least 60 percent of bank debt receives approximately a full recovery. We offer circumstantial evidence that equityholders sometimes initiate bankruptcy. In passing, we offer evidence about other determinants of firm-level recovery rates (and thus, approximately, about determinants of bankruptcy threshold values of assets). We find more limited evidence than has appeared in previous studies of an important role for what are often presumed to be proxies for bargaining frictions, such as number of debt instruments outstanding, time in bankruptcy, or time in default before bankruptcy.

Our model offers some insight into why average recoveries are relatively low, a fact that has been difficult to comfortably reconcile with structural models of default. In models with an exogenous threshold, a common assumption for the threshold has been the asset value boundary between solvency and insolvency, in which case firm-level recovery should be not far from 100 percent.[5] Jumps in asset value (Zhou, 2001), accounting uncertainty (Duffie and Lando, 2001), liquidation

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[3] Bourgeon and Dionne (2007) extend the Hackbarth et al. (forthcoming) model to allow banks to adopt a mixed strategy in which renegotiation is sometimes refused ex-post in order to raise debt capacity ex-ante.

[4] As noted previously, we use the terms "banks," "bank debt" and "bank loans" as a convenient shorthand for senior debt with strong covenants. Such terminology only modestly represents historical patterns of debt structure in the U.S. Corporate bank loans frequently were most senior in firms' debt structures and had the strongest covenants. Privately placed bonds sometimes had such features, and publicly issued bonds rarely did. Very recently, a larger share of risky loans has been issued without strong covenants.

[5] Practitioner models of default often assume an exogeneous boundary of this sort, e.g. Moody's KMV Credit-Monitor (Crosbie and Bohn, 2003).

costs (Fan and Sundaresan, 2000; Mella-Barral, 1999) or (closely-related) asset specificities that imply a large reduction in value when assets are transferred to new owners (Baird and Jackson, 1988) no doubt play a role, but the magnitudes required to produce an average recovery of 48 percent seem implausibly large. In contrast, for reasonable parameter values, a mean recovery rate of 48 percent is broadly consistent with our model, given an empirical mean bank debt share of total debt of 33 to 42 percent.[6] The most likely reason actual average recovery is higher than predicted by a crude application of our model is that we abstract from strategic defaults by equityholders. Because bankruptcy is forced by the first entity with control rights that chooses to do so, any bankruptcies declared by equityholders are likely to have higher average recoveries than those forced by banks (by definition, if equityholders move first, their asset-value threshold for filing is higher than that of the bank).

Our findings imply that debt composition matters for debt pricing and credit risk management. Credit spreads and economic capital charges are roughly linear in expected loss-given-default (one minus the recovery rate), so errors in the specification of recovery are potentially costly.[7] And yet, in models and empirical studies of debt pricing, recovery is almost invariably treated as an afterthought. Expected recovery rates are typically assumed to be homogeneous within very broad debt classes (e.g., senior unsecured bonds). Our results indicate that expected recovery rates ought to be conditioned on debt composition rather than the class label. In applied settings, a simple rule of thumb could be based on the finding in this paper of a one-quarter-percentage-point improvement in recovery per additional percentage point of bank debt share, combined with adjustments for relative seniority of individual debt instruments.

Our assumptions that covenants give creditors rights to call loans to distressed borrowers and that such rights are attached to loans, not bonds, are realistic. Nash et al. (2003) characterize *bond* covenants as restricting financing, investing and restructuring activities. A common feature of such covenants is that they are violated only when the borrower takes a forbidden action, such as selling a large share of its assets.[8] An increase in the borrower's probability of bankruptcy does not by itself trigger a violation. Chava et al. (2004) find that only 4 percent of nonfinancial corporate bonds have a leverage or net worth covenant. In contrast, Carey (1996) and Sufi (2006) find that around 70 percent of bank loan agreements contain financial ratio covenants, such as interest-coverage, debt-to-cash-flow, and leverage ratios, and Carey (1996) offers evidence that such covenants are more likely to appear in loans to riskier borrowers. Dichev and Skinner (2002) and Chava and Roberts (2006) offer evidence that such covenants are customized to be relatively tight, that is, trigger values of ratios are close to those reported by the borrower at the time the loan is made. The very existence of customized covenants in loan contracts is evidence of a role for banks in setting the default boundary–if such covenants did not influence the states of the world in which bankruptcy occurs, why would such effort be expended on crafting them?

There is evidence as well that loan covenants do provide banks with significant control rights over

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[6] Estimates of mean bank debt share depend on how no-bank-debt firms are treated. We suspect that many firms with no bank debt at the time of bankruptcy filing had bank debt not long before, but the bank managed to force payoff of its debt, which in turn forced the firm to file for bankruptcy. Thus, it might be more appropriate to measure debt composition a short time before the bankruptcy date. We are investigating the matter as of this writing.

[7] For example, in Basel II, capital charges under the Internal Ratings-Based approach are proportional to expected loss-given-default (Basel Committee on Bank Supervision, 2005, ¶272).

[8] Some papers that use "covenants" to motivate model assumptions, such as Fan and Sundaresan (2000), focus on the borrower's promise to pay interest and principal on schedule. Legally this is a covenant, but it appears in all U.S. corporate debt contracts and is not viewed as a customizable contract-design feature by practitioners, as are other covenants.

weak borrowers. Loan covenant violations are likely to accompany an increase in the probability of borrower default. Dichev and Skinner report that borrower financial performance is much worse than average in quarters when a net worth covenant is violated. Beneish and Press (1993) and Chava and Roberts (2006) report that resolution of covenant violations commonly includes fees paid to the lenders, increases in interest rates, and incorporation of additional covenants into the credit agreement. Chen and Wei (1993) report that measures correlated with distance-to-default predict whether a covenant violation is resolved by a waiver or by the lender calling the loan.

The existing recovery literature is largely empirical and has related debt characteristics to recoveries (for example, Altman and Kishore, 1996) or has examined sources of systematic variation in recoveries (e.g., Frye, 2000b,a; Altman et al., 2005; Acharya et al., forthcoming).[9] Most papers focus on recoveries to individual debt instruments, which are influenced not only by the value of the firm's assets but by the seniority of the instrument. Among extant empirical studies, only Hamilton and Carty (1999) examine firm-level recovery as we do. They split their sample into firms with and without publicly issued debt and find that the latter have smaller firm-level recoveries on average, which is broadly consistent with our findings. They attribute the difference to larger deadweight costs of bankruptcy due to bargaining frictions associated with more complex capital structures, which is quite different from our explanation. We find that the number of debt instruments involved in a bankruptcy is not correlated with firm-level recovery, although the presence of contractually subordinated debt is associated with smaller recovery. Relative to Hamilton and Carty (1999), we offer a model embedding a different explanation, a larger sample, and more control and other variables in the empirical analysis.

Our model and some comparative statics are presented in Section 1. Section 2 describes data and summarizes some relevant institutional background and statistics. Regression results are presented in Section 3. Concluding remarks follow.

1 Model

We model loan contracts in which covenants permit the bank to foreclose on the borrower and force repayment through the bankruptcy process. In the simplest version of the model, we assume that the bank is effectively able to foreclose at will. We derive the bank's optimal choice of "foreclosure threshold." So long as the borrower's asset value remains above this threshold, the borrower is permitted to continue. Upon first-passage across this threshold, the bank forecloses. In the full version of the model, we recognize that covenant violation is needed for foreclosure. We introduce a contractually-specified "covenant threshold" that serves as an upper bound on the foreclosure threshold and also triggers payment of penalty fees by the borrower to the bank in exchange for forbearance.

Our model is an extension of a model in Black and Cox (1976) for perpetual corporate debt with continuous coupons. These assumptions remove time-dependence in the value of debt, and thereby simplify both the solution of the model and analysis of comparative statics. We also assume there is no restriction on asset sales. When asset sales are restricted, we are led to strategic bankruptcy by equityholders as in Leland (1994) and Leland and Toft (1996). The focus of our model is on the bank's role in initiating bankruptcy, so we therefore assume that assets may be sold freely for the purpose of paying debt coupons. To avoid diversion of assets to equityholders, we assume that

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[9] Schuermann (2005) reviews the literature on recovery rates.

debt contracts specify a fixed dividend rule. The borrower's capital structure is assumed fixed with no possibility of raising new equity or debt.

The baseline model is presented in Section 1.1. This model is identical to the model of Black and Cox (1976) except that the foreclosure boundary is chosen endogeneously by the bank. Comparative statics for the baseline model are explored in Section 1.2. The primary interest here is how the share of bank debt in total firm debt influences the distribution of recoveries at the estate level. In Section 1.3, we extend the model to allow for a stochastic shock to firm value upon bankruptcy. The full model is developed in Section 1.4. This allows for a firm-value boundary above which the bank cannot foreclose and below which the bank receives a waiver fee so long as the bank forbears.

1.1 Baseline model

The firm is financed by debt and equity. Without loss of generality, we assume that the total face value of debt is 1. This unit of debt is divided into a single loan with face value λ and a single class of bonds with face value $1-\lambda$. The bond is junior to the loan, and (for simplicity) only the loan has covenants that permit foreclosure. The loan receives continuous coupon c and the bond receives continuous coupon γ. Equity receives a continuous dividend of $\delta + \rho V_t$, where V_t is the firm's asset value at time t. We take these parameters as nonnegative constants, and assume $0 \leq \rho < r \leq c$. For notation convenience, let $\mathcal{C}$ be the rate of fixed cash outflows per unit time, i.e.,

$$\mathcal{C} = c\lambda + \gamma(1-\lambda) + \delta.$$

To keep the focus on credit risk, we assume riskfree interest rates are fixed at r. The asset value (net of coupons and dividends) follows a geometric Brownian motion with fixed variance σ^2. Under the risk-neutral measure, we have

$$dV_t = V_t((r-\rho)dt + \sigma\, dZ_t) - \mathcal{C}dt \tag{1}$$

In the event of bankruptcy at time t, coupon and dividend payments are frozen. Loan coupons continue to accrue during court proceedings. Settlement occurs after a fixed length of time τ, and the bank gets $\min\{\exp(c\tau)\lambda, V_{t+\tau}\}$.[10] The standard fixed-maturity, zero coupon Merton (1974) formula can be used to price the recovery value at bankruptcy, which is given by

$$B(V) = M(V, \exp(\tau(c-r))\lambda, \sqrt{\tau\sigma^2}) \tag{2}$$

where

$$M(V,D,s) \equiv V\Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right) + D\Phi\left(\frac{1}{s}\log(V/D) - \frac{s}{2}\right) \tag{3}$$

Applying Black and Cox (1976, eq. 18), the valuation equation for the loan satisfies the second-order ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F'' + ((r-\rho)V - \mathcal{C})F' - rF + c\lambda = 0, \tag{4}$$

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[10] If τ is too large, the bank will prefer immediate foreclosure over a riskfree perpetuity at rate c. To avoid this, we assume $c/r > \exp(\tau(c-r))$. This assumption does not bind on our empirical study, as the implied limit on time in bankruptcy would be on the order of 15 to 20 years.

for which the general solution is

$$F(V) = \frac{c\lambda}{r} - A_1 \cdot \psi(V; \alpha, \beta, \zeta) - A_2 \cdot \psi(V; 1-\beta, 1-\alpha, \zeta) \tag{5}$$

where A_1, A_2 are constants that are determined by boundary value conditions. The function ψ is given by

$$\psi(V; a, b, \zeta) = (\zeta V)^{-a} \cdot {}_1F_1\left(a, a+b, -1/(\zeta V)\right) \tag{6}$$

where ${}_1F_1$ is the confluent hypergeometric function. The constants α, β and ζ are given by

$$\begin{aligned} \alpha &= \sqrt{\left(\frac{1}{2} - \frac{r-\rho}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} - \left(\frac{1}{2} - \frac{r-\rho}{\sigma^2}\right) \geq \frac{2(r-\rho)}{\sigma^2} > 0 \\ \beta &= \alpha + 2 - \frac{2(r-\rho)}{\sigma^2} \geq 2 \\ \zeta &= \frac{\sigma^2}{2}\frac{1}{\mathcal{C}} \end{aligned}$$

The lower bounds on α and β are derived in Appendix A.

Let κ denote the foreclosure threshold. Given a choice of κ, the boundary conditions to equation (4) are $F(\kappa) = B(\kappa)$ and $F(\infty) = \lambda c/r$. Given bounds on α and β, it is straightforward to show that $\psi(V; 1-\beta, 1-\alpha, \zeta)$ increases without bound as $V \to \infty$. Therefore, to satisfy the boundary conditions on $F(V)$, we must have $A_2 = 0$. For $\kappa > 0$, the solution to A_1 is

$$A_1 = \left(\lambda\frac{c}{r} - B(\kappa)\right)\frac{1}{\psi(\kappa; \alpha, \beta, \zeta)}$$

which implies

$$F(V; \kappa) = \lambda\frac{c}{r} - \left(\lambda\frac{c}{r} - B(\kappa)\right) \cdot \frac{\psi(V; \alpha, \beta, \zeta)}{\psi(\kappa; \alpha, \beta, \zeta)} \tag{7}$$

where we have written $F(V; \kappa)$ to emphasize the dependence on κ. For the special case of $\kappa = 0$, see Black and Cox (1976, eq. 19).

We now allow the bank to choose κ. For simplicity, we assume in this section that the bank can foreclose at will. At the optimal κ, the bank is indifferent between foreclosing on the borrower and lowering its choice of κ. This will be satisifed if

$$\mathcal{F}(\kappa) \equiv \left.\frac{\partial F(V; \kappa)}{\partial \kappa}\right|_{V=\kappa} = 0 \tag{8}$$

For this model, we have

$$\mathcal{F}(\kappa) = B'(\kappa) - \left(\lambda\frac{c}{r} - B(\kappa)\right)\Xi(\kappa; \alpha, \beta, \zeta) \tag{9}$$

where

$$\Xi(\kappa; \alpha, \beta, \zeta) \equiv \frac{-\psi'(\kappa; \alpha, \beta, \zeta)}{\psi(\kappa; \alpha, \beta, \zeta)} \tag{10}$$

and will henceforth usually be abbreviated as $\Xi(\kappa)$. The first order condition is easily evaluated. The derivative of B is $M_1(y, \exp(\tau(c-r))\lambda, \sqrt{\tau\sigma^2})$, where M_i denotes the partial derivative of M with respect to its i^{th} parameter, which simplifies to

$$M_1(V, D, s) = \Phi\left(-\frac{1}{s}\log(V/D) - \frac{s}{2}\right).$$

The derivative of ψ also simplifies:

$$\psi'(y; a, b, \zeta) = -a\zeta(\zeta y)^{-(a+1)} \cdot {}_1F_1\left(a+1, a+b, -1/(\zeta y)\right) = -a\zeta\psi(y; a+1, b-1, \zeta)$$

where the last equality follows from FWC 07.20.20.0024.01.[11] The optimal κ^* does not have closed-form solution except in the limiting non-stochastic case of $\sigma = 0$ (discussed below). Numerical solution using standard routines for one-dimensional non-linear roots is entirely straightforward.

A finite positive solution to the first order condition always exists. So long as there are positive fixed cashflows to investors other than the bank, there cannot be a corner solution at $\kappa = 0$, because

Proposition 1 $\mathcal{F}(0) = 1 - \frac{c\lambda}{\mathcal{C}}$.

Observe that this expression is strictly positive if $\gamma(1-\lambda) > 0$ or $\delta > 0$. As κ increases towards infinity, $\mathcal{F}(\kappa)$ converges to zero from below, i.e.,

Proposition 2 $\lim_{\kappa\to\infty} \mathcal{F}(\kappa) \nearrow 0$

Proof of these two propositions is outlined in Appendix B. The Intermediate Value Theorem implies that there must be a finite positive κ such that $\mathcal{F}(\kappa) = 0$ and $\mathcal{F}'(\kappa) < 0$.

