Are Indexed Bonds a Remedy for Sudden Stops?*

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Abstract

Recent policy proposals call for setting up a benchmark indexed bond market to prevent 'Sudden Stops.' This paper analyzes the macroeconomic implications of these bonds using a general equilibrium model of a small open economy with financial frictions. In the absence of indexed bonds, negative shocks to productivity or to the terms of trade trigger Sudden Stops through a debt-deflation mechanism. This paper establishes that whether indexed bonds can help to prevent Sudden Stops depends on the "degree of indexation," or the percentage of the shock reflected in the return. Quantitative analysis calibrated to a typical emerging economy suggests that indexation can improve macroeconomic conditions only if the level of indexation is less than a critical value due to the imperfect nature of the hedge provided by these bonds. When indexation is higher than this critical value (as with full-indexation), "natural debt limits" become tighter, leading to higher precautionary savings. The increase in the volatility of the trade balance that accompanies the introduction of indexed bonds outweighs the improvement in the covariance of the trade balance with income, increasing consumption volatility. Additionally, we find that at high levels of indexation, the borrowing constraint can become suddenly binding following a *positive* shock, triggering a debt-deflation.

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1 Introduction

Liability dollarization[1] and frictions in world capital markets have played a key role in the emerging market crises or Sudden Stops of the last decade. Typically, these crises are triggered by sudden reversals of capital inflows that result in sharp real exchange rate depreciations and collapses in consumption. Figures 1, 2, and Table 4 document the Sudden Stops observed in Argentina, Chile, Mexico, and Turkey in the last decade. For example in 1994, Turkey experienced a Sudden Stop characterized by: 10% current account-GDP reversal, 10% consumption and GDP drops relative to their trends, and 31% real exchange rate depreciation.[2]

In an effort to remedy Sudden Stops, Caballero (2002, 2003) and Borensztein and Mauro (2004) propose the issuance of state contingent debt instruments by emerging market economies. Caballero (2002) argues that crises in some emerging economies are driven by external shocks (e.g., terms of trade shocks), and that contrary to their developed counterparts, these economies have difficulty absorbing these shocks due to imperfections in world capital markets. He argues that most emerging countries could reduce aggregate volatility in their economies and cut precautionary savings if they possessed debt instruments for which returns are contingent on the external shocks that trigger crises.[3] He suggests creating an indexed bond market in which bonds' returns are contingent on terms of trade shocks or commodity prices.[4] Borensztein and Mauro (2004) argue that GDP-indexed bonds could reduce the aggregate volatility and the likelihood of unsustainable debt-to-GDP levels in emerging economies. Hence, they argue that these bonds can help these countries avoid pro-cyclical fiscal policies.

This paper introduces indexed bonds into a quantitative general equilibrium model of a small open economy with financial frictions in order to analyze the implications of these bonds for macroeconomic fluctuations and Sudden Stops. The model incorporates financial frictions proposed in the Sudden Stops literature (Calvo (1998), Mendoza (2002), Mendoza and Smith (2005), Caballero and Krishnamurthy (2001), among others). In particular, the economy suffers from liability dollarization, international debt markets impose a borrowing constraint in the small

[1] Liability dollarization refers to the denomination of debt in units of tradables (i.e., hard currencies). Liability dollarization is common in emerging markets, where debt is denominated in units of tradables but partially leveraged on large non-tradables sectors.

[2] See Figures 1 and 2, Table 4 for further documentation of these empirical regularities (see Calvo et al. (2003) and Calvo and Reinhart (1999) for a more detailed empirical analysis).

[3] Precautionary savings refers to extra savings caused by financial markets being incomplete. Caballero (2002) points out that precautionary savings in emerging countries arise as excessive accumulation of foreign reserves.

[4] Caballero (2002) argues, for example, that Chile could index to copper prices, and that Mexico and Venezuela could index to oil prices.

open economy. This constraint limits debt to a fraction of the economy's total income valued at tradable goods prices. As established in Mendoza (2002), when the only available instrument is a non-indexed bond, an exogenous shock to productivity or to the terms of trade that renders the borrowing constraint binding triggers a Fisherian debt-deflation mechanism.[5] A binding borrowing constraint leads to a decline in tradables consumption relative to non-tradables consumption, inducing a fall in the relative price of non-tradables as well as a depreciation of the real exchange rate (RER). The decline in RER makes the constraint even more binding, creating a feedback mechanism that induces collapses in consumption and the RER, as well as a reversal in capital inflows.

Our analysis consists of two steps. The first step is to consider a one-sector economy in which agents receive persistent endowment shocks, credit markets are perfect but insurance markets are incomplete (henceforth frictionless one-sector model). Second, we analyze a two sector model with financial frictions that can produce Sudden Stops endogenously through the mechanism explained in the previous paragraph. The motivation for the first step is to simplify the model as much as possible in order to understand how the dynamics of the model with indexed bond differ from that of the one with non-indexed bond.[6] In this frictionless one-sector model, when the available instrument is only a non-indexed bond with a constant exogenous return, agents try to insure away income fluctuations with trade balance adjustments. Since insurance markets are incomplete, agents are not able to attain full-consumption smoothing, consumption is volatile, and correlation of consumption with income is positive. Furthermore, agents try to self-insure by engaging in precautionary savings. If the return of the bond is indexed to the exogenous income shock only, the insurance markets are only "partially complete." In order to have complete markets, either full set of state contingent assets such as Arrow securities should be available (i.e., there are as many assets as the states of nature) or the return of the bond should be state contingent (i.e., contingent on both the exogenous shock and the debt levels, see Section 3.1 for further discussion). Although indexed bonds partially complete the market, the hedge provided by these bonds are imperfect because they introduce interest rate fluctuations. Assessing whether the benefits (due to hedging) offset the costs (due to interest rate fluctuations) induced by indexed bonds requires quantitative analysis.

[5]See Mendoza and Smith (2005), and Mendoza (2005) for further analysis on Fisherian debt-deflation.

[6]This case can also be used to examine the role of indexed bonds in small open developed economies such as Australia and Sweden, which have relatively large tradables sectors and better access to international capital markets than most emerging market economies.

Our quantitative analysis of the frictionless one-sector model establishes that indexed bonds can reduce precautionary savings, volatility of consumption and correlation of consumption with income only if the "degree of indexation" of the bond (i.e., the percentage of the shock that is passed on to the bonds' return) is lower than a critical value. If it is higher than this threshold (as with full indexation), indexed bonds worsen these macroeconomic variables.

The changes in the precautionary savings is driven by the changes in "natural debt limit." Natural debt limit is the largest debt that the economy can support to guarantee non-negative consumption in the event that income is at its "catastrophic" level almost surely. Agents have strong incentives to avoid attaining levels of debt lower than natural debt limit, since these debt levels lead to infinitely negative utility in case of catastrophic income levels. In other words, by imposing this natural debt limit endogenously, agents ensure that non-positive consumption levels are attained with zero probability. The degree of indexation has a significant effect on determining the state of nature that defines catastrophic level of income, and whether implied natural debt limit is higher or lower than the case without indexation. With higher degrees of indexation, natural debt limit can be determined at a positive shock, because for example, if agents receive positive income shocks forever, they will receive higher endowment income but they will also pay higher interest rates. In the numerical analysis part, we find that for high values of the degree of indexation, the latter dominates the former, leading to higher natural debt limits. Higher natural debt limit creates stronger incentives for agents to save because, the amount of debt that agents would like to avoid will be higher.

The effect of indexation on consumption volatility can be analyzed by decomposing the variance of consumption. (Consider the budget constraint of such an economy $c_t = \exp(\varepsilon_t) - b_{t+1} + (1 + r + \varepsilon_t)b_t$ where b is bond holdings. Using this budget constraint, $var(c_t) = var(y_t) + var(tb_t) - 2cov(tb_t, y_t)$). On one hand, for a given income volatility, indexation increases the covariance of trade balance with income (since in good (bad) times indexation commands higher (lower) repayments to the rest of the world), which lowers the volatility of consumption. On the other hand, indexation increases the volatility of trade balance (due to introduction of interest rate fluctuations), which increases the volatility of consumption. Our analysis suggests that at high levels of indexation, increase in the variance of trade balance dominates the increase in the covariance of trade balance with income, which in turn increases consumption volatility.

This tradeoff is also preserved in the two sector model with financial frictions. In addition, in this model, the interaction of the indexed bonds with the financial frictions leads to additional benefits *and* costs. Specifically, when indexed bonds are in place, negative shocks can result in a relatively small decline in tradable consumption; as a result, the initial capital outflow is milder and the RER depreciation is weaker compared to a case with non-indexed bonds. The cushioning in the RER can help to contain the Fisherian debt-deflation process. While these bonds help relax the borrowing constraint in case of negative shocks, this time, an increase in debt repayment following a *positive* shock can lead to a larger need for borrowing, which can make the borrowing constraint suddenly binding, triggering a debt-deflation. Quantitative analysis of this model suggests, once again, that the degree of indexation needs to be lower than a critical value in order to smooth Sudden Stops. With indexation higher than this critical value, the latter effect dominates the former, hence lead to more detrimental effects of Sudden Stops. We also find that the degree of indexation that minimizes macroeconomic fluctuations and impact effect of Sudden Stops depends on the persistence and volatility of the exogenous shock triggering Sudden Stops, as well as the size of the non-tradables sector relative to its tradables sector; suggesting that the indexation level that maximizes benefit of indexed bonds needs to be country specific. Because an indexation level that is appropriate for one country in terms of its effectiveness at preventing Sudden Stops may not be effective for another and may even expose to higher risk of facing Sudden Stops.

Debt instruments indexed to real variables (i.e., GDP, commodity prices, etc.) have not been widely employed in international capital markets.[7] As Table 3 shows, only a few countries issued this type of instrument in the past. In the early 1990s, Bosnia and Herzegovina, Bulgaria, and Costa Rica issued bonds containing an element of indexation to GDP; at the same time, Mexico and Venezuela issued bonds indexed to oil. Since the late 1990s, Bulgaria has already swapped a portion of its debt with non-indexed bonds. France issued gold-indexed bonds in the early 1970s, but due to depreciation of the French Franc in subsequent years, the French government bore significant losses and halted issuance.[8] Although problems on the demand side have been emphasized in the literature as the primary reason for the limited issuance of indexed bonds, the supply of such bonds has always been thin, as countries have exhibited little interest in issuing them. Our results may also help to understand why it has been the case: countries may have been reluctant due to the imperfect hedge that these bonds provide.

[7]In terms of hedging perspective CPI-indexed bonds may not provide a hedge against income risks, since inflation is pro-cyclical.

[8]The French government paid 393 francs in interest payments for each bond issued, far more than the 70 francs originally planned (Atta-Mensah (2004)).

Several studies have explored the costs and benefits of indexed debt instruments in the context of public finance and optimal debt management.[9] As mentioned above, Borensztein and Mauro (2004) and Caballero (2003) drew attention to these instruments as possible vehicles to provide insurance benefits to emerging countries. Moreover, Caballero and Panageas (2003) quantified the potential welfare effects of credit lines offered to emerging countries. They modelled a one-sector model with collateral constraints where Sudden Stops are exogenous. They used this setup to explore the benefits of these credit lines in terms of smoothing Sudden Stops, interpreting them as akin to indexed bonds. This paper contributes to this literature by modelling indexed bonds explicitly in a dynamic stochastic general equilibrium model where Sudden Stops are endogenous. Endogenizing Sudden Stops reveals that the efficacy of indexed bonds in terms of preventing these crises depends on whether the benefits due to hedging outweigh the imperfections introduced by these bonds. Depending on the structure of indexation, we show that they can potentially amplify the effects of Sudden Stops.[10]

This paper is related to studies in several strands of macro and international finance literature. The model has several features common to the literature on precautionary saving and macroeconomic fluctuations (e.g., Aiyagari (1994), Hugget (1993)). The paper is also related to studies exploring business cycle fluctuations in small open economies (e.g., Mendoza (1991), Neumeyer and Perri (2005), Oviedo (2005), Uribe and Yue (2005)) from the perspective of analyzing how interest rate fluctuations change affect macroeconomic variables. In addition to the papers in the Sudden Stops literature, this paper is also related to follow up studies to this literature, including Calvo (2002), Durdu and Mendoza (2005), and Caballero and Panageas (2003), which investigate the role of relevant policies in terms of preventing Sudden Stops. Durdu and Mendoza (2005) explore the quantitative implications of price guarantees offered by international financial organizations on emerging market assets. They find that these guarantees may induce moral hazard among global investors, and conclude that the effectiveness of price guarantees depends on the elasticity of investors' demand as well as whether the guarantees are contingent on debt levels. Similarly, in this paper, we explore the potential imperfections that can be introduced by the issuance of indexed bonds, and derive the conditions under which such a policy could be effective in preventing Sudden Stops.

