Optimal taxes on fossil fuel in general equilibrium

Preliminary work

John Hassler

IIES, Stockholm University

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- Natural scientists not fully equipped to answer questions like:
    - what is the feedback from the climate to the economy? Adaptation, mitigation and technical change.
    - what should countries do individually and collectively? Effective and efficient policies.
- Despite this, natural scientists dominate the scene.

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- The economic model is non-standard and very difficult to understand.
- Urgent need for macro model of climate - economy interaction. Should build on standard, but modern neoclassical macro theory.

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- Well known workhorse – straightforward to extend to risk, endogenous technology, frictions ....

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- Dynamics are assumed to be linear:

$$\begin{bmatrix} M_{A,t} \\ M_{U,t} \\ M_{L,t} \end{bmatrix} = \Phi \begin{bmatrix} M_{A,t-1} \\ M_{U,t-1} \\ M_{L,t-1} \end{bmatrix} + \begin{bmatrix} E_{t-1} \\ 0 \\ 0 \end{bmatrix}$$

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- Flows between $M_{A,t}$ and $M_{U,t}$ large relative to stocks. Steady state relation achieved quickly (a few decades).
- Flow to and from lower oceans very slow. Half-life of deviation from steady state several hundreds perhaps 1000 of years.

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- costs of damages depends on the economy – adaptation, technical change, insurance markets, migration...., requires a macro model.

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- Disregard uncertainty, regional heterogeneity, adaptation and technical change, and
- simplify climate/coal circulation model drastically to get some analytical results.

The model

- Planning problem

$$\max_{\{C_t,K_{t+1},E_t,R_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U\left(C_t\right)$$

$$\begin{aligned}
C_t + K_{t+1} &= F\left(K_t, , E_t, E_{t-1}, .., E_0, A_t, A_t^e\right) \\
&\quad + (1-\delta) K_t - Q\left(R_{t+1}, R_t, A_t^r\right) \\
R_{t+1} &= R_t - E_t, \\
R_0 &= \bar{R},
\end{aligned}$$

where E_t is oil use, R_t is remaining oil in ground, Q is extraction cost, A_t, A_t^e, A_t^r are technology trends.

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- Cost depends on R_t and oil is extracted in order of extraction cost (easily shown to be optimal). Then,

$$Q\left(R_{t+1}, R_t, A_t\right) = \frac{1}{A_t^r} \int_{R_{t+1}}^{R_t} dq\left(R_s\right).$$

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- Simplify climate externality drastically – current emissions and accumulated emissions affect GDP separably.

$$Y_t = D(E_t) S\left(R_t\right) F\left(K_t, N_{t,} E_t, A_t, A_t^e\right)$$

where

$$D\left(E_t\right) > 0, D'\left(E_t\right) < 0 \text{ and } S\left(R_t\right), S'\left(R_t\right) > 0.$$

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- The climate damage is a pure externality.

Optimality conditions for planner

- FOC for K_{t+1} yields a standard Euler equation:

$$\frac{U'(C_t)}{U'(C_{t+1})\beta} = \frac{\partial Y_{t+1}}{\partial K_{t+1}} + 1 - \delta = D(E_{t+1})S(R_{t+1})F_K(t+1) + 1 - \delta$$

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- FOC for R_{t+1} yields

$$\begin{aligned} & U'(C_t)\left(Y_t \frac{D'(E_t)}{D(E_t)} + \varepsilon_t\right) \\ = \quad & \beta U'(C_{t+1})\left(Y_{t+1}\left(\frac{D'(E_{t+1})}{D(E_{t+1})} + \frac{S'(R_{t+1})}{S(R_{t+1})}\right) + \varepsilon_{t+1}\right), \end{aligned}$$

where $\varepsilon_t \equiv D(E_t)S(R_t)F_E(t) - \frac{q(R_{t+1})}{A_t^r}$ is the marginal value of oil net of extraction cost but excluding externalities (the competitive price of oil).

