

The Role of the Output Gap in the Monetary Policy Transmission Mechanism

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1 Introduction

When demand rises above supply, an output gap may open up, indicating an upward pressure on inflation. In monetary policy models it is common to see this effect captured by a Phillips curve where an output gap term is linked to inflation. That the Phillips curve is so common, though, belies that in practice it can be difficult to find an output gap measure that both shows a systematic relation to inflation on past data and also that can be used to produce a satisfactory forecasts of future inflation. And the practical difficulties to be overcome in estimating output gaps are more challenging in developing countries where output data is less timely and less reliable.¹

In this paper I follow others² in defining potential output *as the level of output in the flexible-price state*: the level of output that would hold if

* This paper represents the views and analysis of the author and should not be thought to represent those of the Bank of England or the Monetary Policy Committee.

¹Using a cross country panel data set, Boyd and Smith (2001) find that the source of much estimation error in the transmission mechanism in developing countries arises from difficulties in measuring important unobservables such as the output gap.

²For definitions and implications of flexible-price output concept, see Astley and Yates (1999), McCallum (2000a, 2001), McCallum and Nelson (1998a) and Neiss and Nelson (2001, 2002) and Smets and Wouter (2001). This can be contrasted for example, with the definition of Nelson (1991) for whom full capacity output is output in the absence of rigidities in adjusting *the capital stock*; hence a flexible-capital measure of potential output.

prices were not flexible in nominal terms not just now, but also in the past and in the expected future. However the actual state of most countries is characterised by the inflexible adjustment of nominal variables such as prices and wages. The flexible-price equilibrium is therefore a counterfactual state and cannot be observed; it has to be estimated by combining theory and data.

As the flexible-price economy refers to an absence of nominal rigidities, In this paper, I derive a micro-founded understanding of flexible-price level of output by taking a monetary policy model, identifying which are the parameters that characterise nominal rigidities and switching them off. The solution of this restricted version of the model provides us with the flexible-price path of output (and other variables).

Economists also use concepts of equilibrium that refer to an absence of both real rigidities and nominal rigidities. For example the balanced growth steady state is defined by the absence of both physical capital adjustment costs and nominal rigidities. Microeconomic textbooks such as Varian's Intermediate Microeconomics refer to a short-run state when the capital stock is held entirely fixed. In the paper, I argue that it is important to be clear which concept we are referring to when we estimate the equilibrium values of a variable, because it can make a great practical difference. It can take very many more years to revert to a flexible-capital stock disequilibrium than to adjust to a disequilibrium from the flexible-price state.³

This is also about interpretation; each measure of disequilibrium should tell us about one particular combination of the underlying shocks of observable data. As I shall explain below, the reason why monetary policy economists are interested in the flexible-price state of output is because that is the concept that should be most appropriate in determining the contribution of shocks that matter for monetary policy objectives.

I then highlight what the possible uses of output gap measurement for monetary policy are — as an indicator of underlying shocks and as a means of being more precise about the short-term output objective. I then show that concentrating on only the expected output gap in objectives can be misleading even in many standard models. The output gap should therefore not be thought of as summarising all that matters about expected output

³I am thinking of the process of economic development itself as a real disequilibrium. There are institutional rigidities in successfully importing productive capital and technology into developing countries, and it can take very many decades to overcome those rigidities.

costs to the central bank: even if it could be well measured, it should be used with care.

1.1 What is the output gap?

The aim of this chapter is to clarify what the output gap means and why it is needed, and then link that to the practice of measurement. The purpose is therefore **not** to survey or evaluate the many different techniques for estimating the output gap that are available in the literature. Of course, as different measures tend to give different results, a model builder who wants to work with an output gap measure will have to choose among these techniques. But I would argue that much is to be gained by first understanding what in essence a measure of the output gap aims to capture, before going on to evaluate why or why not any particular strategy might work for a particular country or given data set.

From the outset I base my analysis squarely on one concept of the output gap. Here I adopt the premise that, for monetary policy purposes, the output gap should be thought of as the deviation of actual from the flexible-price level of output, where the flexible-price level of output is that would hold if there were no costs to adjusting nominal variables.

The output gap is often described in other terms in the literature. Sometimes reference is made to the output gap as the deviations of actual from 'full-capacity output'; or deviations from the 'non-inflationary level of output', and sometimes, deviations from 'trend output'.

For example Nelson (1991) provides a formal definition of an output gap concept that differs from the flexible-price concept that we adopt. He describes the output gap as deviations from the level of output that would hold in the absence of rigidities in adjusting *the capital stock*. His *flexible-capital measure* is similar to the microeconomic definition of the short run as a state where the capital stock is taken to be fixed.⁴ Here I argue that only the flexible-price output gap concept is aimed directly at what matters for monetary policy: the deviation of actual output from the long-run level where capital is free to adjust is a different measure of output disequilibrium to what we are interested in building an understanding of the monetary policy transmission mechanism.

⁴See for example Varian (2002).

1.2 Why are monetary policymakers interested in the output gap?

Why then are monetary policymakers (at least in principle) interested in measuring the deviation of actual output from its flexible-price level? A primary monetary policy interest in output gap measurement is *the purpose of shock identification*. As price developments resulting from nominal shocks can be very different to responses to 'real-side' shocks (shocks that affect flexible-price output), reference to a flexible-price output measure can in principle help in minimising output losses of pursuing price stability.⁵ The potentially useful feature of the flexible-price state for this purpose is that it is defined by an absence of nominal rigidities: real decisions are independent of nominal values. In particular, as nominal shocks will leave flexible-price output untouched, the hope is that monetary policymakers can identify nominal shocks by observing what happens to an accurate measure of flexible-price output.

In formalisations of the monetary policy problem, it is typical to see the objectives of monetary policy stated in terms of both current and expected deviations of inflation from target and the *output gap*. A separate, deeper, question I then tackle is whether monetary policy policymakers can go on to focus on just the volatility of the output gap rather than that of output as a whole when capturing output costs in monetary policy objectives. Would a flexible-price measure of potential output serve also for *the purpose of more sharply defining objectives*? I show that a key assumption that is needed here is nominal neutrality — that actual output will on average be equal to its flexible-price level when we abstract from nominal shocks — implying that is no danger of permanently affecting actual output with nominal monetary policy actions. But what I also show is important is a stronger restriction that shocks that move flexible-price output are expected to affect the short-run path of actual output by the same proportion, leaving their proportionate difference — the output gap — untouched. By example, I show that many models in use (such as those with significant non-linearities due to built-in financial frictions) do not satisfy these latter restrictions.

⁵Note that this is consistent with the consensus view that routine monetary policy actions cannot affect flexible-price output.

1.3 What does this mean for the practical use of the output gap in understanding monetary transmission?

Whether it is the output gap rather than output that matters for objectives, our ultimate interest in measuring flexible-price output is that, in principle at least, it can help us to understand what shocks are happening to output and inflation and how monetary policy actions should be lined up against them. It follows that central banks are interested in forecasting and not just estimating flexible-price output. The distinction between estimating a model and forecasting with it is important. Forecasting involves a broader campaign to model the flexible-price level of output and not just to derive a data series of past values for it.

It follows from this that to measure the output gap one may need to look beyond output data itself. Not only output but also other macroeconomic variables have their flexible-price values; values that are consistent with no nominal stickiness in the economy. For example, the real exchange rate has a fundamental equilibrium value; the unemployment rate has a NAIRU; and the equilibrium real interest rate, a 'Wicksellian'⁶ value.⁷ Much monetary policy research has proposed that deviations of other variables from their flexible-price values can inform us about inflationary pressure. I suggest that it is useful to understand how flexible-price output is related to the flexible-price values of other variables, if only because there may be more data available on these other parts of the economy. To take us further in this direction, I develop a small, dynamic general equilibrium model that distinguishes between the flexible-price and actual economy. The model helps us to understand the link between the output gap and the 'real disequilibria'⁸ (the difference between the actual values and the flexible-price values) of other variables, as well as bringing out some practical messages for output gap measurement.

⁶Wicksell (1958) distinguished between the real rate of return on new capital, the "natural rate of interest" and the actual market rate of interest. The natural rate was a "certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them." See also Woodford (2000).

⁷For current papers measuring the equilibrium rate of interest, see for example Laubach and Williams (2001), Neiss and Nelson (2001) and Chadha and Nolan (2002).

⁸For an explanation of the term, see Astley and Yates (1999).

2 Formal definitions of the output gap and the monetary policy problem

2.1 A definition of flexible-price output and the role of nominal rigidities

The output gap is the difference between actual output and its potential level, and to define the output gap one must define potential output. Our favoured definition of potential output is *the flexible-price level of output*: the hypothetical level of GDP that would hold at time t if nominal variables, such as wages, prices or the nominal exchange rate, were always fully flexible now and in the past, and would be so in the future.⁹ Understanding the output gap helps identify the appropriate monetary policy stance; i.e. that is consistent with achieving the monetary policy target without incurring excessive output costs.

Our aim in this section is to formalise these intuitions about the purpose of potential output measurement in monetary policy forecasting. Later on in this section I derive analytic solutions for a simple theoretical model of an economy from micro-foundations to demonstrate our understanding of the output gap. But here I summarise the transmission mechanism in a more general form; describing it as the reduced-form solutions for the log of real output (y_t); inflation (π_t); all the other endogenous variables ($\mathbf{Z}_{1t}, \mathbf{Z}_{2t}$) as a function of their past values; the current, past and expected future values of the exogenous variables ($\mathbf{q}_{1t}, \mathbf{q}_{2t}$) and the nominal interest rates (i_t).

