Strategic interaction between monetary policies in open economies under flexible exchange rates

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Abstract

This paper fills a gap in the literature regarding strategic interactions of monetary policies between two countries. It develops an open economy model that integrates the supply, demand and foreign exchange channels of monetary policy transmission. The paper shows that when the traded goods are relative substitutes, the two central banks compete over a favorable exchange rate. However, if the home and foreign goods are relative complements, the central banks compete over a favorable interest rate. We find that gains in competitiveness, which are due to a foreign wage increase, disappear when we take account of monetary policy interactions. Furthermore, we find that domestic economic performance improves in foreign central bank conservativeness.

Keywords: Flexible Exchange Rates, Wage Bargaining, Monopolistic Competition.

JEL Classification: E24, E31, E41, E42, E52, E58, F31, F41, F42

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1 Introduction

There is a burgeoning literature on the strategic interaction of monetary policy in the presence of unionized labor market institutions.[1] While the literature investigates the effects of *domestic* central bank (CB) conservativeness on economic activity, it does not investigate *foreign* CB conservativeness. If the change of the chairman of the Federal Reserve from Greenspan to Bernanke represents a policy shift, then that may impact upon the world economy. Therefore, it is important to understand the effects of a CB's degree of conservativeness on other countries. However, most of the recent literature considers a closed economy setting and therefore lacks interactions between national monetary authorities. Only Coricelli, Cukierman and Dalmazzo (2004) introduce an open economy model, which is open only within a monetary union, and therefore it also does not allow for interactions between the monetary union's central bank and foreign CBs. However, in practice monetary policies of different countries interact with each other and these interactions can lead to different macroeconomic outcomes than those predicted by closed economy models or by international trade models that do not consider endogenous monetary policy. In particular, exchange rate movements may reverse because of the optimal intervention of the CB. Moreover, competitive gains in trade due to a foreign wage increase, are washed away by the interactions of monetary policies.

This paper develops a model of strategic interactions among labor unions, monopolistically competitive firms and two central banks of two countries with open economy. We consider two CBs that simultaneously set the money supply to balance optimally the burden between inflation and unemployment, in a Coricelli, Cukierman and Dalmazzo (2004) type model, which we shall refer to as CCD. Recent models incorporate a supply and/or a demand channel of the monetary policy transmission mechanism.[2] This paper introduces a third channel of transmis-

[1] See Skott (1997), Cukierman and Lippi (1999), Guzzo and Velasco (1999, 2002), Lawler (2000), Lippi (2002), Bratsiotis and Martin (1999), Soskice and Iversen (1998, 2000), Coricelli, Cukierman and Dalmazzo (2004, 2005).

[2] The supply side view is represented by Skott (1997), Cukierman and Lippi (1999), Guzzo and Velasco (1999,

sion, an exchange rate channel. Thus our open economy setting integrates three channels: the supply, demand and exchange rate channels of the monetary transmission mechanism.

The paper finds that the mechanism of strategic interaction between two CBs is as follows. Domestic and foreign monetary policies interact through a floating exchange rate and a common real interest rate. One of these two channels of interaction is stronger depending on the relative degree of substitution of the goods traded between the countries. First, if the two countries' goods are relative substitutes, monetary expansion by one CB induces a chain of monetary expansions at home and abroad in order to bid down the value of their own currency, to depreciate it with the aim to increase output. The CBs compete in setting a favorable exchange rate until interest rates fall sufficiently and higher output is attained in both countries. Second, if goods are relative complements, the exchange rate plays a less important role than the real interest rate in affecting output. Then the CBs compete over a favorable real interest rate, i.e. monetary expansion by one CB induces contraction by the other CB. This is because expansion reduces the world real interest rate and that imbalances the existing equilibrium abroad: inflation increases and unemployment falls. Therefore the foreign CB contracts the money supply to restore equilibrium. This further induces a chain of domestic expansions and foreign contractions until the foreign currency appreciates sufficiently to boost home output but without causing inflation abroad.

We find that monetary policy always tightens against a foreign wage increase, if the two countries' goods are relative substitutes, because of imported inflation and competitive gains in employment. Thus, the mere existence of a second CB generates unemployment fears and makes labor unions less aggressive. A surprising result is that competitive gains at home from higher wages abroad are washed away by the home CB that tightens to reduce imported inflation and

2002), Lawler (2000) and Lippi (2002). On the other hand, the demand side view is represented by Soskice and Iversen (1998, 2000). Both supply and demand side views are integrated in Coricelli, Cukierman and Dalmazzo (2004, 2005) (CCD). See also Cukierman (2004) for a survey of the models.

therefore exports by both countries fall.

A central result is that CB conservativeness improves economic performance for the foreign country. When goods are relative complements, a conservative CB directly threatens foreign labor unions with unemployment by tightening to curb imported inflation and, as a consequence, by raising the world real interest rate. When the goods are relative substitutes, a conservative CB coerces the foreign CB to keep a tight monetary policy as well, in order to avoid the depreciation of the foreign currency and therefore inflation in the foreign country. Under such threats of unemployment, foreign labor unions are less aggressive in their wage demands and the foreign economy enjoys lower inflation and unemployment rates.

The structure of the paper is as follows. Section 2 introduces the ingredients of the model. Section 3 builds up three markets. Section 4 describes the reactions of the CBs and section 5 investigates the reactions of labor unions. The last section is a summary of results. Some of the calculations and proofs are provided in the Appendix.

2 The Model

2.1 The timing of the game

The interactions among the central banks (CBs), firms and labor unions evolve in a three stage game as follows:

Stage 1. *Labor unions set nominal wages*

Stage 2. *The CBs simultaneously set the money supply in two countries*

Stage 3. *Firms set prices*

This timing of the game is justified by the following.[3] In stage 1, nominal wages are fixed in contracts over the entire period of the game, so that they cannot change in response to money

[3]In choosing this timing we follow CCD (2004, 2005).

balances and prices. Monetary policy is more flexible, as it reacts in stage 2 to wages, but not as flexible as prices, that are set in stage 3 by firms in response to any change in the economy.

The game is solved with backward induction. In the third stage of the game we consider the problem of firms given wages and the money supply. In the second stage money balances are endogenized while wages are still considered given. In the first stage wages are endogenized as well and the general equilibrium is revealed.

2.2 Price indices

There are two countries of size $s_1 \in (0,1)$ and $s_2 = 1 - s_1$, respectively, where s_1 and s_2 measure the relative number of firms which equals the relative size of the population that exist in the two countries. Each firm is the single producer of a differentiated product indexed $j \in [0, s_c]$ for country $c = 1, 2$.

There are n_c labor unions in country c, indexed $i = 1, ..., n_c$, and they represent the labor force at several firms in the country. Each labor union i in country c negotiates a common wage level for their members, w_{ic}. The national wage rate is defined as a geometric average, $w_c = \frac{1}{n_c} \sum_{i=1}^{n_c} w_{ic}$ where w_{ic} is the log of the wage of labor union i. The n_c labor unions in country c are assumed to be of equal size. Then one labor union provides work force to a number of s_c/n_c firms. The continuum of firms that employ labor from labor union i are in the interval $j \in [(i-1)\frac{s_c}{n_c}, i\frac{s_c}{n_c}]$.

The producer price index (PPI) is defined as the aggregate of all the prices of goods produced within country c, each weighted equally. Since there is a variety s_c of goods produced in country c, the producer price index is defined as

$$\bar{p}_c = \frac{1}{s_c} \int_0^{s_c} p_{jc} dj,$$

where p_{jc} is the log of the price of good j produced in country c. Prices of goods of foreign

origin are not reflected in the PPI. In the model the PPI is also the same as the export price, because all goods produced domestically are exported as well. On the other hand, the consumer price index (CPI) is the price of a basket of both domestic and foreign goods consumed within the country. Individuals in both countries have the same preferences. An individual consumes a continuum of s_1 goods that originate from country 1 at an average price of $\bar{p}_1$ and s_2 goods that originate from country 2 and are sold at an average price of $\bar{p}_2$. Thus the consumer price index in country 1 is the average of prices of all goods with domestic or foreign origin,

$$p_1 = s_1\bar{p}_1 + s_2\left(\bar{p}_2 + e\right) \tag{1}$$

where e represents the log of the exchange rate. The exchange rate is defined as the price of a currency unit of country 2 in terms of the currency of country 1.

