Uncertainty, Information Acquisition and Price Swings in Asset Markets*

Antonio Mele
London School of Economics

Francesco Sangiorgi
Stockholm School of Economics

First draft: May 4, 2009. This draft: December 14, 2010.

Abstract

This paper studies asset markets where Knightian uncertainty about the fundamentals can be mitigated through costly information acquisition. In these markets, investors' information choices can be strategic complements, resulting in multiple equilibria, history-dependent prices, and large price swings occurring after small changes in uncertainty. Our model makes a number of predictions about the market response to uncertainty shocks, including crashes, followed by sustained rallies and price overshoots, and switches in information regimes, which the model generates due to the information complementarities. Our model highlights uncertainty as a new channel for episodes of extreme price volatility and media frenzies.

*We thank Amil Dasgupta, Simon Gervais, Paolo Ghirardato, Christian Julliard, Emre Ozdenoren, Joël Peress, Laura Veldkamp, Michela Verardo and Kathy Yuan for discussions, and seminar participants at Amsterdam Business School, Berkeley-Haas, Bocconi University, Cass Business School, City University of Hong Kong, University of Hong Kong, USI Lugano (Institute of Finance), Warwick Business School, brown bags at our institutions, the 2010 Financial Intermediation Research Society, and the 2010 Western Finance Association, for comments. The usual disclaimer applies.

1. Introduction

One standard assumption in financial economics is that rational decision makers are able to figure out the probability distribution of the events that affect asset prices. "Ambiguity," or Knightian uncertainty, is an alternative way to describe the information context where agents operate. In a world of Knightian uncertainty, some events cannot be assigned an obvious probability distribution. The literature on the impact of Knightian uncertainty and ambiguity aversion on asset prices is expanding at a fast pace (e.g., Epstein and Wang, 1994, 1995; Uppal and Wang, 2003; Maenhout, 2004; Cao, Wang and Zhang, 2005; Caballero and Krishnamurthy, 2008; Epstein and Schneider, 2008; Hansen and Sargent, 2008; Leippold, Trojani and Vanini, 2008; Ozsoylev and Werner, 2008; Anderson, Ghysels and Juergens, 2009; Caskey, 2009; Easley and O'Hara, 2009; Gagliardini, Porchia and Trojani, 2009; Bossaert, Ghirardato, Guarneschelli and Zame, 2010; Epstein and Schneider, 2010). Two standard assumptions in this literature are that (i) investors are symmetrically informed about the asset payoffs or that, alternatively, (ii) investors with less information do not attempt to resolve their uncertainty by learning from the observed price or do not consider a market for information.

In this paper, we consider a market where investors are ex-ante uninformed about the expected value of the asset fundamentals, and display ambiguity aversion: in formulating decisions about portfolio holdings and information acquisition, agents fear extreme events and worst-case scenarios. Our departure from the previous analyses of Knightian uncertainty in financial markets and macroeconomics is the assumption that the very same agents might resolve their ambiguity, by purchasing information. Those who indeed do so, pay a (constant) cost, as in Grossman and Stiglitz (1980). Those who choose to remain uninformed, instead, cannot entirely resolve their uncertainty about the fundamentals, even after having learned about the equilibrium asset price.

In this market, the value of information is higher than in markets without ambiguity, such as that in Grossman and Stiglitz (1980). In spite of this property, we show that agents who are informed and agents who remain uninformed and, hence, ambiguity averse, coexist, in equilibrium. In fact, we show that a multiplicity of equilibria may occur, as a result of strategic complementarities in the process of information acquisition: the larger the mass of informed agents, the higher the incentives to become informed. Complementaries in information acquisition are at the root of many interesting properties our model generates, such as non-Markovian prices, market crashes and varying levels of informational efficiency, including media frenzies, media neglects, and episodes of extreme volatility. These properties are, of course, in common with other models that feature strategic complementarities in information acquisition (e.g., Froot, Scharfstein and Stein, 1992; Veldkamp, 2006; Barlevi and Veronesi, 2000, 2008; Chamley, 2008; García and Strobl, 2008; Hellwig and Veldkamp, 2009). However, the economic rationale behind our results is quite distinct.

In a market with aversion towards uncertainty, information acquisition is driven by two opposing forces. On the one hand, there is a standard strategic substitutability effect, by which an increase in the number of informed agents leads to more informative prices, which reduces the incentives to acquire information. On the other hand, it is well-known since at least Dow and Werlang (1992) that in the presence of uncertainty aversion, there is an interval of prices within which the agents neither buy nor sell short the asset. In our model, the price distribution is determined in equilibrium, and so is the extent of the agent's market participation. When nobody purchases information, the probability the price falls in the non-participation region is necessarily zero, as the uninformed investors always have to trade to clear the market. This probability is, instead, positive in the presence of informed investors, who step in to clear the market when the uninformed do not trade. Accordingly, in the equilibrium of our model, the extent of market participation of the uninformed investors decreases as the mass of informed agents increases. This reduced market participation leads asset prices to be misaligned from the fundamentals, even more so than in markets without ambiguity, to the entire benefit of the informed agents. Therefore, in the presence of ambiguous fundamentals: (i) information is more valuable than in a market without ambiguity; and (ii) as the mass of informed agents increases, investors buy information that others have, to avoid being hurt from reduced market participation.