Writing $\mathbf{X}_{rt} \equiv (\{y_s\}_{s=-\infty}^{t-1}, \{\mathbf{Z}_{1s}\}_{s=-\infty}^{t-1})$ as the vector of all past values of endogenous real variables and $\mathbf{X}_{nt} \equiv (\{\pi_s\}_{s=-\infty}^{t-1}, \{\mathbf{Z}_{2s}\}_{s=-\infty}^{t-1})$ as the vector of all past values of all endogenous nominal values, the transmission mechanism in reduced form and conditional on interest rates is given by 1 to 6:

$$y_t = f_y(\varphi, \mathbf{X}_{rt}, E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^{\infty} \mid \mathbf{X}_{nt}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^{\infty}, E_{\varphi,t} \{i_s\}_{s=t}^{\infty}); \quad (1)$$

⁹This definition differs from that of Woodford (2004), Chapter 4, who allows for past nominal rigidities to determine current potential output. McCallum (2001) argues that if the capital is assumed to be given when prices are determined, the capital stock will always be equal to its flexible-price value. However Casares and McCallum (2001) show that the capital stock need not be taken as fixed in when deriving the microfoundations of price-setting. Investment and capital will depend on whether prices are flexible. See also the appendix to Neiss and Nelson (2001).

$$\pi_t = f_\pi (\boldsymbol{\varphi}, \mathbf{X}_{rt}, E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^\infty \mid \mathbf{X}_{nt}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^\infty, E_{\varphi,t} \{i_s\}_{s=t}); \quad (2)$$

$$\mathbf{Z}_{1t} = \mathbf{f}_{z1} (\boldsymbol{\varphi}, \mathbf{X}_{rt}, E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^\infty \mid \mathbf{X}_{nt}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^\infty, E_{\varphi,t} \{i_s\}_{s=t}); \quad (3)$$

$$\mathbf{Z}_{2t} = \mathbf{f}_{z2} (\boldsymbol{\varphi}, \mathbf{X}_{rt}, E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^\infty \mid \mathbf{X}_{nt}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^\infty, E_{\varphi,t} \{i_s\}_{s=t}); \quad (4)$$

$$\mathbf{q}_{1t} = \mathbf{f}_{q1} (\boldsymbol{\theta}_1, E_{\theta_1} \{\mathbf{q}_{1s}\}_{s=-\infty}^\infty); \quad (5)$$

and

$$\mathbf{q}_{2t} = \mathbf{f}_{q2} (\boldsymbol{\theta}_2, E_{\theta_2} \{\mathbf{q}_{2s}\}_{s=-\infty}^\infty). \quad (6)$$

The system described by 1 to 6 is a general formulation of the transmission mechanism and leaves out many interesting details. But it serves for the purpose of defining flexible-price output because it emphasises the difference between real and nominal sides of the economy. Defining nominal variables as those that can only be measured in contemporary units of domestic currency, either in levels or as rates of change, I designate \mathbf{Z}_{2t} as the vector of current values of all nominal endogenous variables except for the inflation rate (the objective) and the nominal interest rate (the policy instrument). Variables which are not nominal are all designated as real and \mathbf{Z}_{1t} is the vector of any other real endogenous variables apart from output (the objective) in the system. \mathbf{q}_{1t} is the vector of real exogenous variables and \mathbf{q}_{2t} is the vector of nominal exogenous variables.¹⁰ $E_{\varphi,t} z_{t+s}$ denotes the conditional expectation of a variable, z_{t+s} , formed at time t using the information set which is conditional on parameters $\boldsymbol{\varphi}$. I use this notation to be explicit about rigidities in the updating of information sets that are used to make expectations. Also note that all variable are defined in logs or as rates of change, apart from interest rates.

Nominal rigidities are what matter in defining flexible-price output. To emphasise this, I have partitioned the set of parameters of the system, $\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2)$

¹⁰We explain below in which sense these variables are exogenous.

and $\theta = (\theta_1, \theta_2)$, into either one of two categories.¹¹ (θ_2, φ_2) describes the set of parameters that describe only the costs of adjustment of nominal variables in the economy; these are the parameters that make agents care about nominal values. (θ_1, φ_1) refers to all other parameters. That I can separate parameters into two disjoint sets means that at least conceptually I can distinguish the parameters that imply only nominal rigidities from other parameters, even if as I shall show, theories of nominal rigidities often also imply real rigidities. The state of there being no costs of nominal adjustment is defined by parameter values of $(\theta_2, \varphi_2) = (\overline{\theta_2}, \overline{\varphi_2})$.

We only now need to determine the policy instrument to complete our description of the economy. The nominal interest rate, i_t , is set to minimise the central bank's loss function, subject to the central bank's understanding of the transmission mechanism and whilst taking the public's expectations of the sequence of interest rates as given.¹² The central bank's one-period loss function is assumed to be composed of a weighted average of the conditional variances of output and inflation, with the inflation variance being measured around a positive target rate. The infinite horizon objective is then minimise the sum of current and future one-period loss functions:¹³

$$E_{\varphi,t} \{i_s\}_{s=t}^{\infty} \text{ minimises } \sum_{s=t}^{\infty} E_{\varphi,s} (y_s)^2 + \lambda E_{\varphi,s} (\pi_s - \pi_L)^2, \quad (7)$$

given equations 1 to 6.

That output matters in objectives is a crucial assumption in our discussion of the output gap, but not a controversial one. Svensson (2001, page 65) argues that in practice, all inflation-targeting central banks are concerned, and should be concerned, about short-run output losses as well as inflation

¹¹A strict definition would be that a parameter, as opposed to a variable, does not change according to a given process. Even though it can change and the change can be analysed by comparative statics, the best predictor of its future values is always its past value. See Hoover (2001), page 171.

¹²We are assuming that the only time-consistent policy is the discretionary policy. The central bank takes the public's expectations of its own behaviour as given when setting its instrument.

¹³Woodford (2002), Chapter 6, follows a more rigorous approach in deriving the central bank objectives in terms of inflation and output gap volatility from starting point of the representative consumer's utility function. See also McCallum (1986), Section 5 for a discussion of when consumption variability incurs excessive welfare costs.

stabilisation. What is perhaps, more controversial is that I am assuming a quadratic loss function, where inflation and output are separable in objectives. As I show later, the assumption of a quadratic loss function matters if I want to reformulate objectives in terms of the output gap and inflation rather than output and inflation.¹⁴ Note also, that we are allowing for rigidities in the processing of information in the central bank’s forecast; we argue later that this can constitute an important type of nominal rigidity.

Monetary policy decisions are made under uncertainty, so I have included the underlying sources of uncertainty —both real and nominal shocks — that make the system described by 1 to 7 stochastic in \mathbf{q}_{1t} and \mathbf{q}_{2t} . For example a surprise shift in the target rate of inflation in an inflation-targeting regime that is unrelated to any other developments captured in the model would be a member of \mathbf{q}_{1t} in our characterisation, and a sudden change in weather conditions would constitute a member of \mathbf{q}_{2t} .¹⁵¹⁶

The factors that make up \mathbf{q}_{1t} and \mathbf{q}_{2t} are described as being “underlying” or “deep” in the sense that they are exogenous to the system. To be more precise, I am assuming that the parameters that govern the processes affecting each member of \mathbf{q}_{1t} and \mathbf{q}_{2t} do not depend on the parameters that affect any of the other variables in the system, either directly or through the influence of any other variable.¹⁷ Given our categorisation that each parameter must be either associated with nominal rigidities or not, this implies that parameter sets, $(\theta_1, \varphi_1, \theta_2, \varphi_2)$ are all disjoint with each other. In particular, the real exogenous variables are assumed not to depend on nominal rigidities, even in the short run.

Even if nominal shocks are unrelated to real shocks in this deep sense, their entangled effects on endogenous variables may be all that is visible to agents. As they are unable to fully discern which is which, agents’ *expectations* of real and nominal shocks can covary. This would constitute a nominal rigidity in the spirit of Lucas (1972) and Friedman (1968). There

¹⁴Recently al-Nowaihi and Stracca (2002) have discussed the implications of non-standard central bank loss functions.

¹⁵It could be argued that, strictly speaking, we need to develop a better understanding of what constitutes a superexogenous monetary policy shock (McCallum, 1999).

¹⁶We assume that the members of \mathbf{q}_{1t} and \mathbf{q}_{2t} that are stochastic follow zero mean-reverting distributions. That does not mean that all variables in the system are stationary, members of \mathbf{q}_{1t} and \mathbf{q}_{2t} could affect endogenous variables that are random walks.

¹⁷The relevant concept here is super exogeneity. See Hoover (2001) page 172, for example, for a definition.

could also be rigidities in the updating of information sets that agents use to make expectations of future nominal variables (Ball, 2000, Mankiw and Reis, 2001; Sargent, 1999;¹⁸ Sims, 2002). It takes resources to process and transmit economic information accurately, and so agents will not always take new releases of information at face value. Instead they prefer to rely to some extent on what they assumed in the past. For example the central bank may announce a shift in the inflation target but it may take time for agents to update their views as to monetary policy strategy in place. The slow convergence to a new regime could constitute a significant nominal rigidity as in Sargent(1999) chapter 3.¹⁹

With the actual economy described, I can now define what I mean by the flexible-price economy. Flexible-price output was said to be the level of output that would hold if there were no costs of adjustment of nominal variables in the past, at the current time and in the expected future. We assume that the time t value of flexible-price output (as with the flexible price value of other variables) exists and is unique and can then be written as:

$$y_t^* = f_y(\bar{\varphi}, \mathbf{X}_{rt}^*, E_{\bar{\varphi},t} \{\mathbf{q}_{1s}^*\}_{s=-\infty}^{\infty}); \quad (8)$$

where $\bar{\varphi} = (\varphi_1, \bar{\varphi}_2)$.