2.3 Parity conditions

We assume that purchasing power parity (PPP) holds. The economy is open for trade, therefore arbitrage in each good j insures that its price domestically and abroad is the same, adjusted with the exchange rate, $p_{j1} = p_{j2} + e$. That means that the consumer's basket of goods sells for the same price in both countries, once we adjust it with an exchange rate, i.e. in logs

$$p_1 = p_2 + e,$$

where p_1 and p_2 represent the logs of the consumer price index expressed in the currency of country 1 and in the currency of country 2, respectively. We assume covered interest parity, which together with PPP implies that real interest rate parity holds, $r_1 = r_2 = r$.[4]

[4] See Appendix 7.1.

3 Markets

3.1 The goods' market

The demand for a good is a function of its relative price and the real interest rate. Following CCD (2004), we assume that demand for product j produced in country c and consumed in both countries is

$$Y_{jc}^{d} = \left(\frac{P_{jc}}{P_c}\right)^{-\eta} H\left(r\right),$$

where $\eta > 1$ is the elasticity of demand with respect to its relative price and we assume that $H\left(r\right)$ is a decreasing function of the real interest rate. We specialize the function $H\left(r\right)$, which allows us to obtain linear equations in the model. Let

$$H\left(r\right) \equiv \exp\left(-\rho r\right),$$

where $\rho > 0$ is the semielasticity of demand with respect to the real interest rate. Then the log of the demand function can simply be written as

$$y_{jc}^{d} = -\eta\left(p_{jc} - p_c\right) - \rho r. \tag{2}$$

Aggregate demand per capita is obtained by integrating (2) over consumers j in country c and dividing it by s_c,

$$y_{c}^{d} = -\eta\left(\bar{p}_c - p_c\right) - \rho r. \tag{3}$$

The supply side of the goods' market is represented by firm j in country c that hires its labor force from union i. The firm's production function exhibits decreasing returns to labor,

$$Y_{ijc}^{s} = L_{ijc}^{\alpha}, \tag{4}$$

where $\alpha \in (0,1)$ and superscript s stands for supply. Firms face monopolistic competition. The real profit of firm j is

$$\Pi_{ijc} = \frac{P_{ijc}}{P_c} Y^d_{ijc} - \frac{W_{ic}}{P_c} L_{ijc}. \tag{5}$$

where W_{ic} is the nominal wage received in labor union i, country c. By substituting (2) and (4) into (5) the problem of the firm becomes

$$\max_{P_{ijc}} \left(\frac{P_{ijc}}{P_c}\right)^{1-\eta} \exp\left(-\rho r\right) - \frac{W_{ic}}{P_c} \left[\left(\frac{P_{ijc}}{P_c}\right)^{-\eta} \exp\left(-\rho r\right)\right]^{1/\alpha}.$$

The firm takes the general price level P_c, the wage at which it hires the workforce W_{ic} from union i as given. Calculating the first order conditions and taking logarithms yields

$$p_{ijc} - p_c = \theta + \frac{1}{\alpha + \eta(1-\alpha)} \left[\alpha(w_{ic} - p_c) - (1-\alpha)\rho r\right], \tag{6}$$

where $\theta \equiv \alpha / \left[\alpha + \eta\left(1-\alpha\right)\right] \log\left[\eta/\alpha\left(\eta - 1\right)\right] > 0$, and lowercase letters denote the logarithms of the corresponding uppercase letters. The relative price level of goods produced in country c, $\bar{p}_c - p$ is obtained after averaging equation (6) over country c,

$$\bar{p}_c - p_c = \theta + \frac{1}{\alpha + \eta(1-\alpha)} \left[\alpha(w_c - p_c) - (1-\alpha)\rho r\right]. \tag{7}$$

The higher the real wage in a country and the lower real interest rates, the larger is the relative price of the goods produced in that country.

3.2 Labor markets

Demand for labor generated by firms is found by inverting the production function (4). Using (2) we obtain labor demand by firm j from union i,

$$l_{ijc}^{d} = \frac{1}{\alpha}\left[-\eta(p_{ijc} - p_c) - \rho r\right]. \tag{8}$$

Labor demanded by a firm is directly related to the demand for its product. Thus labor demand is decreasing in the relative price of the good and the real interest rate. The unemployment rate faced by labor union i is defined as[5]

$$u_{ic} = l_0 - l_{ic}^{d},$$

where l_0 is the log of the labor supply per union. Labor demand per union l_{ic}^{d} is determined by averaging equation (8) over firms that employ labor belonging to union i. Then the unemployment rate faced by union i becomes

$$u_{ic} = l_0 + \frac{1}{\alpha}\left[\eta\left(p_{ic} - p_c\right) + \rho r\right]. \tag{9}$$

The unemployment rate of a union is an increasing function of the demand for the goods produced with their workforce. Country specific unemployment can be found by aggregating equa-

[5]This definition is made on the basis of a linear approximation to the unemployment rate around the point $u = 0$. The unemployment rate is

$$u = \frac{L - L_0}{L_0} = \frac{\exp l}{\exp l_0} - 1$$

where $l = \log L$ and the unemployment rate is approximately

$$\begin{aligned} u &\simeq \left(\frac{\exp l_0}{\exp l_0} - 1\right) + \frac{\exp l_0}{\exp l_0}\left(l - l_0\right) \\ &= l - l_0 \end{aligned}$$

tion (9) over country c, i.e. $u_c = \Sigma_{i=1}^{n_c} u_{ic}/n_c$, which yields

$$u_c = l_0 + \frac{1}{\alpha}\left[\eta\left(\bar{p}_c - p_c\right) + \rho r\right]. \tag{10}$$

The higher the relative price of the goods produced in country c and the real interest rate, the higher is the unemployment rate in the country.

3.3 National money markets

We assume the following demand function for money, which is related to the classical quantity theory of money,

$$M_c^d = P_c K_c\left(i_c\right) Y_c^{\delta}, \tag{11}$$

where M_c^d is the per capita demand for nominal money balances in country c and Y_c is the per capita real income of country c. The parameter δ is the real income elasticity of money demand and Y_c^{δ} represents the component of the transactions demand for money.

$K_c\left(i_c\right)$ is interpreted as the fraction of planned expenditures that the public desires to hold in the form of money balances, also known as Cambridge K or the inverse of the velocity of money, when $\delta = 1$. $K_c\left(i_c\right)$ is a decreasing function of the nominal interest rate, i.e. $K_c'\left(i_c\right) < 0$. To obtain a tractable linear demand function for goods, we assume that

$$K_c\left(i_c\right) = K_c \exp\left(-\beta i_c\right),$$

where $\beta > 0$ is the semielasticity of money demand with respect to the nominal interest rate, i.e. it shows the percentage change that occurs in the money demand due to a one point change in the interest rate. The money market equilibrium condition is $M_c^s = M_c^d = M_c$, or from equation

(11) the equilibrium condition is simply

$$M_c = P_c K_c \exp\left(-\beta i_c\right) Y_c^{\delta}.$$

Taking the log of the money market equilibrium condition yields

$$m_c = p_c + k_c - \beta i_c + \delta y_c. \tag{12}$$

3.4 Unemployment and inflation

The equilibrium unemployment rate that emerges given money balances and wages is

$$u_c = \xi_0 - \xi_{c1}\left(m_c - k_c + \beta\pi_c^e - w_c\right) + \xi_{c2}\left(m_{-c} - k_{-c} + \beta\pi_{-c}^e - w_{-c}\right), \tag{13}$$

where the coefficients $\xi_0 > 0$, $\xi_{c1} > 0$ and ξ_{c2} are given in (44) in the Appendix. Note that the sign of ξ_{c2} is positive if $\eta \geq \rho/\beta$, otherwise negative. The price level given money balances and wages is

$$p_c = \chi_0 + \chi_{c1}\left(m_c - k_c + \beta\pi_c^e\right) - \chi_{c2}\left(m_{-c} - k_{-c} + \beta\pi_{-c}^e - w_{-c}\right) + \chi_{c3} w_c. \tag{14}$$

where the coefficients $\chi_0 > 0$, $\chi_{c1} > 0$, $\chi_{c2} > 0$, $\chi_{c3} > 0$ are given in (45) in the Appendix.

