On-the-Job Search and the Cyclical Dynamics of the Labor Market*

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Abstract

We develop a business cycle model with search and matching frictions in the labor market and show that on-the-job search generates substantial amplification and propagation. Rising search by employed workers in an expansion amplifies the incentives of firms to post vacancies. By keeping job creation costs low for firms, on-the-job search amplifies exogenous shocks. In our calibration, this allows the model to generate fluctuations of unemployment, vacancies, and job-to-job transitions whose magnitudes are close to the data, and leads output to be highly autocorrelated. On-the-job search implies higher-order serial correlation that is absent from the standard search and matching model.

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1 Introduction

Job-to-job transitions are an important component of labor market dynamics and have attracted renewed attention in the literature. The movements of workers from one job to the next without an intermittent unemployment spell can be interpreted as the outcome of on-the-job search of the employed. From the perspective of a search and matching model of the labor market this has the attractive feature that it expands the set of potential job seekers from which firms can draw. In the standard model, it is only the unemployed that search, while in a framework with on-the-job search they are complemented by already employed workers. In an economic upturn, rising search activity by employed workers expands the pool of potential hires for firms, in addition to those searching from unemployment.

We show in this paper that introducing on-the-job search into an otherwise standard search and matching model affects labor market dynamics in a quantitatively significant manner. The empirical background for our study is the observation by Hall (2005), Shimer (2005) and Costain and Reiter (2008) that the search and matching model along the lines of Mortensen and Pissarides (1994) has difficulty explaining the cyclical dynamics of the labor market. Specifically, the standard framework underpredicts the volatility of vacancies and unemployment. The mechanism at the root of this shortcoming is that workers' bargained share in the returns to job creation depends strongly on labor market tightness, which rises quickly with falling unemployment in an expansion. We argue that on-the-job search offers a resolution to this issue because it keeps labor market tightness more stable and thus the bargaining position of workers low. Thus incentives for firms to post vacancies remain high.[1]

To develop this argument, we present a general equilibrium business cycle model with labor market frictions and search by employed and unemployed workers. Search on the job is motivated in a straightforward manner by the presence of two types of jobs, which differ in terms of profitability and thus the returns to working. Workers in low-wage ('bad') jobs search in order to gain employment in high-wage ('good') jobs. Good job vacancies can be matched with employed and unemployed job seekers, whereas firms in the bad job sector only hire unemployed workers. Wages are determined by Nash bargaining for each matched job-worker unit and are continuously renegotiated. We calibrate the model to match salient long-run features of job and worker flows.

Our model can match the observed volatility of the vacancy-unemployment ratio. At the

[1] Both Hall (2005) and Shimer (2005) explore real wage rigidity as a solution to this shortcoming. When wages do not adjust to rising returns in an expansion, firms' incentives to create new jobs are kept high. Hagedorn and Manovskii (2008), Fujita and Ramey (2005), and Rotemberg (2006), among others, explore alternative mechanisms.

same time, the ratio of vacancies to unemployed *and* employed job seekers is substantially less volatile. It is the latter that is the determinant of wage dynamics as both firms and workers take into account that the presence of an expanded set of job seekers, namely the employed, affects the probability of workers finding a job and firms finding a worker. In other words, the value of a filled vacancy, and thus the willingness to post more, is higher the more potential workers there are. This view of the expanded pool of potential hires is related to the results in Andolfatto (1996). He calibrates the model to an employment rate of 57%, so that the pool of job searchers is an enormous 43%. The standard calibration in the literature is usually around a tenth of this number. Andolfatto finds that his model can match the labor market volatilities remarkably well. In our framework, the counterpart to this large pool of searchers are employed workers engaging in on-the-job search. The key insight we offer in this paper is that, irrespective of the exact calibration of the relevant pools, labor-market participation decisions, be they of employed job seekers or of marginally attached workers, is crucial for understanding labor market dynamics.

We also show that our framework delivers a strong internal propagation mechanism. Employed workers' search activity responds strongly to a positive aggregate shock in order to take advantage of the increased availability of good employment opportunities. Job-to-job flows increase substantially. But as search on the job rises, and wage and hiring cost increases are muted, the incentive to create vacancies remains high. The corresponding fall in unemployment is large. This is achieved even though productivity shocks are of plausible magnitude and wages are, a priori, fully flexible. Through the channel of replacement hiring, as workers in bad jobs leave for good jobs, small aggregate impulses engender large and long-lasting responses of output and employment. We show that this propagation is intricately linked with the mechanism that keeps job creation high.

Our paper is closely related to recent work that has introduced on-the-job search in dynamic stochastic general equilibrium models of the business cycle. Tasci (2007) develops a model where on-the-job search is motivated by uncertainty about the quality of an employer-worker match. Workers in low-quality match are motivated to search in order to improve their joint productivity and thus their wage. In expansions, job-to-job transitions are rising since there is a larger pool of workers that desire to move on and are also more likely to be contacted by firms. His model thus delivers the same degree of amplification and propagation as our benchmark, albeit with a different mechanism. We therefore regard Tasci's work as highly complementary to ours.

Van Zandweghe (2010) studies the effect of on-the-job search on inflation dynamics in a two-sector labor market model similar to ours. He finds that the propagation mechanism engendered by on-the-job search amplifies the output response to monetary policy shocks, but reduces the variability of the inflation response. His paper uses a different timing assumption for when new matches become productive. Interestingly, the persistence and volatility pattern in a model with on-the-job search is strongly affected by this, which would not be the case for the simple search and matching model.

Finally, Krause and Lubik (2006) compare the a simple two-sector model of the labor market with the standard search and matching model and find that they deliver essentially identical aggregate dynamics. This pattern is broken by the introduction of on-the-job search in a similar manner as in this paper. The implication is that on-the-job search facilitates the creation of high-quality jobs in a cyclical upswing. There is also a substantial labor literature on on-the-job search that mostly focuses on steady state behavior. We discuss the relationship of this research to ours in Section 7.

The paper proceeds as follows. The next section gives a brief discussion of the relevant evidence on the dynamic behavior of the labor market, in particular the quit rate. Section 3 lays out the model and characterizes the steady state. Section 4 gives the calibration details. The results of the dynamic simulation of the model are presented in section 5. Section 6 contains further discussion of the role of search intensity. We also assess the robustness of our benchmark model by introducing varying search intensity of the unemployed. Section 7 relates the findings to the literature, while section 8 concludes.

2 The Empirical Background

This section documents the cyclical behavior of key indicators of labor market activity in search and matching models, specifically vacancies, unemployment, and labor market tightness for the U.S. labor market and their relation to productivity, output, employment, and real wages. While we use labor market data from 1950 until 2009, some series cover only a shorter period. For instance, the time series on average hourly earnings which we use as our measure of the real wage (deflated by the CPI) is only available from 1964 on. All series are available from the website of the U.S. Bureau of Labor Statistics (www.bls.gov), except the series on quits, which has been compiled from the Employment and Earnings publication of the BLS. This series, however, is only available up to 1982, when it was discontinued. For the period from the first quarter of 2001 on we use the quit rate reported in the Job Openings and Labour Turnover Survey (JOLTS) from the BLS. Vacancies are constructed from the index of help-wanted advertisements in the 50 largest metropolitan areas, which is compiled by the Conference Board. All variables are quarterly and, where appropriate, detrended using the HP-filter, with the smoothing parameter set to 1600.

The dynamics of vacancies and unemployment follow a familiar pattern. Figure 1 shows vacancies that are highly procyclical whereas unemployment is strongly countercyclical; that is, the two variables exhibit a Beveridge curve with a contemporaneous correlation of -0.89. This pattern implies that a measure of labor market tightness, the vacancy-unemployment ratio, is also highly procyclical. Table 1 presents the standard deviations and cross-correlations of the variables of interest. Real wages are procyclical, the degree of which depends on the time period considered.[2] Particularly the 1970s feature a highly procyclical real wage, while from the 1980s on it appears almost acyclical. In fact, for the full sample, the correlation between output and real wages is 0.40, whereas from 1982 onward it is merely 0.19. For consistency with the theoretical model, we take output per worker as a measure of labor productivity, which has a correlation with output of 0.63.

One of the central variables for the argument considered in this paper is the rate of job-to-job mobility and quits, which are the outcome of on-the-job search activity. A long time series on worker mobility and quits is contained in the BLS labor turnover series for the manufacturing sector from 1926 to 1981, which we use from 1950 on. We follow Blanchard and Diamond (1990) by making two adjustments based more recent numbers. First, quit rates in manufacturing tend to be lower than in the entire economy and therefore need to be adjusted upwards. We use Fallick and Fleischman's (2004) results based on the CPS data. They find an economy-wide average monthly quit rate of 2.6%. Some caution may be mandated since the data cover only one upswing and one mild downturn. A long-run average which includes a severe contraction might yield somewhat lower rates. Secondly, not all quits are job-to-job flows. Fallick and Fleischman (2004) suggest that job-to-job quits are about half of total quits, while Blanchard and Diamond (1990) postulate 40 percent.

The standard deviation of the adjusted quit series can be found Table 1, based on the sample up to the end of 1982. It is worth noting that the quit rate is ten times as volatile as GDP and about 50 percent more volatile than unemployment.[3] The correlation of the quit rate with output is a very high 0.88. Figure 2 shows that the quit rate appears to comove with the vacancy index, especially between about 1955 and 1975. In fact, the detrended series of vacancies and the quit rate for the whole period have a correlation of 0.94. Figure 2 also depicts the quit rate available from JOLTS for completeness. The correlation pattern of the JOLTS-based series is virtually identical to the labor turnover series. however, it is

[2] These results are not reported, but available from the authors.

[3] See Petrongolo and Pissarides (2001) for evidence on the relative magnitudes of different quit flows.

much less volatile. The standard deviation of the JOLTS variable is only 4.96 vs. 10.06. Sample size may be an issue here, but the discrepancy may also reflect the influence of the Great Moderation in aggregate volatilities from the mid-1980s. An analysis of the changing pattern of these statistics over the full sample period is, however, beyond the scope of this paper.

3 A Business Cycle Model with On-the-Job Search

Time is discrete and infinite, and the economy is populated by a representative household, homogeneous workers and heterogeneous firms. There are two types of firms, labeled 'good' and 'bad', which differ in their costs of creating new jobs. In the presence of labor market frictions, these costs generate rents which give rise to differences across jobs in the value of being employed.[4] These differentials motivate workers in low-wage jobs to search for employment in high-wage jobs. All workers in low-wage jobs search on the job with varying intensity that is determined endogenously. Unemployed workers direct their search to either good jobs or bad jobs, according to the respective returns to search. Workers in good jobs have no incentive to search as it is costly and does not offer any improvements over their current returns to employment. We first characterize labor and product markets, and the aggregate household problem. We then discuss the optimal choices by firms and workers in this environment.