Note that flexible-price output is independent of the past, current or expected future values of the nominal monetary policy instrument, inflation, and other nominal variables.²⁰ Property 8 of the flexible-price economy would follow from any standard theory as to what constitutes a nominal rigidity. For example, in the micro-founded model of nominal rigidities such as those described in Mankiw and Romer (1994a), when I abstract from all costs of adjusting nominal variables, agents' decisions over real variables, such as how much consumption or investment to undertake, would only depend on real factors such as real prices and be otherwise independent of any nominal

¹⁸See also Soderstrom and Sargent (2002).

¹⁹Our set-up leads us to conceive of these as distinct from rigidities in updating on real variables, even if, as we show, the theories of imperfect updating of information do not as yet explain any different treatment between real and nominal variables.

²⁰Andersen (1994) describes the flexible-price state as being characterised by a 'scaling up or down of all nominal variables [that] leaves real variables (relative prices and quantities) unaffected.' This could be equivalent to our definition that is applied to the reduced form, but we would first have to be explicit as to what the scaling up or down of a nominal variable (in levels or differences?) would mean.

values.

We can also derive the flexible-price levels for all the other economic variables in the economy that are consistent with this definition of flexible-price output:

$$\pi_t = f_\pi \left(\bar{\varphi}, \mathbf{X}_{rt}^*, E_{\bar{\varphi},t} \{ \mathbf{q}_{1s}^* \}_{s=-\infty}^\infty \mid \mathbf{X}_{nt}^*, E_{\bar{\varphi},t} \{ \mathbf{q}_{2s}^* \}_{s=-\infty}^\infty, E_{\bar{\varphi},t} \{ i_s^* \}_{s=t}^\infty \right); \quad (9)$$

$$\mathbf{Z}_{1t} = \mathbf{f}_{z1} \left(\bar{\varphi}, \mathbf{X}_{rt}^*, E_{\bar{\varphi},t} \{ \mathbf{q}_{1s}^* \}_{s=-\infty}^\infty \right); \quad (10)$$

$$\mathbf{Z}_{2t} = \mathbf{f}_{z2} \left(\bar{\varphi}, \mathbf{X}_{rt}^*, E_{\bar{\varphi},t} \{ \mathbf{q}_{1s}^* \}_{s=-\infty}^\infty \mid \mathbf{X}_{nt}^*, E_{\bar{\varphi},t} \{ \mathbf{q}_{2s}^* \}_{s=-\infty}^\infty, E_{\bar{\varphi},t} \{ i_s^* \}_{s=t}^\infty \right); \quad (11)$$

$$\mathbf{q}_{1t}^* = \mathbf{q}_{1t}; \quad (12)$$

and

$$\mathbf{q}_{2t}^* = \mathbf{f}_{q2} \left(\bar{\theta}, E_{\bar{\theta}_2} \{ \mathbf{q}_{2s}^* \}_{s=-\infty}^\infty \right); \quad (13)$$

where $\bar{\theta} = (\bar{\theta}_1, \bar{\theta}_2)$.

As with output, the flexible-price levels of all other real variables are independent of current, future and past values of the nominal variables. Also we can see that the real exogenous variables ($\mathbf{q}_{1t} = \mathbf{q}_{1t}^*$) will always be equal to their actual values because they do not depend on whether there are nominal rigidities or not. The nominal exogenous variables depend on nominal rigidities, and \mathbf{q}_{2t} hence does not necessarily equal \mathbf{q}_{2t}^* .

Interest-rate setting in the flexible-price world is somewhat simpler than in the actual world, given the additively separable loss function. As the central bank will seek to minimise only what it can control, the central bank will only try to minimise the conditional variance of the rate of inflation about its target path.

Hence I can write that

$$E_{\bar{\varphi},t} \{ i_s^* \}_{s=t}^\infty \text{ minimises } \sum_{s=t}^\infty E_{\bar{\varphi},s} (\pi_s^* - \pi_L)^2, \quad (14)$$

given equations 8 to 13.

2.2 Why do we care about the output gap?

It is straightforward to answer to our question about why monetary policy makers want to measure flexible-price output, given the assumptions I have used so far. As flexible-price output is independent of nominal shocks and driven only by real shocks, measuring flexible-price output offers one route to identifying ‘real’ disturbances. As with many other instruments in the central bank’s toolkit; this would suggest that the purpose of calculating flexible-price output is to provide useful ‘conditioning’ information in forecasting future output and inflation movements; information that identifies what drives future output and inflation movements.

But flexible-price output is often held in regard as being more than just one of the many indicators that central banks produce uses to inform its forecast. It is said to be also valuable because it can make the policy objective on output more precise: given an accurate forecast of the flexible-price level, instruments can be set to minimise only the expected volatility of future output relative to the forecasted flexible-price level. This is formalised in descriptions of monetary policy models where the monetary policy objective function, or that instrument rule, has terms in the output gap rather than the level of growth rate of output.

In this section, I ask what assumptions on our model of the transmission mechanism can be used to justify this focus on the output gap in place of actual output in monetary policy objectives? These answers are important because they give us some guide as to what assumptions would be needed if I want a model of the transmission mechanism that features the output gap rather than output in its objectives. To summarise our findings, I show that two properties of our model are sufficient: that long-run nominal neutrality holds and that the output gap is independent of real shocks.

2.2.1 Long-run nominal neutrality

Long-run nominal neutrality is an assumption that determines the values that variables take when I abstract from nominal shocks only. More precisely, it requires that the parameters that describe the costs of nominal adjustment are such that the expected values of all variables conditional only on the real exogenous uncertainty are their flexible-price values.²¹

²¹This assumption establishes only that the economy will converge to the unique long-run nominal neutral equilibrium. But we have not specified how long it would take for

Let z_t denote any member of the set of variables of the model: $(z_t \in (y_t, \pi_t, \mathbf{Z}_{1t}, \mathbf{Z}_{2t}, \mathbf{q}_{1t}, \mathbf{q}_{2t})')$ and let z_t^* denote the flexible price value of z_t . In terms of our set-up, I can write that if the economy displays long-run nominal neutrality, then the parameters in $(\boldsymbol{\theta}_2, \boldsymbol{\varphi}_2)$ are such that

$$z_t^* = E[z_t | E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^{\infty}]$$

for any $z_t \in (y_t, \pi_t, \mathbf{Z}_{1t}, \mathbf{Z}_{2t}, \mathbf{q}_{1t}, \mathbf{q}_{2t})'$ ²² (15)

where $E[z_t | I_t]$ denotes the expectation of z_t conditional on given values of the stochastic variables I_t and where the solution for z_t would be given from equations 1 to 7. As $\mathbf{q}_{1t}, \mathbf{q}_{2t}$ are the only sources of stochastic uncertainty, 15 describes expectations conditional on the stochastic distribution of \mathbf{q}_{1t} , and hence takes expectations across the nominal sources of uncertainty only.

If the equations in the model of the transmission mechanism were linear, then nominal neutrality would require that both *static* and *dynamic homogeneity* holds in all equations. To define static and dynamic homogeneity, I first must categorise all expressions involving nominal variables in our model into either dynamic and level terms. A dynamic term is one which will have the same order of difference stationarity as the inflation target; e.g. interest rates, wage inflation, nominal GDP growth, nominal exchange rate depreciation. A level term is one which has the same order of difference stationarity as the long-run domestic price (log) level; and so one more degree of difference stationarity than the inflation target. Static homogeneity means that the sum of the coefficients on all level nominal variables on the righthandside of the equation of interest must be equal to the sum of the coefficients on all level nominal variables on the lefthandside. Dynamic homogeneity means that the sum of the coefficients on all dynamic nominal variables on the righthandside of the equation must be equal to the sum of the coefficients on all dynamic nominal variables on the lefthandside. If I conduct experiments where the interest rate shifts temporarily, then for nominal neutrality I do not want any real variables to be affected. For this to happen, every equation in our model must satisfy static homogeneity. If I conduct experiments where the interest rate shifts permanently- a permanent disinflation, for example- then for nominal neutrality I do not want any real variables to be affected

variables to be expected to return to their nominal neutral values following a nominal shock.

permanently. For this to happen, every equation in our model must satisfy dynamic homogeneity.

What long-run nominal neutrality implies is that after abstracting from the current and past and expected future nominal uncertainty the expected value of output is its flexible-price value.²³ We have already assumed that a unique flexible-price state exists; neutrality implies that in the absence of nominal shocks, the economy will by itself converge to the flexible-price state. A crucial property of models that display long-run nominal neutrality follows: monetary policy-makers can aim to minimise the variance of output, given that the expected value of output (an expectation that is conditional on real uncertainty only) is invariant to different settings of the nominal monetary policy instruments.

2.2.2 Independence of the output gap and flexible-price output

But long-run nominal neutrality by itself is not enough to justify focus only on the volatility of the output gap rather than output as a whole over the monetary policy horizon. It may be that the transmission of monetary policy itself can depend on what real shocks are expected to hit the economy. Thus it is also important to think about when and when not the expected output gap is independent of expected real shocks.

One set of assumptions that ensures independence between the gap and real shocks is that the model (as it is written in equations 1 to 7; in logs and as a reduced form) is

1. linear in the endogeneous variables;
2. additively separable in the exogenous variables;
3. and such that the parameters determining the role of the endogenous variables and real variables in the system are not those that characterise nominal rigidities²⁴

²³Grandmont (1989), page 5 presents this assumption as 'when demand shocks lead to multipliers that do not rely on such supply-side effects.' He presents a model where neither long-run nominal neutrality nor independence holds, because of increasing returns to scale and endogenous expectations-driven fluctuations.

²⁴This is a sufficient assumption, but not a necessary one. Weaker conditions would refer to the bounds on the error in approximating the system with functions of the form 16. See Woodford (2004), Chapter 6.

