Job Competition Over the Business Cycle

Andri Chassamboulli*

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Abstract

The incidence of unemployment and its consequences both at the aggregate and the individual level has received considerable attention. However, little has been done in explaining why typically joblessness, and the burden of recessions falls more heavily on the lower end of the skill distribution. This paper examines the consequences of a vertical type of skill-mismatch, that takes the form of workers accepting transitorily jobs they are over-qualified for, and continue searching while employed for more suitable jobs, thereby influencing the labor market prospects of lower skill groups. I develop a matching model with endogenous skill requirements of jobs, and heterogeneous workers, which allows for job finding rates to vary across skill and over the business cycle. The model explains why the burden of unemployment falls disproportionably at the lower segment of the labor market, and is consistent with the well established evidence that match quality and job-to-job transitions are procyclical.

*Department of Economics, Central European University and National Bank of Hungary (e-mail: chassamboulli@ceu.hu). I am greatly indebted to John Haltiwanger, Jeff Smith, John Shea, and Michael Pries for their suggestions and advice.

1 Introduction

Search and matching theory has become a dominant framework for the analysis of labor market dynamics over the business cycle. While skill heterogeneity and on-the-job search has already been introduced into the standard search matching model, to explain cyclical changes in the quality of job-worker matches, the standard assumption that remains is that all workers can perform any type of job, regardless of their skill type. In turn, this implies a unique matching rate for all skill groups. Therefore, existing models offer a characterization of the cyclical behavior of worker flows in terms of average or representative values, but overlook important differences across skill groups. Typically, the unemployment rate of the less skilled is higher and increases more in downturns.[1] Skilled workers have relatively higher exit rates from unemployment, and relatively higher propensity to search on the job.[2] Apart from overlooking observed differences in the cyclical behavior of different skill groups, the assumption of identical matching probabilities fails to capture the search externalities and across-skill spillover effects that arise when workers of different skill compete for the same type of jobs.

The model in this paper allows for the matching rates to differ across skill groups. In the economy I examine, firms open vacancies for either high-productivity jobs, which have high skill requirements, or low-productivity jobs, with lower skill requirements. High-skill workers are best suited for high-skill jobs, but they also qualify for low-skill jobs, whereas, low-skill workers qualify only for low-skill jobs. Some unemployed high-skill workers accept transitorily low-skill jobs and search on the job for high-skill jobs, thereby influencing the employment prospects of low-skill workers. I demonstrate that a cyclical pattern in the matching behavior of high-skill workers, i.e., of downgrading to lower job levels to avoid unemployment, and upgrading to higher job levels by on-the-job search, can explain why the unemployment rate of the more skilled remains relatively low, and why high-skill employment is less sensitive to business cycles. In addition, in contrast to the conventional belief that the more skilled crowd out the lower skilled when competing for jobs, I find that by accepting low-skill job offers, high-skill workers actually improve the chances low-skill

[1] Evidence for the U.S. can be found for instance in Topel (1993), Juhn *et al.* (2002), and Moscarini (1996). Manacorda and Petrongolo (1999), give evidence both for the U.S. and several European countries, including Britain, France, Germany, Italy, the Netherlands, and Spain. Moreover, van Ours and Ridder (1995), give evidence for the Netherlands.

[2] See e.g., Blau and Robins (1990); Pissarides and Wadsworth (1994); Belzil (1996); Beach and Kaliski (1987).

workers find jobs.

There are several reasons to expect that skilled workers are relatively more likely to be on-the-job seekers are opposed to unemployed job seekers. First, they qualify for a wider range of job types, thus, they are relatively more capable in finding transitory jobs, as opposed to remain unemployed until a suitable job offer comes along. As documented in Nagypal (2004), while total separations decline with education, the fraction of job-to-job flows in total separations rise with education. Moreover, the fraction of quits that lead to a direct transition into a new job increases with education, suggesting that temporary employment in lower job levels and upgrading via on-the-job search is more prominent among the higher skilled.[3] There is also direct evidence of over-education phenomena at the higher end of the skill distribution. Hecker (1992) and Shelley (1994), find that in 1990, 17.9% of college graduates in the US were employed in “high-school” type jobs. Graduate over-education measures in the same range can also be found for many European countries including the UK and Spain.[4]

Some have linked these phenomena to skill-biased technological shocks and the crowding out of workers at the lower segment of the labor market, but little attention has been paid to their cyclical implications.[5] The cyclical behavior of skill-mismatch has only been emphasized in Barlevy (2002). Motivated by the well established evidence that match quality, and job-to-job transitions are procyclical, the notion formalized in Barlevy is that mismatched workers have more difficulty moving into better jobs in recessions, as firms open fewer vacancies per job seeker.[6] However, by not accounting for asymmetries in the matching technology, important across-skill interactions entailed in cross-skill matching have been overlooked.

In this paper, when high-skill workers accept transitorily low-skill jobs, they influence

[3]The finding of Bowlus (1995) that the quality of matches falls during recessions more evidently in white collar than blue collar activities, gives additional support to the view that noisier allocations of workers across jobs are more likely to occur at the higher segment of the labor market.

[4]See e.g., Green *et al.* (1999), Oliver and Raymond (2003),....

[5]For direct evidence on over-education phenomena and the “crowding out” of lower educated workers see e.g., Teulings and Koopmanschap (1989), Bewley (1995,1999), Gautier (1998) and Gautier *et al.* (2002).

[6]Evidence that job quality is procyclical can be found for example in Bowlus (1995), Davis *et al.*(1996), Bils (1985), Shin (1994), Bowlus *et al.* (2002), and Liu (2003). These studies proxy job quality either by job duration, since bad matches are likely to be abandoned faster, or as reflected in wages, since bad matches are of lower productivity. The view that match quality is procyclical is also confirmed by surveys that workers are more likely to report underutilized during recessions(see e.g., Akerlof *et al.* (1988), and Acemoglu (1999)).

the labor marker conditions for low-skill workers in two ways. On the one hand, by accepting transitorily low-skill jobs, they increase the profits of low-skill vacancies, as vacancies with low-skill requirements can be filled relatively faster. This in turn, encourages firms to downgrade the skill-mix of vacancies, thus rasing the chances low-skill workers find jobs, especially in periods of high unemployment, when the arrival rate of unemployed to vacant jobs is higher. On the other hand, since over-qualified workers are likely to abandon low-skill jobs sooner, by crowding out low-skill workers, they lower the profits of low-skill jobs, and discourage firms from opening low-skill vacancies.

The asymmetric nature of the matching technology together with the differences in the productivity of high- and low-skill jobs, imply that changes in aggregate conditions have different consequences on the two types of jobs. Moreover, shifts in the composition of job seekers over the business cycle, affect the profits of jobs unevenly. For instance, a rise in the fraction of low-skill unemployed job seekers, lowers job creation costs at the lower segment of the market, while a rise in the fraction of over-qualified job seekers, lowers job creation costs at the higher segment of the labor market. Consequently, firms respond to aggregate shocks by adjusting not only the number of vacancies opened, but also the skill mix of vacancies.

A calibration of the model to match several facts of the U.S. labor market, revealed that the so much lower unemployment rate of college graduates compared to the unemployment rate of those with less than college education, can be sustained by roughly 18% on average of the former being employed in jobs that require less than college education. This figure is well in line with evidence on mismatch rates among college graduates, reported above. Moreover, consistent with the evidence, recessions in the model hurt low-skill employability relatively more. The consequences of negative aggregate shocks are partially alleviated by shifting the vacancy mix towards the more productive type of jobs, which are high-skill jobs in the model. While both types of workers suffer reductions in their chances of finding jobs, as firms open fewer vacancies per job seeker, low-skill workers suffer in addition from the shift in the vacancy mix towards high-skill vacancies. Despite the upgrading in the skill mix of vacancies, recessions entail a higher number of misallocated high-skill workers, in line with evidence that match quality is procyclical. As job finding rates are lower in recessions, a higher number of high-skill unemployed refuges to temporary employment in low-skill jobs, while upgrades to high-skill jobs happen more frequently in booms, when job finding rates rise.

Surprisingly enough, I find that low-skill workers are better off when high-skill refuge to temporary employment in jobs below their skill level. The negative crowding out effect that lowers the average quality of low-skill jobs is small relative to the positive impact of a higher effective matching rate for firms with low-skill vacancies. The willingness of high-skill workers to accept low-skill jobs and the resulting higher search activity at both segments of the labor market maintains a higher incentive for firms to open vacancies in both sectors. Hence, both high- and low-skill employment is higher when cross-skill matching occurs. In addition, high-skill employment is higher not only because of on-the-job searchers, but also because the number of suitably matched high-skill workers is higher.

Consequently, by occupying transitorily low-skill jobs, high-skill workers do not shift the burden of unemployment on low-skill workers in recessions. The model with cross-skill matching exhibits more cyclical employment growth overall, but less cyclical low-skill relative to high-skill employment growth. When both high- and low-skill workers occupy low-skill jobs, in periods of rising unemployment, firms with low-skill vacancies benefit disproportionably from a higher arrival rate of job seekers. This in turn, moderates the negative impact of recessions on the relative profitability of low-skill vacancies. However, it also limits the scope of upgrading the vacancy mix in recessions, which acts as an insulating mechanism that keeps the number of vacancies per job seeker high. As firms react more eminently by cutting down the number of vacancies per job seeker, as opposed to shifting the vacancy mix towards high-skill vacancies, recessions with cross-skill matching have a more moderate impact on low-skill employability relative to high-skill employability, but a stronger negative impact on employability overall.

Overlooking the evidence that document high mismatch rates at the upper end of the skill distribution, one could argue that the employment of the higher skilled is less cyclical, not because they refuge to temporary employment below their skill level, but due to several other reasons. These include, lower separation rates for the higher skilled, or larger productivity gains at the higher segment of the labor market, which imply that high-skill jobs are relatively more abundant. An illustrative simulation of the model assuming that workers can distinguish the types of vacancies before applying, and thus can target only the jobs they are best suited for, showed that this is not the case. This illustrative simulation accounts for the much higher separation rates of the lower skill groups, and focuses on the "best case" scenario for the conditions in the high-skill sub-market relative to the conditions in the low-skill sub-market. Still, the simulation revealed that the model with directed search requires an unrealistic wage premium for college graduates, just for their job finding rate to be higher than that of workers with less than college education, as empirically observed. In order to match the so much lower unemployment rate of college graduates, the required log wage premium is roughly equal to 90%, while in the data is equal to 50%. Unless there are significant differences in the matching technology at the higher end of the labor market, this finding also suggests that employment at the higher end of the skill distribution remains high, due to temporary employment in lower job levels and upgrading via on-the-job search.

The rest of the paper is organized as follows. Section 2 outlines the stochastic model in which aggregate productivity fluctuates over time. Section 3 defines a steady steady equilibrium and uses analytic results to provide a more rigorous intuition for the results of the stochastic model that follow. Section 4 analyzes the stochastic model outlined in section 2. I calibrate and numerically solve the model and discuss its implications. In section 5, I consider the implications of the model without cross-skill matching, in which workers can direct their search towards the jobs their best suited. Section 6 gives a brief description of related literature, and finally section 7 concludes

2 The Model

2.1 Main Assumptions

The labor force is composed by two types of workers: a fraction δ is low-skill (l) and the remaining $(1 - \delta)$ is high-skill (h). Similarly, vacancies can be either high-skill (h) or low-skill (l), but the mix is determined endogenously. High-skill workers qualify for both types of vacancies, whereas low-skill workers can only perform low-skill jobs. Accordingly, a low-skill worker can be either employed and producing in a low-skill job or unemployed and searching, while a high-skill worker can be in any if the following three states: employed and producing in a high-skill job, unemployed and searching, and employed and producing in a low-skill job, but simultaneously searching for another job. I label a worker in the latter state as over-qualified job seeker.

Each firm has at most one job, which can be either vacant and searching for candidates or filled and producing. The mass of each type of vacancy is determined endogenously by a free-entry condition. The exogenous component of job destruction follows a Poisson process with arrival rate s_j, which is assumed to be specific to each type of worker. Notice that even if the arrival rate s is common to both types, the effective job destruction rate of low-skill jobs is higher due to on-the-job search by over-qualified workers. Whenever a match is destroyed the job becomes vacant and bears a maintenance cost c_l, specific to its type.

Wages are chosen to divide the surplus of a match between a worker and a firm at each point in time, according to their relative bargaining powers, in line with Nash bargaining. With γ being the workers'bargaining power, a share γ of the surplus goes to the workers, while a share $1-\gamma$ goes to the firm. I also make the standard assumptions that workers are risk neutral, the interest rate r is constant, and that time is discrete.

The productivity of each match is assumed to be the product of a stochastic aggregate component y, and a match specific component α_{ij}, when a job of type $j = (h, l)$ is filled by a worker of type $i = (h, l)$. The aggregate component is assumed to follow a discrete-state Markov process. The vector of possible aggregate productivity realizations is given by $\bar{y}$ and the elements of the transition matrix Π are given by $\pi_{ij} = prob\{y' = \bar{y}_j \setminus y = \bar{y}_i\}$.

When unemployed the worker enjoys a utility flow b_j, which can be interpreted as the opportunity cost of working.[7] The condition that ensures a match is formed in equilibrium is simply that workers are more productive when employed than when unemployed, i.e., $y\alpha_{ij} > b_j$, $\forall\, i, j$, which as it will be clarified below, ensures that the surplus each match generates is positive. As long as $y\alpha_{hl} > b_h$, it is optimal for unemployed high-skill workers to accept low-skill jobs, since they retain their chances of finding a high-skill job by continuing to search while employed. I assume for simplicity that the rate at which workers meet vacancies is the same regardless of whether the workers is employed or not.