4 National Monetary Policy

4.1 The two central banks' problems

In this section we turn to the second stage of the game and we look at the optimization problem of each country's CB. Here money balances are endogenized, however, the nominal wage is still

considered given. The reaction functions of the two CBs determine money supply concomitantly. Each CB chooses the money supply, m_c, given nominal wages, inflation expectations and the foreign CB's money supply choice, m_{-c}. The CB anticipates the formation of the price level and the unemployment rate according to the mechanisms that lead to equations (14) and (13). The CB dislikes both inflation and unemployment, described with a quadratic loss function following Kydland and Prescott (1977) and Barro and Gordon (1983). The two CBs' objective functions are

$$\min_{m_c} u_c^2 + I_c \pi_c^2, \tag{15}$$

for $c = 1, 2$, where u_c is the unemployment rate of country c, I_c is the degree of conservativeness of the CB of country c, $\pi_c = p_{c,t} - p_{c,t-1}$ is the current inflation rate and $p_{c,t-1}$ is the log of the price level in the previous period, assumed to be a constant and therefore it can be simplified to zero without loss of generality. The first-order conditions are

$$-\xi_{c1} u_c + I_c \chi_{c1} \pi_c = 0, \tag{16}$$

for any country c, where we used equations (13) and (14). We write (16) in the form

$$\mathbf{AM} + \mathbf{BW} + \mathbf{D} = \mathbf{0}, \tag{17}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} \\ \tilde{b}_{21} & \tilde{b}_{22} \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \ \mathbf{W} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

with elements

$$\begin{aligned} a_{c1} &= \xi_{c1}^2 + I_c \chi_{c1}^2, \ a_{c2} = -\left(\xi_{c1}\xi_{c2} + I_c \chi_{c1}\chi_{c2}\right), \ \tilde{b}_{c1} = \xi_{c1}^2 + I_c \chi_{c1}\chi_{c3}, \\ \tilde{b}_{c2} &= \xi_{c1}\xi_{c2} + I_c \chi_{c1}\chi_{c2}, \ d_c = \chi_{c1}\chi_0 - \xi_{c1}\xi_0 + a_{c1}\left(\beta \pi_c^e - k_c\right) + a_{c2}\left(\beta \pi_c^e - k_c\right), \end{aligned}$$

for $c = 1, 2$. From (17) the money supply is

$$\mathbf{M} = \mathbf{A}^{-1}(\mathbf{BW} + \mathbf{D}) \tag{18}$$

Let us denote $\begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix} \equiv \mathbf{A}^{-1}\mathbf{B}$. A coefficient $\mu_{c,-c} = \frac{dm_c}{dw_{-c}}$ denotes the monetary response of the CB to wages, expressed as the elasticity of the money supply of country c with respect to the wage in country $-c$.

4.2 Reactions of monetary policy to wages

The reaction of monetary policy to wages is a central question regarding the interaction with labor unions because labor unions fear adverse reactions by the CB. The recent literature investigates this issue in CCD (2004) for the case of a monetary union and in CCD (2005) for the case of a closed economy. It is found that the CB may react to an increase in domestic wages by either increasing or lowering the money supply, depending on whether the CB is relatively accommodative or conservative. Here we extend this result to the open economy.

There is a critical value of CB conservativeness I_c^* below which the CB accommodates the money supply, $\mu_{c,c} > 0$, and above which the CB contracts the money supply in response to domestic wage increases, $\mu_{c,c} < 0$.[6] A domestic wage rise produces higher unemployment and higher inflation. The CB dislikes both unemployment and inflation, but has the choice to reduce one of them at the expense of the other via the Phillips curve. A relatively populist CB chooses to reduce unemployment by increasing the money supply, even if inflation rises. The opposite holds for a relatively conservative CB. Next, we characterize the reactions of a CB with respect to a *foreign* wage rise.

[6] For a proof see Appendix 7.4.1.

Proposition 1 *The central bank reaction to foreign wages depends on whether relative prices or real money balances affect the demand for goods more, η vs. $\frac{\rho}{\beta}$, given the set of conditions Ω in (53).*
(i) If $\eta > \frac{\rho}{\beta}$, for any level of conservativeness the CB contracts the money supply in response to foreign wage increases, $\mu_{c,-c} < 0$.
(ii) If $\eta < \frac{\rho}{\beta}$, the CB accommodates foreign wage increases if relatively populist, i.e. $\mu_{c,-c} > 0$, if $I < I^{**}$, and contracts if relatively conservative, i.e. $\mu_{c,-c} < 0$, if $I > I^{**}$.

Intuitively, the CB's reaction depends on whether unemployment increases or decreases due to a foreign wage increase. There are two opposing effects of foreign wages on home unemployment:
(i) a substitution effect, according to which consumers shift demand towards the cheaper goods, that are produced with lower labor costs (wages). That decreases unemployment at home.
(ii) a real money balances effect, according to which imported inflation, triggered by higher foreign wages, raises the price level at home and reduces real money balances, increases the real interest rate, reduces demand at home and increases home unemployment.

Whether the substitution effect or the real money balances effect is stronger, depends on the elasticity of demand with respect to prices η versus the elasticity of demand with respect to real money balances ρ/β. That determines whether unemployment in the home country falls or rises following a foreign wage increase. If $\eta > \rho/\beta$, the substitution effect dominates and unemployment at home falls while inflation rises. Then the CB never needs to accommodate. If $\eta < \rho/\beta$, the real money balances effect dominates and unemployment at home rises along with inflation. Then the CB accommodates, if relatively populist and contracts if relatively conservative. Figure 1 shows an illustration of a CB's reactions to domestic and foreign wages as a function of conservativeness.

Monetary policy tightens stronger against any wage increase (and accommodates less),

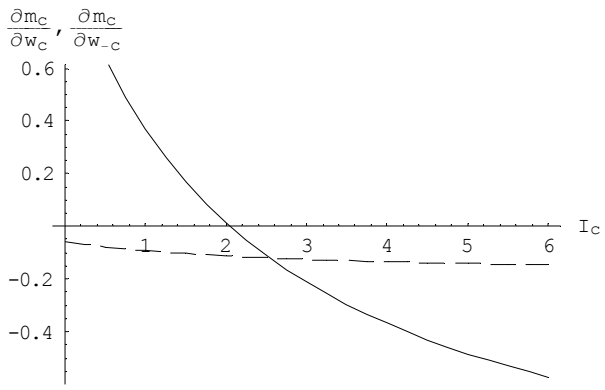

Figure 1: Reactions to domestic and foreign wages by a central bank as a function of its conservativeness. Note: solid line: $\frac{\partial m_c}{\partial w_c}$, dashed line: $\frac{\partial m_c}{\partial w_{-c}}$. The values of the parameters are $\alpha = 1/2$, $I_{-c} = 1$, $\eta = 2$, $s_c = 1/2$, $\rho = 1$, $\beta = 1$, $\delta = 1$.

the more conservative is any of the CBs, if goods are relative substitutes, i.e. $\frac{d\mu_{c,c}}{dI_c} < 0$, $\frac{d\mu_{c,-c}}{dI_c} < 0$ and if $\eta > \frac{\rho}{\beta}$ then also $\frac{d\mu_{c,c}}{dI_{-c}} < 0$ and $\frac{d\mu_{c,-c}}{dI_{-c}} < 0$. Intuitively, a more conservative CB puts more weight on reducing inflation at the expense of unemployment. Then the CB accommodates less and contract more against domestic wage increases. This result for domestic wages carries through for foreign wages as well. The reason is that a foreign wage rise generates inflation in the home economy too, via imports. Thus the more conservative the CB, the more it contracts against foreign wages to reduce *imported* inflation at the expense of unemployment. Moreover, monetary policy tightens stronger against any wage increase, if foreign conservativeness is higher. The reason is that higher foreign conservativeness yields monetary tightening abroad and that depreciates the home currency. A depreciated currency implies a higher price level in home currency, but it also implies competitive advantage for the home country and unemployment falls, given that $\eta > \rho/\beta$, i.e. given that the impact of the exchange rate is stronger on demand than that of real money balances. With lower unemployment and higher price level, the home CB has room and incentives for further monetary tightening.