Our main variable of interest in this paper is total recovery as a share of total debt claims. Assuming that bonds do not accrue interest in default, the total claim of debtholders at emergence is $\lambda \exp(\tau c) + (1-\lambda)$.[12] Measured by post-default market price, recovery rates for all debtholders, the bank and the bondholders are given by

$$\begin{aligned} R &= \frac{M(\kappa^*, \exp(\tau(c-r))\lambda + \exp(-\tau r)(1-\lambda), \sqrt{\tau\sigma^2})}{\lambda \exp(\tau(c-r)) + (1-\lambda)\exp(-r\tau)} && \text{(11a)} \\ R_\ell &= \frac{M(\kappa^*, \exp(\tau(c-r))\lambda, \sqrt{\tau\sigma^2})}{\lambda \exp(\tau(c-r))} && \text{(11b)} \\ R_b &= R - (R_\ell - R)\frac{\lambda \exp(\tau c)}{1-\lambda} && \text{(11c)} \end{aligned}$$

respectively. Note that we express recovery rates as a share of the present discounted value of the legal claim. This definition of recovery is known as the Recovery of Face Value (RFV) convention (see Schönbucher, 2003, §6), and cleaves most closely to practice in bankruptcy court and accounting treatment.

In our data, recovery is measured at emergence. As this recovery is obtained under the physical measure, we must replace r in equation (2) with the drift μ under the physical measure and accrue to the date of emergence. Thus, we have

$$\begin{aligned} R^e &= \frac{M(\exp(\tau\mu)\kappa^*, \exp(\tau c)\lambda + (1-\lambda), \sqrt{\tau\sigma^2})}{\lambda \exp(\tau c) + (1-\lambda)} && \text{(12a)} \\ R^e_\ell &= \frac{M(\exp(\tau\mu)\kappa^*, \exp(\tau c)\lambda, \sqrt{\tau\sigma^2})}{\lambda \exp(\tau c)} && \text{(12b)} \\ R^e_b &= R^e - (R^e_\ell - R^e)\frac{\lambda \exp(\tau c)}{1-\lambda} && \text{(12c)} \end{aligned}$$

for all debtholders, the bank and the bondholders, respectively.

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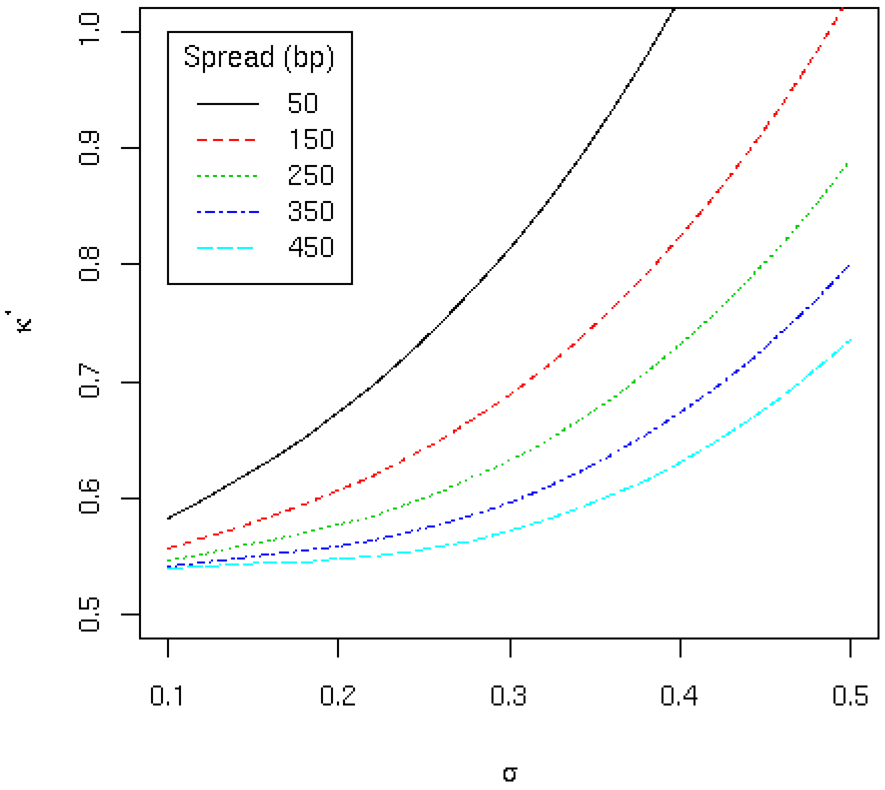
[11] FWC refers to the website `functions.wolfram.com`.

[12] It is typically observed in practice that the claim on a defaulted *loan* accrues interest at the contractual rate while in bankruptcy, whereas the claim on a defaulted *bond* does not.

1.2 Comparative statics for the baseline model

Comparative statics are by no means straightforward even in the most parsimonious versions of the model, and so we resort to numerical exercises. Consider first the influence of volatility σ and loan coupon c on the optimal foreclosure threshold. Rough intuition suggests that κ^* should increase with σ. All else equal, higher σ reduces the post-default market value of the senior debt, and so the bank should foreclose earlier to protect its recovery. Intuition suggests as well that κ^* should decrease with c. All else equal, higher c increases the value of the cash stream from the loan relative to the value obtained by foreclosing, and so the bank should be more willing to forbear. Figure 1 confirms these intuitions over the range of empirically plausible values of σ and c. See Table 1 for the range of σ values found in the Moody's KMV universe and a subsample of the largest North American firms.

Figure 1: Effect of coupon and volatility on foreclosure threshold



Spread is $c - r$, measured in basis points. Parameters: $r = 0.05$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

Table 1: Asset volatility in the Moody's KMV universe

	Full sample	Large NA*
Mean	0.33	0.18
Median	0.25	0.16
5th percentile	0.05	0.09
95th percentile	0.88	0.29

*: Large NA sample consists of North American nonfinancial firms with $1 billion or more in market value of assets and a public rating by Moody's or S&P.

The implications for expected recovery rates at emergence are depicted in Figure 2.[13] We see a monotonic decrease in the loan recovery rate (R^e_ℓ) with c, but a U-shaped relationship with σ. When σ is low and c is high, the "continuation value" of the cashflow stream is relatively high, so aggressive foreclosure is costly. Furthermore, much of the benefit to increasing the foreclosure threshold is not captured by the bank, but rather spills over to the bondholders. Thus, at low levels of σ, the bank allows its recovery to slip as σ increases. As σ reaches higher levels increase, the continuation value of the loan's coupon stream is smaller and smaller relative to the principal at risk. This consideration eventually dominates (despite spillovers), and then R^e_ℓ increases with σ.

The impact of spillovers on the recovery rate for bonds can be dramatic. When the loan spread is 150 basis points and debt is equally-split between loans and bonds, the expected bond recovery rate at $\sigma = 0.1$ is under 17%. At $\sigma = 0.5$, the expected recovery rate is over 69%.

We turn now to the influence of bank share λ on the optimal foreclosure threshold and recoveries at emergence. We expect the bank's choice of κ^* to increase with its share of total debt. This intuition is confirmed in Figure 3, which shows a roughly linear (slightly concave) relationship. Somewhat more complicated is the influence of λ on recoveries. As shown in the upper left panel of Figure 4, expected loan recoveries are decreasing in λ (though always quite high). As λ increases, spillovers to bondholders decrease, and so the bank has an incentive to forebear to capture additional coupon revenue. Correspondingly, recoveries for bondholders are typically increasing in λ. Most importantly, we find total recoveries increasing in λ in the bottom panel.

It should be emphasized that the comparative statics explored above need not hold in all regions of the parameter space. In particular, the predicted effects of σ and c may be reversed when σ is very low, c is high, and λ is high. When the volatility of asset returns is low, the behavior of the model is similar to that of the non-stochastic case, for which the optimal foreclosure threshold converges to the present discounted value of the legal claim at emergence, i.e.,

Proposition 3 $\lim_{\sigma\to 0} \kappa^* = \lambda \cdot \exp(\tau(c - r))$

Proof is given in Appendix C. This implies that κ^* is increasing in c at low σ values. We also find (numerically) that κ^* is decreasing in σ for low σ when c and λ are high. These relationships are shown in Figure 5. Observe that the "normal" relationships are restored as we move towards lower spreads and higher volatility. Comparative statics for total recoveries show a similar pattern of reversals for low σ and high c and λ.

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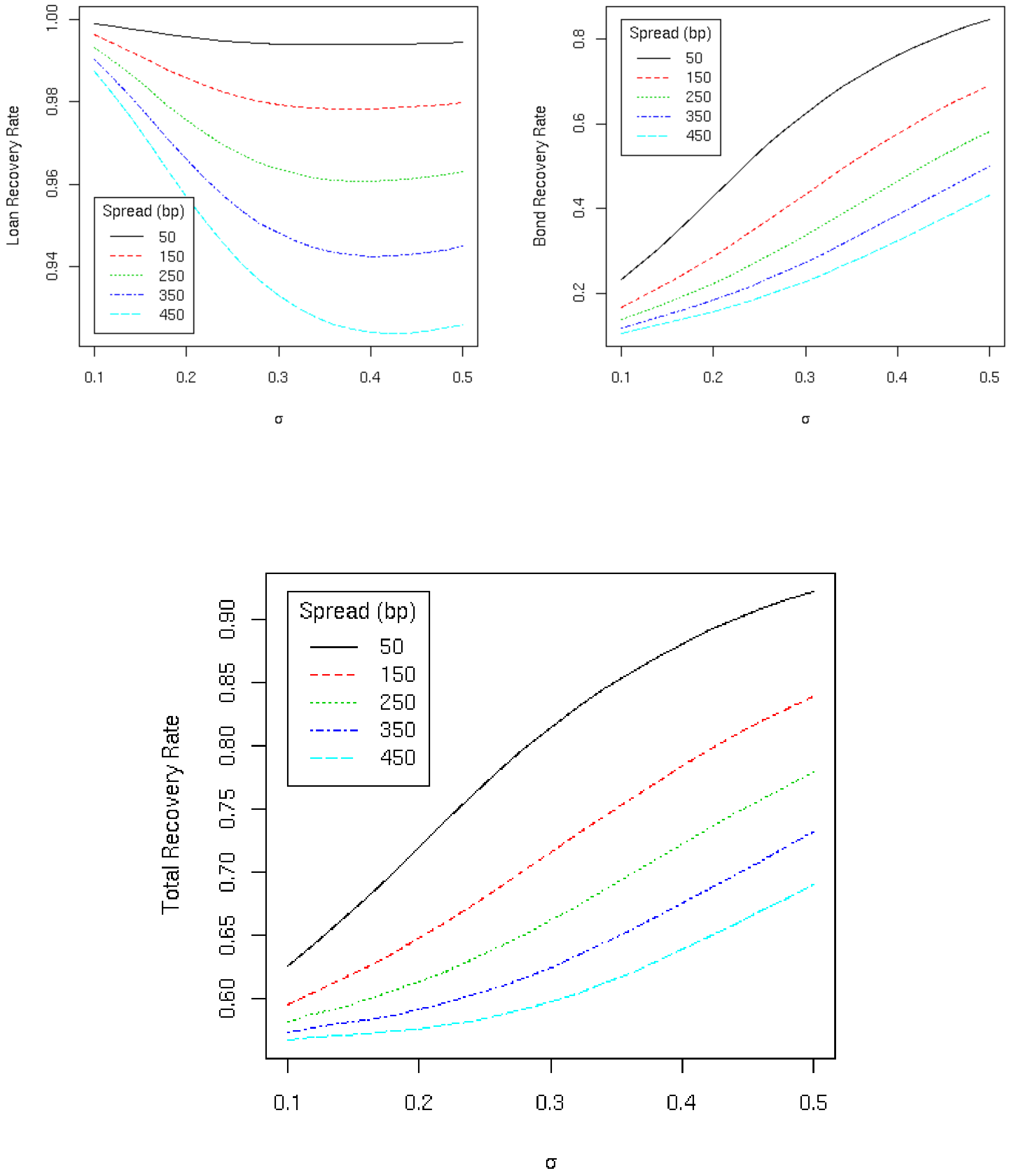
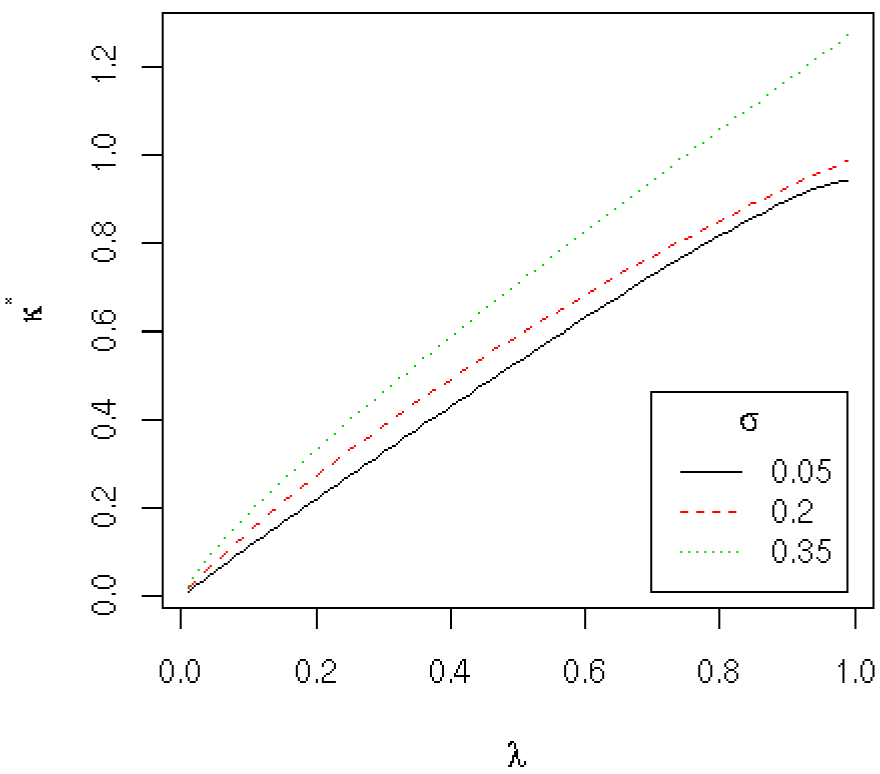
[13] Patterns in recovery rates measured by post-default market prices are qualitatively quite similar to those of the corresponding recovery rates at emergence.