An Hotelling type rule

- Combining the two FOCs, yields

$$\frac{\varepsilon_{t+1} + Y_{t+1}\left(\frac{D'(E_{t+1})}{D(E_{t+1})} + \frac{S'(R_{t+1})}{S(R_{t+1})}\right)}{\varepsilon_t + Y_t \frac{D'(E_t)}{D(E_t)}} = \frac{\partial Y_{t+1}}{\partial K_{t+1}} + 1 - \delta \equiv \rho_{t+1}.$$

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- Flow externality increases return unless it grows faster than ε_t.

Decentralization – individuals and good producers

- A representative individual solves

$$\max \sum_{t=0}^{\infty} \beta^t U\left(\rho_t K_t + \Pi_t^f + \Pi_t^e + T_t - K_{t+1}\right)$$

$$s.t.\ C_t + K_{t+1} \;=\; \rho_t K_t + \Pi_t^f + \Pi_t^e + T_t,$$

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$$U'\left(C_t\right) = \beta \rho_{t+1} U'\left(C_{t+1}\right).$$

- Perfect competition in goods production, implying that the price of the resource is given by its marginal product and the rental price of capital is

$$\rho_t = \frac{\partial Y_t}{\partial K_t} + 1 - \delta$$

  implying that the social Euler equation is satisfied.

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- A representative atomistic oil extraction firm owns a small share of all oil wells with all remaining extraction costs.
- Profit maximization implies selling oil with lower extraction costs first.
- For pedagogical reasons, we introduce an *ad-valorem* tax τ_t and a *per unit* tax θ_t.

Oil producers' problem

- Profits are $\sum_{s=t}^{\infty} \Gamma_t^s \left(\left(p_s^e - \theta_s \right) \left(r_s - r_{s+1} \right) - Q\left(r_{s+1}, r_s; A_s \right) \right) \left(1 - \tau_s \right)$, $s.t. r_t \geq 0$ where r_t is individual remaining resources and $\Gamma_t^s = \prod_{j=0}^{s} \frac{1}{\rho_{t+j}}$ is the discount rate.

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- Private rate of return to keeping a marginal unit of oil should coincide with alternative investment return.

Results 1

Proposition *A constant ad-valorem tax has no impact on equilibrium fossil fuel consumption.*

- This is easily seen from $\frac{(\varepsilon_{t+1}-\theta_{t+1})\left(1-\tau^{e}_{t+1}\right)}{(\varepsilon_t-\theta_t)(1-\tau^{e}_{t})} = \rho_{t+1}$.

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- Note: falling (increasing) *ad valorem* taxes increases (reduces) return to keeping oil in ground.
- **Caveat:** If oil is non-essential, all oil might not be used in the long run. How much can be affected by a constant tax.

Results 2

Falling tax rates

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- Falling θ_t/ε_t can equivalently be implemented with falling τ and $\theta = 0$). Clearly, satisfied if there is only a stock externality $(D' = 0)$.

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Results 2

Falling tax rates

- Typically, the social return to keeping oil in ground is higher than private, requiring taxes relative to price minus cost to fall. Intuition: the externality is postponed.
- **Proposition**: *Tax rates are falling, i.e.,* $\theta_t/\varepsilon_t > \theta_{t+1}/\varepsilon_{t+1}$,if

$$\frac{D'\left(E_{t+1}\right)}{D\left(E_{t+1}\right)} + \frac{S'\left(R_{t+1}\right)}{S\left(R_{t+1}\right)} > \frac{\varepsilon_{t+1}}{\varepsilon_t}\frac{Y_t}{Y_{t+1}}\frac{D'\left(E_t\right)}{D\left(E_t\right)}$$

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- Satisfied unless externality grows very quickly.