The asset price swings our model generates, arise as the outcome of a coordination problem. Consider an asset market where uncertainty about the fundamentals is small. In this market, the incentives to become informed are low, and so is the number of informed agents. Next, suppose that uncertainty increases. For example, some exogenous developments might lead to widen the set of possible scenarios affecting the asset expected payoffs. As the market undergoes such developments, the number of agents who purchase information may stay constant or increase in a continuous fashion, but only up to some critical point, where the market for information enters a media frenzy: the number of agents who desire to acquire information becomes suddenly very high. The critical point occurs precisely when information complementarities kick in: as the number of informed agents increase, information becomes more desirable, to an extent where the market experiences a change in regime characterized by a jump in the number of informed investors. In this new, informationally more efficient market, the uncertainty premium is much lower, and the asset price promptly increases as a result, although then, it may precipitate again, following an uncertainty shrinkage sufficiently large to trigger new information complementarities where information is not desirable by any agent, and the market for information dries up. The market we analyze, then, may be cycling around media frenzies, media neglects, and discrete price changes, as a result of changes in uncertainty.

A multi-period extension of our model suggests an explanation for the rebounds and overshoots we observe in the data. The explanation combines the information demand channel of our basic model with a friction in the market for information. After an uncertainty shock, the market crashes: although agents rush to acquire new information, the market for information cannot entirely satisfy the new demand but with some delay, which leads to a sudden, but temporary, increase in the uncertainty premium induced by the uninformed investors. But as soon as information delays are absorbed, strategic complementarities channel a sustained process of information acquisition, leading to a rapid reduction in the uncertainty premium and, then, a substantial market rally. Our model predicts price overshoots, again, precisely because of the information complementarities: whilst information delays dissipate and demand for new information is satisfied, the market leans towards a new regime, where the asset price is informationally more efficient and, hence, even higher than before the shock.

The paper is organized as follows. In the next section, we develop the model. Section 3 describes the process of information acquisition and Section 4 analyzes the properties of the model, such as information complementarities, multiple equilibria and price swings. Section 5 presents the multi-period market. Section 6 concludes. The Appendices contain details omitted from the main text.

2. Model

2.1. Agents and assets

We consider a market for a risky asset, with payoff equal to $f = \theta + \epsilon$, where $\theta \sim N(\mu, \omega_\theta)$ and $\epsilon \sim N(0, \omega_\epsilon)$. As in Grossman and Stiglitz (1980), the market is populated by a continuum of agents, with a fraction λ of informed agents and a fraction of $1 - \lambda$ of uninformed agents. Informed agents observe θ at cost $c > 0$. The asset supply is $z \sim N(\mu_z, \omega_z)$ and prevents information to be fully revealed in equilibrium. A riskless asset is also available for trading, which is in perfectly elastic supply, and yields a rate of return equal to zero. All agents have negative exponential utility, with constant absolute risk aversion τ.

Our point of departure from Grossman and Stiglitz (1980) is the assumption that all agents are ex-ante uncertain about the expected value of the fundamentals. Although they are unable to assess what μ is, they believe it belongs to some interval, $\mu \in [\underline{\mu}, \bar{\mu}]$, where for some $\Delta\mu \geq 0$, we assume that $\underline{\mu} = \mu_0 - \frac{1}{2}\Delta\mu$ and $\bar{\mu} = \mu_0 + \frac{1}{2}\Delta\mu$. We set $\mu_0 = 0$. The length of this interval, $\Delta\mu$, measures the degree of ambiguity that the investors face in the market. We assume that agents display ambiguity aversion in that they have maxmin expected utility, as formalized by Gilboa and Schmeidler (1989) (see below). We initially take the value of λ as given, although a fundamental purpose of the paper is to determine this value endogenously, as a result of the information acquisition process. In Section 4, we comment on a variant of the model solved in Appendix C, where uncertainty about μ cannot be resolved, in that paying a cost $c > 0$ would only lead to shrink the degree of ambiguity from $\Delta\mu$ to $\xi\Delta\mu$, with $\xi \in (0, 1)$.

2.2. Informed agents

By observing the realization of θ, informed agents resolve their ambiguity straight away, and choose portfolio holdings so as to maximize,

$$v_I(\theta) = E\left(-e^{-\tau W_I} \middle| \theta, p\right),$$

where $W_I = (f - p)\, x_I - c$, p is the observed asset price and, finally, x_I is the asset demand, given by:

$$x_I(\theta, p) = \frac{E(f|\,\theta, p) - p}{\tau Var(f|\,\theta, p)} = \frac{\theta - p}{\tau \omega_\epsilon}.$$

Naturally, while informed agents are able to dissipate their uncertainty about θ, they cannot eliminate risk: conditionally upon θ, the fundamentals, f, are still normally distributed with expectation θ and variance ω_ϵ, as in Grossman and Stiglitz (1980).