Figure 2: Effect of coupon and volatility on recovery



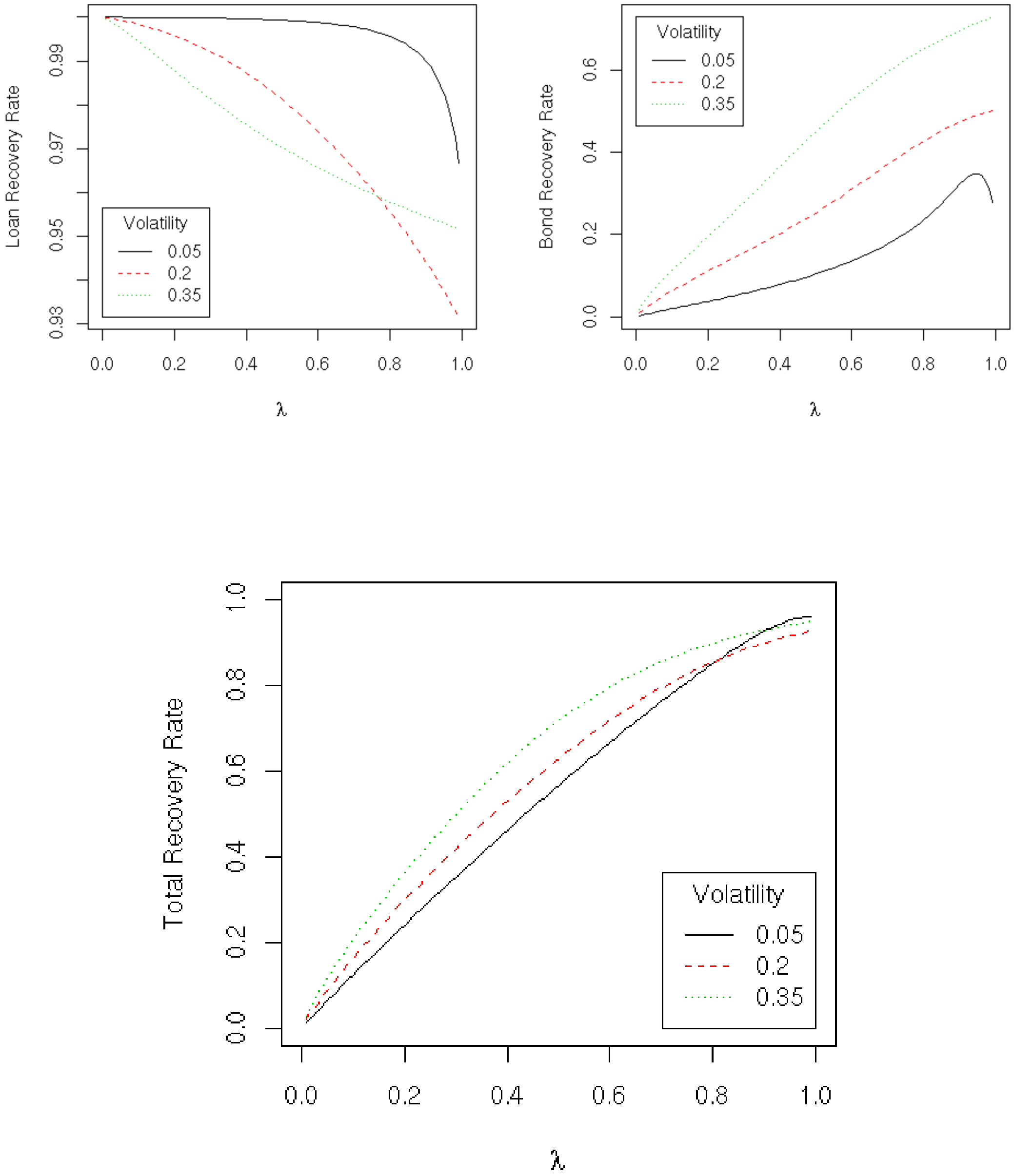
Recovery rates at emergence. Spread is $c - r$, measured in basis points.
Parameters: $r = 0.05$, $\mu = 0.1$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

Figure 3: Effect of debt composition on foreclosure threshold



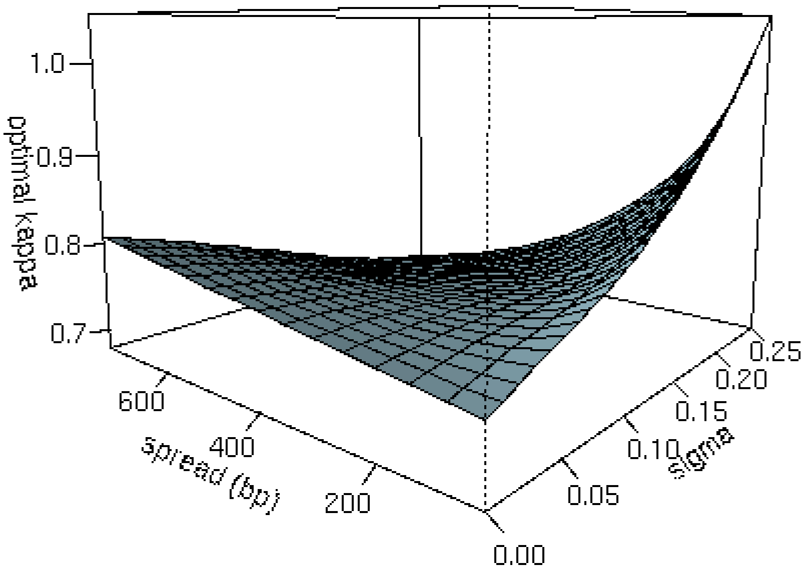
Parameters: $r = 0.05$, $c = 0.07$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

Figure 4: Effect of debt composition on recovery



Recovery rates at emergence. Parameters: $r = 0.05$, $\mu = 0.1$, $c = 0.07$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

Figure 5: Nonmonotonicity in effect on foreclosure threshold



Spread is $c - r$, measured in basis points. Parameters: $r = 0.05$, $\lambda = 0.75$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

1.3 Extension: Stochastic bankruptcy cost

The event of foreclosure can often impart a shock to asset value. Besides the legal costs associated with bankruptcy, franchise value might be sacrificed and certain contracts might be invalidated at foreclosure. In some cases, bankruptcy can help the firm escape a crippling labor contract or pension liability, so the shock need not be negative. We extend the model of the previous section to allow for a foreclosure shock.

We model bankruptcy costs as a multiplicative shock to asset value that is realized immediately following foreclosure by the bank. We assume that the shock is distributed logNormal(χ, η^2). The recovery value $B(V)$ is now

$$B(V) = M(\exp(\chi + \eta^2/2)V, \exp(\tau(c-r))\lambda, \sqrt{\tau\sigma^2 + \eta^2}) \tag{13}$$

It is only through altering the recovery value that χ and η affect the optimal choice of κ.

For this extended model, Proposition 1 generalizes to:

Proposition 1' $\mathcal{F}(0) = \exp(\chi + \eta^2/2) - \frac{c\lambda}{\mathcal{C}}$.

Proposition 2 holds without change. Therefore, the optimal κ^* is always finite but the corner solution $\kappa^* = 0$ may arise when $\chi < 0$. When χ is negative and large in magnitude, it becomes too costly (in terms of foregone interest revenue) to protect recoveries, and so κ^* goes to zero. More formally, in Appendix D we show

Proposition 4

$$\lim_{\chi \to -\infty} \kappa^* = 0.$$

For shocks with large positive mean, the asymptotic foreclosure threshold also goes to zero. As χ grows very large, the bank is able to obtain full recovery at a lower and lower foreclosure thresholds. In Appendix D we show

Proposition 5

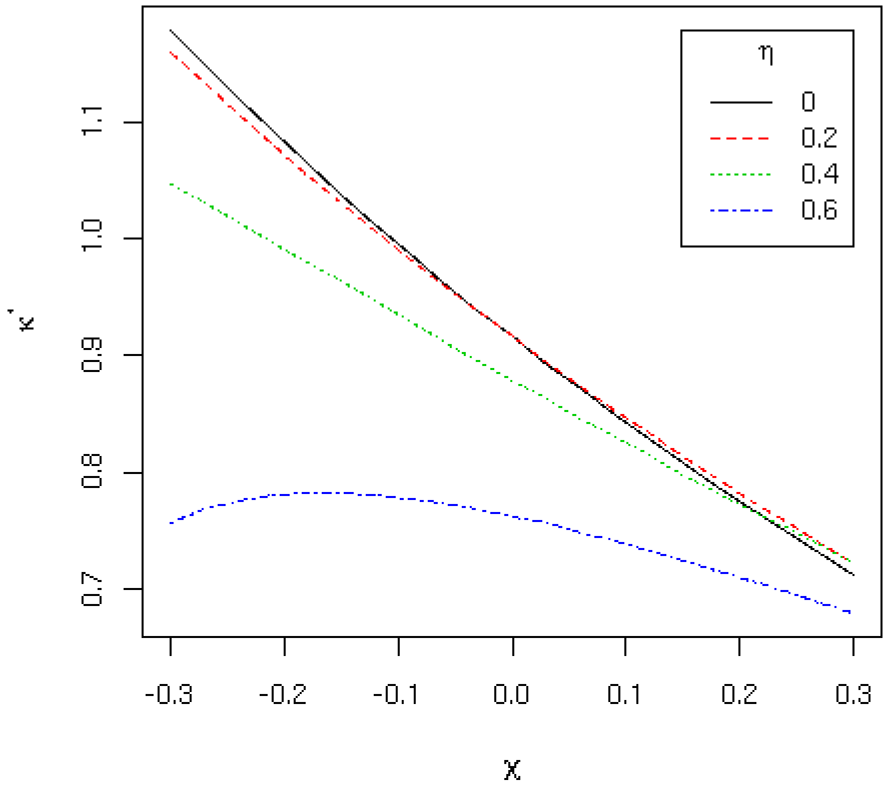
$$\lim_{\chi \to \infty} \kappa^* = 0.$$

Thus, the effect of χ on κ^* is hump-shaped.

The effects of both shock parameters on κ^* are explored numerically in Figure 6. At low values of volatility η, we see that κ^* declines with χ at moderate values of χ. The intuitive explanation is that an increase in the expected value of the shock implies that expected recovery on the loan can be kept constant with a lower foreclosure threshold. When χ is large and negative, however, there is a stronger incentive to forbear in order to delay the expected loss to asset value. The larger is η, the larger the spillover of recoveries to the bondholders, and so the greater the net benefit of forbearance. Thus, the "turning point" in the relationship between χ and κ^* shifts to the right as η increases.

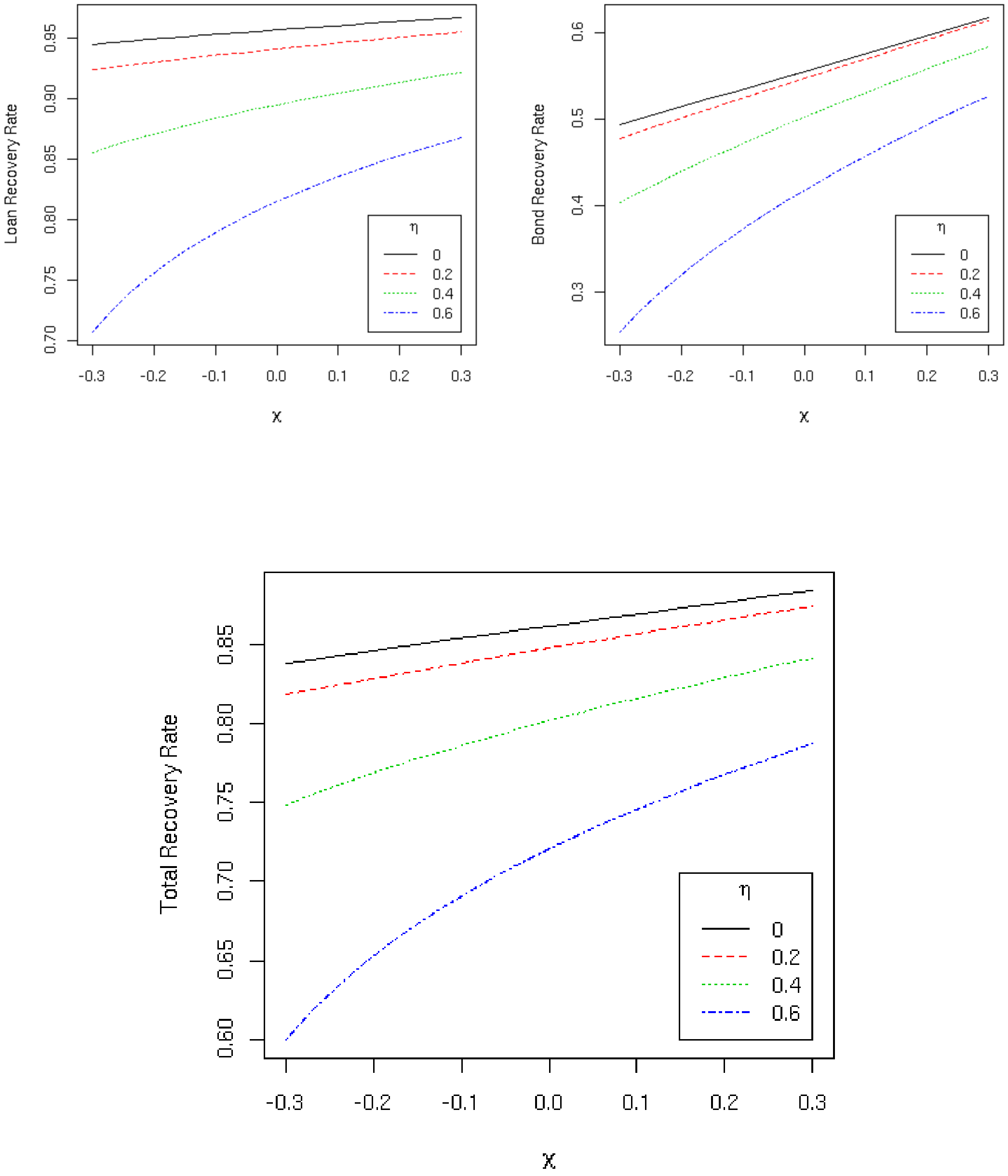
The effect on recoveries at emergence is seen in Figure 7. Recovery is strictly increasing in χ, and is steepest at low levels of χ and high η due to the increasing κ^* in that region. Bondholders share in the benefit of increasing χ, so total recovery increases with χ as well. Comparing the loan recovery panel of Figure 7 with that of Figure 2, we see that this extension to the baseline model allows for materially lower loan recovery rates.

Figure 6: Effect of bankruptcy cost on foreclosure threshold



Parameters: $r = 0.05$, $\sigma = 0.3$, $\lambda = 0.75$, $c = 0.07$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$

Figure 7: Effect of bankruptcy cost on recovery



Recovery rates at emergence. Spread is $c - r$, measured in basis points.
Parameters: $r = 0.05$, $\mu = 0.1$, $\sigma = 0.3$, $\lambda = 0.75$, $\gamma = 0.08$, $\delta = \rho = 0$, $\tau = 1$.

1.4 Covenant boundary and waiver fees

In this section, we introduce a finite covenant boundary ν. Whenever $V_t \leq \nu$, the borrower is considered to be in violation of covenants and the bank has an option to foreclose at will. Whenever $V > \nu$, covenants are satisfied and the bank cannot foreclose. Loan contracts may specify a fee to be paid to the bank when a covenant violation is waived, and in other cases something similar might be achieved by renegotiation at the time of covenant violation. For simplicity, we assume that a waiver penalty of w is added to the coupon rate c whenever $\kappa < V \leq \nu$. If the bank forecloses, the penalty rate no longer applies, so the legal claim λ accrues at the original coupon c during bankruptcy proceedings.

To maintain clarity in notation, we mark with a check any parameter that pertains under $V > \nu$, and mark with a hat any parameter that pertains under $V \leq \nu$. Thus, $\check{c}$ is the normal coupon rate, and $\hat{c} = \check{c} + w$ is the penalty coupon rate. (Think "smile" for the normal state and "frown" for the violation state.) We allow for the possibility that the contract requires lower dividend payments to equityholders when $V \leq \nu$, so similarly distinguish $\hat{\delta} \leq \check{\delta}$ and $\hat{\rho} \leq \check{\rho}$. All other fundamental parameters are fixed across the two regimes, but derived parameters such as α, β and ζ vary and so are marked with checks and hats. For fixed κ, the loan price is

$$F(V) = \begin{cases} \hat{F}(V) & \text{if } V \leq \nu, \\ \check{F}(V) & \text{if } V > \nu. \end{cases} \tag{14}$$

where $\hat{F}(V)$ and $\check{F}(V)$ are solutions to equation (4) under the two parameter regimes. It is important to recognize here that the $\hat{F}$ and $\check{F}$ functions differ from equation (7) because the relevant boundary conditions are not the same.

For the moment, take default boundary κ as fixed. The lower boundary value for $\hat{F}$ is $\hat{F}(\kappa) = \check{B}(\kappa)$, where B is marked with a check rather than a hat because the accrual rate in bankruptcy is $\check{c}$. The upper boundary value for $\check{F}$ is $\check{F}(\infty) = \lambda\check{c}/r$. Two additional boundary restrictions are required to provide the upper boundary of $\hat{F}(V)$ and lower boundary of $\check{F}(V)$ where they join at covenant threshold $V = \nu$. These are given by the smooth pasting conditions, $\hat{F}(\nu) = \check{F}(\nu)$ and $\hat{F}'(\nu) = \check{F}'(\nu)$. As V is driven by a diffusion, passage across the threshold at ν is an accessible event, which implies that F must be continuous at $V = \nu$. Dixit (1993, §3.8) provides a no-arbitrage argument for continuity in the first derivatives.

