February 8, 2006

The Interaction of Labor Markets and Inflation: Micro Evidence from the International Wage Flexibility Project

William T. Dickens, Lorenz Goette, Erica L. Groshen, Steinar Holden, Julian Messina, Mark E. Schweitzer, Jarkko Turunen, and Melanie Ward[1]

---

[1]William Dickens is a Senior Fellow in Economic Studies at The Brookings Institution (wdickens@brookings.edu), Lorenz Goette is a an Assistant Professor at the University of Zurich (lorenz@iew.unizh.ch), Erica Groshen is Assistant Vice President in the Research and Statistics Group of the Federal Reserve Bank of New York, (erica.groshen@ny.frb.org), Steinar Holden is a Professor of Economics in the Department of Economics, University of Oslo and a Research Fellow of CESifo (steinar.holden@econ.uio.no), Julian Messina is an economist with the European Central Bank (DG-Research), and a Research Fellow with the IZA (jmessina@unisa.it), Mark Schweitzer is Assistant Vice President and Economist in the Research Department of the Federal Reserve Bank of Cleveland (mark.e.schweitzer@clev.frb.org), Jarkko Turunen is an Economist with the European Central Bank (jarkko.turunen@ecb.int), and Melanie Ward is Senior Economist at the European Central Bank and Research fellow at IZA (melanie.ward-warmedinger@ecb.int).This paper reports the results of the International Wage Flexibility Project. The International Wage Flexibility Project country team members are Cedric Audenis, Richard Barwell, Thomas Bauer, Petri Bockerman, Holger Bonin, Pierre Biscourp, Ana Rute Cardoso, Francesco Devicienti, Orrietta Dessy, John Ekberg, Tor Eriksson, Bruce Fallick, Ernst Fehr, Nathalie Fourcade, Seppo Laaksonen, Michael Lettau, Pedro Portugal, Jimmy Royer, Mickael Backman, Kjell Salvanes, Paolo Sestito, Alfred Stiglbauer, Uwe Sunde, Jari Vainiomaki, Marc van Audenrude, William Wascher, Rudolf Winter-Ebmer, Niels Westergaard-Nielson, and Josef Zuckerstaetter. We are grateful to the European Central Bank, IZA, Federal Reserve Bank of New York and the Volkswagen Foundation for their generous support. Philippe Moutot, and Francesco Mongelli provided helpful guidance as our contacts at the ECB. The discussants and participants at the IWFP conference held in Frankfurt in June 2004 provided useful input that helped shape the project. Finally, Daniel Egel and Rebecca Vichniac wrote most of the programs used by the country teams and consulted extensively with them on the implementation of the IWFP protocol. Their contribution to the project goes well beyond what is normally acknowledged as excellent research assistance. The views expressed in this article are those of the authors and do not reflect the views of the Federal Reserve System, The European Central Bank or any other organization with which authors or team members are affiliated.

The adoption of explicit or implicit inflation targets by many central banks, and the low stable rates of inflation that have ensued, raise the question of how inflation affects market efficiency. The goal of the International Wage Flexibility Project (IWFP)—a consortium of over forty researchers with access to micro level earnings data for 16 countries—is to provide microeconomic evidence on the costs and benefits of inflation in the labor market. The results are intended to inform researchers as well as monetary and regulatory policymakers who are interested in labor markets or the impact of inflation. We study three market imperfections that may cause the rate of inflation to affect labor market efficiency. They are:

- Nominal wages may be rigid downward because of the presence of substantial resistance to *nominal* wage cuts, most often attributed to money illusion, fairness considerations, nominal minimum wages or nominal contracts (Keynes 1936; Slichter and Luedicke 1957; Tobin 1972; Akerlof, Dickens and Perry 1996). Under low inflation, such rigidity means that more workers have wage freezes and fewer workers experience wage cuts than would be the case otherwise, leading to increased wage pressure and higher unemployment. In this case, higher inflation could ameliorate the negative consequences of nominal wage rigidity.

- Alternatively, if the resistance to wage cuts is informed and concerted enough to prevent *real* wage cuts, for example owing to the impact of union bargaining or wage indexation, wages may exhibit downward real rigidity. In this case, higher inflation would not ameliorate the problems associated with downward rigidity. Only higher productivity growth, by providing scope for higher growth in real wages, can reduce the unemployment caused by a real wage floor.

- Finally, market participants may have incomplete information about inflation and productivity changes—the fundamental drivers of aggregate nominal wage growth. Thus, wage changes may be subject to uncertainty-induced errors or adjustment lags (Groshen and Schweitzer 1999, 2000). If wages behave like goods' prices under uncertainty, higher inflation will be associated with more frequent wage changes, higher search costs, and greater uncertainty about the future path (Sheshinski and Weiss 1977; Friedman 1977; Vining and Elwertowski 1976). Conversely, low inflation will minimize the costs associated with uncertainty-induced errors and adjustment lags.

In short, inflation can "grease" the wheels of economic adjustment in the labor market by relieving the constraint imposed by downward *nominal* wage rigidity, but not if there is also substantial downward *real* wage rigidity. At the same time, inflation can throw "sand" in the wheels of economic adjustment by degrading the value of price signals. A number of recent studies suggest that wage rigidity is much more important for business cycles and monetary policy than previously believed (see Erceg, Henderson and Levin, 2000, Smets and Wouters, 2003, and Hall, 2005). Thus, our results on how wage rigidity and other labor market imperfections vary between countries and how they are affected by the rate of inflation should be of considerable value in formulating monetary policy and conducting related research.

To investigate these imperfections, the IWFP convened thirteen country teams plus a central analysis team that computed wage change distributions locally and then analyzed them jointly. The country teams have access to large panels of European or US data on individuals' wage or earnings. The countries include most of Europe (large and small, north and south, euro and non-euro area countries). In all, the IWFP applied a common protocol to 31 distinct panel datasets that contain over 27 million workers' wage changes, resulting in a unique cross-country panel dataset with 360 dataset-years of wage rigidity measures.

The paper proceeds as follows. First, we briefly review the empirical literature. Next, we describe our data and empirical approach. We then establish that wage changes show substantial dispersion that rises with the rate of wage inflation, as predicted by grease and sand effects. Next, we examine histograms of wage changes for the particular asymmetries and spikes that are characteristic of downward real and nominal wage rigidity. This yields estimates of the prevalence of both types of wage rigidity for each dataset and year that we then analyze for insight into the causes and consequences of those rigidities. Finally, we examine the linkage between wage change dispersion and inflation for evidence of errors and adjustment lags (sand effects).

I. Previous studies and the IWFP approach

The IWFP unites and advances three largely distinct strands of research on wage changes. Of these, only downward nominal wage rigidity has been extensively studied before. Furthermore, with few exceptions these studies focus on evidence for a single country, so they cannot take advantage of cross-country comparisons to measure or explain wage rigidities.

a. Downward nominal wage rigidity – grease effects

Taken at face value, the many studies of U.S. and Canadian wages show conflicting evidence of the extent of downward nominal wage rigidity (see reviews in Camba-Mendez, García and Rodríquez-Palenzuela, 2003 and Holden, 2004). Almost all rigidity studies examine distributions of wage changes over several years. They gauge wage rigidity using distributional measures such as the concentration of wage freezes (often called the "spike at zero"), the asymmetry caused by the "missing" wage cuts that have been constrained to be wage freezes, and the extent to which more inflation reduces the asymmetry and the height of the spike. Inflation's relaxation of the constraints imposed by downward nominal wage rigidity is often called the "grease" effect.

On close examination, the apparently conflicting evidence is largely due to a different treatment of mismeasurement in wages. Wages can be measured inaccurately for several reasons. Reporting error is likely to bias micro data measures of rigidity towards finding too little rigidity, because it produces spurious variability in wage changes and false wage "cuts".[2] Similarly, many studies use earnings data divided by hours worked to construct a measure of wages. Earnings include fluctuations in other parts of compensation (for example, overtime work paid at bonus rates) that can disguise the rigidity of the base wage. Data on hours worked may also contain errors that can also be a source of spurious fluctuations in a constructed measure of wages.

The subset of studies (including Altonji and Devereux 2000; Akerlof, Dickens and Perry 1996 and Gottschalk forthcoming) on the U.S. that focuses on the base wage[3] and take proper account of reporting errors are quite consistent. These papers find a clear pattern of substantial resistance to nominal wage cuts in the U.S.[4] Specifically, they find a large number of workers receiving wage freezes and very few workers whose wages are cut in nominal terms.

International comparisons offer a key route for investigation of the relative importance, causes and consequences of rigidity. However, differences in data and methods among independent micro studies can confound attempts to compare rigidities among countries. Three studies using high quality British data (Barwell and Schweitzer 2005, Nickel and Quintini 2003 and Smith 2000) find considerably less resistance to nominal wage cuts in the U.K. than in the U.S. or Canada. Fehr and Goette (2005) use

---

[2] A validation study of the Current Population Survey, which uses questions very similar to those in the surveys analyzed for the IWFP, find that only about 55% of people correctly report their earnings (Bound and Krueger, 1991).

[3] Some have argued that the more comprehensive measures of compensation are appropriate for studying rigidity. We believe a focus on base wages is appropriate because if base wages are highly rigid (in either real or nominal terms), then circumventing those effects by varying other types of compensation is likely to be costly. Furthermore, many changes in other aspects of compensation are not voluntary, such as when a hike in insurance premiums raises employer costs for the same package of benefits. Such changes may occur, but they may not mean that employers have the ability to make such changes to what employees receive. Finally, Lebow et al.'s (2003) study of the U.S. Employment Cost Index, finds no evidence that firms circumvent rigidity in base wages by changing other types of compensation.

error correction techniques on administrative and survey data, along with personnel data, in Switzerland and find considerable downward nominal wage rigidity—more similar to the level observed in the U.S. than in the U.K. Two studies of cross-country variation during the mid 1990s using the European Community Household Panel (Dessy 2005; and Knoppik and Beissinger 2005) find that nominal rigidity varies considerably across countries. Finally, using industry level data for 19 OECD countries, Holden and Wulfsberg (2005) also find significant downward nominal wage rigidity that is more prevalent when unemployment is low, union density is high, and employment protection is strict.

In a study using U.S. data, Akerlof, Dickens and Perry (1996) assess the impact of downward nominal wage rigidity on unemployment by estimating Phillips curves that include a term representing the wage effects of rigidity. The inclusion of this term reveals evidence of a long run trade-off between inflation and unemployment at very low rates of inflation (less than 3%). They find that only during the Great Depression was inflation sufficiently low in the U.S. for downward nominal wage rigidity to increase unemployment by more than a percentage point. By contrast, Fortin (1996), Djoudad and Sargent (1997), and Dickens (2001) find large effects in Canada in the 1990s when inflation was low for an extended period. However, application of this method to several European countries (Dickens 2001) does not provide consistent evidence of a long-run trade-off between inflation and unemployment as would be expected if downward

[4] Other consistent support is found in studies that use interviews of market participants (see Kaufman 1984; Blinder and Choi 1990; and Bewley 1999) or analyze personnel files (see Altonji and Deveraux 2000 and Wilson 1999) which are less prone to error than survey data.

nominal rigidity were important in wage setting. One explanation for this result could be the presence of real rigidity in these countries.

b. Downward real wage rigidity

There has been little study of downward real wage rigidity in a formulation that parallels downward nominal wage rigidity, as is done here. Nevertheless, many European observers believe that their labor market institutions (such as high unionization, wage indexation, and coordinated bargaining) protect the *real* wages of workers, rather than their *nominal* wages. Helping to fill this void, several recent micro data studies that use a methodology developed for an earlier phase of the IWFP find varying degrees of downward real rigidity in the U.K., Finland, Italy and other European countries.[5]

In addition, there have been several attempts to use macro data to assess a related, but distinct concept, namely the extent to which real and nominal wage changes are insensitive to unemployment or the business cycle (for example, see Alogoskoufis and Manning 1988; Layard, Nickell, and Jackman 1991; and Blanchard and Summers 1986).[6] If widespread, the real rigidity that we investigate here could produce insensitivity to fundamentals, although it is not necessary for such insensitivity.

c. Adjustment lags and errors--sand effects

Few studies have examined the degree to which increased inflation distorts wage signals in labor markets. Instead, studies have emphasized such problems in product markets. In the only exception of which we are aware, Groshen and Schweitzer (1999

---

[5] See Barwell and Schweitzer (2004), Bauer, Bonin, and Sunde (2003), Böckerman, Laaksonen and Vainiomäki (2003), Dessy (2005), and Devicienti, Maida and Sestito (2005). The methodology used in these studies is not used here because some identifying assumptions proved invalid in some of the countries.

and 2000) note that both downward nominal wage rigidity and inflation uncertainty (that rises with the level of inflation) imply that the dispersion of wage changes should increase with inflation. Increasing inflation should reduce the concentration of wage changes at zero that is caused by downward nominal wage rigidity, while more errors in wage setting will raise the dispersion of wage changes regardless of the effects of rigidity. Groshen and Schweitzer find increasing variance of wage changes with increased inflation and implement a method for disentangling the two effects in explaining that relationship.

d. Design of the IWFP

The IWFP has developed a new approach to measuring these three imperfections – downward nominal rigidity (grease), downward real rigidity, and sand effects – that is based on the different effects they have on the dispersion and symmetry of wage changes and takes careful account of the biases introduced by measurement error. The features for which we test are summarized Table 1. Each row lists an imperfection and its predicted effects on the distribution of wage changes.

The first key prediction (column 2) is that grease and sand effects both imply that the dispersion of wage changes rises with inflation (under downward nominal rigidity, firms are less constrained; if there are informational problems, they make more mistakes or have lagged adjustments). By contrast, in the case of real rigidity, higher inflation may simply raise the mean wage change without affecting the dispersion of wages—although it could raise dispersion if higher inflation leads to more disagreement about the expected

[6] Another related branch of the literature measures insensitivity of individual real wages to local unemployment (see Blanchflower and Oswald, 1994 and 2005 and Sanz-de-Galdeano and Turunen, 2006).

rate of inflation. Then wage setters expecting high inflation would have higher wage growth than those who expect low inflation.

Second, with regard to symmetry and spikes (column 3 of Table 1), both nominal and real downward rigidity should lead to high concentrations of workers with nominal or real wage freezes (that is, with wage changes equal to zero or to expected inflation, respectively) and correspondingly fewer workers with increases below those rates. By contrast, more errors or lags in wage setting will increase the variance of the observed wage change distribution, but there is no reason to believe they will affect the distribution's symmetry.

II. Empirical approach and data

a. Empirical approach

The empirical approach used by the IWFP (called "distributed micro analysis") has country teams apply common analytical protocols to datasets for their countries locally. They use their expertise with the relevant data, history and institutions, while observing the confidentiality restrictions under which they are granted access to the information. Statistics generated by the protocols for each dataset are used by teams as the basis of country-specific analysis and also collected into a panel dataset for combined analysis.

This strategy has several virtues. First, heterogeneous country environments provide important variation for analysis of the impact of policy and institutions. Second, the application of common, flexible protocols allows for better comparison among countries than is typically available for meta-analysis of micro results. Third, the use of multiple data sources provides insight into the impact of data characteristics on estimated outcomes. Fourth, researchers who are familiar with the data, the country, and its institutions are best able to apply a protocol to the data properly and to interpret the results appropriately. Since the application of the protocol to a particular dataset and country often requires some adjustment, collaboration between the project directors and the country teams ensures a good balance between the demands of uniformity and respect for local variation. Finally, local application of the protocols allows inclusion of confidential data that would be inaccessible otherwise.

b. Data

The first goal of the IWFP was to examine the relative importance of sand and grease effects across a number of European countries and the U.S. As such, availability of data appropriate to this task was the main determinant of participation in the study. The countries included, and the broad characteristics of the datasets used, are described in Table 2. To augment the datasets analyzed by country teams, the central analysis team obtained access to the European Community Household Panel (ECHP), which adds another 12 datasets and 3 additional countries.[7]

The 31 datasets analyzed for the project cover over 27 million wage changes and are diverse with respect to source, coverage, years, industrial sector, occupations covered, and definitions of variables of interest (see Table 2). The many differences among these types of data add richness as well as potentially confounding factors to the analysis below.

[7] The ECHP is a longitudinal database drawn from a survey of households in 15 EU countries; it includes detailed information about individual characteristics, including earnings (for more information on the first three waves of ECHP data see Peracchi 2002). Wages are reported as net earnings (including overtime pay and bonuses) in the previous month (except for France and Finland, where net earnings are derived from reported gross data using a net/gross ratio). We exclude the series for Spain, Luxembourg and Sweden due to data limitations.

The three main sources of data are employment registers, household surveys and employer surveys. An employment register (maintained by a government for the administration of taxes and/or benefits) covers all workers in a specified universe and has minimal reporting error. Some country teams work with random samples drawn from the registers, while others analyze the entire census. Household surveys sample from the universe of all workers, but typically rely on respondent recall, and so they are subject to both sampling and reporting error. By contrast, employer salary surveys typically cover all workers in the occupations and firms in their purview and draw their data from payroll records, but vary considerably in how many occupations or firms they cover. The employer surveys in the IWFP are particularly comprehensive because they are conducted by national employer associations and are used extensively for policy and managerial purposes.

Datasets also vary in terms of the compensation measures available. Some datasets have base wages. However, most wage information in the IWFP is based on monthly or annual labor earnings (that is, including base wage, overtime pay, and bonuses). In those cases, we use a proxy for base wage: earnings divided by the best available measure of hours worked. Hours worked information is available for most of the data sources.

Samples are restricted to workers who did not switch employer in order to concentrate on rigidity in ongoing employment relationships. In addition, large outliers in wage changes[8] are excluded as they likely reflect wage reporting errors or unidentified job changes.

The time periods covered by the different datasets vary, with some starting in the early 1970s and others running through the beginnings of the 2000s, with an average of twelve years per dataset. In total, there are 360 dataset-year observations, including observations from multiple datasets for twelve countries: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Portugal, Sweden, Switzerland, and the U.K.

III. The dispersion of wage changes

We begin with the simple scatter plot of the standard deviation of log (percentage) wage changes against the rate of wage inflation for each year for each dataset in our study shown in Figure 1a. Both the magnitude and range of these standard deviations are remarkable. To some extent, the magnitude and range are artifacts of a high average level of measurement error and of variation across datasets in the extent of error. But as Figure 1b shows, even when the average standard deviation for each dataset is subtracted from the standard deviations for the years observed (which should remove persistent differences due to dataset measurement error characteristics), there is still substantial variation.

Further, the linear relationships plotted in the two graphs suggest that inflation plays a role in determining the extent of variation, as we would expect. The magnitude of inflation's impact on wage change dispersion is modest. A two standard deviation rise in

[8] Increases of more than 60% in wage data or 100% in annual income data and cuts of more than 35% in wage data or 85% in income data were eliminated.

inflation (5.7 percentage points) raises the dispersion of wage changes by about half of a standard deviation (or 2.1 percentage points).

Histograms of wage changes offer a way to identify the grease, real rigidity and sand effects more directly. For example, Figure 2a presents the histogram of percentage wage changes for wage earners in the U.S. in 1988. It has four noteworthy features:

- The histogram illustrates the substantial variation in wage changes among individuals that was shown to be common across all countries and years in Figures 1a and 1b.
- There is a large concentration of workers at exactly zero wage change (that is, with wage freezes) suggesting the presence of downward nominal wage rigidity.
- The histogram reveals notable asymmetry; the mean wage change is 1.2 percentage points greater than the median. This asymmetry is largely due to the absence of workers with wage cuts and the piling up of workers with wage freezes. If the workers with wage freezes are spread among the wage-cut bins in proportion to the workers who actually received wage cuts, the difference between the mean and the median drops to only 0.4 percentage points. The boxes above the distribution to the left of the median show the reflection of the upper tail of the distribution. It is clear that a substantial number of workers are missing from the lower (wage-cut) tail; they are concentrated in the wage-freeze spike at zero instead.
- The distribution of wage changes shown would not be Gaussian or normal even if the wage-freeze spike at zero and the missing wage-cuts were ignored. The distribution is notably more peaked and has somewhat fatter tails than does a normal distribution with the same median and standard deviation.