3.1 The Labor Market

The process of matching workers and firms is subject to frictions, represented by a matching function, which determines the number of per period matches of job searchers and vacancies. The matching function has constant returns to scale and is homogeneous of degree one. We assume that the functional form of the matching function is the same for both job types and searchers.[5] For good jobs, the total number of new matches between vacancies and searching workers each period is given by:

$$m_t^g = m(v_t^g, u_t^g + e_t), \tag{1}$$

where v_t^g is the measure of good job vacancies, u_t^g the measure of unemployed workers searching for good jobs, and $e_t = s_t n_t^b$ is the measure of efficiency units of search by

[4] In this respect, the model is similar to Pissarides (1994) and Acemoglu (2001). The key elements of the model are the heterogeneity of jobs and the endogeneity of search intensity by employed workers.

[5] This assumption is based on empirical reasoning, e.g., Blanchard and Diamond (1990). However, these estimates typically ignore the presence of job-to-job flows. For a thorough discussion of the potential biases see Petrongolo and Pissarides (2001).

employed job seekers n_t^b, who search with intensity s_t.

Unemployed job seekers are assumed to search with fixed search intensity (equal to one)[6]. For bad jobs, the number of matches between vacancies and unemployed workers is:

$$m_t^b = m(v_t^b, u_t^b). \tag{2}$$

Note that unemployed workers search in distinct pools for jobs. They have to decide to which type of job they devote their search effort. Worker mobility implies that the returns to search for either job type are equalized.

Define $\theta_t^g = v_t^g/(u_t^g + e_t)$ and $\theta_t^b = v_t^b/u_t^b$ as measures of labor market tightness in the matching markets for good jobs and bad jobs, respectively. Vacancies are filled with the corresponding probabilities $q_t^g \equiv m_t^g/v_t^g = m\left(1, 1/\theta_t^g\right)$ and $q_t^b \equiv m_t^b/v_t^b = m\left(1, 1/\theta_t^b\right)$. For searching workers, the probabilities of finding a good or bad vacancy are given by $p_t^g \equiv m_t^g/(u_t^g + e_t) = m\left(\theta_t^g, 1\right)$ and $p_t^b \equiv m_t^b/u_t^b = m\left(\theta_t^b, 1\right)$. An employed worker's probability of being matched with a good job is $s_t p_t^g$ with p_t^g taken as given by the worker. Note that employed job seekers and unemployed job seekers cause congestion for each other in the market for good jobs.[7] We will show below that this feature is the main driver of the business cycle dynamics in the model.

The evolution of employment in good and bad jobs is governed by the equations:

$$n_{t+1}^g = (1-\rho)[n_t^g + m_t^g], \tag{3}$$

$$n_{t+1}^b = (1-\rho)[n_t^b + m_t^b - p_t^g s_t n_t^b], \tag{4}$$

where ρ is the exogenous separation rate for new hires and existing employment relationships. It is identical for both types of jobs.[8] The separation rate comprises both job destruction events and separations of workers for reasons other than quits to another employer. The last term in the second equation can also be expressed as $p_t^g s_t n_t^b = e_t/(u_t^g + e_t) m_t^g$, which is the fraction of new good matches that go to employed searchers. Aggregate unemployment equals $u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b = 1 - n_t$.

In order to determine wages, we assume that a worker and a firm split the joint surplus that their match generates in fixed proportions. The surplus of job type i is given by $S_t^i = J_t^i - V_t^i + W_t^i - U_t^i$, where J_t^i is the value of a filled job for firms, V_t^i the value of a

[6] This assumption will be relaxed below. We show that this has no substantive implications for the results.

[7] This observation is consistent with empirical evidence, see, for example, Burgess (1995), but also the discussion in Petrongolo and Pissarides (2001). In Pissarides' (1994) model with on-the-job search, workers cannot direct their search and are randomly matched across good and bad vacancies.

[8] Allowing for differing job destruction rates for good jobs would not change the basic mechanism of the model.

vacancy, W_t^i is the return of working to a worker, and U_t^i is the value of unemployment. The wage is such that workers obtain a share $W_t^i - U_t^i = \eta S_t^i$, with bargaining weight $0 < \eta < 1$. Firms receive the remainder $J_t^i = (1-\eta) S_t^i$. Wages are determined by taking the search intensity of workers as given, while search intensity itself is chosen by workers taking as given the current wage. Contracts are renegotiated each period.

3.2 Firms and Product Markets

The cost of creating a job is represented by a flow cost of posting a vacancy, c^g for good firms, and c^b for bad firms, where $c^g > c^b$. Production of a (representative) firm of type $i = g, b$ is given by the constant returns to scale technologies:

$$y_{it} = A_t n_t^i, \tag{5}$$

where A_t is aggregate productivity and n_t^i is employment in sector i. Output of good and bad firms is combined in a final goods sector according to the aggregator function:

$$y_t = y_{bt}^{\alpha} y_{gt}^{1-\alpha}. \tag{6}$$

The two intermediate goods, y_{gt} and y_{bt}, are sold at competitively determined prices, $P_{gt} = (1-\alpha)\,(y_{gt}/y_t)^{-1}$ and $P_{bt} = \alpha\,(y_{bt}/y_t)^{-1}$. The price of aggregate output serves as numeraire. In the model, both types of jobs coexist in equilibrium.[9]

3.3 The Aggregate Household

We use a representative household to construct the discount factor that governs intertemporal decisions of workers and firms. We follow Merz (1995) in assuming that workers are members of a large family which pools income and then redistributes it equally to all members. The family ensures that all workers, employed and unemployed, participate in the labor market. Thus, the optimization problem of a representative household is:

$$\max_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\tau} - 1}{1-\tau}, \tag{7}$$

subject to the aggregate resource constraint:

$$c_t = y_t - h_t, \tag{8}$$

[9] A similar product market structure is used by Acemoglu (2001). It can be interpreted as representing either differences across industries or differences across firms within industries. Evidence by Parent (2000), among others, indicates that a large fraction of job-to-job transitions are within industries. This is suggestive of intra-industry differences of jobs motivating worker mobility. Additional evidence comes from Albaek and Sorensen (1998), who find that flows of workers in upturns typically are from small firms to large firms.

where $0 < \beta < 1$ is the household's discount factor, and $\tau > 0$ is the inverse of the intertemporal elasticity of substitution. c_t is consumption, y_t is aggregate production and $h_t = c^g v_t^g + c^b v_t^b$ are the aggregate hiring (or job creation) costs incurred by firms. From the household's problem we can construct the implied stochastic discount factor $\beta_{t+1} = \beta c_{t+1}^{-\tau}/c_t^{-\tau}$, which firms and workers use to evaluate their activities.

3.4 Job Creation, Search Intensity, and Wages

The optimal choices by firms and workers are governed by asset values. The asset values of the two types of jobs filled with a worker are given by the Bellman equations:

$$J_t^g = P_{gt}A_t - w_t^g + E_t\beta_{t+1}\left[(1-\rho)J_{t+1}^g + \rho V_{t+1}^g\right], \tag{9}$$

$$J_t^b = P_{bt}A_t - w_t^b + E_t\beta_{t+1}\left[(1-\rho)(1-p_t^g s_t)J_{t+1}^b + (\rho + (1-\rho)p_t^g s_t)V_{t+1}^b\right]. \tag{10}$$

w_t^i, $i = g, b$ are the wages paid, E_t is the expectation operator conditional on the information set at time t. Jobs survive into the next period at the rate $(1-\rho)$, and are destroyed otherwise. Bad jobs face the additional risk of workers leaving to good jobs. A higher search intensity by an employed worker reduces the likelihood $(1-p_t^g s_t)$ of the job remaining filled in the next period.

The value V_t^i of a vacancy for either good or bad jobs, $i = g, b$, is:

$$V_t^i = -c^i + E_t\beta_{t+1}\left[(1-\rho)q_t^i J_{t+1}^i + (1-(1-\rho)q_t^i)V_{t+1}^i\right]. \tag{11}$$

A vacancy is filled and produces in the next period with probability $(1-\rho)q_t^i$. It remains unfilled with probability $(1-(1-\rho)q_t^i)$. Free entry implies that the values of vacancies are driven to zero at any point in time, i.e., $V_t^g = V_t^b = 0$, for all t. Solving the asset equations for vacancies then yields the two job creation conditions:

$$\frac{c^g}{q_t^g} = (1-\rho)E_t\beta_t J_{t+1}^g, \quad \text{and} \quad \frac{c^b}{q_t^b} = (1-\rho)E_t\beta_t J_{t+1}^b. \tag{12}$$

The equations relate the cost of a posted vacancy to the expected benefit.

Turning to workers, the asset values of employment in good and bad jobs are, respectively:

$$W_t^g = w_t^g + E_t\beta_{t+1}\left[(1-\rho)W_{t+1}^g + \rho U_{t+1}\right], \tag{13}$$

$$W_t^b = \max_{s_t}\left(w_t^b - k(s_t) + E_t\beta_{t+1}\left[(1-\rho)(1-s_t p_t^g)W_{t+1}^b + (1-\rho)s_t p_t^g W_{t+1}^g + \rho U_{t+1}\right]\right). \tag{14}$$

$k(s_t)$ denotes the strictly convex cost of search intensity s_t, with $k(0) = 0$, $k' > 0$, and $k'' > 0$. The higher the search intensity, the more likely a worker is matched with a good job. Convexity of the effort function guarantees uniqueness of the optimal search effort. Search intensity is chosen by the worker, taking the wage as given. We assume that firms cannot directly observe the search effort of workers. However, firms anticipate the optimal choice that workers will make in equilibrium.[10]

The optimal search intensity is:

$$k'(s_t) = \frac{\eta}{1-\eta} p_t^g \left(\frac{c^g}{q_t^g} - \frac{c^b}{q_t^b} \right), \tag{15}$$

where we used the fact that $c^i/q_t^i = (1-\rho)E_t\beta_t J_{t+1}^i = (1-\rho)E_t\beta_t[W_{t+1}^i - U_{t+1}](1-\eta)/\eta$ from bargaining (see section 3.1). Search intensity is increasing in the probability of finding a good job, and in the difference between the value of good and bad jobs. If $c^g/q_t^g \leq c^b/q_t^b$ no search on the job would take place. The factor $\eta/(1-\eta)$ reflects the fact that workers obtain only a share of the value of a job.

The asset values of unemployed search for jobs of type $i = g, b$ are

$$U_t^i = z + E_t\beta_{t+1}[(1-\rho)p_t^i W_{t+1}^i + (1-(1-\rho)p_t^i)U_{t+1}^i]. \tag{16}$$

From worker mobility, we know that $U_t^g = U_t^b = U_t$, for all t. Setting the asset values equal, and using the bargaining equations yields $(1-\rho)p_t^g E_t\beta_t J_{t+1}^g = (1-\rho)p_t^b E_t\beta_t J_{t+1}^b$. Inserting the job creation condition results in:

$$p_t^g \frac{c^g}{q_t^g} = p_t^b \frac{c^b}{q_t^b} \qquad \Longleftrightarrow \qquad \theta_t^g c^g = \theta_t^b c^b. \tag{17}$$

Thus, the relative labor market tightness for both types of jobs are exactly proportional to the relative costs of job creation. Note that in percentage terms labor market tightness in both sectors move together perfectly and are, in fact, identical.