We can write these set of assumptions as requiring that

$$\begin{aligned}
& f_z(\boldsymbol{\varphi}, \mathbf{X}_{rt}, E_{\varphi,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^{\infty} | \mathbf{X}_{nt}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^{\infty}, E_{\varphi,t} \{i_s\}_{s=t}^{\infty}) \\
= & g_z(\boldsymbol{\varphi}_1, \mathbf{X}_{rt} | \mathbf{X}_{nt}) + h_{rz}(\boldsymbol{\varphi}_1, E_{\varphi_1,t} \{\mathbf{q}_{1s}\}_{s=-\infty}^{\infty}) \\
& + h_{nz}(\boldsymbol{\varphi}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^{\infty}); \tag{16}
\end{aligned}$$

where f_z is the function determining the variable z_s , ($z_s \in (y_s, \pi_s, \mathbf{Z}_{1s}, \mathbf{Z}_{2s})$); g_z is a linear function and f_z , g_z and h_z are all well-behaved functions.

Let us denote the deviation of a variable, z , from its flexible price value, z^* , as \tilde{z} and the difference between a function of the actual values of variables, $f_z(\mathbf{x})$, and the same function of their flexible-price values, $f_z(\mathbf{x}^*)$, as $\tilde{f}_z(\mathbf{x})$. Using the independence assumption 16, I can subtract our expression for the actual value of a variable from that for the flexible-price value to show that the gap in the variable is independent of the real exogenous variables:

$$\begin{aligned}
\tilde{z} = & g_z(\boldsymbol{\varphi}_1, \tilde{\mathbf{X}}_{rt} | \tilde{\mathbf{X}}_{nt}) \\
& + h_{nz}(\boldsymbol{\varphi}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^{\infty}) - h_{nz}(\bar{\boldsymbol{\varphi}}, E_{\bar{\varphi},t} \{\mathbf{q}_{2s}^*\}_{s=-\infty}^{\infty}). \tag{17}
\end{aligned}$$

Applying this to output, 18 shows that the output gap, $\tilde{y}_t = y_t - y_t^*$, is independent of the real exogenous variables:

$$\begin{aligned}
\tilde{y}_t = & g_y(\boldsymbol{\varphi}_1, \tilde{\mathbf{X}}_{rt} | \tilde{\mathbf{X}}_{nt}) \\
& + h_{ny}(\boldsymbol{\varphi}, E_{\varphi,t} \{\mathbf{q}_{2s}\}_{s=-\infty}^{\infty}) - h_{ny}(\bar{\boldsymbol{\varphi}}, E_{\bar{\varphi},t} \{\mathbf{q}_{2s}^*\}_{s=-\infty}^{\infty}). \tag{18}
\end{aligned}$$

Similarly the past, current and expected future deviations of all other endogenous variables from their flexible-price values are independent of real sources of uncertainty and only driven by the stochastic nominal shocks, given our assumption 16.

Comparing the equation for the output gap, 18, with that of flexible-price output, 8, I can see that now each is driven by an entirely different source of uncertainty: flexible-price output is driven by real shocks only and the output gap, by nominal shocks only. The expected distributions for the two variables are hence statistically independent, and for example, the conditional covariance between the two will be zero.

Now if I turn our attention back to the loss function of the central bank, the conditional variance in actual output can be decomposed in terms of the conditional variances of the output gap, and flexible-price output and the covariance between the two:

$$\begin{aligned}
& E_{\varphi,t} \sum_{s=t}^{\infty} (y_s - y_s^*)^2 + E_{\varphi,t} \sum_{s=t}^{\infty} (y_s^*)^2 + E_{\varphi,t} \sum_{s=t}^{\infty} (y_s - y_s^*) (y_s^*) \\
& + \lambda E_{\varphi,t} \sum_{s=t}^{\infty} (\pi_s - \pi_L)^2.
\end{aligned} \tag{19}$$

As the covariance of flexible-price output and the output gap is zero and as the central bank cannot affect the variance of flexible-price output, I can now write the objectives of the central bank in terms of the conditional variance of the output gap only:

$$E_{\varphi,t} \sum_{s=t}^{\infty} (y_s - y_s^*)^2 + \lambda E_{\varphi,t} \sum_{s=t}^{\infty} (\pi_s - \pi_L)^2. \tag{20}$$

The objective of the central bank can therefore be equivalently written in terms of the output gap rather than output providing the model of the economy is such that nominal neutrality holds and that the model satisfies properties 1 to 3 above. Clearly this role of potential output measurement is distinct from the goal of measuring output gaps to provide conditioning information on the expected future inflation and output. Its purpose is to simplify the output objective over the uncertain horizon. The supposed advantage of concentrating on the gap rather than output would simply be that the unconditional variance of the output gap (assuming that errors in measuring and predicting it are not too large) is less than that of actual output.

3 Implications

To summarise, I will now run through what I shown with our general framework in the previous section.

3.0.2.1 What is flexible-price output? First, I have shown how the concept of flexible-price output is defined by the absence of nominal rigidities, currently, in the past and in the expected future.

3.0.2.2 Why do we want to measure flexible-price output? Second, I discussed why flexible-price output measurement is important for monetary policy. It can both serve as a shock identification device and also it can sharpen objectives. But there are important conditions which our economies and hence the monetary policy models that capture them must satisfy for both to be true.

These two implications were derived from the previous section. But there are other, more practical, implications from the framework that I can now turn out.

3.0.2.3 Theory versus data in measuring potential output The schema above points to two routes to forecasting flexible-price output. The first, more data-based method, follows from the long-run nominal neutrality assumption 15. Neutrality implies that if we were able to measure over a long enough (or more precisely, informative enough) data set for nominal shocks to cancel out, and providing that I correctly condition on the effects of real shocks, the average value of variables would be equal to the flexible-price values. This suggests that we can use these correctly conditional averages as estimates of the process for potential output. If we can project forecasts of the effects of real shocks into the policy horizon, this method can be used to derive a forecast for potential output.

The other, more theory-based approach, would be to build a model of our economy in which we have separately identified the nominal rigidities inherent in the economy. The path of output that would follow when we solve the model after setting the nominal rigidity parameters to values consistent with no nominal rigidities would then describe flexible-price output. With nominal effects absent, the forecast would leave us with having to model real shocks and their dynamic effects.

In principle both methods can be consistent with each other. And both crucially depend on an accurate understanding and assumptions about real shocks and effects. However the first method builds on the assumption that the effects of nominal rigidities 'cancel' out within the sample of data that we have, whilst the second would favour using more theory on sticky nominal

adjustments.

Broadly speaking the data-based method lends itself to using statistical methods to deal with nominal shocks. The emphasis is here on estimating the output gap through utilising only a few assumptions about how the cyclical nature of the output gap differs from that of potential output. Typically only a single time series of GDP or other output data is used.

The danger with these data-based methods is that estimates of the output gap based on single time-series estimations are highly sensitive to what we assume about the cyclical properties of potential output, as Quah (1986) formally demonstrated. They can also depend on what sample of data is used. For this reason, output gap researchers following data-based techniques frequently find that the most recent measured output gap is highly sensitive to seemingly innocuous changes in the technique used.

This need not be as disheartening as it first sounds: many of the econometric estimates that policymakers work with are sensitive to the assumptions that are used to derive them. But what is particularly disconcerting with data-based potential output forecasting is that methodological differences that drive results tend to have little economic content.

The second theory-based route relies more on imposing theory in estimating the output gap and correspondingly placing less weight on the single time series of output data. A seemingly inevitable and indeed desirable side-effect of bringing more theory on board is that the output gap is estimated utilising information on what is happening to inflation, even to the extent that potential output is estimated jointly in a system with the Phillips curve. According to the more theory-based approaches, a system that explains real output and inflation (and possibly other important variables) is estimated and identified, and the scrutiny is directed towards explaining how underlying shocks affect inflation, output and other variables differently.²⁵

The framework in the previous section explained that the theories that matter when we want to separate actual from potential (flexible-price) output movements are those that refer to nominal rigidities. It follows that the theory-based approach to measuring potential output depends critically on our ability to quantify theories of nominal rigidities. The major challenge here is to be able to separately identify and understand short-run fluctuations in flexible-price output from those due to nominal shocks.²⁶ Both types of

²⁵See, for example Adams and Coe (1990), Kuttner (1994), Astley and Yates (1999), Blanchard and Quah (1989), Bayoumi and Sterne (1993) and Haltmaier (1996).

²⁶In the literature, reservations have been expressed as whether currently available theo-

fluctuations can be present at a similar frequency in the data. For example developing countries which tend to be more dependent on the production conditions of particular commodities and therefore more sensitive to temporary supply-side shocks caused by weather conditions. Local circumstances must then matter in drawing out differences between the fluctuations in output and inflation that are due to nominal rigidities alone and those that are due to supply-side shocks.

3.0.2.4 The balanced growth path versus flexible-price output I have conceived of the output gap as the deviation from flexible-price output. Many monetary policy researchers follow the real business cycle literature in considering the output gap in terms of deviations from a *balanced-growth path equilibrium*, which is often referred to as a steady state. The difference is that the balanced-growth equilibrium is itself an equilibrium of the flexible-price economy, one in which there are not only no nominal rigidities but in addition with the restriction that, when we abstract from the uncertainty in real exogenous variables, all real variables are expected to be either constant or grow at a constant rate.

When compared to what is required to model the flexible-price state, it seems relatively straightforward to calculate and forecast the balanced-growth state. The defining assumption of constant growth rates of output and capital and labour supply lend themselves easily to producing forecasts for these variables, once we assume that technical progress is either a smooth exponential process or a random walk.²⁷ Modelling the (more general) flexible-price state would require us to separately identify and quantify real frictions: the adjustment costs that would affect real variables even in the absence of nominal rigidities.