Earlier seminal studies that in financial innovation literature such as Shiller (1993) and Allen

[9]See, for instance, Barro (1995), Calvo(1988), Fischer (1975), among others

[10]Krugman (1998) and Froot et al. (1989) emphasize moral hazard problems that GDP indexation can introduce. Here, we point out other adverse effects that indexation can cause even in the absence of moral hazard.

and Gale (1994) analyze how creation of new class of "macro markets" can help to manage economic risks such as real estate bubbles, inflation, recessions, etc. and discusses what sorts of frictions can prevent the creation of these markets. This paper emphasizes possible imperfections in global markets, and points out under which conditions issuance of indexed bonds may not improve macroeconomic conditions for a given emerging market.

The rest of the paper proceeds as follows. The next section describes the full model environment. Section 3 presents the quantitative results of the frictionless one-sector model, and the two-sector model with financial frictions. We conclude and offer extensions in Section 4.

2 Model

In this section, we describe the general setup of the two sector model with financial frictions. The model with non-indexed bonds is similar to Mendoza (2002). Foreign debt is denominated in units of tradables and imperfect credit markets impose a borrowing constraint that limits external debt to a share of the value of total income in units of tradables (which therefore reflects changes in the relative price of non-tradables that is the model's RER).

Representative households receive a stochastic endowment of tradables and non-stochastic endowment of non-tradables, which are denoted $\exp(\varepsilon_t)y^T$ and y^N, respectively. $\exp(\varepsilon_t)$ is a shock to the world value of the mean tradables endowment that could represent a productivity shock or a terms-of-trade shock. In our model, $\varepsilon \in \mathcal{E} = [\varepsilon_1 < ... < \varepsilon_m]$ (where $\varepsilon_1 = -\varepsilon_m$) evolves according to an m-state symmetric Markov chain with transition matrix $\mathcal{P}$. Households derive utility from aggregate consumption (c), and maximize Epstein's (1983) stationary cardinal utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \exp\left[-\sum_{\tau=0}^{t-1} \gamma \log(1 + c_t) \right] u(c_t) \right\}. \tag{1}$$

Functional forms are given by:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{2}$$

$$c_t(c_t^T, c_t^N) = \left[\omega (c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu} \right]^{-\frac{1}{\mu}}. \tag{3}$$

The instantaneous utility function (2) is in constant relative risk aversion (CRRA) form with an inter-temporal elasticity of substitution $1/\sigma$. The consumption aggregator is represented in constant elasticity of substitution (CES) form, where $1/(1+\mu)$ is the elasticity of substitution between consumption of tradables and non-tradables and where ω is the CES weighing factor. $\exp\left[-\sum_{\tau=0}^{t-1}\gamma\log(1+c_t)\right]$ is an endogenous discount factor that is introduced to induce stationarity in consumption and asset dynamics. γ is the elasticity of the subjective discount factor with respect to consumption. Mendoza (1991) introduced preferences with endogenous discounting to quantitative small open economy models, and such preferences have since been widely used.[11]

The households' budget constraint is:

$$c_t^T + p_t^N c_t^N = \exp(\varepsilon_t)y^T + p_t^N y^N - b_{t+1} + (1 + r + \phi\varepsilon_t)b_t \tag{4}$$

where b_t is current bond holdings, $(1+r+\phi\varepsilon_t)$ is the gross return on bonds, and p_t^N is relative price of non-tradables. The indexation of the debt works as follows. Consider a case in which there are high and low states for tradables income. The return on the indexed bond is low in the bad state and high in the good one, but the mean of the return remains unchanged and equal to R.[12] When households' current bond holdings are negative, (i.e., when households are debtors) they pay less (more) in the event of a negative (positive) endowment shock. The standard assumption on modelling bond's return is to assume that indexation is one-to-one; i.e., the return of indexed bond is $1 + r + \varepsilon_t$ (see for example Borensztein and Mauro (2004)). Here, we consider a more flexible setup by assuming a flexible degree of indexation by introducing a parameter $\phi \in [0, 1]$, which measures the degree of indexation of the bond. In particular, the limiting case $\phi = 0$ yields the benchmark case with non-indexed bonds, while $\phi = 1$ is the full-indexation case. Notice that ϕ affects the variance of the bond's return (since $var(1 + r + \phi\varepsilon_t) = \phi^2 var(\varepsilon_t)$). As ϕ increases, the bond provides a better hedge against negative income shocks, but at the same time it introduces additional volatility by increasing the return's variance. As explained below, there is a critical degree of indexation beyond which the distortions due to the increased volatility of returns outweigh the benefits that indexed bonds introduce. In our quantitative experiments, we will characterize the value of ϕ; at which, the bond's benefits are maximized.

To simplify notation, we denote bond holdings as b_t regardless of whether bonds are non-indexed or indexed. As mentioned above, when ϕ is equal to zero, the bond boils down to a

[11] See Schmitt-Grohé and Uribe (2003) for other specifications employed for this purpose.

[12] Although return is indexed to terms of trade shock, our modeling approach potentially sheds light on the implications of RER indexation, as well. In our model, the aggregate price index (i.e., the RER) is an increasing function of the relative price of non-tradables (p^N), which is determined at equilibrium in response to endowment shocks.

non-indexed bond with a fixed gross return $R = 1+r$. This return is exogenous and equal to the world interest rate. When ϕ is greater than zero, it is an indexed bond with a state contingent return; i.e., it (imperfectly) hedges income fluctuations.

In addition to the budget constraint, foreign creditors impose the following borrowing constraint, which limits debt issuance as a share of total income at period t not to exceed κ:

$$b_{t+1} \geq -\kappa \left[\exp(\varepsilon_t) y^T + p_t^N y^N\right]. \tag{5}$$

The borrowing constraint takes a similar form as those used in the Sudden Stops literature in order to mimic the tightening of the available credit to emerging countries (see for example, Caballero and Krishnamurthy (2001), Mendoza (2002), Mendoza and Smith (2005), Caballero and Panageas (2003)). As Mendoza and Smith (2005) explain, although these types of borrowing constraints are not based upon a contracting problem between lenders and borrowers, they are realistic in the sense that they resemble the risk management tools used in international capital markets, such as Value-at-Risk models employed by investment banks.

The optimality conditions of the problem facing households are standard and can be reduced to the following equations:

$$U_c(t)\left(1 - \frac{\nu_t}{\lambda_t}\right) = \exp\left[-\gamma \log(1 + c_t)\right] E_t \left\{\frac{(1 + r + \phi\varepsilon_t)p_t^c}{p_{t+1}^c} U_c(t+1)\right\} \tag{6}$$

$$\frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu} = p_t^N \tag{7}$$

along with the budget constraint (4), the borrowing constraint (5), and the standard Kuhn-Tucker conditions. ν and λ are the Lagrange multipliers of the borrowing constraint and the budget constraint, respectively. U_c is the derivative of lifetime utility with respect to aggregate consumption. p_t^c is the CES price index of aggregate consumption in units of tradable consumption, which equals $\left[\omega^{\frac{1}{\mu+1}} + (1-\omega)^{\frac{1}{\mu+1}} (p^N)^{\frac{\mu}{\mu+1}}\right]^{\frac{1+\mu}{\mu}}$. Equation (6) is the standard Euler Equation equating marginal utility at date t to that of date $t+1$. Equation (7) equates the marginal rate of substitution between tradabales consumption and non-tradables consumption to the relative price of non-tradables.

3 Quantitative Analysis

We explore the model's dynamics in two steps. First, we examine the role that indexed bonds play in a standard one-sector model in which the problem of liability dollarization is excluded and there is no borrowing constraint. Then we introduce the two frictions back as in the complete model described above in order to examine the role that indexed bonds can play in reducing the adverse effects of liability dollarization and preventing Sudden Stops.

3.1 The frictionless one-sector model

In the frictionless one-sector version of the model, single indexed bond with returns indexed to the exogenous shock is not able to complete the market but just partially completes it by providing the agents with the means to hedge against fluctuations in endowment income. If we call $(1+r+\phi\varepsilon)b_t$ financial income, the underlying goal to complete the market would be to keep the sum of endowment and financial incomes constant and equal to the mean endowment income, i.e., $\exp(\varepsilon_t)y^T + (1+r+\phi\varepsilon)b_t = y^T$. Clearly, we can keep this sum constant only if the bond's return is state contingent (i.e., contingent on both the exogenous shock and the debt stock, which requires $R_t(b,\varepsilon) = \frac{(1-\exp(\varepsilon_t))}{b_t/y^T}$) or agents can trade Arrow securities (i.e., there are as many assets as the number of state of nature). Hence, indexed bond introduces a tradeoff: on one hand it hedges income fluctuations but on the other hand it introduces interest rate fluctuations. In order to analyze the overall effect of indexed bond, we solve the model numerically. The dynamic programming representation (DPP) of the household's problem in this case reduces to:

$$\begin{aligned} V(b,\varepsilon) = \max_{b'} \left\{ u(c) + (1+c)^{-\gamma} E\left[V(b',\varepsilon')\right] \right\} \quad s.t. \\ c^T = \exp(\varepsilon)y^T - b' + (1+r+\phi\varepsilon)b. \end{aligned} \tag{8}$$

Here, the endogenous state space is given by $\mathcal{B} = \{b_1 < ... < b_{NB}\}$, which is constructed using $NB = 1,000$ equidistant grid points. The exogenous Markov process is assumed to have two states for simplicity: $\mathcal{E} = \{\varepsilon_L < \varepsilon_H\}$. Optimal decision rules, $b'(b,\varepsilon) : \mathcal{E} \times \mathcal{B} \rightarrow \mathcal{R}$, are obtained by solving the above DPP via a value function iteration algorithm.

3.1.1 Calibration

The parameter values used to calibrate the model are summarized in Table 1. The CRRA parameter σ is set to 2, the mean endowment y^T is normalized to one, and the gross interest rate is set to the quarterly equivalent of 6.5%, following the values used in small open economy RBC literature (see for example Mendoza (1991)). The steady state debt-to-GDP ratio is set to 35%, which is inline with the estimate for the net asset position of Turkey (see Lane and Milesi-Ferretti (1999)). The elasticity of the subjective discount factor follow from euler equation for consumption evaluated at steady-state:

$$(1+\bar{c})^{-\gamma}(1+r) = 1 \Rightarrow \gamma = \log(1+r)/\log(1+\bar{c}). \tag{9}$$

The standard deviation of the endowment shock is set to 3.51% and the autocorrelation is set to 0.524, which are the standard deviation and the autocorrelation of tradable output for Turkey given in Table 4.

Table 1: Parameter Values

σ	2	relative risk aversion	RBC parametrization
y^T	1	tradable endowment	normalization
σ_ε	0.0351	tradable output volatility	Turkish data
ρ_ε	0.524	tradable output autocorrelation	Turkish data
R	1.0159	gross interest rate	RBC parametrization
γ	0.0228	elasticity of discount factor	steady state condition

Using the "simple persistence" rule, we construct a Markovian representation of the time series process of output. The transition probability matrix $\mathcal{P}$ of the shocks follows:

$$\mathcal{P}(i,j) = (1-\rho_\varepsilon)\Pi_i + \rho_\varepsilon \mathcal{I}_{i,j} \tag{10}$$

where $i, j = 1, 2$; Π_i is the long-run probability of state i; and $\mathcal{I}_{i,j}$ is an indicator function, which equals 1 if $i = j$ and 0 otherwise, ρ_ε is the first order serial autocorrelation of the shocks.