For a clear contrast, consider Figure 2b, which shows a wage change histogram for Belgium in 1979. While it initially looks similar to Figure 2a, the horizontal axis reveals that the spike is located in the range of 4 to 5%, rather than at zero in a year when price inflation was 4.5%. In this diagram, there are almost no “extra” wage freezes (that is, almost no spike at zero) and no evidence of a lack of wage cuts compared to low wage growth. If Figure 2a suggests downward nominal wage rigidity, Figure 2b suggests downward real wage rigidity and shows how the presence of strong downward real wage rigidity can make downward nominal rigidity less relevant.

Analysis of the 360 dataset-year histograms in the IWFP yields strong, consistent evidence that wage changes are not normally distributed. Wage change distributions consistently show the following features:

- Too many people have wage freezes, creating a large **spike at zero**.
- Workers' wage changes are tightly clustered around the median change, making the distributions much **more peaked** than the normal.
- More people have more extreme wage changes than predicted by normality (that is, the distributions have **fat tails**).

Appendix A details the analysis that reveals these features and also shows that fitted two-sided Weibull distributions provide a much better fit for wage changes than do fitted normal distributions. These important regularities underlie the design of the protocol used by the IWFP.

IV. Methodology

This section describes the methodology used to assess histograms for evidence of the three effects under investigation. First we explain the construction of some simple measures of downward real and nominal rigidity and of inflation-induced errors and lags in wage adjustment. We then discuss the potential problems that measurement error causes in these metrics. Many of the statistical measures used in past studies may be subject to important biases caused by errors associated with data collection procedures. Finally, we describe a new two-step estimation procedure developed by the IWFP to address these biases and render results more comparable across different datasets.

a. Simple measures of downward nominal and real wage rigidity

To measure real or nominal wage rigidity in a particular year, we can construct metrics that provide answers to two distinct questions:

- What fraction of workers who would normally receive a wage cut in the absence of downward rigidity will instead receive a real or nominal wage freeze?
- What is the impact of the rigidity on the average nominal wage change in that year?

We begin by positing that the "notional" percentage wage change distribution, i.e. the distribution of wage changes that would prevail in the absence of downward rigidity, is symmetric. Thus, in the absence of rigidities, the mean wage change equals the median wage change. In fact, this appears to be the case. We find symmetry in the wage-change distribution between zero and two times the median when inflation is high in countries with little or no real rigidity. As we will see later, a model that assumes that the only deviations from symmetry are due to downward real and nominal rigidity does a very good job of describing the actual wage change distribution in nearly every country in nearly every year.

To answer the first question, in the case of downward nominal wage rigidity we proceed as follows. Suppose a fraction n of workers is protected from wage cuts by downward nominal rigidity. The fraction of workers whose wages do not change will equal the fraction of workers in the left tail of the notional distribution below zero times n. This means that we can estimate the fraction n as the ratio of the number of workers with a wage freeze to the total at risk of a cut (that is, those receiving no wage change plus those receiving a cut).

By how much does nominal rigidity raise the average wage in the true distribution above that of the notional distribution? Assuming that the median wage change is greater than or equal to zero (as it is in every year in every country in all of our datasets) downward nominal wage rigidity will not affect the median wage change, but it will raise the mean. Assuming that all workers who would have received wage cuts are equally likely to be subject to downward nominal wage rigidity, the mean will increase by the absolute value of the average wage cut times the fraction of workers receiving no wage change. This measure, which has been called the "wage sweep-up," has been used to estimate the impact of downward nominal wage rigidity.

Turning to downward real rigidity, we begin by noting that if the expected rate of inflation is below the median notional wage change (as is likely if productivity is growing), downward real wage rigidity will also raise the mean without affecting the median wage change. Thus (all else equal), the rise in the mean wage change due to the combined effect of downward nominal and downward real wage rigidity is equal to the difference between the mean and the median. So, we can estimate the wage impact of downward real wage rigidity by subtracting the impact of downward nominal rigidity (that we estimate as described above) from the difference between the mean and the median.

Finally, we need a measure of the fraction of the workforce susceptible to downward real wage rigidity that is analogous to our measure of downward nominal wage rigidity. Unfortunately, not all agents in the labor market share the same expected rate of inflation, so we will not see the extreme spikes we observe around zero to identify nominal rigidity. Thus, we take an indirect approach: backing out the number from the wage impact.

The wage impact of downward real wage rigidity is equal to the fraction of workers receiving real wage freezes multiplied by how much their wages were affected. Thus, dividing our estimate of the wage effect of real rigidity by the difference between expected inflation and the average wage change of workers who received a raise of less than expected inflation yields an estimate of the fraction of workers who receive real wage freezes.[9] To arrive at the fraction of people potentially subject to real rigidity (r) we divide that fraction by the fraction of workers at risk for a real wage freeze; the fraction of workers with notional wage changes less than the expected rate of inflation. To take into account the variation across agents in the expected rate of inflation we assume that half of the people who receive real wage freezes will receive wage changes above the average expected inflation and half will receive wage changes below this value. Thus, the fraction at risk equals the fraction of people receiving less than our estimate of expected inflation plus half the people who we believe are affected by downward real wage rigidity. We compute

(1) $$r = \mathrm{f} / \mathrm{A} = \mathrm{f}\,(1 - \tfrac{1}{2} r) / \mathrm{B} = \mathrm{f} / (\mathrm{B} + \tfrac{1}{2}\mathrm{f}),$$

since $\mathrm{B} = (1 - \tfrac{1}{2} r)\,\mathrm{A}$,

where r is the fraction of workers at risk for downward real rigidity, f is the fraction of workers receiving real wage freezes, A is the fraction of workers with notional wage changes below the average expected rate of inflation, and B is the fraction of workers observed to have wage changes lower than the expected rate of inflation.

---

[9] We estimate expected inflation from a simple linear regression of the current year's inflation on the

b. Inflation-induced adjustment lags and errors

Examination of histograms also yields evidence of the presence of inflation-induced lags and errors in wage changes. Workers with above-median wage raises are normally not affected by downward real or nominal rigidity. All else held equal, we would expect more errors or a greater frequency of errors to be associated with higher variance of the notional wage change distribution. Downward rigidity will distort the variance in the lower tail of the distribution, but usually the upper tail will be unaffected. We thus compute the standard deviation of the notional wage change distribution as our measure and construct it by taking the square-root of the average squared difference between the median and observations above the median in our observed wage change histograms.

c. Issues with the simple measures and description of alternative MMM estimates

The simple measures described above are potentially subject to several problems:

- Wages may be mismeasured due to response errors or, if wages are imputed from earnings and hours, due to contamination with other components of compensation or with inaccurate information on hours worked.
- The measure of downward nominal rigidity may be confounded with the effects of symmetric nominal rigidity due to rounding errors or employers' unwillingness to give very small wage changes of either sign.
- The spike at zero in annual income data may be smaller than that for wage changes since people must go two years without any wage changes for their annual income to show no change.
- Our estimate of expected price inflation may not match the expectations of wage-setting agents.

To deal with these issues, the IWFP created a set of comparable estimates that correct for measurement error, allow for symmetric nominal rigidity, model the link

previous year's inflation.

between wage determination and annual income, and estimate the expected rate of inflation from the observed wage change data.

This section briefly describes this new method for estimating rigidity, which we call the Mixed Method of Moments (MMM) approach. For full detail, see Appendix B. The method has two main elements: a correction for measurement error (that estimates distributions of "true" wage changes from observed ones) and estimation of rigidities (by comparing true wage changes with notional ones).

Our correction technique can be used on a variety of datasets without strong assumptions about the distribution of wage changes.[10] Consider two histograms: one showing the distribution of true wage changes and one showing the distribution of observed wage changes, where measurement error may pollute the observed data. Given the distribution of measurement error, one can write a transformation matrix that converts the histogram of true wage changes into the histogram of observed wage changes. If that transformation matrix were known, the histogram of the true distribution could be recovered by multiplying the vector of observed frequencies times the inverse of the transformation matrix. This relationship amounts to a set of moment conditions equal to the number of cells minus 1in the true wage change histogram.

In order to obtain this transformation matrix, we must assume a form for the distribution of the measurement error and estimate its parameters. The intuition behind our method for estimating the parameters of the error distribution is straightforward. Measurement error in wage levels introduces negative serial correlation into wage

---

[10] Previous approaches to correcting for measurement error make strong functional form assumptions about the distribution of true wage changes (Altonji and Devereux, 2000; Fehr and Goette, 2005), or require

changes, and that negative serial correlation can be related directly to the frequency and variance of errors. Given an estimate of the true wage change distribution, and the distribution of errors, predictions can be made about the frequency with which people who receive wage changes larger than one value in one period will receive wage changes smaller than a lower value in the next period or vice versa. These relations yield several additional moments that allow us to identify the parameters of the error distribution.

The large number of parameters in the true wage change distribution makes the generalized method of moments (GMM) approach impractical for estimating the error correction model. Instead, given guesses of the parameters of the error distribution, we solve for the true wage change distribution from the observed wage change distribution, concentrate it out of the remaining moment conditions, and use GMM to estimate the remaining parameters of the error distribution.

Two features of this estimator are noteworthy. First, the estimator makes only very limited assumptions about distribution of true nominal wage changes (i.e., that it can be represented by a histogram with 76 or 186 cells, depending on the type of dataset). Therefore, the estimator is not biased towards finding one particular form of wage rigidity over another. Second, application of this correction strategy to IWFP datasets provides convincing evidence that the process works as intended.[11] For example, the U.S. PSID and the ECHP are survey datasets where wages are reported with a great deal of error, which our method detects. A few of our datasets (notably the Finnish employer

---

high-frequency data on wage changes (Gottschalk, forthcoming) that are not available in most of the countries we study.

[11] In addition, Gottschalk's (forthcoming) method for estimating true wage changes also yields an error and a true wage series for each individual in his dataset. We tested all of our assumptions on those two series and could not reject any of them.

survey data and the German administrative data) accurately measure a base wage concept and our method yields negligible corrections in these cases.

After correcting for measurement error, we use GMM to fit a simple model of wage changes to the error-corrected wage- or income-change histograms for each dataset year.[12] We assume that, in the absence of rigidity, log wage changes have a symmetric two-sided Weibull notional distribution (see Appendix A for further explanation of the choice of the two-sided Weibull distribution). We estimate all three parameters of the notional distribution in each year, as well as productivity increases and the mean and standard dispersion of expected inflation.

We use that notional distribution and expected inflation estimates to produce alternative estimates of the extent of downward nominal wage rigidity (n) and of downward real wage rigidity (r), as well as estimates of the dispersion of notional wage changes that we can examine for evidence of errors (sand effects) in wage setting. The procedure also produces an estimate of the extent of symmetrical nominal rigidity that we do not consider here.

V. Rigidity estimates

When we apply the methods described above to estimating rigidity for each dataset for each year, we find a great deal of variation across time and datasets. Before proceeding with further analysis, we first compare the results obtained from the simple and MMM methods and examine the validity of the variation in wage rigidity.

[12] The IWFP has experimented with a number of other methods for identifying differing degrees of rigidity (Dickens and Goette 2004) requiring less restrictive assumptions. The other methods were judged inferior

Note that each method produces estimates of the extent of downward nominal wage rigidity (*n*) and of downward real wage rigidity (*r*). In concept, these measures vary between 0 and 1, where 0 indicates perfect flexibility (no one is constrained) and 1 indicates full rigidity (all workers are potentially constrained).

a. Comparison of simple and MMM cross-country estimates and other validation exercises

Focusing first on cross-country evidence, the two sets of estimates provide similar results. Country averages of the two sets of estimates are closely correlated, more so for the measures of real wage rigidity (with a correlation coefficient of 0.68) than the measures of nominal wage rigidity (0.55). On average across all countries, the MMM measure points to somewhat higher nominal rigidity (with an estimated fraction of 0.36 workers being potentially subject to nominal rigidity) than the simple measure (0.28), and somewhat lower real rigidity (0.27 versus 0.25 for the simple measures). This is to be expected as the MMM estimates correct for measurement error which causes an underestimation of the true size of the spike at zero (the basis for our simple estimate of *n*). On the other hand, measurement error has almost no effect on the difference between the mean and median wage change, which is the basis of our estimate of the fraction of workers who are potentially subject to downward real wage rigidity.

Regressions (not reported here) of the two sets of estimates of *r* and *n* on country indicators show that country effects are jointly significant in both sets, even with controls

---

in that deviations from this more restrictive method seemed to result from shortcomings of the alternative methods rather than from failures of the distributional assumptions critical to the method used here.

for a full set of dataset characteristics.[13] Dataset characteristics explain an important part (27 percent) of the variation in the simple measure of nominal wage rigidity, while they explain only a minor part (less than 5 percent) of the variation in the MMM measure of nominal rigidity (*n*). This suggests that the error correction procedure does a good job of removing the influence of dataset characteristics when measuring nominal rigidity. In the case of real rigidity, dataset characteristics explain less than 5 percent of the variation in both cases.

For further validation, we can compare these measures to those from other studies and between different datasets in our study. We find that the MMM estimates of nominal wage rigidity are positively correlated with measures from two other cross-national studies (Knoppik and Beissinger 2005, and Holden and Wulfsberg, 2005) that use different methodologies to estimate the extent of downward nominal wage rigidity (Figure 3). It is worth noting that the correlation between our simple measures of nominal rigidity and the results in Knoppik and Beissinger (2005), also not corrected for measurement error, is even larger (0.86) than the correlation with the MMM measures shown in the Figure 3.

In countries where we have *r* and *n* estimates from the ECHP and another dataset, the two are strongly positively correlated (0.53 for both *r* and *n*) when we correct for measurement error using MMM. Correlations using our simple measures are weaker (-0.12 for *r* and 0.26 for *n*). This provides further evidence of the impact of data characteristics on the simple estimates of rigidity, since ECHP data differs in a number of

---

[13] The dataset characteristics include the average autocovariance of wage changes and indicator variables for the following: census vs. sample, survey vs. administrative records, earnings vs. wage data, whether

ways from most of the other datasets (see Table 2). However, since the paired estimates cover different time periods, and in some cases different types of workers, we do not expect a perfect correlation.

In sum, we conclude that the MMM measures detect patterns of rigidities that are similar to the simple measures and those obtained in previous studies, but preferable because they are less prone to bias from measurement error. Overall, we consider these results to be very supportive of the reliability of our MMM country average estimates. Thus, the analysis below uses only the preferred MMM estimators. As a check, we have performed all the exercises below for the simple measures and find similar, albeit usually weaker effects in each case.

b. Cross-country comparisons of rigidity

We find considerable variation in the extent of both real and nominal rigidity across countries when we average the MMM measures for each country across all datasets and time (see Figure 4). Estimates of the fraction of workers potentially affected by downward nominal wage rigidity (*n*) range from 9% in Germany to 66% in Portugal, while the comparable range for real rigidity (*r*) is 3% in Greece to 52% in Sweden. Furthermore, countries with higher nominal rigidity tend to show less real rigidity (correlation=-0.25). Overall, the cross-country variation points to a possibly important role of differences in institutional characteristics of the labor market as a determinant of the extent of rigidity.

---

the country team ran the annual earnings protocol vs. the wage protocol, whether the data was drawn

c. Time-series variation in rigidity estimates

The MMM estimates of rigidity vary considerably over time for many countries. We expect this time-series variation to be of considerable interest in future work. For now, we focus on the implications of the time-series variation for validation of our cross-country means.

Validation of the variation of our country estimates over time is difficult; nevertheless, they do receive support in a number of cases. Our ability to validate systematically is limited because we have no comparable alternative cross-country studies and because there is little or no overlap between the time periods covered by the different datasets for the same countries in our study.

However, some of the notable changes that we observe in our measures of rigidity happen at the same time that important institutional changes in the particular countries occur. For example, our estimates of the fraction of workers affected by real wage rigidity in the U.S. declines significantly from the 1970s to the mid-1980s and 1990s. This decline corresponds to the decline in the role of unions and pattern bargaining in U.S. wage setting (Blanchflower and Freeman, 1992).

In addition, we find that the MMM estimates of *r* have no detectable relationship to inflation, as should be the case if wage-setting institutions change slowly. By contrast, under some specifications MMM estimates of *n* have a weakly statistically significant relationship to the level of inflation that begs further investigation.[14]

from the ECHP, and whether hours worked were available.

[14] Such a relationship could arise if the error correction procedure was sensitive to the rate of inflation and over- or under- compensated when inflation is low.

Finally, we note that for several countries our measures of real rigidity show volatility over short periods of time that seems implausible. The likely explanations for these anomalies suggest strategies to correct them. We will implement these modifications in the next phase of our analysis in order to exploit the time-series variation in our sample. For now, we focus on country averages, which are robust to these issues.

VI. Correlates of rigidity

We now explore whether our measures of wage rigidity are associated with labor market institutions that are suspected sources of wage rigidity. We consider the following six labor market institutions: strictness of employment protection legislation (EPL), union density, collective bargaining coverage, whether minimum wage or wage indexation legislation is in place, and the degree of corporatism (an index of bargaining coordination and centralization). Our most robust results are for measures of unionism.

Figures 5 and 6 show scatter plots of country averages of n and r against six measures of labor market institutions. We see that the index of EPL has a weak, statistically insignificant positive correlation with both measures of wage rigidity. The corporatism index—which is a summary measure of centralization and coordination bargaining structures—has a statistically insignificant negative correlation with both nominal and real rigidity.

Our measure of indexation is weakly negatively correlated with nominal rigidity and positively correlated with real rigidity, though neither relationship is statistically significant.[15] The weakness of both results is not surprising since all indexation regimes in our sample provide only partial coverage of the economy. Also, some countries such as Finland and France experience relatively high real rigidity without ever having had wage indexation clauses in place.

The figures also show that countries with higher ratios of minimum wages to average wages have modestly higher levels of nominal rigidity and lower levels of real rigidity. However, this result seems to be driven by the contrast between countries with substantial collective bargaining but no minimum wages, and those with substantial minimum wages. This suggests that the relationship is only a reflection of the much stronger and more robust correlation between unionism and rigidity.

The strongest results are for union coverage and union density. Both measures of unionism are negatively correlated with nominal rigidity and positively correlated with real rigidity (all significant at the 10 percent level in a one-tailed test). We speculate that union representation raises workers' awareness of the path of their real wages and gives them the bargaining power to protect their real wages. Accordingly, workers become less concerned with nominal wage changes. Alternatively, unions may effectively move their members from a regime where their nominal wages were protected to one where they are guaranteed some nominal wage increase. For example, they may threaten to work less efficiently under the protection of an incompletely specified nominal wage contract. Under these circumstances we would estimate a lower rate of nominal rigidity.

[15] For the indexation estimate we exclude Switzerland, which has a prevalent but unusual form of wage indexation. Many Swiss employment contracts require a new round of wage negotiations if prices have increased by a certain amount since the contract period began. However, the outcome of these negotiations is open, so indexation is not automatic (see Blattner et al. 1993). If we include Switzerland, real rigidity has a weak negative correlation with indexation.

The results of our correlation analysis are reinforced by regressions of our time-varying rigidity measures on measures of labor market institutions. Here we also include time and country dummies. Union density has a particularly consistent and statistically significant positive relationship with the real rigidity measure.

VII. Are these rigidities important?

We now consider the consequences of wage rigidity. If downward *nominal* rigidity causes some unemployment at very low rates of inflation, small increases in inflation can reduce this joblessness, creating the grease effect. However, in cases where it is *real* rigidity that reduces employment, and many workers are potentially affected, the grease effect of inflation can be mitigated. To examine the relationship between wage rigidity and unemployment we turn to the general equilibrium model of Akerlof, Dickens and Perry (1996) that motivates a Phillips curve relation of the form

(2) $\pi_t = \pi_t^e + c - a\, U_t + b\, \mathbf{S}_t + x_t + \varepsilon_t.$

In this model, π_t is the rate of price inflation at time t, π_t^e is the expected rate of price inflation, c is a constant, U_t is the unemployment rate, $\mathbf{S}_t$ represents the wage effects of rigidity, x_t is the effects of supply shocks and ε_t reflects other unobserved factors affecting the rate of inflation. This relationship implies that

(3) $U_t = \mathbf{S}_t\, b/a + [c + x_t - (\pi_t - \pi_t^e)]/a + \mu_t.$

The constant b is equal to 1 plus the average mark-up of prices over labor costs. Typical estimates of the coefficient a place it between 0.2 and 1.0, so we expect the impact of $\mathbf{S}_t$ on unemployment to be greater than 1.0.