But even if they are easier to produce, forecasts based on balanced growth assumptions may be inappropriate. That is because over the horizons that monetary policy operates, the actual economy can be far from the balanced growth path. In particular, during many decades of economic development, growth seems to be punctuated by intermittent shifts and phases that take

retical understanding of macroeconomic dynamics, many of which have real business cycle models as a distant ancestors, are adequately designed to quantify the potential output definition that is relevant for monetary policy (McCallum, 2000).

²⁷For example, Church et al (2000) compare the effect of developments in technical progress as described by three different models of the UK economy.

longer than a few years to work through and which can be explained by disequilibrium dynamics.

Typical examples of factors that especially impact on developing countries and make the process of development unsteady include:

a) the production conditions of primary and agricultural commodities, which may have to do with climate;

b) administered price changes which, like any other relative price movement, influence the composition of the consumers' basket and can induce changes in real income;²⁸

c) changes in the world prices of important imported inputs and exported goods;

d) shifts in world demand for exports;

e) and international financial markets' appetite for investment in emerging market economies.

We can be of course be cheered by reminding ourselves that the first-round effects of many of these real or these supply-side processes are observable (Pesaran and Smith, 1999). For example, we know about current weather conditions and may even have reliable forecasts of future weather. For primary commodity producing countries, forecasts of the world price of their exports are also available, as are oil prices for oil-importing manufacturers, for example. Institutions such as the IMF and the OECD produce forecasts of US GDP, and world real interest rate and emerging market risk premium can to some extent be measured from available data.

But that task becomes more complicated when these flexible-price developments can feed through to have 'second-round effects' on price and output. For example if the primary commodity export is produced under a government monopoly, the government may alter its spending plans when the world price for that good changes.²⁹ If this then threatens price stabil-

²⁸See for example Fischer (1981) for references and a discussion of the theory of relative price movements and inflation and Mohanty and Klau (2000) for a survey of evidence in emerging market economies.

²⁹Tracing the effect of export price movements on output or domestic prices can require careful modelling. An export price shock will only boost real output in so far as it boosts export volumes; and production conditions (supply elasticities) must be crucial here. For example, the nominal wage bargain and labour supply in the export sector may also depend on the terms of trade (Bean, 1986). Following the export price shocks, there could even be a deflationary effect on prices if foreign investors aim to take advantage of any rise in profits and the exchange rate appreciates in response to the ensuing capital inflow. For an example of a developing country experience see Perera's (1984) account of Sri Lanka's

ity, a monetary policy response may be appropriate, making the modelling of the second-round effect a crucial issue. Often the second-round effects of even those shocks that have very apparent first-round effects may have to be traced as they reverberate through the transmission mechanism and it may require a model to do so.

One useful tactic could be to decompose potential output into its inputs by using an assumption as to how they are combined in production. This is the essence of the production function approach to potential output measurement.³⁰ The advantage is that we can separate out the effect of technical progress on output from the contributions of flexible-price values of marketable inputs, say labour and capital. By isolating technical progress and assuming that is exogenous, it may then be adequately modelled by a smooth trend or random walk.

But this still begs the question of how to determine the flexible-price quantities of the other production inputs, in particular capital and labour. Time-series approaches may not help if the quantities of these inputs are away from the balanced growth path in much of the sample and if the slow adjustment to the balanced growth path is not exogenous but rather depends on other economic factors.

To begin with, investment dynamics can be related to other economic factors, because of the sunk costs and lumpiness involved.³¹ For example, if firms can vary the intensity with which they use that capital to maximise profits (Basu (1995), Neiss and Nelson (2001), Chadha and Nolan (2002)) the path that the flexible-price capital stock takes to return to the balanced growth state can depend on output and employment adjustment.

A separate important consideration is that as physical capital investment has to be financed in imperfect capital markets, factors that shape the access to financial capital and bank credit may become important inputs for the running of the firm. The state of the financial system and its interaction with macroeconomic variables can affect flexible-price output especially if these financial markets are subject to significant imperfections. The flexible-price prices and quantities of financial flows may require explicit treatment if the flexible-price capital stock is to be modelled.³²

response to fluctuations in the tea price. Cufer, Mahadeva and Sterne (2000) discuss the second-round effects of administered price changes in Slovenia.

³⁰See Torres and Martin (1997) for an example.

³¹Cabellero (1999) surveys the modelling of investment.

³²A separate issue as to whether credit and financial market imperfections matter in the

Analogously, in the flexible-price state, labour supply (in heads or hours or effort) may also depend on factors that determine the wage bargainers' choices over the share of real wages, such as labour market legislation and unemployment benefits.³³ But more generally the different components of flexible-price labour input (working population, hours, effort and human capital) can be affected by war, immigration, demographics, health, development in the informal sector and education for example. These circumstances really do seem to matter for a monetary policy understanding of the developments of labour input in developing countries, and should deserve some consideration in our models.

As for other inputs, the intensity of usage of intermediate inputs in modern systems of roundabout production can vary with macroeconomic factors (Basu, 1994). And models of imperfect competition stress how barriers to entry and other real market structure factors or preferences (and not just monetary factors) affect the share of profit above costs, even in the flexible-price world.

To summarise, I have shown that by interpreting potential output as output in the counterfactual flexible-price state, we can derive important lessons about forecasting potential output in developing countries. One ben-

flexible-price state is whether they increase or decrease the sensitivity of investment and output to nominal shocks.

In a simple model of a developed economy, Bean et al (2002) demonstrate that if financial frictions make the model nonlinear in endogenous factors (in our set-up, this would represent a departure from assumption 16), then the transmission of monetary policy can be amplified. Real shocks will also distort the transmission of nominal shocks onto inflation and output.

In developing countries, financial frictions can sometimes act to diminish rather than amplify transmission.

Montiel (1991) and Green and Murinde (1992) discuss monetary transmission when unofficial curb markets for foreign exchange and loans co-exist with financially repressed (with a low real interest rate) formal markets. See also Burkett and Vogel (1992) for a discussion of how investment decisions are made in these financially repressed economies. Kamin et al (1995) page 44 provide a broader discussion of credit market imperfections in emerging markets and Meltzer (1995, 2001) surveys the implications of these imperfections for monetary transmission more generally. Borio (1995) and Borio and Fritz (1996) provide some evidence from industrialised countries. Finally, many theories of economic growth now emphasise the role of institutions (and investors' perceptions of a country's institutions), and not just the initial level of capital, in affecting the incorporation of financial capital originating from abroad and thus the flows of capital in and out of national borders.

³³See Layard et al.(1991), Nickell (1996), Manning (1993) for example.

efit is that the forecasted values of real variables in the flexible-price state can more realistically display non-constant growth rates. But we would also be encouraged to make and defend assumptions about the dynamics of the flexible-price economy: tackling the implications of rich features such foreign investment; financial market development; government pricing; agriculture production and trade conditions; and labour supply, to refer to but a few important aspects. In many cases, there would seem to be much to gain from tackling the dynamic effects of these important developmental rigidities explicitly .

3.0.2.5 The flexible-price values of other real variables The flexible-price level of output is related to the flexible-price levels of other real variables such as unemployment, money velocity, or the real exchange rate. As with output, the deviations of actual from flexible-price level of these other real variables can in principle tell us about nominal shocks independent of the effects of real disturbances if long-run neutrality holds and if the model is additively separable in real exogenous factors.

1. In Chapter 3 of this volume, and also Woodford (1999), Neiss and Nelson (2001) and Chadha and Nolan (2002), *the equilibrium or Wick-sellian real interest rate* refers to where the real interest rate would be if prices were flexible. The gap between this, equilibrium real rate and the actual real rate, may tell us about the output gap, and also more directly about where real interest rates should be heading.
2. *The fundamental equilibrium exchange rate* (and related equilibrium exchange rate concepts) tell us what the real exchange rate would be if prices were flexible, and hence where the real exchange rate should be heading.
3. Sbordone (2000), Galí and Gertler, (1999); Galí, (2000,2002) and Batini et al., (2000) for example have argued that *the mark-up of prices on unit labour costs* may tell us about where inflation will head.
4. *The deviation of unemployment from its natural rate* should provide valuable information about wage, and presumably inflationary pressure (Ball and Mankiw, 2002; Budd et al., 1988 and Nickell, 1996).³⁴

³⁴A typical assumption used to identify the natural rate in these models is that inflation

5. *An unsustainable balance of trade deficit* may be used as a proxy for inflationary pressure when both excessive import growth and inflation are symptoms of excess demand. In some economies, especially developing countries with fixed exchange rates, a sustainable trade balance becomes an intermediate objective in its own right, as it determines the sustainability of the exchange rate regime.³⁵
6. Finally, and most importantly for traditional developing country central bank models, *the deviation of the velocity of money from the value that is compatible with growth being sustainable and inflation being at target* can, under certain conditions, tell us about whether policy should be tightened or not. As Polak (1998) and Mussa and Savastano (1999) explain the modelling of this real disequilibria in velocity is at the heart of the IMF Monetary Programming Framework. Explicit recognition is made that real, structural factors can affect the flexible-price velocity, such as remonetization and financial sector reform, and these are distinguished from inflationary shifts in velocity (De Broek et al, 1997).

The advantage of using these measures is that the data on these variables may be more readily available than GDP data. That is obviously an important motivation for central banks in developing countries. However there are also reasons why we should be cautious about the use of these measures in isolation.