3.1.2 Simulation Results

We report long run values of the key macroeconomic variables, such as mean bond holdings that is a measure of precautionary savings, volatility of consumption, correlation of consumption with income, which measures to what extend income fluctuations affect consumption fluctuations, and serial autocorrelation of consumption which measures the persistence of consumption, of the model to highlight the effect of indexation on consumption smoothing in Table 5. Without indexation ($\phi = 0$), mean bond holdings are higher than the case with perfect foresight (-0.35) (which is an implication of precautionary savings), volatility of consumption is positive, and consumption is correlated with income.

Now we analyze how the results change when we index debt repayments to endowment shocks. As Table 5 reveals, when the degree of indexation is in the [0.015, 0.25) range, households engage in less precautionary savings (as measured by the long run average of b) and the standard deviation of consumption declines relative to the case in which there is no indexation. Moreover, in this range, correlation of consumption with GDP falls slightly and its serial autocorrelation increases slightly. These results suggests that when the degree of indexation is in this range, indexation improves these macroeconomic variables from the consumption smoothing perspective. However, when the degree of indexation is greater than 0.25, these improvements reverse. In the full-indexation ($\phi = 1$) case, for example, the standard deviation of consumption is 4.8%, four times the standard deviation in the no-indexation case. The persistence of consumption also declines at higher degrees of indexation. The autocorrelation of consumption in the full indexation case is 0.886, compared to 0.978 in the no-indexation case and the high of 0.984 in the case where $\phi = 0.10$. Not surprisingly, the ranking of welfare is in line with the ranking of consumption volatility, as the last row of Table 5 reveals. However, the absolute values of the differences in welfare are quite small.[13]

The above changes are driven by the changes in the ability to hedge income fluctuations with indexed bonds. This hedging ability is affected by the degree of indexation because the degree of indexation alter the incentives for precautionary savings. In particular, it has a significant effect on determining the state of nature that defines the "catastrophic" level of income at which household reach their natural debt limits. The natural debt limit (ψ) is the largest debt that the economy can support to guarantee non-negative consumption in the event that income remain at its catastrophic level almost surely, i.e.,

$$\psi = -\frac{\exp(-\varepsilon)y^T}{r}. \tag{11}$$

[13]As pointed out by Lucas (1987), welfare implications of altering consumption fluctuations in these type of models are quite low.

With non-indexed bond, catastrophic level of income is realized at state of nature with the negative endowment shock. When the debt approaches to the natural debt limit, consumption approaches zero, which leads to infinitely negative utility. Hence, agents have strong incentives to avoid holding debt levels lower than natural debt limit. In order to guarantee positive consumption almost surely in the event that income remains at its catastrophic level, agents engage in strong precautionary savings. An increase (decrease) in this debt limit strengthens (weakens) the incentives to save, since the level of debt that agents would try to avoid would be higher (lower). With indexation, the natural debt limit can be determined at either negative or positive realization of the endowment shock, depending on which yields the lower income (determines the catastrophic level of income). To see this, notice that using the budget constraint, when the shock is negative, we derive:

$$c_t \geq 0 \Rightarrow \exp(-\varepsilon)y - b_{t+1} + b_t(1 + r - \phi\varepsilon) \geq 0 \Rightarrow \psi_L \geq -\frac{\exp(-\varepsilon)y}{r - \phi\varepsilon},\ if\ r - \phi\varepsilon > 0. \tag{12}$$

Notice that for the ranges of values of ϕ where $r - \phi\varepsilon < 0$, Equation 12 yields an upper bound for the bond holdings; i.e., $\psi_L \leq -\frac{\exp(-\varepsilon)y}{r-\phi\varepsilon}$). Hence, in this range, negative shock will not play any role in determining the natural debt limit. Again using the budget constraint, positive endowment shock implies the following natural debt limit:

$$c_t \geq 0 \Rightarrow \exp(\varepsilon)y - b_{t+1} + b_t(1 + r + \phi\varepsilon) \geq 0 \Rightarrow \psi_H \geq -\frac{\exp(\varepsilon)y}{r + \phi\varepsilon}. \tag{13}$$

Combining these two equations, we get:

$$\psi = \begin{cases} \max\{-\frac{\exp(-\varepsilon)y}{r-\phi\varepsilon}, -\frac{\exp(\varepsilon)y}{r+\phi\varepsilon}\},\ if\ \phi < r/\varepsilon \\ -\frac{\exp(\varepsilon)y}{r+\phi\varepsilon},\ if\ \phi > r/\varepsilon. \end{cases} \tag{14}$$

Further algebra suggest that when $\frac{1-\varepsilon}{1+\varepsilon} < \frac{r-\phi\varepsilon}{r+\phi\varepsilon}$ or $\phi < r$, natural debt limit is determined at state of nature with a negative endowment shock and in this case, $\partial\psi/\partial\phi < 0$, i.e., increasing the degree of indexation decreases the natural debt limit or weakens the precautionary savings incentive. However if $\frac{1-\varepsilon}{1+\varepsilon} > \frac{r-\phi\varepsilon}{r+\phi\varepsilon}$ or $\phi > r$, $\partial\psi/\partial\phi > 0$, i.e., increasing the degree of indexation increases the natural debt limit or strengthens the precautionary savings incentive.

In Table 6, we numerically calculate these natural debt limits as functions of the degrees of indexation, along with the corresponding returns in both states ($R^i_t = 1+r+\phi\varepsilon_t$) and confirm the analytical results derived above. When the degree of indexation is less than 0.0159, the natural debt limit is determined by the negative shock and decreases (i.e., the debt limit becomes looser) as we increase ϕ. When ϕ is greater than 0.0159, it is determined by the positive shock and increases (i.e., the debt limit becomes tighter) as we increase ϕ (we print the corresponding limits darker in the table). In the full-indexation case, for example, this debt limit is -20.09, whereas the corresponding value is -61.49 in the non-indexed case. In other words, in the full-indexation case, positive endowment shocks decrease the catastrophic level of income to one third of the value in the non-indexed case. This in turn sharply strengthen precautionary savings motive.

In order to understand the role of indexation on volatility of consumption, we perform a variance decomposition analysis. Higher indexation provides a better hedge to income fluctuations by increasing the covariance of the trade balance ($tb = b' - R_t^i$b) with income (since in good (bad) times agents pay more (less) to the rest of the world). However, higher indexation also increases the volatility of the trade balance. In order to pin down the effect of indexation on these variables, we perform a variance decomposition using the following identity:

$$var(c^T) = var(y^T) + var(tb) - 2cov(tb, y^T).$$

In Table 7, we present the corresponding values for the last two terms in the above equation for each of the indexation levels.[14] Clearly, both the variance of the trade balance and the covariance of the trade balance with income monotonically increase with the level of indexation. However, the term $var(tb) - 2cov(tb, y^T)$ fluctuates in the same direction as the volatility of consumption, suggesting that at high levels of indexation the rise in the variance of the trade balance offsets the improvement in the co-movement of the trade balance with income, i.e., the effect of increased fluctuation in interest rate dominates the effect of hedging provided by indexation. Hence, consumption becomes more volatile for higher degrees of indexation.

To sum up, when the degree of indexation is higher than a critical value (as with full-indexation), the precautionary savings motive is stronger and the volatility of consumption is higher than in the non-indexed case. These results arise because the natural debt limit is lower at higher levels of indexation and because the increased volatility in the trade balance far outweighs the improvement in the co-movement of the trade balance with income.

These results suggest that in order to improve macroeconomic variables, the indexation level

[14]Since the endowment is not affected by changes in the indexation level, its variance is constant.

should be low. When ϕ is lower than 0.25, agents can better hedge against fluctuations in endowment income than when ϕ is at higher levels. In this case, the precautionary savings motive is weaker, the volatility of consumption is smaller, and consumption is more persistent. When ϕ is in the $[0.10, 0.25]$ range, the correlation of consumption with income approaches zero and the autocorrelation of consumption nears unity. These values resemble the results that could be attained in the full-insurance scenario, and suggest that partial indexation is optimal.

The results using a frictionless one-sector model shed light on the debate about the indexation of public debt. Our findings in this section suggest that the hedge indexed bonds provide is imperfect and that indexation of the debt in a one-to-one fashion may not improve macroeconomic variables. However, partial indexation could prove beneficial by mimicking outcomes that would arise under full insurance.

3.2 Two Sector Model with Financial Frictions

When we introduce liability dollarization and a borrowing constraint, the DPP of the household's problem becomes:

$$\begin{aligned} V(b,\varepsilon) &= \max_{b'} \left\{ u(c) + (1+c)^{-\gamma} E\left[V(b',\varepsilon')\right] \right\} \quad s.t. \\ c^T &= \exp(\varepsilon) y^T - b' + (1+\phi\varepsilon) R b \\ c^N &= y^N \\ b' &\geq -\kappa \left[\exp(\varepsilon) y^T + p^N y^N\right]. \end{aligned} \tag{15}$$

As in the previous one-sector model, the endogenous state space is given by $\mathcal{B} = \{b_1 < ... < b_{NB}\}$, and the exogenous Markov process is assumed to have two states: $\mathcal{E} = \{\varepsilon_L < \varepsilon_H\}$. Optimal decision rules, $b'(b,\varepsilon) : \mathcal{E} \times \mathcal{B} \rightarrow \mathcal{R}$, are obtained by solving the above DPP.

3.2.1 Solving the Model

We solve the stochastic simulations using value function iteration over a discrete state space in the [-2.5, 5.5] interval with 1,000 evenly spaced grid points. We derive this interval by solving the model repeatedly until the solution captures the ergodic distribution of bond holdings. The endowment shock has the same Markov properties described in the previous section. The solution procedure is similar to that in Mendoza (2002). We start with an initial conjecture for the value function and solve the model without imposing the borrowing constraint for each coordinate

(b, ε) in the state space, and check whether the implied b' satisfies the borrowing constraint. If so, the solution is found and we calculate the implied value function that is then used as a conjecture for the next iteration. If not, we impose the borrowing constraint with equality and solve a system of non-linear equations defined by the three constraints given in the DPP (15) as well as the optimality condition given in Equation (7). Then, we calculate the implied value function using the optimal b', and iterate to convergence.

3.2.2 Calibration

We calibrate the model such that aggregates in the non-binding case match the certain aggregates of Turkish data. In addition to the parameters used in the frictionless one-sector model, we introduce the following parameters, the values of which we summarize in Table 2.: y^N is set to 1.3418, which implies a share of non-tradables output in line with the average ratio of the non-tradable output to tradable output in between 1987-2004 for Turkey; μ is set to 0.316, which is the value Ostry and Reinhart (1992) estimate for emerging countries; the steady state relative price of non-tradables is normalized to unity, which implies a value of 0.4027 for the CES share of tradable consumption (ω), calculated by using the condition that equates the marginal rate of substitution between tradables and non-tradables consumption to the relative price of non-tradables (Equation (7)). The elasticity of the subjective discount factor (γ) is recalculated including these new variables in the solution of the non-linear system of equations implied by the steady-state equilibrium conditions of the model given in Equation (9). κ is set to 0.3 (i.e. households can borrow up to 30% of their current income), which is found by solving the model repeatedly until the model matches the empirical regularities of a typical Sudden Stop episode at a state where the borrowing constraint binds with a positive probability in the long run.

Table 2: Parameter Values

μ	0.316	elasticity of substitution	Ostry and Reinhart (1992)
y^N/y^T	1.3418	share of NT output	Turkish data
p^N	1	relative price of NT	normalization
κ	0.3	constraint coefficient	set to match SS dynamics
ω	0.4027	CES weight	calibration
γ	0.0201	elasticity of discount factor	calibration

3.2.3 Simulation Results

The stochastic simulation results are divided into three sets. In the first set, which we refer to as the frictionless economy, the borrowing constraint never binds. In the second set of results, which we refer to as the constrained economy, the borrowing constraint occasionally binds and households can issue only non-indexed bonds. In the last set, which we refer to as the indexed economy, borrowing constraint occasionally binds but households can issue indexed bonds.