Let f_ν be the value of the loan at ν. Solution to $\check{F}(V)$ proceeds exactly as for the baseline model, except that the lower boundary is $\check{F}(\nu) = f_\nu$. This implies

$$\check{A}_1 = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\psi(\nu; \check{\alpha}, \check{\beta}, \check{\zeta})} = \left(\lambda\frac{\check{c}}{r} - f_\nu\right)\frac{1}{\check{\psi}_1(\nu)}$$

where for convenience we define

$$\check{\psi}_1(y) = \psi(y; \check{\alpha}, \check{\beta}, \check{\zeta}).$$

We similarly define for the violation state

$$\begin{aligned} \hat{\psi}_1(y) &= \psi(y; \hat{\alpha}, \hat{\beta}, \hat{\zeta}) \\ \hat{\psi}_2(y) &= \psi(y; 1 - \hat{\beta}, 1 - \hat{\alpha}, \hat{\zeta}) \end{aligned}$$

The boundary conditions for $\hat{F}(V)$ lead to simultaneous linear equations

$$\begin{aligned}\hat{A}_1 \cdot \hat{\psi}_1(\kappa) + \hat{A}_2 \cdot \hat{\psi}_2(\kappa) &= \lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\\ \hat{A}_1 \cdot \hat{\psi}_1(\nu) + \hat{A}_2 \cdot \hat{\psi}_2(\nu) &= \lambda\frac{\hat{c}}{r} - f_\nu.\end{aligned}$$

which has solution

$$\begin{aligned}\hat{A}_1 &= \frac{1}{\hat{\Delta}}\left(\hat{\psi}_2(\nu)\left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right) - \hat{\psi}_2(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right)\\ \hat{A}_2 &= \frac{1}{\hat{\Delta}}\left(-\hat{\psi}_1(\nu)\left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right) + \hat{\psi}_1(\kappa)\left(\lambda\frac{\hat{c}}{r} - f_\nu\right)\right)\end{aligned}$$

where $\hat{\Delta}$ is the determinant

$$\hat{\Delta} \equiv \hat{\psi}_1(\kappa)\hat{\psi}_2(\nu) - \hat{\psi}_1(\nu)\hat{\psi}_2(\kappa)$$

Finally, we impose $\hat{F}'(\nu) = \check{F}'(\nu)$ to pin down f_ν as

$$f_\nu = \frac{\lambda\frac{\check{c}}{r}\check{\Xi}(\nu) + \lambda\frac{\hat{c}}{r}\hat{\Xi}(\kappa,\nu) - \left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right)\hat{\Xi}(\nu,\nu)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{15}$$

where the two-variable Ξ function extends the one-variable function as

$$\hat{\Xi}(a,b) \equiv \frac{1}{\hat{\Delta}}\left(\hat{\psi}_2(a)\hat{\psi}_1'(b) - \hat{\psi}_1(a)\hat{\psi}_2'(b)\right)$$

Two examples are shown in Figure 8. The solid curve is $F(V)$. The points $(\kappa^*, \check{B}(\kappa^*))$ and $(\nu, F(\nu))$ are marked with circles. Observe that F need not be monotonic in V. If the waiver fee is high enough, then the loan is most valuable when covenants are in violation while V is still not too close to the default boundary. In this case, F peaks between κ and ν, and $\check{F}$ converges to its asymptotic value from above rather than from below.

The dashed curves are lower and upper bounds derived from the baseline model. The value of the loan must be no less than the value of a loan in which parameters are held fixed at $c = \check{c}$, $\delta = \check{\delta}$, and $\rho = \check{\rho}$, and where the initial condition is a value of $\check{B}(\kappa)$ at $V = \kappa$. Similarly, $F(V)$ can be no greater than the value of a loan in which parameters are held fixed at $c = \hat{c}$, $\delta = \hat{\delta}$, and $\rho = \hat{\rho}$, for the same initial condition. Therefore,

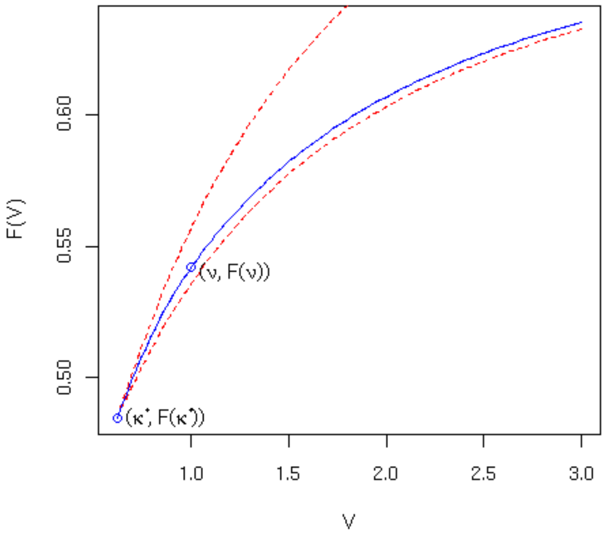
$$F^{lower}(V) \leq F(V) \leq F^{upper}(V)$$

where

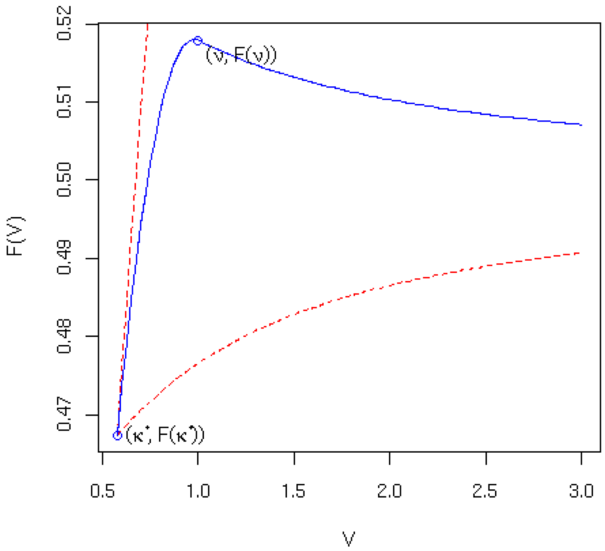
$$\begin{aligned}F^{lower}(V) &= \lambda\frac{\check{c}}{r} - \left(\lambda\frac{\check{c}}{r} - \check{B}(\kappa)\right)\cdot\frac{\check{\psi}_1(V)}{\check{\psi}_1(\kappa)}\\ F^{upper}(V) &= \lambda\frac{\hat{c}}{r} - \left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right)\cdot\frac{\hat{\psi}_1(V)}{\hat{\psi}_1(\kappa)}\end{aligned}$$

Observe that F clings to its upper bound at very low V (where the violation state parameters are the dominant influence), and converges to its lower bound as V tends to infinity (where the normal state parameters dominate).

Figure 8: Loan value and bounding functions



(a) Small waiver fee ($\check{c} = 0.07, \hat{c} = 0.08$)



(b) Large waiver fee ($\check{c} = 0.05, \hat{c} = 0.10$)

Solid blue line is $F(V)$, dashed red lines are upper and lower bounds from baseline model.
Parameters: $\nu = 1$, $r = 0.05$, $\mu = 0.1$, $\sigma = 0.3$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$.

To complete the solution of our model, we solve for the optimal κ^* using the first order condition (8), and find

$$\mathcal{F}(\kappa) = \check{B}'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right)\hat{\Xi}(\nu,\kappa) + \left(\lambda\frac{\hat{c}}{r} - f_\nu(\kappa)\right)\hat{\Xi}(\kappa,\kappa) \tag{16}$$

where we have written $f_\nu(\kappa)$ to emphasize the dependence of f_ν on κ. As κ^* is constrained to the interval $[0,\nu]$, corner solutions must be checked. Otherwise, numerical solution for κ^* is straightforward.

We can rearrange equation (16) to emphasize its relationship to the FOC for the baseline model. We substitute in equation (15) to arrive at

$$\mathcal{F}(\kappa) = \check{B}'(\kappa) - \left(\lambda\frac{\hat{c}}{r} - \check{B}(\kappa)\right)\left(\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)}\right) + \lambda\frac{w}{r}\frac{\check{\Xi}(\nu)\hat{\Xi}(\kappa,\kappa)}{\check{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} \tag{17}$$

When the penalty state does not alter contractual parameters (i.e., $w = 0$, $\hat{\delta} = \check{\delta}$, and $\hat{\rho} = \check{\rho}$), then $\check{\Xi}(\nu) = \hat{\Xi}(\nu)$. Some tedious algebra can verify that

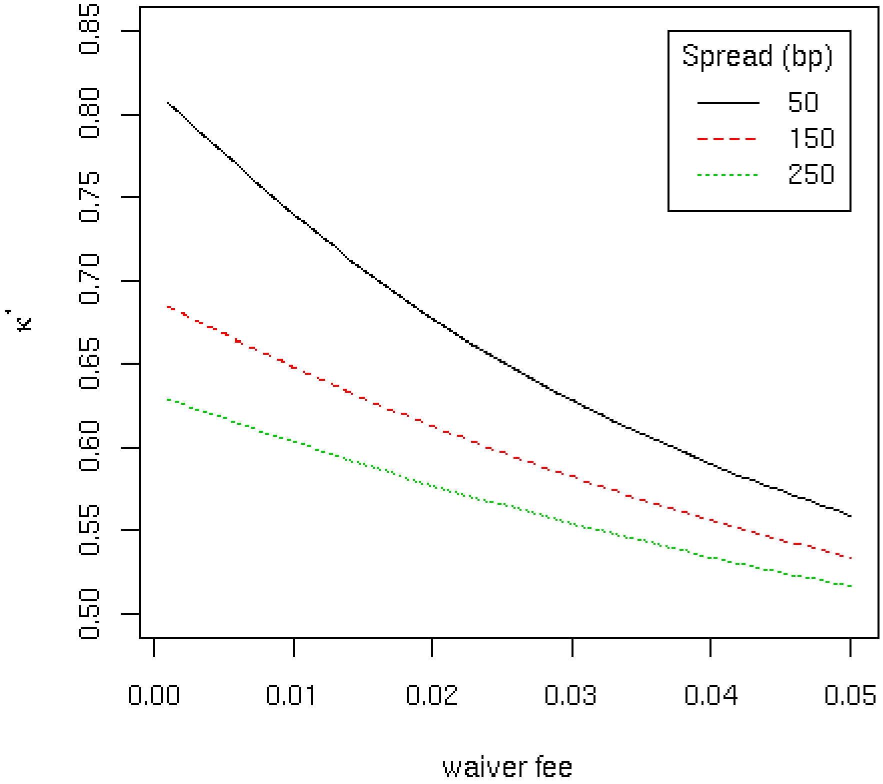
$$\hat{\Xi}(\nu,\kappa) - \frac{\hat{\Xi}(\nu,\nu)\hat{\Xi}(\kappa,\kappa)}{\hat{\Xi}(\nu) + \hat{\Xi}(\kappa,\nu)} = \hat{\Xi}(\kappa)$$

in which case equation (17) reduces to equation (9).

Figure 9 explores the dependence of the optimal foreclosure boundary on the waiver fee $w = \hat{c} - \check{c}$ and the normal state spread $\check{c} - r$. We find that κ^* decreases with w over this range of parameters. As the waiver fee is received by the bank only until foreclosure (or a return to the "normal" state $V > \nu$), an increase in the waiver fee increases the bank's incentive to forbear. It should be noted, however, that the effect of w on κ^* is reversed when σ is very low and λ is high. This is similar to the pathological case discussed in Section 1.2, and the graphical depiction is quite similar to that of Figure 5 (with w in place of $c - r$ on one axis). We would not expect to see such cases arise in practice, as the borrower would get no lenience benefits in exchange for including waiver fees in a loan contract when these unusual parameter values pertain.

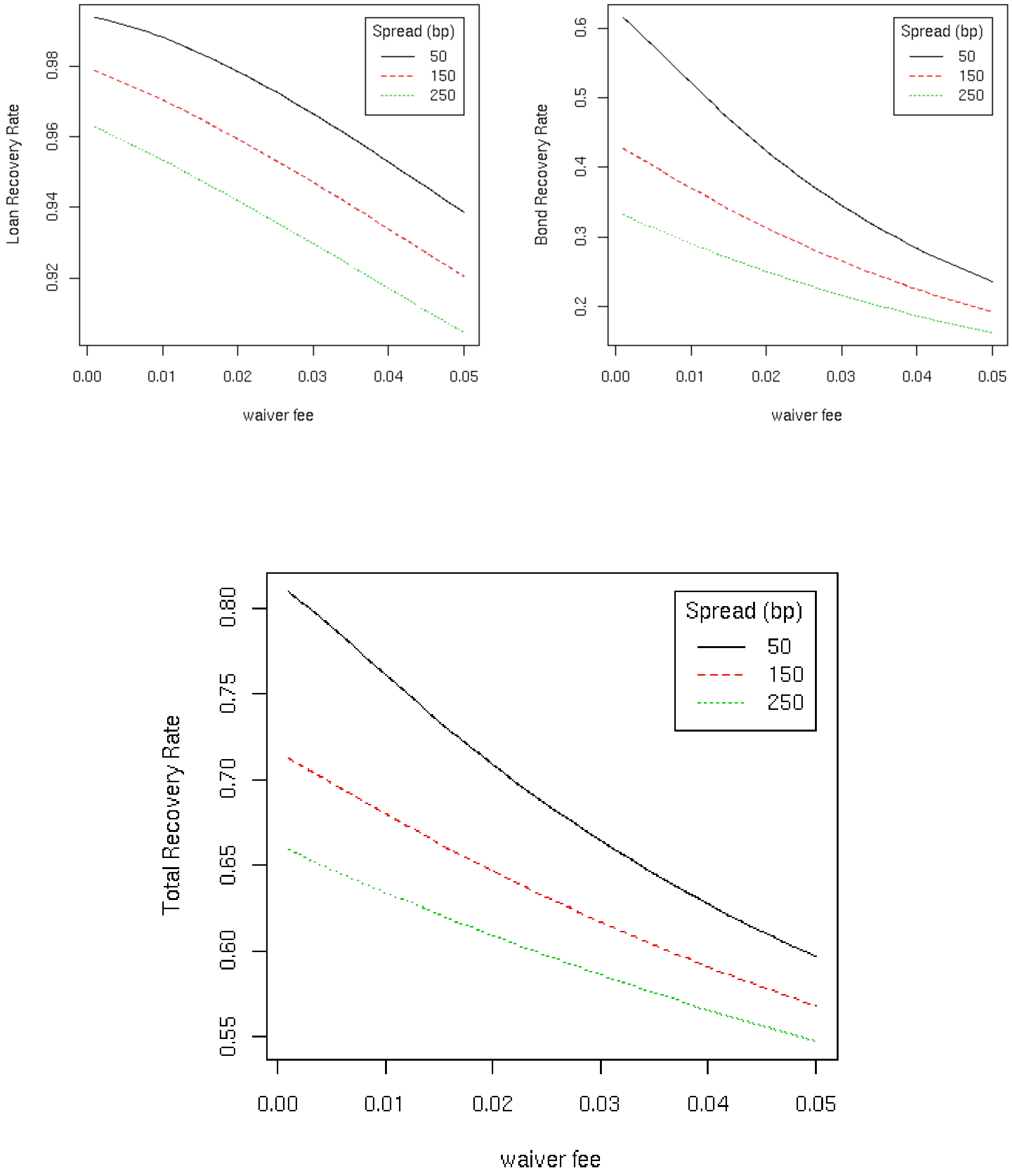
Finally, in Figure 10, we explore the effect on recovery. As we would expect, the higher is w, the lower is the recovery rate for both debt classes. The effect shown on the loan's loss given default can be quite large on a relative basis, even if not terribly large on an absolute basis.

Figure 9: Effect of waiver fee on foreclosure threshold



The "spread" in the legend is $\check{c} - r$, expressed in basis points. Parameters: $r = 0.05$, $\sigma = 0.3$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$, $\nu = 1$.

Figure 10: Effect of waiver fee on recovery



Recovery rates at emergence. The "spread" in the legend is $\check{c} - r$, expressed in basis points. Parameters: $r = 0.05$, $\sigma = 0.3$, $\lambda = 0.5$, $\gamma = 0.08$, $\delta = \rho = 0$, $\chi = \eta = 0$, $\tau = 1$, $\nu = 1$.

2 Empirical Strategy, Data and Measures

For realistic parameter values, the model implies that the bankruptcy threshold value of assets and firm-level recovery rates are increasing in the share of bank debt in total debt, decreasing in the coupon interest rate on bank debt, and increasing in the volatility of the firm's asset value. We provide evidence about all three relationships, but we focus most attention on bank debt share. We examine parameter estimates from variants of a regression of the form

$$R = a_0 + a_1\lambda + a_n\text{Controls} + \epsilon$$

where R is the ultimate recovery rate on all debt of the firm taken together ("firm-level" recovery), λ is the share of bank debt in total debt, and Controls is a vector of control variables and other variables of interest such as loan coupon interest rate. The null hypothesis of most interest usually is $a_1 = 0$, which is in the spirit of existing structural models of default: If the bankruptcy threshold level of assets is exogenous or is chosen by equityholders to maximize the value of their claims, we would expect no relationship between bank debt share and recovery rate. In contrast, if our model is at least partly realistic, we expect $a_1 > 0$. If our model explains all bankruptcy decisions and recoveries, we would expect $a_1 = 1$.