4.3 Reactions of monetary policy to monetary policy abroad

In an open economy a CB takes into account the monetary policy response from the foreign CB when forming its own monetary policy. Here we identify the precise mechanism that drives the CBs to implement their equilibrium monetary policies.

The interaction between the CBs is intermediated by the real interest rate and the exchange rate. A monetary expansion by the domestic CB has simultaneously two effects: (1) a fall in the real interest rate and (2) an appreciation of the foreign currency. Abroad the first effect causes a imbalance between unemployment and inflation by decreasing unemployment and rising inflation. It distorts the optimal balance between unemployment and inflation for the foreign CB, which therefore responds with contraction. The second effect, the appreciated foreign currency, causes an increase in unemployment due to weaker competitiveness and lower inflation. This imbalance is in the opposite direction and the foreign CB responds with expansion. Depending on which of the two effects is stronger, the response of the foreign CB can be either contraction or expansion.

The best response function of CB1 to CB2's money supply choice is obtained by rearranging equation (16) to express m_c as a function of m_{-c},

$$m_c = \phi_{c0} + \phi_{c1}(m_{-c} - w_{-c}) + \phi_{c2} w_c, \tag{19}$$

for $c = 1, 2$, where $\phi_{c0} \equiv -d_c/a_{c1}$, $\phi_{c1} \equiv -a_{c2}/a_{c1} \in (0, 1)$ if $\eta > \rho/\beta$, and otherwise ϕ_{c1} may be negative. The coefficient $\phi_{c2} \equiv -\tilde{b}_{c1}/a_{c1}$, is negative if the CB in country c is sufficiently conservative and positive otherwise.

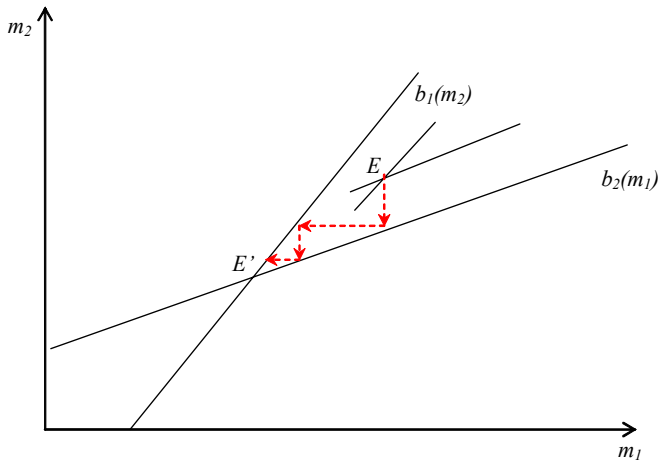

Figure 2: Positively sloped central bank reaction functions for relatively substitutable goods. A monetary contraction triggers further contractions.

4.3.1 Exchange rate competition

We assume that the goods are relative substitutes, i.e. $\eta > \rho/\beta$. In Figure 2 we draw the best response functions from equation (19) in this case. A rise in w_1 shifts CB2's best response function $b_2(m_1)$ downwards, because $\phi_{21} \in (0,1)$ and it shifts $b_1(m_2)$ upwards if we assume that CB1 is sufficiently conservative, i.e. $\phi_{12} < 0$. Then the new equilibrium point becomes E'. The adjustment to the new equilibrium involves consecutive monetary contractions by both CBs.

The exchange rate channel leads to the following interactions between CBs as they react to a wage increase by labor unions. Suppose, nominal wages increase in the home country. In the foreign country that yields higher imported inflation and lower unemployment, due to competitive gains. Then the foreign CB tightens to rebalance inflation and unemployment. Tightening, however, depreciates the home currency. Depreciation yields a higher price level in the home country and lower unemployment. When one of the CBs finds an optimal balance between unemployment and inflation, its monetary policy moves the exchange rate and that

creates an imbalance between unemployment and inflation in the other country. The home CB then contracts the money supply to rebalance unemployment and inflation optimally. The home monetary contraction appreciates the home currency. Appreciation induces lower foreign unemployment due to competitive gains and higher foreign price level, due to a relatively weaker foreign currency. Therefore, the foreign CB contracts the money supply again.

The demand-supply channel limits the extent of contractions. The two monetary policies tighten until the world real interest rate rises sufficiently to moderate inflation in both countries and therefore the optimal balance between inflation and unemployment is reached in both countries. Note that the real interest rate helps balancing unemployment and inflation in both countries at the same time, unlike the exchange rate, which imbalances it abroad.

4.3.2 Interest rate competition

Next we turn to the case when the goods are relative complements, i.e. $\eta < \rho/\beta$. Now $\phi_{c1} \in (-\infty, 0)$. Figure 3 provides an example of downward sloping reaction functions. A rise in w_1 shifts the foreign reaction curve upward because $\phi_{c1} < 0$, and the home reaction curve may shift in either direction, depending on conservativeness. Adjustment to the new equilibrium involves now consecutive contractions by CB1 and expansions by CB2.

When $\eta < \frac{\rho}{\beta}$, goods are relative complements and the monetary authority has a stronger effect on aggregate demand through the interest rate than through the exchange rate. Therefore the foreign CB reacts to the adverse impact of interest rates in the other country.

Let us assume that the home CB contracts against a domestic wage increase. The real interest rate rises internationally and unemployment rises while inflation moderates abroad as well. The foreign CB reacts to rebalance unemployment and inflation by accommodation. That way it counteracts the interest rate rise by the home CB. The foreign CB thus accommodates because it 'dislikes' the high interest rate set by the home CB. It's accommodation raises the

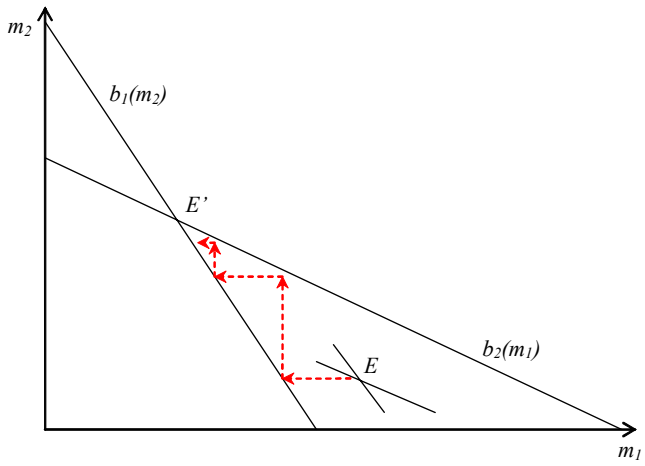

Figure 3: Downward sloping central bank reaction functions when goods are relatively complements. Monetary tightening triggers monetary expansions abroad and further tightening at home.

interest rate and creates an imbalance in the home country, to which the home CB responds by contracting again. The foreign CB, on the other hand is compelled to accommodate again to counteract the adverse effect to its economy. A chain of contractions by the home CB and accommodations by the foreign CB emerges, which eventually converges to the new equilibrium. The factor that limits the extent of contractions by the home CB (accommodations by the foreign CB) is the continuous appreciation of the home currency during this process. Appreciation reduces inflation at home and increases output in the foreign country, and the new equilibrium is reached.