Our results that compare the frictionless and constrained economies are analogous of those presented by Mendoza (2002). Hence, here we just emphasize the results that are specific and crucial to the analysis of indexed bonds and refer the interested reader to Mendoza (2002) for further details. Since at equilibrium, the relative price of non-tradables is a convex function of the ratio of tradables consumption to non-tradables consumption, a decline in tradables consumption relative to non-tradables consumption due to a binding borrowing constraint leads to a decline in the relative price of non-tradables, which makes the constraint more binding and leads to a further decline in tradables consumption.

Figure 3 shows the ergodic distributions of bond holdings. The distribution in the frictionless economy is close to normal and symmetric around its mean. Mean bond holdings are -0.299, higher than the steady state bond holdings of -0.35; this reflects the precautionary savings motive that arises as a result of uncertainty and the incompleteness of financial markets. The distribution of bond holdings in the constrained economy is shifted right relative to that of the frictionless economy. Mean bond holdings in the constrained economy are 0.244, which reflects a sharp strengthening in the precautionary savings motive due to the borrowing constraint.

Table 8 presents the long-run business cycle statistics for the simulations. Relative to the frictionless economy, the correlation of consumption with the tradables endowment is higher in the constrained economy. In line with this stronger co-movement, the persistence (autocorrelation) of consumption is lower in the constrained economy.

Behavior of the model can be divided into three ranges. In the first range, debt is sufficiently low that the constraint is not binding. In this case, the response of the constrained economy to a negative endowment shock is similar to that of the frictionless economy, and a negative endowment shock is smoothed by a widening in the current account deficit as a share of GDP. There is also a range of bond holdings in which debt levels are too high. In this range, the constraint always binds regardless of the endowment shock. However, at more realistic debt levels where the constraint only binds when the economy suffers a negative shock, the model with non-indexed bond roughly matches the empirical regularities of Sudden Stops. This range, which we call the "Sudden Stop region" following Mendoza and Smith (2005), corresponds to the 218-230th grid points.

In Figure 4, we plot the conditional forecasting functions of the frictionless and constrained economies for tradables consumption, aggregate consumption, the relative prices of non-tradables, and the current account-GDP ratios, in response to a one-standard deviation endowment shock. These forecasting functions are conditional on the 229th bond grid, which is one of the Sudden Stop states and has a *long-run probability* of 0.47%, and they are calculated as responses of these variables as percentage deviations from the long-run means of their frictionless counterparts.[15]

As these graphs suggest, the response of the constrained economy is dramatic. The endowment shock results in a 4.1% decline in tradable consumption. That compares to a decline of only 0.9% in the frictionless economy. In line with the larger collapse in the tradables consumption, the responses of aggregate consumption and the relative price of non-tradables are more dramatic in the constrained economy than in the frictionless economy. While households in the frictionless economy are able to absorb the shock via adjustments in the current account (the current account deficit slips to 1.4% of GDP), households in the constrained economy cannot due to the binding borrowing constraint (the current account shows a surplus of 0.02% of GDP). These figures also suggest that the effects of Sudden Stops are persistent. It takes more than 40 quarters for these variables to converge back to their long-run means.

Figures 5, 6, and 7 compare the detrended conditional forecasting functions of the constrained economy with that of the indexed economy to illustrate how indexed bond can help smooth Sudden Stop dynamics (the degrees of indexation are provided on the graphs).[16] As Figure 5 suggests, when the degree of indexation is 0.05, indexed bonds provide little improvement over the constrained case; indeed, the difference in the forecasting functions is not visible. When indexation reaches 0.10, however, the improvements are minor yet noticeable. At this degree of indexation, aggregate consumption rises 0.11%, tradables consumption rises 0.24%, the relative price of non-tradables increases 0.30%.

With increases in the degree of indexation to 0.25 and 0.45, the initial effects are relatively small. Figure 6 suggests that the improvements in tradables consumption are close to 1% and 1.8% when the degrees of indexation are 0.25 and 0.45, respectively. Figure 7 suggests that

[15]Bond holdings on this grid point are equal to -0.674, which implies a debt-to-GDP ratio of 30%.

[16]These forecasting functions are detrended by taking the differences relative to the frictionless case.

when the degree of indexation gets higher, 0.7 and 1.0 for example, tradables consumption and aggregate consumption fall below the constrained case after the fourth quarter and stay below for more than 30 quarters despite the initially small effects of a negative endowment shock. In other words, degrees of indexation higher than 0.45 in an indexed economy imply more pronounced detrimental Sudden Stop effects than in a constrained economy.

Table 9 summarizes the initial effects of both a negative and a positive shock conditional on the same grid points used in the forecasting functions. When indexed bonds are in place, our results suggest that if the degree of indexation is within [0.05, 0.25], indexed bonds help to smooth the effects of Sudden Stops. As Table 9 suggests, when the degree of indexation is 0.05, indexed bonds provide little improvement. As we increase the degree of indexation, the initial impact of a negative endowment shock on key variables gets smaller. In this case, debt relief accompanies a negative endowment shock, and this relief helps to reduce the initial impact of a binding borrowing constraint. Hence, the depreciation in the relative price of non-tradables is milder, which in turn prevents the Fisherian debt-deflation.

Table 9 also suggests that although the smallest initial impact of a negative endowment shock occurs when the degree of indexation is unity (full-indexation), this level of indexation has significant adverse effects if a positive shock realizes. In this case, households must pay a significantly higher interest rate over and above the risk-free rate. Although the constrained economy is not vulnerable to a Sudden Stop when there is a positive endowment shock, agents in such an economy face a Sudden Stop due to a sudden jump in debt servicing costs.

Hence, our analysis suggests that household face a tradeoff when they engage in debt contracts with high degrees of indexation. If the households are hit by a negative endowment shock, highly indexed bonds can allow them to absorb the shock without suffering severely in terms of consumption. Such a shock might trigger a Sudden Stop if households were to borrow instead via non-indexed bonds (the initial effects are closest to the frictionless case when the degree of indexation is one). However, if they receive a positive endowment shock, the initial effects are larger in the indexed economy (where the degree of indexation equals 1) than in the constrained economy (e.g., the impact on tradable consumption jumps from -1.1% to -6.7%). Analyzing the results in columns 3-9, we conclude that degrees of indexation in the [0.45, 1.0] interval lead to stronger Sudden Stop effects. If we take the average of initial responses across the high and the low states in this range of values, we find that the minimum of these averages is attained when the degree of indexation is 0.25, which suggests that households with concave utility functions would attain a higher utility with this consumption profile than ones achieved with indexation levels higher than 0.25.

In Figure 8, we plot the time series simulations of the frictionless, constrained, and indexed economies. These simulations are derived first by generating a random exogenous endowment shock process using the transition matrix, $\mathcal{P}$, and then by feeding these series into each of the respective economies. On the top left graph, the dotted line is the tradable consumption series for the frictionless economy. The solid line is the series for the constrained economy. As the graphs reveal, although patterns of consumption in each economy mostly move together, there are cases (around periods 2000, 3600, 6500, 8800), where we observe sharp declines in constrained economy. These declines correspond to Sudden Stop episodes. In these cases, a consecutive series of negative endowment shocks make the constraint binding, which in turn triggers a debt-deflation that ultimately leads to a collapse in consumption.

When the return is indexed and the degree of indexation is 0.05 (top right graph), the volatility of consumption is noticeably lower than in the constrained case, and collapses in consumption during Sudden Stop episodes are milder. When we increase the degree of indexation to 0.45, however, there is a significant increase in the volatility of consumption, and there are more frequent collapses. When the degree of indexation is 1.0 (due to space limitations, we leave out the figures associated with other degrees of indexation), we observe a spike in volatility and much more frequent and sizeable collapses in consumption. These simulations illustrate that when indexation is full, the effect on consumption can be significantly negative, furthermore that indexation can yield benefits in terms of consumption volatility only if the degree of indexation is quite low.

Table 8 suggests that in addition to the tradeoff of gains in the low state versus losses in the high state, there is also a short run versus long run tradeoff with respect to issuing indexed bonds with high degrees of indexation. With higher indexation levels, indexed bonds can generate substantial short-run benefits, but also introduce more severe adverse effects in the long run; i.e., consumption volatility and its co-movement with income increase with greater degrees of indexation. Consistent with our findings in the frictionless one-sector model, the value of indexation that minimizes the co-movement of consumption with GDP and yields more persistent consumption is low (in the range of [0.05, 0.1] for this calibration). These results also suggest that, depending on the objectives, the optimal degree of indexation level may vary. As we illustrated before, the level of indexation that would minimize the effect of Sudden Stops is in the [0.25, 0.45] interval, whereas the one that minimizes long-run fluctuations is in the [0.05, 0.1] range. However, regardless of whether we would like to smooth Sudden Stops or long-run fluctuations, full-indexation is undesirable.

3.3 Sensitivity Analysis

This section presents the results of analysis aimed at evaluating the robustness of our results to several variations in model parameterization. Due to space limitations, for the first three sensitivity analysis we present result of the the frictionless one-sector model. These results are summarized in Table 10.

We first analyze the robustness of the results to changes in the number of exogenous state variables. For this analysis, we use a seven-state Markov chain that maintains the same autocorrelation and standard deviation of the shock as in the previous setup. Note that the simple persistence rule can be employed only if the number of exogenous state variables is two. In order to create the transition matrix with seven exogenous states, we employ the method described in Tauchen and Hussey (1991). The first block in Table 10 presents key long-run statistics, which are nearly identical to the ones presented in Table 5; in fact, for a given indexation level, the statistics are the same out to two decimal points. Hence, we conclude that our results are robust to the number of state variables used in the Markov process.

Second, we increase the standard deviation of the exogenous endowment shock to 4.5%. As Table 10 suggests, when bonds are not indexed, the precautionary savings motive is stronger, consumption is more volatile, and consumption displays greater correlation with income when we increase variation in the magnitude of the exogenous endowment shock. Comparing Table 10 with Table 5 for the indexed case, we conclude that the optimal indexation level that minimizes long-run macroeconomic fluctuations is in the [0.05, 0.1] interval in the former case, whereas it is in the [0.1, 0.25] interval in the latter one. In other words, the optimal degree of indexation decreases with increases in the volatility of the exogenous endowment shock.

Next, we evaluate the changes in results that arise when we lower the autocorrelation of the endowment shock. Compared to the baseline results given in Table 5, with an endowment shock autocorrelation of 0.4, agents engage in less precautionary savings. Furthermore, consumption volatility and its co-movement with income are lower. When indexed bonds are in place, the lower the persistence of the shock, the higher the degree of indexation that would minimize the co-movement of consumption with income. For instance, when the indexation is 0.1, the correlation of consumption with income is 0.07 when the autocorrelation of the shock is 0.4. By comparison, at the same indexation level, the correlation of consumption with income is 0.017 when the autocorrelation is 0.524.

As a final robustness check, we examine the effect of having a larger non-tradables sector. The results are summarized in Table 11. We set the y^N/y^T ratio to 1.6, which implies that the degree of openness of the country is lower than in the baseline case. Not surprisingly, the model in this case captures the empirical regularities of an economy with less financial integration. In particular, consumption is more volatile than in the baseline case (for instance, the volatility of the tradables consumption in the frictionless economy increases to 1.6%, compared to the baseline value of 1.5%), and the co-movement of consumption with income is stronger (the correlation of tradables consumption with income in the frictionless economy increases to 0.75 from the baseline value of 0.69). When we compare the initial responses of each of these economies to a one-standard-deviation endowment shock, the response of the constrained economy with a higher share of non-tradable output is stronger than that of the one with baseline parameters, which suggests that the debt-deflation process is more severe in the former economy. This result is consistent with the empirical evidence on the relationship between the degree of openness and the severity of Sudden Stops (see Calvo et al. (2003)). In order to compare the optimal indexation levels across different parameterizations, we compare the average responses of these economies in the high and the low states to a one-standard-deviation endowment shock. These results suggest that the minimum average response is attained when the degree of indexation is 0.25, which is the same degree of indexation in the baseline results. However, this result depends on the coarseness of the indexation intervals with which we are solving the problem. Economic intuition suggests that lower financial integration would require higher indexation levels to smooth exogenous shocks better.