Since high-skill workers are best suited for high-skill jobs, their productivity is higher when they are suitably matched than when are over-qualified. This implies $y\alpha_{hh} - b_h > y\alpha_{hl} - b_h$. Moreover, I assume that high-skill workers are at least as productive as low-skill workers in low-skill jobs, so that $\alpha_{hl} \geq \alpha_{ll}$. However, the productivity of over-qualified workers net of unemployment benefits, is lower than that of correctly matched low-skill workers. The underlying assumption is that $b_h > b_l$ so that $y\alpha_{ll} - b_l \geq y\alpha_{hl} - b_h$. This assumption assumption is convenient because it ensures without any additional restrictions in the parameter space that firms with low-skill vacancies are better off hiring low-skill instead of over-qualified workers, since the latter are more likely to quit. If instead I assumed that the net productivity of over-qualified workers is higher than correctly allocated low-skill workers, I would have to specify additional restrictions to ensure that the negative quit effect dominates the positive productivity effect.[8] Finally, since low-skill workers do not

[7]Since there is no government or any form of taxation in the model, I avoid naming b_j as unemployment benefit, which in reality would only be one of the factors that determine b_j. A variety of additional factors could influence a worker's opportunity cost of working, including the value attributed to leisure, spousal income, and the value of home production.

[8]In section 4 the model is calibrated to match several U.S. facts. In calibrating the values for productiv-

have the minimum required skills to perform high-skill jobs, the underlying assumption is that $y\alpha_{lh} - b_l \leq 0$.

2.2 Matching and Timing

Firms and workers meet each other via a matching technology $m(v, z)$, where $v = v_h + v_l$ is the number of high- and low-skill vacancies, and $z = u_h + u_l + e_{hl}(1 - s_h)$ is the number of job seekers; u_h and u_l denote the number of high- and low-skill unemployed, and $e_{hl}(1 - s_h)$ the number of over-qualified job seekers who survive separation. The function $m(\cdot, \cdot)$ is strictly increasing in its arguments, and exhibits constant returns to scale. This allows me to write the flow rate at which workers meet vacancies as $m(\theta)$, where $\theta = \frac{v_h + v_l}{u_h + u_l + e_{hl}(1-s)}$ captures the degree of labor market tightness.

I assume that workers cannot distinguish ex-ante the vacancy types. Therefore, they cannot direct their search towards a specific type of vacancy. Consequently, low-skill workers encounter low-skill vacancies with probability per unit of time that is proportional to the fraction of low-skill vacancies. Similarly, high-skill workers encounter low- and high-skill vacancies with a probability per unit of time that is proportional to the fraction of low- and high-skill vacancies, respectively. Assuming that $\eta = \frac{v_l}{v_l + v_h}$, the effective matching rate of low-skill workers is $\eta m(\theta)$, while over-qualified workers relocate into high-skill jobs at rate $(1 - \eta)m(\theta)$. Unemployed high-skill workers accept both high- and low-skill jobs, thus their effective matching rate is $m(\theta)$.

The timing within a period is as follows. Let $e = \{e_{hh}, e_{hl}, e_{ll}\}$ be the distribution of employed workers across types of matches, at the beginning of period t. At this point, the realizations of aggregate state become common knowledge to all agents. After the aggregate state is determined, agents produce. Subsequently, some of the existing matches are exogenously destroyed and vacancies are posted by firms to ensure zero profits. Finally, search takes place. Based on the the matching rates specified above, some over-qualified workers switch to high-skill jobs and some unemployed workers find jobs, leading to the following distribution of employed workers at the beginning of period $t + 1$

$$
\begin{aligned}
e'_{ll} &= e_{ll}(1 - s_l) + \eta m(\theta) \left[\delta - e_{ll}(1 - s_l)\right] \\
e'_{hh} &= e_{hh}(1 - s_h) + (1 - \eta) m(\theta) \left[1 - \delta - e_{hh}(1 - s_h)\right] \\
e'_{hl} &= e_{hl}(1 - s_h) + \eta m(\theta) \left[1 - \delta - (e_{hl} + e_{hh})(1 - s_h)\right]
\end{aligned}
$$

ities, and opportunity costs of working, I do not impose any restrictions. I choose the parameters implied by the data. Still, the calibrated values are consistent with these assumptions.

$$-(1-\eta)m(\theta)e_{hl}(1-s_h) \quad (1)$$

The rate at which a firm meets a job seeker of any type is equal to $q(\theta) = m(1, \frac{1}{\theta})$, which is decreasing in θ and exhibits the standard properties: $lim_{\theta\to 0}^{q(\theta)} = lim_{\theta\to\infty}^{\theta q(\theta)} = \infty$, and $lim_{\theta\to\infty}^{q(\theta)} = lim_{\theta\to 0}^{\theta q(\theta)} = 0$. To specify the effective matching rates of vacancies, it is convenient to define the fraction of low-skill unemployed and the fraction of unemployed job seekers as

$$\varphi = \frac{u_l}{u_l + u_h}$$
$$\psi = \frac{u_l + u_h}{u_l + u_h + e_{hl}(1-s_h)}$$

Low-skill vacancies match only with unemployed job seekers. An over-qualified worker has no incentive to change employer unless the new employer offers a high-skill job. Accordingly, some firms with low-skill vacancies meet over-qualified workers who refuse to match. Therefore, low-skill vacancies match with low-skill workers at rate $\psi\varphi q(\theta)$, and with high-skill workers at rate $\psi(1-\varphi)q(\theta)$. Likewise, employers with high-skill vacancies do not hire the low-skill workers they meet. Consequently, high-skill vacancies match only with either over-qualified or unemployed high-skill workers, and thus, their effective matching rate can be written as $(1-\psi\varphi)q(\theta)$.

2.3 Value Functions

To describe the value functions I let $x = \{y, e\}$ denote the vector of state variables, and adopt the following notation. U_i is the value of unemployment, V_j is the value of a vacancy to the firm, W_{ij} is the value of employment to the worker, and J_{ij} is the value of a filled job to the firm. In all cases, i denotes the type of worker and j the type of job. Moreover, in what follows, the primes denote the values next period, and $\beta = \frac{1}{1+r}$ the discount factor.

2.3.1 Workers' Values

The value of unemployment to a low-skill worker satisfies

$$U_l(x) = b_l + \beta E_{x'|x}\left[\eta(x)m\left(\theta(x)\right)W_{ll}(x') + \left(1-\eta(x)m(\theta(x))\right)U_l(x')\right] \quad (2)$$

The interpretation of this expression is straightforward. The value of unemployment to a low-skill worker is equal to payoff in current period, b_l, plus the present value of the expected value next period. The latter is given by the probability the worker finds a job, $\eta m(\theta(x))$, times the value of employment in a low-skill job, $W_{ll}(x')$, plus the probability the worker remains unemployed, $(1 - \eta m(\theta(x)))$, times the corresponding value, $U_l(x')$. The rest of the value functions take a similar form. Each one is equal to the flow output or cost of the corresponding state, and the present value of the expected value next period. The expectations operator $E_{x'|x}$ depends on the transition matrix of aggregate productivity Π, and the transition equations described in (1).

Given that high-skill workers accept both types of jobs, the value of unemployment to a high-skill worker satisfies

$$U_h(x) = b_h + \beta E_{x'|x} \left[\begin{array}{c} m(\theta(x))[\eta(x)W_{hl}(x') + (1 - \eta(x))W_{hh}] \\ +(1 - m(\theta(x)))U_h(x') \end{array} \right] \tag{3}$$

The values of being employed in high- and low-skill jobs,to high- and low-skill workers, respectively, satisfy

$$W_{hh}(x) = w_{hh}(x) + \beta E_{x'|x} \left[s_h U_h(x') + (1 - s_h) W_{hh}(x') \right] \tag{4}$$

$$W_{ll}(x) = w_{ll}(x) + \beta E_{x'|x} \left[s_l U_l(x') + (1 - s_l) W_{ll}(x') \right] \tag{5}$$

The value of being employed in a low-skill job to a high-skill worker is given by

$$W_{hl}(x) = w_{hl}(x) + \beta E_{x'|x} \left[\begin{array}{c} s_h U_h(x') + (1 - s_h) W_{hh}(x') \\ (1 - s_h)(1 - \eta(x)) m(\theta(x)) \left[W_{hh}(x') - W_{hl}(x') \right] \end{array} \right] \tag{6}$$

where $w_{ij}(x)$ denotes the wage rate in each case. The value of being over-qualified incorporates in addition the expected gain from on-the-job search. This is given by the last term in the bracket, which is interpreted as follows: given that the match survives job destruction with a probability $(1 - s_h)$, the worker meets a high-skill vacancy with a probability $(1 - \eta(x))m(\theta(x))$, and obtains $[W_{hh}(x') - W_{hl}(x')]$ from switching jobs.

2.3.2 Firms' Values

The values of high- and low-skill jobs filled by the suitable type of worker satisfy

$$J_{hh}(x) = y\alpha_{hh} - w_{hh}(x) + \beta E_{x'|x} \left[s_h V_h(x') + (1 - s_h) J_{hh}(x') \right] \tag{7}$$

$$J_{ll}(x) = y\alpha_{ll} - w_{ll}(x) + \beta E_{x'|x} \left[s_l V_l(x') + (1 - s_l) J_{ll}(x') \right] \tag{8}$$

while the value of a low-skill job filled by an over-qualified worker incorporates in addition the loss due to endogenous separations.

$$J_{hl}(x) = y\alpha_{hl} - w_{hl}(x) + \beta E_{x'|x} \left[\begin{array}{c} s_h V_l(x') + (1 - s_h) J_{hl}(x') \\ -(1 - s_h)(1 - \eta(x)) m(\theta(x)) [J_{hl}(x') - V_l(x')] \end{array} \right] \tag{9}$$

This is captured by the term $(1-s_h)(1-\eta(x))m(\theta(x))[J_{hl}(x')-V_l(x')]$. If the match is not exogenously destroyed, the worker continues searching on the job, in which case the match is endogenously destroyed with a probability $\eta m(\theta)$. Finally, the values of opening high- and low-skill vacancies are given by

$$V_h(x) = -c + \beta E_{x'|x}\left[(1-\psi\varphi)q(\theta(x))(J_{hh}(x') - V_h(x')) + V_h(x')\right] \tag{10}$$

$$V_l(x) = -c + \beta E_{x'|x}\left[\begin{array}{c} \psi\varphi q(\theta(x))[J_{ll}(x') - V_l(x')] \\ +\psi(1-\varphi)q(\theta(x))[J_{hl}(x') - V_l(x')] + V_l(x') \end{array}\right] \tag{11}$$

2.3.3 Surpluses

Given that the worker and the firm share the surplus in fixed proportions with γ being the worker's share, the wage $w_{ij}(x)$, satisfies the following Nash bargaining conditions

$$\begin{aligned} W_{ij}(x) - U_j(x) &= \gamma S_{ij}(x) \\ J_{ij}(x) - V_i(x) &= (1-\gamma)S_{ij}(x). \end{aligned} \tag{12}$$

where S_{ij} denotes the surplus of the match, and is defined as

$$S_{ij}(x) = W_{ij}(x) + J_{ij}(x) - U_i(x) - V_j(x) \tag{13}$$

The surplus each match generates, reflects the value to the worker and the firm, net of the values each of them would obtain if they remained unmatched. Substituting the value functions together with the Nash bargaining conditions in (12), into the surplus expression above yields

$$S_{ll}(x) = y\alpha_{ll} - b_l + \beta E_{x'|x}[(1-s_l)S_{ll}(x') - \gamma\eta(x)m(\theta(x))S_{ll}(x')] \tag{14}$$

$$S_{hh}(x) = y\alpha_{hh} - b_h + \beta E_{x'|x}\left[\begin{array}{c} (1-s_h)S_{hh}(x') - \gamma(1-\eta(x))m(\theta(x))S_{hh}(x') \\ -\gamma\eta(x)m(\theta(x))S_{hl}(x') \end{array}\right] \tag{15}$$

$$S_{hl}(x) = y\alpha_{hl} - b_h + \beta E_{x'|x}\left[\begin{array}{c} (1-s_h)S_{hl}(x') - s_h\gamma(1-\eta(x))m(\theta(x))S_{hh}(x') \\ -\gamma\eta m(\theta(x))S_{hl}(x') \\ -(1-s_h)(1-\eta(x))m(\theta(x))S_{hl}(x') \end{array}\right] \tag{16}$$

The surplus of a low-skill job filled by a low-skill worker, $S_{ll}(x)$, takes the standard form. The first term gives the productivity of the match net of the opportunity cost of working, b_l. The first term in the bracket gives the expected surplus given that the match survives to the next period, and the second term the cost to the worker for giving up search.

Once employed, the worker gives up the opportunity to match with a low-skill vacancy with a probability $\eta(x)m(\theta(x))$ and gain a share γ of the resulting surplus $S_{ll}(x')$. Therefore, the value of this opportunity is subtracted from the surplus. When it comes to the surplus of a high-skill job, $S_{hh}(x)$, the only difference is that when matched with a high-skill job, high-skill workers give up searching for both high- and low-skill jobs. Consequently, the additional term $\gamma(1-\eta(x))m(\theta(x))S_{hh}(x')$, which reflects the value of the opportunity to match with a high-skill vacancy, is also subtracted from the surplus.