2.3. Uninformed agents

The uncertainty about the expected value of the fundamentals, μ, leads the uninformed agents to choose portfolio holdings, so as to maximize,

$$v_U(p) = \min_\mu E_\mu\left(-e^{-\tau W_U} \middle| p\right) = -e^{-\tau \min_\mu E_\mu(W_U|p) + \frac{1}{2}\tau^2 var(W_U|p)}, \tag{1}$$

where $W_U = (f - p)\, x_U$, x_U is the asset demand, and $E_\mu(\cdot)$ is defined to be the expectation operator taken under the assumption $E(\theta) = \mu$. The criterion underlying Eq. (1) is the celebrated maxmin expected utility representation of aversion to Knightian uncertainty, introduced by Gilboa and Schmeidler (1989).

We conjecture that for every pair (θ, z), the equilibrium price function is $P(\theta, z)$. We look for an equilibrium in which the uninformed agents sell the asset when the price is sufficiently high and buy the asset when the price is sufficiently low, in a sense to be made precise below. As we shall show, this search process leads to a simpler problem, in which the uninformed agents' concern is to determine the expectation of the fundamentals in the states of nature in which they buy and sell. Accordingly, let us introduce the following notation,

$$E^{\text{buy}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)\equiv E_{\underline{\mu}}\left(f|\,P\left(\cdot,\cdot\right)=p\right),\quad E^{\text{sell}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)\equiv E_{\bar{\mu}}\left(f|\,P\left(\cdot,\cdot\right)=p\right).$$

We conjecture that the solution to the uninformed agents' problem is,

$$x_U\left(p,P\left(\cdot,\cdot\right)\right)=\begin{cases}\dfrac{E^{\text{buy}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)-p}{\tau Var\left(f|\,P\left(\cdot,\cdot\right)=p\right)}, & \text{for } p<E^{\text{buy}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)\\ 0, \quad \text{for } p\in\left[E^{\text{buy}}\left(f|\,P\left(\cdot,\cdot\right)=p\right),E^{\text{sell}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)\right] & \\ \dfrac{E^{\text{sell}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)-p}{\tau Var\left(f|\,P\left(\cdot,\cdot\right)=p\right)}, & \text{for } p>E^{\text{sell}}\left(f|\,P\left(\cdot,\cdot\right)=p\right)\end{cases}\tag{2}$$

In words, the uninformed agents do not participate in the market if the observed equilibrium price does not take a sufficiently favorable value. This value has to be such that the agents believe that in the worst-case scenario, they can actually make profits, on "average." In particular, the uninformed agents enter the market as buyers (sellers) when the price realization, p, is less (larger) than the agents' worst-case scenario expectation of the asset value, conditional upon p. Hence, the decision to participate involves a fixed-point problem, in which the expectation of the asset value, conditional on the price realization, is equal to the very same price realization, in equilibrium,

$$E^{\text{buy}}\left(f|\,P\left(\cdot,\cdot\right)=\underline{p}\right)=\underline{p}\quad\text{and}\quad E^{\text{sell}}\left(f|\,P\left(\cdot,\cdot\right)=\bar{p}\right)=\bar{p}.\tag{3}$$

The uninformed agents do not participate in the asset market if the equilibrium price realization, p, is such that $p\in[\underline{p},\bar{p}]$. Naturally, the cutoffs $\underline{p}$ and $\bar{p}$ are endogenous, and we shall verify that in equilibrium, they satisfy $\underline{p}<\bar{p}$.

2.4. Equilibrium

We conjecture that the equilibrium price function is, $P\left(\theta,z\right)=P\left(s\left(\theta,z\right)\right)$, where $s\left(\theta,z\right)$ is the compound signal, defined as,

$$s\left(\theta,z\right)=\frac{\lambda}{\tau\omega_\epsilon}\theta-\left(z-\mu_z\right).\tag{4}$$

From the market clearing condition,

$$(1-\lambda)\, x_U\,(p, P\,(\cdot)) + \lambda x_I\,(\theta, p) = z, \tag{5}$$

we easily see that the compound signal is observationally equivalent to the equilibrium price. Therefore, the equilibrium in this market is also one in which uninformed agents condition the expectation of the asset value on the compound signal.

We have:

Proposition I. *The equilibrium price is piecewise linear in the compound signal,*

$$P\,(s) = \begin{cases} \underline{a} + bs, & \text{for } s < \underline{s} \\ a + \dfrac{\tau\omega_\epsilon}{\lambda} s, & \text{for } s \in [\underline{s}, \bar{s}] \\ \bar{a} + bs, & \text{for } s > \bar{s} \end{cases} \tag{6}$$

for some constants $\underline{a}, \bar{a}, a, b$ *given in Appendix A. The threshold values for the compound signal,* $\underline{s}, \bar{s}$, *satisfy:*

$$\underline{s} = \frac{\lambda}{\tau\omega_\epsilon}\underline{\mu} + \frac{\omega_s}{\omega_z}\mu_z, \quad \bar{s} - \underline{s} = \frac{\lambda}{\tau\omega_\epsilon}\Delta\mu,$$

where ω_s *is the variance of* s *in Eq. (4). Finally, we have that* $\underline{p} < \bar{p}$, *where the expressions for* $\underline{p}$ *and* $\bar{p}$ *are given in the Appendix A.*