The sensitivity analysis presented in this section suggests that the optimal indexation level depends on the properties of the exogenous shock, including its persistence and its volatility. Hence, the optimal degree of indexation needs to be country specific, since it is highly likely that each emerging country receives shocks with different statistical properties.The findings of this paper suggest that while indexed bonds might aid many countries in averting or at least mitigating the effects of Sudden Stops in emerging markets, an indexation level appropriate for one country might not be optimal for another.

4 Conclusion

Recent policy proposals argue that indexing the debt of emerging markets could help prevent the sudden reversals of capital inflows accompanied by real exchange rate devaluations that were typical of the emerging market crises of the last decade. This paper explores the quantitative implications of this policy in a DSGE model. Debt is denominated in units of tradables, and international lenders impose a borrowing constraint that limits debt to a fraction of national income. The benchmark model with non-indexed bonds and credit constraints features Sudden Stops as an equilibrium outcome that results from a debt-deflation process, the feedback mechanism between liability dollarization and the borrowing constraint that operates through the relative price of non-tradables.

We conducted our quantitative experiments to evaluate the effects of indexed bonds in two steps. First, we studied the effects of bonds indexed to output in a canonical one-sector small open economy model with varying degrees of indexation. We found that the introduction of indexed bonds partially completes the insurance market in such an economy, and whether they help to reduce precautionary savings, the volatility of consumption, and the correlation of consumption with income depends on the degree of indexation of the bond. When this degree is higher than a critical threshold (as with the full indexation for example), indexation can, in fact, make agents worse off. Because increase in the variance of trade balance (due to higher interest rate fluctuations) outweighed the improvement in the covariance of trade balance with income, which then led to higher volatility of consumption; and natural debt limits became tighter, which then led to an increase in precautionary savings.

In the second step, we analyzed the role of indexed bonds in smoothing Sudden Stops and RER fluctuations. We found that indexed bonds can reduce the initial capital outflow in the event of an exogenous shock that otherwise trigger a Sudden Stop in an economy with only non-indexed bonds. Indexed bonds can in turn reduce the depreciation in the RER and break the Fisherian debt-deflation mechanism. However, once again, the benefit of these bonds depends critically on the degree of indexation. When the level of indexation is lower than a critical value, indexed bonds weaken Sudden Stops. If indexation is higher than this critical value, although indexed bonds can provide some temporary relief in the event of a negative shock, the initial improvement is short lived. Moreover, in the event of a positive shock, the economy is vulnerable to a Sudden Stop even though such a shock would never trigger a Sudden Stop in an economy in which household facing borrowing constraints can only issue non-indexed bonds. Because in this case, positive shock commands higher repayment, which increases the need for larger borrowing, this in turn can make the borrowing constraint suddenly binding, and triggering a debt-deflation.

To conclude, contrary to the existing proposals, bonds on which the return is indexed in a one-to-one fashion (i.e., full-indexation) will not necessarily provide benefits to emerging countries. However, an indexed bond with an optimal degree of indexation can help these countries smooth Sudden Stops. This optimal value depends on the persistence and the volatility of the exogenous shocks a given country experiences, as well as the size of the country's non-tradables sector relative to the its tradables sector (i.e., the openness of the country). Hence, in terms of policy implications, our analysis reveals that the degree of indexation is a key variable that should optimally be chosen in order to smooth Sudden Stops, and furthermore that this value should be country specific.

In our analysis, we assumed that investors are risk-neutral and that indexing debt repayments would not require them to obtain more country specific information. It may be the case that indexed returns may affect investors' incentives to collect more country specific information. The implications of introducing risk-averse investors or informational costs in a dynamic setup are left for future research. The model can also be used to explore the implications of indexation to relative price of non-tradables, or to CPI, but it is left for further research. Analyzing if trading in option or futures markets can help emerging countries for mitigating Sudden Stops is an avenue of research. This would require a richer model, and it is left for further research, as well. Another avenue for future research could be analyzing the implications of indexed bonds on default probabilities. In order to carry out such an analysis, indexed bonds could be introduced into "willingness to pay" models such as those of Eaton and Gersovitz (1980) and Arellano (2004).

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Table 3: Previous Attempts of Indexed Bonds

	Date Issued	Indexation Clause	Note
Argentina	1972-1989	CPI	
Australia	1985-1988	CPI	
Bosnia and Herzegovina	1990s	GDP	Issued as part of Brady Plan, VRRs
Brazil	1964-	CPI	
Bulgaria	1990s	GDP	Issued as part of Brady Plan, VRRs
Colombia	1967-	CPI	
Costa Rica	1990s	GDP	Issued as part of Brady Plan, VRRs
Chile	1956-	CPI	
Israel	1955-	CPI	
France	1973	Gold	Debt servicing cost increased significantly due to depreciation of French Franc against gold
	1970s	Oil	Petro-bonos
Mexico	1990s	Oil	Issued as part of Brady Plan, VRRs
	1989-	CPI	
Turkey	1994-	CPI	
UK	1975-	CPI	
Venezuela	1990s	Oil	Issued as part of Brady Plan, VRRs

Sources: Borensztein and Mauro (2004), Campell and Shiller (1996), Kopcke and Kimball(1999), Atta-Mensah (2004).

Table 4: Business Cycle Facts for Emerging Countries

Variable:x	$\sigma(x)$	$\sigma(x)/\sigma(Y)$	$\rho(x)$	$\rho(x,Y)$	Sudden Stop	Sudden Stop relative to std.
Argentina					2002:1-2	
GDP (Y)	4.022	1.000	0.865	1.000	-12.952	3.220
tradables GDP	4.560	1.134	0.667	0.923	-15.100	3.311
nontradables GDP	3.977	0.989	0.894	0.990	-12.169	3.060
consumption	4.475	1.113	0.830	0.975	-17.063	3.813
real exchange rate	15.189	3.777	0.754	0.454	-48.177	3.172
CA/Y	0.916	0.228	0.837	-0.802	1.353	1.476
Chile					1998:4-1999:1	
GDP (Y)	2.093	1.000	0.731	1.000	-4.492	2.147
tradables GDP	1.833	0.876	0.473	0.762	-5.068	2.764
nontradables GDP	2.520	1.204	0.796	0.961	-4.840	1.921
consumption	4.184	1.999	0.748	0.898	-8.410	2.010
real exchange rate	0.007	0.003	0.649	0.372	-0.019	2.578
CA/Y	3.302	1.578	0.352	-0.512	10.932	3.311
Mexico					1994:4-1995:1	
GDP (Y)	2.261	1.000	0.799	1.000	-7.440	3.290
tradables GDP	2.682	1.186	0.712	0.921	-8.976	3.347
nontradables GDP	2.189	0.968	0.832	0.978	-6.178	2.822
consumption	4.222	1.867	0.841	0.973	-11.200	2.653
real exchange rate	8.627	3.816	0.726	0.599	-32.844	3.807
CA/Y	0.698	0.309	0.831	-0.475	2.220	3.180
Turkey					1994:1-2	
GDP (Y)	3.695	1.000	0.667	1.000	-10.383	2.001
tradables GDP	3.511	0.950	0.524	0.962	-10.925	3.112
nontradables GDP	4.021	1.088	0.680	0.982	-10.007	2.489
consumption	4.134	1.119	0.746	0.919	-10.098	2.443
real exchange rate	9.110	2.465	0.675	0.602	-31.630	3.472
CA/Y	2.744	0.743	0.633	-0.591	9.704	3.375

Source: Argentinean Ministry of Finance (MECON), Bank of Chile, Bank of Mexico, Central Bank of Turkey, International Financial Statistics. The data cover periods 1993:Q1-2004:Q4 for Argentina, 1986:Q1-2001:Q3 for Chile, 1987:Q1-2004:Q4 for Mexico, 1987:Q1-2004:Q4 for Turkey. Data are quarterly seasonally adjusted real series. GDP and consumption data are logged and filtered using an HP filter with a smoothing parameter 1600. Real exchange rates are calculated using the IMF definition ($RER_i = NER_i \times CPI_i/CPI_{US}$ for country i).

Table 5: Long Run Business Cycle Statistics of the One-Sector Model

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
$E(b)$	-0.328	-0.349	-0.355	-0.385	-0.428	-0.042	0.522	1.458	2.026
$\sigma(cons)$	1.243	1.242	1.240	1.236	1.209	1.474	2.119	3.291	4.731
$\sigma(tb/y)$	3.486	3.516	3.527	3.590	3.674	4.211	4.820	5.724	6.755
$\rho(cons,y)$	0.186	0.160	0.151	0.097	0.017	-0.311	-0.409	-0.381	-0.304
$\rho(tb/y,y)$	0.936	0.937	0.937	0.939	0.945	0.943	0.916	0.849	0.752
$\rho(cons)$	0.978	0.980	0.980	0.981	0.984	0.909	0.870	0.876	0.886
$\rho(tb/y)$	0.549	0.549	0.548	0.546	0.541	0.542	0.562	0.601	0.646
$welfare$	n.a.	0.0025	0.0034	0.0090	0.0146	-0.0032	-0.0092	-0.0120	-0.0136

Note: Standard deviations are in percent of the mean. Welfare gains are in percent and relative to the non-indexed model.

Table 6: Returns and Natural Debt Limits

	Degree of Indexation (ϕ)								
	0.00	0.01	0.015	0.05	0.10	0.25	0.45	0.70	1.0
$R^i(L)$	1.016	1.016	1.015	1.014	1.012	1.007	1.000	0.991	0.981
$R^i(H)$	1.016	1.016	1.016	1.018	1.019	1.025	1.032	1.040	1.051
$NDL(L)$	**-61.487**	**-62.182**	**-62.894**	-68.503	-78.431	-138.754	5440.508	106.131	48.760
$NDL(H)$	-64.517	-63.819	-63.136	**-58.642**	**-53.262**	**-41.767**	**-32.434**	**-25.353**	**-20.089**

Note: First two rows are the corresponding gross returns in each states. In the last two rows, the implied natural debt limits are printed bolder.

Table 7: Variance Decomposition Analysis for Consumption

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
$\sigma(cons)$	1.243	1.242	1.240	1.236	1.209	1.474	2.119	3.291	4.731
$var(tb)$	12.241	12.463	12.540	13.008	13.638	17.707	22.903	31.959	44.788
$cov(tb, y)$	11.508	11.620	11.660	11.897	12.248	13.929	15.365	16.724	17.364
$var(tb)$ $-2cov(tb, y)$	-10.775	-10.777	-10.781	-10.792	-10.857	-10.147	-7.827	-1.488	10.061

Table 8: Long Run Business Cycle Statistics of the Two-Sector Model

	Degree of Indexation (ϕ)							
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
$E(b)$	-0.299	0.244	0.122	0.276	0.594	1.599	2.328	2.516
$\sigma(c^T)$	1.530	1.268	1.251	1.389	1.851	2.835	3.914	5.266
$\sigma(c)$	0.775	0.638	0.631	0.697	0.923	1.392	1.889	2.508
$\sigma(p^N)$	2.026	1.682	1.660	1.845	2.467	3.804	5.291	7.162
$\sigma(tb/y)$	1.534	1.467	1.491	1.610	1.799	2.113	2.398	2.755
$\rho(c^T, y)$	0.687	0.663	0.636	0.567	0.609	0.773	0.875	0.930
$\rho(c, y)$	0.687	0.664	0.637	0.567	0.608	0.770	0.870	0.924
$\rho(p^N, y)$	0.687	0.663	0.636	0.567	0.609	0.774	0.877	0.933
$\rho(tb/y, y)$	0.512	0.648	0.646	0.548	0.290	-0.141	-0.404	-0.580
$\rho(c^T)$	0.986	0.971	0.976	0.967	0.953	0.926	0.911	0.907
$\rho(c)$	0.986	0.971	0.976	0.967	0.953	0.925	0.909	0.903
$\rho(p^N)$	0.986	0.971	0.976	0.967	0.953	0.927	0.912	0.909
$\rho(tb/y)$	0.581	0.546	0.540	0.546	0.572	0.609	0.631	0.661

Note: The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraint and indexed bonds (with given degrees of indexation). Standard Deviations are in percent.