Finally, wages paid in good and bad jobs are, respectively:

$$w_t^g = \eta P_{gt} A_t + (1-\eta)z + \eta c^g \theta_t^g \tag{18}$$

[10] There is no role for the wage in reducing the likelihood of workers quitting, because of the timing structure of the model and the nature of bargaining. Wages are continuously renegotiated so that current wages have no implications for wages paid next period, which will be newly negotiated. However, next period's payments are what motivates search activity this period. If firms could commit to wages for more than a period, then adjusting today's wage would have an effect on search intensity and thus quitting. We exclude this possibility. This also allows us to determine the wage as an outcome to Nash bargaining, because the bargaining set is convex. The need to determine the wage as the outcome of bargaining with alternating offers thus does not arise. See Shimer (2006) for a discussion of the relevant issues.

and

$$w_t^b = \eta P_{bt} A_t + (1-\eta)(z + k(s_t)) + \eta\left((1-s_t)c^g \theta_t^g\right). \tag{19}$$

The equations are derived from the bargaining relationship $(1-\eta)(W_t^i - U_t) = \eta J_t^i$, using the respective asset equations and the job creation conditions. The second equation makes use of equation (17). The wage compensates the worker for the incurred search cost $k(s_t)$ and compensates the firm for the increased likelihood of separation due to the workers search effort s_t. Note that we assume that wages in previous jobs are not part of the outside options of a worker.

4 Calibration and Model Solution

We proceed by linearizing the equation system around the non-stochastic steady state. The resulting linear rational expectations model is then solved by the method described in Sims (2002). To evaluate the cyclical properties of the model we assign numerical values to the structural parameters. The calibration is somewhat more complicated than in the standard model as some parameters in our model do not have easily identifiable counterparts in aggregate labor market data. Moreover, since pertinent information is not available for some parameters, we have to compute these indirectly from the steady-state values of quantifiable endogenous variables. The calibration is summarized in Table 2.

We choose a separation rate of $\rho = 0.1$. Following the argument in Den Haan et al. (2000), this value captures both exogenous job destruction and quits into unemployment as well as movements out of the labor force. We set the unemployment rate to 12%, i.e., $u = 0.12$. It is chosen higher than that commonly observed in the data to take into account workers that are only loosely attached to the labor force, such as discouraged workers or workers that are engaged in home production. Once the opportunity arises, these (potential) workers participate in the matching market.[11]

We set the steady-state job-to-job transition rate to 0.06. In our model, this corresponds to the term ep_g/n, i.e. the number of workers in bad jobs who move on to good jobs relative to total employment. This number is derived from the data on average job-to-job quits over the sample period as reported above. When combined with the dynamics of employment, this implies a ratio of job-to-job movements to total hires of 54%, which we regard to be at the high end of the empirically plausible range.

For the matching function, we choose a Cobb-Douglas functional form that is identical

[11] This argument is based on Blanchard and Diamond (1990).

in both sectors, so that $m_g = M_g v_g^{1-\mu} (u_g + e)^\mu$ and $m_b = M_b v_g^{1-\mu} u_b^\mu$. Similarly to the literature, the elasticity parameter is calibrated as $\mu = 0.4$.[12] The level parameters M_g, M_b are computed to imply an economy-wide firm matching probability of 0.7, which is a commonly used value in the literature. This leads to $M_g = M_b = 0.6$. The corresponding steady state sectoral matching rates, that is, the probability that a firm in the good or bad sector finds an employee, are 0.77 and 0.63, respectively.

Job heterogeneity is generated by differences in the job creation costs $c_g > c_b$. Crucial as these parameters are, it is also not trivial to pin them down. We let our choice be motivated by the following considerations. First, job creation costs consist of costs for recruitment, training, and unused capital, which are likely to be proportional to the capital intensity. In fact, Acemoglu (2001) links creation costs to capital intensities in service and manufacturing sectors. We thus impose that job creation costs for good firms are four times as large as for bad firms, which is on the order of magnitude of the difference between the capital intensity of average high-wage and low-wage jobs. Secondly, even though job creation costs can be treated as scale parameters, they should not be out of line with the general steady state implications of the model. Specifically, they cannot be so large as to substantially reduce aggregate GDP below production. Setting $c_g = 0.16$ and $c_b = 0.04$ results in 5% of output used in job creation activities and obeys the first criterion. Furthermore, we impose that sectoral prices are roughly equal in steady state which implies a share $\alpha = 0.4$ of production derived from bad jobs. Together with the differential in job creation costs, this implies that wages are higher in good jobs.

The costs of searching on the job are assumed to be strictly increasing and convex in the search intensity. We use $k(s) = \kappa s^\sigma$, where $\kappa > 0$, $\sigma > 1$. In our benchmark calibration we choose $\sigma = 1.1$. We regard highly elastic search as the most plausible case, based on the following reasoning. First, there may be increasing returns to search as argued by Rotemberg (2006). *S*econdly, the model tries to explain data generated by search at both the intensive and extensive margins.[13] We also note that Merz (1995) chooses a value of one. Since this is one of our main parameters of interest, we will present and discuss the implications of variations in the search elasticity below. The scale parameter κ is not chosen independently, but is computed implicitly to be consistent with the calibrated steady state. We find $\kappa = 0.04$.

[12] Empirical estimates of this elasticity parameter are biased when there is on-the-job search (see Petrongolo and Pissarides, 2001, for the estimation). We are aware of no empirical study of the matching function that takes on-the-job search into account.

[13] Christensen et al. (2005) estimate a search model with intensive and extensive search on the job and report a search elasticity of 2.

The parameters describing the household are standard. We choose a coefficient of relative risk aversion $\tau = 1$, and a discount factor $\beta = 0.99$. The worker's share in the surplus of the match is $\eta = 0.5$. This follows the convention in the literature, which is largely agnostic about this value. Hagedorn and Manovskii (2008) have demonstrated recently that small values of η are needed for matching the volatilities of unemployment and vacancies. We do not follow this calibration, however, since we demonstrate in this paper that on-the-job search alone is sufficient for capturing labor dynamics.

Similar reasoning also applies to the value of the outside option of the worker, whereby a high value of z, close to the marginal product of labor, results in high volatility of tightness (see Hagedorn and Manovskii, 2008). We partially avoid making a stand on this parameterization since we back out the utility value z from the model's steady state conditions to be consistent with our calibrated unemployment rate of 12%. We find that $z = 0.39$, which is below wages in both sectors.

Finally, we need to calibrate the shock process. The (logarithm of the) aggregate productivity shock is assumed to follow an AR(1) process with coefficient $\rho_A = 0.90$. As is common in the literature we choose an innovation variance such that the baseline model's predictions match the standard deviation of U.S. GDP, which is 1.62% over the sample period. Consequently, the standard deviation of technology is set to $\sigma_\varepsilon = 0.0049$.

Based on this calibration, we find that in the non-stochastic steady state equilibrium about 30% of jobs are in the bad, that is, low-wage sector, and that search intensity s is about one third. In other words, 10% of the labor force are effectively searching on the job in any time period. Although we lack independent information on this number, we regard it as not outlandishly implausible. A relatively low number of unemployed workers look for good jobs (1.3%), while the remainder of the unemployed (10.7%) search for bad jobs. This is the result of an endogenous response of the unemployed to the competition for good jobs that they face with employed seekers. Vacancies relative to the labor force (which is normalized to one) are 7.5 percent for good jobs, and 15.6 percent for bad jobs. The resulting match probabilities for workers with, respectively, a good or a bad job are $p_g = 0.43$ and $p_b = 0.67$. Similarly, the flow of new good matches per period is 0.057 and for new bad matches 0.092. The larger amount of bad matches reflects the fact that the workers moving from bad to good jobs are replaced at the industry level.[14] We finally note that wages for good jobs are slightly higher than for bad jobs, the difference being roughly 4%.

[14] The flows in the bad job sector can be interpreted as either reflecting replacement hiring at the firm level, or as job destruction in some firms, while others expand, holding total industry employment steady.

5 Model Analysis

We first discuss the business cycle statistics generated from simulating the model, followed by a characterization of the economy's response to a productivity shock. We then analyze in detail the sources of the model's propagation and amplification mechanism.

5.1 Business Cycle Properties

We report labor market variables of interest in Table 3. Since the variance of the technology shock was calibrated to match the standard deviation of U.S. GDP we only evaluate the model's predictions based on relative volatilities. We find that, in general, the variables in the model are only slightly less volatile than in the data, in particular, vacancies, unemployment, and labor market tightness. This lends support to the assertion that a model with on-the-job search can explain the Shimer (2005) finding that a standard search and matching model cannot replicate the observed volatility of unemployment and vacancies.

The volatility of aggregate unemployment u_t and vacancies v_t, which we compute as simple sums of the sectoral variables, are roughly as volatile as those in the data, although both statistics still fall somewhat short of the data. Model-implied labor market tightness is an order of magnitude more volatile than output and does not quite come close to the data, but it is a substantial improvement over the standard calibration of the simple search and matching model. Our model also captures the high volatility of the quit rate in the data extremely well. We will see below in the robustness section that this is largely the result of a highly responsive search intensity. The elastic supply of additional searchers holds the ratio of vacancies to unemployment and employed search relatively stable. At the same time, it keeps the incentives high for firms to post vacancies.

We also note the large discrepancy between the volatility of the standard measure of aggregate tightness $\theta_t = v_t/u_t$ and an alternative measure that includes on-the-job seekers; to wit, $\widehat{\theta}_t = v_t/(u_t + e_t)$. The standard deviation of the latter is an order of magnitude smaller than that of the standard measure, but still two and a half times more volatile than output. We do not report a corresponding measure in the actual data, since on-the-job search activity would be difficult to observe. We can infer it implicitly from the outcome of on-the-job search, namely job-to-job transitions, but this would require a more elaborate empirical approach.

This discrepancy highlights the key contribution of the paper: On-the-job search can explain the observed high variability of the vacancy-unemployment ratio, while at the same time the tightness variable relevant for wage determination is much less volatile. In other words, the standard tightness measure is misleading in the sense that it does not properly reflect the variable that is guiding workers' and firms' choices. This also suggests that on-the-job search has a significant effect on the model's propagation mechanism, when compared to the standard framework, as tightness is the driving force behind firms' vacancy posting decisions and wage setting outcomes. We will discuss this issue in more detail below.

Finally, we find that the aggregate wage, the size-weighted average of the sectoral wages, is substantially less volatile than in the data (0.19 vs. 0.65). The low volatility of the former is due to the relative smoothness of the tightness variable which is an important determinant for wage determination, see Eqs. (18)-(19). We do not want to push this interpretation too far, but one could interpret this finding as endogenously generated inertia in the absence of any ad-hoc wage stickiness mechanism.