The wage rigidity variable ($\mathbf{S}_t$) equals the amount by which nominal wages of those constrained by downward rigidity are higher as a result of wage rigidity, multiplied by their share of the wage bill. For each dataset for each year, we approximate $\mathbf{S}_t$ by estimating the extent to which wage rigidity raised average wage changes, as compared to the notional wage change distribution for that dataset-year.[16]

We estimate equation (2) and equation (3). In estimating equation (2) we assume that the expected rate of inflation is equal to the previous period's inflation and that the effect of supply shocks can be captured by indicator variables for specific events (oil shocks) or for all years. In estimating equation (3), we implicitly assume that expectation errors are orthogonal to $\mathbf{S}_t$ by excluding inflation or expectations from the regression. We assume that the expected rate of inflation is equal to the previous period's inflation and that the effect of supply shocks not orthogonal to the rate of unemployment in the period can be captured by indicator variables for specific events. Other shocks become regression error.

We run two specifications for each equation. All specifications include dataset-specific intercepts. The second specification also includes year-specific intercepts to control for common supply shocks. Table 3 presents the results.

[16] To be explicit, we compute a numerical estimate of the average notional log wage change conditional on the wage change being negative. We also compute the average log wage change conditional on it being less than our estimate of the average expected rate of inflation. The latter average wage change is multiplied by a smoothed estimate of the fraction of the workforce potentially subject to downward real wage rigidity (r). Similarly, the former average wage change is multiplied by a smoothed estimate of the fraction of the workforce potentially affected by downward nominal rigidity times one minus the fraction potentially subject to real rigidity ($[1-r]n$). These are summed to obtain to obtain an approximation of $\mathbf{S}_t$. This is only an approximation because the effects of rigidity can accumulate over time if a large fraction of the workforce is affected by the rigidity (see Akerlof, Dickens and Perry). This normally unimportant effect can be strong during extended spells of low inflation or deflation such as the U.S. Great Depression.

In all cases the estimate of the unemployment impact (*b/a*) is statistically significantly greater than zero at least at the 0.1 level, and in all but one case, it is significantly greater than 1. When we separately estimate the effects of real and nominal rigidity (in regressions not reported here), some of the estimates are not statistically significantly greater than zero, but we can never reject the hypothesis that the unemployment effect of a rigidity measure is greater than 1. Nor can we reject the hypothesis that the coefficients on the two measures of the wage impact of rigidity are equal. Also, when we estimate *b* independent of *a* in equation (2) (regressions not reported), it is always less than the predicted minimum of 1, but we can never reject the hypothesis that it is greater than 1.

Taken together, these results provide moderately strong evidence that inflation can lower unemployment in the presence of downward *nominal* wage rigidity (that is, of grease effects), but does not lower unemployment in the presence of downward *real* wage rigidity.

VIII. Adjustment lags and errors (sand effects)

We turn now to the effects of inflation on adjustment lags and errors in wage setting. Here we try to isolate *unintended* variation in wage changes by estimating the effect of inflation on the notional dispersion of wage changes.

Our metric of the dispersion of wage changes is the standard deviation of the MMM notional wage changes distribution (from which the impact of wage rigidities and reporting errors has been removed). To check validity, we can compare these estimates to the standard deviation of wage changes above the median (where the impact of rigidities should be absent). The correlation between the two measures is indeed high, at 0.90.

To assess the relationship between this variability and inflation, we note that over time, nominal wage growth will reflect price growth plus productivity growth. Thus, aggregate nominal changes contain three components: expected inflation, inflation surprises and productivity growth. Accordingly, Table 4 reports the relationship between the standard deviation of log wage changes above the median and inflation expectations and these three sources of nominal wage growth, all in quadratic form.[17] The coefficients on the first-order terms of all three factors are similar in magnitude and statistically significant at the 5 percent level (one-tail test), and there is little evidence that these effects taper off, as the coefficients on the quadratic terms are not statistically significant. These results are consistent with the proposition that higher inflation distorts price signals, but do not suggest that unexpected inflation causes more variation than expected inflation.

To gauge the size of misallocation effects of inflation on the labor market, we estimate regressions similar to equations (2) and (3) that include the MMM estimate of the standard deviation of the notional wage change distribution as an additional independent variable, both with and without the measures of rigidity effects. Coefficients were small and statistically insignificant in nearly all specifications.[18] Of course, the costs of inflation need not show up as unemployment, and it is difficult to assess the magnitude of the effects without a detailed structural model of firm wage setting.

---

[17] We measure expected inflation as a three year moving average of realized inflation rates. Several other expectations measures produced similar coefficient estimates, but the standard errors were larger.

IX. Conclusion

The International Wage Flexibility Project has investigated three ways in which labor market imperfections interact with inflation. First, moderate inflation in the presence of resistance to *nominal* wage cuts can "grease" the wheels of relative wage adjustment to ongoing shocks and thus improve economic efficiency. Second, widespread resistance to *real* wage cuts can also raise unemployment rates, but in this case inflation provides no relief. Third, inflation can cause distortions in relative wages that lead to costly resource misallocations, thus throwing "sand" in the wheels of economic adjustment. While the first effect has been studied extensively, especially in the U.S., the other two have not.

The IWFP investigates these three effects simultaneously using 31 panel datasets covering over 27 million wage changes for individual workers in 16 European countries and the U.S., incorporating the expertise, data access, and analysis contributed by 13 country teams. We find considerable variation in wage changes among workers in the same country and year. The variation increases with inflation, as we would expect if either downward nominal rigidity (grease effects) or inflation-induced adjustment lags and errors (sand effects) were important.

Applying a new estimator for the prevalence of these three effects, we find evidence of both types of rigidity in nearly every country. Estimates of the fraction of workers potentially affected by downward nominal wage rigidity (n) range from 9% in Germany to 65% in Portugal while the comparable range for real rigidity (r) is 3% in

[18] The coefficients on the rigidity effects variables are virtually unaffected by the inclusion of this variable and no substantive conclusions are altered.

Greece to 52% in Sweden. Furthermore, there is some evidence that countries with higher nominal rigidity tend to show less real rigidity.

Our technique and wealth of datasets enable us to explore the impact of data features on empirical estimates of wage rigidity and compare our results with those of previous studies. These new measures of downward nominal rigidity are strongly correlated with other recent measures derived from industry or individual data.

Examination of the causes of downward nominal and real wage rigidity suggests an important role for the extent of unionization and collective bargaining coverage. Both show a consistently positive relationship with the extent of downward real wage rigidity and a negative association with downward nominal wage rigidity. This finding suggests that collective bargaining focuses workers' attention on real wages and gives them some ability to resist real wage cuts. Other institutional variables that we examined had weaker relationships with our measures of rigidity.

These differences in rigidity across countries may translate into differences in unemployment. Measures of the "wage impact" of downward nominal and real wage rigidity (that is, the extent to which workers' wages are affected in a particular year) are positively related to unemployment with statistically significant coefficients of about the size predicted by theory. The estimated effects are large. A one percentage point increase in the average wage relative to the notional average wage due to rigidity will cause about a 1 to 3 percentage point increase in the unemployment rate.

Finally, we find evidence that the dispersion of notional wage changes rises with inflation (expected or not) and productivity growth. This rise in notional dispersion is consistent with the view that inflation causes more adjustment lags and errors in wage setting. In addition, we find no evidence of unemployment effects from degradation of price signals, so any costs imposed may be on productivity rather than on jobs.

Our results show that inflation's interaction with labor markets is multifaceted. From a monetary policy standpoint, the beneficial grease effects of inflation (stemming from downward nominal wage rigidity) that we and others detect are only part of the story. Another impact of inflation is likely to be detrimental: we find evidence of inflation-induced errors and lagged adjustments, raising the possibility that high inflation causes misallocation of resources. Third, we find that real rigidities are common in many European countries, and that one source may be high unionization. To the extent that a country's high unemployment stems from real rigidities, it can be tackled only by addressing underlying determinants of the rigidities, not by adjusting monetary policy.

References

Akerlof, George A., William T. Dickens, and George L. Perry (1996). "The Macroeconomics of Low Inflation." *Brookings Papers on Economic Activity 1*, 1-75.

Alogoskoufis, George S. and Alan Manning (1988). "On the Persistence of Unemployment," *Economic Policy*, 7, 428-469.

Altonji, Joseph G., and Paul J. Devereux (2000). "The Extent and Consequences of Downward Nominal Wage Rigidity." In S. Polachek (ed.). *Worker Well-Being*, # 7236, Elsevier.

Baker, George., Michael Gibbs and Bengt Holmstrom (1994). "The Wage Policy of a Firm," *Quarterly Journal of Economics* 109, 921-955.

Barwell, Richard and Mark E. Schweitzer (2005). "The Incidence of Nominal and Real Wage Rigidity in Great Britain: 1978–1998." Federal Reserve Bank of Cleveland Working Paper 05-08.

Bauer, Thomas, Holger Bonin and Uwe Sunde (2003): "Real and Nominal Wage Rigidities and the Rate of Inflation: Evidence from West German Micro Data", *IZA working paper* 959.

Bewley, Truman F., (1999). *Why Do Wages Not Fall During a Recession?* Harvard University Press.

Blanchard, Olivier J. and Lawrence H. Summers (1986). "Hysteresis and the European Unemployment Problem." In S. Fischer (ed.), *NBER Macroeconomics Annual*, MIT Press:Cambridge, MA.

Blanchflower, David and Richard Freeman (1992). "Unionism in the U.S. and in Other Advanced OECD Countries", *Industrial Relations* 31, pp. 56-79.

Blanchflower, David and Oswald, Andrew (1994): *The Wage Curve*, MIT press, Cambridge Massachusetts.

Blanchflower, David and Oswald, Andrew (2005): "The Wage Curve Reloaded" NBER Working Paper 11338.

Blattner, N., W. Hess, M.Jeger and C. Klingen (1993). "Lohnindexierung Formen Verbreitung und Wirkungen", Verlag Rüegger, Chur, Switzerland.

Blinder, Alan S. and Don H. Choi (1990). "A Shred of Evidence on Theories of Wage Stickiness," *Quarterly Journal of Economics*, 105: 4 (November), pp. 1003-1015.

Bound, John and Alan B. Krueger (1991). “The Extent of Measurement Error in Longitudinal Earnings Data: Do Two Wrongs Make a Right?” *Journal of Labor Economics* (9)1: 1-24.

Böckerman, Petri, Seppo Laaksonen and Jari Vainiomäki (2003). “Who Bears the Burden of Wage Cuts? Evidence from Finland During the 1990s.” working paper, University of Tampere.

Bullard, James, and John W. Keating (1995). “The Long-run Relationship between Inflation and Output in Postwar Economies.” *Journal of Monetary Economics 36*, 477-496.

Camba-Mendez, Gonzalo, Juan García, and Diego Rodríguez Palenzuela (2003). “Relevant Economic Issues Concerning the Optimal Rate of Inflation.” Background Studies for the ECB’s Evaluation of its Monetary Policy Strategy, European Central Bank.

Dessy, Orietta (2005). “Nominal Wage Rigidity in Europe: Estimates, Causes and Consequences.” Mimeo, University of Milan.

Devicienti, Francesco, Agata Maida, and Paolo Sestito (2005). “Nominal and Real Rigidity: Micro Based Measures and Implications,” Mimeo, University of Torino.

Dickens, William T. (2001). “Comment on Charles Wyplosz, Do We Know How Low Inflation Should Be?” in Alicia G. Herrero, Vitor Gaspar, Lex Hoogduin, Julian Morgan, and Bernhard Winkler eds. *Why Price Stability? Proceedings of the Frist ECB Central Banking Conference, November 2000, Frankfurt Germany* ECB: Frankfurt.

Dickens, William T. and Lorenz Goette (2004). “Notes on Estimating Wage Rigidity,” Brookings mimeo.

Dickens, William T. and Lorenz Goette (2005). “Estimating Wage Rigidity for the International Wage Flexibility Project,” Brookings mimeo.

Djoudad, R., and Thomas C. Sargent (1997). “Does the ADP Story Provide a Better Phillips Curve for Canada?” Working Paper, Economic Studies and Policy Analysis Division, Department of Finance, Ottawa, October.

EIRO (2005) – The European industrial relations observatory online - http://www.eiro.eurofound.eu

Elmeskov, Jorgen, John P. Martin and Stefano Scarpetta (1998). "Key Lessons for Labour Market Reforms: Evidence from OCED Countries' Experiences", *Swedish Economic Policy Review*, 5, 205-252

Elsby, Michael (2004). "Evaluating the Economic Significance of Downward Nominal Wage Rigidity." Mimeo, London School of Economics.

Erceg, C.J. and Henderson, D.W. and Levin, A.T. (2000) Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics 46*, 281–313.

Fehr, Ernst and Lorenz Goette (2005). Robustness and Real Consequences of Nominal Wage Rigidity. *Journal of Monetary Economics* 52, 779-804.

Fortin, Pierre (1996). "The Great Canadian Slump," *Canadian Journal of Economics* 29, November, 761-87.

Friedman, Milton (1977). "Nobel Lecture: Inflation and Unemployment," *Journal of Political Economy*, vol. 85, no. 3, pp. 451-472.

Gottschalk, Peter (forthcoming). "Downward Nominal Wage Flexibility: Real or Measurement Error?" *Review of Economics and Statistics*.

Groshen, Erica and Mark Schweitzer (1999). "Identifying Inflation's Grease and Sand Effects in the Labour Market in Martin Feldstein, ed. *The Costs and Benefits of Price Stability*, (Chicago: Univ. of Chicago Press).

Groshen, Erica L. and Mark E. Schweitzer (2000). "The Effects of Inflation on Wage Adjustments in Firm-Level Data: Grease or Sand?" Federal Reserve Bank of New York Staff Report No. 9, January 1996 (Revised version: June 2000).

Hall, Robert E. (2006). "Employment fluctuations with equilibrium wage stickiness." Forthcoming in *American Economic Review*.

Holden, Steinar (2004). "Wage Formation Under Low Inflation". In *Collective Bargaining and Wage Formation - Challenges for an European Labour Market*, H. Piekkola and K. Snellman (eds.), Springer-Verlag, pp 39-58.

Holden, Steinar and Fredrik Wulfsberg (2005). Downward Nominal Wage Rigidity in the OECD. Mimeo, available at http://folk.uio.no/sholden/#wp

Kaufman, Roger T. (1984). "On Wage Stickiness in Britain's Competitive Sector." *British Journal of Industrial Relations* 22, 101-112.

Keynes, John Maynard (1936). *The General Theory of Employment, Interest and Money*.

Knoppik, Christoph and Thomas Beissinger (2005). "Downward Nominal Wage Rigidity in Europe: An analysis of European Micro Data from the ECHP 1994-2001." IZA DP # 1492.

Layard, Richard, Stephen Nickell, and Richard Jackman (1991). *Unemployment: Macroeconomic Performance and the Labour Market.* Oxford University Press.

Lebow, David E., Raven E. Saks, Beth A. Wilson (2003). “Downward Nominal Wage Rigidity. Evidence from the Employment Cost Iindex*.” Advances in Macroeconomics* vol. 3, Issue 1, article 2.

Nickell, Stephen J. and Glenda Quintini (2003). “Nominal Wage Rigidity and the Rate of Inflation.” *The Economic Journal 113*, 762-781.

OCED (2004). "Wage Setting Institutions and Outcomes" Chapter 3 in the *OECD Employment Outlook*.

Peracchi, F. (2002). “The European Community Household Panel: A Review”, *Empirical Economics* 27, pp. 63-70.

Reinsdorf, Marshall (1994). “New Evidence on the Relation between Inflation and Price Dispersion,” *American Economic Review*, vol. 84, no. 3, pp. 720-731.

Sheshinski, Eytan and Yoram Weiss (1977). “Inflation and Costs of Price Adjustment,” *Review of Economic Studies*, vol. 44, no. 2 pp. 287-303.

Slichter, Sumner and Heinz Luedicke (1957). “Creeping Inflation—Cause or Cure?” *Journal of Commerce*, pp. 1-32.

Sanz-de-Galdeano, Anna and Turunen, Jarkko (2006): “The Euro Area Wage Curve,” forthcoming in *Economics Letters*.

Smets, F. and Wouters, R. (2003). “An estimated dynamic stochastic general equilibrium model for the Euro area.” *Journal of the European Economic Association* 1, 5, 1123–1175.

Smith, J.C. (2000). “Nominal Wage Rigidity in the United Kingdom,” *Economic Journal* 110(462): C176-C195.

Stigler, George J. and James K. Kindahl (1970). *The Behavior of Industrial Prices*, National Bureau of Economic Research General Series No. 90 (New York: Columbia University Press).

Tobin, James (1972). “Inflation and Unemployment,” *American Economic Review*, vol. 62, no. 1, pp. 1-18.

Vining, Daniel R. and Thomas C. Elwertowski (1976). “The Relationship between Relative Prices and the General Price Level,” *American Economic Review*, vol. 66, no. 4, pp. 699-708.