Table 9: Initial Responses to a One-Standard-Deviation Endowment Shock

	Non-Indexed		Degree of Indexation (ϕ)					
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
A) Negative Shock								
tradable consumption	-0.907	-4.126	-4.007	-3.888	-3.531	-3.056	-1.657	-1.748
aggregate consumption	-0.384	-1.780	-1.728	-1.676	-1.520	-1.312	-0.706	-0.745
relative price of non-tradables	-1.197	-5.398	-5.244	-5.090	-4.626	-4.007	-2.179	-2.299
B) Positive Shock								
tradable consumption	-0.291	-1.095	-2.019	-2.138	-2.494	-2.970	-4.369	-6.691
aggregate consumption	-0.120	-0.464	-0.862	-0.913	-1.068	-1.275	-1.887	-2.919
relative price of non-tradables	-0.387	-1.444	-2.653	-2.808	-3.274	-3.895	-5.714	-8.716

Note: The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraint and indexed bonds (with given degrees of indexation). Initial responses are calculated as percentage deviations relative to the long-run mean of the frictionless economy.

Table 10: Sensitivity Analysis of the One-Sector Model

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
I. seven-state markov chain									
$E(b)$	-0.320	-0.345	-0.351	-0.369	-0.371	-0.083	0.548	1.459	1.968
$\sigma(cons)$	1.246	1.245	1.244	1.243	1.258	1.487	2.147	3.319	4.776
$\rho(cons, y)$	0.182	0.154	0.144	0.079	0.031	-0.301	-0.410	-0.378	-0.293
$\rho(cons)$	0.970	0.971	0.971	0.974	0.982	0.906	0.869	0.870	0.869
II. σ_ε=0.045									
$E(b)$	-0.315	-0.335	-0.343	-0.359	-0.295	-0.017	0.908	1.741	2.064
$\sigma(cons)$	1.567	1.566	1.566	1.560	1.576	1.919	2.899	4.372	6.226
$\rho(cons, y)$	0.208	0.173	0.160	0.085	-0.046	-0.270	-0.357	-0.307	-0.230
$\rho(cons)$	0.983	0.987	0.988	0.991	0.974	0.927	0.892	0.893	0.898
III. ρ_ε=0.4									
$E(b)$	-0.335	-0.357	-0.361	-0.398	-0.477	-0.300	0.180	0.918	1.637
$\sigma(cons)$	1.074	1.069	1.068	1.060	1.034	1.202	1.462	2.229	3.351
$\rho(cons, y)$	0.178	0.157	0.152	0.112	0.070	-0.167	-0.361	-0.367	-0.301
$\rho(cons)$	0.966	0.968	0.969	0.970	0.975	0.944	0.865	0.865	0.885

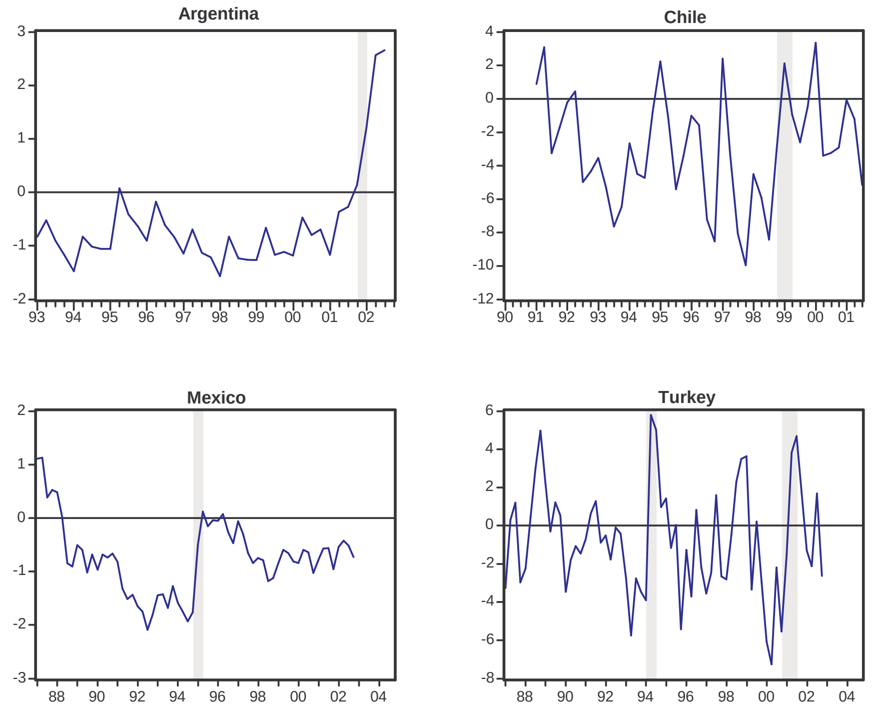
Note: Resulting transition matrix for seven-state markov chain is approximated using the method described in Tauchen and Hussey (1991). Standard deviations are in percent.

Table 11: Sensitivity Analysis of the Two-Sector Model: Higher Share of Non-tradable Output

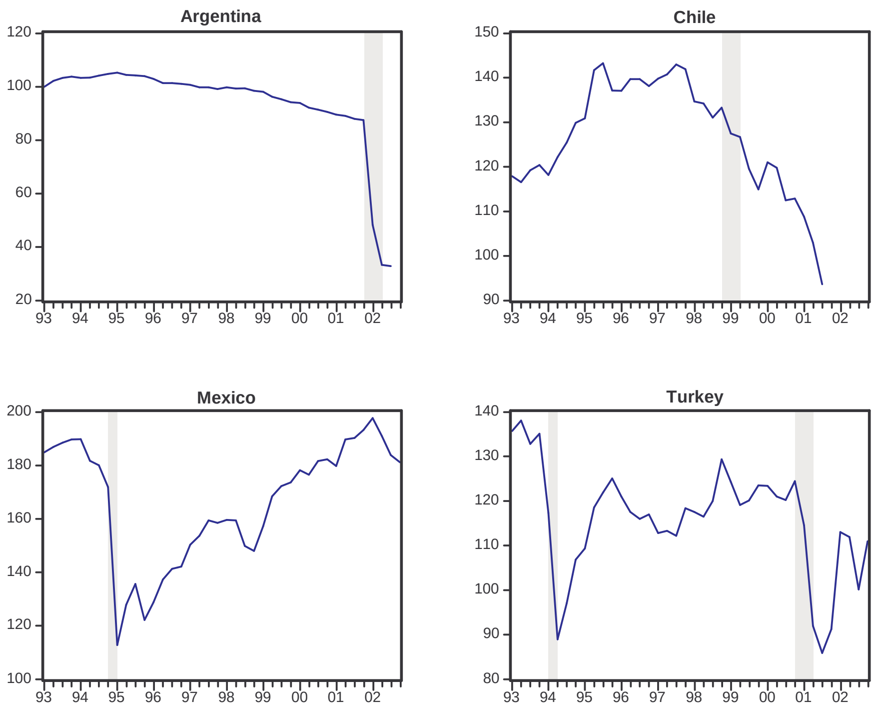
	Degree of Indexation (ϕ)							
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
I. Long run statistics								
$E(b)$	-0.290	0.258	0.084	0.682	0.667	1.739	2.399	2.551
$\sigma(c^T)$	1.590	1.306	1.261	1.639	1.957	2.919	3.956	5.300
$\sigma(c)$	0.822	0.671	0.649	0.836	0.994	1.457	1.941	2.565
$\sigma(p^N)$	2.105	1.734	1.672	2.182	2.609	3.920	5.351	7.211
$\rho(c^T, y)$	0.749	0.716	0.691	0.664	0.714	0.844	0.913	0.951
$\rho(c, y)$	0.750	0.718	0.692	0.664	0.713	0.841	0.908	0.945
$\rho(p^N, y)$	0.749	0.716	0.691	0.664	0.714	0.845	0.915	0.953
$\rho(c^T)$	0.987	0.975	0.975	0.973	0.956	0.931	0.914	0.909
$\rho(c)$	0.987	0.976	0.976	0.974	0.956	0.930	0.911	0.905
$\rho(p^N)$	0.987	0.975	0.975	0.973	0.957	0.932	0.915	0.911
II. Initial Responses								
*A)*Negative Shock								
tradable consumption	-1.036	-4.254	-4.122	-3.991	-3.596	-3.070	-1.608	-1.623
aggregate consumption	-0.395	-1.655	-1.603	-1.551	-1.395	-1.187	-0.616	-0.622
relative price of non-tradables	-1.366	-5.565	-5.395	-5.224	-4.711	-4.025	-2.115	-2.135
*B)*Positive Shock								
tradable consumption	-0.420	-2.029	-2.156	-2.292	-2.686	-3.213	-4.675	-7.074
aggregate consumption	-0.157	-0.780	-0.818	-0.883	-1.037	-1.244	-1.823	-2.788
relative price of non-tradables	-0.557	-2.666	-2.985	-3.010	-3.525	-4.211	-6.111	-9.208

Notes: y^N/y^T ratio is set to 1.6 in this analysis. Standard deviations are in percent of the mean. The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraint and indexed bonds (with given degrees of indexation).