The simulation also yields strong predictions with respect to contemporaneous correlations. First and foremost is the Beveridge curve, the negative correlation of unemployment and vacancies over the business cycle. In U.S. data this correlation is -0.95, which we are able to replicate fairly closely.[15] We also match the negative comovement of unemployment with all other aggregate variables of interest. For instance, the unemployment rate is highly negatively, though not perfectly, correlated with the job-to-job transition rate. When an adverse technology shock raises unemployment, search intensity falls due to a declining job finding probability. Workers are less likely to engage in on-the-job search so that relatively fewer workers in bad jobs move on to better ones. Interestingly, our two measures of labor market tightness are perfectly correlated on account of the strong comovement of search intensity with GDP. We also note the very high procylicality of job-to-job quits in terms of the correlation with output. A noteworthy exception is the high correlation of wages and on the job search in the model, in contrast to the data.

5.2 Impulse Responses

We illustrate the influence of on-the-job search on the model dynamics by using the impulse responses reported in Figures 3 and 4. Consider a positive, one percent shock to aggregate productivity. On impact, aggregate output rises with productivity, followed by a protracted hump-shaped increase until peaking three quarters after the initial shock. At the same time, vacancies and labor market tightness for both job types rise. Since the probability of finding good jobs is now higher, search intensity, and thus the effective number of on-the-job searchers, e, increases (see Eq. (15)). Vacancies in the bad jobs sector rise proportionally

[15] For their model, Mortensen and Pissarides (1994) report a correlation of only -0.26. See also the interesting discussion in Shimer (2005).

more than those in the good sector because firms anticipate the future flows of workers to better jobs, which will then have to be replaced in the next period.

Aggregate unemployment does not move on impact since the timing of the model is such that new matches become productive only in the period after which they were formed. Unemployment starts declining persistently in the period after the shock, while it follows the same hump-shaped pattern as aggregate output. Note that sectoral search activity of the unemployed does react immediately.[16] Unemployed searchers face increased competition from employed searchers who raise their search intensity. They therefore redirect their search activity to low quality jobs. The congestion effect from on-the-job search effectively crowds out unemployed searchers from the good sector. In our benchmark version, we assume that the unemployed search with fixed intensity. We show below that the results of the paper go through when the unemployed can also vary their search intensity.

In the periods after the initial shock, good vacancies start to fall quickly from their impact level, while the adjustment in bad vacancy postings is more pronounced. This is due to the fact that as employment rises for both job types, more workers leave bad jobs, which requires rising replacement hiring. Overall, the hiring rate rises for several more quarters. Furthermore, search intensity continues to rise because the fall in unemployment increases the chances of the employed to be matched with good jobs even more. Employment in the good sector rises because the inflow of new workers exceeds the outflow from job destruction.

Even though the standard measure of labor market tightness $\theta = v/u$ is highly volatile, wages rise by much less than without on-the-job search. The reason is that the measures of labor market tightness relevant for the workers' outside options that enter wage bargaining, are substantially less volatile, as can be seen in Figure 3. The wage in bad jobs rises by less, however, because of higher search intensity. While search has a positive impact on the present value of the match for workers, it reduces the value of the match to firms.

We see from the impulse responses that changes in productivity have persistent effects, indicating that search on the job adds substantial propagation to the model. Similarly, employment has a hump-shaped response. This is not caused per se by the heterogeneity of jobs in the economy. Analysis of the model without employed search, as in Krause and Lubik (2006), show that the impulse responses of that model are very similar to those of a standard one-sector model, such as those by Andolfatto (1996) or Merz (1995).

[16] u_g and u_b are not state variables, but jump variables. these variables should not be thought of as the stock of unemployed searching for jobs in the respective sectors, but rather as the degree of search activity directed towards each sector.

5.3 Inspecting the Mechanism with On-the-Job Search

We now dig deeper into the mechanism that generates the results of the model. We show that on-the-job search modifies the standard model both in terms of amplification and propagation of productivity shocks in a qualitatively and quantitatively significant manner.

The key to understanding the model's dynamics is the definition of labor market tightness in the good jobs sector when workers can search on the job:

$$\theta_t^g = \frac{v_t^g}{u_t^g + e_t}, \tag{20}$$

where $e_t = s_t n_t^b$ is the effective number of on-the-job searchers. Without on-the-job search, tightness is defined as $\theta_t^g = v_t^g / u_t^g$. The inclusion of on-the-job searchers expands the pool of potential hires that a job-posting firm is confronted with. Other things being equal, on-the job search reduces the responsiveness of tightness to movements in vacancies v_t^g and sectoral search u_t^g, which, in turn, dampens movements in wages, see Eqs. (18)-(19). This keeps the firm's incentive to post vacancies up, as less of its surplus is eaten up by corresponding wage increases. Hence, vacancy posting is more volatile with on-the-job search than in the standard model.

The key mechanism of the model is as follows. Consider a persistent increase in aggregate productivity which raises the expected value of a job in both sectors. This stimulates vacancy creation, in a manner identical to the propagation mechanism of shocks in the search and matching model as encapsulated in the job creation condition:

$$\frac{c^i}{m}\left(\theta_t^i\right)^{\mu} = (1-\rho) E_t \beta_t J_{t+1}^i, \quad i = g, b. \tag{21}$$

What is different is how sectoral tightness reacts. Note that on impact neither aggregate nor sectoral employment moves since they are pre-determined, nor does aggregate unemployment u_t. However, the unemployed can alter their direction of search. Figure 3 shows, that relative to the steady state, the number of the unemployed u_t^g searching in the good sector falls substantially, while it rises in the bad sector. This is driven by the competition with employed searchers for good jobs. Search for low quality jobs is relatively more attractive since the pool of potential searchers in that sector is smaller.

We now log-linearize the tightness equation around the steady state levels of its variables (we denote $\widetilde{x}_t = \log x_t - \log x$):

$$\widetilde{\theta}_t^g = \widetilde{\vartheta}_t^g + \frac{e}{u^g + e}\left(\widetilde{u}_t^g - \widetilde{e}_t\right), \tag{22}$$

where we have defined $\widetilde{\vartheta}_t^g = \widetilde{v}_t^g - \widetilde{u}_t^g$. Tightness in the good sector can be decomposed into a tightness measure $\widetilde{\vartheta}_t^g$ that takes into account only the search activity of the unemployed and a component that captures the congestion effect from on-the-job search. Recall that $e_t = s_t n_t^b$, so that in steady state $e = sn^b$ and $\widetilde{e}_t = \widetilde{s}_t + \widetilde{n}_t^b$. We now assume for sake of the argument that search intensity is fixed, i.e. $\widetilde{s}_t = 0$, and $\widetilde{e}_t = \widetilde{n}_t^b$. The robustness section below looks more closely at the importance of time-varying search intensity of the employed.

The decline in u_t^g (and thus $\widetilde{u}_t^g < 0$) drives a wedge between the two measures of tightness so that $\widetilde{\theta}_t^g < \widetilde{\vartheta}_t^g$. Specifically, on-the-job-search dampens the movements of the inclusive measure θ_t^g. This implies that firm's incentives for vacancy creation are higher than they otherwise would be since wages do not increase by as much. Note that the arbitrage condition (17) implies $\widetilde{\theta}_t^g = \widetilde{\theta}_t^b$ and $\widetilde{p}_t^g = \widetilde{p}_t^b$. Since movements in $\widetilde{\theta}_t^g$ are dampened by the on-the-job searchers, search activity in the bad sector has to rise ($\widetilde{\theta}_t^b = \widetilde{v}_t^b - \widetilde{u}_t^b$) to maintain equality between the job-finding probabilities and, by implication, between the expected values of a job $E_t J_{t+1}^i$ in both sectors. Furthermore, as those who quit for good jobs will have to be replaced, additional vacancies are posted, which can be seen from the protracted response of $\widetilde{v}_t^b$ in Figure 3.

In subsequent periods this mechanism is amplified through the rise in effective search as employment in the bad sector increases, $\widetilde{e}_t = \widetilde{n}_t^b > 0$. This effect can be seen in the lower panel of Figure 4. Tightness increases by even more than on impact resulting in a characteristic hump-shaped response pattern. The amplification channel is present in the specification without varying search intensity. Once we allow for an endogenous choice of s_t, there is additional feedback. Eq. (15) implies that $\widetilde{s}_t = \frac{1}{\sigma-1}\widetilde{\theta}_t^g$. As tightness rises, so does search intensity, which increases the wedge between $\widetilde{\theta}_t^g$ and $\widetilde{\vartheta}_t^g$ by even more.

Our model with on-the-job search also has a striking implication for the labor market propagation mechanism. This insight is depicted in Figure 5, which shows impulse response functions of aggregate output for various (nested) specifications of our benchmark model. The dotted line depicts the case without on-the-job search, that is, a model with a two-sector, good job/bad job structure as in Krause and Lubik (2006). The complete lack of endogenous propagation is clearly evident as the response essentially tracks the dynamic path of a highly persistent productivity shock.[17] The middle (dash-dotted) line shows the adjustment path in our model with a fixed search intensity, while the solid line replicates

[17] It is this inability of the search and matching model that has been widely discussed in the literature (see, for instance, Den Haan et. al., 2000).

the aggregate output response from Figure 3. In both cases, output exhibits a pronounced hump-shaped pattern, which is apparently generated by the presence of on-the-job search. The difference between the two top responses stems from the endogenous response of search intensity to rising labor market tightness, as given by the optimality condition for s_t, Eq. (15). This difference, however, only has an amplification effect. Clearly, on-the-job search changes the model's propagation mechanism.

In technical terms, the response without on-the-job search follows a typical first-order autoregressive process, which carries over from the AR(1)-productivity shock.[18] Hump-shaped dynamics, on the other hand, require autoregressive terms of at least second order. These are provided by the stock of workers that engage in on-the-job search. The employment equation in the good sector can be linearized as follows:

$$\widetilde{n}^g_{t+1} = (1-\rho)\widetilde{n}^g_t + \rho(1-\mu)\widetilde{\theta}^g_t + \rho\frac{u^g}{u^g+e}\widetilde{u}^g_t. \tag{23}$$

The definition of tightness θ^g_t implies (assuming $\widetilde{s}_t = 0$ for simplicity) that $\widetilde{u}^g_t = \widetilde{n}^b_t + \frac{u^g+e}{e}\left(\widetilde{\theta}^g_t - \widetilde{\vartheta}^g_t\right)$. Substituting this expression yields:

$$\widetilde{n}^g_{t+1} = (1-\rho)\widetilde{n}^g_t + \rho(1-\mu)\widetilde{\theta}^g_t + \rho\frac{u^g}{u^g+e}\widetilde{n}^b_t + \rho\frac{u^g}{e}\left(\widetilde{\theta}^g_t - \widetilde{\vartheta}^g_t\right). \tag{24}$$

Good employment has intrinsic dynamics via its own lag, but is also 'driven' by endogenous processes $\widetilde{\theta}^g_t$, $\left(\widetilde{\theta}^g_t - \widetilde{\vartheta}^g_t\right)$, and $\widetilde{n}^b_t$.