The surplus of a low-skill job filled by an overqualified worker, $S_{hl}(x)$, takes a slightly different form. Over-qualified workers search on-the-job. Unless the job is destroyed before a high-skill job comes along, on-the-job search is as effective as off-the-job search. The chances of meeting a high-skill vacancy when they are employed are the same as when they are unemployed. Accordingly, only a fraction s_h of the corresponding value is subtracted, and this is captured by the second term in the bracket. The third term in the bracket represents the cost of giving up the opportunity to match with a low-skill vacancy, as in (14) and (15). Finally, the last term in the bracket represent the cost to the firm for employing a worker who is searching on the job. Given that the match survives to the next period, with a probability $(1-\eta(x))m\left(\theta(x)\right)$, the over-qualified worker quits to a high-skill job, in which case $S_{hl}(x')$ is lost.

By just looking at the surplus expressions above, it can be easily verified that an increase in the meeting rate $m(\theta)$ lowers the surpluses of all jobs, as in the standard model. Intuitively, a higher meeting rate raises the workers value of unemployment. This in turn lowers the surplus of jobs, because firms need to compensate the worker for giving up being unemployed. It is also straightforward to verify that upgrading the skill composition of vacancies (i.e., lowering η), raises $S_{ll}(x)$, but lowers $S_{hh}(x)$, as long as $S_{hh}(x') \geq S_{hl}(x')$. The intuition is similar; when high-skill vacancies are relatively more abundant, high-skill workers can more easily ovoid temporary employment in low-skill jobs, which generate lower surplus and thus offer lower wages. This in turn raises the value of unemployment for high-skill workers, thus lowering the surplus of high-skill jobs. The opposite holds for low-skill workers; when high-skill vacancies are relatively more abundant, they have more difficulty finding jobs. Therefore, the value of unemployment is lower for low-skill workers, and therefore, the surplus is higher.

The impact of a fall in η on $S_{hl}(x)$ is more cumbersome to determine. On the one hand, with high-skill vacancies relatively more abundant, over-qualified workers can more easily upgrade to high-skill jobs. Hence, as endogenous quits are more likely, the surplus declines. On the other hand, since over-qualified workers search on the job, by accepting low-skill jobs, they only give up the opportunity to match with a low-skill vacancy. When η is lower, the value of this opportunity is also lower. Therefore, the surplus of the job is higher. The overall impact depends on which of the two effect dominates, making it difficult to establish it analytically.

2.4 Equilibrium

Given free entry, $V_i(x) = 0$ should be satisfied in equilibrium. Therefore, $E_{x'|x}V_i(x) = 0$ must also hold in equilibrium. Applying these conditions to (10) and (11) together with the Nash bargaining conditions in (12) yields the following free-entry conditions for low- and high-skill vacancies, respectively

$$(1-\gamma)\beta E_{x'|x}\left[(\psi\varphi S_{ll}(x') + \psi(1-\varphi)S_{hl}(x'))\right] = \frac{c_l}{q(\theta(x))} \tag{17}$$

$$(1-\gamma)\beta E_{x'|x}[(1-\psi\varphi)S_{hh}(x')] = \frac{c_h}{q(\theta(x))} \tag{18}$$

The free-entry conditions are such that in equilibrium, the expected profit from filling a vacancy (left hand side) is equal to the costs of keeping the vacancy unfilled (right hand side), and implicitly define $\theta(x)$ and $\eta(x)$.

More formally, the equilibrium is given by a vector $\{\theta, \eta\}$ that for each realization of aggregate state, y, and distribution of employment, $e = \{e_{hh}, e_{hl}, e_{ll}\}$, satisfies the following: (i) the three types of matches are formed voluntarily, i.e., $y\alpha_{hh} > b_h$, $y\alpha_{ll} > b_l$, and $y\alpha_{lh} > b_h$; (ii) the two free entry conditions in (17) and (18) are satisfied so that the values of maintaining low- and high-skill vacancies are zero; and (iii) the state variables e_{hh}, e_{hl}, and e_{ll} are determined by the set of flow equations (1). With the characterization of the equilibrium I complete the description of the model.

Before digging deeper into the mechanisms that generate the results of the model a few words are in line regarding the properties of the equilibrium. First, notice uniform changes in the expected profits of all types of vacancies require offsetting changes in market tightens, θ, while unequal changes in the expected profits of high- and low-skill vacancies require adjustments in the equilibrium value of η (i.e., adjustments in the skill mix of vacancies) to keep the values of both types of vacancies equal to zero.

Observe also that unlike the standard model, shifts in the skill composition of job seekers affect the two sectors unevenly, thus altering the skill composition of vacancies opened. An increase in the fraction of unemployed job seekers, ψ, (or equivalently, a decline in the fraction of over-qualified job seekers) raises the expected surplus of low-skill jobs, while it lowers the expected surplus of high-skill jobs. It follows that an increase in the fraction of over-qualified job seekers induces firms to open relatively more high-skill vacancies, making it more difficult for low-skill workers to find suitable jobs. Moreover, when $S_{ll}(x') - S_{hl}(x') > 0$, it can be easily verified by rearranging terms in (17) that an increase in the fraction of high-skill job seekers (i.e., a reduction in $\psi\phi$), lowers the expected surplus of low-skill vacancies, but raises the expected surplus of high-skill vacancies. Hence, if over-qualified workers generate lower surplus than low-skill workers, a rise in the fraction of high-skill job seekers, discourages firms from opening low-skill vacancies.[9]

3 Steady State

In this section I first solve for a unique steady state equilibrium, and then I illustrate the impact of a permanent decline in aggregate productivity y on market tightness θ and skill composition of vacancies as captured by η. The proofs of the results presented in this section are given in the Appendix. The purpose of this analytic exercise is to provide a more rigorous intuition for the results of the numerical analysis that follow. Evidently, this exercise is limited, because it does not provide insights into the dynamic associated with shocks.[10] The task of characterizing the dynamic responses of variables to temporary shocks is taken in subsequent sections.

To keep calculations tractable, I consider the case $s_h = s_l = s$, $b_h = b_l = b$, and $\alpha_{hl} = \alpha_{ll}$. As it will become clear below, this choice of parameters ensures that high-skill workers are better off when they are suitably matched as opposed to over-qualified, and that firms with low-skill vacancies are better off hiring low- as opposed to high-skill workers.

[9]To establish these results analytically one has to prove that η lowers the expected surplus of low-skill vacancies, as captured by the left-hand-side of (17), so that when the relative surplus of high-skill jobs rises, firms respond by lowering η. However, as mentioned earlier, this is not a straightforward task. Although an increase in η lowers $S_{ll}(x)$, the impact on $S_{hl}(x)$, can go either way. The illustrative steady-state exercise that follows specifies parameter restrictions, which ensure that firms respond by shifting the vacancy mix towards high-skill vacancies, when high-skill vacancies become relatively more profitable. Moreover, the simulations of the calibrated stochastic model that follow, confirm this result.

[10]A steady state analysis can at most be illustrative in this model; it cannot stand alone. The presence of asymmetric matching and on-the-job search requires that the endogenous variables depend on the distribution of employment across types of matches in a complicated non-monotonic way, making it difficult to establish that the model exhibits global asymptotic stability. In the simulations that follow, for any initial distribution, the endogenous variables always converge to the same values, suggesting that the system is globally stable. However, global stability cannot be guaranteed analytically.

Assuming continuous time, the steady state free entry conditions along which the value of opening a vacancy is equal to zero, are given by the set of equations below.

$$(1-\gamma)[\psi\varphi S_{ll} + \psi(1-\varphi)S_{hl}] = \frac{c_l}{q(\theta)} \tag{19}$$
$$(1-\gamma)(1-\psi\varphi)S_{hh} = \frac{c_h}{q(\theta)} \tag{20}$$

where

$$S_{ll} = \frac{y\alpha_{ll} - b}{(r+s+\gamma\eta m(\theta))} \tag{21}$$
$$S_{hl} = \frac{y\alpha_{ll} - b}{(r+s+\gamma\eta m(\theta) + (1-\eta)m(\theta))} \tag{22}$$
$$S_{hh} = \frac{(y\alpha_{hh} - b)}{(r+s+\gamma(1-\eta)m(\theta))} - \gamma\eta m(\theta)S_{hl} \tag{23}$$

By just looking at (21) and (22) one can be easily verify that $S_{ll} \geq S_{hl}$, so that low-skill jobs are better off hiring low- as opposed to high-skill workers. In addition, by rearranging terms in (23) after substituting for S_{hl}, it is easy to show that $S_{hh} \geq S_{hl}$, when $\alpha_{hh} \geq \alpha_{hl}$. Therefore, over-qualified workers have an incentive to search on-the-job. Sufficient parameter restrictions to ensure the steady state equilibrium is unique, are:

i) $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)}\left[\frac{\delta}{(1-\delta)} + \frac{2\gamma}{\gamma+1}\right] \geq \frac{c_l}{c_h}$

ii) $\gamma \geq \frac{1}{2}$ and $\delta \geq \frac{1}{2}$

iii) $\frac{(y\alpha_{ll}-b)}{(y\alpha_{hh}-b)} \leq \gamma$

The first condition ensures that $\psi\phi$ decreases when θ increases, and is sufficient to establish that the value of low-skill vacancies declines with θ. Conditions *ii*) and *iii*) ensure that a higher η increases the surplus of low-skill vacancies (left-hand-side of (19)), but lowers the surplus of high-skill vacancies (left-hand-side of (19)).[11] Therefore, if for some exogenous reason the surplus of high-skill jobs increases relative to the surplus of low-skill jobs, η must decline, for the free-entry conditions to be satisfied in equilibrium. Under these conditions, the free-entry conditions (19) and (20) have opposite slopes in the [η, θ] plane, and the equilibrium is characterized by the intersection of the two loci as shown in Figure 1

[11] An increase in η lowers value of unemployment to high-skill workers, U_h, and increases the value of unemployment to low-skill workers, U_l. Since the value of unemployment is subtracted from the surpluses of jobs, a rise in η increases S_{hl} and lowers S_{ll}, both of which enter the left hand side of (19) positively. Therefore, the impact of a rise in η on (19) is not clear-cut. Notice also from (23) that a fraction of S_{hl} is subtracted from S_{hh}. Therefore, the impact on the surplus of high-skill vacancies is not straightforward either. Condition *ii*) ensures that the decline in S_{ll} dominates the increase in S_{hl}, so that the left-hand-side of (19) declines. Condition *iii*) ensures that positive impact of the fall in U_h on S_{hh}, dominates the negative impact of the rise in S_{hl}, so that left-hand-side of (20) increases.

Notice that a reduction in y lowers the surpluses of both types of jobs. Therefore, both loci shift down in response to a rise in y, and the equilibrium value of θ declines. Intuitively, when aggregate productivity is low, each job is proportionally less productive, thus firms post fewer vacancies per job seeker. The impact on η depends on which of the two types of jobs is hurt the most. In other words, it depends on which of the two loci shifts down by more. To determine this, I first take the ratio of the low-skill free-entry condition to the high-skill free-entry condition, as below,

$$\frac{\psi\varphi}{(1-\psi\varphi)}\frac{S_{ll}}{S_{hh}} + \frac{\psi(1-\varphi)}{(1-\psi\varphi)}\frac{S_{hl}}{S_{hh}} = \frac{c_l}{c_h} \tag{24}$$

and then evaluate the derivative with respect to y. As proven in the appendix this derivative is positive. A reduction in aggregate productivity has a stronger negative impact on the value of low-skill vacancies. Therefore, the free-entry condition for low-skill vacancies shifts down relatively more so that both η and θ decline, as illustrated in Figure 2. However, θ declines less than it would, if the skill mix of vacancies remained unchanged. The reason low-skill jobs are hurt the most, is simply that the net productivity of low-skill jobs, $(y\alpha_{ll}-b)$ is lower than the net productivity of high-skill jobs, $(y\alpha_{hh}-b)$. As a consequence, at lower values of y, the percentage gap between the productivity of the job and the opportunity cost of employment declines more for low- than for high-skill jobs, pushing the relative surplus of high-skill jobs up.[12]

Consequently, the burden of a permanent reduction in aggregate productivity falls more heavily on low-skill workers. The reduction in θ implies that high-skill workers have more difficulty finding vacancies, because $m(\theta)$ declines. However, in addition to the reduction in $m(\theta)$, low-skill workers bear the reduction in η. Hence, they suffer a higher reduction in their matching rate relatively to high-skill workers, implying a relatively higher increase in low-skill unemployment in recessions.

As can be verified in the results presented in the appendix, the higher the difference between the productivity of high- and low-skill jobs, the larger the decline in the relative profitability of low-skill jobs in recessions. Hence, the larger the decline in η, since the low-skill locus shifts down by relatively more. This leads to the conclusion that the higher

[12] It is important to point out that an additive aggregate productivity shock (i.e. $y + a_{ij}$ instead of $y\alpha_{ij}$) would imply an even higher increase in the relative surplus of high-skill vacancies and thus an even higher increase in η. Moreover, this result is not sensitive to the assumption that b is the same for both types of workers. Assuming that high-skill workers generate b_h while unemployed and low-skill workers generate b_l while unemployed, the same result would still hold as long as the net productivity of high-skill jobs $(y\alpha_{hh} - b_h)$ is greater than the net productivity of low-skill jobs $(y\alpha_{ll} - b_l)$.

the difference in the profits of high- and low-skill vacancies, the more heavily the burden of recessions falls on low-skill employability.

A conclusion regarding the impact of a fall in y on the number of over-qualified high-skill workers cannot be reached based on this analytic result alone. A fall in y implies that high-skill workers encounter low-skill vacancies less frequently, as $\eta m(\theta)$ declines. However, if the rise in high-skill unemployment due to the fall in $m(\theta)$ is sufficiently high, then the number of over-qualified workers may still rise. Moreover, the transition to a new steady state following a negative productivity shock, may involve a rise in the fraction of unemployed job seekers, which as mentioned earlier, encourages firms to open relatively more low-skill vacancies. Hence, the downgrading in the vacancy mix, together with the rise in unemployment, may entail higher over-qualification rates in recessions.