- First, as these concepts are linked to the GDP gap, some of them — for example the equilibrium velocity of money demand—will themselves depend on having a reliable estimate of potential output (IMF,1996).
- Many of these concepts are dogged by the same measurement problems as would be faced in calculating the output gap. These measurement problems are common because the task is similar: we are trying to

is constant in the flexible-price state, hence the term "non-accelerating inflationary rate of unemployment". But Nickell (1988) argues against automatically linking a stable level of inflation with unemployment being at its natural rate in macroeconomic models.

³⁵Assessing the sustainability of fiscal deficits (both sovereign and private sector) is an important aspect of macroeconomic modelling on developing countries. As Hagemann (1999) points out, in his explanation of the IMF methodology, we need to distinguish the role of different shocks in affecting the forecasted balance when we assess the sustainability of a country's policy mix. Separating out the role of monetary factors by forecasting the public deficit in the flexible-price state is one such, potentially useful, identification.

measure different aspects of a hypothetical flexible-price economy for which no direct data exist! For example in using the balance of trade, how do we know that the growth of imports is excessive? Even large trade deficits can be sustainable: reflecting that the economy is importing capital to grow and catch up with its trading partners, or perhaps satisfying the greater consumption needs of its younger population.

- Finally if it is the output gap we are interested in, and if we have to enable any of these concepts to proxy the output gap, then we should make sure that the link between the two is clearly thought out. It is important to be aware that these links are not automatic and are conditional on what shocks are expected to be taking place. Excess demand does not always imply inflationary pressure: a large trade deficit detracts from the inflationary impact of excessive domestic demand on the GDP output. Similarly, even if unemployment falls well below the natural rate, that need not always be matched by a rise in the output gap. The fall in unemployment could reflect firms changing the way their staff work, perhaps with more or less overtime. Finally, measures of demand pressure based on real money aggregates are only appropriate if the money demand function and the effect of interest rates and money on consumption — that together link this monetary proxies to the output gap — are stable. As financial markets are liberalised, these monetary relationships may become unpredictable and other ways of guessing at potential output may have to be sought out.

All real disequilibria concepts relate to how far the economy is from where it will be if prices were flexible, and all these concepts are linked to the output gap. And they can certainly be useful, especially when GDP data is scarce. But they should not be seen as automatic indicators of demand pressure. The detective work of explaining what is causing recent and projected changes in any of these indicators and how they are linked cannot be neglected,³⁶ just as it can't be when we look at the aggregate GDP output gap.

³⁶There are many examples of papers that show how the real disequilibria of different variables can be related. For example see Fischer's (1988) study of disinflation in a small open economy or Joyce and Wren-Lewis (1990) linking between real exchange rate and labour market behaviour. Phelps (1999) links short-run fluctuations in flexible-price unemployment to difference in asset price evaluations.

4 A Dynamic General Equilibrium Model

In order to explain the implications 3.0.2.1 to 3.0.2.5 of the previous section, it may be helpful to demonstrate them in a simple model of the transmission mechanism derived from microfoundations. The model is simplified because it describes a closed economy, with only one asset and allows only for labour as the only marketable input into production. Population is fixed and I abstract from capital accumulation. Monetary policymakers only care about inflation and nominal rigidity arises only from an imperfect updating of the information used in setting wages. These assumptions are unrealistic and difficult to justify in developing countries. But I shall use them because they serve to demonstrate what more complicated and more appropriate models will also show.

4.1 Microeconomic foundations

There are four categories of economic decisions made in this economy. Workers choose leisure, consumption and asset holdings; firms choose employment and production; the central bank chooses nominal interest rates and wage-setters choose the nominal wage rate. We shall take each in turn, and use the first-order conditions to construct a general equilibrium model of the actual economy, which can be compared with its flexible-price state.

4.1.0.6 Consumption and leisure The objective of the i^{th} representative consumer/worker is to maximise his utility over an infinite horizon, where the infinite horizon utility function is given as

$$U_{it} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{(C_{is})^{1-\vartheta}}{1-\vartheta} + \mu_s \frac{(1-N_{is})^{1-\vartheta}}{1-\vartheta} \right) \quad (21)$$

with $\vartheta < 1$. N_{it} is the time spent in formal employment by the i^{th} representative worker/consumer, as opposed to leisure or informal employment. The real net income from his investments in the only asset is $(\frac{1+i_t-1}{P_t} B_{it-1} - \frac{B_{it}}{P_{it}})$ and earnings from formal employment is $(\frac{W_t}{P_t} N_{it})$.

The return (in terms of utility) of an hour of leisure or informal employment depends on technical progress, as in Correa, et al. (1995). We can write this as

$$\mu_t = A_t^{-(1-\vartheta)} \quad (22)$$

where A_t is technological progress.

The budget constraint of the i^{th} representative worker/consumer is therefore written as

$$P_t C_{it} = W_t N_{it} - B_{it} + (1 + i_{t-1}) B_{it-1}. \quad (23)$$

We can write the Lagrangean for this problem as

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{(C_{is})^{1-\vartheta}}{1-\vartheta} + \frac{(\mu_s (1 - N_{is}))^{1-\vartheta}}{1-\vartheta} \right) + \sum_{s=t}^{\infty} \lambda_s (P_s C_{is} - W_s N_{is} + B_{is} - (1 + i_{s-1}) B_{is-1}). \quad (24)$$

The first-order conditions give us an expression for labour supply,

$$\mu_s (1 - N_{is})^{-\vartheta} = (C_{is})^{-\vartheta} \frac{W_t}{P_t}; \quad (25)$$

an intertemporal equation for consumption,

$$C_{it} = E_t \left(\left(\frac{1 + i_t}{\beta} \right) \frac{P_t}{P_{t+1}} \right)^{\frac{1}{\vartheta}} C_{it+1}; \quad (26)$$

and the budget constraint,

$$P_t C_{it} = W_t N_{it} - B_{it} + (1 + i_{t-1}) B_{it-1}. \quad (27)$$

There are L identical consumer/workers and the supply of labour in heads is exogenous. Aggregating across them gives us the following expression for aggregate consumption and labour effort:

$$C_t = E_t \left(\left(\frac{1 + i_t}{\beta} \right) \frac{P_t}{P_{t+1}} \right)^{\frac{1}{\vartheta}} C_{t+1}; \quad (28)$$

$$1 - N_t = \frac{C_t}{L_t} \left(\frac{1}{\mu_t} \frac{W_t}{P_t} \right)^{-\frac{1}{\vartheta}}; \quad (29)$$

and

$$P_t C_t = W_t N_t L - B_t + (1 + i_{t-1}) B_{t-1}. \quad (30)$$

4.1.0.7 Monetary policy setting Our simple assumption about monetary policy is that the policymaker chooses to set the nominal rate of interest such that *expected* rate of inflation is always equal to a target path, ignoring any output losses whatsoever.³⁷

$$E_t p_{t+1} = p_t + \overline{\text{target}}_t. \quad (31)$$

The choice of policy rule is dictated by convenience rather than realism. Building in a more realistic rule that is in some sense optimal, given the transmission mechanism, or that allows for concern over short-run output volatility would certainly imply different solutions. But it would not alter the qualitative messages that this example manages to send out.

The targeted path of inflation is assumed to follow an autoregressive process about a constant mean rate, so that the planned readjustment to the long-run target ($\overline{\text{target}}$) following a shock is gradual:

$$\overline{\text{target}}_t = (1-\alpha)\overline{\text{target}} + \alpha \overline{\text{target}}_{t-1} + e_{nt}; \quad (32)$$

where e_{nt} is independently normally distributed with mean of zero and a variance of σ_n^2 .

4.1.0.8 Firms Turning to production, there is one firm that produces all domestic consumption. The owner of this firm chooses to maximise profits using a simple production function that is linear in the only input of production, effective labour (LN_t):

$$Y_t = A_t (LN_t). \quad (33)$$

Technological progress (A_t) follows a random walk with drift in logs, that I can write as:

$$a_t = a_{t-1} + e_{rt}; \quad (34)$$

where e_{rt} is normally distributed with a mean of zero and with a variance of σ_r^2 .

4.1.0.9 Information and expectations To close the model, I need to specify what information is available to agents at each moment in time. *The broadest information set* constructed at time t comprises the values of all

³⁷In what follows, lower case values indicated natural logs, except for interest rates.

variable dated at time t or earlier, including the values of the shocks e_{rt} and e_{nt} .³⁸

For the large part, the broadest information set is what agents use when making decisions at time t . For example central banks use this set in setting interest rates, and in appendix 6.1.1 I show that this implies a solution for inflation as³⁹

$$p_t - p_{t-1} = -e_{nt} + \overline{\text{target}} + \sum_{j=0}^{\infty} \alpha^j e_{n,t-1-j}. \quad (35)$$

The inflation solution shows that current inflation depends negatively on the current shock to the target: as policy makers aim to raise expected inflation in line with the a sudden ratchet up in its target, current inflation will fall. However if these changes in target are persistent ($\alpha > 0$), shocks will not be immediately reversed, and current inflation will also depend positively on past shocks.

4.1.0.10 Nominal wage setting Not *all* decisions are made with the broadest information set, though. The only nominal rigidity that characterises this economy arises from an imperfect updating of information that firms and workers use when they make expectations of the marginal revenue product in determining the nominal wage rate. Following Mankiw and Reis (2001), the updating of information on real variables is costly and is hence updated towards the broadest set available at the exogenous geometric rate of ζ_1 . On the nominal variables, updating takes place at the rate of ζ_2 .⁴⁰

³⁸This would have to be generalised if we wanted to allow for pre-announced shocks etc.

³⁹We assume that $E_t p_{t+T} = E_{t-1} p_{t+T}$, for a terminal date that is far enough into the future.

⁴⁰For example assume that information is updated at a rate ρ towards the broadest information set in forming expectations of a variable z_{t+s} at time t .