Figure 1 depicts the equilibrium price in Proposition I. The solid line is the price schedule arising in the presence of ambiguity, $\Delta\mu > 0$. The dashed line is the benchmark price in the Grossman and Stiglitz (1980) model. In the top panel, the proportion of informed agents is $\lambda = 0.2$, while in the bottom panel, $\lambda = 0.5$. In equilibrium, the uninformed agents' portfolio choice, as formalized in Eq. (2), reflects the expected returns in the worst-case scenarios: the uninformed agents buy when $s < \underline{s}$ (sell when $s > \bar{s}$), but less aggressively than they would do in the absence of ambiguity. Such a pessimistic behavior leads to a price lower (higher) than the benchmark for low (high) realizations of the compound signal, s. As the proportion of informed agents, λ, increases, the price impact of uninformed (and ambiguity averse) agents is reduced, and so is the extent of this price impact, as illustrated by Figure 1.

When the compound signal, s, lies within the range $[\underline{s}, \bar{s}]$, the uninformed agents do not participate in the market. Proposition I tells us that the non-participation region, $\bar{s}-\underline{s}$, is proportional to the size of the ambiguity in the market, $\Delta\mu$. The proportionality factor, $\frac{\lambda}{\tau\omega_\epsilon}$, is the total risk-bearing capacity of the informed agents, defined as the mass of informed agents, λ, times their trading aggressiveness, $\frac{1}{\tau\omega_\epsilon}$. As the informed risk-bearing capacity increases, prices move towards fundamentals. It now takes more extreme realizations of the compound signal, s, for prices to be favorable enough and induce uninformed agents to trade, in equilibrium. Therefore, the non-participation region widens.

The non-participation region is proportional to $\Delta\mu$ for the following reasons. Consider the comparative statics of a change in $\underline{\mu}$ and $\bar{\mu}$. If $\underline{\mu}$ increases, $E^{\text{buy}}\left(f|\, P\left(\cdot,\cdot\right)=p\right)$ increases as well, for each price realization p, but then the threshold equilibrium price at which the agent does not buy the asset, $\underline{p}$, has to increase, by the fixed point problem defined by Eqs. (3). This requires that $\underline{s}$ increase. A similar reasoning leads to the conclusion that as $\bar{\mu}$ decreases, $\bar{s}$ does necessarily have to decrease as well.

Finally, this market exhibits a feature about the information transmitted by the price. By observing the price, the uninformed investors learn about the fundamentals, in that they experience a reduction in their initial uncertainty about the expected payoffs:

$$0 < E_{\bar{\mu}}\left(f|\, P\left(\cdot\right)=p\right) - E_{\underline{\mu}}\left(f|\, P\left(\cdot\right)=p\right) < E_{\bar{\mu}}\left(f\right) - E_{\underline{\mu}}\left(f\right) \equiv \bar{\mu} - \underline{\mu}, \tag{7}$$

for each price realization p (see Appendix A). However, such a shrinkage in uncertainty is incomplete, as the first inequality in (7) reveals. In other words, the price does not reveal all the information informed investors have, only a portion of it.