Figure 1: Sudden Stops in Emerging Markets

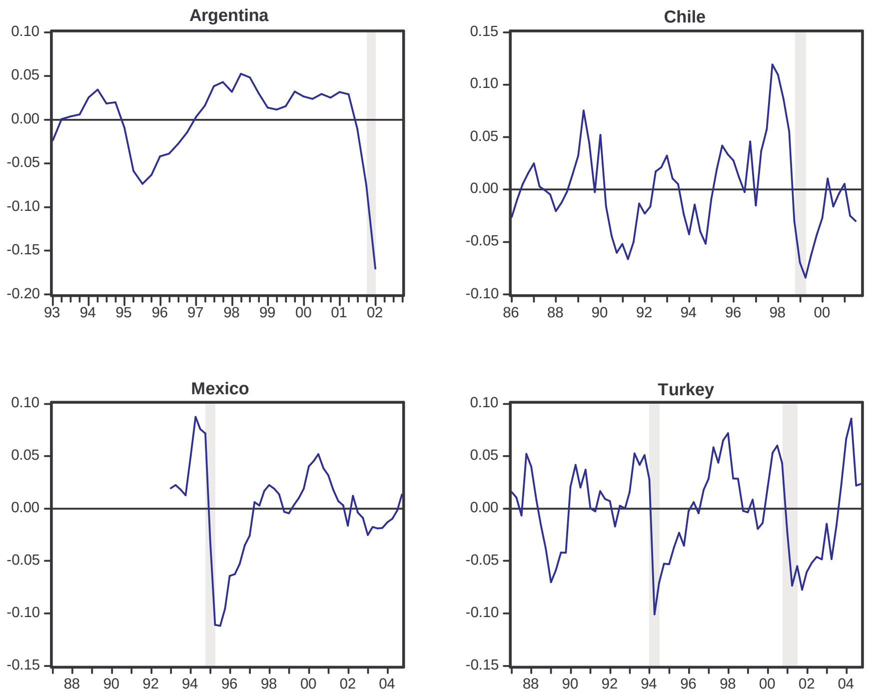


a. Current Account-GDP Ratio

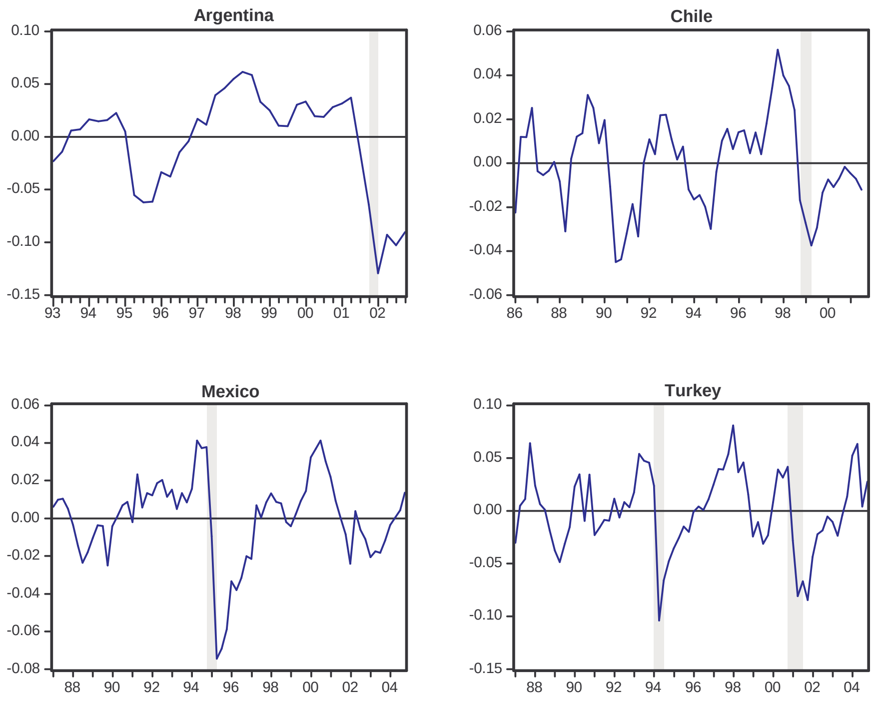


b. Real Exchange Rate

Figure 2: Deviations from Trend in Consumption and Ouput



a. Consumption



b. Output

Figure 3: Long Run Distributions of Bond Holdings in Non-Indexed Economies

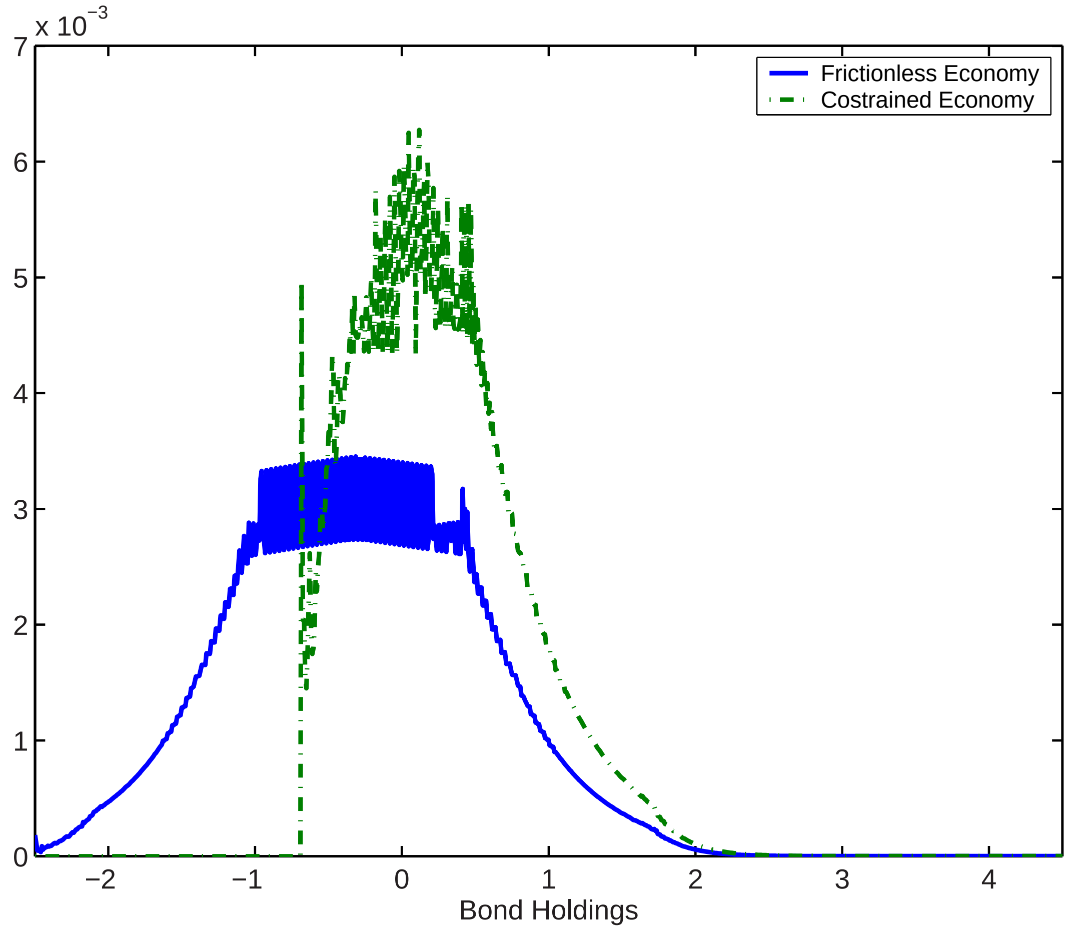
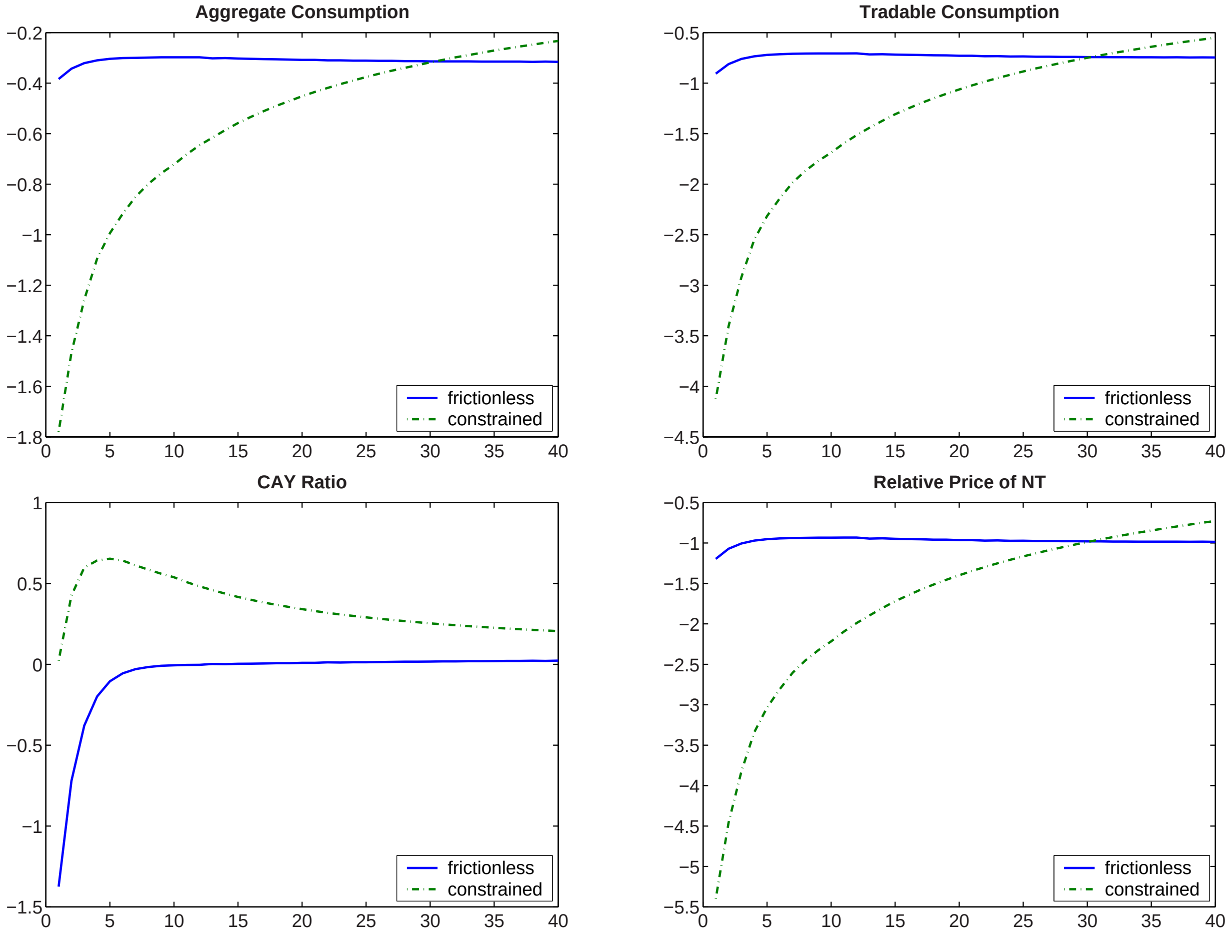
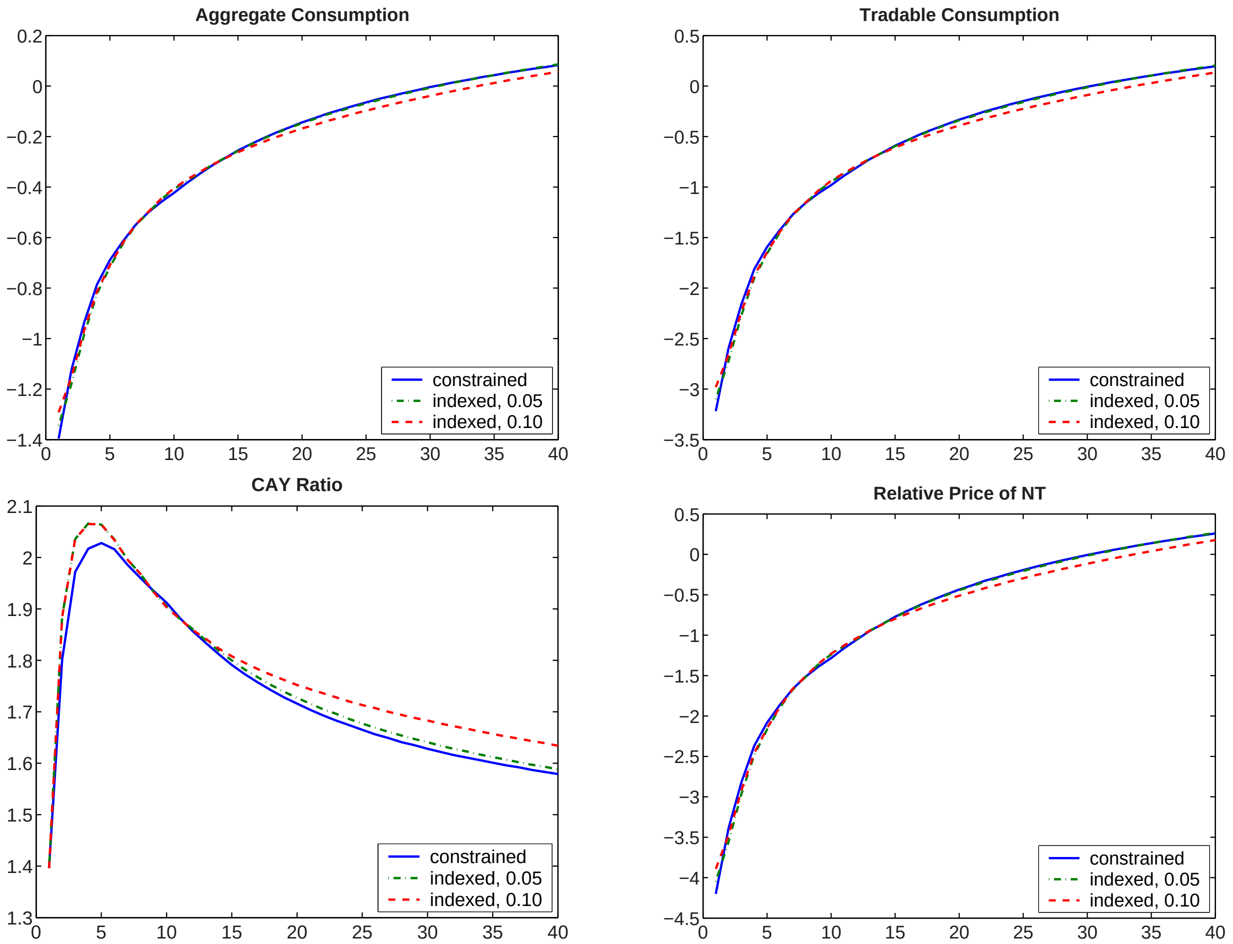