The model also implies that the label applied to a debt instrument ("bank loan," "bond," etc.) is immaterial—what matters to the default boundary is the share of debt with financial covenants. We present evidence consistent with the model's implication that instrument-level recoveries to such debt should be bunched near 100 percent.

As noted previously, we assume that bank debt share is exogenous around the time of bankruptcy. However, in earlier periods, bank debt share seems likely to be an important choice variable for the firm because it influences the states of the world in which bankruptcy occurs. Thus, bank debt share might reasonably be interpreted not as important in its own right, but as a summary representation of the characteristics of the firm at the time it made debt structure decisions. Such an interpretation is consistent with our model and with a view that future research on the determinants of debt structure is likely to be fruitful. Such research may be aided by evidence about the sensitivity of the relationship between recovery rates and bank debt share to other firm and bankruptcy characteristics. Moreover, prior studies have suggested a number of possible predictors of recovery rates. Our measurement strategies differ from those of prior literature, permitting us to provide additional perspective on extant results. For both reasons, we include in empirical specifications a fairly large number of variables that, strictly speaking, are outside our model. We examine robustness of results to inclusion or exclusion of such variables.

2.1 Data

The primary data are from Standard and Poor's LossStats database, which tracks ultimate recovery for each debt instrument outstanding at default for each firm in the database. For example, suppose a firm defaulted and declared bankruptcy on 1 June 1998, that it emerged from bankruptcy exactly one year later, and that the firm's debt on the bankruptcy date consisted of a single bank loan and a single bond issue. Suppose that at emergence, the holders of the loan and bond received a mixture of cash and debt obligations of the emerging firm in compensation for their claims. The database records:

- The market value of such compensation at the time of emergence, separately for each pre-bankruptcy debt instrument.

- The identity and some characteristics of the firm and of its experience in bankruptcy, such as the court which handled the case.
- Characteristics of each debt instrument, such as original-issue amount, amount outstanding at default, coupon interest rate, whether the instrument is subordinated or secured, and the priority class to which the instrument is assigned by the bankruptcy court.

The database has information for the complete debt structure of each firm. It does not have information about equity or preferred stock claims and their recoveries, nor about accounts-payable or other liabilities (discussed further below).

Although the database includes defaults and distressed restructurings that did not involve bankruptcy, in this paper we use only data for bankruptcies. Expected outcomes of bankruptcy are likely to influence bargaining and outcomes in non-bankruptcy situations. We wish to understand bankruptcies before attempting to analyze other situations.

S&P obtains LossStats data primarily by analyzing SEC filings and bankruptcy court documents. Values of compensation received at emergence are gathered from a variety of sources. S&P attempts to capture all defaults by firms with more than $50 million of debt outstanding on the date of default, but inclusion in the database is subject to availability of information. The data begin with defaults in 1987, but coverage is more complete in recent years because of the creation of electronic recordkeeping systems at U.S. courts and the SEC. For defaults in the early years of the sample, records may be unobtainable. Moreover, in populating the database in earlier years, S&P focused on obtaining data for defaults involving relatively large amounts of debt. Smaller defaults are more likely to appear if the firm filed its bankruptcy petition with one of the major bankruptcy courts, such as those in New York or Delaware. Almost all the firms are U.S. firms, and most had publicly issued debt or equity outstanding at default.

The release of the database that we use ends in late 2006, but bankruptcies appear in the database only after they are resolved (because only then can ultimate recovery be determined). This raises the possibility of bias: Firms that take a long time to emerge from bankruptcy are more likely to be omitted from our analysis. A common supposition is that the debt of such firms tends to have smaller recovery. Rather than complicating estimation by including corrections for censoring, we demonstrate robustness of results to dropping from the sample those firms with bankruptcy dates in later years. Mean time in bankruptcy is 14 months, regardless of how many trailing bankruptcies are dropped, and the longest bankruptcy took a bit less than six years to resolve. In obtaining most results, we drop the ten bankruptcies dated in 2005 and 2006 that had appeared on LossStats as of the release date, but we also drop all bankruptcies after 1997 and find all results to be robust.

We matched LossStats observations to entries in Compustat (to obtain financial statement variables and ratings), Moody's KMV CreditMonitor (to obtain asset volatility measures), and to Loan Pricing Corporation's Dealscan database (to obtain information about loan covenants and other loan characteristics). In creating and cleaning variables, we used a variety of sources to learn about details of bankruptcies or debt structure, especially SEC filings and Moody's Bond Record. The date of Compustat balance-sheet and income-statement variables is the latest fiscal year-end date that precedes the bankruptcy date. Where the available fiscal year-end data is more than 1.1 years before the bankruptcy date, we eliminate the firm from the Compustat-matched subsample. Dates of asset volatility measures are as described below. Dealscan provides loan characteristics at the time the loan was originated.

Table 2 presents mean, median, minimum and maximum values for many of the variables that appear in the analysis below, for the full sample (645 usable observations) and for the Compustat-matched subsample (373 observations). Most variables are described in more detail below. Average firm-level recovery is not far from 50 percent, but individual-firm recoveries range widely, with the best outcome being a gain of 64 percent of the amount of the claim and the worst being a total loss. (As discussed further below, gains can occur because of fluctuations in the market value of the firm between the date that cash and liabilities of the firm are allocated to claimants and the date of emergence. Less than 5 percent of observations have firm-level recovery rates greater than 100 percent.) A rough measure of firm size is the total amount of debt claims, which varies widely, but on average sample firms are fairly large, with median total debt claims near \$300 million and median total assets around \$450 million. The median firm had approximately a zero net worth at the fiscal year-end before filing (BookLeverageRatio$\approx$ 1, computed as liabilities/assets) and had four debt instruments outstanding. Mean time in bankruptcy was 1.2 years (14 months) and median time was one year.

On average, bank debt represents about one-third of all firm debt, and ranges from none to all. 23 percent of sample firms had no bank debt as of the filing date. Although our model admits such firms, they may be unusual, so we include dummy variables in regressions for such firms and also for firms with all bank debt.

About 60 percent of Compustat subsample firms had an S&P rating at the fiscal year-end before filing, and only about one-sixth of such firms were rated BB or better (very few were rated investment-grade).

2.2 Why firm-level and ultimate recovery?

We examine firm-level recovery partly because it is a proxy for the value of the firm's assets at emergence from bankruptcy and partly because it is a natural measure for examining how characteristics of the firm and the economic environment affect recovery rates. Recoveries on individual debt instruments are non-linear functions of firm-level recovery. Using instrument-level recoveries to analyze firm-level influences would require adequate controls for the non-linear payoff properties of individual instruments, which have not yet been developed.

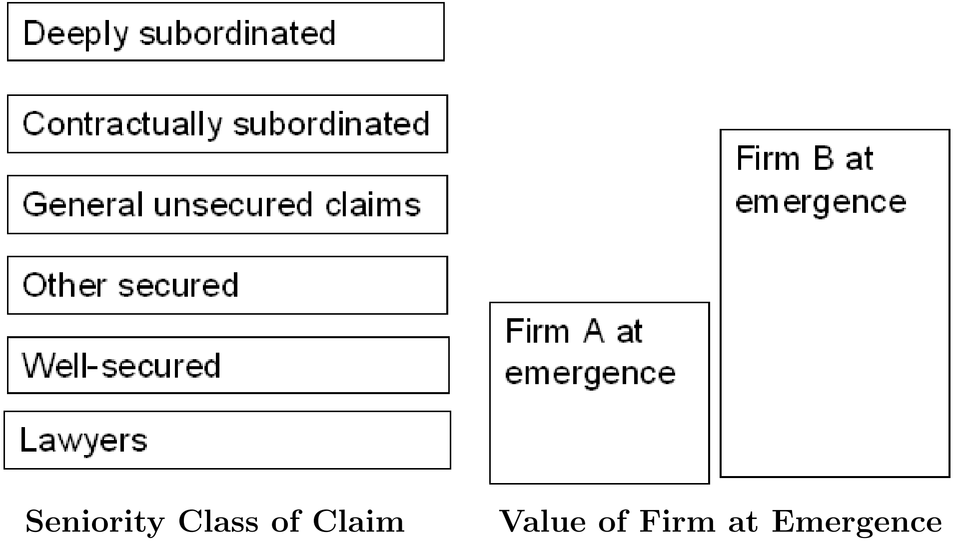
To see this, note that individual debt instruments of a bankrupt firm, like any corporate debt, are contingent claims on the value of the firm at emergence from bankruptcy. However, payoff properties of the claims are specified by bankruptcy law rather than the pre-bankruptcy contractual terms of the instrument. The court uses the rules of absolute priority to rank-order debt instruments into classes. Assets are allocated to each class in order of priority until assets are exhausted. For example, in Figure 11, "well secured" claims on a bankruptcy involving Firm A would receive a complete recovery, but "other secured" claims would be only partly in the money. In contrast, if firm value at emergence corresponded to that shown for Firm B, only "deeply subordinated" claims would be totally out of the money. Thus, a debt instrument of a bankrupt firm is similar to a collar option written on the value of the firm: It has two "strike values" and can be out of the money, receive part of its claim, or all of its claim. To learn the influence of firm-level factors on firm-level recovery by analyzing individual-instrument recoveries, it would be necessary to control for the non-linearities in the payoff functions. Unfortunately, the "strike values" on defaulted debt (their true seniority) are not adequately captured by labels like "senior" or "subordinated" which have been used as control variables in literature to date. A subject for future research is construction of satisfactory empirical models of individual instrument recovery.

Table 2. Sample summary statistics

Data are for all bankruptcies in the LossStat database with bankruptcy filing dates before 2005. Debt claim amounts and assets-of-firm are in millions of dollars. Compustat data are as of the most recent fiscal year-end date preceding the bankruptcy filing date, except that data for year-end dates more than 1.1 years prior to the filing date are eliminated. Number of debt instruments is the number of separate debt obligations of the firm at the time bankruptcy is filed, whereas number of priority classes is the number of different class labels assigned by the court that are shown for debt instruments in LossStats.

	Full Sample				Compustat Subsample			
Variable	Mean	Median	Min	Max	Mean	Median	Min	Max
NumberOfBankruptcies			645				373	
RecoveryRateInPct	50	48	0	164	51	50	0	138
AmountOfClaims$Mil	681	291	12	32869	791	358	12	32869
TotalBookAssets$Mil					1833	456	2	103914
LoanIntRateSpreadPct	2.71	2.68	0.03	10.50	2.64	2.63	0.03	6.75
Debt Structure								
BankDebtShareAsFrac	0.33	0.28	0.00	1.00	0.32	0.28	0.00	1.00
NoBankDebtDummy	0.23	0.00	0.00	1.00	0.23	0.00	0.00	1.00
AllBankDebtDummy	0.06	0.00	0.00	1.00	0.05	0.00	0.00	1.00
SecuredDebtShareInPct	0.44	0.41	0.00	1.00	0.42	0.39	0.00	1.00
AllSubDebtDummy	0.05	0.00	0.00	1.00	0.06	0.00	0.00	1.00
NoSubDebtDummy	0.38	0.00	0.00	1.00	0.35	0.00	0.00	1.00
SubDebtShareInPct	0.31	0.24	0.00	1.00	0.32	0.25	0.00	1.00
Frictions								
YearsInBankruptcy	1.23	1.01	0.05	5.79	1.23	0.94	0.09	5.79
YearsFromPlanToEmerge	0.48	0.33	0.01	3.82	0.46	0.33	0.04	3.82
YearsInDefaultPreFile	0.31	0.08	0.00	3.15	0.27	0.06	0.00	2.83
PrePackagedBRDummy	0.26	0.00	0.00	1.00	0.27	0.00	0.00	1.00
NumberDebtInstruments	4.46	4.00	1.00	55.00	4.92	4.00	1.00	55.00
NumberPriorityClasses	2.20	2.00	1.00	16.00	2.30	2.00	1.00	16.00
Bad Actors								
FraudDummy	0.05	0.00	0.00	1.00	0.06	0.00	0.00	1.00
FiledAgainWithin5YrsDummy	0.04	0.00	0.00	1.00	0.06	0.00	0.00	1.00
Presiding Court Dummies								
Court_CA	0.06	0.00	0.00	1.00	0.07	0.00	0.00	1.00
Court_NY	0.19	0.00	0.00	1.00	0.20	0.00	0.00	1.00
Court_DE	0.36	0.00	0.00	1.00	0.36	0.00	0.00	1.00
Court_IL	0.03	0.00	0.00	1.00	0.02	0.00	0.00	1.00
Court_TX	0.08	0.00	0.00	1.00	0.08	0.00	0.00	1.00
Compustat Variables								
NonIntangAssetToTotal					0.87	0.97	0.20	1.00
BookLeverageRatio					1.20	1.00	0.25	5.50
OpIncomeToAssets					0.03	0.05	-0.25	0.25
AcctsPayableToTotLiabs					0.09	0.07	0.00	0.30
PPE_ToAssets					0.39	0.37	0.00	0.96
RatedBBOrBetter					0.09	0.00	0.00	1.00
RatedSingleB					0.30	0.00	0.00	1.00
RatedCCC					0.15	0.00	0.00	1.00
RatedWorseThanCCC					0.09	0.00	0.00	1.00
Most Industies Not Shown								
BubbleFirmDummy	0.08	0.00	0.00	1.00	0.08	0.00	0.00	1.00
UtilityDummy	0.02	0.00	0.00	1.00	0.02	0.00	0.00	1.00

Figure 11: Debt instruments as collar options



In the case of Firm A, the value of the firm at emergence is below the lower strike for the General Unsecured Claims class, so this class (and those junior to it) finish out-of-the-money and recover nothing. In the case of Firm B, asset value at emergence is above the upper strike for this class, so General Unsecured claimants enjoy a full recovery, and further increases in firm value would not improve their payout.

Figure 12 illustrates the different properties of instrument- and firm-level recovery. The hump-shaped line plots the kernel-smoothed empirical distribution of firm-level recovery at emergence. The U-shaped line plots the empirical distribution of recovery for individual debt instruments. Unsurprisingly given the nature of the instruments, their distribution is bimodal with peaks at or near out-of-the-money (zero recovery) and full recovery. The firm-level data in the figure were constructed from the instrument-level data, so differences in sample selection do not drive shapes of the curves.[14]

We examine ultimate recovery, defined as recovery received at emergence from bankruptcy, because it corresponds most closely to the payoff on debt of a bankrupt firm and because of data limitations. Much of the empirical literature has examined recovery-at-default, proxied by the secondary market trading price of defaulted debt instruments approximately 30 days after default. One problem is that price data are not available for many instruments, making it impossible to construct firm-level measures of recovery-at-default for most bankruptcies. Another problem is that the trading price soon after default embeds all of (a) market estimates of the present discounted value of the firm at emergence (ultimate firm-level recovery); (b) estimates of the seniority class to which the instrument will be assigned by the court; and (c) risk premiums in discount rates applied to the payoffs. The latter factors are very interesting subjects for future research, but since we are most interested in the value of the firm, using ultimate recovery is preferable.

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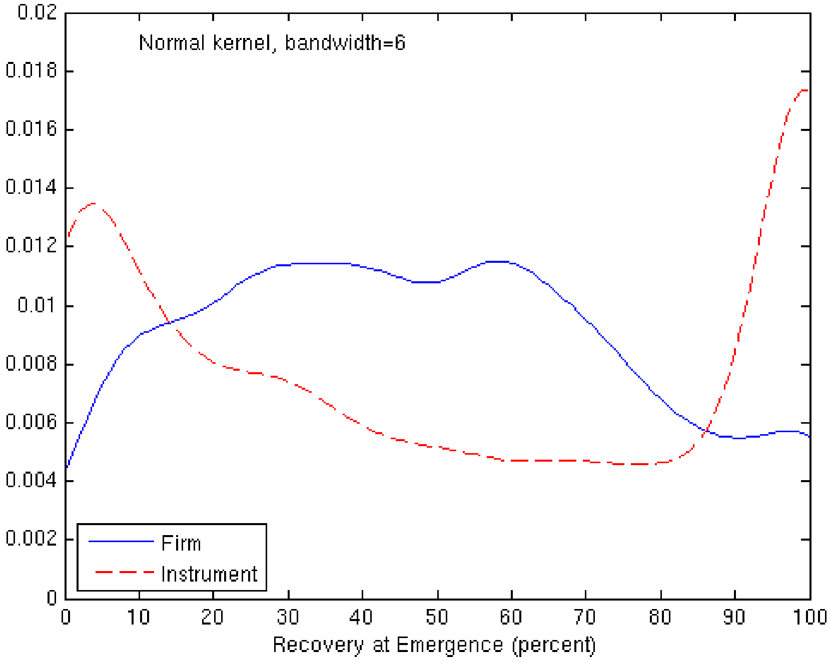
[14] Because instrument-level regressions in the literature to date use equally weighted observations, there is potential for unintended correlations between explanatory variables and payoff properties of the instruments to affect results.