4.4 Exchange rate movements due to strategic monetary policies

One of the mechanisms through which monetary policy affects the economy is the exchange rate channel. It is of interest therefore to examine the reactions of exchange rates to wages when

there is an optimal monetary policy in both countries. A wage increase alone, given constant money supply, depreciates the currency of the respective country in equation (40), because higher nominal wages translate into higher (export) prices $\bar{p}_1$, which shifts demand away from domestic goods. Then, with falling output, the transactions' demand for domestic currency diminishes, yielding a depreciation of the exchange rate. Such a depreciation restores somewhat output for country 1, although not entirely.

As we show in the proposition below, the original depreciation may be reversed, when the CBs react, so the exchange rate eventually appreciates, if the home CB is rather conservative. Substituting equation (18) into (40) we obtain the exchange rate that emerges taking account of the CBs reactions,

$$\begin{aligned} e &= \left[\phi_e\left(\mu_{11}-\mu_{21}\right)+1-\phi_e\right]w_1-\left[\phi_e\left(\mu_{22}-\mu_{12}\right)+1-\phi_e\right]w_2 \\ &\quad +\phi_e\left(\mu_{10}-\mu_{20}-k_1+k_2+\beta\pi_1-\beta\pi_2\right). \end{aligned} \tag{20}$$

In fact, the exchange rate reaction to a wage increase can be in both directions, depending on the relative conservativeness of the two countries. For example in the special case of two equal countries, the home currency depreciates following a wage increase at home, if the home CB is not more conservative than the foreign CB $I_1 \leq I_2$ and given Ω. More generally we find:

Proposition 2 *A wage increase causes the depreciation of the domestic currency if there is no CB intervention. However, if both CBs react optimally, the exchange rate appreciates, provided that the home CB is sufficiently conservative for a given foreign conservativeness and given the set* Ω.

Intuitively, it is obvious that the more conservative the home CB, the stronger are home contractions and the more populist the foreign CB, the stronger are foreign expansions, and therefore, the stronger the home currency. A relatively conservative home CB (given foreign

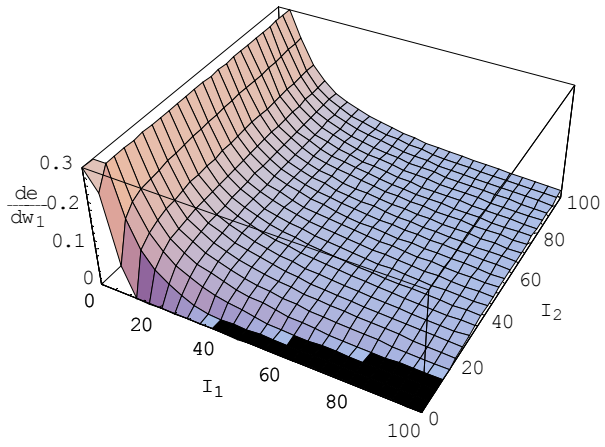

Figure 4: Exchange rate reaction to wages as a function of conservativeness: the home currency depreciates following a wage increase at home except for the dark area, where it appreciates. Parameter values: $\alpha = 1/2$, $\eta = 2$, $s_c = 1/2$, $\delta = 1$, $\rho = 1$, $\beta = 1$.

conservativeness), contracts the money supply to counter inflation and induces appreciation, which reverses the original impact of the wage rise. A sufficiently conservative foreign CB (given home conservativeness), however, contracts the foreign money supply to counter imported inflation and that generates the depreciation of the home currency. An example is given in Figure 4.

4.5 Inflation and unemployment with endogenous money supply

We have seen above that the intervention of the CBs may reverse the original reaction of the exchange rate to a wage increase. Therefore it is of interest whether monetary policy makes a difference about the impact of wages on inflation and unemployment. In the absence of monetary policy intervention, a home wage increase generates gains in competitiveness in the foreign country, if traded goods are relative substitutes, therefore unemployment falls abroad.

As it turns out, the proposition below shows that in this case the optimal monetary policy reverses the effect of wages on foreign unemployment.

Using equation (18) in equations (14) and (13) we obtain the price level and unemployment as functions of wages. The functions exhibit the following properties:

Proposition 3 *Given the set Ω, and taking account of the reaction of the central banks, an increase in wages in one country increases inflation and unemployment in both countries.*

It is intuitive that higher wages raise unemployment and inflation domestically. If firms face higher labor costs, then they raise prices and inflation increases. Then, firms sell less, produce less, which increases unemployment. The intervention of the CB balances the burden between inflation and unemployment and both inflation and unemployment increase proportionally.

Abroad, on the other hand, foreign inflation and foreign unemployment face different effects. Inflation rises due to demand shifting towards foreign goods and due to exported inflation. In the absence of CB intervention, unemployment abroad falls, if $\eta > \rho/\beta$, suggested by equation (13), due to a shift in demand towards the country with cheaper labor costs. However, optimal monetary policy balances the rising inflation and falling unemployment abroad by contracting the foreign money supply. Then, unemployment rises abroad as well, along with a tempered rise in inflation. Thus monetary intervention reverses the impact of wages on foreign unemployment. The CB's optimal rule is to balance inflation proportionally to unemployment, as the CB's first order condition suggests. Thus inflation and unemployment change in the same direction if the CB intervenes. If the traded goods are complements, then a home wage increase reduces foreign output along with home output, even without CB intervention. With CB intervention, inflation and unemployment increase proportionately.

5 Labor Unions

5.1 The labor union's problem

Next we turn to the optimization problem of labor unions, the third stage of the game. Labor unions anticipate the reaction of firms and CBs to their wage demands, and take wages set by other labor unions as given. Unions set nominal wages so as to minimize an objective function that shows preference for higher real wages and dislike for unemployment.[7]

$$\min_{w_{ic}} \left\{ -2 \left(w_{ic} - p_c \right) + A u_{ic}^2 \right\}, \tag{21}$$

where $w_{ic} - p_c$ is the real wage of labor union i, and the parameter A measures the relative weight of unemployment versus the real wage. The first order conditions

$$-\frac{d \left(w_{ic} - p_c \right)}{dw_{ic}} + A u_{ic} \frac{du_{ic}}{dw_{ic}} = 0,$$

provide n_c equations for each country, and thus determine simultaneously nominal wages w_{ic} for all i, c. To simplify notation, we denote the total elasticity of the real wage of a union with respect to the union's nominal wage as

$$Z_{wc} = \frac{d \left(w_{ic} - p_c \right)}{dw_{ic}}.$$

Similarly, let Z_{uc} denote the total elasticity of the unemployment rate of a union with respect to the union's nominal wage

$$Z_{uc} = \frac{du_{ic}}{dw_{ic}}.$$

[7] Unions also may be averse to inflation. One of the main reasons is that pensions of workers are not indexed and that union members, as consumers, dislike inflation in general. In this paper we abstract from the inflation aversion of labor unions.

Our aim is to reflect differences between countries and monetary regimes. Therefore heterogeneity within a country does not play a role, and we can assume that all firms and unions within a country are identical, that is, there is symmetry within a country. Then the first order conditions of the labor unions' minimization problem for the two countries can be rewritten as

$$-Z_{wc} + AZ_{uc}u_c = 0, \tag{22}$$

where the elasticities Z_{wc} and Z_{uc} are derived in (48) and (52) in the Appendix.

5.2 Equilibrium unemployment and inflation

We calculate unemployment by rearranging equation (22). The equilibrium unemployment rate for each country is

$$u_c^{Float} = \frac{Z_{wc}}{AZ_{uc}}, \tag{23}$$

where the upper index $Float$ is added on u_c for clear reference to the floating exchange rate regime. To reach this result we used the assumption that all labor unions are equal within a country, thus $u_{ic} = u_c$. The inflation rate is obtained by rearranging the first order condition of the CB's optimization problem, equation (16), to yield

$$\pi_c^{Float} = \frac{\xi_{c1}u_c^{Float}}{\chi_{c1}I_c}, \tag{24}$$

The inflation rate is thus a positive function of the unemployment rate. The following proposition establishes the effects of the two CBs' preferences on equilibrium unemployment and inflation rates.