For now it is enough to note that aggregate shocks have uneven consequences on the two types of workers. Firms face the choice of which type of vacancy to open and how many vacancies to open. As the relative profits of the two types of jobs change with changes in aggregate productivity, firms respond accordingly by changing the vacancy mix. Adjusting the vacancy mix towards more high-skill vacancies acts as an insulating mechanism that limits the scope of cutting down on the number of vacancies opened, in response to negative aggregate shocks. That is, by shifting the burden of recession on low-skill employability, firms keep the overall number of vacancies per job seeker higher than what it would be if the vacancy mix remained unchanged. Given that high-skill workers qualify for both a wider range of job types, and search on the job is manageable, high-skill employability is less vulnerable to changes in the vacancy mix. On the contrary, low-skill workers who qualify only for low-skill jobs, are subject to unfavorable shifts in the vacancy mix in recessions.

4 The Stochastic Model

I now proceed with characterizing the stochastic version of the model outlined in section 2. I first describe the calibration of the model, and I subsequently simulate the model and describe the dynamic evolution of key variables: high- and low-skill exit rates from unemployment, job-to-job transition rate, over-qualification rate, and high- and low-skill unemployment rates. The calibration of the model is summarized in Table 1 (to be added), and the results of the simulations are summarized in Figure 5.

4.1 Calibration

I consider the high-skill type as representing workers who hold at least a college degree. I therefore set the proportion of high-skill workers to d=0.25, which based on the March CPS Annual Demographic Survey Files for the period from 1964 to 2003, equals the average proportion of US labor force that holds a college degree or more. I choose the model period to be one quarter and therefore set the discount rate to r=0.012. For the matching function I make the standard choices. I assume a Cobb Douglas functional form so that $m = z^a v^{1-a}$ and choose an elasticity parameter $a = 0.4$, which lies at the lower range of estimates reported in Petrongolo and Pissarides (2001). I also make a standard choice for the worker's bargaining power. I assume that workers and firms split the surplus equally, i.e., $\gamma = 0.5$.

Following the literature, I select values for the separation rates, s_h and s_l, which are higher than the empirical measures of transition rates from employment to unemployment, to take into account workers who exit the labor force, but whose behavior is similar to those counted as unemployed.[13] Blanchard and Diamond (1990) show that in the US, the "want-a-job" pool in the stock of those not in the labor force is roughly equal to the stock of unemployed. Moreover, they document that only half of the average flow into employment comes from unemployment, with the other half coming from people classified as not in the labor force, signifying that "out of the labor force" job seekers also take part in matching. Assuming that all people classified as out of the labor force participate in the matching process sets an upper bound to the value of the separation rate, which can be computed by adding together the flows from employment to unemployment and out of the labor force. A lower bound can be computed by looking only at flows to unemployment, assuming that only those classified as unemployed search for jobs. To calculate these upper and lower bounds, I use the monthly estimates of transition rates from employment to unemployment and out of the labor force, for college and non-college graduates, reported in Nagypal (2004). After converting the monthly estimates into quarterly frequencies, I find that s_h should lie

[13] Since Clark and Summers (1979) it became eminent that the distinction between the pool of unemployed and the pools of those out of the labor force is fuzzy, with many workers going back an forth between the two states. This observation has been emphasized more recently in Blanchard and Diamond (1990), and Cole and Rogerson (1999), among others, and encouraged empirical research on the "true" job finding rate, which accounts for non-unemployed job seekers. See for instance, Hall (2005). Moreover, a number of studies that calibrate search matching models, take this observation into account. See for instance, Krause and Lubk (2006a, 2006b),and Den Haan et al. (2000).

in the range [0.013-0.041] and s_l in the range [0.032-0.077].[14] I chose to set $s_h = 0.03$ and $s_l = 0.07$, which puts more weight on low-skill separations, and results in an average separation rate in the model of 0.06, which is line with CPS estimates of Hall (2005), when roughly half of the flows from employment to out of the labor force are flows into a job seeking state.[15]

For the parameter values for job creation costs I construct an upper bound as follows. According to Hamermesh (1993), in 1990 average recruitment and training costs in the US represent about on-sixth of average annual labor earnings. Moreover, the job creation costs cannot be too large relative to aggregate output in the model. The standard upper bound in the literature is 5% of output devoted in job creation activities. Based on these two observations, I set kl=0.13 and kl=0.22, which are roughly equal to one third of quarterly low- and high-skill wages, respectively, when the latter are suitably matched with high-skill jobs. With these parameter values, the overall vacancy costs simulated by the model are 5% of simulated output.

I next turn to the calibration of high- and low-skill productivities, α_{hh} and α_{ll}, the productivity of over-qualified high-skill workers, α_{hl}, and the opportunity costs of working, b_h and b_l. These parameters are selected to match statistics from the simulated data to empirical measures of, i) wage differences between college educated and non-college educated workers, ii) wages differences between over-qualified and correctly matched workers, ii) average job finding rate, and iv) unemployment rates of workers with college and less than

[14] As far as I know, estimates of separation rates by education can only be found in Fallick and Fleischman (2001) who uses the basic monthly survey of the CPS for the period between February 1994 and December (2000), and Nagypal (2004) who expands the period to January 2004. Using average employment shares by education, computed from the March CPS Annual Demographic Survey files, and Nagypal's estimates, I find the average monthly flows from employment to unemployment as a share of employment to be approximately 0.6% for college graduates and 1.4% for workers without a college degree. When adding also the flows from employment to out of the labor force the corresponding measures are approximately 1.9% and 3.6%, respectively. By counting paths in a probability tree, the probability of observing someone who had a job a quarter ago not having a job, can be computed as: $s_m[(1-f_m)^2+f_m s_m]+(1-s_m)[s_m(1-s_m)+s_m(1-f_m)]$ were s_m is the monthly separation rate and and f_m the monthly job finding probability. I am grateful to Bruce Fallick for providing me with his estimates of monthly job finding rates by education, and thus enabling me to calculate the quarterly upper and lower bounds of separation rates.

[15] Note that the assumed separation rates do not account for job-to-job transitions. For high-skill workers job-to-job transitions in the form of upgrading to higher job levels, are endogenous in the model. For low-skill workers they are not. Still, since the focus of the analysis is unemployment differences across skill-groups, I choose not to include them in separation rates, because workers who directly move into a new job are not accounted as unemployed.

college education. To match these statistics, I set $\alpha_{ll} = 0.4$, $\alpha_{hh} = 0.68$, and $\alpha_{hl} = 0.6$. The values for the opportunity costs of employment are set to $b_h = 0.53$, and $b_l = 0.28$, which are less than the simulate average high- and low-skill wages, respectively. Below I discuss my choice of relevant targets.

I begin with my choice of target for the wage difference between workers with college and less than college eduction. Based on the March CPS, Autor et al. (2007) find that the college-plus to high school log wage premium (i.e. the average log wage ratio of college to high school graduates) ranges form 0.4 to 0.65 in the period between 1963 to 2005. This implies an average log wage premium of approximately 0.5.[16] The low-skill group in the model is not restricted to high-school graduates only; it also includes workers with some college education and workers with less than high-school education. However, with no more than 0.25 of employed non-college graduates having some college education, I consider an average log wage premium of 0.5 as a fair target.

The productivity of mismatches is unobservable and rather difficult to infer due to the lack of direct empirical evidence. My choice of this parameter was guided by evidence on wage differentials between overeducated and correctly matched workers. For the US, Sicherman (1991) finds that overeducated workers earn more than their co-workers who are not overeducated, but less than similar workers with the same level of schooling that work in jobs that require their actual level of schooling (i.e. correctly allocated workers). In particular, the wage rate of overeducated workers is on average 5% lower than that of correctly allocated worker. Considering this as a lower bound for the the wage difference, I choose the value of α_{hl} that implies that the wage of over-qualified workers is 10% lower than the wage of correctly allocated high-skill workers.

The job finding rate I choose to target, incorporates out of the labor force job seekers, in line with my choice of separation rates. I make use of the Hall (2005) estimates who incorporates this group into the group unemployed. Hall took advantage of the expanded unemployment rate series, available from the BLS starting in 1994, which includes those classified as discourage workers who want a job but believe a job is unavailable for several reasons, and those marginally attached to the labor force, who would indicate a likelihood of returning to the labor force in the near future. The series was approximated for earlier years, by regressing the expanded series to the standard unemployment rate for the years 1994 through 2004 and using the fitted value for the years before. After constructing the expanded series, the job finding rate was calculated as the ratio of new hires to the number

[16] Estimates in the same range can also be found in Wheeler (2005), and Baldwin and Gain (2000).

of job seekers, as measured by the expanded unemployment rate series. For the period from 1964 to 2003 the estimated monthly job finding rate averages to 0.28, which works out to an average quarterly job finding rate of about 0.6.

Consistent with my choice of separation and job finding rates, the targeted unemployment rates are higher than the official empirical measures, to take into account workers classified as out of the labor force who participate in the matching process. Unfortunately, the series of marginally attached or discouraged workers in the BLS is not available by education. Therefore, a similar methodology as in Hall (2005) of imputing the expanded unemployment rate series for earlier years using the years after 1994, cannot implemented to construct expanded unemployment rate series by education. Instead, guided by the Blanchard and Diamond (1990) finding that the want-a-job group is roughly equal to the number of unemployed, I approximate the expanded unemployment rates as $\frac{2u}{u+l}$, were u is the number of unemployed and l is the labor force. Based on the March CPS Annual Demographic Survey files from 1964 to 2003, this calculation yields an average unemployment rate of 0.044 for college graduates, and 0.114 for workers with less college education. The resulting average unemployment rate of 0.10 in the model is consistent with the figure in Hall (2005).

Finally, I turn to the calibration of the aggregate productivity process. I approximate through a 9-state Markov chain the quarterly deviations of U.S. GDP for the period between 1964 to 2003, from a linear trend. The estimated autocorrelation coefficient of the standard AR(1) model is 0.9139 and the standard error of the innovation is 0.0084. A recent strand of literature, has documented that the standard model, along the lines of Mortensen and Pissarides (1994), can explain the magnitude of cyclical changes in unemployment only by assuming implausibly large productivity shocks.[17] The reason is that for reasonable calibrations, the magnitude of fluctuations in the vacancy-unemployment ratio is small relative to the fluctuations in the data. The present model, which incorporates asymmetric matching and on-the-job search performs considerably better in this dimension.[18] Still, as it

[17]See e.g., Hall (2005), Shimer (2005) and Constain and Reiter (2005), Pries(2005), Krause and Lubik(2006), Hagedorn and Manovskii (2005).

[18]On-the-job search as an amplification mechanism has been emphasized in Krause and Lubik (2006). However, in a model with homogeneous workers. Pries (2005) shows that when the standard model is extended to allow for worker heterogeneity it exhibits greater volatility. To my knowledge none of the existing studies have incorporated two-sided heterogeneity, asymmetric matching, and on-the-job search at the same time. –I need to document how much better this model performs relative to others in terms of standard deviations—

will be discovered below, it underpredicts the volatility of unemployment. The focus of this paper is on the relative responses of high- and low-skill unemployment rates to the same underlying shock process. Moreover, the magnitudes of the fluctuations in employment are higher when the magnitude of fluctuations in the vacancy-job seekers ratio is higher, but the qualitative implications are still the same. Therefore, the results discussed below are not sensitive to this caveat of the model.

Before I proceed with describing the results of the simulations, note that with regard to surplus differences across jobs, the calibrated stochastic model is similar to the steady-state model in the previous section. With the calibrated productivity values, high-skill workers are more productive than low-skill workers, not only when employed in high-skill jobs, but when employed in low-skill jobs as well. However, given that the calibrated value of b_h is higher than the value of b_l, the net productivity of over-qualified workers is lower than the net productivity of low-skill workers. Hence, in the simulations that follow, over-qualified workers generate lower surplus than suitably matched low-skill workers, despite the much higher separation rate of the latter. In addition, the surplus of high-skill jobs, is by far higher than the surplus of low-skill jobs.

4.2 Simulations

With all the parameter values assigned, I use the free entry conditions given by equations (17) and (18) to find the state-contingent market tightness $\theta(x)$ and fraction of low-skill vacancies $\eta(x)$. I then simulate the model as follows: first, I generate a sequence of random aggregate state realizations; then, starting with the first realization of aggregate state, and an initial distribution of employment $e = \{e_{hh}, e_{hl}, e_{ll}\}$, I use the flow equations in (1) to compute the new distribution of employment at the beginning of the next period; and then I repeat. At the end of each period, I record the values of the variables of interest along the sequence of aggregate state realizations.