Then the time t expectations of z , $E_{t-k,\rho} z_{t+s}$, are given by $E_{t-k,\rho} z_{t+s} = \sum_{k=0}^{\infty} \rho^k E_{t-k} z_{t+s}$; where $E_{t-k} z_{t+s}$ denotes expectations of z_{t+s} , using the broadest set available at time $t-k$.

We can write the time t nominal wage rate in logs as:

$$w_t = (1 - \varsigma_1) \sum_{k=0}^{\infty} \varsigma_1^k E_{t-k} (y_t - l_t - n_t) + (1 - \varsigma_2) \sum_{k=0}^{\infty} \varsigma_2^k E_{t-k} (p_t). \quad (36)$$

Note that the parameter ς_2 by itself determines the degree of nominal rigidity in this model; if and only if $\varsigma_2 \neq 0$ will nominal values by themselves bear on real variables.

Rewriting 36 yields

$$w_t = p_t + y_t - l_t - n_t + (1 - \varsigma_1) \left(\sum_{k=0}^{\infty} \varsigma_1^k [E_{t-k} (y_t - l_t - n_t) - (y_t - l_t - n_t)] \right) + (1 - \varsigma_2) \left(\sum_{k=0}^{\infty} \varsigma_2^k [E_{t-k} (p_t) - p_t] \right). \quad (37)$$

37, 34 and 76 from the appendix imply that:

$$w_t = p_t + y_t - l_t - n_t + (1 - \varsigma_1) \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right) + \frac{(1 - \varsigma_2)}{1 - \alpha} \left(\sum_{k=1}^{\infty} \varsigma_2^k \sum_{i=0}^{k-1} \alpha^i e_{n,t-i} \right) \quad (38)$$

38 shows us that the real wage differs from effective labour productivity because of real rigidities and real shocks (the moving average term in the errors in predicting labour productivity) on one hand, and nominal rigidities and nominal shocks (the moving average error term in predicting the price level) on the other.

4.2 The model of the actual economy

Leaving aside the equation that determines bonds,⁴¹ and taking a linear approximation of labour supply,⁴² I can now rewrite the model of the actual economy in logs as

$$c_t = y_t; \quad (39)$$

$$-n_t = y_t - l_t - \frac{1}{\vartheta} (\ln(\mu_t) + w_t - p_t); \quad (40)$$

$$\ln(\mu_t) = -(1 - \vartheta) a_t; \quad (41)$$

$$y_t = a_t + l_t + n_t; \quad (42)$$

$$a_t = a_{t-1} + e_{rt}; \quad (43)$$

$$E_t p_{t+1} = p_t + \overline{\text{t arg et}} + e_{nt}; \quad (44)$$

$$i_t = \overline{\text{t arg et}} + e_{nt} + \vartheta E_t (y_t - y_{t+1}) + \ln \beta; \quad (45)$$

and

$$w_t = p_t + a_t - (1 - \varsigma_1) \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right) + \frac{(1 - \varsigma_2)}{1 - \alpha} \left(\sum_{k=1}^{\infty} \varsigma_2^k \sum_{i=0}^{k-1} \alpha^i e_{n,t-i} \right). \quad (46)$$

Assuming that the exogenous value of labour supply in heads is \bar{l} , I also have

$$l_t = \bar{l}. \quad (47)$$

⁴¹This would be written as

$$\begin{aligned} \exp(b_t) &= \exp(w_t + n_t + l_t) - \exp(p_t + y_t) \\ &\quad + (1 + i_{t-1}) * \exp(b_{t-1}). \end{aligned}$$

⁴²We are assuming that $-n_t \approx 1 - N_t$, which would be accurate only if a small proportion of hours were spent at work. This approximation is not innocuous; we discuss its ramifications in section 4.5 below

4.3 The model of the flexible-price economy

Following implication 3.0.2.1, the flexible-price state of the economy is as the above but with $\varsigma_2 = 0$. The flexible-price economy can be written as:

$$c_t^* = y_t^*; \quad (48)$$

$$-n_t^* = y_t^* - \bar{l} - \frac{1}{\vartheta} (\ln(\mu_t) + w_t^* - p_t^*); \quad (49)$$

$$y_t^* = a_t + \bar{l} + n_t^*; \quad (50)$$

$$p_t^* = p_t; \quad (51)$$

$$i_t^* = \overline{\text{target}} + e_{nt} + \vartheta E_t (y_t^* - y_{t+1}^*) + \ln \beta; \quad (52)$$

$$w_t^* = p_t^* + a_t - (1 - \varsigma_1) \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right); \quad (53)$$

and

$$l_t^* = \bar{l}. \quad (54)$$

We can combine 49,50 and 53 to solve for the flexible-price effective labour supply as:

$$-2n_t^* = a_t - \frac{1}{\vartheta} \left(\ln(\mu_t) + a_t - (1 - \varsigma_1) \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right) \right);$$

or, by rearranging, as

$$n_t^* = -\frac{a_t}{2} \left(1 - \frac{1}{\vartheta} \right) + \frac{1}{2\vartheta} \ln(\mu_t) - \frac{(1 - \varsigma_1)}{2\vartheta} \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right). \quad (55)$$

Substituting 55 into 50 produces an expression for flexible-price output:

$$y_t^* = a_t + \bar{l} - \frac{a_t}{2} \left(1 - \frac{1}{\vartheta} \right) + \frac{1}{2\vartheta} \ln(\mu_t) - \frac{(1 - \varsigma_1)}{2\vartheta} \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right). \quad (56)$$

Using 22 and rearranging gives us

$$y_t^* = a_t + \bar{l} - \frac{(1 - \varsigma_1)}{2\vartheta} \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right). \quad (57)$$

The solutions for flexible-price output in our example economy demonstrates the implications laid out in Section 3.0.2.4.

First, the flexible-price output need not be near its balanced growth path value, which is $(a_t + \bar{l})$, because real rigidities mean that the lagged effects of past productivity shocks still matter.

Second, all other real variables also have their flexible-price values, which convey similar but not the same information as flexible-price output. For example the flexible-price real interest rate is given in 52 by $r_t^* \equiv i_t^* - \bar{t} \arg e_t - e_{nt}$. Although it is related to the same real shocks that drives flexible-price output, the dynamics between the two differ. Also in this closed economy with monetary policy entirely directed at inflation, the flexible-price interest rate depends on intertemporal preference for current consumption, flexible-price output does not.

4.4 The output gap

Following a similar set of substitutions on the actual economy, I can show that actual output is given by

$$\begin{aligned}
y_t &= a_t + \bar{l} - \frac{(1 - \varsigma_1)}{2\vartheta} \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right) \\
&\quad + \frac{(1 - \varsigma_2)}{2\vartheta(1 - \alpha)} \left(\sum_{k=1}^{\infty} \varsigma_2^k \sum_{i=0}^{k-1} \alpha^i e_{n,t-i} \right). \tag{58}
\end{aligned}$$

Taking expectations of 58, conditional on real shocks, I note that

$$\begin{aligned}
&E [y_t \mid \{e_{rs}\}_{s=-\infty}^{\infty}] \\
&= a_t + \bar{l} - \frac{(1 - \varsigma_1)}{2\vartheta} \left(\sum_{k=1}^{\infty} \varsigma_1^k \sum_{s=1}^k e_{r,t-k+s} \right); \\
&= y_t^*. \tag{59}
\end{aligned}$$

providing that $\varsigma_2 < 1$.

Clearly the important linear restriction that ensures long-run nominal neutrality (implication 3.0.2.2) in this model is simply that the updating of information on nominal variables is convergent. If there are no more nominal

shocks, the information set used in wage setting will converge to the broadest set.

The output gap is given by subtracting 56 from 58:

$$\begin{aligned}
\tilde{y}_t &= \frac{(1 - \varsigma_2)}{2\vartheta(1 - \alpha)} \left(\sum_{k=1}^{\infty} \varsigma_2^k \sum_{i=0}^{k-1} \alpha^i e_{n,t-i} \right); \\
&= \frac{(1 - \varsigma_2)}{2\vartheta(1 - \alpha)} \left(\sum_{i=0}^{\infty} \frac{(\varsigma_2)^{i+1} \alpha^i}{1 - \varsigma_2} e_{n,t-i} \right); \\
&= \frac{\varsigma_2}{2\vartheta(1 - \alpha)} \left(\sum_{i=0}^{\infty} (\varsigma_2 \alpha)^i e_{n,t-i} \right).^{43} \tag{60}
\end{aligned}$$

Comparing 56 and 60, I can see that the output gap is driven only by nominal shocks and is therefore independent of flexible-price output, that is only driven by real shocks as explained in implication 3.0.2.2.

Note also that the smoothness of the output gap is affected both by the smoothness of the underlying monetary policy shock (α), and the rate of information updating (ς_2). This demonstrates that the cyclical properties of the output gap depend on the degree of the nominal rigidity.

4.5 Path-dependence of the output gap

We can also adapt this model to describe the circumstances when assumption 16 does not hold.

There are at least two plausible sets of circumstances under which the covariance of flexible-price output and the output gap may not be zero:

1. There may be non-linearities in the economy. One important reason for non-linear effects in the transmission mechanism is the presence of significant financial market frictions. Under endogenous models of financial market frictions, the spreads between interest rates on two assets are non-linearly related to the quantities of financial assets, physical capital, and thus to output.
2. A second reason is that the parameters that determine real rigidity may be related to parameters that determine nominal rigidity by economic theory. For example it may be that the information updating costs on real variables are restricted to be always equal to those on nominal

variables, unlike our version of the Mankiw and Reis model. Other forms of nominal rigidity that are common in the literature would could make the output gap dependent on real shocks.