Wilson, Beth Anne, (1999). “Wage Rigidity: A Look Inside the Firm,” April 16, FEDS Working Paper No. 99-22 **http://ssrn.com/abstract=165588**

Figure 1a

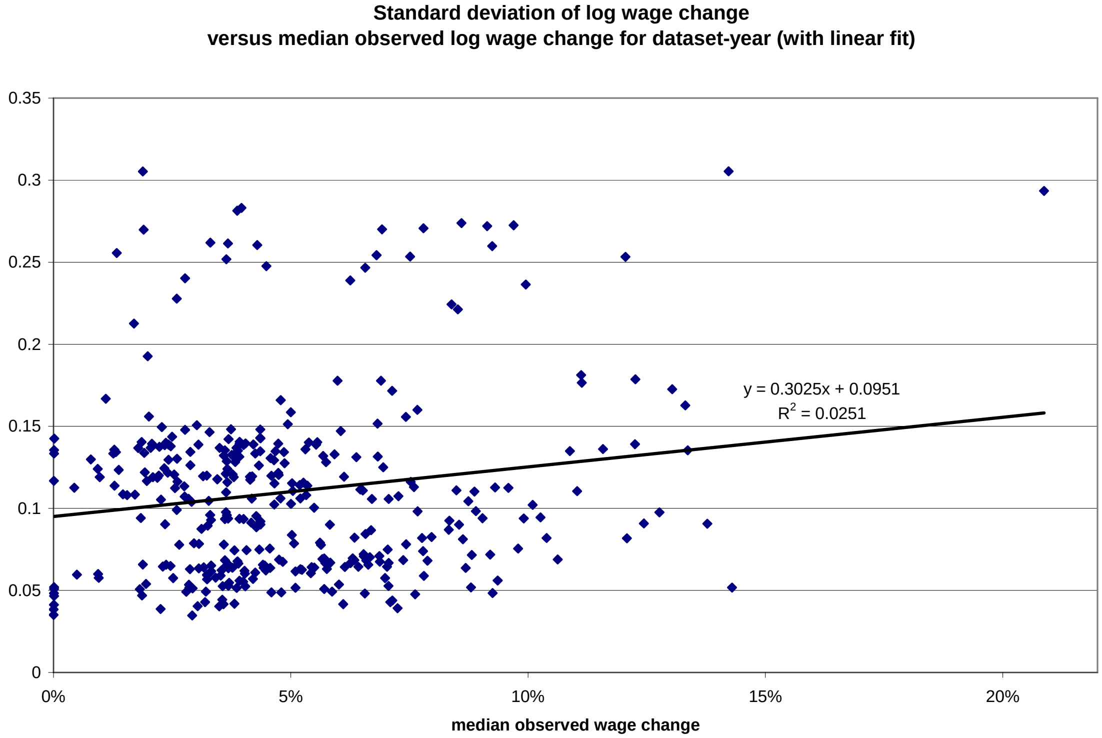
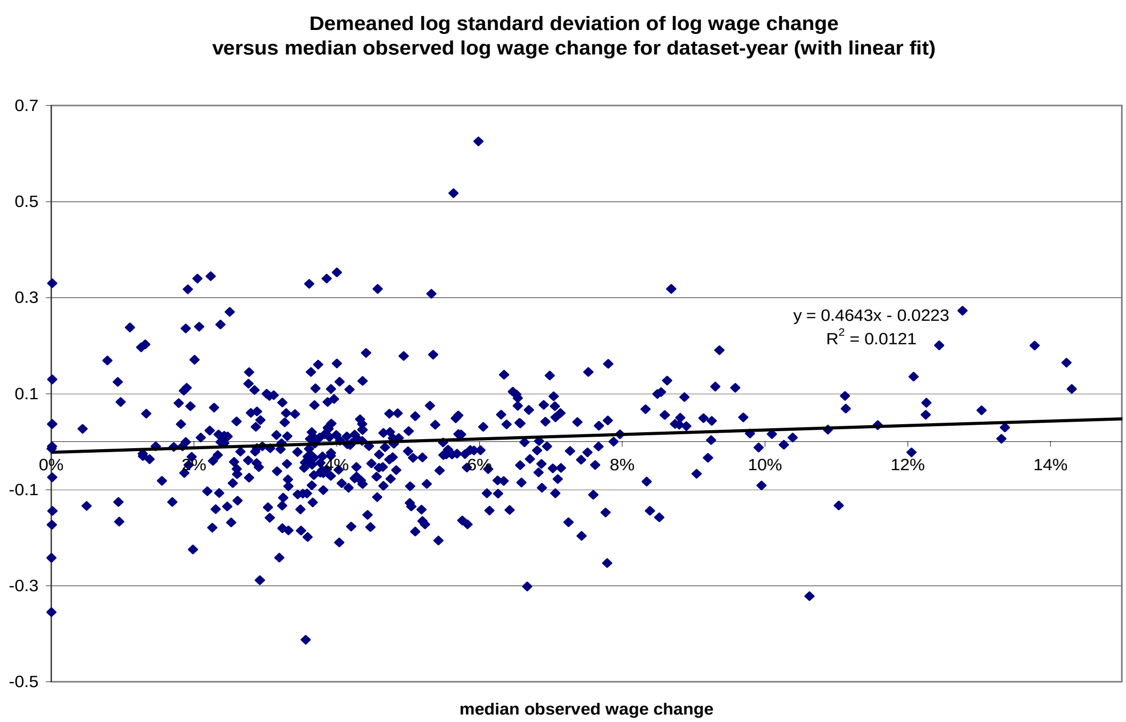


Figure 1b



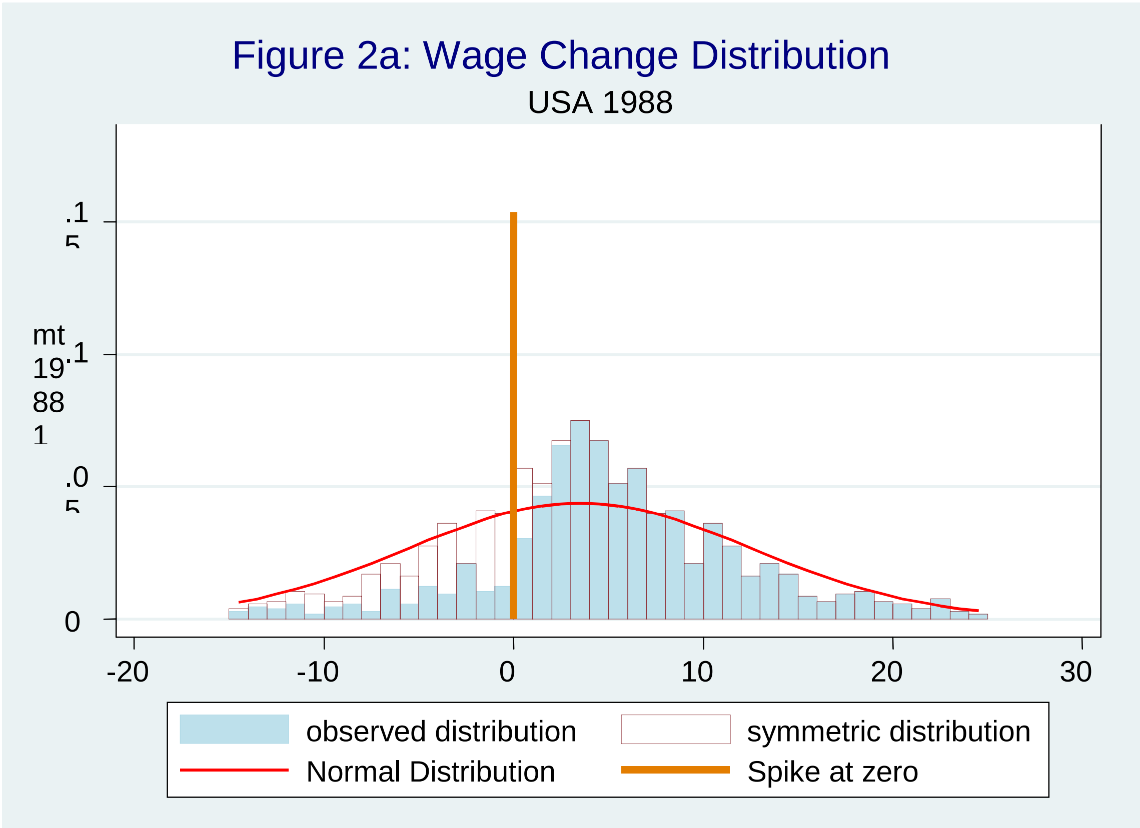


Figure 2a: Wage Change Distribution
USA 1988
mt 1988 1
.15
.1
.05
0
-20
-10
0
10
20
30
observed distribution
symmetric distribution
Normal Distribution
Spike at zero

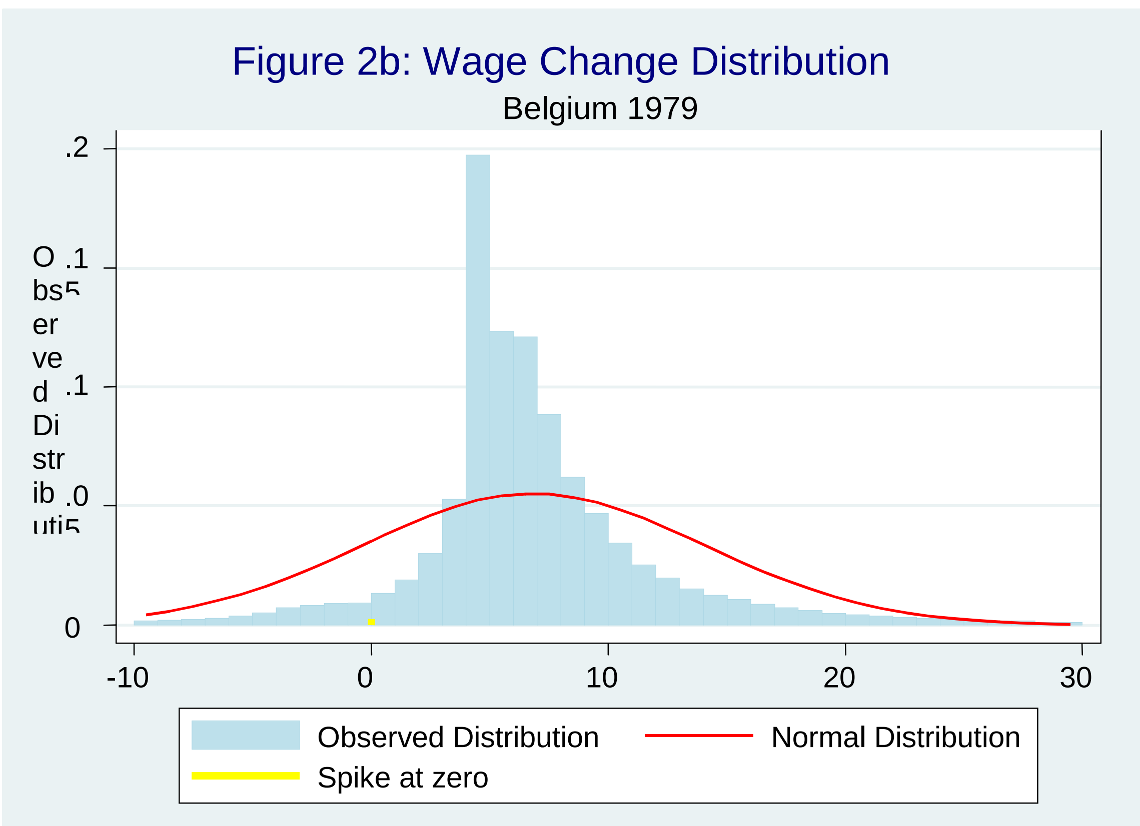


Figure 2b: Wage Change Distribution
Belgium 1979
Observed Distributi
.2
.15
.1
.05
0
-10
0
10
20
30
Observed Distribution
Normal Distribution
Spike at zero

Figure 3

Comparison of MMM country measures of downward nominal rigidity with measures in previous studies

p values in parentheses

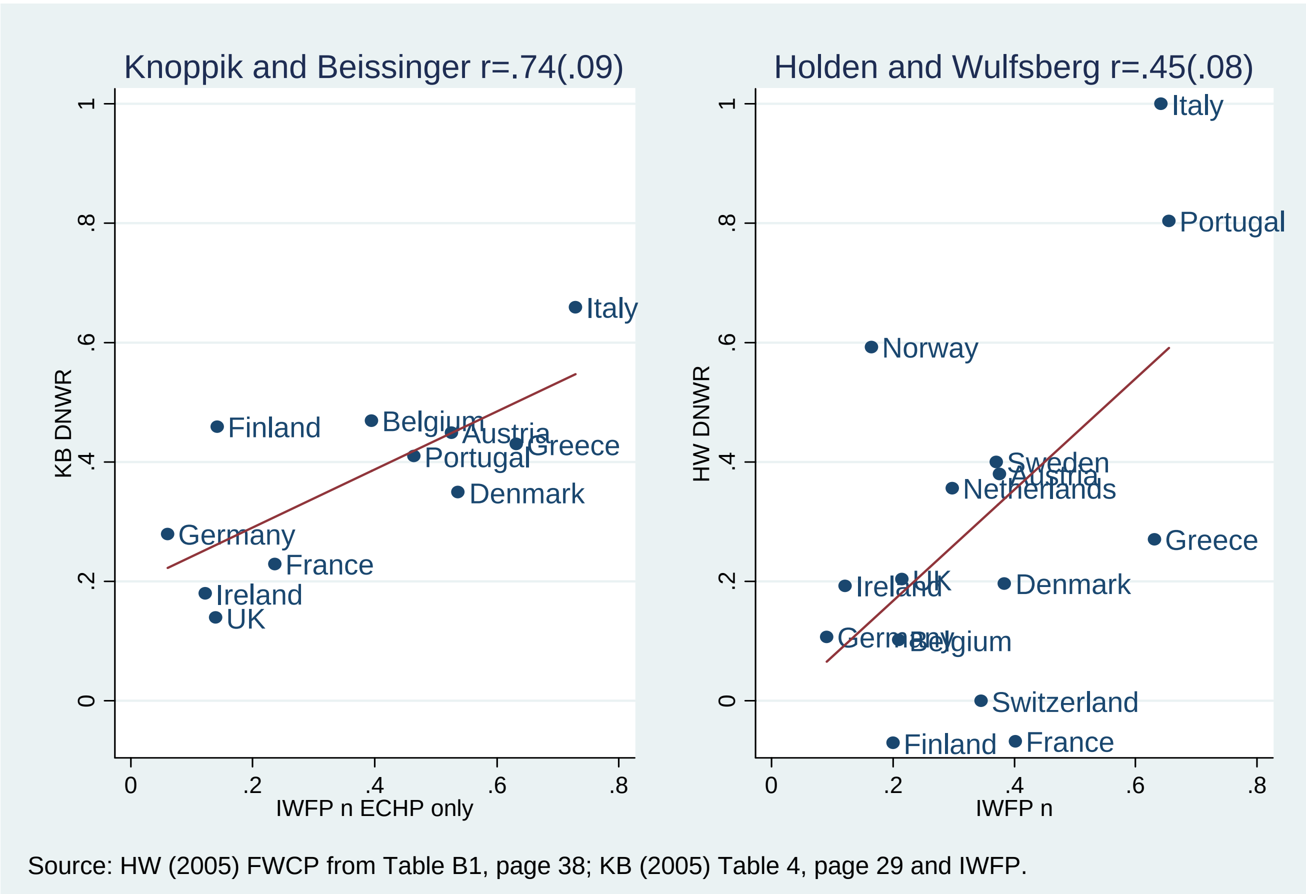
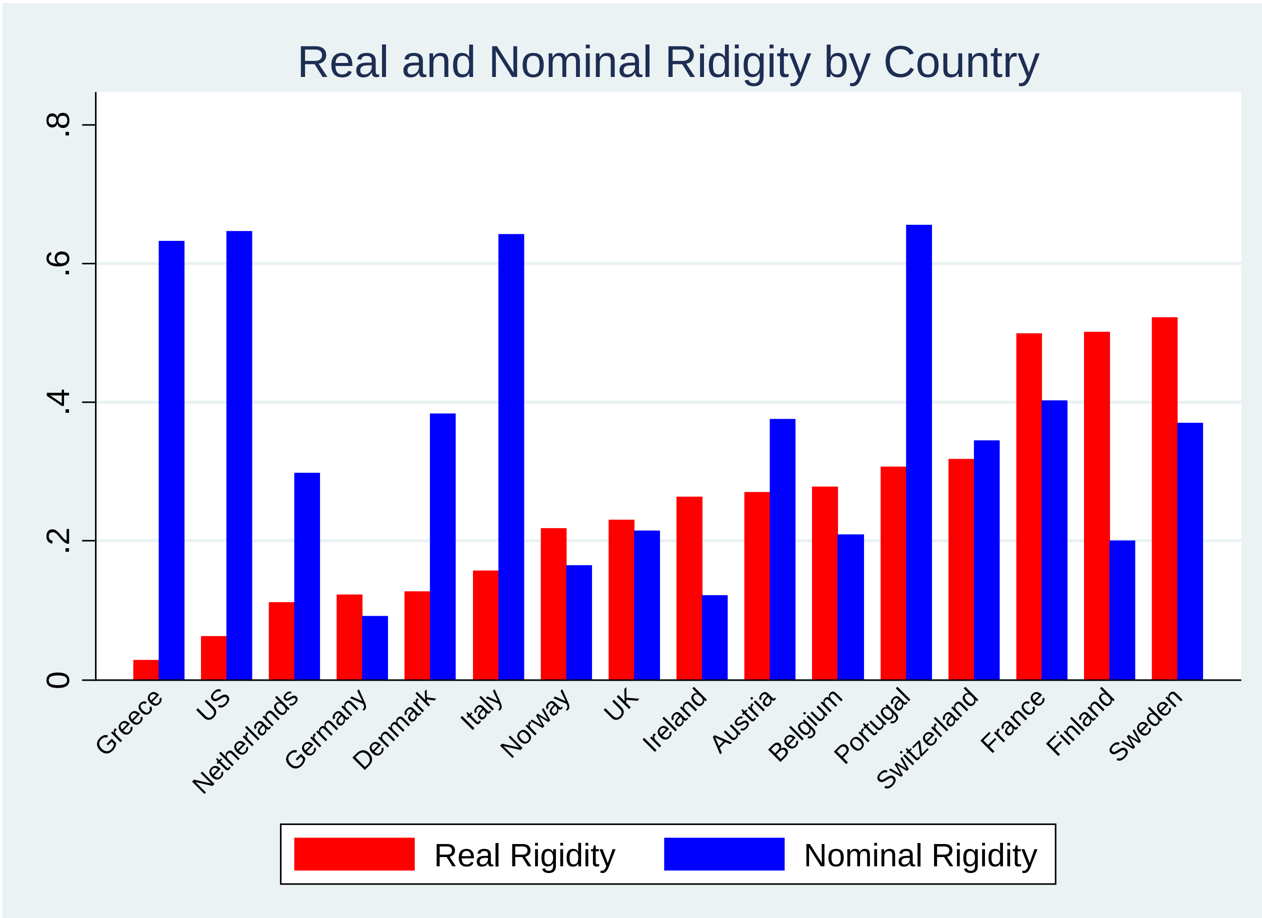


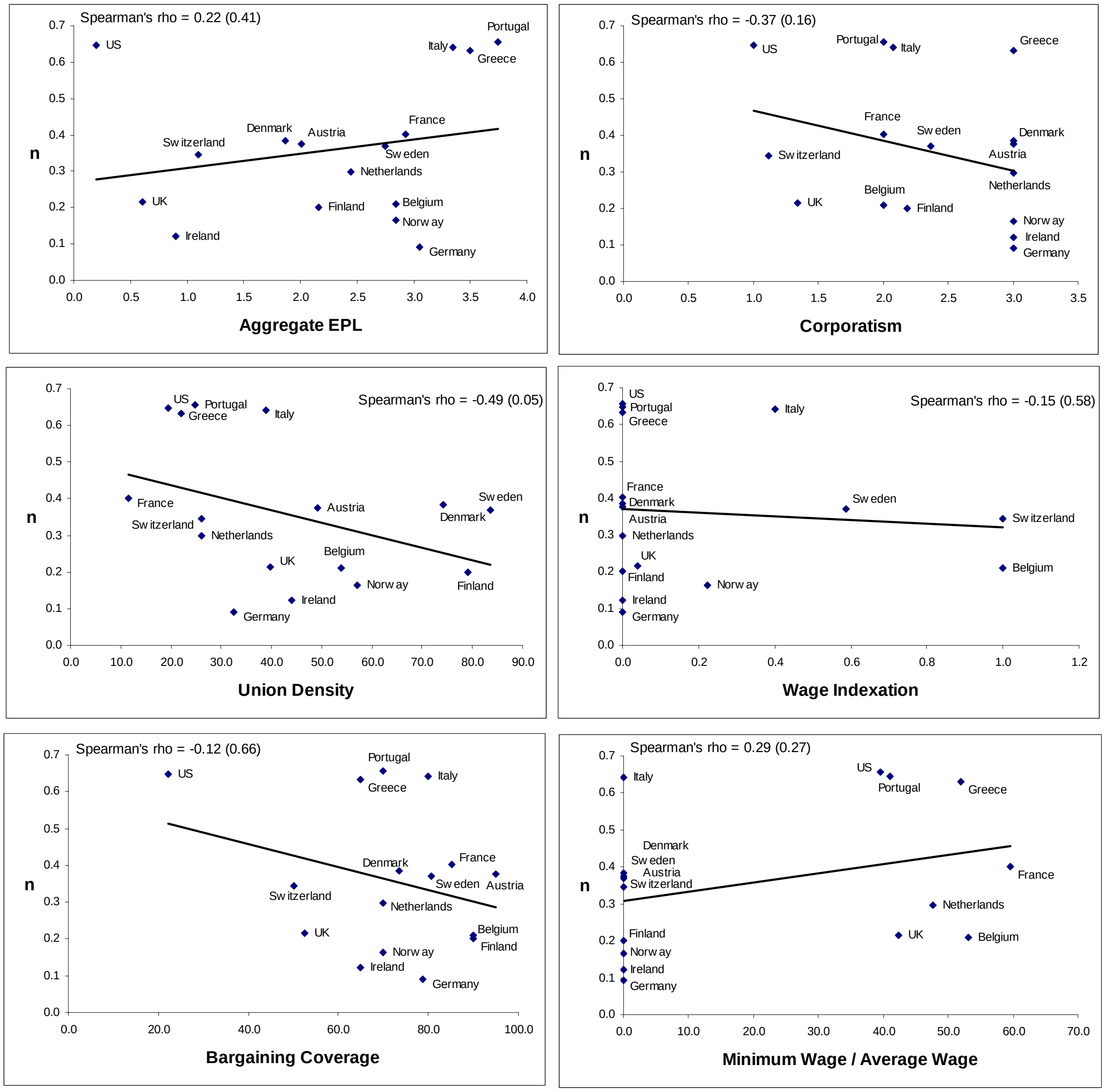
Figure 4



Notes:
These are the MMM estimates of *r* (the prevalence of downward real wage rigidity) and *n* (the prevalence of downward nominal wage rigidity), averaged across all dataset-years for each country.
By construction, the measures range from 0 (where no one is subject to the rigidity) to 1 (where all workers are potentially affected).