Results 3

Decentralization

- When $S'(R) = 0$, social return = private return on oil is

$$\frac{\varepsilon_{t+1} + Y_{t+1}\frac{D'(E_{t+1})}{D(E_{t+1})}}{\varepsilon_t + Y_t\frac{D'(E_t)}{D(E_t)}} = \frac{\varepsilon_{t+1} - \theta_{t+1}}{\varepsilon_t - \theta_t}$$

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- Again, tax equals marginal (now dynamic) externality cost.

An analytical example

- Cobb-Douglas production, $F\left(K_t, E_t, A_t\right) = A_t K_t^{\alpha} E_t^{\gamma}$, log utility, full depreciation, yields the usual $\frac{\alpha Y_{t+1}}{K_{t+1}} = \frac{C_{t+1}}{\beta C_t}$. Also assuming no extraction costs, this is solved by $C_t = \left(1 - \alpha\beta\right) Y_t$

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- Using this and $\varepsilon_t = \gamma \frac{Y_t}{E_t}$ implies that optimal resource use must satisfy

$$\frac{1}{\beta} = \frac{\frac{D'(E_{t+1})}{D(E_{t+1})} + \frac{S'(R_{t+1})}{S(R_{t+1})} + \gamma \frac{1}{E_{t+1}}}{\frac{D'(E_t)}{D(E_t)} + \frac{\gamma}{E_t}}$$

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- With no externalities, this is solved by setting $E_t = (1 - \beta) R_t$, i.e., a share $(1 - \beta)$ of the *remaining* stock of the resource is consumed each period.

Specific externalities

- Let us now consider two particular examples for the externality. First, assume that $D(E) = e^{-\gamma_D E}$ and $(R) = e^{\gamma_S R}$, implying

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- Together with $R_{t+1} = R_t - E_t$, and $K_{t+1} = \alpha\beta A_t e^{\gamma_S R_t - \gamma_D E_t} K_t^{\alpha} E_t^{\gamma}$, this determines the optimal allocation.

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- When instead $\gamma_D = 0$, we get $t_t = \gamma_S \frac{\beta}{1-\beta} Y_t$ where we used the fact that $\rho_{t+1} \frac{Y_t}{Y_{t+1}} = \frac{1}{\beta}$. Of course, the tax/price ratio also here falls over time at rate $1/\beta$.

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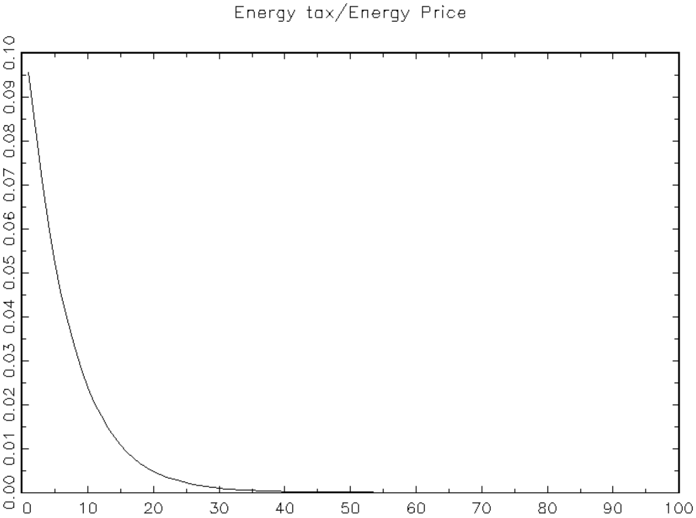
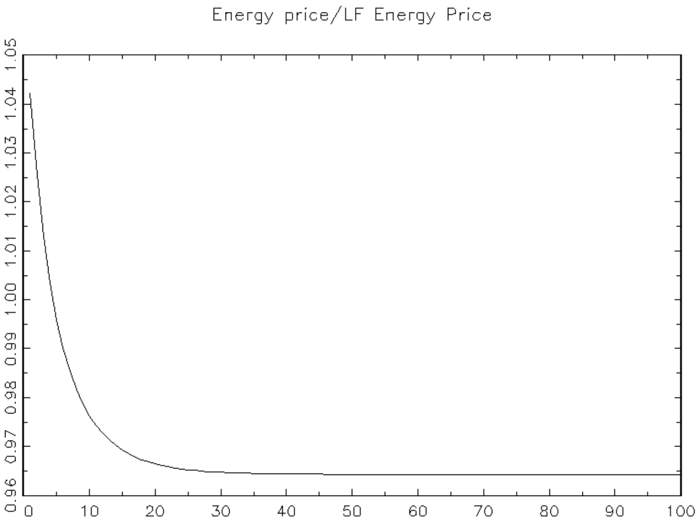
- We set $\alpha = 0.7$ and $\gamma = 0.1$. γ_D is set to .25 and γ_S to 0.01, which generate an externality at period 1 of close to 4%.
- Technology growth and discount rate to 10% $\Rightarrow$ a period is 5 years.