Based on the above calibration, on average 85% of vacancies are low-skill. The average matching rate is 0.86 for low-skill vacancies and 0.5 for high-skill vacancies, resulting in average matching rate for firms of 0.8. Therefore, workers meet low-skill vacancies more frequently with 0.59 being the average rate by which workers meet low-skill vacancies and 0.11 the average rate by which workers meet high-skill vacancies. It follows that the significantly lower unemployment rate of low-skill workers can be sustained by some high-skill workers refuging to temporary employment in low-skill jobs. The simulation yields that on average 18.4% of college graduates are over-qualified. This finding is well in line with available evidence. Hecker (1992) and Shelley (1994), find that in 1990, 17.9% of college graduates were employed in "high-school" type jobs. Over-education measures in the same range can also be found for many European countries including the UK and Spain. For instance, Green *et al.* (1999) find that just over 20% of graduates in the UK are genuinely over-educated for their jobs. Oliver and Raymond (2003) show that the proportion of over-educated college graduates in Spain was 21% in 1998.[19]

The resulting average job-to-job flows as a share of employment and total separations are 0.02 and 0.4 respectively. These measures are comparable, but still lower than the corresponding monthly estimates for college graduates, reported in Nagypal (2004), of approximately 0.02, and 0.53, respectively. This is not is not puzzling; on the contrary, this is what one should expect given that the model captures only transitions to higher job levels (i.e. upgrading to jobs with higher skill requirements), and overlooks transitions to jobs of the same level, while such a distinction is not done in the data. Moreover, some caution may be mandated since the data in Nagypal cover only the period from February 1994 to January 2004 in which the U.S. economy has experienced an expansion and only a mild recession. A longer series would cover additional recessions, and the severe contraction at the beginning of the 80s. Therefore, it would probably yield lower averages. Consequently, the available empirical measures can only serve as an upper bound to the simulated measures. This leads me to conclude that the job-to-job quit rate in the model is reasonable, given that it captures only upward transitions, and the calibrated productivity process in the model corresponds to the longer period from 1964 to 2003.

I now turn to the cyclical behavior of the simulated series. To illustrate how accurately the cyclical behavior of the unemployment rates in the model, matches the data, I simulate the model along a series of aggregate productivity realizations that replicates the U.S. GDP log deviations from trend for the period from 1964 to 2003. As shown in Figure 3, with

[19]For over-education measures in Europe in the same range see also....Unfortunately, other empirical measures of over-qualification rates are hard to find for the US, especially referring to college graduates alone. Barlevy(2002) also measures mismatch rates using a question in the PSID asking employed workers whether they have been thinking of getting a new job. However, he measures mismatch rates overall, and not by education. He finds that the fraction of employed workers thinking of getting a new job ranges from 9.6% in 1967 to 17.8% in 1984. In the model, on average 18.6% of employed high-skill workers are over-qualified. I find the simulated over-qualification rate to be well in line with Barlevy's estimates. As I also argue at the introduction, given their higher propensity to find better jobs, and their ability to perform a wider range of job types, skilled workers are more likely to be employed as opposed to unemployed job seekers. Hence, it is reasonable to expect that their mismatch rates are above the average.

9 productivity states the artificial series matches quite well the empirical series. I then compare the simulated unemployment rates to the empirical unemployment rates for the same period.

The empirical series traced in Figure 4 refers to the expanded yearly unemployment rates for college educated and non-college educated workers, constructed as described above. Obviously, the unemployment rate of workers with less than college education fluctuates more than the unemployment rate of college graduates. For the former, the standard deviation of the differences between it and a yearly trend equals 0.021 with differences from trend ranging from -0.307 to 0.0590; the standard deviation of the differences between the unemployment rate of college graduates and a yearly trend is 0.0086, with differences from trend ranging from -0.0126 to 0.0187. Hence, the unemployment rate of workers with less than college education is 2.5 times more volatile than the unemployment rate of college graduates, as measured by the ratio of the standard deviations of differences from trend.

The simulated quarterly high- and low-skill unemployment rates are reported in Figure 5. In line with the data, both unemployment rates are strongly countercyclical, but the low-skill unemployment exhibits much more volatility. Figure 6 compares the deviations from a yearly trend of the empirical unemployment rates to the deviations from a yearly trend of the simulated unemployment rates, after averaging over quarters. Clearly, the magnitude of the simulated deviations is smaller. As mentioned earlier, this corresponds to the general failure of the matching model to match the empirical volatilities. But, in terms of relative volatilities the model performs quite well. The standard deviation of the differences from the low-skill unemployment rate and a trend, is 2.6 times higher than the corresponding standard deviation of the high-skill unemployment rate series, compared to 2.5 times in the data.

The behavior of exit rates from unemployment is also traced in Figure 5. As expected the matching rate, $m(\theta)$, which corresponds to the rate by which high-skill workers exit from unemployment, is procyclical. The rate by which low-skill workers exit unemployment depends also on he skill mix of vacancies. The insights from the analytic exercise above carry over to the stochastic version of the model.[20] In downturns firms open fewer vacancies of both types, but relatively fewer low-skill vacancies. Therefore, the skill mix of vacancies moves countercyclically; firms upgrade the skill mix of vacancies in recessions and downgrade in booms. Although not obvious to the naked eye, the low-skill exit rate from unemployment

[20] mention that this contradicts the findings of Krause and Lubik that good jobs are more abundant in booms.

exhibits relatively higher volatility, reflecting the countercyclical behavior of the vacancy mix. The standard deviation of log deviations from a quarterly trend is 0.0370 for the low-skill, and 0.0331 for the high-skill exit rate.

The model is also consistent with the evidence that job-to-job transitions are procyclical, which as emphasized in Barlevy (2002) explains the observed procyclicality in match quality. As shown in Figure 5, despite the countercyclical behavior of the skill mix of vacancies, which implies that workers are relatively more likely to meet high- as opposed to low-skill vacancies in recessions, both of these regularities are present in this model. A higher fraction of high-skill workers relocates into high-skill jobs in booms, while the over-qualification rate (i.e., the fraction of over-qualified high-skill workers) moves countercyclically, with a lag of two quarters relative to high-skill unemployment. The lag is not surprising, since it takes time for unemployed workers to arrive to jobs, and time for over-qualified workers to find suitable jobs.

Overall, the model is consistent with empirical regularities on worker flows and match quality over the business cycle. Moreover with an average of 18.4% of college graduates being temporarily employed in jobs that require less than college education, the model is in line with observed differences in the cyclical behavior of unemployment rates of college graduates and workers with less than college education. Sections 4.3 and 4.4 that follow, go deeper into the underlying mechanisms that drive the results, and analyze the the consequences of over-qualification on low-skill employability.

4.3 Responses to Aggregate Productivity Shocks

This section goes deeper into the mechanisms underlying dynamic responses of variables to aggregate productivity shocks. I demonstrate the consequences of a fall in aggregate productivity from $y = 1.0346$ to $y = 1$. This switch represents a reduction of approximately 1.5 standard deviation in output. To illustrate the various effects I simulate the model with $y = 1.0346$ until the endogenous variables converge to a stable value. I then set $y = 1$ and simulate the effects, assuming that once the negative shock arrives it persists for 25 periods.[21] The results of this exercise are summarized in Figure 7.

I begin by the conventional result that in recessions firms open fewer vacancies per job seeker. On impact, the number of vacancies and meeting rate decline. The number of vacancies rises afterwards as rising arrival rates of job seekers to firms, encourages more

[21] Despite the assumed sample path, the value functions used in the simulations assume aggregate productivity obeys the calibrated AR(1) process as described earlier.

job openings, but never reaches its initial level. The exit rates from unemployment follow a similar pattern; they decline on impact, and subsequently recover partial of the initial decline, reflecting the rise in the number of job seekers. However, both the initial decline is lower and the recovery in exit rate is higher for high-skill workers.

On impact, the composition of job seekers shifts towards more unemployed and fewer over-qualified ($\psi\varphi$ and $\psi(1-\varphi)$ increase), reflecting the drop in exit rates. Moreover, the fraction of high-skill job seekers ($1-\psi\varphi$) declines. This shift in the composition of job seekers entails a higher matching rate for firms with low-skill vacancies, and a lower matching rate for firms with high-skill vacancies. Still, it is not sufficient to encourage firms to downgrade the skill mix of vacancies. The fraction of low-skill vacancies declines on impact, reflecting the "scale" effect, which as emphasized in section 2, makes high-skill jobs relatively more profitable in recessions. Despite the subsequent increase in vacancies, the fraction continues to decline. The upturn in the number of high-skill vacancies, is higher than the upturn in the number of low-skill vacancies. This is because of the subsequent increase in the fraction of over-qualified (or equivalently, decline in the fraction of unemployed) job seekers, which reinforces the scale effect, by making high-skill vacancies even more attractive to firms. At the onset of the recession, the fraction of over-qualified high-skill workers declines slightly, reflecting the upgrading in the vacancy mix. But rises afterwards as rising number of high-skill unemployed arrive to to firms with low-skill vacancies, and converges to a higher level.

It follows that the burden of recessions falls more heavily on low-skill workers, not only because the relative profitability of low-skill vacancies is lower in recessions, but also because of the rise in the number of over-qualified job seekers, who congest the low-skill market. Over-qualified workers lower the chances low-skill workers find jobs in recessions, both directly, by making it more difficult for low-skill workers to locate jobs, and indirectly by lowering the profits of low-skill jobs, and discouraging firms from opening low-skill vacancies. However, one has to keep in mind that congestion at the lower segment of the labor market is inevitable in recessions even if high-skill workers refuse the low-skill jobs they encounter. In this case, instead of over-qualified job seekers, a higher number of unemployed high-skill job seekers who refuse to match, congests the low-skill segment. Hence, it not apparent that low-skill employability improves when high-skill workers refuse low-skill jobs.

4.4 The consequences of cross-skill matching

The question I ask in this section, is whether low-skill workers are better off when high-skill workers reject the low-skill jobs they encounter, and search only off the job. I also investigate whether in the absence of cross-skill matching recessions would have a more moderated impact on low-skill employability relative to high-skill employability. Surprisingly enough, the answer is not. Indeed, as shown above, recessions hurt low-skill employability relatively more, when cross-skill matching occurs, because high-skill workers exit rates from unemployment are not vulnerable to changes in the vacancy mix, while low-skill workers suffer in addition the congestion effects of rising over-qualification rate. However, as will be shown below, when high-skill workers refuse the low-skill jobs they encounter, the models exhibits less cyclical employment growth overall, but relatively more cyclical low-skill employment growth.

When high-skill workers accept low-skill job offers, they affect the profits of low-skill jobs in two ways. First, by raising the effective matching rate of firms with low-skill vacancies, they raise their profits. Firms with low-skill vacancies are better off hiring low- instead of high-skill workers since the latter are more likely to quit, and thus generate lower surplus. Nevertheless, they are still better off when an over-qualified worker fills the vacancy instead of the vacancy remaining unfilled. This positive impact can also be interpreted as lowering the vacancy costs for low-skill firms, and is captured by the second term in the free-entry condition (17), which would be absent if high-skill workers refused to match with the low-skill jobs they meet. When both types of workers accept low-skill jobs, low-skill vacancies are filled faster.

The second consequence on the profits of low-skill jobs is negative, and arises when high-skill workers crowd out low-skill workers from low-skill jobs. Crowding out occurs when the number of over-qualified high-skill workers increases at the cost of a lower number of correctly matched low-skill workers, who instead remain unemployed. In particular, over-qualified workers push low-skill workers into unemployment by congesting the low-skill market, thus making it harder for firms to locate low-skill workers. When this occurs, the profits of low-skill vacancies may decline, as the decline in the "quality" of low-skill jobs outweighs the positive impact of a higher effective matching rate. As a consequence, firms post relatively fewer low-skill vacancies, thus lowering the chances low-skill workers exit unemployment even further. Looking at the free-entry condition for low-skill vacancies, the crowding out effect translates into a reduction in $\psi\phi$ due to a rise in the number of overqualified workers e. Naturally, when high-skill workers accept low-skill jobs u_h is lower. Hence, even if u_l is higher, $\psi\phi$ may decline if e_{hl} is sufficiently high.

On the other hand, when high-skill workers refuse the low-skill jobs thy encounter, firms with low-skill vacancies do not suffer the negative quality effect arising from overqualified high-skill workers crowding out low-skill ones. But in the meantime, they do not benefit from the arrival of unemployed high-skill workers either. Instead, the high-skill unemployed congest the low-skill market, making it more difficult for firms to fill vacancies.

The question that follows is whether the negative crowding out effect, dominates the positive effect of a higher effective matching rate. To answer this question I simulate the calibrated model assuming that high-skill workers who meet low-skill vacancies refuse to match and compare the results to the case cross-skill matching occurs, as above.[22] The results of this comparison are summarized in Figure 8, where I compare the responses to the same negative shock as in the previous section, of the model with to the model without cross-skill matching.

The comparison reveals that by accepting transitorily low-skill jobs high-skill workers improve the employment prospects of low-skill workers. The low-skill exit rate from unemployment is is higher, suggesting the the crowding out effect is relatively small. Indeed, with the presence of on-the-job searchers, the skill composition of job seekers shifts towards the high-skill workers, despite the fact that high-skill unemployment is lower. Hence, firms with low-skill vacancies have more difficulty locating suitable workers, while firms with high-skill vacancies benefit from higher arrival rates of suitable workers. It follows that the fraction of low-skill vacancies is lower when cross-skill matching occurs. But still, low-skill workers' exit rate from unemployment is higher, because there more vacancies available per job seeker.

As it turns out, when high-skill workers occupy transitorily low-skill jobs, firms open more vacancies of both types. Firms open more low-skill vacancies, because the benefit of a higher effective matching rate, outweighs the negative quality effect of crowding out. But

[22]Plugging $e = 0$ into the free-entry conditions (17) and (18), and setting the second term in (17) equal to zero bring us to the free-entry conditions for the model without cross-skill matching. To isolate the effects of cross-skill matching at the lower segment of the labor market, I assume that firms with high-skill vacancies do not internalize the fact that high-skill workers refuse the low-skill jobs they meet. The underlying assumption is that high-skill workers refuse low-skill jobs because of some idiosyncratic reasons, that are unknown to the employers. Such reasons could be for instance high on-the-job search costs. Hence, employers who hire high-skill workers assume that these workers can refuge to temporary employment in low-skill jobs if the opportunity arises.

firms open more high-skill vacancies also, because they benefit from a higher arrival rate of suitable workers. Overall, the the willingness of high-skill workers to accept low-skill jobs, and the resulting higher search activity at both segments of the labor market, maintains a higher incentive for firms to open vacancies, thus facilitating exit from unemployment. Therefore, both the rate by which workers meet high-skill vacancies and the rate by which workers meet low-skill vacancies is higher. It follows that both the number of over-qualified and the number of correctly allocated high-skill workers is higher in the model with cross-skill matching, as illustrated in Figure 9.