Figure 5

Nominal Rigidity vs. Institutions



<u>Sources and definitions:</u>

Aggregate EPL: OECD (2004), Index of the strictness of employment protection legislation, Categorical variable coded 0 to 6, where 6 is most restrictive.

Corporatism: Elmeskov, Martin and Scarpetta (1998), Wage-bargaining corporatism index, summary measure of collective bargaining structures of centralization and coordination, Categorical variable coded 1= low to 3 =high.

Union Density: Elmeskov, Martin and Scarpetta (1998), The proportion of workers who are members of a trade union, in percent.

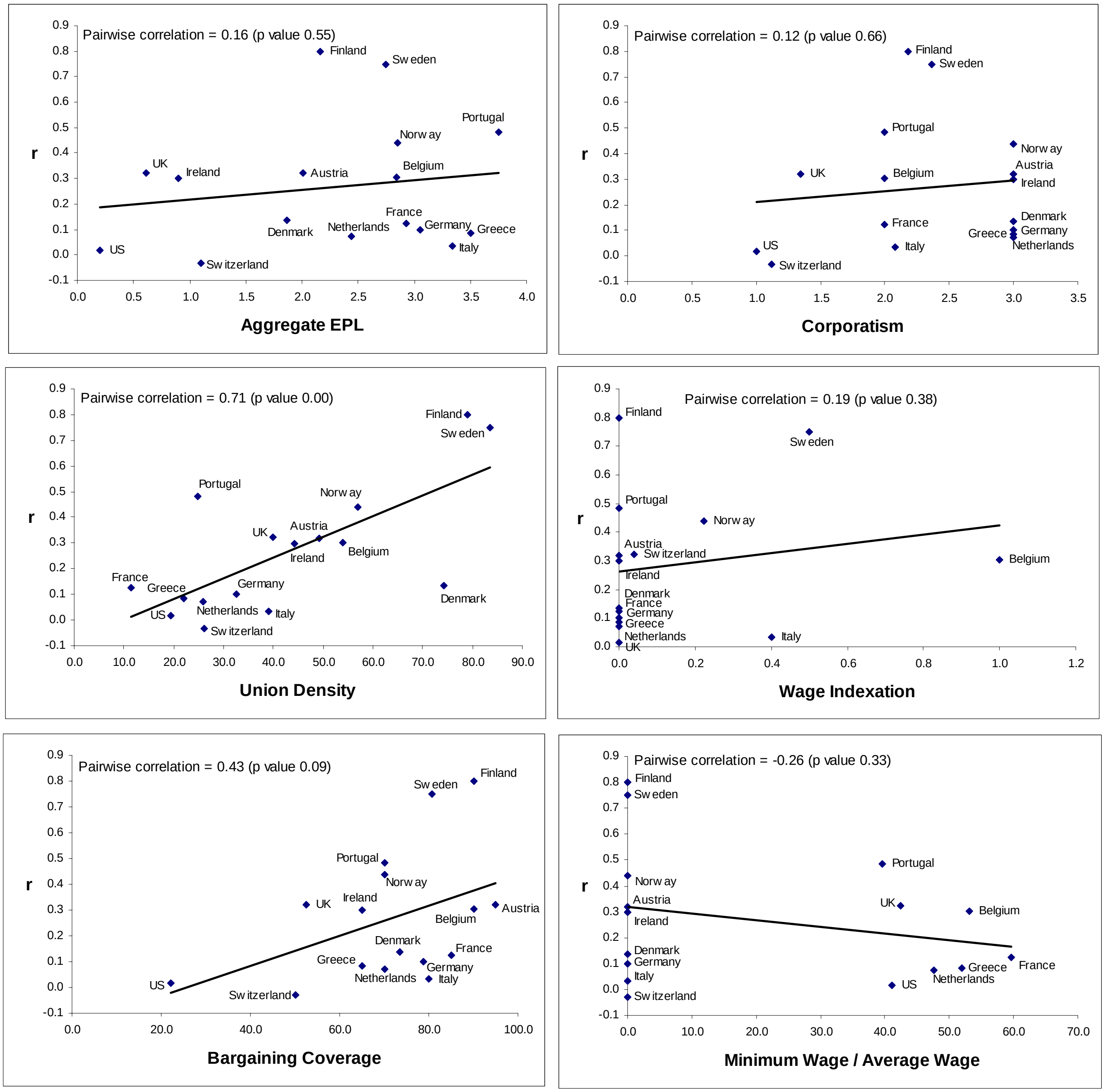
Wage Indexation: Checchi and Lucifora (2002), Categorical variables coded 0 to 1, where 1 represents the presence of automatic wage indexation clauses for most sectors, updated for Belgium and extended to include Greece and the United States.

Bargaining Coverage: OECD (2004), the extent to which salaried workers are subject to union-negotiated terms and conditions of employment, in percent, extended to include Greece and Ireland.

Minimum Wage/Average Wage: Elmeskov, Martin and Scarpetta (1998), Ratio of gross statutory minimum wage relative to average wage; supplemented by the Kaitz measure in Checchi and Lucifora (2002) for Greece, Portugal and the US; set equal to 0 for countries without minimum wages.

Figure 6

Real Rigidity vs. Institutions



<u>Sources and definitions:</u>
See notes to Figure 5.

Table 1

Three Labor Market Interactions with Inflation Examined by the IWFP

Interaction: Underlying imperfection	**Will the dispersion of wage changes rise with inflation?**	**Will wage change distribution show asymmetry and spikes?**
Grease effect: Downward nominal wage rigidity	**Yes:** more inflation reduces the number of nominal wage freezes and allows more wage changes below the mean wage change	**Yes:** skewed right, nominal wage freezes cause spike at zero wage change
Real rigidity: Downward real wage rigidity	**No:** if inflation expectations are widely shared, the entire distribution shifts with expected inflation **Yes:** if coverage is sparse or disagreement about expectations rises with inflation	**Yes:** skewed right, real wage freezes cause high concentration of wage changes around the expected inflation rate
Sand effect: Adjustment lags and errors	**Yes:** inflation adds more errors and lags to the variation in firms' wage changes	**No:** errors and lags are assumed to be symmetric

Note: These effects contrast with a fully flexible wage-setting regime that is assumed to produce wage changes that are symmetrical and unaffected by the rate of inflation.

Table 2

IWFP Dataset Characteristics

	Total	Average	
Countries	16		
Datasets	31	Datasets per country	1.9
Data-set-years	360	Years per dataset	11.6
Wage changes observed	27.6 million	Observations per dataset-year	76 thousand

Country	Dataset	Years	Source	Earnings or wages	Hours	Firm identifiers?
1. Austria	Social Security	1972-1998	Register	Annual earnings	No	Yes
2. Belgium	Social Security	1978-1985	Register	Annual earnings	No	Yes
3. Denmark	Statistics Denmark	1981-1999	Register	Annual earnings	No	Yes
4. Finland	Finnish Service Sector Employers	1990-2001	Employer survey	Wages	Yes	Yes
	The Confederation of Finnish Industry and Employers (Manual)	1985-2000	Employer survey	Wages	Yes	Yes
	The Confederation of Finnish Industry and Employers (Non-manual)	1985-2000	Employer survey	Wages	Yes	Yes
5. France	La Déclaration Automatisé des Salaires (DADS)	1976-1980, 1984-1989, 1991-2000	Register	Annual earnings	Yes	No
	French Labor Survey	1994-2000	Household survey	Earnings	Yes	No
6. Germany	Institut für Arbeitsmarkt und Berufsforschung (IAB)	1975-1996	Register	Earnings	No	No
7. Italy	Istituto Nazionale per la Previdenza Sociale (INPS)	1985-1996	Register	Annual earnings	No	Yes
8. Norway	Norwegian Confederation of Business and Industry (Blue Collar)	1987-1998	Employer survey	Wages	Yes	No
	Norwegian Confederation of Business and Industry (White Collar)	1981-1997	Employer survey	Wages	Yes	No

Table 2, continued

IWFP Dataset Characteristics

Country	Dataset	Years	Source	Earnings or wages	Hours	Firm identifiers?
9. Portugal	Quadros de Pessoal	1991-2000	Employer survey	Wages	Yes	Yes
10. Sweden	Swedish Enterprises (Blue Collar)	1979-1990, 1995-2003	Employer survey	Wages	Yes	Yes
	Swedish Enterprises (White Collar)	1995-2003	Employer survey	Wages	Yes	Yes
11. Switzerland	Social Insurance Files (SIF)	1988-1999	Register	Annual earnings	No	No
	Swiss Labor Force Survey	1992-1999	Household survey	Wages	No	No
12. U.K.	National Employment Survey	1976-2000	Employer survey	Earnings	Yes	No
13. U.S.	Employment Cost Index*	1981-2003	Employer survey	Earnings	Yes	Yes
	Panel Study of Income Dynamics	1970-1997	Household survey	Wages	Yes	No
14. Various** Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and the U.K	European Community Household Panel	1993-2001	Household survey	Earnings	Yes	No

Notes:
*Not individual data. Not used in analysis of wage rigidity.
**Suitable ECHP data for the analysis are available for Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and the United Kingdom, available years vary somewhat by country.

Table 3

Combined Effect of Downward Nominal and Real Wage Rigidity on Unemployment

	Dependent variable			
	Unemployment		**Change in inflation**	
Unemployment effect (b/a) (standard error)	1.26 (0.35)	0.90 (0.32)	2.99 (1.28)	2.90 (2.09)
p for null hypothesis $0 \geq b/a$ (one tail test)	0.00	0.01	0.01	0.08
Controls for year included (x_t)?	no	yes	no	yes

Notes:
The equations estimated are as follows:

$$\pi_t = \pi_t^e + c - a\, U_t + b\, \mathbf{S}_t + x_t + \varepsilon_t$$

$$U_t = \mathbf{S}_t\, b/a + [c + x_t - (\pi_t - \pi_t^e)]/a + \mu_t$$

where
- π_t = price inflation at time t
- π_t^e = expected price inflation at time t
- c = a constant
- U_t = unemployment rate at time t
- $\mathbf{S}_t$ = estimated wage effects of rigidity = MMM estimate of the rise in the average wage change caused by rigidities in that year (that is, the increase in the nominal wages received by those constrained by downward nominal or real rigidity as a result of wage rigidity, multiplied by their share of the wage bill in year t)
- x_t = effects of supply shocks controlled for by annual intercepts
- ε_t = other unobserved factors affecting the rate of inflation, and
- μ_t = other unobserved factors affecting the unemployment rate.

Dataset intercepts are also included in all specifications.
N=360.

Table 4

Effect of Inflation and Productivity Growth on the Dispersion of Notional Wage Changes

Independent variables	**Dependent variable: MMM estimate of underlying standard deviation of log wage adjustments**
Expected inflation	-.053 (-0.42)
Expected inflation^2	.003 (0.28)
Inflation surprise	.165 (1.74)
Inflation surprise^2	-.009 (-0.20)
Productivity growth	.170 (1.64)
Productivity growth^2	-.020 (-1.18)
Within-group R^2	*0.0202*

Notes:
Regressions include dataset specific intercepts.
T-statistics in parentheses.
Expected inflation is generated using an MA3 process.
N=360.

Appendix A. How Are Wage Changes Distributed?

A-I. Tests of normality

We find strikingly consistent evidence across countries and time that wages are quite prone to nominal freezes across years, and if not, are more likely to have wage changes that either cluster around the median or are more extreme than would be predicted if wage changes had a normal distribution. The normal distribution is a reasonable baseline for comparison because the central limit theorem tells us that any phenomena influenced in an additive fashion by a large number of independent factors will tend to be normally distributed, no matter how the factors affecting it are distributed. However, for wage changes, a two-sided Weibull distribution consistently fits much better.

Figure A-1 shows a typical distribution (for the US in 1997) of wage changes (in percentage points) plotted along with a normal distribution fit to the actual distribution[19]. The bars show the concentration of observations in cells that are one percentage point in width (for example, from -5% to -4%, +7% to +8%, etc.), with three exceptions that isolate wage freezes. The exceptions are the cell centered at zero and the two cells immediately adjacent, which have the following boundaries: -1.0% to -0.017%, -0.017% to +0.017%, and +0.017% to +1.0%.

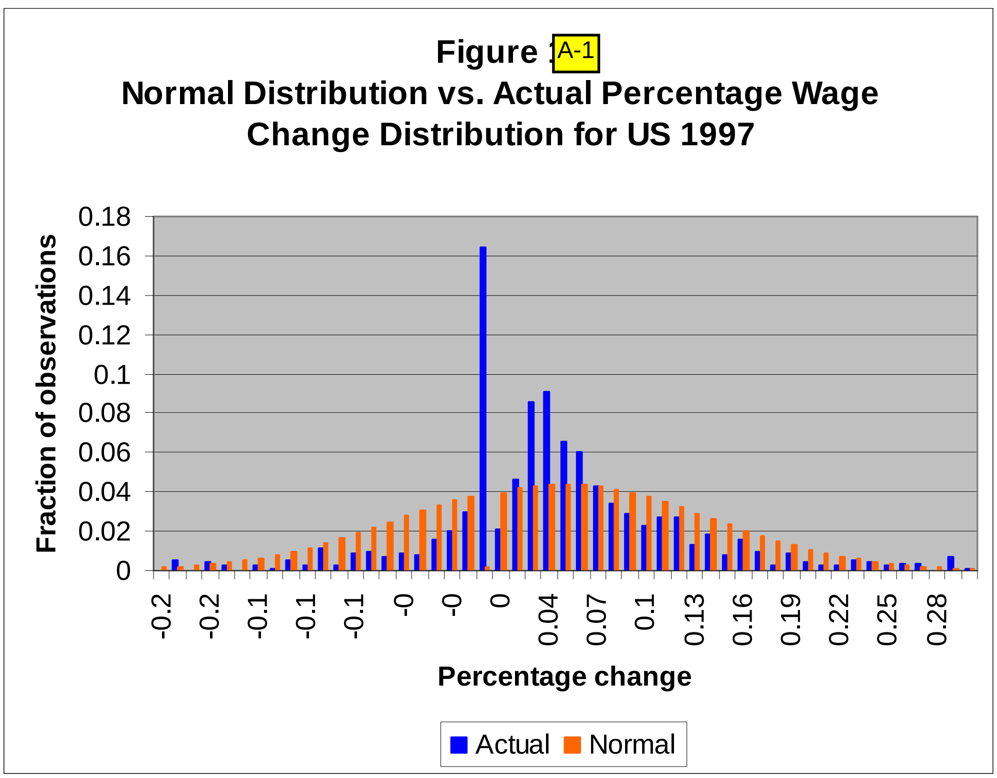


Figure A-1
Normal Distribution vs. Actual Percentage Wage Change Distribution for US 1997

---

[19] The mean of the normal is set equal to the median of the actual distribution and the standard deviation is set equal to the square-root of the average squared deviation of observations above the median from the median (or the standard deviation computed using only observations from the upper-tail).Our reasons for doing this are explained below.

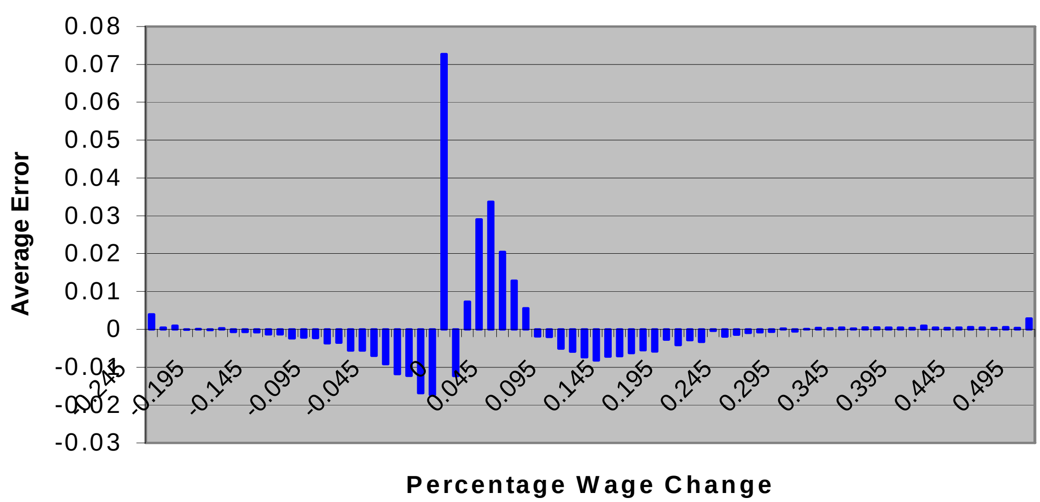
As can be seen in Figure A-1, wage changes are not normally distributed. There are three key divergences from the normal distribution.

1. Too many people had wage freezes, creating a large **spike at zero**. If the distribution were normal, very few changes would fall in the narrow (-0.017% to +0.017%) interval around zero, instead of the large share observed.
2. Workers' wage changes are tightly clustered around the median change. This clustering makes the distribution much **more peaked** than the normal.
3. Although one cannot see this in figure 1, more people have extreme wage changes than predicted by normality. That is, the distribution has **fat tails**. The absolute errors are not large, but the proportional errors are huge because the predicted fractions become essentially zero long before the actual fractions do.

These divergences also apply to nearly every case in the IWFP. Of 269 annual wage change distributions[20] observed, we fail to reject the normal distribution at the 0.001 level in all but one case, where there were relatively few observations. The median X^2 is 239,102 with 75 degrees of freedom.[21] Figure A-1 and the possibility of downward rigidities suggest that the distribution is closer to normal on the upper tail. However, when we fit the normal only to values above the median, we still reject normality in all but five cases. The median X^2 statistic is 26,245 (with approximately 45 degrees of freedom).[22]

Strikingly, the pattern of divergences from normality is quite consistent across the

Figure A-2
Average Error 269 Wage Change Distributions
Actual minus Predicted from Normal Distribution



---

[20] These are the distributions that can be constructed without using annual earnings data.
[21] The mean (median) sum of squared errors is 0.039 (0.019).
[22] The mean (median) sum of squared errors is 0.035 (0.009).

disparate IWFP datasets. Figure A-2 shows the average error in each cell for all 269 wage change distributions. We see the under prediction of the cells immediately around the median and the over prediction of cells in the intermediate ranges, as well as under prediction of the extreme tails and the significant tendency to under predict the spike at zero.

A-II. The Two-Sided Weibull

If the distribution of wage changes is not normal, what alternative distribution yields a better approximation? One possibility is the 2-sided Weibull distribution, which has three parameters. This distribution nests the two-parameter, two-sided exponential distribution as a special case when the parameter determining peakedness is equal to 1, but allows a more peaked distribution with fatter tails as its third parameter declines. When applied to entire wage change distributions, the 2-sided Weibull fits wage changes considerably better than the normal. The quality of the fit improves even more dramatically when we look only at the upper tail. Despite very large sample sizes, we now fail to reject the distribution at the 0.001 level in 86 cases or 31% of the time. The median X^2 drops to 916, while the mean and median sums of squared errors fall to 0.007 and 0.001, respectively. The tendency to over predict observations in cells just above the median and under predict those in the intermediate range is nearly gone. In contrast to the fit of the upper tail, there are still notable problems to the fit of the lower tail. There is still under prediction of values just below the median and over prediction of values more than 4 percentage points below the median.