Optimal tax

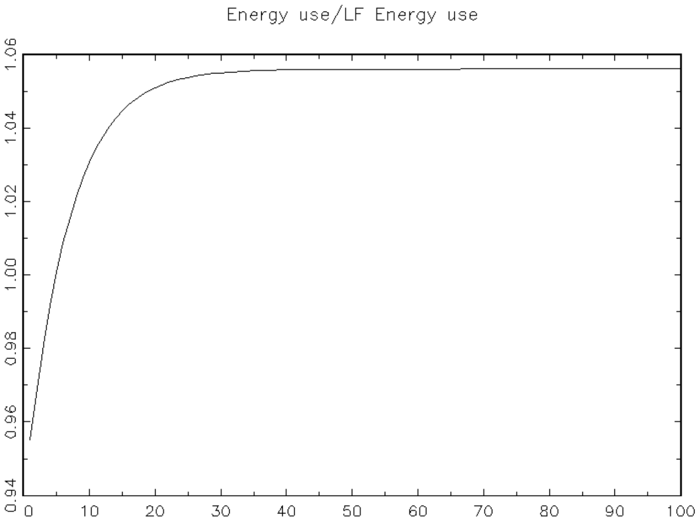


Energy tax/Energy Price
0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10
0 10 20 30 40 50 60 70 80 90 100

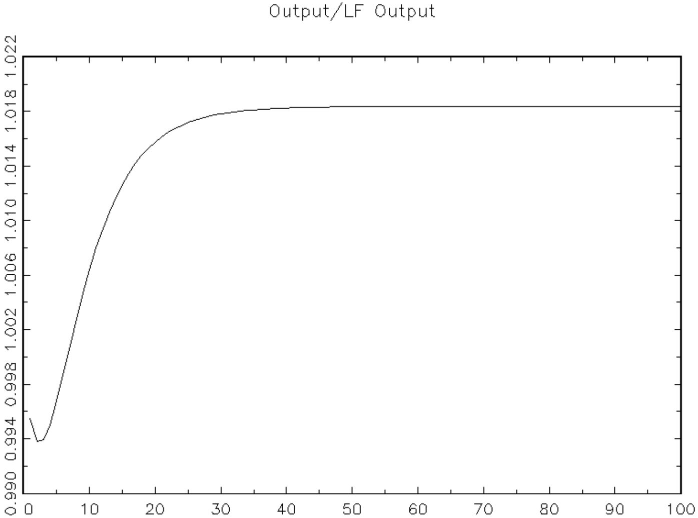
Equilibrium oil price



Oil use


Energy use/LF Energy use
0.94
0.96
0.98
1.00
1.02
1.04
1.06
0
10
20
30
40
50
60
70
80
90
100