Figure 4: Conditional Forecasting Functions in Response to a One-Standard-Deviation Negative Endowment Shock



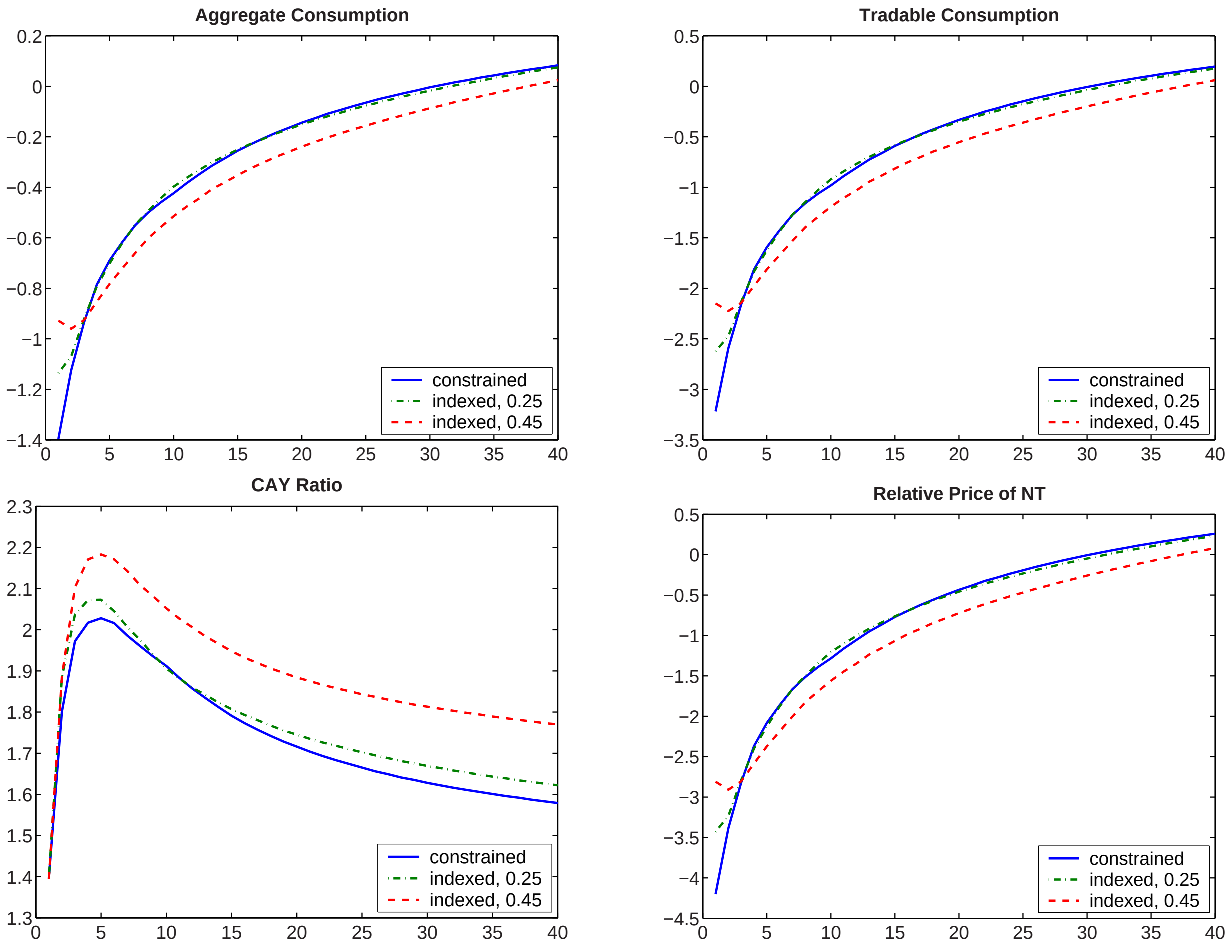
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30%. Solid and dashed lines are forecasting functions of the frictionless, and constrained economies, respectively.

Figure 5: Conditional Forecasting Functions in Response to a One-Standard-Deviation Negative Endowment Shock



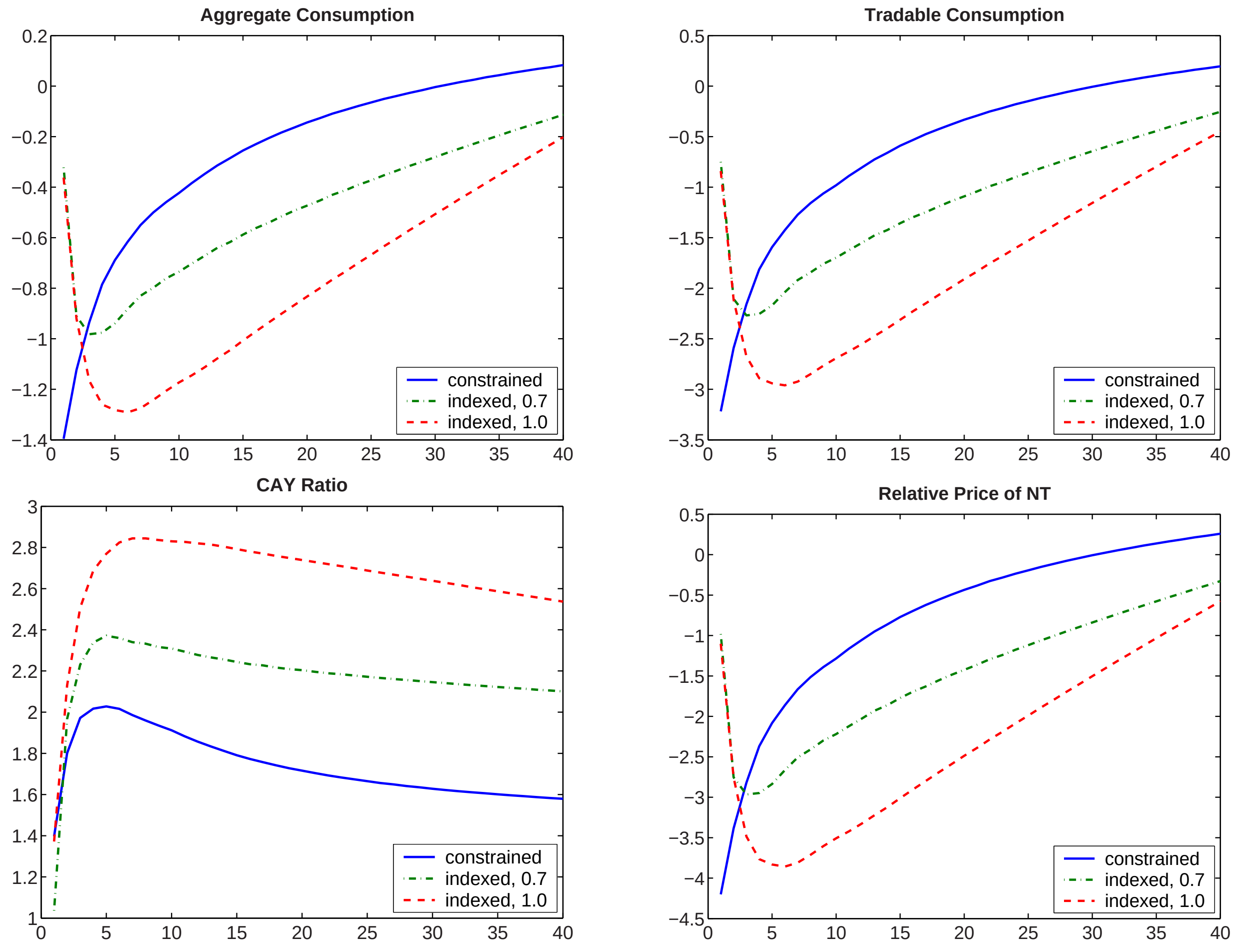
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30%.

Figure 6: Conditional Forecasting Functions in Response to a One-Standard-Deviation Negative Endowment Shock



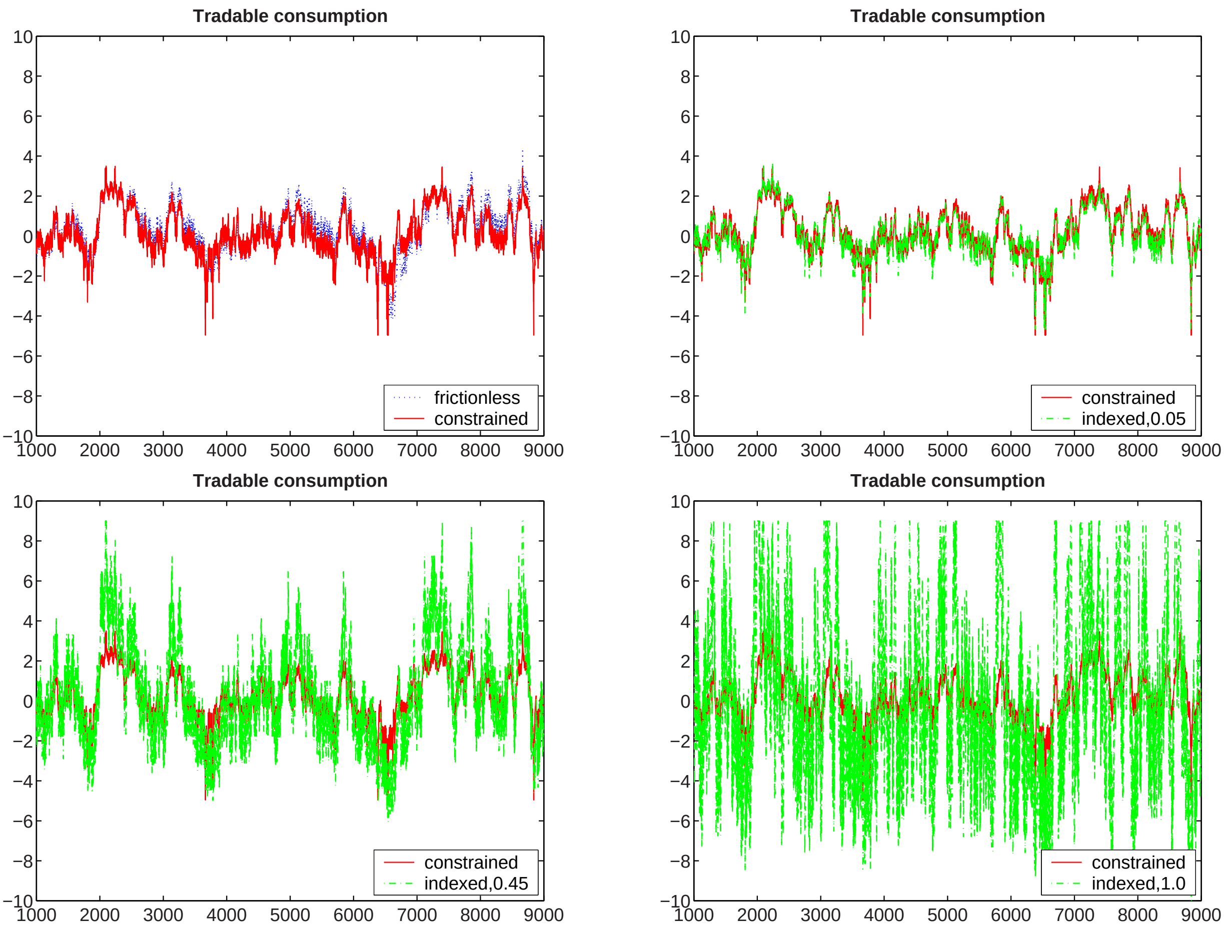
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30%.

Figure 7: Conditional Forecasting Functions in Response to a One-Standard-Deviation Negative Endowment Shock



Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30%.

Figure 8: Time Series Simulation



Note: Consumptions are in percentage deviations from their corresponding means. The first 1,000 periods have been excluded from these graphs to focus on the data which are independent of initial conditions.