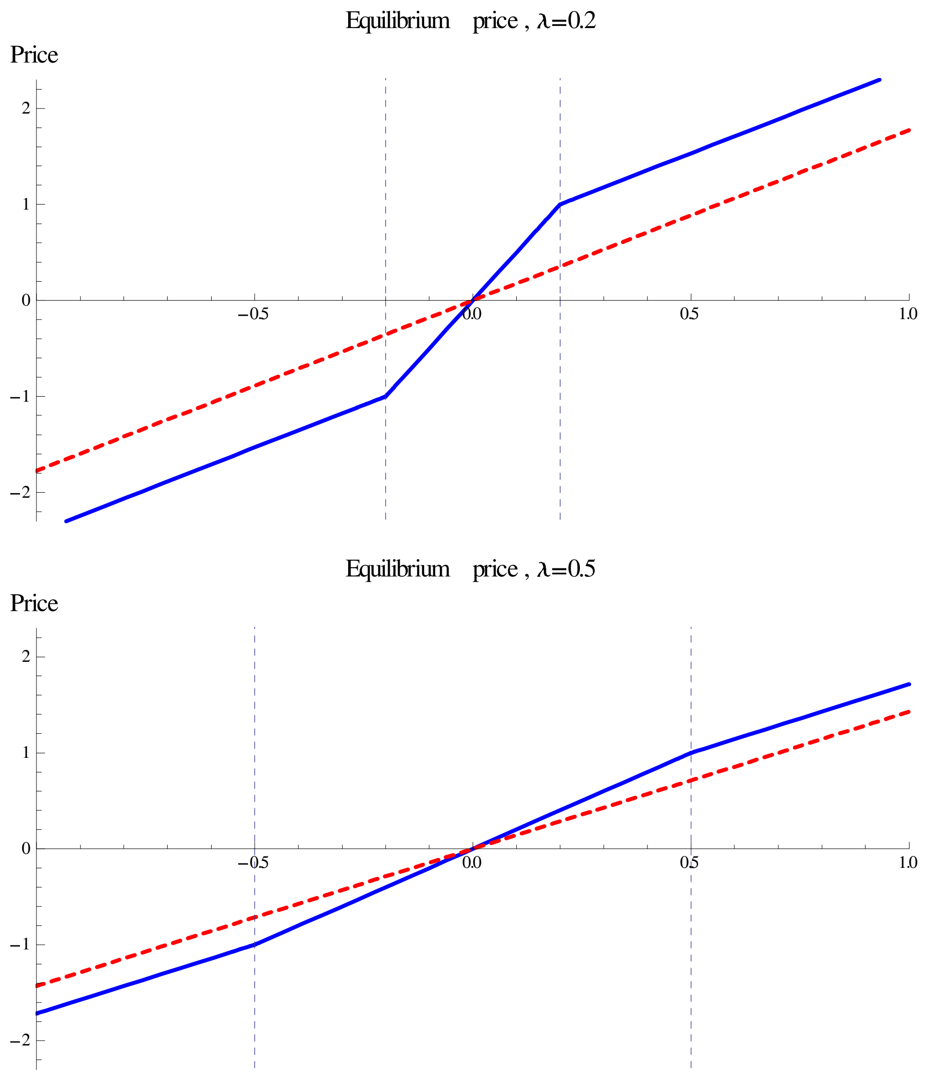

FIGURE 1. This picture depicts the equilibrium asset price in Proposition I, as a function of the compound signal, s. Both panels compare the price function with the Grossman-Stiglitz linear function (the dashed line), which arises in the absence of ambiguity in the market, $\Delta\mu = 0$. The region delimited by the vertical dashed lines is where ambiguity averse agents do no participate. Parameters values are $\Delta\mu = 2$, $\omega_\theta = \omega_\epsilon = \omega_z = \tau = 1$, and $\mu_z = 0$. In the top panel, the proportion of informed agents, $\lambda = 0.2$, and in the bottom panel, $\lambda = 0.5$.

3. Information acquisition

This section analyzes how ambiguity affects the incentives to acquire fundamental information, and solve for the endogenous fraction of informed agents, λ. As in Grossman and Stiglitz (1980), all agents need to evaluate ex-ante expected utilities, before deciding whether to become informed or not. However, the process of information acquisition differs from that in Grossman and Stiglitz, in that *all* agents are ex-ante ambiguity averse, which leads them to assess future events at the worst-case scenarios.

3.1. Uninformed agents

The ex-ante expected utility for a would-be uninformed agent is:

$$\mathcal{U}_U(\lambda) = \min_{\mu} E_\mu \left[v_U \left(s \left(\theta, z \right) \right) \right], \tag{8}$$

where $v_U(s)$ is the interim utility for the uninformed agents, defined as

$$v_U(s) = -e^{-\tau \mathcal{C}_U(s)}, \quad \mathcal{C}_U(s) = \min_{\mu} E_\mu \left(W_U | \, s \right) - \frac{1}{2} \tau var \left(W_U | \, s \right).$$

By Eq. (4), the compound signal s is normally distributed, with mean $\mu_s(\mu)$ and variance ω_s, where,

$$\mu_s(\mu) \equiv \frac{\lambda}{\tau \omega_\epsilon} \mu.$$

In Appendix B, we provide a closed-form expression for the unconditional expectation of the interim utility:

$$E_\mu \left[v_U(s) \right] = \int_{-\infty}^{\infty} v_U(t) \, d\Phi \left(t; \mu_s(\mu), \omega_s \right),$$

where $\Phi(\cdot; \mu, \omega)$ denotes the cumulative function of a normal variate with mean μ and variance ω.

Figure 2 depicts the interim utility, $v_U(s)$, and the density function $d\Phi$ of the compound signal, s. The interim utility achieves its minimum in the non-participation region, where the interim certainty equivalent $\mathcal{C}_U(s)$ is flat at zero. Moreover, it is monotonically increasing, and symmetric, as the compound signal moves away from the non-participating thresholds $\underline{s}$ and $\bar{s}$. The next proposition provides the solution to the problem in Eq. (8):

Proposition II. *Let $\mu_z \geq 0$. Then, the ex-ante expected utility of the uninformed agents, $\mathcal{U}_U(\lambda, \mu)$,* *is minimized at,*

$$\mu_U(\lambda) = \min\left\{\frac{\tau\omega_\epsilon}{\lambda}\frac{\omega_s}{\omega_z}\mu_z, \bar{\mu}\right\}.$$