Figure 12: Distribution of Recovery at Firm-level and Instrument-level



Kernel estimate of density of recovery at emergence. The “firm” curve is for total recovery on all debt, whereas the “instrument” curve is for all individual debt instruments observed in our data.

2.3 Recovery measures

We normalize recovery cash flows by amount-owed in order to work with recovery rates. Our instrument-level measure of recovery is R_i/D_i, where R_i is the total dollar amount of the recovery received by holders of instrument i and D_i is the total amount owed according to the terms of the debt contract and the rules of bankruptcy. Our firm-level measure is R/D, where R is the sum of recoveries on all of the firm's debt instruments and D is the sum of amounts owed. Thus, firm-level recovery rates are weighted averages of the recovery rates on the firm's individual obligations.

LossStat's recovery measures embody some practices and assumptions that seemed potentially problematic to us. We used raw cash flow amount, type, and date information in LossStats to produce measures that we believe are more suitable for our purposes. Our measures are highly correlated with those of S&P (Pearson correlations are between 0.97 and 0.99), and our results are robust to use of measures based on a wide variety of assumptions, including S&P's. Detailed information about variable construction and data cleaning is in an appendix available from the authors.

Our recovery measure is as of the date of emergence. Conceptually, recovery as of the date of bankruptcy would be preferable but, as noted previously, direct observation of recovery at bankruptcy is infeasible in the case of firm-level recovery. Discounting our at-emergence measure back to the bankruptcy date requires assumptions about discount rates. The appropriate value of such rates for defaulted firms is a controversial subject in the credit risk literature. We checked robustness by producing measures discounted using risk-free rates and risk free rates plus spreads of 250 and 500 basis points, respectively. Our results are robust to use of any of the measures, apart from a modest across-the-board reduction in average recoveries, perhaps because both average time in bankruptcy and the standard deviation of such times are small relative to the cross-sectional variation on recovery rates at emergence. In passing, we examined the correlation between recovery rates at emergence and the return on the Wilshire 500 Total Return Index during the periods that firms were in bankruptcy and found an economically and statistically zero relationship. This would seem to indicate that any systematic risk in variations in firm value during bankruptcy are dominated by cross-sectional variation in firm value at the time of bankruptcy.

The dollar value received by debtholders at emergence sometimes exceeds the amount of debtholders' claims. Some such cases may arise because our measure of claims is imperfect, and some arise because time elapses between filing of the firm's plan of reorganization and emergence from bankruptcy. If the value of the firm increases sharply during this interval, or if the court's estimate of value as embodied in the plan of reorganization is too small, debtholders may receive some value that would have gone to equityholders (or other deeply subordinated claimants) in a world of instantaneous action and perfect information. Our results are robust to dropping all observations with firm level recovery greater than 100 percent.

2.4 Non-debt claims

At emergence from bankruptcy, firm value is allocated not only to holders of pre-bankruptcy debt claims, but also to pay administrative costs of the bankruptcy, to pay taxes, to other claims such as accounts payable, and to repay debtor-in-possession (DIP) loans, if any.

Many bankrupt firms obtain superpriority debtor-in-possession (DIP) loan commitments, usually from banks, which almost always are repaid in full if any balances are outstanding. However, funds are generally not drawn under such facilities. The facilities help the firm to continue operations by assuring trade creditors that the firm will not experience a liquidity problem while in bankruptcy. Only a few DIP loans appear in our data (which includes only loan commitments for which balances were outstanding at bankruptcy or emergence), and in only one case were new balances added after the bankruptcy was filed. Thus, our results are not affected by the superpriority status of DIP loans.

As noted, our data report only recoveries on debt, so our firm-level recoveries represent a lower-bound estimate of the value of the firm's assets at emergence. In effect, we assume that the sum of non-debt claims experiences the same recovery rate as that for the sum of debt claims. For example, accounts payable are usually treated as "general unsecured claims" or "senior debt." Other things equal, it would seem that a larger share of accounts payable in total liabilities should reduce the dollar amount of our measure of firm-level recovery. We check robustness by using the shares of different types of non-debt and non-equity claims in total liabilities as predictors and find that only accounts payable predicts our measure of recovery, as discussed further below.[15]

In the U.S., most subordination is contractual.[16] Structural subordination refers to cases where debt is a claim on a holding company and the debt is not guaranteed by subsidiary operating companies. Holding company debtholders are not legal claimants in the operating company bankruptcies and will receive a recovery only if the holding company's equity interest in the subs is worth something at emergence (or if the holding company has other assets). Thus, structurally subordinated debtholders often lose everything or almost everything. Because we are interested in recovery to the firm as a whole, without regard to the structure of the firm, we have identified cases of related-company bankruptcies in LossStats and have combined each set of related entities into a single simulated entity. There are six such cases. Results are robust to use of uncombined data.

2.5 Bank debt share

We use two measures of the share of bank debt in total debt. The first includes any debt that LossStats' broad debt type variable describes as a "Line of Credit," "Revolving Credit," or "Term Loan" (examples of other common classifications are "Subordinated Bonds," "Senior Unsecured Bonds," etc.). We examine LossStats' more detailed description of each instrument and remove from the bank debt category any instrument that does not appear to be bank debt (for example,

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[15] We largely ignore deviations from absolute priority. Deviations that involve transfers from one group of debtholders to another are immaterial because we examine firm-level recovery. However, deviations that involve payoffs to equityholders will reduce firm-level debt recovery. Previous studies imply that such deviations, while frequent, are usually relatively small. They are a source of noise in our empirical work.

[16] At issuance, the indenture for a subordinated debt instrument specifies the existing debt instruments to which the new debt is subordinated. At emergence from bankruptcy, holders of the subordinated debt promise to make side-payments of their recovery to holders of the debt to which theirs is subordinated, up to the point at which the recipients' bankruptcy claims are fully satisfied. Leaving aside the subordination agreement, subordinated debt is just another general unsecured claim, that is, it is "senior unsecured debt." The subordination agreement is a private contract that is typically enforced and implemented by the bankruptcy court as part of the agreed-upon plan of reorganization, but if the bankruptcy court does not enforce it, separate lawsuits for enforcement of the agreement must be litigated in other courts.

Often accounts payable and other general unsecured claims are not included in the list of debt to which the instrument is subordinated. Thus, in some cases, if the gross recovery received by subordinated debtholders is not exhausted by the contractual side-payments, subordinated debtholders may have positive recoveries even if some senior claimants do not have a full recovery. This is not a violation of absolute priority. None of these details of contractual subordination are material for our firm-level recovery estimates, but they are material for instrument-level analysis.

loans from suppliers or parents). Results are robust to including or excluding such debt from the bank debt category.

Some loans in our sample do not have financial covenants. We match instruments classified as bank debt with the LPC Dealscan database and other sources to identify those with financial covenants. We also attempt to identify those with "borrowing base" features, which most frequently apply to lines of credit and which limit the borrower's outstanding balance to some fraction of relatively liquid collateral, such as 80 percent of accounts receivable balances. Such features help the lender to achieve a good recovery rate. However, they do not confer any rights to accelerate the maturity of the loan, so in the absence of financial covenants such loans would not be "bank" debt for the purposes of this paper. We are not able to find information for all loans, so we construct three variables: The share of the firm's debt that is loans with covenants, the share that is without covenants, and the share for which covenant status is unknown. We construct a similar set of three variables for borrowing base features. As of this writing, construction of such variables is still in progress.

3 Empirical Results

We first focus on tests of the model's main predictions, then on evidence that equityholders sometimes choose a bankruptcy threshold above the bank's threshold, and then on results for auxiliary and control variables.

3.1 Bank debt share

Table 3 reports estimates from simple ordinary least-squares models of firm-level recovery (parameter estimates and statistical significance are similar when produced by Tobit estimation or when all observations with recovery rates above 100 percent are dropped (not tabulated)). The main variable of interest is the share of bank debt in total debt at the time of filing of bankruptcy. Bank debt share is an economically and statistically significant predictor of recovery, with a 1 percentage point increase in share associated with about a one-quarter percentage point increase in recovery rate, other things equal (bank debt share is measured as a fraction, so the regression coefficient is the change in recovery per 100 percentage point change in share, or 24 rather than 0.24). Other debt structure variables include dummies for firms with no bank debt and all bank debt at filing, the share of debt that is secured, the share that is contractually subordinated, and dummies for firms with all subordinated debt and no subordinated debt. The all-bank-debt dummy coefficient is statistically significant at the 10 percent level and the point estimate implies a 10 percentage point increase in average firm level recovery is associated with such cases. Coefficients on the all-subordinated-debt dummy and subordinated debt share variables have p-values just above 0.10 and coefficients near -10, implying substantially worse recoveries for firms with large amounts of subordinated debt. We can offer no confident interpretation of these auxiliary debt structure variable results. The all-bank-debt and subordinated debt variable results are not very robust to changes in sample period. We speculate that some of these variables may be standing in for unmeasured firm characteristics, such as the firm experiencing a leveraged buyout sometime before bankruptcy, but these are subjects for future research.[17]

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[17] That the secured debt share variable is not significant is not particularly surprising given that collateral merely gives one class of claimants priority over other classes. However, liens might protect assets from dissipation by the

Table 3. Main regressions

The dependent variable in OLS regressions, with p-values based on conventional standard errors, is the firm-level recovery rate at emergence. The shares of bank, subordinated and secured debt are the fractions of each type of debt outstanding at default. The utility dummy indicates regulated public utilities, such as natural gas delivery companies. Court dummies identify the location of the court that supervised the bankruptcy. The omitted court is “all others.” Industry dummies are based on a judgmental collapsing of industry codes provided by S&P into sixteen categories, all of which are included in regressions reported in all but column 3, but only utility, telecom, computer, and airline are shown to save space. Others are statistically insignificant. The last column shows results when the sample is restricted to bankruptcies filed in the years 1987-97.

	(1)		(2)		(3)		(4)		(5)	
Independent	Base case		No debt struc.		Only debt struc.		Only bank share		1987-97 Only	
Variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	58.76	<.0001	62.84	<.0001	44.79	<.0001	48.42	<.0001	49.72	0.0002
Share bank debt	**24.43**	**0.0004**			**12.98**	**0.0474**	**36.06**	**<.0001**	**36.27**	**0.0029**
No bank debt dummy	-3.45	0.4309			-9.49	0.0121			0.74	0.9172
All bank debt dummy	10.13	0.0914			11.66	0.0600			-4.48	0.7085
Share secured debt	-2.29	0.5884			7.54	0.0663			-3.11	0.6475
All sub debt dummy	-10.44	0.1042			-5.94	0.3684			-6.91	0.4053
No sub debt dummy	2.50	0.5278			3.06	0.4211			0.98	0.8903
Share sub debt	-10.63	0.1115			-4.99	0.4391			-22.93	0.0251
Bankruptcy year dummies:										
1987-88	1.29	0.8860	-0.64	0.9480			-1.09	0.9047	2.19	0.8169
1989	-11.99	0.1659	-15.09	0.1076			-15.00	0.0873	-7.36	0.4260
1990	-11.34	0.1040	-12.63	0.0942			-14.09	0.0462	-10.57	0.1431
1991	-1.54	0.7978	-2.03	0.7559			-1.89	0.7567	1.93	0.7541
1992	1.01	0.8734	2.06	0.7635			-0.47	0.9420	1.28	0.8423
1994	-6.01	0.4343	-2.15	0.7964			-3.39	0.6634	-2.27	0.7742
1995	2.48	0.7283	5.75	0.4520			7.94	0.2668	5.94	0.4353
1996	-2.58	0.7267	8.85	0.2629			2.13	0.7746	-1.44	0.8568
1997	-8.61	0.2843	-2.65	0.7591			-5.07	0.5300	-9.49	0.2814
1998	-12.54	0.1026	-5.09	0.5372			-9.55	0.2172		
1999	-7.96	0.1992	-1.32	0.8427			-5.10	0.4139		
2000	-14.67	0.0157	-4.33	0.5028			-11.88	0.0519		
2001	-16.11	0.0049	-8.03	0.1862			-13.60	0.0175		
2002	-15.01	0.0093	-4.50	0.4607			-10.72	0.0623		
2003	-0.35	0.9550	12.21	0.0586			3.62	0.5525		
2004	-2.90	0.6762	9.27	0.2085			1.52	0.8263		
Time in bankruptcy	0.34	0.8217	0.20	0.9019			0.64	0.6743	1.57	0.4877
Time from plan to emerge	-5.02	0.0622	-5.80	0.0471			-4.52	0.0980	-6.41	0.1083
Time in default pre-filing	-0.38	0.8741	-3.31	0.1974			-0.88	0.7165	-2.22	0.4898
Prepackaged bankruptcy	5.84	0.0433	3.42	0.2735			6.11	0.0374	6.87	0.1787
Number debt instruments	0.13	0.6620	0.13	0.6910			0.24	0.4367	-0.52	0.5009
Number priority classes	0.21	0.8480	0.55	0.5617			0.19	0.8328	3.32	0.0883
Fraud dummy	1.79	0.7292	1.72	0.7584			-0.38	0.9418	-7.98	0.4908
Filed again within 5 yrs dum	-4.33	0.4264	-6.13	0.2971			-4.98	0.3653	3.78	0.6302
Court dummies:										
California	-2.99	0.5466	-6.98	0.1904			-4.11	0.4109	-4.63	0.4994
New York	-5.48	0.0865	-6.29	0.0680			-6.93	0.0316	-6.83	0.1691
Delaware	-5.93	0.0403	-6.93	0.0262			-6.92	0.0177	-6.05	0.2361
Illinois	-3.54	0.5946	-3.80	0.5967			-4.57	0.4972	-6.81	0.5488
Texas	-3.11	0.4736	-4.15	0.3778			-2.92	0.5067	-3.96	0.5275
Selected industry dummies:										
Bubble-firm dummy	-16.56	0.0067	-19.92	0.0025			-15.79	0.0106		
Utilities	26.41	0.0007	22.31	0.0081			28.45	0.0003	41.48	0.0002
Telecom	-5.58	0.3523	-11.58	0.0692			-3.52	0.5587	10.78	0.6011
Computer	-4.40	0.3422	-12.48	0.0123			-6.67	0.1555	1.77	0.8215
Airline	-7.62	0.3978	-18.45	0.0560			-6.15	0.5000	0.18	0.9875
Number observations	644		644		644		644		255	
Adjusted R-squared	**0.27**		**0.13**		**0.15**		**0.24**		**0.28**	

Columns 2 and 3 report results when debt structure variables are dropped and when all other variables are dropped, respectively. As measured by adjusted R-squared, it is striking that debt structure variables account for more of the variation in recovery rates than do all the other variables combined (and the other variables, or variations on them, have been the main focus of the literature to date). In column 3, the drop in value of the bank debt share coefficient to 13 appears to be due to correlations between bank debt share and the other debt structure variables in the absence of the year dummies and the public utility dummy. When those variables are added to the specification shown in column 3, the bank debt share coefficient has a value of 22 and is highly statistically significant (not tabulated).

Column 4 reports results when the only included debt structure variable is bank debt share but with all control variables included. The bank debt share coefficient is 36. If the regression includes only bank debt share as an independent variable, its coefficient is 34 and has a p-value of $< .0001$, and the adjusted R-squared is 0.12, not far below the adjusted R-squared in column 3, where all debt structure variables but no controls are included (not tabulated). Thus, our characterization of a one percentage point increase in bank debt share as being associated with a one-quarter-percentage-point increase in recovery is arguably somewhat conservative. Results in column 4 might be the basis of an argument for a one-third-for-one impact.