Proposition 4 *Labor unions set wages such that equilibrium unemployment and inflation are (i) decreasing in home CB conservativeness* I_c

(ii) decreasing in foreign CB conservativeness I_{-c}

The intuition for the first part of this result is the following. The more conservative the CB the stronger its reaction against wage increases. Monetary contractions reduce demand and lead to a higher unemployment rate. Then, fearing a rise in unemployment, labor unions are more reluctant to raise wages. Lower wages generate lower unemployment and lower inflation rates.

CCD (2005) assume that labor unions are averse to both unemployment and inflation. Therefore they obtain that intermediate ranges of conservativeness are best for economic performance. In this model we assume that labor unions are not averse to inflation, which is reflected in their objective function (21). Therefore, labor unions' inflation fears as a moderating factor does not operate here. Accommodation does not reduce wage demands as in CCD (2005), because it does not create inflation fears. In this model only contractions create fears, in particular unemployment fears and thus the more conservative the CB, the lower are wage demands.

Why is *foreign* conservativeness better for the home country? A higher foreign conservativeness is associated with a stronger foreign currency and a weaker home currency. This implies that the price level in home currency is higher and unemployment is lower, if traded goods are relative substitutes. Second, depreciation implies competitive advantage for the home country and lower unemployment if $\eta \geq \rho/\beta$. A higher price level and lower unemployment induce tightening from the home CB. Thus an increase in foreign conservativeness generates home monetary tightening. Tighter domestic monetary policy, in turn, threatens labor unions with unemployment and moderates wages.

If goods are relative complements, a conservative foreign CB represents a threat to home labor unions because it raises the world real interest rate against imported inflation caused by a home wage increase. Then, labor unions moderate wages because of the unemployment fears that are created directly by the foreign CB, instead of intermediated via the home CB.

Thus we have shown that the more conservative the home or the foreign CB, the better it is for home economic performance in terms of inflation and unemployment. As an illustration, that means that Germany realized a welfare loss due to a less conservative economic partner, Italy. Germany enjoyed, however, the conservativeness of the Bank of England. This provides a reason that Germany and Italy joined the EMU and not the UK.

6 Conclusions

This paper contributes to the recent literature along three major lines. First, the paper considers strategic interactions between a home and a foreign central bank (CB). Most of the recent literature deals with a closed economy and it does not reveal the mechanism of interaction between CBs. Second, this paper considers monetary policy reactions to foreign wage inflation and threats leveraged against foreign labor unions mainly through the exchange rate. The recent literature is confined to domestic interactions between monetary policy and labor unions. In fact, the recent depreciation of the dollar against the euro may have contributed to the recent wage freezes and longer working hours in Germany and France. Third, while the literature investigates the issue of domestic central bank conservativeness, the problem of foreign CB conservativeness remains the focus of this paper.

The two CBs interact through the exchange rate and the real interest rate. The mechanism of interaction differs according to the relative degree of substitution of goods traded between the countries. First, if the goods traded between the two countries are relative substitutes, the two CBs compete over a favorable exchange rate. A monetary expansion in country 1 appreciates the foreign currency and induces the foreign CB to expand its money supply in order to avoid a reduction in exports and output. The foreign monetary expansion appreciates the home currency and induces further monetary expansion at home and further expansionary reactions from abroad. As the two money supplies expand, the real interest rate falls in

both countries and boosts their output to the desired level. Therefore the expansionary policies converge to an equilibrium.

A monetary contraction in country 1 induces monetary contraction in country 2 in order to avoid the depreciation of its currency, which triggers inflation. A competition to appreciate one's own currency emerges in order to avoid imported inflation. In this process the world real interest rate rises, which reduces both countries' inflation rates.

Second, if the goods traded between the two countries are relative complements, the two CBs compete over a favorable real interest rate. A monetary expansion in country 1 reduces the world real interest rate and causes 'overheating' in country 2. Therefore CB2 tightens and raises the real interest rate again. Then CB1 expands further and CB2 contracts further in response, until the exchange rate depreciates sufficiently to boosts output in country 1 but without raising inflation in country 2.

Monetary policy reacts to wages and generates fears of unemployment for labor unions, which influences their wage demands. We find that the CB tightens the money supply against a domestic wage increase if the CB is relatively conservative and expands it if relatively populist. This result is similar to the one obtained for the case of a closed economy by CCD (2005) and for a monetary union by CCD (2004). Here however, in addition to the home CB, the foreign CB exercises a further threat against labor unions in the home country. If the goods traded between the two countries are relative substitutes, the foreign CB contracts the money supply in reaction to a wage increase in the home country. The reason is that a wage increase at home creates imported inflation abroad and foreign unemployment falls due to relatively cheaper foreign labor costs. If the goods between the two countries are relative complements, home goods sell well along with foreign goods, and a wage rise creates unemployment both abroad and at home. Then foreign monetary policy accommodates a home wage rise, if the foreign CB is relatively populist and tightens if it is relatively conservative.

There are two results regarding an open economy that reverse in the presence of monetary

policy. First, in the absence of monetary intervention, a wage rise determines the depreciation of the domestic currency. This is due to lower transactions demand for the home currency as output shrinks following the wage rise. However, the reactions of the CBs may revert the direction of the exchange rate change. If the domestic CB is sufficiently conservative given foreign conservativeness, the currency appreciates.

Second, in the absence of monetary intervention, a wage rise produces competitive benefits abroad, e.g. higher foreign output and foreign exports, due to relatively lower production costs abroad. However, an optimal intervention by the CBs washes away such competitive gains from the foreign country. The reason is that the wage rise inflicts imported inflation abroad, against which foreign monetary policy tightens to an extent that foreign exports and output actually fall.

Equilibrium unemployment and inflation are lower the higher is the degree of domestic CB conservativeness, supporting previous findings by CCD (2004) for the case when labor unions are not inflation averse. The reason is that tighter monetary policy creates unemployment fears which moderates labor unions' wage demands, yielding lower unemployment and inflation. This is interpreted by CCD (2005) as an instance of the Lucas critique. That is, the more the CB prefers to alleviate unemployment at the expense of inflation by exploiting the Phillips curve, the higher unemployment in fact becomes.

We find that a higher degree of *foreign* CB conservativeness is beneficial for the home country as well, as it moderates labor unions in the home country. This may happen in two ways. First, if the goods are relatively substitutes, the driving mechanism is the foreign exchange channel. Higher foreign conservativeness is associated with a weaker home currency. Depreciation produces a higher home price level and a lower unemployment rate, due to competitive gains. Then the home CB has incentives and room to tighten its monetary policy, which threatens labor unions with unemployment and unions moderate wages. Thus the conservativeness of the foreign CB affects the tightness of the home CB's policy. That generates fears of unemployment

and restrains labor unions. Second, if the goods are relative complements, home labor unions fear to rise wages because that creates imported inflation abroad against which a conservative foreign CB tightens monetary policy. Foreign monetary tightening raises the world real interest rate and threatens home labor unions with unemployment. Hence the foreign CB's conservativeness directly affects the behavior of labor unions in the home country. In both cases, the conservativeness of the foreign CB moderates home wage demands and the home country also benefits from improved economic performance.

A point of interest for further research is represented by a multi-period version of this model. In our static model expected inflation remains unaffected by monetary policy, because it is determined in the next period, while a multi-period model would account for effects on expected inflation.