The employment equation in the bad sector can be linearized as:

$$\widetilde{n}^b_{t+1} = (1-\rho)(1-p^g s)\widetilde{n}^g_t + \rho(1-\mu)\widetilde{\theta}^b_t + (\rho + (1-\rho)p^g s)\,\widetilde{u}^b_t. \tag{25}$$

We can substitute this equation into the employment equation for the good sector and find:

$$\begin{aligned} \widetilde{n}^g_{t+1} &= [(1-\rho)(1-p^g s) + (1-\rho)]\,\widetilde{n}^g_t - (1-\rho)^2(1-p^g s)\widetilde{n}^g_{t-1} + \\ &\quad + \left(\text{terms in } \widetilde{\theta}^g_t \text{ and } \widetilde{\theta}^g_{t-1}\right) + \left(\text{terms in } \widetilde{\vartheta}^g_t \text{ and } \widetilde{\vartheta}^g_{t-1}\right). \end{aligned} \tag{26}$$

This semi-reduced from therefore implies that sectoral employment has intrinsic dynamics through its own lags of second order. Moreover, the driving terms of employment, namely labor market tightness in the two sectors are determined by the respective job-creation conditions. These are expectational difference equations which can be solved out as functions of

[18] Lubik (2010) shows that the standard (one-sector) search and matching model with AR(1)-productivity shocks implies a reduced-form specification for output that is an AR(2). However, because of the lack in endogenous propagation the second autoregressive root is small. Consequently, the model delivers dynamics that are very close to an AR(1). It can also be shown that the two-sector model without on-the-job search easily aggregates to the one-sector set-up, and that, hence, aggregate dynamics are identical (see Krause and Lubik, 2006).

the exogenous aggregate productivity process, which results in a reduced-form specification for tightness of autoregressive order one. Consequently, the driving processes would add even more autoregressive dynamics.

All these elements combined guarantee that employment exhibits higher-order dynamics that result in hump-shaped adjustment path. The ultimate source of this pattern is the specification of the employment ladder in our model. Employed searchers enter the matching function of another sector and thereby provide strong endogenous propagation through this simple accounting mechanism. In a robustness check below, we show that this mechanism is still present when we modify the model to include endogenous search intensity of the unemployed. We therefore conclude that on-the-job search as modeled in this multi-sector set-up would have to play a central role in explaining aggregate business cycle dynamics.

In order to highlight this argument we compare our model to the autocorrelation patterns in U.S. data. Figure 6 depicts the autocorrelation functions of U.S. GDP growth rates over the period 1950:1-2009:1 and for the three model specifications discussed above. The lack of propagation in the model without on-the-job search is well documented by a flat autocorrelation function around zero. The benchmark model, on the other hand, captures U.S. output dynamics remarkably well, even slightly overpredicting the first-order autocorrelation. But even when search intensity is constant, the autocorrelation pattern by far outperforms the standard model without on-the-job search.[19]

6 Discussion and Robustness

We now discuss the robustness of the results with respect to aspects of the calibration and to a number of extensions, specifically the calibration of the search elasticity and endogenous intensity of unemployed search.

6.1 The Role of Search Intensity

Why does the cyclicality of job-to-job quits change the behavior of the economy so dramatically? On the one hand, rising search effort raises good firms' incentives to post vacancies. Without employed searchers, the creation of good jobs is constrained by the fall in the number of unemployed searchers and the strong rise in wages. On the other hand, the increasing availability of good jobs further encourages on-the-job search. A small rise in productivity leads to large changes in the incentives to search and posting vacancies, which explains that

[19] Inclusion of capital is likely to further increase the autocorrelation of output in addition to that achieved by on-the-job search.

unemployment falls substantially even though competition with employed job seekers rises. Only slowly do these incentives fall back to their steady state levels.

The role of search intensity can be illustrated by varying the elasticity of search effort. The results of this exercise are depicted in Figure 7. We plot the standard deviations of measures of labor market tightness and the quit rate against the elasticity parameter σ in the search cost function.

As σ approaches one from above, the quit rate and labor market tightness become exceedingly volatile. Since the responsiveness of search costs to changing search effort declines, the volatility of job-to-job quits rises. Even though the standard and our modified measures of labor market tightness, $\theta = v/u$ and $\widehat{\theta} = v/(u+e)$, are almost perfectly correlated, their volatility is strikingly different. While the former is very responsive to changes in σ, the latter is barely affected. The reason is that as unemployment falls, employed search rises, keeping the incentives for vacancy creation high after a favorable aggregate shock. The theoretical counterpart in our model, $v^g/(u^g+e)$, behaves similarly. As is evident from the impulse responses, the presence of time-varying on-the-job search activity leads to persistent movements of output after shocks to technology. The elasticity of search is, however, only partially responsible for the propagation mechanism in the model. Even with fixed search intensity, productivity shocks are still amplified and propagated in a hump-shaped manner as Figure 5 illustrates.

Figure 7 also shows how our calibration of $\sigma = 1.1$ manages to target both the standard deviation of the quite rate and the observed vacancy-unemployment ratio θ. This comes at the price of an on-the-job-search inclusive measure of tightness that is not very volatile, which results in wage movements that are too smooth relative to U.S. data. Moreover, such a high degree of search elasticity may be empirically doubtful. We regard this issue therefore far from settled.

6.2 Endogenous Search Intensity of the Unemployed

The key mechanism in our model is the increasing flow and search activity of employed job seekers. At the same time, the search intensity of unemployed workers is fixed. We show in this section that this assumption is immaterial for our results. Conceivably, as unemployed search intensity rises, their incentives to search for good jobs stay high. They would thus compete more strongly with the employed searchers from the bad sector. In our on-the-job search framework, however, the mechanism that expands search on the job is the fall in unemployment. If unemployed workers were to search more intensively, the unemployment pool would deplete even faster. This would then further amplify the importance of search for employed workers.

It is fairly straightforward to include endogenous search intensity of the unemployed in our model. The asset value of unemployment is modified to introduce a search cost which is convex in search intensity:

$$U_t^i = z - k(s_t^{ui}) + E_t\beta_{t+1}[(1-\rho)p_t^i s_t^{ui} W_{t+1}^i + (1-(1-\rho)p_t^i s_t^{ui})U_{t+1}^i], \tag{27}$$

where s_t^{ui} denotes the search intensity in sector i. The first order condition for search intensity is:

$$k'(s_t^{ui}) = \frac{\eta}{1-\eta}(1-\rho)p_t^i E_t\beta_t\left[W_{t+1}^i - U_{t+1}^i\right]. \tag{28}$$

As before, arbitrage between sectors implies that $s_t^{ui} = s_t^u$. This also implies that $c^g\theta_t^g = c^b\theta_t^b$. The optimal search intensity of the unemployed can then be written as:

$$k'(s_t^u) = \frac{\eta}{1-\eta}p_t^g\frac{c^g}{q_t^g} = \frac{\eta}{1-\eta}c^g\theta_t^g. \tag{29}$$

As expected, increases in tightness in either sector would lead the unemployed to search more intensively and the probability of finding a job p_t^i increases.

How does this affect the search intensity of the employed? We can use the optimality condition (15) and divide the two expressions. This yields:

$$\frac{k'(s_t^e)}{k'(s_t^u)} = \left[1 - \left(c^b/c^g\right)^{1-\mu}\right]. \tag{30}$$

This condition implies that the search intensity of the unemployed and of on-the-job searchers move in fixed proportions.[20] In a somewhat loose sense, employed job seekers respond to rising search intensity of employed by increasing their intensity as well. Key to the model's propagation mechanism via firms' job posting decision is not how the pool of searchers is composed, but that it is expanding in upturns. In other words, varying search intensity of the unemployed does not affect the basic propagation mechanism of the model as highlighted in the previous section. Endogenous search intensity provides an additional feedback mechanism that can amplify the adjustment pattern, but not overturn it.

[20] If the cost functions are identical, then obviously $\widetilde{s}_t^e = \widetilde{s}_t^u$. For different cost functions, we can always find combinations of the level parameters κ^i and elasticity parameters σ^i such that the modified model produces the same aggregate dynamics as our benchmark model. In the end, it is an empirical question as to what these parameters are and whether job search of the employed or the unemployed is more costly at the margin.

7 Relation to Previous Work

Earliest precursors of our model with on-the-job search are the contributions by Pissarides (1994) and Mortensen (1994). The former develops a model that shares with ours the presence of two different job types, 'good' and 'bad' with ours, and features random search for jobs. The existence of idiosyncratic productivity draws in a match generates heterogeneity in worker productivities across jobs. This implies a threshold in the tenure of workers above which workers do not switch jobs, because starting wages in good jobs are lower than the wage in bad jobs. In this model employed job search reduces the volatility of unemployment, and would therefore not aid in understanding the unemployment and vacancy volatility found in the data.

Mortensen (1994) simulates a stochastic version of the Mortensen and Pissarides (1994) model, with the addition of on-the-job search. The presence of employed search helps in explaining the negative correlation between job creation and destruction. The model also features a procyclical quit rate, with workers being randomly matched to the most productive jobs. Both Pissarides and Mortensen do not explore prediction of their models for the joint dynamics of vacancies, unemployment and job-to-job flows or the effects on wages. Finally, the two papers have exogenous interest rates and prices, shutting down general equilibrium effects, which affect the dynamics of vacancies and unemployment. Neither of these papers considers these dynamics quantitatively.

In many other models in the labor market literature, employed search is mainly varied at the extensive margin, instead of search intensity as in our framework. Search is made costly, however, through the payment of a lump sum. Pissarides (2000) is an early example for this modeling strategy. Jobs differ by idiosyncratic productivity levels, drawn from a continuous distribution. With workers choosing whether to search or not, this implies two thresholds in terms of productivity. Below the higher threshold workers have an incentive to search for better employment, participating in the common matching market. New matches start at the highest possible productivity. Below the second threshold, the joint value of the match with the firm is below the parties' outside option, leading to job destruction. Since all jobs are created at the highest possible productivity level, vacancies are the same for employed and unemployed workers. The key difference to our model is the search at the intensive margin, and the persistent difference between job types. Including persistent idiosyncratic shocks in a business cycle model of this type comes, however, at considerable computational costs.