To illustrate the effects of non-linearity note that when I assumed that $-n_t \approx 1 - N_t$, I acknowledged that this linear approximation that would be accurate only if a small proportion of hours were spent in formal employment. Taking logs of the original equation without using this approximation gives us

$$\ln(1 - \exp(n_t)) = c_t - l_t - \frac{1}{\vartheta} \left(\ln\left(\frac{1}{\mu_t}\right) + w_t - p_t \right). \quad (61)$$

Rearranging and substituting in from 22

$$\ln(1 - \exp(n_t^*)) = \frac{a_t}{\vartheta} + n_t^* - \frac{1}{\vartheta} (w_t^* - p_t^*). \quad (62)$$

Using 61 and 62 instead of 40 and 49 would lead us to depart from a world in which the path of output gap to its new equilibrium would not depend on real shocks. Although the only real exogenous variable in this model in logs (a_t) still enter this new model additively, the disequilibria in employment, $n_t - n_t^*$, and hence the output gap no longer just depends on nominal shocks.

We could similarly allow real exogenous uncertainty to enter non-linearly, and I would find that the output gap is no longer purely driven by nominal factors. For example if the parameter ϑ were not a fixed parameter but instead a real exogenous stochastic variable reflecting, say structural changes in hours of labour supply, then the output gap would be again be affected by these real shocks.

We can also illustrate the importance of our assumption of independence in the parameters determining real and nominal rigidity with a simple example. Instead of 36, assume that the time t nominal wage rate (in logs) is autoregressive:

$$w_t = \nu (y_t - l_t - n_t + p_t) + (1 - \nu) w_{t-1}. \quad (63)$$

The stickiness in wages, $0 < \nu < 1$, now determines the degree of nominal rigidity in the system.

Rewriting 63 gives us an expression for the real wage

$$\begin{aligned}
w_t - p_t &= -(1 - \nu) p_t + \sum_{i=1}^{\infty} \nu (1 - \nu)^i p_{t-i} + \sum_{i=0}^{\infty} \nu (1 - \nu)^i (y_{t-i} - l_{t-i} - n_{t-i}) \\
&= \sum_{i=1}^{\infty} \nu (1 - \nu)^i (p_{t-i} - p_t) + \sum_{i=0}^{\infty} \nu (1 - \nu)^i a_{t-i}. \tag{64}
\end{aligned}$$

The real wage depends on past inflation as well as the history of technical progress when $\nu < 1$. What expression 64 also shows that the rate of updating on nominal variables is related to the rate of updating on real variables. Now when I subtract the flexible-price wage, $w_t^* - p_t^* = y_t^* - l_t^* - n_t^* = a_t$, from the actual wage, the real disequilibrium in wages is shown to be dependent on both nominal and real shocks:

$$\tilde{w}_t - \tilde{p}_t = \sum_{i=1}^{\infty} \nu (1 - \nu)^i (p_{t-i} - p_t) + \sum_{i=0}^{\infty} \nu (1 - \nu)^i (a_{t-i} - a_t). \tag{65}$$

As the disequilibria in the real wage affects hours worked, the output gap will also depend on productivity shocks under this nominal rigidity.

We have shown that assumption 16 is not 'weak'; there are good reasons why the convergence path of the output gap, and hence the expected short-run costs of monetary policy actions, depend on what is happening to the flexible-price economy. Path-dependence could be a feature in many standard monetary policy models, although it remains an empirical issue as to quantitative significance. If it is important then the implication is that the central bank should forecast and understand potential output as well as the output gap when formulating policy, because even if potential output is independent of monetary policy actions, it can itself affect the output gap.

5 Conclusions

In this chapter I detailed the problems in measuring the output gap and assessed the costs and benefits of different strategies for dealing with this in formulating monetary policy. We began by explaining what potential output is supposed to capture, making reference to a counterfactual state of the economy in which there are no nominal rigidities. Local circumstances and data issues *do* seem to matter in the practice of potential output measurement, and according to our discussion of what potential output means,

they *should* matter. A key message is that as the potential output process is driven by structural economic factors, different strategies will work best in different environments –there is no globally successful technique.

The purpose of the output gap measurement is to separate out the role of demand from supply-side shocks in affecting output movements, and inform policy about the trade-off between output and inflation. What I have also shown is that even if the output gap can be well measured, there are good reasons to believe that if referred to by itself it can mislead about expected output trade-off. Even if central banks cannot affect the flexible-price economy with their systematic monetary policy actions, what is happening to flexible-price economy can impinge on the expected path of the output gap, and so we have to be aware of what is happening to both and how they interact. For example financial fragility can affect the expected output gap, because when firms and consumers are excessively indebted to banks, high real interest rates can lead to large output losses than otherwise. This explains why central banks devote resources to thinking about the role of productivity shocks and financial market imperfections in monetary policy transmission. They do not only want to forecast the supply-side, they also want to understand its interaction with the effects of nominal frictions.

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6.1 Appendix

6.1.1 Solving for Inflation

The monetary policy set-up is sufficiently simple so as to make the rational expectation solution of the model straightforward.⁴⁴ Taking logs of 31 and

⁴⁴Explicit solutions to some more complicated monetary policy set-ups in this framework may be derived using the analysis set out Gourieroux and Montfort (1990), chapter 12.

solving forward T periods, we have:

$$\begin{aligned}
p_t &= E_t p_{t+1} - t \arg et_t; \\
&= E_t p_{t+2} - E_t t \arg et_{t+1} - t \arg et_t; \\
&= E_t p_{t+T} - E_t \sum_{s=0}^{T-1} t \arg et_{t+s}.
\end{aligned} \tag{66}$$

Subtracting a similar expression for the time $t - 1$ log of the price level, and using

$$t \arg et_{t+s} = \alpha^{s+1} t \arg et_{t-1} + \sum_{j=0}^s \alpha^j [(1-\alpha) \overline{t \arg et} + e_{n,s+t-j}]; \tag{67}$$

gives the general solution for the rate of inflation as

$$\begin{aligned}
p_t - p_{t-1} &= - \sum_{s=0}^{T-1} (E_t t \arg et_{t+s} - E_{t-1} t \arg et_{t+s}) \\
&\quad + E_{t-1} t \arg et_{t-1} + E_t p_{t+T} - E_{t-1} p_{t+T}; \\
&= - \sum_{s=0}^{T-1} \left(E_t \sum_{j=0}^s \alpha^j [e_{n,s+t-j}] - E_{t-1} \sum_{j=0}^s \alpha^j [e_{n,s+t-j}] \right) \\
&\quad + E_{t-1} t \arg et_{t-1} + E_t p_{t+T} - E_{t-1} p_{t+T}.
\end{aligned} \tag{68}$$

We have assumed that agents follow rational expectations and that only time t and earlier variables are known with certainty at time t . We also assume that time $t + T$ is far enough forward such that $E_t p_{t+T} = E_{t-1} p_{t+T}$.

We can then write 68 as:

$$p_t = p_{t-1} - e_{nt} + t \arg et_{t-1}. \tag{69}$$

32 implies that

$$\begin{aligned}
t \arg et_{t-1} &= \alpha^{-\infty} t \arg et_0 + \sum_{j=0}^{\infty} \alpha^j [(1-\alpha) \overline{t \arg et} + e_{n,t-1-j}]; \\
&= \overline{t \arg et} + \sum_{j=0}^{\infty} \alpha^j e_{n,t-1-j}.
\end{aligned} \tag{70}$$

We can then write 69 as:

$$p_t = p_{t-1} - e_{nt} + \overline{t \arg et} + \sum_{j=0}^{\infty} \alpha^j e_{n,t-1-j}. \tag{71}$$

6.1.2 Solving for the Expectational Error for Inflation

We now wish to derive the expectation error in forecasting current prices when the broadest information set that was available k periods earlier is used.

To begin with, note that 32 implies that

$$t \arg et_{t+s} = \alpha^{s+k} t \arg et_{t-k} + \sum_{j=0}^{s+k-1} \alpha^j [(1-\alpha) \overline{t \arg et} + e_{n,s+t-j}]. \quad (72)$$

Substituting 72 into 66, I can write the price level as

$$p_t = E_t p_{t+T} - E_t \sum_{s=0}^{T-1} \left(\alpha^{s+k} t \arg et_{t-k} + \sum_{j=0}^{s+k-1} \alpha^j [(1-\alpha) \overline{t \arg et} + e_{n,s+t-j}] \right) \quad (73)$$

Taking expectations k periods earlier gives

$$E_{t-k} p_t = E_{t-k} p_{t+T} - E_{t-k} \sum_{s=0}^{T-1} \left(\alpha^{s+k} t \arg et_{t-k} + \sum_{j=0}^{s+k-1} \alpha^j [(1-\alpha) \overline{t \arg et} + e_{n,s+t-j}] \right). \quad (74)$$

Subtracting 73 from 74, I have

$$\begin{aligned} E_{t-k} p_t - p_t &= E_t \sum_{s=0}^{T-1} \sum_{j=0}^{s+k-1} \alpha^j e_{n,s+t-j} - E_{t-k} \sum_{s=0}^{T-1} \sum_{j=0}^{s+k-1} \alpha^j e_{n,s+t-j}; \\ &= \sum_{s=0}^{T-1} \sum_{j=s}^{s+k-1} \alpha^j e_{n,s+t-j}; \\ &= \sum_{s=0}^{T-1} \sum_{i=0}^{k-1} \alpha^{i+s} e_{n,t-i}. \end{aligned} \quad (75)$$

Taking T to be ∞ gives an expression that can be substituted to give 38 in the main text:

$$E_{t-k} p_t - p_t = \frac{1}{1-\alpha} \sum_{i=0}^{k-1} \alpha^i e_{n,t-i}. \quad (76)$$