7 Appendix

7.1 Derivation of real interest rate parity

We assume that covered interest rate parity holds, due to free international capital flows.

$$f - e = i_1 - i_2,$$

where f is the log of the forward exchange rate and i_1 and i_2 are the nominal interest rates in country 1 and 2 respectively. There is extensive empirical evidence showing that covered interest rate parity is found in the data, as surveyed by Taylor (1995). With rational expectations, the forward exchange rate equals the expected future exchange rate. It implies, therefore, that uncovered interest parity holds,

$$E_t\{e_{t+1}\} - e_t = i_{1,t} - i_{2,t},$$

where $E_t\{e_{t+1}\}$ is the future exchange rate expected for period $t+1$ given information at period t. Here we can replace the exchange rate by its definition under PPP, $e = p_1 - p_2$. It yields

$$E_t\{p_{1,t+1}\} - E_t\{p_{2,t+1}\} - p_{1,t} + p_{2,t} = i_{1,t} - i_{2,t},$$

or equivalently,

$$\pi_1^e - \pi_2^e = i_1 - i_2, \tag{25}$$

where $\pi_c^e = E_t\{p_{c,t+1}\} - p_{c,t}$ is the expected inflation rate for the next period. The Fisher parity gives the real interest rate as the difference between the nominal interest rate and expected inflation $r_c = i_c - \pi_c^e$. The Fisher parity and (25) lead to the equalization of real interest rates across countries, i.e. real interest rate parity holds, $r_1 = r_2 = r$. Thus we have shown that combining uncovered interest rate parity, PPP and the Fisher parity implies that real interest parity holds as well.

7.2 Derivation of unemployment and inflation

Let us define the terms of trade T as the price of the basket of goods produced in country 1 relative to that of country 2 expressed in the same currency, i.e. $T = \bar{p}_1 - \bar{p}_2 - e$. The system of equations composed of the firm's pricing equation (7), aggregate demand (3) and the LM curve (12) can be rewritten as

$$s_2 T = \theta + \frac{1}{\nu}\left[\alpha\left(w_1 - p_1\right) - \left(1-\alpha\right)\rho r\right], \tag{26}$$

$$-s_1 T = \theta + \frac{1}{\nu}\left[\alpha\left(w_2 - p_1 + e\right) - \left(1-\alpha\right)\rho r\right], \tag{27}$$

$$y_1 = -\eta s_2 T - \rho r, \tag{28}$$

$$y_2 = \eta s_1 T - \rho r, \tag{29}$$

$$m_1 = p_1 + k_1 - \beta\left(r + \pi_1^e\right) + \delta y_1, \tag{30}$$

$$m_2 = p_1 + k_2 - e - \beta\left(r + \pi_2^e\right) + \delta y_2, \tag{31}$$

where $\nu = (\alpha - \alpha\eta + \eta)$. These are six equations in six variables, $T,\ p_1,\ r,\ e,\ y_1,\ y_2$. Next we transform the system (26)-(31) by subtracting the equation for country 1 from the respective equation for country 2.

$$T = \frac{\alpha}{\nu}\left(w_1 - w_2 - e\right), \tag{32}$$

$$y_1 - y_2 = -\eta T, \tag{33}$$

$$m_1 - m_2 = e + (k_1 - k_2) - \beta\left(\pi_1^e - \pi_2^e\right) + \delta\left(y_1 - y_2\right). \tag{34}$$

The three equations solve for three variables: $T,\ e,\ y_1 - y_2$.Then, we transform the system (26)-(31) by summing up (averaging) equations of partner countries. The equation of country 1 is weighted by s_1 and that of country 2 by s_2 and summed up,

$$0 = \theta + \frac{1}{\nu}\left[\alpha\left(w - p\right) - (1 - \alpha)\,\rho r\right], \tag{35}$$

$$y = -\rho r, \tag{36}$$

$$m = p + k - \beta r - \beta\pi^e + \delta y. \tag{37}$$

These three equations solve for three variables: $p,\ y,\ r$, where $p = s_1 p_1 + s_2 p_2$, $w = s_1 w_1 + s_2 w_2$, $y = s_1 y_1 + s_2 y_2$, and $m = s_1 m_1 + s_2 m_2$. Note that p is the common price level, w is the average wage and m is an index of the average money supply denominated in a common currency unit (CCU). This aggregates all economies into one and loses country specific variables. In order to obtain the national price level in a certain country's denomination, p has to be adjusted with

the exchange rate,

$$p_1 = p + s_2 e, \tag{38}$$

$$p_2 = p - s_1 e. \tag{39}$$

Substituting (32) into (33) and the latter into (34), and expressing the exchange rate yields

$$e = \phi_e \left[m_1 - m_2 - k_1 + k_2 + \beta \left(\pi_1^e - \pi_2^e \right) \right] + \left(1 - \phi_e \right) \left(w_1 - w_2 \right), \tag{40}$$

where $\phi_e \equiv \frac{\alpha - \alpha\eta + \eta}{\alpha - \alpha\eta + \eta + \alpha\delta\eta} \in (0, 1)$ is a weight. The exchange rate is a weighted average of the relative percentage difference between two countries' money supplies and wages. Next we obtain the terms of trade by substituting the above into (32),

$$T = -\frac{\alpha\phi_e}{\nu} \left(m_1 - m_2 - w_1 + w_2 - k_1 + k_2 + \beta\pi_1^e - \beta\pi_2^e \right). \tag{41}$$

It shows that the terms of trade is affected by the relative money supply and relative wages in equal weight. Rearranging equation (35), we express the common price level denominated in CCU as,

$$p = \frac{\theta\nu}{\alpha} - \frac{1 - \alpha}{\alpha}\rho r + w. \tag{42}$$

This shows that producers set prices based on labor costs w and the demand they face, which in turn is affected by the real interest rate r. Producers raise prices when faced with higher demand or higher wages. Substituting equations (35) and (36) into (37), we obtain the equilibrium real interest rate,

$$r = \frac{\alpha}{\rho\tau} \left(\frac{\theta\nu}{\alpha} + w - m + k - \beta\pi^e \right), \tag{43}$$

where $\tau = 1 - \alpha + \alpha \left(\beta/\rho + \delta \right)$ and $\frac{\alpha}{\tau}$ is the semi-elasticity of real interest rate with respect to money balances. It shows that the real interest rate, which is common over the two countries,

is determined by the combined money supplies of both countries relative to aggregate wages. A country has a weight of its size s_c in affecting the real interest rate. Substituting equation (41) and (43) into (28), rearranging, and then substituting the resulting output y_c into $u_c = l_0 - \frac{1}{\alpha} y_c$, yields (13) where

$$\xi_0 \equiv l_0 + \frac{\nu\theta}{\alpha\tau},\ \xi_{c1} \equiv \frac{\eta\left(1-\alpha+\alpha\delta\right)+\eta\alpha\beta/\rho\left(1-s_c\right)+\alpha s_c}{\tau\left(\nu+\alpha\delta\eta\right)},\ \xi_{c2} \equiv \frac{\alpha\left(\beta\eta/\rho-1\right)s_{-c}}{\tau\left(\nu+\alpha\delta\eta\right)}, \tag{44}$$

and where

$$\tau \equiv 1-\alpha+\alpha\left(\beta/\rho+\delta\right),\ \nu \equiv \alpha+\left(1-\alpha\right)\eta.$$

Substituting (43) into (42), and (40) into (38) yields after some rearrangement (14), where

$$\begin{aligned} \chi_0 &\equiv \frac{\nu\theta\left(\beta/\rho+\delta\right)}{\tau},\ \chi_{c1} \equiv \frac{\left(1-\alpha\right)\left(\nu+\alpha\delta\eta\right)+\alpha\left(1-s_c\right)\gamma}{\tau\left(\nu+\alpha\delta\eta\right)}, \\ \chi_{c2} &\equiv \frac{\alpha\left(\delta\tau+\gamma s_c\right)}{\tau\left(\nu+\alpha\delta\eta\right)},\ \chi_{c3} \equiv \frac{\alpha\gamma s_{-c}}{\tau\left(\nu+\alpha\delta\eta\right)}. \end{aligned} \tag{45}$$

and where

$$\gamma \equiv \alpha\left(\beta/\rho+\delta\right)+\left(1-\alpha\right)\beta\eta/\rho,\ g \equiv \tau\left(\nu+\alpha\delta\eta\right).$$