A second class of models with on-the-job search consider the possibility of endogenous wage distributions arising in the presence of frictions. However, these models are primarily steady state models, and are based on wage posting. That is, wages do not respond to shocks and are not renegotiated. Burdett and Mortensen (1998) explore the link with inter-industry and firm-size wage differentials. Cahuc et al. (2006) estimate such a model and show how it accounts for a steady state distribution of wages. Christensen et al. (2005) estimate a similar model with endogenous intensity of search. We do not know of any example in the literature that analyses dynamic stochastic general equilibrium versions of these models with a focus on business cycle analysis.[21]

We offer a final thought on the literature that confronts the Mortensen and Pissarides (1994) model with the data. It typically focuses on the performance of the model along the dimension it was designed to explain, namely the behavior of job creation and destruction. For example, Cole and Rogerson (1999) find that the model performs well if the steady-state unemployment rate is high. Den Haan et al. (2000) achieve plausible job flows by modeling endogenous job destruction along with capital. As mentioned, Hall (2005) and Shimer (2005) are the first to consider the ability of the search and matching framework to quantitatively match the cyclical behavior of unemployment and vacancies. In all papers, the performance of the model is enhanced by an assumption that reduces the cyclicality of hiring costs or wages. In our model, it is the presence of employed search.

8 Conclusion

We have presented a model of labor market and aggregate dynamics and in which search on-the-job plays a crucial role. We show that it is possible to explain the joint dynamics of vacancies, unemployment, and productivity without resorting to any imperfection other than search and matching frictions. In particular, we do not require wages to be rigid in order to bring the model closer to the data. Instead, increased search effort by employed workers is sufficient to dampen the movements in labor market tightness and to keep the costs of job creation more stable for firms. Consequently, wages are less volatile, and incentives to post vacancies remain high. Unemployed workers' incentives to direct search to jobs where they do not compete with employed searchers further amplify these effects.

The model delivers a rich description of the labor market over the business cycle. Booms are times which allow employed workers to upgrade into better jobs, while opening jobs for unemployed workers, albeit of lower quality. The reallocation of labor to more productive

[21] See also Shimer (2005) who reports that no such analysis has been conducted.

units is facilitated by direct job-to-job transitions, rather than requiring movements of workers through the unemployment pool. One fundamental reason for worker mobility is the heterogeneity of jobs which gives rise to persistent differences in the returns to workers. The creation of good jobs is amplified by the rising intensity of search by employed workers.

The propagation mechanism that the model implies has important implications for business cycle analysis. In response to a positive productivity shock, output displays a marked hump-shaped pattern, which is considered a stylized fact in the empirical macroeconomics literature.. A higher match probability induces employed workers to search for better jobs. This feeds back into the incentives for firms to continue posting vacancies for a protracted period. Falling unemployment further reduces the competition for good jobs and keeps incentives for search high. Interestingly, we obtain a propagation of shocks that is similar to Den Haan et al.(2000), even though we do not include capital or a variable job destruction rate.

However, the findings are not meant to rule out an potentially important role for (real) wage rigidity. Hall (2005) and Shimer (2005) suggest this as a solution to the empirical difficulties they identified with Mortensen-Pissarides model. Also in our model, wage rigidity would further amplify the cyclical response of vacancies, unemployment and job-to-job flows. Hall (2005) has made an interesting advance modeling wage setting based on social norms, which allows wages even for new hires to be rigid. In previous work, we applied this idea in a monetary business cycle model with search frictions (Krause and Lubik, 2007). Van Zandweghe (2010) combines these elements in a model with on-the-job search similar to ours.

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Appendix: The Equation System

1. Job creation conditions:

$$\begin{aligned}
\frac{c^g}{q_t^g} &= (1-\rho)E_t\beta\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}\left[P_{g,t+1}A_{t+1} - w_{t+1}^g + \frac{c^g}{q_{t+1}^g}\right], \\
\frac{c^b}{q_t^b} &= (1-\rho)E_t\beta\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}\left[P_{b,t+1}A_{t+1} - w_{t+1}^b + (1-p_{t+1}^g s_{t+1})\frac{c^b}{q_{t+1}^b}\right].
\end{aligned}$$

2. Wage determination:

$$\begin{aligned}
w_t^g &= \eta P_{g,t}A_t + (1-\eta)z + \eta p_t^g\frac{c^g}{q_t^g}, \\
w_t^b &= \eta P_{b,t}A_t + (1-\eta)(z+\kappa s_t^\sigma) + \eta(1-s_t)p_t^g\frac{c^g}{q_t^g}.
\end{aligned}$$

3. Optimal search intensity:

$$\kappa\sigma s_t^{\sigma-1} = \frac{\eta}{1-\eta}p_t^g\left(\frac{c^g}{q_t^g} - \frac{c^b}{q_t^b}\right).$$

4. Evolution of employment:

$$\begin{aligned}
n_{t+1}^g &= (1-\rho)\left(n_t^g + m_t^g\right), \\
n_{t+1}^b &= (1-\rho)\left(n_t^b + m_t^b - p_t^g s_t n_t^b\right).
\end{aligned}$$

5. Unemployment:

$$u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b.$$

6. Employed searchers:

$$e_t = s_t n_t^b.$$

7. Matching functions:

$$\begin{aligned}
m_t^g &= m(v_t^g, u_t^g + e_t) = M_g(v_t^g)^{1-\mu}(u_t^g + e_t)^\mu, \\
m_t^b &= m(v_t^b, u_t^b) = M_b(v_t^b)^{1-\mu}(u_t^b)^\mu.
\end{aligned}$$

8. Firm and worker match probabilities:

$$\begin{aligned}
&q_t^g = m_t^g/v_t^g, \quad q_t^b = m_t^b/v_t^b\ , \\
&p_t^g = m_t^g/(u_t^g + e_t), \quad p_t^b = m_t^b/u_t^b\ .
\end{aligned}$$

9. Arbitrage condition:

$$p_t^g \frac{c^g}{q_t^g} = p_t^b \frac{c^b}{q_t^b}.$$

10. Sectoral and aggregate output:

$$y_{g,t} = A_t n_t^g, \quad y_{b,t} = A_t n_t^b \ ,$$

$$y_t = y_{b,t}^{\alpha} y_{g,t}^{1-\alpha}.$$

11. Prices:

$$\begin{aligned} P_{g,t} &= (1-\alpha) \left(\frac{y_{g,t}}{y_t} \right)^{-1}, \\ P_{b,t} &= \alpha \left(\frac{y_{b,t}}{y_t} \right)^{-1}. \end{aligned}$$

12. Aggregate consumption:

$$c_t = y_t - c_g v_{g,t} - c_b v_{b,t}.$$

13. Tightness:

$$\theta_t^g = \frac{v_t^g}{u_t^g + e_t}, \quad \theta_t^b = \frac{v_t^b}{u_t^b} \ .$$

14. Aggregate technology:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{At}.$$

Table 1: U.S. Business Cycle Statistics

Standard Deviation

Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
1.68	0.65	0.88	0.80	8.71	8.39	15.99	10.06

Cross-Correlations

	Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
Y	1	0.40	0.82	0.63	-0.87	0.88	0.88	0.88
W	–	1	0.11	0.57	-0.25	0.32	0.29	0.45
N	–	–	1	0.11	-0.90	0.84	0.87	0.86
Y/N	–	–	–	1	-0.31	0.41	0.37	0.40
U	–	–	–	–	1	-0.89	-0.95	-0.90
V	–	–	–	–	–	1	0.98	0.88
θ	–	–	–	–	–	–	1	0.89
QR	–	–	–	–	–	–	–	1

Notes: The statistics are computed from HP-filtered data with a smoothing parameter of 1,600. The standard deviations are measured relative to that of GDP.

Table 2: Model Parameters and Calibration

Parameter	Value	Description
μ	0.4	Match Elasticity
M_g	0.6	Level Parameter
M_b	0.6	Level Parameter
c^g	0.16	Good Job Creation Cost
c^b	0.04	Bad Job Creation Cost
ρ	0.1	Separation Rate
σ	1.1	Search Elasticity
η	0.5	Nash Bargaining Share
α	0.4	Output Aggregator Elasticity
β	0.99	Discount Factor
τ	1	Intertemporal Substitution Elasticity
u	0.12	Steady State Unemployment Rate
ζ	0.06	Steady State Quit Rate
z	0.39	Unemployment Benefit
κ	0.04	Search Cost Function Parameter

Table 3: Benchmark Simulation

Standard Deviations

Y	W	N	$\frac{Y}{N}$	U	V	θ	θ_n	QR
1.62	0.19	0.56	0.27	6.09	5.43	11.17	2.57	10.05

Cross-Correlations

	Y	W	N	$\frac{Y}{N}$	U	V	θ	QR
Y	1	0.83	0.99	0.97	-0.99	0.93	0.99	0.96
W	-	1	0.81	0.86	-0.75	0.97	0.88	0.94
N	-	-	1	0.54	-1.0	0.87	0.97	0.92
Y/N	-	-	-	1	-0.54	0.84	0.87	0.96
U	-	-	-	-	1	-0.87	-0.97	-0.92
V	-	-	-	-	-	1	0.96	0.99
θ	-	-	-	-	-	-	1	0.98
QR	-	-	-	-	-	-	-	1

Notes: The statistics were computed as follows. We simulated the model 500 times by drawing realizations from the innovation of the productivity shock. The sample length was 200 periods. We then computed the statistics above for each simulations and averaged. The standard deviations are measured relative to that of GDP.