While these tendencies are not particularly notable in figure A-3 (the fit for the US in 1997), they are in figure A-4, which shows the 2-Sided Weibull plotted against the actual distribution for the United Kingdom in 1985. The prevalence of wage freezes is weak in this distribution, but there is a notable concentration of wage adjustments in the three cells just below the median and a paucity of wage changes below those rates. Inflation in the UK had been in the range of 4 to 5% in the previous two years and rose to 6% in 1985, so it is not unreasonable to assume that the spikes at those values in the graph reflect wage increases equal to agents expected rates of inflation or real wage freezes.

Figure A-3
2-Sided Weibull vs. Actual for US 97

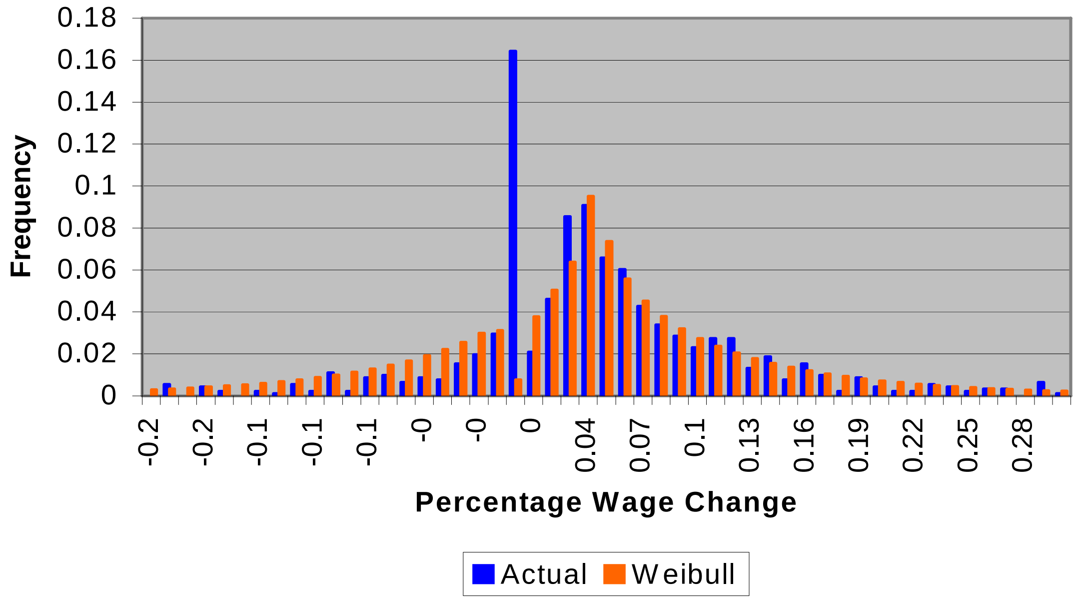
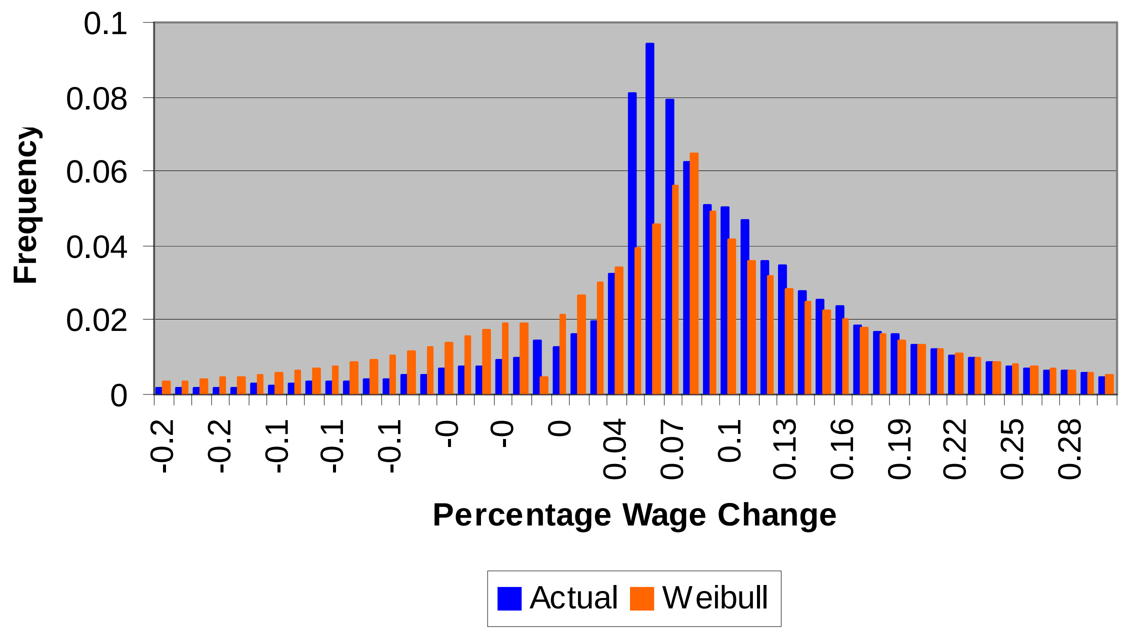


Figure A-4
2-Sided Weibull vs. Actual for UK 1988



Appendix B. Estimating Wage Rigidity for the International Wage Flexibility Project Using the Mixed Method of Moments (MMM) Estimator

William T. Dickens, The Brookings Institution

Lorenz Goette, University of Zurich

Introduction

Nearly all previous attempts to use cross national data to assess the causes and consequences of wage rigidity have relied on estimates derived from macro data.[23] Typically these studies have estimated Phillips curve relationships and interpreted differences in the sensitivity of inflation to unemployment as indicative of the extent of wage rigidity (the less sensitive the more rigid wages are). Such an approach assumes wage rigidity takes the form of slow adjustment of wages to economic fundamentals. However, several other sources of wage rigidity have been suggested. Downward nominal wage rigidity only slows adjustment under certain circumstances and is likely due to different causes than generalized slow adjustment. Minimum wages or national bargains are yet other kinds of rigidity, and menu costs one more. The presence of one or more of these other types of rigidity could complicate or invalidate studies attempting to measure slow adjustment using macro data.

The initial phase of the International Wage Flexibility Project suggests an alternative to studies that use macro data. Twelve country teams have access to micro data on individual earnings. Examination of distributions of individual earnings changes for the 12 countries for a number of years suggests that it may be possible to identify the presence and importance of a number of different types of rigidity using micro data on individual wage changes. Wage change distributions differ considerably across countries and even across time within some countries. It is possible that if we can find ways to summarize and quantify these differences in the wage change distribution that we will be able to identify measures of the extent of different types of wage rigidity and to relate them to their causes and consequences in a cross-country-time-series analysis. Doing this may contribute significantly to our understanding of how wage rigidity affects economic outcomes and the institutional sources of rigidity.

However, before we can measure wage rigidity using wage change distributions we need an accurate measure of the distribution of wage changes. In nearly all the datasets available to the IWFP the observed measure of earnings is distorted either by reporting and recording error, by an absence of accurate data on time worked to compliment earnings data, or by a divergence between the concept of wages we wish to measure and what is available (for example average hourly earnings including overtime when we would want base wage). To deal with these problems we need a way to

[23] See Holden and Wulfsberg (2004) for a recent exception. Several other efforts related to this project have been circulated in recent years as well.

transform the observed wage change distribution into an estimate of the true distribution without errors.

Section B-I of this appendix describes an estimator for a semi-non-parametric representation of the underlying wage change distribution and for the parameters of the error process, and an estimator for the variance covariance-matrix of the parameters and of rigidity measures based on them. We dub this estimator a mixed method-of-moments estimator or MMM. The likelihood function for the problem is computationally intractable, given available resources. There is certainly more information available than needed for a just identified method-of-moments estimator, but estimation of the model by generalized-method-of-moments would be impractical given the computing resources available to the IWFP teams. The MMM estimator solves the problem of computational complexity by concentrating out the estimates of the parameters of the true wage change distribution and the one time varying parameter of the error distribution. Given the other parameters of the model it is possible to solve for these moments exactly. We then use an iterative process to minimize a GLS distance measure for the remaining moments which are much fewer in number and a function of only three parameters.

With these new estimates of the true wage change distribution we turn to the task of estimating the extent of different types of rigidity. We have developed four different ways to measure the extent of rigidity using distributions of wage changes. What is needed is some way of knowing what the distribution of wages would look like in the absence of rigidity and then that can be compared to the actual distribution. Candidate methods for establishing the counterfactual of what the distribution would look like in the absence of rigidity are:

1. assume a particular form for the distribution and estimate a model of the true wage change distribution based on it,
2. assume that the effects of rigidity are seen only below the median and that the distribution would be symmetric in the absence of rigidity,
3. assume that the notional wage change distribution is fixed over time except for a changing mean,
4. assume that in the absence of changes in the extent of rigidity certain aspects of the wage change distribution are constant (such as skew, kurtosis, smoothness).

We tried all four, but settled on the first as providing the most reliable estimates.

The methods described in sections B-I and B-II will not work if the proxy for wages is annual earnings divided by annual hours. Unless the typically annual wage change takes place at the same time as the survey, a year's income reflects two different wages. This causes two problems. First, it induces a positive correlation between the change in earnings in one year and in the next year which violates one of the important assumptions of the error correction model. Second, the change in income from one year to the next confounds two changes in wages. So, for example, someone would have to go two years without a wage change in order to have no change in annual income for two years (unless the wage change was synchronized with the period of observation for income). Section B-III describes methods for adapting both our error correction model and the method that estimates rigidity using an ideal distribution to the case where we only observe annual income.

Finally, a note at the end of the appendix presents a list of the notation used in the first three sections, definitions of variables and parameters and a list of the page in the text where each is defined.

B-I. MMM Estimator for True Distribution

Our first attempt to develop parametric rigidity estimates using ML was not completely successful. That estimator could identify the extent of measurement error only because we assumed that we knew the form of the distribution of the notional wage changes and the measurement error. However, the non-normality of the notional wage change distribution – and probably also the error distribution – made this identification suspect. In particular, the identification of the extent of measurement error in the ML estimator probably depends on conditions on high order moments that are very sensitive to treatment of observations in the tails of the distribution. It would be good if this could be avoided.

While it is not possible to estimate the model without making some use of distributional assumptions, it is possible to estimate the true wage change distribution semi-non-parametrically with relatively innocuous distributional assumptions if we can assume that the only source of auto-correlation in wage changes is measurement error. Such an assumption allows us to identify measurement error variance without distributional assumptions. Only relatively minor distributional assumptions are then necessary to separately identify the frequency of the error and the variance of the error when it is made.

The assumption that all auto correlation in changes in log wages is due to measurement error is suggested by the findings of Abowd and Card (1989) who show that the best characterization of the stochastic process generating individual wages in US panel data is an ARIMA(0,1,1) – a process which is MA1 in first differences. Measurement error with no serial correlation added to a random walk will generate this sort of process. Since all the covariance in wage changes is due to the MA1 process, the assumption that the measurement error is the only source of serial correlation in wage changes is tantamount to assuming that any observed wage change that goes away within a year was an error. This is probably a reasonable point of departure for attempts to estimate the true wage change distribution. Calculations based on Bound and Krueger's (1999) estimates of the extent of measurement error in US survey data suggest that error could be sufficient to explain all the negative correlation between wage changes observed in datasets such as the PSID.[24]

---

[24] Their estimated ratio of noise to signal plus noise in wage changes for men is 85% of our estimate for the PSID for 1987-89 (Table 6 "Classical measurement error"). Bound and Krueger also find a significant positive correlation in the errors in adjacent periods and a negative correlation between errors and wage levels for men. Both correlations are inconsequential for women. The presence of a positive correlation in the errors would lead us to underestimate the true extent of wage variance. Their point estimates suggest an underestimate of about 20 percent. If errors in levels are also correlated with changes in wages our estimated true wage distributions likely have more variance than the true distribution. Also, if this correlation exists the total variance of wage changes would be less than the sum of the signal and noise variance. Bound and Krueger don't report the variance of log wage changes in their data, but under the assumption that wages are a random walk it is possible to estimate the variance as the variance of the signal minus the covariance of the signal in adjacent years. The ratio of the noise to this variance is 60%

Further, we have now analyzed data produced by Gottschalk (forthcoming). He uses a regression discontinuity model in a dataset with multiple wage observations a year for each person to separate true wage changes from errors.[25] We have computed the auto-correlation of his estimated true wage changes and find a statistically insignificant value of .006. If the assumption that true wage changes are uncorrelated is incorrect it probably isn't off by much.

A Description of the Estimator

The estimator works as follows. The underlying true change in log wage distribution is assumed to be a discrete. The log wage change variable can take one of 76 values from -.245 to .495 in steps of .01 or it can take the value zero. The histogram of observed wage changes for each pair of years in the data is calculated with the number of histogram cells in each year equal to the number of elements in the underlying discrete distribution. If wages are measured with error that is uncorrelated from year to year, the error will produce a negative covariance between wage changes in adjacent years. The magnitude of this covariance will depend on the frequency and the variance of the errors. We make an initial guess at the two parameters determining the frequency of errors and use them, and the auto-covariances, to compute the implied variance of the error in each period.

With an initial guess at the parameters of the error distribution, the estimates of the variance of the error distribution, and an assumption that errors, when made, are drawn from a two-sided Weibull distribution[26], it is possible to compute the fraction of observations associated with each element of the true (discrete) change-in-log-wage distribution that can be expected to be found in each cell of the observed histogram of log wage changes. These fractions can be arranged into a matrix with each row representing the fraction of the observations associated with each element of the discrete true change distribution that is expected to be found in each cell of the histogram of observed changes. One can then represent the expected number of observations in each cell of the histogram as the result of multiplying that matrix times the vector which contains the fraction of observations at each point in the discrete true change distribution. By inverting the matrix and multiplying it times the vector of the fraction of observations in each cell of the histogram one can estimate the fraction of observations generated by each element of the discrete true change-in-log-wage distribution.

Now we need some way to check our original guess about the frequency of errors and the curvature of the error distribution. When a large positive error is made it will at first cause a large increase in the observed wage. If errors are not correlated from one year to the next, the next year after a large increase due to an error there will likely be a large fall in the reported wage. Using the estimated true wage distribution and the

---

greater than our estimate of this value. Although this is much larger than our estimates, it is for total income which is probably measured less precisely than hourly wage (our measure in the PSID). The large differences between the error processes for men and women suggest that the model should be estimated separately by sex, but when we tried this in several IWFP datasets we found no practical or statistically significant differences between men and women.

[25] This method isn't available to us since our country teams have only annual wage observations.

[26] The Weibull distribution has support on interval $(0,\infty]$. The two-sided Weibull has support on the entire real number line excluding 0. The density is given by $d(x)=b^{-a}\,|x|^{a-1}\exp[-(|x|/b)^{a}]$.

parameters of the error distribution it is possible to compute the predicted number of such changes from one year to the next at several points in the distribution (we call people who make such moves “switchers”). We use an iterative process to minimize a quadratic distance measure of the difference between the actual and predicted fraction of people “switching” from high to low, or low to high, as a function of the remaining parameters.

In theory we could have any number of such “switcher” moments however the cost of computing them is very high. As a compromise between the efficiency from the addition of more such moments and computational speed we chose to use two switcher moments, one centered around zero (with a switcher being someone who is on different sides of zero in the two periods) and one around 1% (with a switcher being someone who whose wage change is above 1% in one of the two periods and below it in the other). These values were chosen because they showed a fair amount of variation over the sample which was not as highly correlated as other values tried (for example zero and plus or minus 2% around zero).

Notation and Assumptions

[The note at the end of this appendix contains a list of all the variable names used in these notes, their meaning, and the point in the notes where they are defined.]

To begin, we will assume that the following process generates observed wages:

$$(1) \quad w_{it} = w_{it-1} + e_{it}$$

$$(2) \quad w^{o}_{it} = w_{it} + \eta^{'}_{it} \quad where \quad \eta^{'}_{it} = \begin{cases} 0 & if \;\; \mu_{it} > 0 \;\; or \;\; if \;\; \tau_{i} > 0 \\ \eta_{it} & otherwise \end{cases}$$

w_{it} is the natural log of the actual wage for person i at time t, and w^o_{it} is the natural log of the wage for that person at time t observed in the data. The random variable η_{it} will be assumed to be an i.i.d. two-sided Weibull with mean zero and parameters b_t and a, μ_{it} is assumed to be an i.i.d. random variable with a uniform distribution over the interval $[-c, (1-c)]$, and τ_i is assumed to be an i.i.d. random variable with a uniform distribution over the interval $[p-1, p]$[27]. We, will also assume that the e_{it}s are i.i.d. and drawn from a discrete distribution that can take one of K known values which we designate as the K length vector q. The probability that any e_{it} is equal to each of the K q_js is represented as the K vector m_t* and is *not* assumed to be known. Since the sum of the elements of m_t* equals one, only K-1 of them need be estimated. We denote the vector with the last element dropped m_t. We will denote the true wage change $d_{it}=w_{it}-w_{it-1}$ and observed wage change $d^o_{it}=w^o_{it}-w^o_{it-1}$ and the vector of such observations in time period t as d^o_t.

The data used to estimate the model will be log wages for T+1 sequential periods. We will denote the first period for which wage data are available period 0 and the last period T. Change in log wages will be computed between overlapping pairs of periods (ie. period 0 to period 1 and period 1 to period 2). Let N_t denote the number of individuals for

[27] This structure for the error – the two-sided Weibull with a fraction of people never making errors – was chosen to match the distribution of estimated errors in Gottschalk’s data. His estimated errors had a distinctly peaked distribution and showed some auto-correlation in the probability of an error that was simply accounted for by having a group of people who didn’t make errors.

whom data on wage changes are available from period t-1 to t, and $N_{t,t+1}$ will denote the number of people with wage change observations between periods t-1 and t and periods t and t+1. We will assume that the process generating missing data is random so that no bias is imparted by ignoring missing data in computing sample moments.

Define l and u as K vectors of upper and lower limits to categories of a histogram, chosen in advance, for the observed wage change data. Define the K-1 length column vector g_{it} to have zeros in all positions except j where $u_j > d^o_{it} \geq l_j$ for j =1 to K-1 (the sum of the g vectors across observations yields the frequency count of observations in the cells of a histogram). Define U_{tj} and L_{tj} as pairs of upper and lower limits for defining switchers (there will be (T-1)Q with Q being the number of pairs per year). Define h_{itj} as equal to 1 if $d^o_{it} > U_{tj}$ and $d^o_{it+1} < L_{tj}$ or if $d^o_{it} < L_{tj}$ and $d^o_{it+1} > U_{tj}$ and zero otherwise (a one indicating that the observation is a "switcher" in year t).

Assuming $b_0 = b_1$, and $b_{T-1} = b_T$[28] we can define a mixed method of moments estimator for a, p, c, the T-1 b_ts, and the (K-1)T m_ts as the values of those parameters that solve:

$$(3)\quad \begin{aligned} &0 = v_1 - E(v_1 \mid a,c,p,b_t \ \forall t = 1 \ to \ T-1 \quad and \quad m_t \ \forall t = 1 \ to \ T) \\ &and \\ &\min_{w.r.t.\,a,c,p} \quad (v_2 - E(v_2 \mid v_1,a,c,p))'\hat{\Lambda}_{v_2}(v_2 - E(v_2 \mid v_1,a,c,p)) \end{aligned}$$

where v_1, is the KT-1 column vector of sample moments that stacks the T vectors

$$(4)\quad v_{1t} = \begin{bmatrix} \sum_{i=1}^{N_t} g_{it} / N_t \\ \sum_{i=1}^{N_{t,t+1}} \frac{(d^o_{it} - \bar{d}_t)(d^o_{it+1} - \bar{d}_{t+1})}{N_{t,t+1} - 1} \end{bmatrix} \quad t = 1 \ to \ T-1 \qquad v_{1T} = \left[\sum_{i=1}^{N_T} g_{iT} / N_T \right],$$

v_2 is Q(T-1) and stacks the T-1 vectors

$$v_{2t} = \begin{bmatrix} \sum_{i=1}^{N_t} h_{it1} / N_{t,t+1} \\ \vdots \\ \sum_{i=1}^{N_t} h_{itQ} / N_{t,t+1} \end{bmatrix},$$

[28] We need a covariance of wage changes to identify b. We restrict the first and last pairs of b to be equal in order to allow us to use all the years of data on wage changes we have available. If we didn't do this we would have to restrict estimation of the true wage distribution to years t+1 through T-1 in order to have enough covariances to identify all the bs. When we tried this for the PSID results were essentially unchanged

$\overline{d}_t = \sum_{i=1}^{N_t} d^o_{it} / N_t$ and $\hat{\Lambda}_{v_2}$ is the sample covariance matrix of v_2.