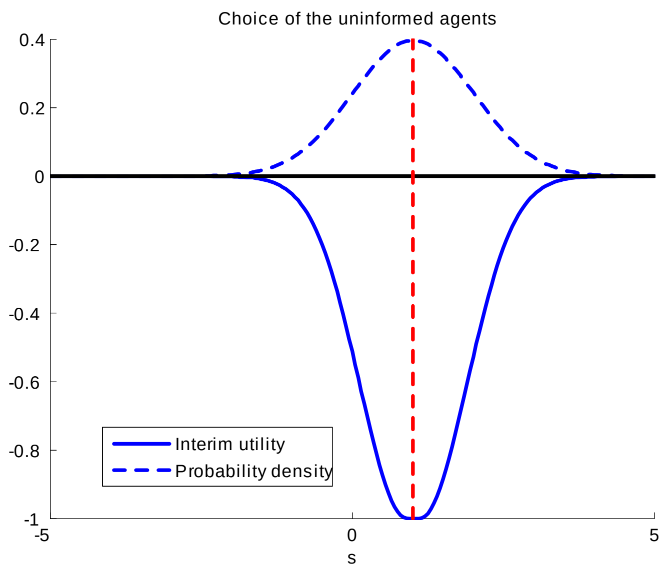

FIGURE 2. This picture depicts the identification and assessment of the worst-case scenario made by uninformed agents. The worst-case scenario occurs over the non-participation region, $[\underline{s}, \bar{s}]$, where the interim utility attains its minimum. Accordingly, the interim utility is given the largest probability weight at $\hat{s} = \frac{1}{2}(\underline{s} + \bar{s})$. The vertical dashed line connects the probability density to the interim utility at the point $\hat{s}$. Parameters values are $\Delta\mu = 2$, $\omega_\theta = \omega_\epsilon = \omega_z = \tau = 1$, $\lambda = 0.1$, and $\mu_z = 1$. The resulting value of $\hat{s}$ is 1.01.

The economic mechanism underlying Proposition II is the following. The uninformed agents attach the largest probability to the occurrence of the worst events, and choose μ in such a way that the expected value of the signal, $\mu_s(\mu)$, is as close as possible to the midpoint in the non-participation region, $\hat{s} = \frac{1}{2}(\underline{s} + \bar{s})$. Naturally, $\mu_U(\lambda)$ is increasing in the average asset supply, μ_z: following an increase in μ_z, for markets to clear, the probability the uninformed agents enter as buyers (sellers) must increase (decrease) and as a result, the non-participation region shifts to the right.

3.2. *Informed agents*

The ex-ante expected utility for a would-be informed agent is,

$$\mathcal{U}_I\left(c,\lambda\right)=\min_{\mu}E_{\mu}\left[v_I\left(\theta,s\left(\theta,z\right)\right)\right], \tag{9}$$

where $v_I\left(\theta,s\right)$ is the interim utility for any informed agent, defined as

$$v_I\left(\theta,s\right)=-e^{-\tau\left(\mathcal{C}_I\left(\theta,s\right)-c\right)},\quad \mathcal{C}_I\left(\theta,s\right)=\frac{1}{2}\frac{\left(\theta-P\left(s\right)\right)^2}{\tau\omega_\epsilon},$$

and the equilibrium price, $P\left(s\right)$, is as in Eqs. (6) of Proposition I.

In Appendix B, we provide a closed-form expression for the unconditional expectation of the interim utility,

$$E_{\mu}\left[v_I\left(\theta,s\left(\theta,z\right)\right)\right]=e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\cdot E_{\mu}\left[\bar{v}_I\left(s;\mu\right)\right], \tag{10}$$

where $\omega_{f|s}$ denotes the variance of the fundamentals, f, conditional on the compound signal s, and $\bar{v}_I\left(s;\mu\right)$ is some negative function defined in Appendix B.

3.3. *The value of information*

An equilibrium with endogenous information acquisition is defined in the usual way, as the fraction of informed agents, $\lambda^*\in[0,1]$, that makes any agent ex-ante indifferent whether to be informed or not, $\mathcal{U}_I\left(c,\lambda^*\right)=\mathcal{U}_U\left(\lambda^*\right)$, or,[1]

$$\frac{\mathcal{U}_I\left(c,\lambda^*\right)}{\mathcal{U}_U\left(\lambda^*\right)}=\underbrace{e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}}_{\text{Grossman-Stiglitz effect}}\cdot\underbrace{\frac{E_{\mu_I}\left[\bar{v}_I\left(s;\mu\right)\right]}{E_{\mu_U}\left[v_U\left(s\right)\right]}}_{\text{Ambiguity aversion effect}}=1, \tag{11}$$

where μ_I and μ_U solve the two problems in Eqs. (8) and (9).

The left hand side of Eq. (11) is the value of information, evaluated at λ^*. It is the product of two terms. The first term is the usual value of information in the Grossman and Stiglitz (1980) model, the benchmark without ambiguity, $\Delta\mu=0$. It summarizes the usual trade-off between the cost of acquiring information and its benefits, in terms of the informational advantage over the uninformed

[1] Non-interior equilibria are defined in the usual way, as $\lambda^*=0$ such that $\mathcal{U}_I\left(c,0\right)<\mathcal{U}_U\left(0\right)$ and $\lambda^*=1$ such that $\mathcal{U}_I\left(c,1\right)>\mathcal{U}_U\left(1\right)$.

fringe. The effect of ambiguity on the incentives to acquire fundamental information is captured by the additional term in Eq. (11), which we label “ambiguity aversion effect.” The next proposition relates ambiguity to the value of information:

Proposition III. *Let $\Delta\mu > 0$. Then, the ratio $\frac{E_{\mu_I}[\bar{v}_I(s;\mu)]}{E_{\mu_U}[v_U(s)]}$ in Eq. (11) is less then one. That is, information is more valuable in a market with ambiguous fundamentals ($\Delta\mu > 0$) than in a market without ambiguity* ($\Delta\mu = 0$).