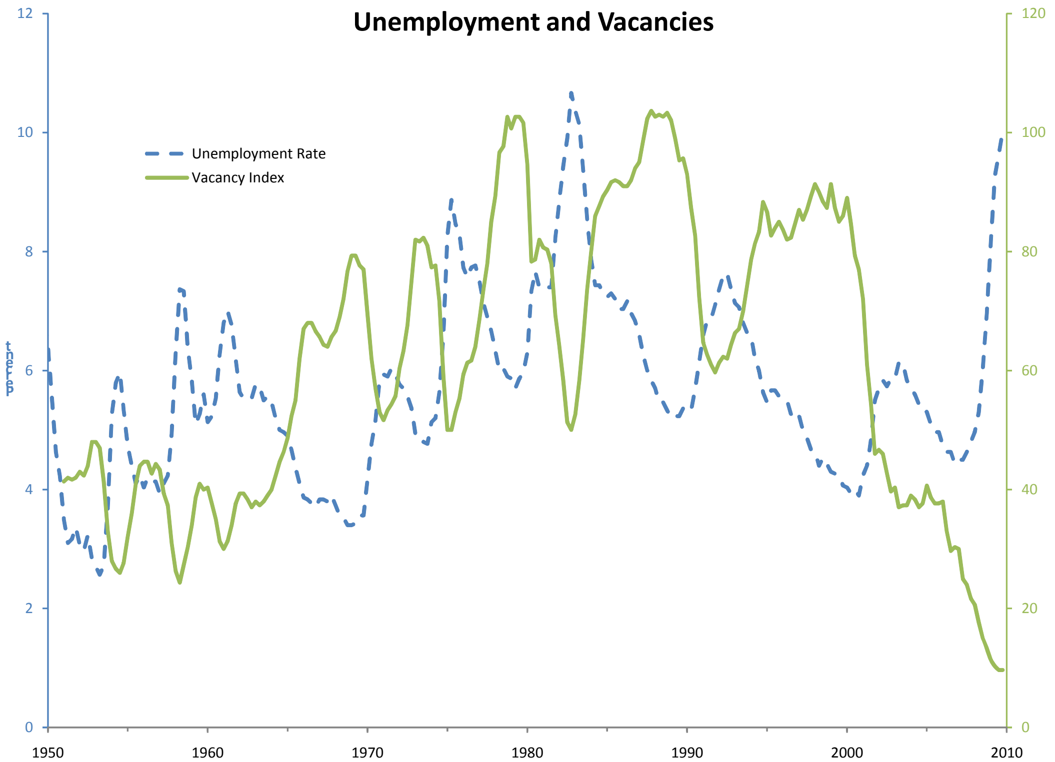

Figure 1: Unemployment and Vacancies

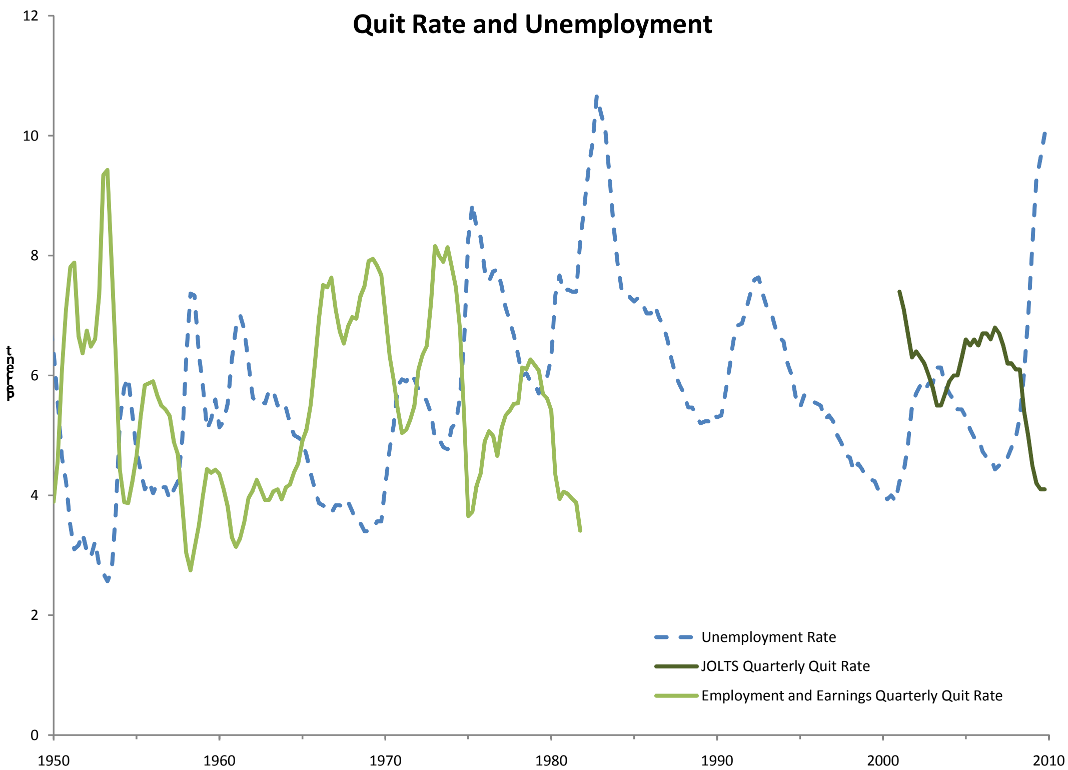

Figure 2: The Quit Rate and Unemployment

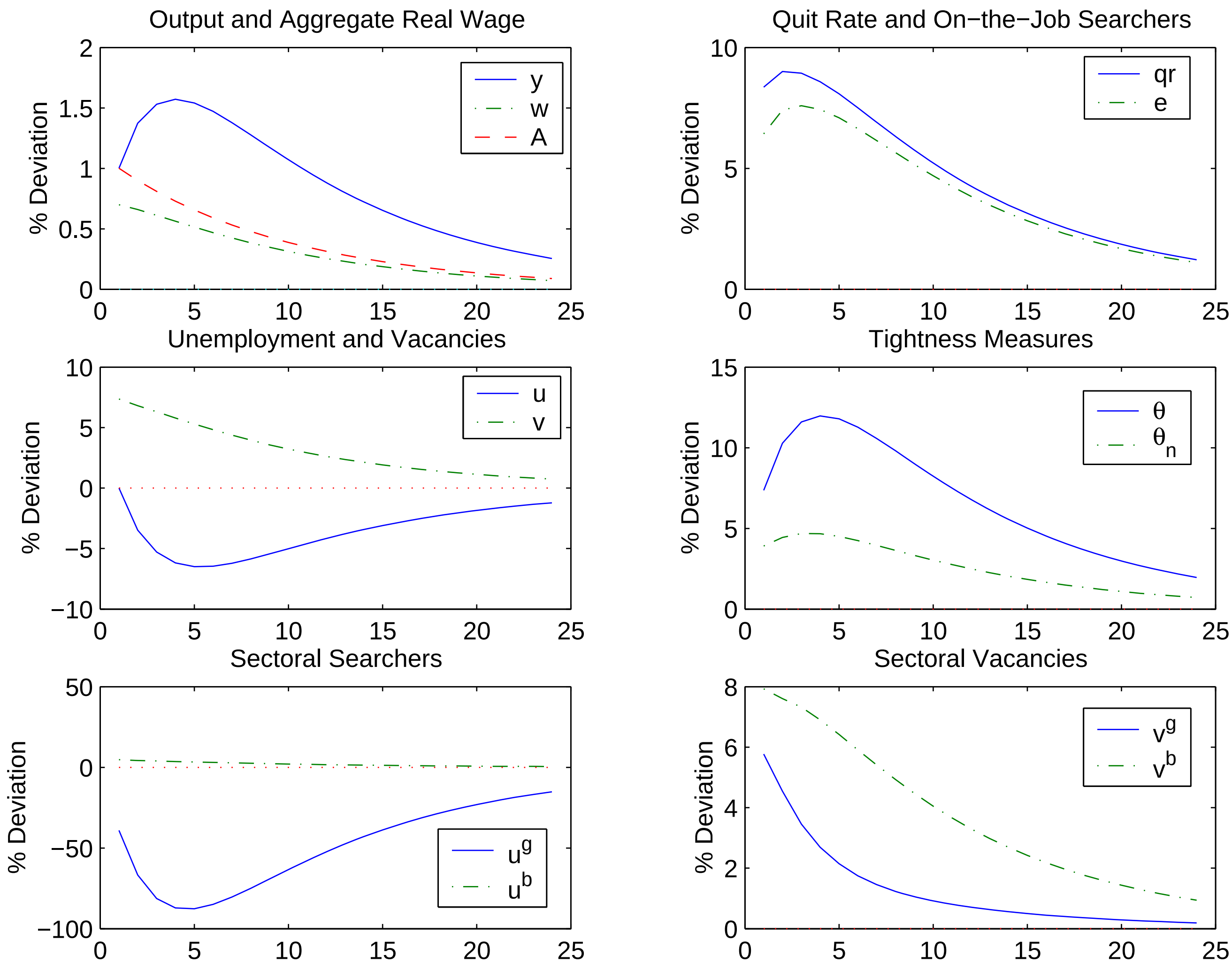

Figure 3: Impulse Response Functions to a 1% Productivity Shock

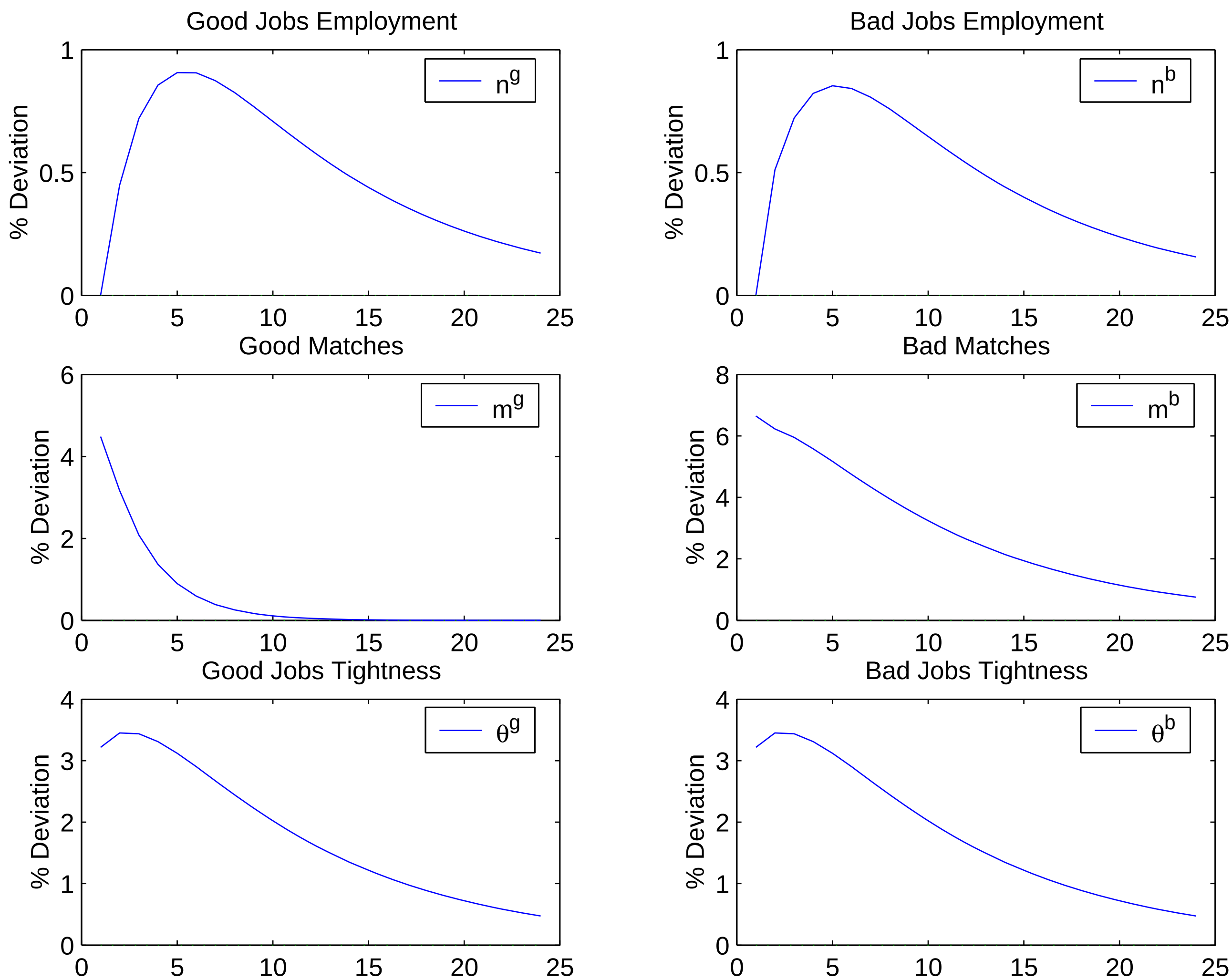