Finally, we will denote $m^o_t = \sum_{i=1}^{N_t} \frac{g_{it}}{N_t}$.

This estimator will be consistent, but it will not be efficient. It is equivalent to GMM with infinite weight on the v_1 moments.

Deriving the Expectations

First note that

$$(5) \quad E(m^o_t) = (R_t - R_{t.K} 1'_{K-1}) m_t + R_{t.K}$$

where R_t is the (K-1)x(K-1) matrix with elements

$$(6) \quad \begin{aligned} R_{tij} &= (1-p)c^2 \left(B_t\left(u_i - q_j\right) - B_t\left(l_i - q_j\right)\right) \\ &+ (1-p)c(1-c)\left[\left(W_t\left(u_i - q_j\right) - W_t\left(l_i - q_j\right)\right)\right. \\ &+ \left.\left(W_{t-1}\left(u_i - q_j\right) - W_{t-1}\left(l_i - q_j\right)\right)\right] \\ &+ [1-(1-p)(2c-c^2)]\, i(u_i \geq q_j > l_i), \end{aligned}$$

$R_{t.K}$ is the (K-1) vector, the elements of which are defined by (6) for i=1,K-1, and 1_{K-1} is a K-1 vector of ones. In (6)

$$B_t(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{x-y} w(z \mid a, b_t) w(y \mid a, b_{t-1}) dz dy,$$

$$W_t(x) = \int_{-\infty}^{x} w(y \mid a, b_t) dy,$$

i(.) is an indicator function which returns the value 1 if the condition in parenthesis is true and zero otherwise, and w(x|a,b) denotes the two-sided Weibull density. In practice we will approximate these integrals (and several more below) using Gauss-Legendre quadrature (Judd 1998, p260).

Next, the covariance is

$$(7) \quad \begin{aligned} & Cov(d^o_{it}, d^o_{it+1}) = E([d^o_{it} - E(d^o_{it})][d^o_{it+1} - E(d^o_{it+1})]) = \\ & E([e_{it} - E(e_{it}) + \eta'_{it} - \eta'_{it-1}][e_{it+1} - E(e_{it+1}) + \eta'_{it+1} - \eta'_{it}]) = -E(\eta'^2_{it}) = \\ & -(1-p)c\, \sigma^2_t = -(1-p)c\, b^2_t \Gamma\left(1 + \frac{2}{a}\right) \end{aligned}$$

where Γ (x) is the gamma function and σ^2_t is the variance of η in year t. We will abbreviate $\Gamma_2=\Gamma\,(1+2/a)$. Finally, the expected value of the fraction of switchers is given by

$$(8)\quad E\left(\sum_{i=1}^{N_{t,t+1}} h_{it}/N_{t,t+1}\right) = m_t^{*}{}' S_t m_{t+1}^{*}$$

where

$$\begin{aligned}(9)\ S_{tij} = {} & (1-p)c^3\left[C_t\left(L_t - q_i, q_j - U_{t+1}\right) + C_t\left(q_i - U_t, L_{t+1} - q_j\right)\right] + \\ & (1-p)c^2(1-c)\left[D_2\left(L_t - q_i, q_j - U_{t+1} \mid a, b_t, b_{t-1}\right) + \right. \\ & \quad \left. D_1\left(L_{t+1} - q_j, q_i - U_t \mid a, b_t, b_{t-1}\right)\right] + \\ & (1-p)(1-c)c^2\left[D_2\left(L_{t+1} - q_j, q_i - U_t \mid a, b_t, b_{t+1}\right) + \right. \\ & \quad \left. D_1\left(L_t - q_i, q_j - U_{t+1} \mid a, b_t, b_{t+1}\right)\right] + \\ & (1-p)(1-c)^2 c\left[W_t\left(\min(L_t - q_i, q_j - U_{t+1})\right) + \right. \\ & \quad \left. W_t\left(\min(L_{t+1} - q_j, q_i - U_t)\right)\right] + \\ & (1-p)c^2(1-c)\left[W_{t-1}(L_t - q_i)W_{t+1}\left(q_j - U_{t+1}\right) + W_{t+1}\left(L_{t+1} - q_j\right)W_{t-1}\left(q_i - U_t\right)\right] + \\ & (1-p)(1-c)^2 c\left[i\left(L_t > q_i\right)W_{t+1}\left(q_j - U_{t+1}\right) + W_{t+1}\left(L_{t+1} - q_j\right)i\left(U_t < q_i\right)\right] + \\ & (1-p)c(1-c)^2\left[W_{t-1}(L_t - q_i)i\left(U_{t+1} < q_j\right) + i\left(L_{t+1} > q_j\right)W_{t-1}\left(q_i - U_t\right)\right] + \\ & [p + (1-p)(1-c)^3]\left[i\left(L_t > q_i\right)i\left(U_{t+1} < q_j\right) + i\left(L_{t+1} > q_j\right)i\left(U_t < q_i\right)\right],\end{aligned}$$

where

$$C_t(x,y) = \int_{-\infty}^{\infty}\int_{\eta_t - x}^{\infty}\int_{\eta_t - y}^{\infty} w(\eta_{t-1} \mid a, b_{t-1})w(\eta_t \mid a, b_t)w(\eta_{t+1} \mid a, b_{t+1})d\eta_{t+1}d\eta_{t-1}d\eta_t,$$

$$D_1(x, y \mid a, b_1, b_2) = \int_{-\infty}^{x}\int_{-\infty}^{\eta_1 + y} w(\eta_2 \mid a, b_2)w(\eta_1 \mid a, b_1)d\eta_2 d\eta_1,\ and$$

$$D_2(x, y \mid a, b_1, b_2) = \int_{-\infty}^{y}\int_{\eta_1 - x}^{\infty} w(\eta_2 \mid a, b_2)w(\eta_1 \mid a, b_1)d\eta_2 d\eta_1.$$

Solving the Model

If we use a 76 element distribution to approximate the true wage change distribution, then we only need a couple of years of data before solving for the 2+TK parameters using standard non-linear estimation procedures becomes unmanageable. However, the form of the problem allows us to solve for the T(K-1) m_ts as functions of the other parameters. Similarly, we can solve for the T-1 b_ts as functions of the covariances and the other parameters. These can then be substituted out of the remaining 3 equations making a solution practical.

Specifically, we can substitute the estimate of *m*

$$(10) \quad m_t^e = (R_t^{-1} - R_{t.K} 1'_{K-1})^{-1} (m_t^o - R_{t.K})$$

for m_t in the remaining 2+T equations in (3) and solve for the *b*s as.

$$(11) \quad b_t = \sqrt{\left(\frac{-\hat{\sigma}^2_{d_t d_{t+1}}}{c(1-p)} \right) \Big/ \Gamma_2}$$

where

$$\hat{\sigma}^2_{d_t d_{t+1}} = \sum_{i=1}^{N_{t,t+1}} \frac{(d_{it} - \bar{d}_t)(d_{it+1} - \bar{d}_t)}{N_{t,t+1} - 1}.$$

This leaves only a, c and p to be found by minimizing the quadratic distance measure. Since the quadrature formula we use to compute the integrals in (6) and (9) causes discontinuities in the derivatives of the objective function we use a modified version of Powell's method, a non-derivative method, to minimize the distance measure. Since we encountered a great deal of difficulty with local minimums we started Powell's method from an extensive grid search on all three parameters.

Variance-Covariance Matrix for Parameters

To find the covariance matrix for the parameters we linearize and stack the solution to the first TK-1 moment conditions and the 3 first order conditions for the minimization in (3) to get TK+2 equations

$$(12) \quad Z(v - v^*) - M(\hat{\beta} - \beta^*) =$$

$$\left[\begin{array}{c:c} I_{TK-1} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{matrix} & \left. \dfrac{\partial E v_2}{\partial \beta_2} \right|'_{\sim \beta_2} \hat{\Lambda}_{v_2}^{-1} \end{array} \right] \left[\begin{array}{c} v_1 - v_1^* \\ \hdashline v_2 - v_2^* \end{array} \right] - \left[\begin{array}{c:c} \dfrac{\partial E v_1}{\partial \beta_1} & \dfrac{\partial E v_1}{\partial \beta_2} \\ \hdashline \left. \dfrac{\partial E v_2}{\partial \beta_2} \right|'_{\sim \beta_2} \hat{\Lambda}_{v_2}^{-1} \dfrac{\partial E v_2}{\partial \beta_1} & \left. \dfrac{\partial E v_2}{\partial \beta_2} \right|'_{\sim \beta_2} \hat{\Lambda}_{v_2}^{-1} \dfrac{\partial E v_2}{\partial \beta_2} \end{array} \right] \left[\begin{array}{c} \hat{\beta}_1 - \beta_1^* \\ \hdashline \hat{\beta}_2 - \beta_2^* \end{array} \right],$$

where

$$\beta_1 = \begin{bmatrix} m_{1,1} \\ \vdots \\ m_{T,K-1} \\ \hdashline b_1 \\ \vdots \\ b_{T-1} \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} a \\ p \\ c \end{bmatrix},$$

and $\left.\frac{\partial E v_2}{\partial \beta_2}\right|_{\sim\beta_2}$ denotes the derivative of the second set of theoretical moments with respect to each element of β_2 holding constant only the other elements of β_2 with the elements of β_1 concentrated out. Using equation (12) we can approximate the variance-covariance matrix of the parameters as

$$(13) \quad \hat{\Omega} = M^{-1} Z \hat{\Lambda} Z' M^{-1}{}'$$

where the matrix Λ can be estimated as

$$\hat{\Lambda}_{jk} = \sum_{i=1}^{N} \frac{(v_{ij} - \bar{v}_j)(v_{ik} - \bar{v}_k)}{(N_j - 1)(N_k - 1)}$$

N_i is the number of observations used to construct the ith moment and N is the total number of individuals in the sample. Missing observations are set equal to their mean so that any observation i with missing values for j or k will contribute nothing to the jkth covariance. The derivatives of the moments with respect to the parameters can be calculated numerically.

B-II. Estimating Rigidity

We have previously proposed the following taxonomy of sources of wage rigidity:

Nominal Rigidity
- Symmetric rigidity at zero due to menu costs
- Asymmetric or downward nominal rigidity or resistance to wage cuts

Real Rigidity
- Resistance to real wage cuts or downward real rigidity
- Insensitivity of real wages to economic conditions

Institutional Rigidity
- Statutory minimum wages, and
- Collectively bargained wages.

For both types of nominal rigidity and downward real rigidity we can develop specific estimates of the fraction of people subject to such rigidity.

Evidence for rigidity will be discerned by comparing the estimated true wage distribution to a counterfactual estimate of what the distribution would look like in the absence of the hypothesized rigidity. We have attempted to approach this problem of constructing a counter-factual in four ways: 1. assume an ideal type and estimate the parameters of the distribution from moments that are assumed not to be affected by rigidity (for example the 90^{th} and 75^{th} percentile of the distribution), 2. assume that the effects of rigidity are seen only below the median and that the distribution would be symmetric in the absence of rigidity, 3. assume that the notional wage change distribution is fixed over time except for a changing mean, 4. assume that in the absence of changes in the extent of rigidity certain aspects of the wage change distribution are constant (such as skew, kurtosis, smoothness). However, symmetry based measures could not be estimated for many years when reasonable measures of the expected rate of inflation put it above the median wage change for several countries. The method of assuming a constant wage change distribution (following Kahn 1997) did not work because in many countries there was insufficient variation in the median of the distribution to allow us to identify the extent of all three types of rigidity. Clear differences emerged between countries in several aspects of the shape of the distribution and these were clearly related to differences in wage setting institutions, but it proved difficult to relate these aspects of the distribution systematically to wage rigidity. Thus we settled on the first method.

Estimates Based on an Ideal Distribution

Examination of both the estimated true distributions we have seen so far in developing the estimator, and an analysis of Gottschalk's estimates of true wages, suggests that wage changes have a distribution that is both more peaked and has fatter tails than the normal. The half of the distribution above the median appears to be well approximated by a Weibull distribution. The lower tail, in countries where real rigidity does not appear to be much of a problem, seems to be a mirror image of the upper tail for those parts that are above zero when the distribution is not affected by real rigidity. Thus it seems reasonable to model the notional wage change distribution as symmetric with the shape of a Weibull.

The Weibull is a three parameter distribution with support on the positive real numbers with cumulative distribution function $P(d<x) = 1-\exp(-([x-\mu]/a_t)^{c_t})$ where μ, a and c are the parameters. If it is assumed that the notional wage change density is symmetric around the mean it could be given by $f_t(x)= .5\, a_t^{1-c_t}\, c_t\, |(x-\mu)/a_t|^{c_t-1} \exp(-(|x-\mu|/a_t)^{c_t})$. The integral of this density (excluding the singularity at $x=\mu$) yields the cumulative wage change distribution for the two-sided Weibull. Denote that cumulative distribution function for period t $F_t(x)$.

We assume that the true wage change is determined from the notional wage change by the following process. Person i's notional wage change in period t d^n_{it} will be a draw from the distribution for period t. From the notional wage change the notional real adjusted wage change will be computed as

$$(14)\quad d_{it}^{r} = \begin{cases} d_{it}^{n} & if \ \ \varepsilon_{it}^{r} > \rho_t \\ \max(d_{it}^{n}, \pi_{it}^{e}) & otherwise \end{cases}$$

where π^e_{it} is a normally distributed random variable with mean π^e_t and variance $\sigma_t^{\pi^e}$ (with cumulative distribution function in period t $\Phi_t(x)$) representing the expected rate of inflation determining this wage, ε^r_{it} is an i.i.d. random variable that is drawn from a uniform distribution on the unit interval, and ρ_t is the probability of an individual being subject to downward real wage rigidity if the notional wage is less than the expected rate of inflation. The true wage change is then determined as

$$(15)\quad d_{it}^{T} = \begin{cases} 0 & \begin{array}{l} if \ (d_{it}^{r} \le 0 \ and \ \varepsilon_{it}^{n} < n_t) \ or \ (-.01 \le d_{it}^{r} \le .01 \ and \ \varepsilon_{it}^{1} < s_{1t}) \\ or \ (-.02 \le d_{it}^{r} < -.01 \ or \ .01 < d_{it}^{r} \le .02 \ and \ \varepsilon_{it}^{2} < s_{2t}) \end{array} \\ d_{it}^{r} & \qquad\qquad\qquad otherwise \end{cases}$$

where ε^n_{it}, ε^1_{it}, and ε^2_{it}, are all uniform distributed random variables with support on the unit interval, n_t is the probability of being subject to downward nominal rigidity if one's notional real adjusted wage is less than zero, and s_{1t} and s_{2t} are the probability of being subject to symmetric nominal rigidity if notional real adjusted wage changes are close enough to zero. Unless the true wage change is zero, it is then rounded to its nearest masspoint in the sequence {-.245, -.235, …, .485, .495}.

Given this model of wage changes we represent the expected mass at each point in m^c_t as

$$(16)\quad E(m_{jt}^{c}) = \begin{cases} j < -2 & (1-n_t)x_{jt} \\ j = -2 & (1-s_{2t})(1-n_t)x_{jt} \\ j = -1 & (1-s_{t1})(1-n_t)x_{jt} \\ j = 0 & \begin{array}{l} s_{2t}x_{2t} + s_{1t}x_{1t} + (s_{1t} + n_t - n_t s_{1t})x_{-1t} \\ + (s_{2t} + n_t - n_t s_{2t})x_{-2t} + n_t \sum_{k=-25}^{-3} x_{kt} \end{array} \\ j = 1 & (1-s_{1t})x_{jt} \\ j = 2 & (1-s_{2t})x_{jt} \\ j > 2 & x_{jt} \end{cases}$$

where

$$(17)\quad x_{jt} = [1-\rho_t + \rho_t \Phi_t(l_j)][F_t(u_j) - F_t(l_j)] + \rho_t[\Phi_t(u_j) - \Phi_t(l_j)]F_t(u_j)\cdot$$

and $u_j = (.01)(j+1)$ and $lj = (.01)j$ for $j<0$ and $u_j = (.01)j$ and $lj = (.01)(j-1)$ for $j>0$.

The x_{jt} represents the fraction of notional real adjusted wages that would fall in interval j of the histogram if there was no downward or symmetric nominal rigidity. This is the fraction of notional wage changes that would fall in the interval $(F_t(u_j)-F_t(l_j))$ times the fraction of people in that interval not affected by real rigidity. Those will include people whose ε^r_{it} is too large as well as those whose $\varepsilon^r_{it}<\rho_t$ but whose expected rate of inflation lies below the lower end of the interval. This is added to the fraction of people subject to real rigidity whose expected rate of inflation falls in the interval and whose notional wage change is in or below the interval.

This model can be estimated by GMM using the estimated m^cs as the observed moments and their covariance matrix as the weights. We estimate the model for each year for each dataset used in the project. The program described below also allows structural break modeling to choose sub-periods over which to estimate the model holding rigidity parameters constant. In some years in some datasets where real rigidity is relatively unimportant the estimator tries to use the real rigidity regime to create a denser upper tail than that predicted by the symmetric Weibull. To avoid this we specify relatively narrow ranges for the expected inflation parameter based on actual inflation and simple predictions of inflation.

B-III. Annual Income Data

When instead of wage observations we observe annual income we have two additional problems. First, if all wage changes aren't exactly synchronized with the period over which income is observed the overlap of the effects of wage changes from one year to the next will cause positive auto-covariance in the observed wage changes which will prevent us from using the error correction method used for wages. This can be seen if we assume that each person gets at most one wage change a year and define income as

$$(18)\qquad y_{it} = x_i\, w_{it-1} + (1-x_i)\, w_{it}$$

where w_{it} is person i's wage at time t and x_i is the fraction of the year that passes before a wage change takes place. With this, if observed income is written as

$$(19)\qquad y^o_{it} = y_{it} + \eta'_{it}$$

the observed change in income from one year to the next will be

$$(20)\qquad d^{y0}_{it} = x_i\, e_{it-1} + (1-x_i)\, e_{it} + \eta'_{it} - \eta'_{it-1}$$

where e_{it} is the innovation in wages in period t and η'_{it} is the observation error in period t. As with wages the presence of the observation error will tend to induce negative autocorrelation in the income data, but the presence of the lagged wage innovation will tend to induce an offsetting positive autocorrelation as long as the wage change isn't synchronized with the wage change (x=0 or x=1).

Using income data produces another problem as well. The confounding of two wage change distributions in each income change distribution makes it impossible to use any of the measures of rigidity proposed in the last section. Here we propose solutions to both problems.