The additional benefits of collecting fundamental information, due to the presence of ambiguous fundamentals, can be better understood by comparing the welfare of both types of agents to a benchmark without ambiguity. First, *for any* realization of the fundamentals, uninformed agents trade lower quantities than if there was no ambiguity (or if they were ambiguity neutral), as explained in Section 2. Therefore, by giving up investment opportunities, uninformed agents experience lower expected utility. Such a welfare reduction is actually reinforced from an ex-ante perspective: while assessing the outcomes arising from being uninformed at the trading stage, agents attach the largest probability weight to those future states in which participation is the lowest, as formalized in Proposition II and illustrated in Figure 2.

Second, informed investors benefit from the price impact of uninformed ambiguity-averse investors, as illustrated in Figure 1: they can buy at lower prices and sell at higher prices, thus making higher profits.

Since the value of information increases in the presence of ambiguity, we immediately obtain the following result on the amount of resources spent on collecting information:

Corollary 1. *Information is purchased by more agents in the presence of ambiguity than in markets without ambiguity.*

4. Information complementarities, multiple equilibria and price swings

Complementarities in information acquisition arise when the incentives to acquire information become stronger with the size of informed agents. This section analyzes conditions leading to this situation, and their implications for the asset price.

4.1. Complementarities in information acquisition

The following proposition identifies sufficient conditions under which ambiguity leads to complementarities in the process of information acquisition:

PROPOSITION IV. *Let $\Delta\mu > 0$. Then, there exists a level of the average asset supply $\bar{\mu}_z > 0$, such that there are complementarities in information acquisition for all $\mu_z > \bar{\mu}_z$.*

As the fraction of informed agents λ increases, there are two opposing forces that affect the incentives to acquire information. The first relates to the standard strategic substitutability effect, which is well-known since Grossman and Stiglitz (1980): more informed trading increases price efficiency, which reduces the informational advantage of the informed agents above the uninformed. This effect is still present in our model, as the first term in Eq. (11) is monotonically increasing in λ. Our analysis uncovers a second effect, specific to ambiguity, and captured by the second term in Eq. (11): the volume of uninformed trading decreases with the mass of informed agents, which makes uninformed agents worse off, ex-ante. Proposition IV shows that the ambiguity aversion effect may dominate the strategic substitutability effect, thereby generating strategic complementarities in the process of information acquisition.

The role the average asset supply, μ_z, plays in generating these information complementarities is subtle. First, note that the informed agents' ex-ante utility is also decreasing in λ, as a reduction in the mass of uninformed agents reduces the extent of the price impact informed agents benefit from (see Figure 1). This effect might counter-balance the net effect of an increase in λ on relative welfare, but it becomes less relevant for larger values of the average asset supply. Consider, for example, the case in which the market is only populated by uninformed investors, in which case the pricing effect of ambiguity aversion is the highest. If the asset supply is sufficiently high, on average, agents will be buyers most of the time. With uninformed investors holding the positive supply and being price setters, prices reflect an ambiguity premium: low expected payoffs, $\underline{\mu}$, translate into low prices. The worst-case scenario for an agent considering, ex-ante, to become informed, then, is that the expected payoffs are indeed low (i.e. $\mu_I = \underline{\mu}$), so that the perceived ambiguity premium (and the benefits from it) vanish. If the ex-ante perceived ambiguity premium is low to start with, then, as λ increases, the shrinkage in the ex-ante utility of the informed investors is weak, compared to the loss in the ex-ante utility of the uninformed. As a result, the ambiguity aversion effect in Eq. (11) strengthens with λ when μ_z is large enough, thereby channelling information complementarities.

Do these results arise by the assumption that informed agents resolve *all* of their ambiguity? It is not the case. In Appendix C, we consider a model where uncertainty cannot be resolved: paying a cost $c > 0$ leads informed agents to reduce their ambiguity from $\Delta\mu$ to $\xi\Delta\mu$, with $\xi \in (0, 1)$. We show that complementarities in information acquisition arise even in this case. This result sheds further light on our findings: because the Grossman and Stiglitz (1980) framework can only lead to strategic substitutability in information acquisition, then, clearly, information complementarities arise in our model because agents are heterogeneous in their ambiguity towards fundamentals.

4.2. Multiple equilibria, crashes and rebounds

Information complementarities may lead to multiple equilibria. As an illustration, Figure 3 displays the value of information, as a function of λ, obtained for two degrees of ambiguity, $\Delta\mu$.[2] The solid line, which corresponds to $\Delta\mu = 1$, leads to three equilibria. Two of these, $\lambda^* = \lambda_U$ and $\lambda^* = \lambda_S$, are interior equilibria: the leftmost equilibrium (λ_U) is unstable, and the rightmost (λ_S) is stable. The third, and stable, equilibrium is that with $\lambda^* = 0$. As $\Delta\mu$ increases, the value of information increases, for each λ, and shifts the leftmost (unstable) equilibria to the left, and the rightmost (stable) equilibria to the right. When $\Delta\mu$ is sufficiently high, there remains one equilibrium only, and stable. The dashed line in Figure 3, which corresponds to $\Delta\mu = 1.30$, depicts an example of such a situation.