Output


Output/LF Output
0.990
0.994
0.998
1.002
1.006
1.010
1.014
1.018
1.022
0
10
20
30
40
50
60
70
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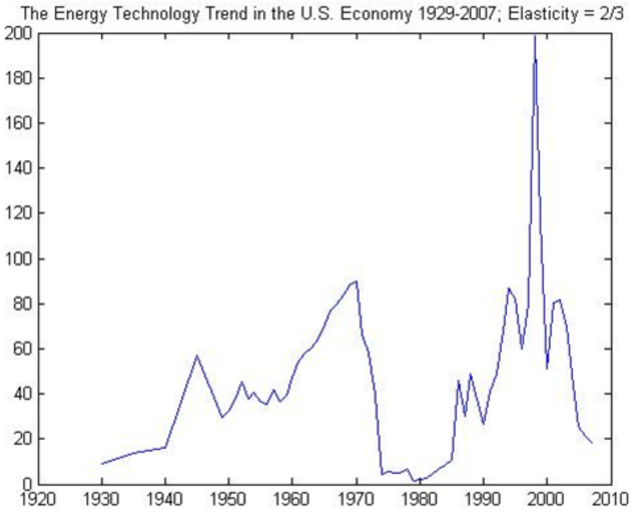
- Assuming

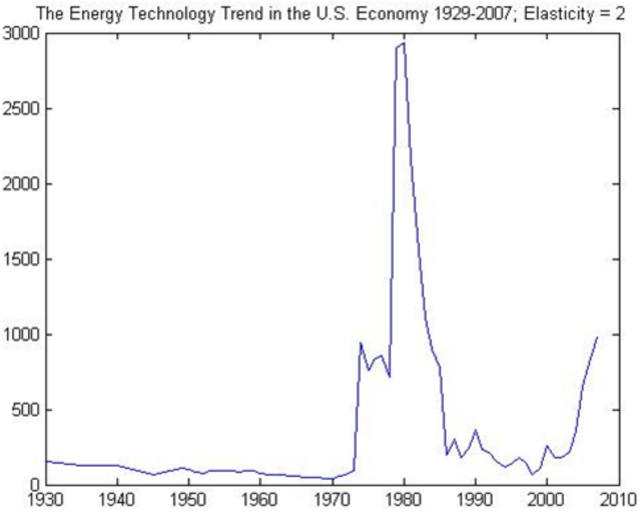
$$P_t^e = \frac{\partial F_t}{\partial E_t} = A_t \left((1-\gamma) \left(\frac{K_t}{E_t} \right)^{\frac{\sigma-1}{\sigma}} + \gamma A_t^{e\,\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \gamma A_t^{e\,\frac{\sigma-1}{\sigma}}$$

and using data on output, capital to energy use and oil prices, we can back out A_t and A_t^e.

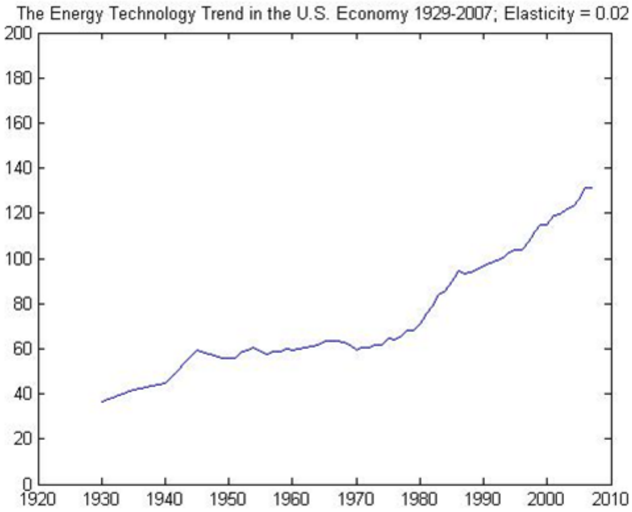
CES – technology trend. Intermediate elasticity



CES – technology trend. High elasticity



CES – technology trend. Close to Leontieff



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- The *timing of* resource use is likely to be distorted in *laissez faire.*
- Time profile of taxes are key! Taxes that decrease relative to oil price likely to be optimal. Can be interpreted as Lindahl externality correction.
- Constant taxes have no effect. Increasing taxes likely to be worse than nothing.