Correcting the Income Change Histogram

The same general correction technique can be used, but another method must be found to estimate the parameters of the error distribution used to compute the correction matrix. For the wage change histogram correction we used the auto-covariance and counts of the number of switchers, but the positive auto-correlation of the income changes makes the computation of the expected number of switchers intractable. However, similar information can be recovered from the auto-covariance of higher order moments. If these are used we must also use higher order moments of the wage changes to identify some higher order moments of the error and wage change distribution. Thus we propose to use the following six moments, constructed for each year and pair of years for which we have data in order to identify the parameters of the error distribution. Those moments and their expected values are (dropping the subscripts and superscripts on d^{yo}_{it} except for those denoting the time period):

$$\mathrm{cov}(d_t, d_{t-1}) = \sum_{j=1}^{12} \omega_j x_j (1-x_j)\sigma^2_{e_{t-1}} - (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} + O(1/N^2)$$

$$\mathrm{var}(d_t) = \sum_{j=1}^{12} \omega_j [x_j^2 \sigma^2_{e_{t-1}} + (1-x_j)^2 \sigma^2_{e_t}] + (1-p)c_t\sigma^2_{\eta_t} + (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} + O(1/N^2)$$

$$\begin{aligned} E((d_t - \bar{d}_t)^2(d_{t-1} - \bar{d}_{t-1})^2) = \sum_{j=1}^{12} \omega_j [& x_j^2(1-x_j)^2\sigma^4_{e_{t-1}} + x_j\sigma^2_{e_{t-1}}\{x_j^3\sigma^2_{e_{t-2}} + x_j(1-p)c_{t-1}\sigma^2_{\eta_{t-1}} \\ & + x_j(1-p)c_{t-2}\sigma^2_{\eta_{t-2}} + 4(1-x_j)(1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\} + (1-x_j)\sigma^2_{e_t} \\ & \{x_j^2(1-x_j)\sigma^2_{e_{t-2}} + (1-x_j)^3\sigma^2_{e_{t-1}} + (1-x_j)(1-p)c_{t-1}\sigma^2_{\eta_{t-1}} \\ & + (1-x_j)(1-p)c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_t\sigma^2_{\eta_t}\{x_j^2\sigma^2_{e_{t-2}} + (1-x_j)^2\sigma^2_{e_{t-1}} \\ & + c_{t-1}\sigma^2_{\eta_{t-1}} + c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\{x_j^2\sigma^2_{e_{t-2}} \\ & + (1-x_j)^2\sigma^2_{e_{t-1}} + c_{t-2}\sigma^2_{\eta_{t-2}}\} + (1-p)c_{t-1}\sigma^4_{\eta_{t-1}}] + O(1/N^2) \end{aligned}$$

$$\begin{aligned} (21) E((d_t - \bar{d}_t)^3(d_{t-1} - \bar{d}_{t-1})) = \sum_{j=1}^{12} \omega_j [& x_j^3(1-x_j)\sigma^4_{e_{t-1}} + x_j(1-x_j)\sigma^2_{e_{t-1}} 3\{(1-x_j)^2\sigma^2_{e_t} + (1-p)c_t\sigma^2_{\eta_t} \\ & + (1-p)c_{t-1}\sigma^2_{\eta_{t-1}}\} - (1-p)c_{t-1}\sigma^4_{\eta_{t-1}} - (1-p)c_{t-1}\sigma^2_{\eta_{t-1}} 3\{x_j^2\sigma^2_{e_{t-1}} \\ & + (1-x_j)^2\sigma^2_{e_t} + c_t\sigma^2_{\eta_t}\}] + O(1/N^2) \end{aligned}$$

$$E((d_t - \bar{d}_t)(d_{t-1} - \bar{d}_{t-1})^3) = \sum_{j=1}^{12} \omega_j [x_j (1-x_j)^3 \sigma^4_{e_{t-1}} + x_j (1-x_j) \sigma^2_{e_{t-1}} 3\{x_j^2 \sigma^2_{e_{t-2}} + (1-p) c_{t-2} \sigma^2_{\eta_{t-2}}$$
$$+ (1-p) c_{t-1} \sigma^2_{\eta_{t-1}}\} - (1-p) c_{t-1} \sigma^4_{\eta_{t-1}} - (1-p) c_{t-1} \sigma^2_{\eta_{t-1}} 3\{x_j^2 \sigma^2_{e_{t-2}}$$
$$+ (1-x_j)^2 \sigma^2_{e_{t-1}} + c_{t-2} \sigma^2_{\eta_{t-2}}\}] + O(1/N^2)$$

$$E(d_t - \bar{d}_t)^4 = \sum_{j=1}^{12} \omega_j [x_j^4 \sigma^4_{e_{t-1}} + x_j^2 \sigma^2_{e_{t-1}} 6\{(1-x_j)^2 \sigma^2_{e_t} + (1-p) c_t \sigma^2_{\eta_t} + (1-p) c_{t-1} \sigma^2_{\eta_{t-1}}\}$$
$$+ (1-x_j)^4 \sigma^4_{e_t} + (1-x_j)^2 \sigma^2_{e_t} 6\{(1-p) c_{t-1} \sigma^2_{\eta_{t-1}} + (1-p) c_t \sigma^2_{\eta_t}\}]$$
$$+ (1-p) c_t \sigma^4_{\eta_t} + 6(1-p) c_{t-1} \sigma^2_{\eta_{t-1}} c_t \sigma^2_{\eta_t} + (1-p) c_{t-1} \sigma^4_{\eta_{t-1}}$$
$$+ O(1/N^2)$$

where the ωs are fraction of the population with wage changes in each month and, assuming that wage changes take place only on the first of each month, $x_j = (j-1)/12$, where all observations are made on the first of the first month. As with the wage change model we assume that errors are made by a fraction (1-p) of the population with probability c_t in period t so that $\sigma^2_{\eta' t} = \sigma^2_{\eta t} (1-p) c_t$. Given our sample sizes we will ignore the terms of order $1/N^2$.

The parameters σ^2_{et}, σ^4_{et} (for t beyond the first period), $\sigma^2_{\eta t}$ and $\sigma^4_{\eta t}$ are themselves functions of other parameters we will estimate. We assume that the errors have a two-sided Weibull distribution so $\sigma^i_{\eta t} = b^i_{\eta t} \Gamma(1+i/a)$ where $b^2_{\eta t}$ and a are the parameters we will be estimating. The parameters σ^2_{et} and σ^4_{et} can be computed recursively given the parameters of the true income change distribution and the initial values of σ^2_{et} and σ^4_{et}

$$(22) \quad \sigma^i_{e_t} = \frac{\sum_{j=1}^{K} m^{e*}_{tj} \left(\frac{u_j - l_j}{2} - \sum_{k=1}^{K} m^{e*}_{tk} \frac{u_k - l_k}{2} \right)^i - \sum_{j=1}^{12} \omega_j x_j^i \sigma^i_{e_{t-1}}}{\sum_{j=1}^{12} \omega_j (1-x_j)^i}.$$

Note that the m^{e*}_ts are themselves functions of $\sigma^2_{\eta t-1}$, $\sigma^2_{\eta t}$, c_{t-1}, c_t and p.

The empirical counterparts to these theoretical moments are:

$$(23) \quad \begin{aligned} \hat{E}((d_t - \bar{d}_t)(d_{t-1} - \bar{d}_{t-1})) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \bar{d}_t)(d_{it-1} - \bar{d}_{t-1}) / N_{t,t-1} \\ \hat{E}((d_t - \bar{d}_t)^2) &= \sum_{i=1}^{N_t} (d_{it} - \bar{d}_t)^2_{it} / N_t \\ \hat{E}((d_t - \bar{d}_t)^2 (d_{t-1} - \bar{d}_{t-1})^2) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \bar{d}_t)^2 (d_{it-1} - \bar{d}_{t-1})^2 / N_{t,t-1} \\ \hat{E}((d_t - \bar{d}_t)^3 (d_{t-1} - \bar{d}_{t-1})) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \bar{d}_t)^3 (d_{it-1} - \bar{d}_{t-1}) / N_{t,t-1} \\ \hat{E}((d_t - \bar{d}_t)(d_{t-1} - \bar{d}_{t-1})^3) &= \sum_{i=1}^{N_{t,t-1}} (d_{it} - \bar{d}_t)(d_{it-1} - \bar{d}_{t-1})^3 / N_{t,t-1} \\ \hat{E}((d_t - \bar{d}_t)^4) &= \sum_{i=1}^{N_t} (d_{it} - \bar{d}_t)^4 / N_t \end{aligned}$$

The second and fourth moment can be computed for each year, and the other four moments can be computed for each pair of years giving 6T-4 moments to compute 2T+6 parameters. If there are inadequate numbers of moments, restrictions can be imposed on the parameters.

To estimate the parameters of the error distribution we minimize the distance measure

$$(24) \qquad (E^* - T^*)' \hat{V}^{-1} (E^* - T^*)$$

with respect to the 2T+6 parameters where E* is a 6T-4 vector of empirical moments, T* is the conforming vector of corresponding theoretical moments, and $\hat{V}$ is the empirical covariance matrix of the moments.

Estimating Rigidity

The fact that each income change reflects two wage changes, unless the period of observation is synchronized with the period of the wage change, makes it impossible to use any of the symmetry based estimates. However, the ideal distribution based estimator can be extended to deal with annual income data.

If we assume that the underlying wage change densities are the same for each month of the year for which an observation is made, that the densities are piecewise uniform in one percentage point change increments (except for a mass point at zero which will be treated in what follows as an infinitesimal interval), and if we continue to assume that wage changes are independent, then the expected fraction of observations of the true income change distribution in each cell of a histogram can be represented as

$$(25) \qquad m^y_t = \Psi\, vec(m_{t-1}\, m_t'),$$

where vec is a function which turns a KxK matrix into a K^2x1 vector and Ψ is a KxK^2 matrix with elements

$$(26) \quad \Psi_{i,K(l-1)+k} = \Psi_{i,l,k} = P[l_i < d^y_t < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l]$$

where *l* and *u* are the upper and lower bounds for cells of a K length histogram. Under the assumption that e_t is piecewise uniform Ψ will depend only on the weights ω thus allowing the representation of m^y_t in (31).

For the case with 75 1-percentage-point-wide cells and a mass point at zero in position 26 of the histogram the elements of Ψ can be computed as

$$(27) \quad \Psi_{i,l,k} = \sum_{j=1}^{12} \omega_j P[l_i < d^y_t < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l, x_j]$$

where

(34) $P[l_i < d_t^y < u_i \mid l_k < e_t < u_k, l_l < e_{t-1} < u_l, x_j] =$

$$
\begin{cases}
0 & \text{if } (u_i - x_j l_l)/(1-x_j) < l_k \text{ or } (l_i - x_j u_l)/(1-x_j) \ge u_k \\
\dfrac{.5}{(u_k - l_k)(u_l - l_l)}\left(\dfrac{u_i - x_j l_l}{1-x_j} - l_k\right)\left(\dfrac{u_i - (1-x_j)l_k}{x_j} - l_l\right) & \text{if } l_k \le (u_i - x_j l_l)/(1-x_j) < u_k \text{ and } (u_i - x_j u_l)/(1-x_j) < l_k \\
\dfrac{1}{u_k - l_k}\left(\dfrac{u_i - x_j\left(l_l + \frac{u_l - l_l}{2}\right)}{1-x_j} - l_k\right) & \text{if } l_k \le (u_i - x_j l_l)/(1-x_j) < u_k \text{ and } l_k \le (u_i - x_j u_l)/(1-x_j) < u_k \\
\dfrac{1}{u_l - l_l}\left(\dfrac{u_i - (1-x_j)\left(l_k + \frac{u_k - l_k}{2}\right)}{x_j} - l_l\right) & \text{if } u_k \le (u_i - x_j l_l)/(1-x_j) \text{ and } (u_i - x_j u_l)/(1-x_j) < l_k \\
1 - \dfrac{.5}{(u_k - l_k)(u_l - l_l)}\left(u_k - \dfrac{u_i - x_j u_l}{1-x_j}\right)\left(u_l - \dfrac{u_i - (1-x_j)u_k}{x_j}\right) & \text{if } u_k \le (u_i - x_j l_l)/(1-x_j) \text{ and } l_k \le (u_i - x_j u_l)/(1-x_j) < u_k \\
1 - \dfrac{.5}{(u_k - l_k)(u_l - l_l)}\left(\dfrac{l_i - x_j l_l}{1-x_j} - l_k\right)\left(\dfrac{l_i - (1-x_j)l_k}{x_j} - l_l\right) & \text{if } l_k \le (l_i - x_j l_l)/(1-x_j) < u_k \text{ and } (l_i - x_j u_l)/(1-x_j) < l_k \\
\dfrac{1}{u_k - l_k}\left(u_k - \dfrac{l_i - x_j\left(l_l + \frac{u_l - l_l}{2}\right)}{1-x_j}\right) & \text{if } l_k \le (l_i - x_j l_l)/(1-x_j) < u_k \text{ and } l_k \le (l_i - x_j u_l)/(1-x_j) < u_k \\
\dfrac{1}{u_l - l_l}\left(u_l - \dfrac{l_i - (1-x_j)\left(l_k + \frac{u_k - l_k}{2}\right)}{x_j}\right) & \text{if } u_k \le (l_i - x_j l_l)/(1-x_j) \text{ and } (l_i - x_j u_l)/(1-x_j) < l_k \\
\dfrac{.5}{(u_k - l_k)(u_l - l_l)}\left(u_k - \dfrac{l_i - x_j u_l}{1-x_j}\right)\left(u_l - \dfrac{l_i - (1-x_j)u_k}{x_j}\right) & \text{if } u_k \le (l_i - x_j l_l)/(1-x_j) \text{ and } l_k \le (l_i - x_j u_l)/(1-x_j) < u_k \\
1 & \text{if } u_k \le (u_i - x_j u_l)/(1-x_j) \text{ and } (l_i - x_j l_l)/(1-x_j) < l_k
\end{cases}
$$

Using the Ψ matrix defined this way it is possible to compute the expected distribution of changes in income given the ideal distribution models of the wage change distribution allowing GMM estimation of the underlying parameters.

To construct the Ψ matrix requires knowledge of the distribution of wage changes over the months of the year. Different IWFP teams used different methods to obtain this information. The Swiss conducted a small labor force survey. Several countries had data on the timing of wage changes in contracts, and others used employer surveys.

To avoid having to estimate parameters for all wage change distributions simultaneously we made the assumption that the relevant wage change distributions for year t and t-1 where identical and estimated the model for each year separately.

Notation for Appendix B

Duplication of variable and parameter names has been avoided in the first two sections, but some duplication was unavoidable in the section on rigidity estimates. The page number in parenthesis after each definition tells where in the text the symbol is defined and first used.

a parameter of the 2 sided Weibull distribution that determines curvature (p4 footnote)

a_t scaling parameter for the Weibull density used to model the true change in log wage distribution (p11)

b_t parameter of the 2 sided Weibull distribution that scales the argument (p4 footnote)

$B_t(x)$ the integral of the product of two two-sided Weibull distributions (p7)

c parameter of the error distribution that is equal to the probability that someone who is prone to errors makes one (p5)

c_t the curvature parameter of the true change in log wage distribution for one of the rigidity estimators (p11)

$C_t()$ function yielding the integral of three two-sided Weibull densities (p8)

d_{it} the change in person *i*s log wage between t and t-1 (d_t is the N_t vector of the d_{it}s) (p6)

d^o_{it} the change in person *i*s observed log wage between t and t-1 (d_t^o is the N_t vector of the d_{it}^o s) (p5)

d^y_{it} the change in person *i*s observed income between t and t-1 (d_t^y is the N_t vector of the d_{it}^y s) (p14)

$D_j()$ function yielding the integral of two two-sided Weibull densities (p8)

e_{it} log wage change for person *i* from period t-1 to period t (p5)

E* a 6T-4 vector of empirical moments used to estimate the income model (p16)

E() expectation operator (p7)

$f_t(x)$ the density for the true change in log wage distribution in period t (p11)

$F_t(x)$ cumulative 2-sided Weibul distribution function for true wage changes in period t (p11)

g_{it} a K-1 length column vector that is zero except in the position corresponding to person *i*s period t wage change (where it equals 1) (p6)

h_{itj} a variable equal to 1 if person i is defined as a switcher by criteria j in period t (zero otherwise) (p6)

i index number normally differentiating individuals (p5)

j index number normally differentiating elements of a vector (p5)

K the number of discrete mass points in the true wage change distribution (p5)

l a K vector of lower bounds for wage change categories (p6)

L_t the number of constraints imposed on the estimated wage change distribution (p8)

L_{tj} The lower limit for the *j*th definition of switchers in period t (p6)

m_t a K-1 vector giving the probability that any e_{it} is equal to each of the first K-1 *q*s (p5)

m_t* a K vector that has m_t as the first K-1 elements and one minus the sum of the other elements as the last element (p5)

m_t^{e*} a K vector that has m^e_t as the first K-1 elements and one minus the sum of the other elements as the last element (p15)
m^c_t the K-1 vector of constrained estimates of *m* (p12)
m^e_t the K-1 vector of estimated *m*s (p9)
m^o_t the K-1 vector of observed *m*s (p7)
m^y_t the K-1 vector of true *m*s for income (p16)
M a matrix of coefficients in a linear expansion of the estimator's normal equations (p9)
N_t the number of observed wage changes between period t and t-1 (p5)
N_j the number of observations used to construct the *j*th moment (p10)
$N_{t,t+1}$ the number of individuals with observed wage changes in both period t and t+1 (p6)
n_t parameter for the fraction of observations affected by DNWR and DRWR in period t (p12)
O(X) an abbreviation for "additional terms of order X". (p14)
p parameter of the error distribution that determines the probability that someone is not prone to reporting errors (p5)
q_j location of the *j*th element of the true wage distribution (p5)
Q the number of ways in which switchers are measured (p6) also the set of cells around the expected rate of inflation in the Kahn estimator (p17)
R_{tij} an element of the KxK matrix R_t whose *ij*th element is the probability that an true wage change of q_j will be observed in the *i*th range of the empirical frequency distribution in period t (when subscripted only with *t* the matrix is K-1xK-1 leaving out the last row and column, when subscripted *t.K* it is a K-1 vector containing the last column) (p7)
S_{tij} an element of the matrix S_t which is used to compute the expected number of switchers in period t (p8)
s_{1t} parameter for the fraction unaffected by menu costs within one percent of zero (p12)
s_{2t} parameter for the fraction of wage changes unaffected by menu costs within two percent of zero but more one percent (p12)
t index number for time period (p5)
T the number of periods for which wage changes are measured (one minus the number of periods for which wages are measured (p5)
T* a 6T-4 vector of theoretical moments used in estimating the income model (p16)
u a K vector of upper bounds for wage change categories (p6)
U_{tj} The upper limit for the *j*th definition of switchers in period t (p6)
v_1 a TK-1 vector of the moments (when a t subscript is present it is the subvector containing only those parameters for year t) (p6)
v_2 a vector with the elements a, p, and α (p6)
w_{it} log of person *i*s true wage in period t (p5)
w^o_{it} log of person *i*s observed wage in period t (p5)
w() two-sided Weibull density function (p7)
W_t(x) integral of the two-sided Weibull from minus infinity to x (p7)
x_i the fraction of the year that passes before person i gets his/her annual wage change (p13)

y_{it} true annual income for person i in year t (p13)
y^o_{it} observed annual income for person i in year t (p13)

Z a matrix of coefficients in the linear expansion of the normal equations for the estimator (p9)

β a vector that stacks all the parameters of the model (subscripts 1 and 2 denote particular subsets of the parameters) (p9)

$\Phi_t(x)$ cumulative normal distribution with mean and variance appropriate for time t (p11)

$\Gamma(x)$ the gamma function (the integral from zero to one with respect to t of the function ln(1/t)^(x-1) (Γ_2 is used to designate $\Gamma(1+2/a)$) (p8)

η_{it} measurement error in the log wage of person i in period t if there is measurement error (p5)
η'_{it} actual measurement error in log wage in period t (p5)
Λ the covariance matrix of the empirical moments (p6)
μ the mean of the true change in log wage distribution (p11)

μ_{it} uniform random variable on $[-c_t,(1-c_t)]$ which determines whether a person prone to errors makes one in period t (p5)

π^e_t parameter for the expected rate of inflation in period t (p11)

ρ_t parameter for the fraction of observations affected by real rigidity (p11)
σ^2_t variance of η (the error when an error is made) (p7)
σ^2_x variance of variable x (p14)
$\sigma^2_{x,y}$ covariance of variables x and y (p9)

τ_i uniform random variable on [-p,(1-p)] which determines whether a person is prone to errors in reporting wages (p5)

ω_j the fraction of people who receive wage changes in month j (p14)

$\Psi_{i,j}$ element i,j of a KxK^2 matrix Ψ used to combine two annual wage change histogram frequency vectors into a single annual income change histogram vector (p16)

Ω covariance matrix of the model parameters (p10)