7.3 Derivation of equilibrium unemployment

The elasticity of real wage with respect to nominal wage is

$$Z_{wc} = \frac{d\left(w_{ic}-p_c\right)}{dw_{ic}} = 1-\frac{dp_c}{dw_{ic}}. \tag{46}$$

Since $w_c = \Sigma_{i=1}^{n_c} w_{ic}/n_c$, its derivative is $\frac{dw_c}{dw_{ic}} = \frac{1}{n_c}$. Using this relationship and equation (14), we obtain

$$\frac{dp_c}{dw_{ic}} = \frac{1}{n_c}\left(\frac{\partial p_c}{\partial w_c}+\frac{\partial p_c}{\partial m_c}\frac{dm_c}{dw_c}+\frac{\partial p_c}{\partial m_{-c}}\frac{dm_{-c}}{dw_c}\right).$$

We obtain the appropriate partial derivatives from equations (14) and (18), and substituting them yields

$$\frac{dp_c}{dw_{ic}} = \frac{1}{n_c}\left[\frac{\alpha\left(\delta\tau+\gamma s_c\right)}{\tau\left(b+\alpha\delta\eta\right)} + \frac{\left(1-\alpha\right)\left(\nu+\alpha\delta\eta\right)+\alpha\left(1-s_c\right)\gamma}{\tau\left(\nu+\alpha\delta\eta\right)}\mu_{c,c} - \frac{\alpha\gamma s_{-c}}{\tau\left(\nu+\alpha\delta\eta\right)}\mu_{-c,c}\right]. \tag{47}$$

Substituting equation (47) into (46) we obtain Z_{wc},

$$Z_{wc} = 1 - \frac{\alpha\left(\delta\tau+\gamma s_c\right)+\left[\left(1-\alpha\right)\left(\nu+\alpha\delta\eta\right)+\alpha\left(1-s_c\right)\gamma\right]\mu_{c,c}-\alpha\gamma s_{-c}\mu_{-c,c}}{n_c\tau\left(\nu+\alpha\delta\eta\right)}. \tag{48}$$

From equation (9) the elasticity of the unemployment rate with respect to the nominal wage of a labor union is

$$Z_{uc} = \frac{du_{ic}}{dw_{ic}} = \frac{1}{\alpha}\left[\eta\frac{d\left(p_{ic}-p_c\right)}{dw_{ic}} + \rho\frac{dr}{dw_{ic}}\right]. \tag{49}$$

The derivative of the relative price can be obtained using equation (6) averaged over firms j that belong to labor union i,

$$\frac{d\left(p_{ic}-p_c\right)}{dw_{ic}} = \frac{1}{\alpha+\eta(1-\alpha)}\left[\alpha\frac{d(w_{ic}-p_c)}{dw_{ic}} - (1-\alpha)\rho\frac{dr}{dw_{ic}}\right]. \tag{50}$$

Let's denote the semi-elasticity of the real interest rate with respect to nominal wage w_{ic} as Z_{rc}. This semi-elasticity is calculated using equation (43),

$$Z_{rc} = \frac{dr}{dw_{ic}} = -\frac{\alpha}{\tau\rho n_c}\left[s_c\left(\mu_{c,c}-1\right)+s_{-c}\mu_{-c,c}\right]. \tag{51}$$

Substituting equation (51) into (50) and the result into (49) yields

$$Z_{uc} = \frac{\eta Z_{wc}+Z_{rc}}{\nu}. \tag{52}$$

Having found the two elasticities Z_{wc} and Z_{uc} we can now calculate the equilibrium level of unemployment as $u_c = \frac{Z_{wc}}{AZ_{uc}}$.

7.4 Proofs

Regarding the methodology used, analytic results are obtained for the symmetric country case when $s_1 = s_2 = 1/2$, $\alpha = 1/2$. In addition to this symmetric case, we derive analytically a number of asymmetric cases to extend the generality of the results, e.g. when $s_1 = 0.4$ and $s_2 = 0.6$, etc. All the cases considered are collected into the set

$$\begin{aligned} \Omega \equiv \{\alpha &\in \{0.1,\ 0.2, ...,\ 0.9\},\ s_1 \in \{0.1,\ 0.2, ...,\ 0.9\},\ s_2 = 1 - s_1, \\ n_c &\in \{1, 2, ...,\ 10\},\ \eta > 1,\ I_1 > 0,\ I_2 > 0,\ \rho > 0,\ \beta > 0,\ \delta > 0\}. \end{aligned} \tag{53}$$

All statements are based on analytic proof with Mathematica 5.1 subject to Ω.

7.4.1 Proof of claim

The CB's reaction to domestic wages as a function of CB conservativeness $\mu_{c,c}(I_c)$ changes sign from positive $\lim_{I_c \to 0} \mu_{c,c} = 1 > 0$ to negative $\lim_{I_c \to \infty} \mu_{c,c} < 0$. Furthermore $\mu_{c,c}(I_c)$ is a monotonic decreasing function in I_c. Therefore the change of sign occurs only once, there exists a unique I_c^* for which $\mu_{c,c}(I_c^*) = 0$. For $I_c < I_c^*$ the CB accommodates, $\mu_{c,c} > 0$, and for $I_c > I_c^*$ the CB contracts the money supply to a domestic wage increase, $\mu_{c,c} < 0$.

7.4.2 Proof of Proposition 1

(i) If $\eta > \frac{\rho}{\beta}$, then the CB's reaction to foreign wages is always negative, $\mu_{c,-c}(I_c) < 0$, because $\mu_{c,-c}(0) < 0$ and because it is monotonic decreasing. (ii) If $\eta < \frac{\rho}{\beta}$, then $\mu_{c,-c}(I_c)$ changes sign from positive $\mu_{c,-c}(0) > 0$ to negative $\lim_{I_c \to \infty} \mu_{c,-c} < 0$. Also, $\mu_{c,-c}(I_c)$ is monotonic

decreasing in I_c. Therefore the change of sign occurs only once, there exists a unique I_c^{**} for which $\mu_{c,-c}\left(I_c^{**}\right)=0$.

7.4.3 Proof of Proposition 2

If there is no CB intervention, from equation (20) we find that $\mu_{11}=\mu_{21}=0$. Then $\frac{de}{dw_1}=1-\phi_e>0$. However, if there is CB intervention, then

$$e_{w1}\equiv\frac{de}{dw_1}=\phi_e\left(\mu_{11}-\mu_{21}\right)+\left(1-\phi_e\right).$$

The function $e_{w1}\left(I_1,I_2\right)$ is monotonic decreasing in I_1 and monotonic increasing in I_2, $\frac{de_{w1}}{dI_1}<0$, and $\frac{de_{w1}}{dI_2}>0$. There is a change of sign from $\lim_{I_1\to 0}e_{w1}=1>0$ to $\lim_{I_1\to\infty}e_{w1}<0$. Then for each I_2 there exists a unique $\tilde{I}_1^*(I_2)$ such that $e_{w1}\geq 0$ if $I_1\leq\tilde{I}_1^*(I_2)$ and $e_{w1}<0$ otherwise.

7.4.4 Proof of Proposition 3

Given the set Ω, the inequalities

$$\frac{du_c}{dw_c}>0 \text{ and } \frac{du_{-c}}{dw_c}>0 \tag{54}$$

hold. Then, equation (16) implies that $\frac{d\pi_c}{dw_c}>0$ and $\frac{d\pi_{-c}}{dw_c}>0$.

7.4.5 Proof of Proposition 4

We use equation (23). Given the set Ω, the following inequalities hold: $du_c/dI_c\leq 0$, and $du_c/dI_{-c}\leq 0$. Then equation (24) implies that also $d\pi_c/dI_c\leq 0$ and $d\pi_c/dI_{-c}\leq 0$.

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