I now turn to the cyclical consequences of cross-skill matching. In Figure 10, I compare the percentage deviation responses to the shock. When high-skill workers accept low-skill jobs, in periods of rising unemployment, firms with low-skill vacancies benefit disproportionably from a higher arrival rate of job seekers, which would otherwise only cause congestion. This in turn, moderates the negative impact of recessions on the relative profitability of low-skill vacancies. However, it also limits the scope of adjusting the vacancy mix towards more high-skill vacancies, which as mentioned earlier, acts as an insulating mechanism that prevents firms from lowering the number of vacancies per job seeker in response to negative shocks. As a consequence, in the model with cross-skill matching, firms respond to negative shocks by lowering the number of vacancies opened more drastically, while keeping the skill-mix of vacancies relatively more stable. As can be verified in the figure, the percentage deviation in the fraction of low-skill vacancies following the decline in productivity is lower, while the percentage deviation in the meeting rate, is higher, when high-skill workers accept low-skill jobs.

As firms react more eminently by cutting down vacancies as opposed to shifting the vacancy mix, recessions with cross-skill matching have a more moderate impact on low-skill employability relative to high-skill employability, but a stronger negative impact on employability overall. The decline in exit rates from unemployment following a negative aggregate productivity shock is higher for both types of workers in the presence of cross-skill matching. A reduction in the meeting rate, introduces more congestion in the economy than a shift in the vacancy mix towards high-skill vacancies. It hurts both high- and low-skill exit rates from unemployment, and in addition, overcrowds the low-skill sector, as higher numbers of high-skill unemployed arrive at low-skill vacancies. On the other hand, an upgrading in the vacancy mix hurts low-skill employability, but does not affect high-skill employability. In contrast, it facilitates the allocation of high-skill workers into the jobs they are best suited for, thus reducing congestion at the lower segment of the labor market. Consequently, cross-skill matching amplifies the response of unemployment to cyclical shocks, exactly because firms have a lower incentive to concentrate reductions in the number of vacancies opened at the lower segment of the market. Evidently, both unemployment rates react more eminently to the negative shock in the model with cross-skill matching, but the high-skill unemployment rate reacts much more.

5 Directed Search

A question that arises is whether high-skill unemployment rate is lower and responds less to economic slowdowns simply because the market of high-skill jobs is tighter (i.e., there are more high-skill jobs available per high-skill job seeker, than low-skill jobs per low-skill job seeker), and not because high-skill workers refuge to temporary employment. To investigate whether this may be the case, I consider the case workers can distinguish the type of job before they apply, and therefore, can target the jobs they are best suited for (i.e., search is directed). In this case, mismatches are not possible, the two sub-markets are separated, and matching in one sub-market is independent of the conditions in the other market. Hence, the number of vacancies per job seeker in each sub-market depends only on the surplus jobs generate in each sub-market. Consequently unemployment rate differences are driven mainly by differences in the surpluses of the two types of jobs.

In the model with directed search a worker of type i, locates a vacancy of the same type at rate $m(\theta_i)$, where $\theta_i = \frac{v_i}{ui}$, and firms locate suitable workers at rate $q(\theta_i)$. Given that cross-skill matching does not occur the free-entry conditions in each market take the following form

$$(1-\gamma)\beta E_{x'|x} S_{ll}(x') = \frac{c_l}{q(\theta_l(x))} \tag{25}$$

$$(1-\gamma)\beta E_{x'|x} S_{hh}(x') = \frac{c_h}{q(\theta_h(x))} \tag{26}$$

and the surplus functions are given by

$$S_{ll}(x) = y\alpha_{ll} - b_l + \beta E_{x'|x}[(1-s_l)S_{ll}(x') - \gamma m(\theta_l)S_{ll}(x')] \tag{27}$$

$$S_{hh}(x) = y\alpha_{hh} - b_h + \beta E_{x'|x}[(1-s_h)S_{hh}(x') - \gamma m(\theta_h)S_{hh}(x')] \tag{28}$$

$$\tag{29}$$

Notice that the composition of job seekers is no longer relevant in determining the number of vacancies posted of each type. The only thing that matters is the cost of filling each type of vacancy and surplus each job generates. In turn, the surplus of each job depends on worker's net productivity and tightness in each sub-market, as reflected in the meeting rates $m(\theta_l)$ and $m(\theta_l)$.

Given this set up, the question I ask is whether productivity differences between high- and low-skill workers as reflected in the wages they earn, is what hidden behind the higher exit rates from unemployment of high-skill workers. To address this question I simulate the model with directed search, searching for the dispersion in the productivities of high and low-skill workers, that can generate the observed unemployment rate differences. To simplify things, I focus on the "best-case" scenario for the profitability of high-skill vacancies. In particular, I assume that the cost of filling high-skill vacancies is equal to the cost of filling low-skill vacancies, and that the opportunity cost of employment is the same for both types. The reasonable case is to assume that these values are higher for high-skill workers, which makes high-skill vacancy creation more costly. Moreover, I keep the lower separation rate for high-skill and higher separation rate for low-skill workers as calibrated in the previous section, which also makes high-skill jobs relatively more profitable. By reverse calibration of the unemployment rate of workers with college and less than college education, I determine the required productivity dispersion, while the rest of the parameters, are as calibrated in section 4.1.

Despite the parameter choices that favor high-skill vacancy creation I find that in the absence of cross-skill matching, the productivity of high-skill net of unemployment benefit, but be six times higher than that of low-skill workers in order for the exit rate from unemployment of the former to be higher than that of the latter, as it is empirically observed. In order for their unemployment rate to be equal to its empirical measure, the net productivity of high-skill workers must be eight times more than the net productivity of low-skill workers. This implies an average college wage premium of approximately 0.9, which is unrealistic. Unless there are large differences in the matching technology between the two submarkets, this finding suggests that temporary employment in "unsuitable" jobs is an important channel through which higher skill groups manage to keep their unemployment rates lower as also suggested by evidence that the propensity to search on the job rises with education.

6 Related Literature

Several modifications to the equilibrium matching model have been made, mainly by allowing for heterogeneity in agents' skills, to explain long run uneven developments in the unemployment rates of different skill groups. Mortensen and Pissarides (1999) examine the consequences of skill-biased technological change on the skill mix of vacancies and thus on unemployment rates by skill. However, they assume ex post perfectly segmented labor markets where skill-mismatches and thus interactions across skill groups never occur. Acemoglu (1999) is the first to introduce terms such as "overskilled" workers into the equilibrium matching model. The skill composition of jobs is endogenous and the unemployment rates of high- and low-skill workers change endogenously depending on the vacancy creation strategy of firms. Firms find it profitable either to create simple jobs that can be performed by both low- and high-skill workers, or to create both simple and complex jobs and search for the appropriate candidates. In the first case, some workers are overskilled for the jobs, thus markets are not ex post segmented. However, by assuming a constant contact rate between unemployed workers and vacancies independent of labor market conditions or skill, Acemoglu does not deal with across-skill spillover effects and search externalities.

The more recent contributions by Albrecht and Vroman (2002), Gautier (2002), and Dolado *et al.* (2003), extend this type of model to include job competition at the lower segment of the labor market. Similar to Acemoglu (1999), in Albrecht and Vroman (2002) high- and low-skill submarkets can endogenously segregate or merge depending on the productivity differential of high- and low-skill jobs. It may be worthwhile for unemployed high-skill workers to accept low-skill jobs, in which case vacancy creation and unemployment in the high-skill market affects job creation and unemployment in the low-skill market. In Gautier (2002) and Dolado *et al.* (2003) over-qualified high-skill workers search on the job. Thus, it is optimal for high-skill workers to accept a low-skill job as long as it pays more than their flow income while unemployed. In this regard, both models are similar to the model in this paper, but Gautier (2002) assumes that high- and low-skill jobs are offered in different markets. Therefore, by searching in the low-skill market high-skill workers congest the low-skill workers, but not the other way around.

The main difference between this paper and the studies mentioned above, is that it adopts a dynamic framework and focuses on the effects of business cycles on overqualification and unemployment across skill. Although the studies above lead the way in addressing the impact of job competition at the lower segment of they labor market, they are limited in that they only perform comparative static exercises on the steady-state equilibrium. Their focus is to explain long-term uneven developments in unemployment rates or wages of different skill groups, in response to either skill biased technological shocks or exogenous increases in the supply of skill. Therefore, they do not deal with across-skill differences in the cyclical patterns of unemployment and over-qualification phenomena, and their connection to job competition.

With regard to the cyclical properties of unemployment and match quality, closer to this paper is the approach of Barlevy (2002), who formalizes what is known as the “sullying” effect of recessions via a business cycle matching model with two-sided skill heterogeneity and on-the-job search. In order to make the role of aggregate shocks more transparent Barlevy’s model adopts a symmetric framework and focuses only on symmetric equilibria in which firms create equal numbers of each type of vacancy. Consequently, even if two-sided heterogeneity is assumed, the model breaks down to a single job finding probability for all skill types, and a uniform unemployment rate. Therefore, it does not deal with across skill interactions and cannot account for observed the uneven cyclical fluctuations in unemployment rates of different skill groups.

Finally, Moscarini (2001), also investigates the quality of worker-job matches in depressed labor markets and finds that a higher cost of waiting for jobs due to low job creation and intense competition for jobs raises workers’ willingness to accept offers that do not provide them with the highest possible value in the market. However, in Moscarini firms observe the characteristics of the applicants and hire only the applicants that “best fit the job”, which are always the more specialized ones by assumption. Consequently, given that more specialized workers have higher chances of beating competing applicants, they are less likely to search randomly for jobs. Hence, “noisier” allocations across jobs are more likely to occur among the less specialized workers.

7 Conclusion

—WILL BE ADDED LATER—

—Describe briefly what the paper does, why important, and main results— —Mention that the model fits also in the literatures that works on amplifying responses to shocks in the matching model—

—NOTE: THE REFERENCES THAT FOLLOW NEED TO BE UPDATED—.