As mentioned previously, we address calendar censoring of the sample by dropping bankruptcies after 2004. Results in column 5, which ends the sample with bankruptcies filed in 1997, is evidence that censoring is not a problem, as there are more than enough years elapsed after 1997 to resolve any bankruptcies begun by 1997. The bank debt share effect in column 5 is stronger than in the full sample. Also of note among results in column 5 is that the coefficient on subordinated debt share is larger and more significant. Due to space limitations in the table, we do not tabulate results for 1998-2004, but in that subsample the subordinated debt variable coefficients are much smaller and have p-values not close to conventional significance levels. The coefficient on bank debt share is 22 in such regressions and remains significant at the 1 percent level. Thus, the bank debt share relationship persists throughout the sample period, but the subordinated debt effect is limited to earlier years.

Our model implies that the relationship between bank debt share and recovery may be approximately linear or somewhat non-linear. In untabulated results, we computed estimates when bank debt share is represented by a spline with two and four linear segments. We cannot reject a hypothesis that the relationship is linear, but standard errors are high enough that a mildly non-linear relationship cannot be ruled out.

3.2 Loan coupon interest rate and volatility

Our model implies a negative correlation between interest rate spreads on bank debt and firm-level recovery. The intuition is that the larger the spread, the more the bank delays forcing the firm into bankruptcy because it hopes to enjoy the spread in the future.[18]

Column 1 of Table 4 reports results when the loan interest rate is included in regressions.[19]

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firm prior to bankruptcy. Apparently such protection is not very material to recovery rates, perhaps because banks take into account the degree of such protection on a case-by-case basis in setting the default boundary.

[18] Of course, such a negative correlation might be predicted by simple structural models with a random but observable default boundary as well, because the market would set loan interest rates to price the lower recoveries expected of firms with exogenously lower default boundaries.

[19] The interest rate is measured as the loan interst rate over LIBOR in percentage points. In our model, riskfree

Table 4. Impact on recovery of bank debt interest rate spread

The interest rate spread on bank debt is the spread over LIBOR as recorded in the LossStat database. Where a firm has multiple bank loans outstanding, the mean spread is used. Other variables are as in previous tables. The second column reports base-case regression results for the subsample for which spreads are available to support comparisons.

Independent Variable	(1) With rate Coeff.	p-value	(2) Base case Coeff.	p-value
Intercept	72.63	<.0001	-68.33	<.0001
Loan interest rate spread	**-2.72**	**0.0375**		
Share bank debt	27.30	0.0027	26.69	0.0035
All bank debt dummy	6.19	0.3762	6.62	0.3465
Share secured debt	-5.77	0.3884	-6.68	0.3200
No sub debt dummy	5.29	0.3502	5.80	0.3075
Share sub debt	-7.12	0.4905	-7.99	0.4413
Bankruptcy year:				
1987-88	-0.89	0.9492	-0.55	0.9689
1989	31.76	0.2792	28.69	0.3304
1990	-16.59	0.2356	-16.35	0.2451
1991	-10.57	0.3184	-10.30	0.3336
1992	-11.98	0.3128	-11.83	0.3213
1994	-1.18	0.9308	-1.17	0.9317
1995	3.21	0.7895	2.66	0.8259
1996	1.19	0.9156	-1.02	0.9277
1997	-8.69	0.4614	-10.38	0.3809
1998	-20.80	0.1091	-21.53	0.0991
1999	-18.49	0.0672	-19.35	0.0567
2000	-20.65	0.0345	-21.70	0.0271
2001	-23.08	0.0148	-23.97	0.0118
2002	-19.21	0.0513	-21.79	0.0269
2003	-1.36	0.8921	-4.94	0.6194
2004	-4.11	0.7088	-9.48	0.3783
Time in bankruptcy	-0.21	0.9223	-0.25	0.9080
Time from plan to emerge	-11.01	0.0189	-11.59	0.0138
Time in default pre-filing	-1.72	0.6657	-1.89	0.6379
Prepackaged bankruptcy	2.23	0.5752	2.34	0.5588
Number debt instruments	0.53	0.1445	0.58	0.1150
Number priority classes	-0.55	0.6683	-0.64	0.6217
Fraud dummy	1.13	0.8756	2.42	0.7377
Filed again within 5 yrs dum	-4.76	0.4975	-3.98	0.5716
Court dummies:				
California	15.13	0.0751	13.97	0.1014
New York	-3.35	0.4503	-2.92	0.5123
Delaware	-6.37	0.1074	-6.64	0.0951
Illinois	-6.16	0.5107	-4.54	0.6285
Texas	-0.23	0.9705	-1.69	0.7869
Selected industry dummies				
Bubble-firm dummy	-24.34	0.0063	-24.95	0.0053
Utilities	14.46	0.1563	17.51	0.0849
Telecom	-5.55	0.5002	-6.99	0.3964
Computer	-4.99	0.4544	-4.97	0.4593
Airline	-8.51	0.6510	-6.20	0.7425
Number observations	343		343	
Adjusted R-squared	0.25		0.24	

Interest rates are not available for all observations, so to permit comparision, column 2 reports results when the base specification is estimated using the same subsample as is used in column 1. The estimated coefficient on the interest rate variable is negative and statistically significant and implies that a one percentage point increase in the interest rate reduces average firm-level recovery rates by almost three percentage points, which seems to us to be a reasonable magnitude. Comparing columns 1 and 2, the bank debt share coefficient is not much affected by the presence of the interest rate variable.

We do not believe that we can measure the volatility of value of the firm's assets properly. The measurement problem is that our model implies that conventional methods of translating equity volatility into asset volatility are mis-specified. Such methods treat equity as an option and implicitly assume a fixed bankruptcy boundary as a strike "price" for the option. One common boundary assumption is the value of assets at which the firm is on the border between solvency and insolvency. Our model implies that the boundary varies across firms, and the boundary itself is determined in part by the true value of asset volatility. That is, the error in conventional empirical measures of asset volatility will be correlated with the boundary value and with the actual value of volatility. Recovery rates and bank debt share are proxies for the boundary value of assets. Thus, any empirical relationship we observe between measured asset volatility and recovery rates or bank debt share will partly reflect the true relationship between asset volatility and recovery and partly the component of asset volatility measurement error that is driven by the difference between the bankruptcy boundary value of assets and the boundary value assumed by the volatility measurement method. We cannot confidently predict the properties of that component of measurement error without developing new measures of asset volatility, and such development is beyond the scope of this paper.[20]

Lacking a good volatility measure, we do not tabulate results. We did experiment with Moody's KMV's measure of asset volatility one year before the bankruptcy date, which is available for less than half of the observations in the full sample. When this variable is included in the base specification, the estimated coefficient is negative and significant at the 10 percent level (the opposite of what our model predicts). We checked for a non-linear relationship by using a three-segment spline. The relationship is negative and insignificant for small and medium-size values of (mismeasured) volatility, but is positive and marginally significant for large values. As a crude and not at all satisfactory control for measurement error, we included both the level of (mismeasured) volatility and an interaction between volatility and bank debt share. The interaction is negative and marginally significant, while the level of volatility is not significant. This last result adds to our concern that Moody's KMV's variable mismeasures asset volatility in our sample, and in a manner that is material to inference about recovery rate relationships (other conventional volatility measures are likely to pose the same problem).

3.3 Loan recovery rates

Taken literally, our model implies that bank debt should almost always receive a high recovery and all other debt should receive a low recovery. The case of other debt is discussed further below, but

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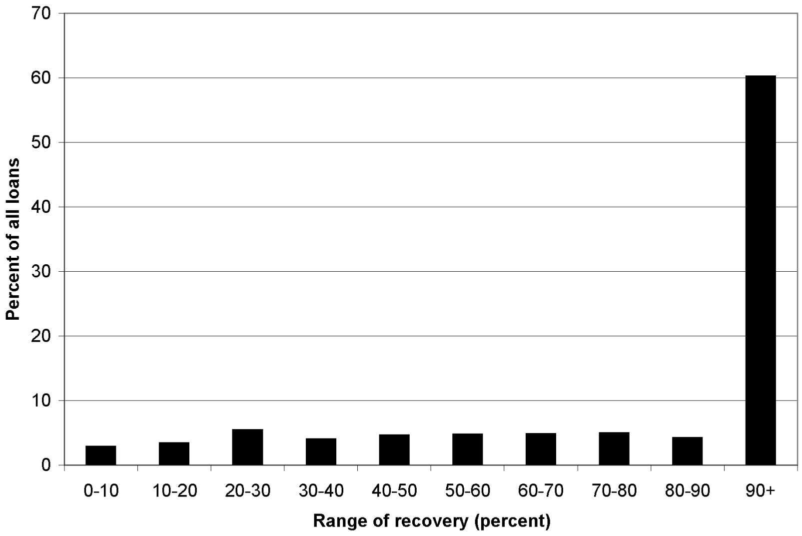
rates are assumed zero, so coupon rates may be interpreted as credit spreads. Where the firm has more than one loan, the mean is used.

[20] We speculate that the error in conventional measures is not too large for firms with asset value far from the bankruptcy threshold. Judging by ratings, however, most firms in our sample were not so far from the threshold even a year or two before bankruptcy was filed, so measurement error may be material for them.

Figure 13 displays the distribution of loan recovery rates. About 60 percent receive a full recovery, with the remainder approximately uniformly distributed across other recovery rates. The uniformity is somewhat surprising. As of this writing, we are still investigating possible explanations, but one likely cause is that the figure is based on a presumption that all debt categorized as "loans" in LossStats has covenants. As noted previously, we are still attempting to identify which actually have covenants. We suspect that many observations with much less than full recovery are no-covenant debt, that is, not bank debt from the standpoint of our model.

Figure 13: Frequency distribution of bank debt recovery rates



3.4 Evidence consistent with firms sometimes voluntarily choosing bankruptcy

As noted previously, casual observation of the news gives a strong impression that firms sometimes declare bankruptcy before banks force them do to so and when they could still make debt payments. We assume equityholders are passive for simplicity. Our goal in this paper is to demonstrate that holders of debt with financial covenants play an important role in bankruptcy decisions and recovery rate determination, not that equityholders play no role. To aid future research that attempts to model both influences, we provide a few pieces of circumstantial evidence that many firms have at least two bankruptcy boundary values of assets, one set by holders of bank debt and one set by those in control of the firm.

First, if bank debt always set the highest boundary, we would expect the coefficient on bank debt share in our regressions to be near one, but it is less than one-half. Some such attentuation toward zero may arise because firm value changes between the time of bankruptcy and emergence.

But observations for firms that declare bankruptcy at asset values higher than the bank's threshold also will attenuate the estimated coefficient toward zero.

If a firm has two thresholds, its own and the bank's, the firm's own threshold is more likely to be the higher of the two the smaller the share of bank debt. As noted previously, we cannot reject a hypothesis that the relationship between bank debt share and recovery is linear. When we split the bank debt share variable into two, one for values of bank debt share below 0.18 (approximately the 25th percentile) and one for values above, coefficient estimates on the two variables are similar and not far from 25. However, the coefficient for the low-bank-debt share variable is not statistically significantly different from zero (p-value 0.40), whereas the other coefficient is (p-value .002) (not tabulated).

Positive residuals from our base-case regression should be positively correlated with cases where the firm's threshold is above the bank's, and material recoveries to nonbank debt should be more likely in such cases. We ran logistic regressions on instrument-level data that included only nonbank debt, with the dependent variable set to one where the recovery rate was greater than 75 percent and using as independent variables the residuals from the base case regression, expressed as two variables: One is the residual if negative and zero otherwise, and one is the residual if positive and zero otherwise. The coefficient on the positive residual variable is positive and significant (p-value .0001) and retains its significance for a variety of choices of recovery rate cutoff value in creating the dependent variable, whereas that for the negative residual variable is only marginally significant (p-value .07) (not tabulated).

3.5 Other auxiliary and control variables

3.5.1 Year and industry

As noted previously, our examination of firm-level measures of ultimate recovery differs from almost all prior studies. Most have examined samples of recoveries to individual debt instruments and some have interpreted results as revealing information about the relationship between firm characteristics and recovery rates. To the extent that our results differ for similar variables, more research may be needed to reveal robust interpretations, because we would expect that firm-level explanations would be revealed most clearly in firm-level regressions. Our purpose is not to criticize previous work, merely to point out in passing some auxiliary results that may be interesting.

Returning to Table 3, industry effects on recovery rates are weak at best, with the exception of Utilities. All regressions include a full set of industry dummies that we created by boiling down S&P's more than 100 industry designations appearing in LossStats to 17 categories (retail is the omitted category in regressions). Coefficients on most of these dummies are never statistically significant and are omitted from tables. Only those industries that have signficant coefficients in some specifications are tabulated. Chief among these are utilities, which are associated with firm-level recovery rates about 30 percent higher than the average of about 50 percent. Like prior researchers, we speculate that the regulated nature of utilities in the United States is responsible. Regulators may play a role in forcing utilities into bankruptcy "early," as they probably have a preference that firms they regulate not become deeply insolvent.

We created a dummy variable for "bubble" firms, which are defined as firms in the telecom, internet, or energy trading sectors that filed for bankruptcy in the year 2000 or later. We classified bubble firms by inspection, as S&P's industry classifications are not always indicative. Coefficient estimates imply that such firms have economically and statistically significantly smaller recovery rates than other firms. We regard this result as consistent with the finding of Acharya et al. (forthcoming) that recoveries are lower for firms whose industry is deeply distressed when bankruptcy is filed. If we omit this variable from specifications, coefficients on the Telecom and Computer industry dummies are often economically and statististically significant (not tabulated), but in the presence of the bubble dummy they usually are not.

Dummies for the year in which bankrutpcy was declared also appear in most regression specifications (1993 is the omitted year; 1987 and 1988 are combined because the number of observations for those years is small). The dummies are intended as controls for cyclical and trend effects. Most coefficients are not significantly different from zero and no trend is evident, but point estimates hint of the possibility of important cyclical effects, with lower average recoveries during recessions.

3.5.2 Time in bankruptcy, frictions, and bad actors

Conventional wisdom holds that deadweight costs of bankruptcy increase with duration of the bankruptcy. Such deadweight costs are likely to reduce debtholder recoveries, so it is striking that we find that the coefficient on a time-in-bankruptcy variable (measured in years) is near zero and not statistically significant (Covitz et al. (2004) also find time in bankruptcy does not predict recovery). Moreover, as shown in Table 3, variables for the number of instruments or the number of priority classes of debt specified by the court, which are often taken as proxies for the severity of bargaining frictions, also do not predict firm-level recovery. Similarly, the time from the first payment default by the firm to the bankruptcy filing date is not predictive of recovery.[21]

The only proxies for bargaining frictions that we find to be significant are a dummy for prepackaged bankruptcies and a variable measuring the elapsed time between filing of the plan of reorganization and its approval. Prepackaged bankruptcies, for which the firm has negotiated a tentative plan of reorganization with creditors before filing, are associated with somewhat higher average firm-level recovery rates. The coefficient on a prepackaged dummy in column 1 of Table 3 is near 6 and is statistically significant. Prior literature typically interprets better recoveries by prepacks as evidence of reduced bargaining frictions. However, in the context of our analysis, the result may arise because prepackaged bankruptcies are likely to be those in which the firm's bankruptcy threshold is above that of the bank's, and such bankruptcies are likely to feature a higher recovery (the firm may not have time to negotiate a prepack if its value has fallen below the bank's threshold). That is, the result may or may not be indicative of lower bargaining frictions.

The coefficient on YearsFromPlanToEmerge, which measures the time elapsed between filing of a plan of reorganization and emergence from bankruptcy, is near -5 and is weakly statistically significant. The median time is about four months, but in some cases the time is much longer (max almost four years). We speculate that an unusually long elapsed time during this stage of the bankruptcy process indicates problems getting claimants to vote to confirm the plan, which is consistent with bargaining problems sometimes being important.

Wang (2007) offers evidence that recovery rates are lower for bankruptcies precipitated at least in part by fraud, and for bankruptcies managed by certain courts. We construct a fraud variable in a manner similar to Wang (2007) (by examining Lynn Lopucki's Bankruptcy Research Database, supplemented by some additional frauds we noticed while cleaning the data). We also used similar sources to identify firms that experienced more than one bankruptcy within five years of that recorded in any given observation (often called "Chapter 22" bankruptcies). In column 1 of Table

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[21] We also find no relationship between time in bankruptcy and bank debt share (not tabulated).