Figure 4: Impulse Response Functions to a 1% Productivity Shock

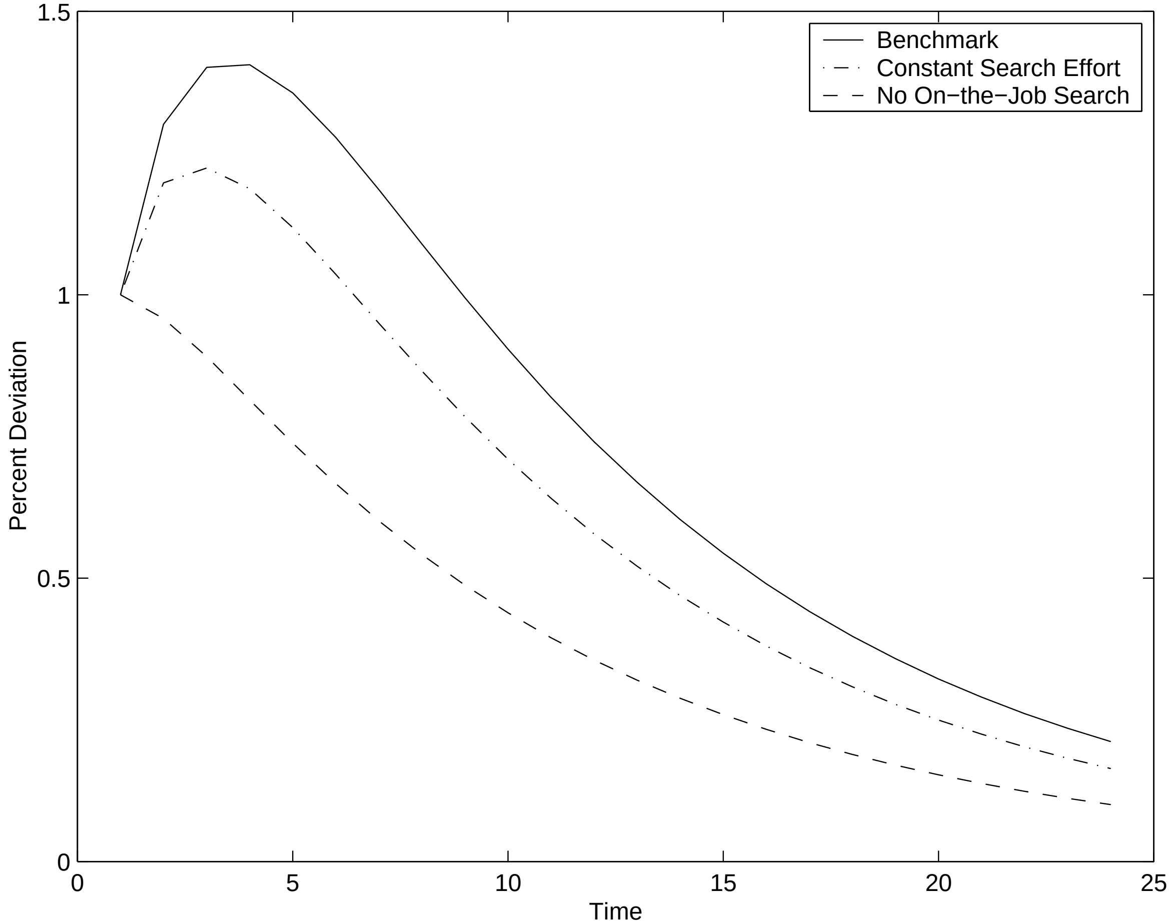

Figure 5: Impulse Responses of Output to a 1% Productivity Shock

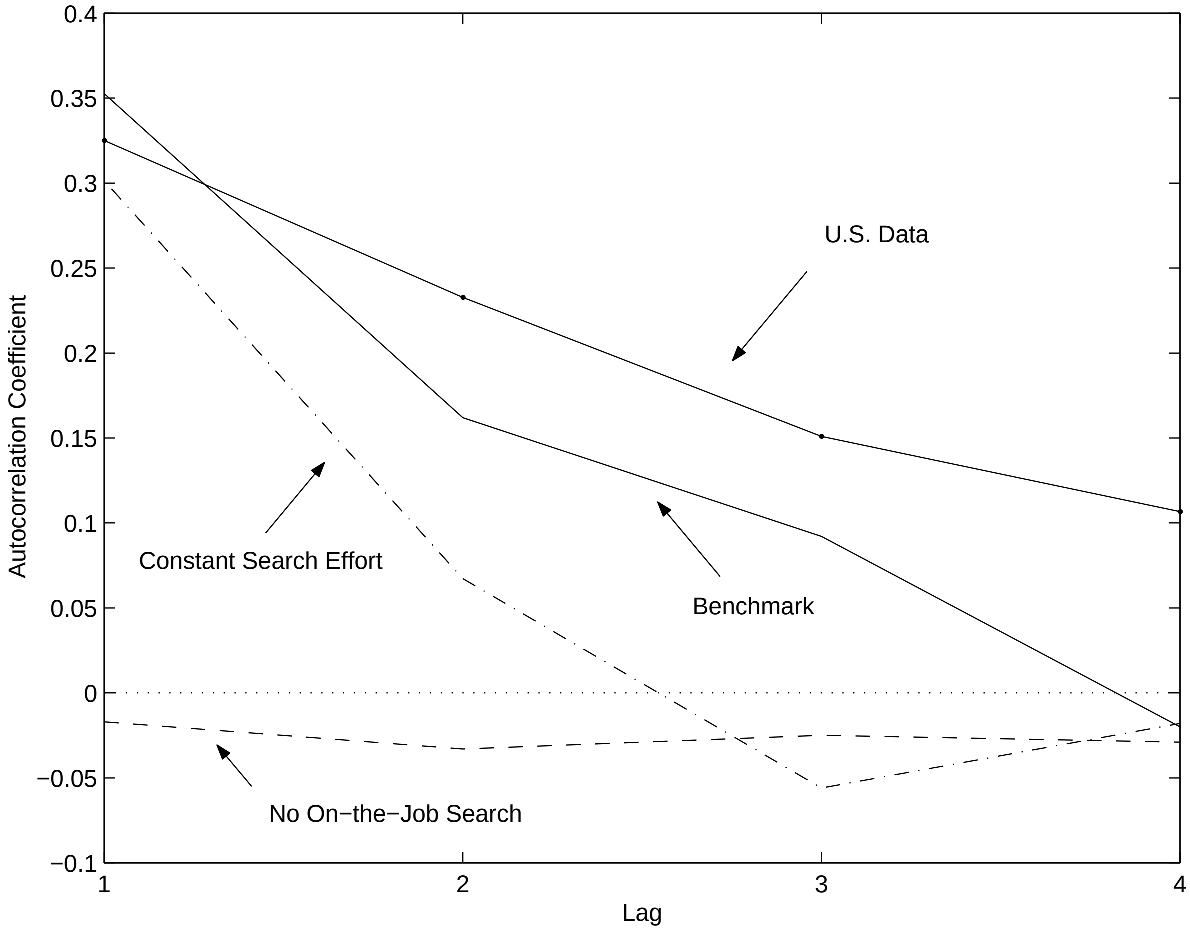

Figure 6: Autocorrelations of Output Growth Rates

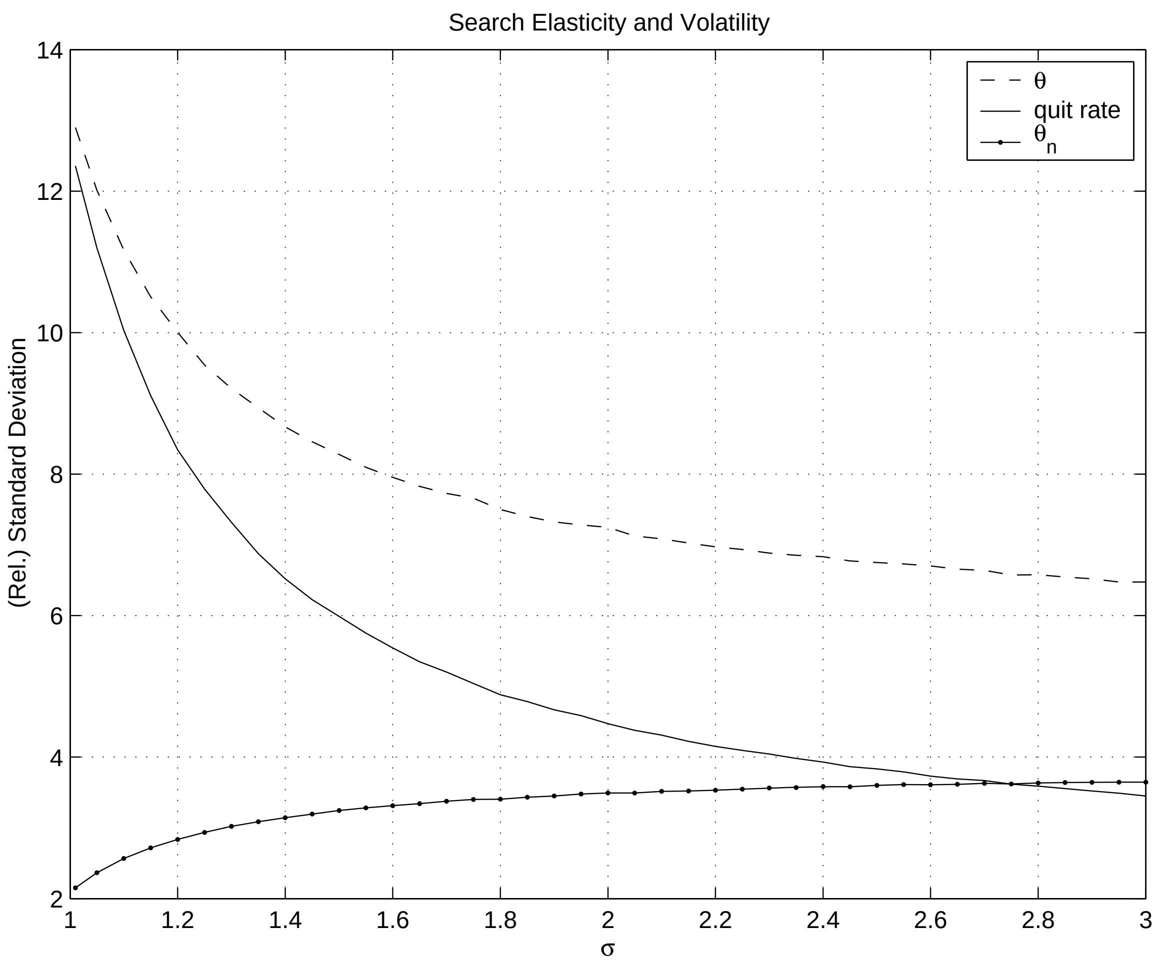

Figure 7: Search Elasticity and Aggregate Volatilities