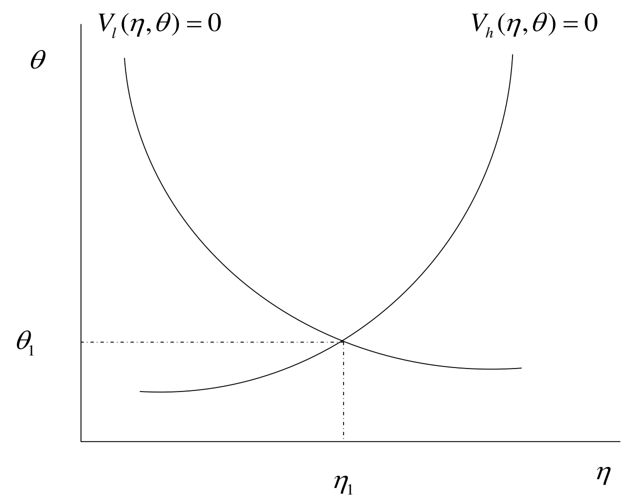

Figure 1: Steady State Equilibrium

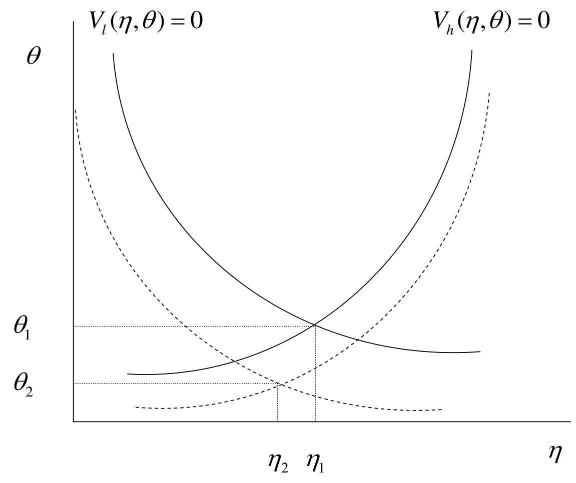

Figure 2: Impact of a negative productivity shock

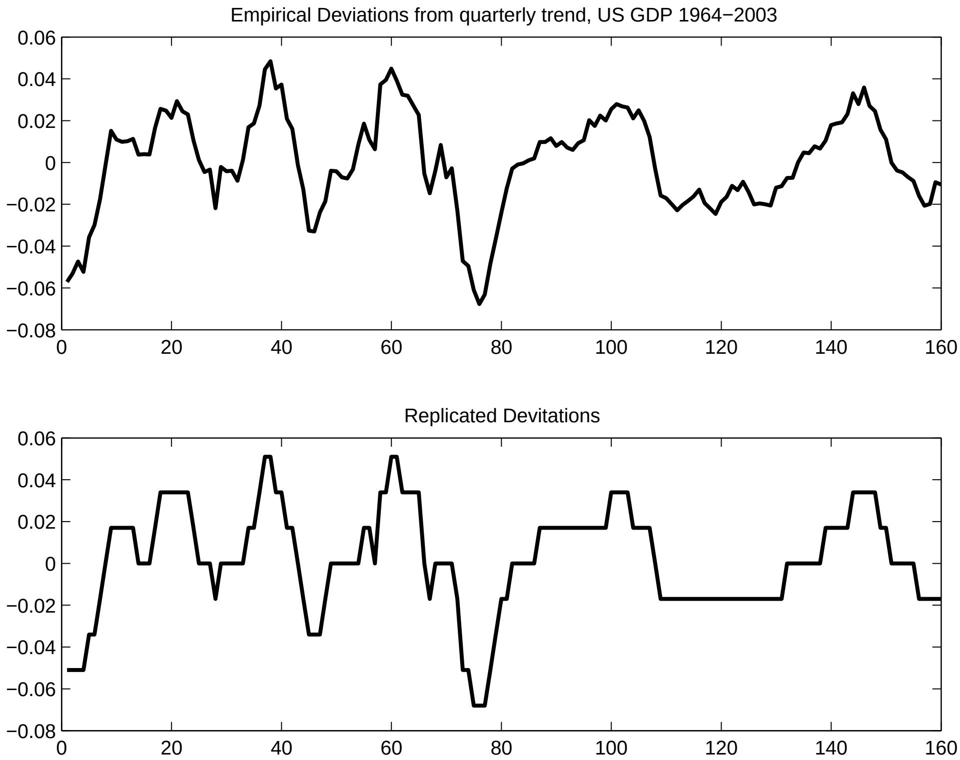

Figure 3: Real and Assumed Log Deviations from Trend

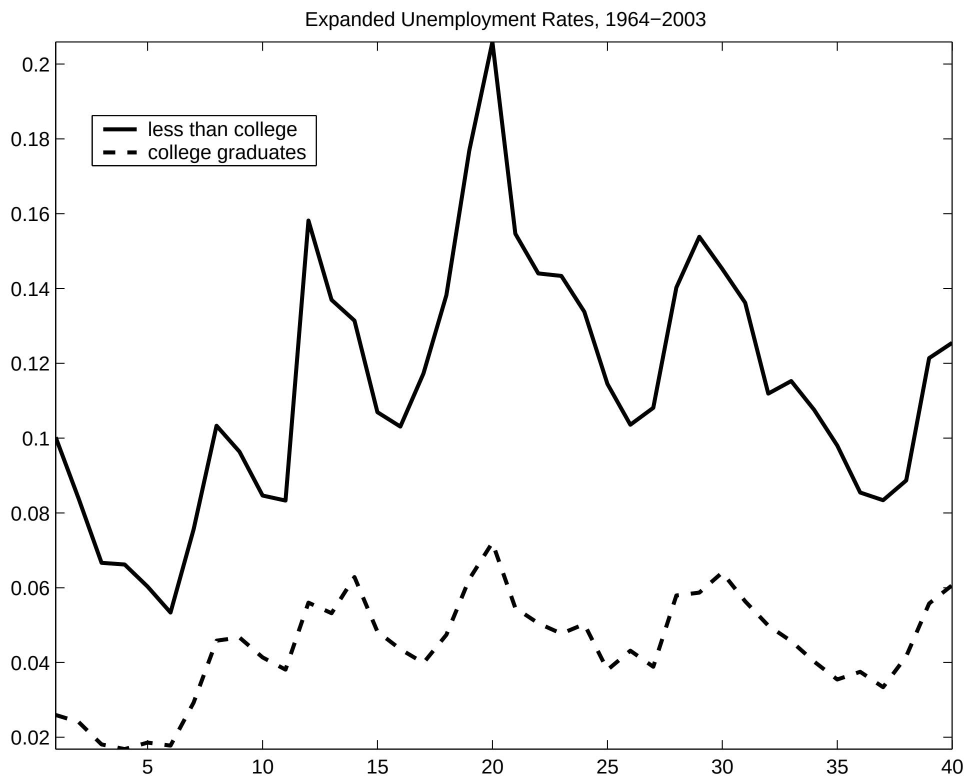

Figure 4: Expanded Unemployment Rates

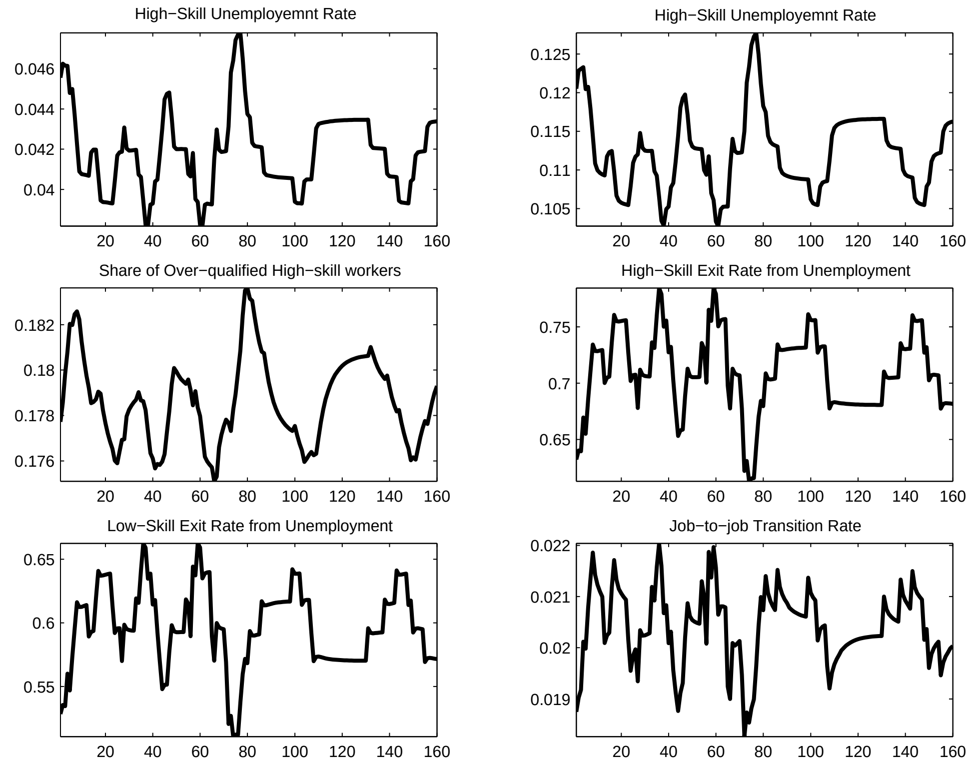

Figure 5: Simulated Series for the period 1964 to 2003

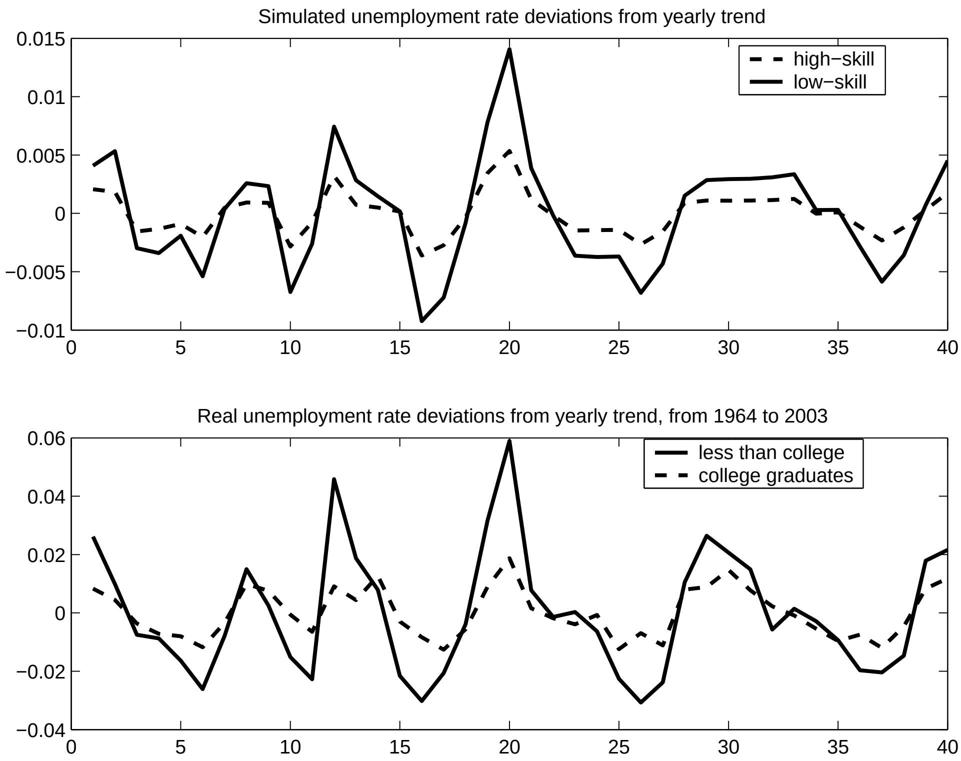

Figure 6: Real and Simulated deviations

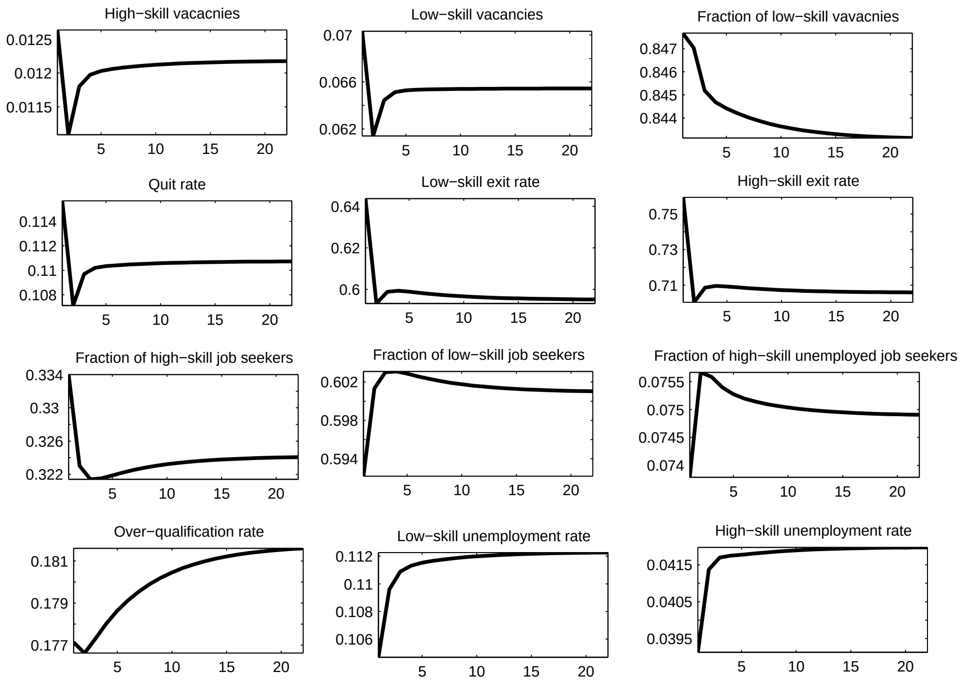

Figure 7: Responses to a negative productivity shock

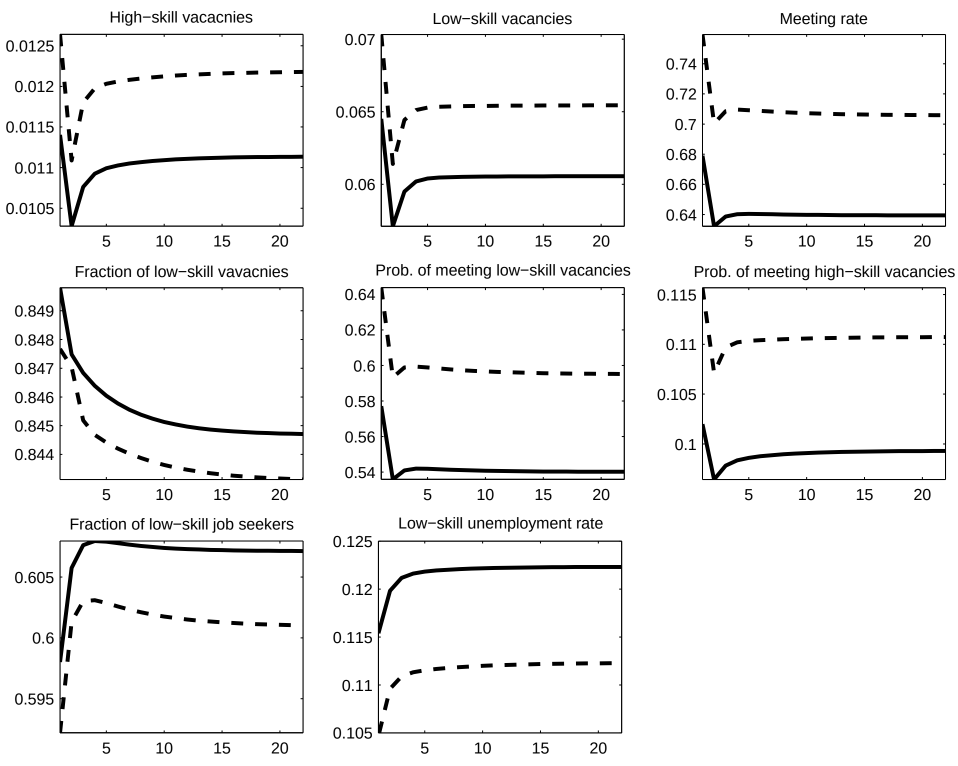

Figure 8: Responses to a negative productivity shock. The dashed line refer to the model with cross-skill matching and the solid lines to the model without cross-skill matching