3 (and other specifications), coefficients on the associated dummy variables are economically small and not statistically significant.

A number of authors, such as Lopucki (2005), have suggested that the efficiency of bankruptcy courts varies, or that firms that are deeply insolvent may be more likely to file in some venues than others. We include dummy variables for each of the bankruptcy courts that handles a substantial volume of bankruptcies in our sample (the omitted category is all other courts). Although coefficient values are uniformly negative for the court dummies, none are very large, and only the Delaware dummy is marginally statistically significant from zero. Moreover, it is possible that in our sample, the dummies act as controls for any selection bias that may result from smaller bankruptcies being more likely to be coded by S&P if they are filed in one of the courts with electronic record systems.

As noted, we report these results to aid future research. We do not claim that interpretations in prior studies are incorrect. However, we do believe that results reported in this subsection indicate that research revisiting issues related to bargaining frictions might be fruitful.[22]

3.5.3 Financial statement variables and credit ratings

Table 5 reports results of OLS regressions for a subsample of bankruptcies by firms for which we were able to find usable data in Compustat. Compustat's balance sheet, income statement, and debt-rating variables are as of the firm's fiscal year-end data prior to the bankruptcy date (firms for which usable fiscal year-end data is more than 1.1 years prior to the bankruptcy date are dropped).

The Compustat subsample affords an opportunity to examine whether other characteristics of the firm are associated with recovery, such as the nature of its assets, its size, or its operating cash flow not long before filing. Estimates imply most such characteristics are not predictive of firm-level recovery, whether debt structure variables are included or not (we examined variables not shown in the table as well and were not able to find any others that are predictive). The borrower's S&P rating at the fiscal year-end before filing also is not significant. Moreover, both the economic magnitude and the statistical significance of the bank debt share variable is maintained in the smaller Compustat subsample, regardless of what other variables are included.

The main exception is the share of total liabilities that is accounts payable. As noted previously, this category includes trade credit extended to the firm, which is likely to be treated by the court as a senior unsecured claim and, especially in the case of small accounts payable, is likely to be paid in full during bankruptcy in order to reduce the number of creditors and to permit the firm to continue operating with normal trade relationships. Because accounts payable are not measured in LossStats, a marginal additional dollar of payables represents an additional claim, and payments to such claims in effect reduce payments to debtholders and our measured firm-level recovery rate. The estimated coefficient on the accounts payable variable in column 2 of Table 5, at -51, implies a reduction of about half a percentage point of recovery rate for each additional percentage point of total liabilities that are accounts payable. Inclusion of the variable does not materially affect the estimated coefficient on bank debt share.

Another exception is leverage. Measured as the ratio book total liabilities to total assets, the coefficient estimate implies a moderate reduction in firm level recovery rate of about five percentage points if leverage increases from its median ratio of 1 to a value of 2.

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[22] We ran regressions similar to those in Table 4 using instrument-level data, adding dummies for the type of debt (subordinated, junior subordinated, etc.), and found that debt structure variables are significant predictors at the instrument-level as well. However, results for control variables such as time in bankruptcy and court dummies are sensitive to details of the specification at the instrument level.

Table 5. Regressions using Compustat-matched subsample

The dependent variable in OLS regressions, with p-values based on conventional standard errors, is the firm-level recovery rate at emergence. All other variables are as in Table 3, except that balance sheet and income statement variables, as well as rating dummies, are from Compustat and are dates as of the firm's last fiscal year-end date before filing bankruptcy for which data are available.

Independent Variable	(1) Base case		(2) Add firm vars		(3) No debt struc		(4) Ratings	
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	64.79	<.0001	64.67	<.0001	60.19	<.0001	55.50	<.0001
Share bank debt	**20.60**	**0.0211**	**27.88**	**0.0024**			**24.00**	**0.0078**
No bank debt dummy	-1.10	0.8407	3.84	0.5226			-2.25	0.6860
All bank debt dummy	15.42	0.0551	16.53	0.0370			14.85	0.0694
Share secured debt	-7.65	0.1951	-9.26	0.1127			-9.03	0.1313
All sub debt dummy	-20.72	0.0076	-20.91	0.0075			-18.91	0.0162
No sub debt dummy	3.16	0.5130	5.70	0.2338			3.65	0.4466
Share sub debt	-12.89	0.1071	-8.12	0.3122			-10.04	0.2099
Firm characteristics								
Log Total Assets			1.03	0.5119	1.94	0.2461	3.44	0.0211
Non-intang. assets/assets			0.05	0.9958	8.17	0.4121		
Book liabs./assets			**-5.14**	0.0400	-7.28	0.0066		
Operating income/assets			19.29	0.1531	21.45	0.1362		
Accts payable/tot liabilities			**-51.05**	0.0171	-31.32	0.1693	-38.64	0.0456
PPE/assets			-6.15	0.4395	-7.95	0.3576		
Ratings								
BB or safer							-4.75	0.3831
B							-5.59	0.1215
CCC							1.01	0.8238
CC or worse							1.62	0.7630
Time in bankruptcy	-1.08	0.5507	-1.51	0.4147	-2.35	0.2444	-2.26	0.2246
Time from plan to emerge	0.75	0.8356	1.64	0.6431	1.17	0.7641	2.03	0.5737
Time in default pre-filing	1.35	0.6926	3.79	0.2965	3.81	0.3381	-0.10	0.9800
Prepackaged bankruptcy	6.62	0.0695	4.91	0.1798	0.32	0.9362	4.76	0.2012
Number debt instruments	0.16	0.6446	0.04	0.9229	-0.02	0.9704	-0.29	0.4624
Number priority classes	0.30	0.8029	1.56	0.3445	1.00	0.4494	0.28	0.8244
Fraud dummy	2.52	0.6696	-4.79	0.4436	-7.90	0.2427	-0.64	0.9153
Filed again within 5 yrs dum	-3.78	0.5194	-4.32	0.4536	-6.31	0.3132	-5.47	0.3525
Court dummies:								
California	5.36	0.3531	6.20	0.2753	3.49	0.5744	6.88	0.2310
New York	-3.00	0.4517	-2.22	0.5808	-2.57	0.5583	-2.38	0.5572
Delaware	-3.10	0.3992	-3.19	0.3797	-4.84	0.2225	-2.10	0.5669
Illinois	-0.68	0.9442	-3.27	0.7383	-7.77	0.4623	-0.44	0.9637
Texas	-1.84	0.7332	0.17	0.9742	-0.94	0.8723	-1.21	0.8217
Selected industry dummies								
Bubble-firm dummy	-14.05	0.0485	-9.91	0.1660	-14.19	0.0681	-12.55	0.0815
Utilities	23.53	0.0276	19.58	0.0673	16.75	0.1514	19.54	0.0688
Telecom	-14.05	0.0460	-17.65	0.0162	-17.26	0.0298	-20.96	0.0043
Computer	-9.99	0.0766	-11.71	0.0435	-21.10	0.0008	-12.74	0.0258
Airline	-20.94	0.0369	-22.44	0.0288	-28.52	0.0107	-27.49	0.0071
Number observations	372		360		360		367	
Adjusted R-squared	**0.34**		0.38		0.24		0.36	

In the spirit of our model, perhaps it is unsurprising that most observable firm characteristics are not strongly associated with recovery. They might be in a world with an exogenous default boundary, but banks can observe such variables and thus can be expected to take values of such variables into account in setting the default boundary in a manner likely to erase any correlation.[23]

Discussion

This paper offers a model and evidence supportive of a hypothesis that private debtholders play an important role in determining the value of assets at which firms declare bankruptcy. In order to protect the recovery they receive, and using the control rights granted by loan covenants, private lenders set a threshold that is higher the larger is their share of the firm's debt. Because asset value at bankruptcy strongly influences the value distributed to claimants at emergence, a higher private debt share is associated with higher ultimate firm-level recovery rates. Our model also sheds light on the long-standing puzzle of relatively low average recoveries on defaulted corporate bonds.

We do not claim that banks always are the effective setters of the default boundary — casual inspection of the news reveals obvious cases of strategic default by equityholders — but banks' role appears to be of substantial empirical importance. Nor do we claim that our empirical evidence applies throughout the world — an implication of our paper is that recovery rates are likely to be quite sensitive to the legal and practical feasibility of the conditional control rights that covenants give creditors and to details of bankruptcy law and practice.

In closing, we offer some suggestions for future research. First, our results suggest that literature on the capital structure decision might be enriched by analysis of the choice of the private debt share of total debt. We assume the share is exogenous, which is reasonable for firms near the bankruptcy threshold, but it is clearly a choice variable for solvent firms. Given that debt structure influences the states of the world in which bankruptcy occurs, the debt composition decision may interact with the leverage decision, but more research is needed to reveal the nature and relevance of such interactions.

Second, modeling of instrument-level recovery, which has been the focus of most empirical work on recovery to date, might be revisited. Combining a model of firm-level recovery with non-linear modeling of the impact of debt instrument seniority might provide more insight than models suggested to date, which are usually linear and ignore debt composition.

Third, in analyzing the extent to which recovery rate risk is systematic, which is important to debt pricing and risk management, we speculate that interpretation of results, and robustness, may be cleaner by doing analysis at the firm level. Moreover, controlling for debt composition may be important because the aggregate distribution of debt composition may vary over the business cycle. For example, if average bank debt share is lowest as cyclical peaks are approached, our results imply one would expect lower average recoveries during recessions. The nature of cycles in debt composition is an open empirical question.

Finally, our paper may help point the way toward resolution of some puzzles implicit in existing literature. Davydenko (2005) finds that while fixed boundary models of default do reasonably well in predicting default rates on average, such models do not perform so well in the cross section. Cross sectional variation in the absence of controls for debt structure is natural in our framework because the boundary differs with bank debt share. Faulkender and Petersen (2006) find that firms with bonds outstanding are considerably more leveraged on average than firms with only private

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[23] We are grateful to Richard Cantor for this point.

debt in their capital structure. We do not examine capital structure decisions, but in our model, a firm that wished to increase leverage while holding its bankruptcy probability fixed could do so by issuing more bonds and no more loans.

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A Parameter bounds and special cases in the baseline model

We document here a number of useful parameter bounds and special cases for the baseline model. First, we have the bounds

Result 1 *For all* $0 \leq \rho \leq r$, $\alpha \geq \frac{2(r-\rho)}{\sigma^2} \geq 0$ *and* $\beta \geq 2$.

To derive this result, observe that we can re-write the equation for α as

$$\alpha - \frac{2(r-\rho)}{\sigma^2} = \sqrt{\left(\frac{1}{2} + \frac{r-\rho}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} - \left(\frac{1}{2} + \frac{r-\rho}{\sigma^2}\right) \geq 0$$

When dividends to equity holders are flat (i.e., $\rho = 0$), the bound is exact and we have

Result 2 *When* $\rho = 0$, $\alpha = \frac{2r}{\sigma^2}$ *and* $\beta = 2$, *and the function* $\Xi(\kappa; \alpha, \beta, \zeta)$ *simplifies as*

$$\Xi(\kappa) = \frac{r \cdot G(1/(\zeta\kappa); \alpha + 1)}{\mathcal{C} \cdot g(1/(\zeta\kappa); \alpha + 1) - (r\kappa - \mathcal{C})G(1/(\zeta\kappa); \alpha + 1)}$$

where $G(z; a)$ *is gamma cdf at* z *for shape parameter* a *and* $g(z; a)$ *is the corresponding density.*

The last result follows from the special cases for the confluent hypergeometric function in FWC 07.20.03.0006.01 and 07.20.03.0004.01.

B Boundary values of the first order condition

We prove these results under the extended model of Section 1.3. For $\kappa \to 0$, we make use of the limit

Result 3

$$\lim_{V\to 0} \Xi(V;\alpha,\beta,\zeta) = \alpha(\beta-1)\zeta = r/\mathcal{C}.$$

This follows from the asymptotic limit (FWC 07.20.06.0009.01)

$$\lim_{z\to\infty} z^{a}{}_1F_1(a,a+b,z) = \frac{\Gamma(a+b)}{\Gamma(b)}$$

and by noting that $\alpha(\beta-1) = 2r/\sigma^2$. It is easily verified that $M(0,D,s^2) = 0$ and $M_1(0,D,s^2) = 1$, which implies that $B'(0) = \exp(\chi + \eta^2/2)$ and $B(0) = 0$.

To prove Proposition 2, observe that $\kappa B'(\kappa) \to 0$ for large κ, but $\kappa\Xi(\kappa) = \alpha + o(\kappa^{-1})$ As $\lambda\frac{c}{r} - B(\kappa)$ is positive and bounded, we have $\kappa\mathcal{F}(\kappa)$ asymptotically negative and bounded.

C Optimal foreclosure threshold in nonstochastic case

In the non-stochastic case ($\sigma = 0$), the stochastic differential equation (1) because an ordinary differential equation with solution

$$V_t = \frac{\mathcal{C}}{r-\rho} + \left(V_0 - \frac{\mathcal{C}}{r-\rho}\right)\exp((r-\rho)t)$$

Assuming $V_0 < \mathcal{C}/(r-\rho)$, then cash outflows will be larger than the returns on assets, so firm value will decay over time. Given a threshold κ, foreclosure will occur at time

$$T_\kappa = \frac{1}{r-\rho}\left(\log(\mathcal{C} - \kappa(r-\rho)) - \log(\mathcal{C} - V_0(r-\rho))\right)$$

The value of the loan is then given by

$$\begin{aligned} F(V_0;\kappa) = \int_0^{T_\kappa} c\lambda\cdot\exp(-ru)du + \exp(-rT_\kappa)\min\{\kappa, \exp(\tau(c-r))\lambda\} \\ = \lambda\frac{c}{r} - \exp(-rT_\kappa)\left(\lambda\frac{c}{r} - \min\{\kappa, \exp(\tau(c-r))\lambda\}\right) \end{aligned}$$

This function is increasing for $\kappa < \lambda\cdot\exp(\tau(c-r))$ and decreasing for all $\kappa > \lambda\cdot\exp(\tau(c-r))$, so the maximum is at the cusp at $\kappa^* = \lambda\cdot\exp(\tau(c-r))$.

The same result can be derived by taking the limit of $\mathcal{F}(\kappa)$ as $\sigma \to 0$. For this, we use the asymptotic formula for the ${}_1F_1$ function in FWC 7.20.06.0008.01 to get the intermediate result

$$\lim_{\sigma\to 0}\Xi(\kappa) = \frac{r}{\mathcal{C} - \kappa(r-\rho)},$$

and the remaining calculations are straightforward. This implies that the comparative statics for κ^* at $\sigma = 0$ will be locally valid for small positive σ as well.

D Asymptotic results for large expected bankruptcy shocks

This appendix shows that $\kappa^* \to 0$ whenever χ is very large in magnitude. When $\chi \to -\infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\exp(\tau(c-r))\lambda, \exp(\chi+\eta^2/2)V_{t+\tau}\} | V_t = \kappa\right] \to 0$$

and

$$B'(\kappa) = \exp(\chi+\eta^2/2)M_1(\exp(\chi+\eta^2/2)\kappa, \exp(\tau(c-r))\lambda, \sqrt{\tau\sigma^2+\eta^2}) \to 0$$

for any fixed κ. Therefore, for χ sufficiently large and negative, $\mathcal{F}(\kappa)$ is dominated by the term $-\lambda\frac{c}{r}\Xi(\kappa)$ which is negative. This pushes us to the corner solution $\kappa^* = 0$.

When $\chi \to \infty$, we have

$$B(\kappa) = \mathrm{E}\left[\min\{\exp(\tau(c-r))\lambda, \exp(\chi+\eta^2/2)V_{t+\tau}\} | V_t = \kappa\right] \to \exp(\tau(c-r))\lambda$$

for any $\kappa \to 0$, so again $B'(\kappa) \to 0$. Therefore, for χ sufficiently large and positive, $\mathcal{F}(\kappa)$ is dominated by the term $-(\lambda\frac{c}{r} - \exp(\tau(c-r))\lambda)\Xi(\kappa) < 0$. This pushes us towards $\kappa^* = 0$, though the corner solution will not be reached for any finite χ.