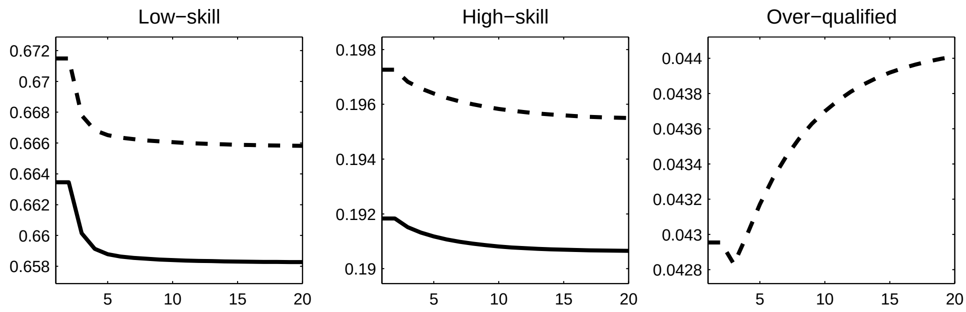

Figure 9: Employment responses to a negative productivity shock

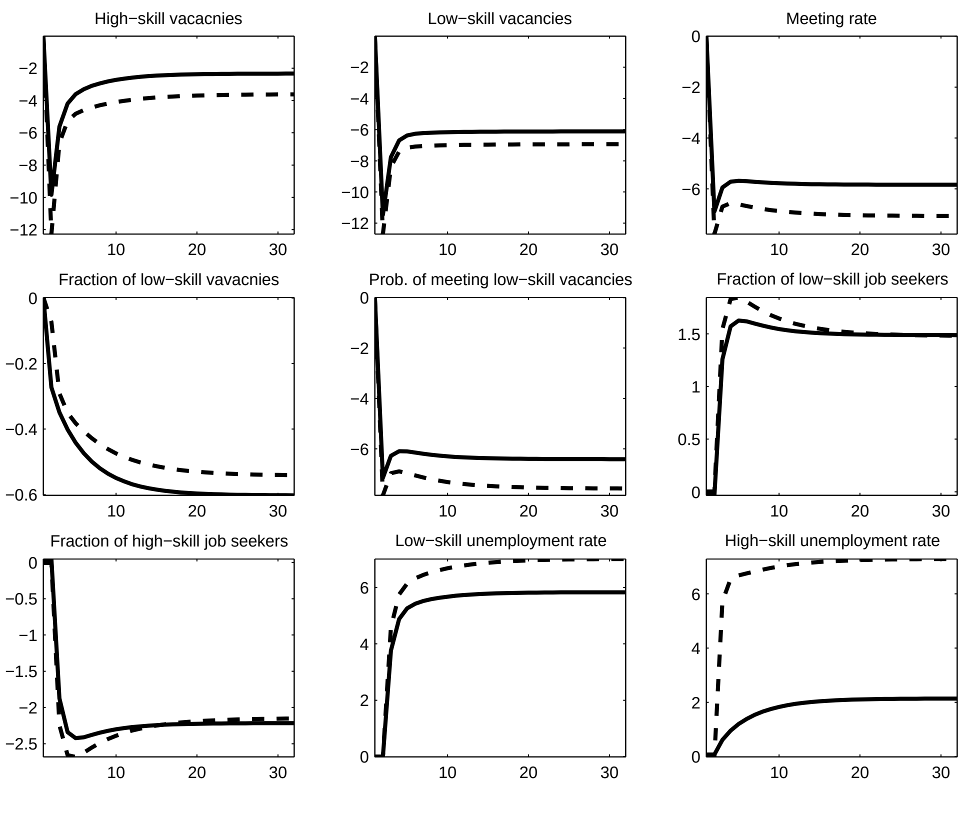

Figure 10: Percentage Deviations in response to a negative productivity shock. The dashed line refer to the model with cross-skill matching and the solid lines to the model without cross-skill matching

APPENDIX

The Steady State Equilibrium

The steady state distribution $\{e_{ll}, e_{hl}, e_{hh}\}$ is constant over time. By equating the flows in to the flows out of each of the three states, the steady state values of $\psi\varphi$ and $\psi(1-\varphi)$ are uniquely determined for a given value of η and θ as follows

$$\psi\varphi = \frac{\delta(s+(1-\eta)m(\theta))}{\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))} \tag{30}$$

$$\psi(1-\varphi) = \frac{(1-\delta)(s+\eta m(\theta))(s+(1-\eta)m(\theta))}{[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))](s+m(\theta))} \tag{31}$$

Substituting these expressions together with the surplus expressions (21) to (23) into the free entry conditions (19) and (20) yields a set of equations in terms of the endogenous variables η and θ, which I denote as $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$. To ensure the existence and uniqueness of a steady state equilibrium I define the parameter conditions under which $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$ intersect only once.

First, I specify the conditions under which, for a given η, an increase in θ lowers has a negative effect on both loci. The corresponding partial derivatives with respect to θ are given by

$$\frac{\partial F_l}{\partial\theta} = \frac{\partial(q(\theta)\psi\varphi)}{\partial\theta}S_{ll} + q(\theta)\psi\varphi\frac{\partial S_{ll}}{\partial\theta} + \frac{\partial(q(\theta)\psi(1-\varphi))}{\partial\theta}S_{hl} + q(\theta)\psi(1-\varphi)\frac{\partial S_{hl}}{\partial\theta} \tag{32}$$

$$\frac{\partial F_h}{\partial\theta} = \frac{\partial(q(\theta)(1-\psi\varphi))}{\partial\theta}S_{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S_{hh}}{\partial\theta} \tag{33}$$

I begin by specifying the conditions that ensure $\frac{\partial F_l}{\partial\theta} \le 0$. The second and last terms in (27) are negative, because $\frac{\partial S_{hl}}{\partial\theta} \le 0$ and $\frac{\partial S_{ll}}{\partial\theta} \le 0$. Moreover,

$$\frac{\partial\psi(1-\varphi)}{\partial\theta} = -\frac{\partial m(\theta)}{\partial\theta}\frac{s(1-\delta)[\delta(1-\eta)(s+(1-\eta)m(\theta))^2+(1-\delta)\eta(s+\eta m(\theta))^s]}{(s+m(\theta))^2[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))]^2} \le 0 \tag{34}$$

Therefore, given that $q'(\theta) < 0$, the second term is also negative. To complete the proof I need to show that the first term is also negative, which requires $\frac{\partial\psi\varphi}{\partial\theta} \le 0$. This derivative is given by

$$\frac{\partial\psi\varphi}{\partial\theta} = \frac{\partial m(\theta)}{\partial\theta}\frac{\delta(1-\delta)s(1-2\eta)}{[(1-\delta)(s+\eta m(\theta))+\delta(s+(1-\eta)m(\theta))]^2} \tag{35}$$

which is negative as long as $\eta \ge \frac{1}{2}$. I therefore proceed with specifying the condition that ensures $\eta \ge \frac{1}{2}$ in equilibrium. The ratio of $F_l(\eta,\theta)$ to $F_h(\eta,\theta)$ yields,

$$\frac{(ya_l-b)}{(ya_h-b)}\left[\frac{\delta\lambda_1(s+(1-\eta)m(\theta))}{(1-\delta)\lambda_2(s+\eta m(\theta))} + \frac{\lambda_1(s+(1-\eta)m(\theta))}{\lambda_3(s+m(\theta))} + \frac{\gamma\eta m(\theta)}{\lambda_3}\right] = \frac{c_l}{c_h} \tag{36}$$

which must be satisfied in equilibrium. One can easily verify that the left hand side of this expression declines both with η and θ. Evaluating it at $\eta = \frac{1}{2}$, and taking the limit when θ goes to infinity, yields

$$\frac{(ya_l - b)}{(ya_h - b)}\left[\frac{\delta}{(1-\delta)} + \frac{2\gamma}{\gamma+1}\right] \tag{37}$$

If the above expression is is greater than $\frac{c_l}{c_h}$, then $\eta \geq \frac{1}{2}$ must hold for the condition in (36) to be satisfied.

I now turn to the conditions under which $\frac{\partial F_h}{\partial \theta} \leq 0$. Given that $\frac{\partial S_{hh}}{\partial \theta} \leq 0$, the second term in (28) is negative. To establish that the first term is also negative requires an additional restriction on the matching technology. Namely, that $\frac{\partial q(\theta)(1-\psi\varphi)}{\partial \theta} \leq 0$. This condition imposes a tighter restriction on the elasticity of $q(\theta)$ than what is standardly assumed. The standard assumption (see e.g., Mortensen and Pissarides, 1994) is that the elasticity of $q(\theta)$ with respect to θ is between -1 and 0. As Dolado *et al.* (2003) also argue, compared to the standard matching model, this is the only additional restriction that needs to be imposed on the matching technology. Moreover, numerical simulations show that this derivative is always positive for values of $\delta \geq \frac{1}{2}$.

I next show that $\frac{\partial F_h}{\partial \eta} \geq 0$ whereas $\frac{\partial F_l}{\partial \eta} \leq 0$, so that the two loci have opposite slopes in the $[\theta,\eta]$ plane. Taking the derivative of $F_l(\eta, \theta)$ and $F_h(\eta, \theta)$ with respect to η yields

$$\frac{\partial F_l}{\partial \eta} = q(\theta)\frac{\partial \psi\varphi}{\partial \eta}(S_{ll} - S_{hl}) + q(\theta)\frac{\partial \psi}{\partial \eta}S_{hl} + q(\theta)\psi\varphi\frac{\partial(S_{ll} - S_{hl})}{\partial \eta} + q(\theta)\psi\frac{\partial S_{hl}}{\partial \eta} \tag{38}$$

$$\frac{\partial F_h}{\partial \eta} = q(\theta)\frac{\partial(1-\psi\varphi)}{\partial \eta}S_{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S_{hh}}{\partial \eta} \tag{39}$$

All but the last term in (38) are negative, because

$$\begin{aligned}
\frac{\partial \psi\varphi}{\partial \eta} &= -\frac{\delta(1-\delta)m(\theta)(2s+m(\theta))}{\chi^2} \leq 0 \\
S_{ll} - S_{hl} &= \frac{(1-\eta)m(\theta)}{\lambda_2\lambda_3} \geq 0 \\
\frac{\partial \psi}{\partial \eta} &= -\frac{(1-\delta)m(\theta)}{\chi^2}\left[\begin{array}{c} (s+(1-\eta)m(\theta))[3\delta(1-\eta)+3(1-\delta)\eta+\delta\eta] \\ +\delta(s+\eta m(\theta))\eta m(\theta) \end{array}\right] \leq 0 \\
\frac{\partial(S_{ll} - S_{hl})}{\partial \eta} &= -\frac{(y\alpha_{ll} - b)m(\theta)}{(\lambda_2\lambda_3)^2}\left[\begin{array}{c} \lambda_2(r+s) + \lambda_3(1-\gamma(1-\eta))m(\theta) \\ +\gamma\eta m(\theta)^2 + (1-\gamma)(1-\eta)m(\theta)^2 \end{array}\right] \leq 0
\end{aligned}$$

where $\lambda_1 = [\delta(s+(1-\eta)m(\theta)) + (1-\delta)(s+\eta m(\theta))]$, $\lambda_2 = (r+s+\gamma\eta m(\theta)+(1-\eta)m(\theta))$, and $\lambda_3 = (r+s+\gamma(1-\eta)m(\theta))$. However, the last term is positive, because $\frac{\partial S_{hl}}{\partial \eta} \geq 0$. But, adding together the third and last term results in a negative term as long as $\gamma \geq \frac{1}{2}$ and $\varphi \geq \frac{1}{2}$. Since the exogenous separation rate is the same for both types of workers, whereas low-skilled workers have lower exit rate from unemployment, they must be over-represented in the pool of unemployed, so that $\varphi \geq \delta$ must hold.[23] Hence, $\delta \geq \frac{1}{2}$ is sufficient to ensure $\varphi \geq \frac{1}{2}$.

As shown above, $\frac{\partial \psi \psi}{\partial \eta} \leq 0$. Therefore, the first term in expression (39) is positive. Finally, by rearranging terms, one can show that sufficient, but not necessary condition for

$$\frac{\partial S_{hh}}{\partial \eta} = \frac{1}{\lambda_2^2 \lambda_3^2} \times \left[\begin{array}{c} (y\alpha_{hh} - y\alpha_{ll})[\gamma m(\theta)\lambda_2\lambda_3 + \gamma^3\eta^2 m(\theta)^3 + \gamma(1-\gamma)\eta(1-\eta)m(\theta)^3] \\ +(y\alpha_{hh} - b)\gamma(1-\gamma)(1-\eta)m(\theta)^2\lambda_2 + (\gamma y\alpha_{hh} - y\alpha_{ll})\gamma\eta m(\theta)^2\lambda_3 \end{array} \right] \tag{40}$$

to be positive is $\frac{(y\alpha_{ll} - b)}{(y\alpha_{hh} - b)} \leq \gamma$. This completes the proof of $\frac{\partial F_h}{\partial \eta} \leq 0$.

The impact of a changes in y on relative surplus of jobs

Here I derive the results discussed in section 3. After substituting the surplus functions into (24), its derivative with respect to y is given by

$$\frac{\partial R}{\partial y} = \left[\frac{b\lambda_3\lambda_1(\psi\varphi\lambda_3 + \psi(1-\varphi)\lambda_2)}{(1-\psi\varphi)\lambda_2[(y\alpha_{hh} - b)\lambda_3 - \gamma\eta m(\theta)(y\alpha_{ll} - b)]^2} \right] (\alpha_{hh} - \alpha_{ll}) \tag{41}$$

which is always positive, because by assumption $(a_{hh} - a_{ll}) \geq 0$.

[23] Note that when $s_h < s_l$ as it is well established empirically, this argument is reinforced.

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