Figure 4, right panel, depicts the unconditional expectation of the equilibrium price, assuming the asset is in positive supply, as a function of the size of ambiguity, when the proportion of agents who acquire information is determined endogenously, as in the left panel. A negative price, i.e. a price discount, reflects the positive expected return that is required by the agents to hold the asset. The figure shows how, for low levels of ambiguity, when the economy is in its "media neglect" regime, an increase in $\Delta\mu$ leads to a larger price discount, as the price reflects the possible occurrence of increasingly more severe worst-case scenarios. As the size of ambiguity gets sufficiently large, and the economy shifts to its "media frenzy" regime, more investors purchase information. This jump in the size of informed agents implies a discrete reduction both in the price impact related to ambiguity aversion and in the conditional risk perceived by the market, reducing the price discount. As a consequence, the average price jumps up. Moreover, within this regime, the equilibrium fraction of informed agents increases with the size of ambiguity, such that higher values of $\Delta\mu$ lead to a lower price discount. Therefore, our model predicts a non-monotonic relation between the degree of Knightian uncertainty and uncertainty premia. Furthermore, the price inherits the same properties as those featured by the proportion of informed agents: it exhibits path-dependence and different

[2] Note that due to negative exponential utility, lower values of the ratio in Eq. (11) mean higher values of information.

jump sizes, according to whether the size of ambiguity is increasing or decreasing.

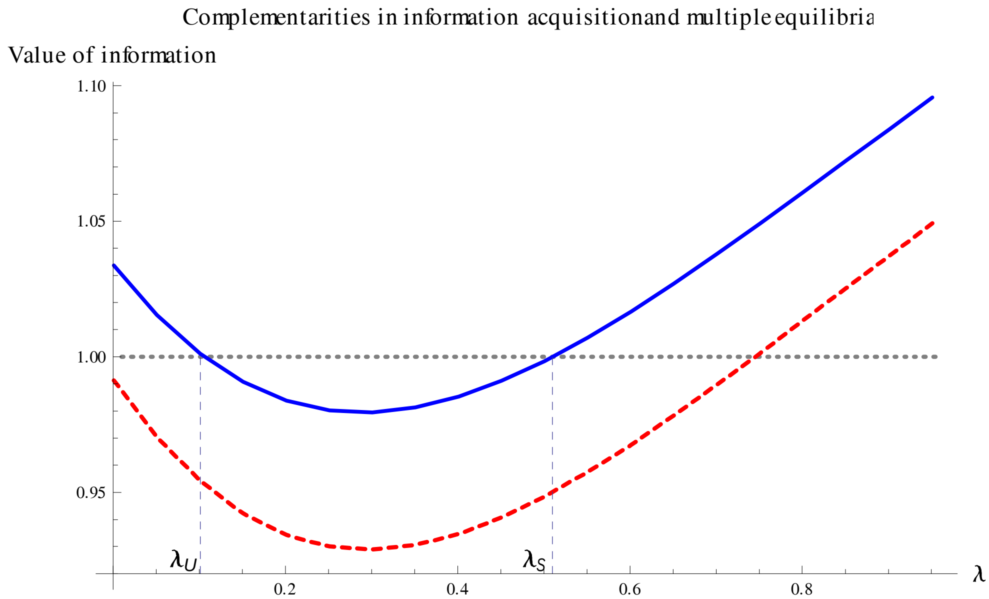

FIGURE 3. This picture depicts the value of information, $\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)}$, as a function of the fraction of informed agents, λ, for a given cost of information, c. Parameters values are $\omega_\theta = \omega_\epsilon = \omega_z = \tau = \mu_z = 1$, and $c = 0.5$. The solid (dashed) line is the value of information for $\Delta\mu = 1$ ($\Delta\mu = 1.30$).

Figure 4 (left panel) depicts the proportion of agents who acquire information, as a function of the size of ambiguity, $\Delta\mu$. We can interpret changes in $\Delta\mu$ as those that result in a repetition of two-period markets, as we further elaborate in the next section. When ambiguity is low, say $\Delta\mu = 0.5$, the market is in its "media neglect" regime. If ambiguity increases, say to 1.30, the proportion of agents who become informed increases by a discrete change: from zero, to nearly 75%, a "media frenzy" regime. As $\Delta\mu$ decreases back to, say, 0.8, the market for information precipitates again. The model, then, generates path-dependence: for any size of ambiguity $\Delta\mu$ between the two vertical dashed lines, the number of informed agents can be either zero or strictly positive, according to the previous values of $\Delta\mu$. Accordingly, given that the value of information increases with $\Delta\mu$, the jump size in the proportion of informed agents is larger when we head towards times of higher uncertainty than when we move back to times of decreased uncertainty.

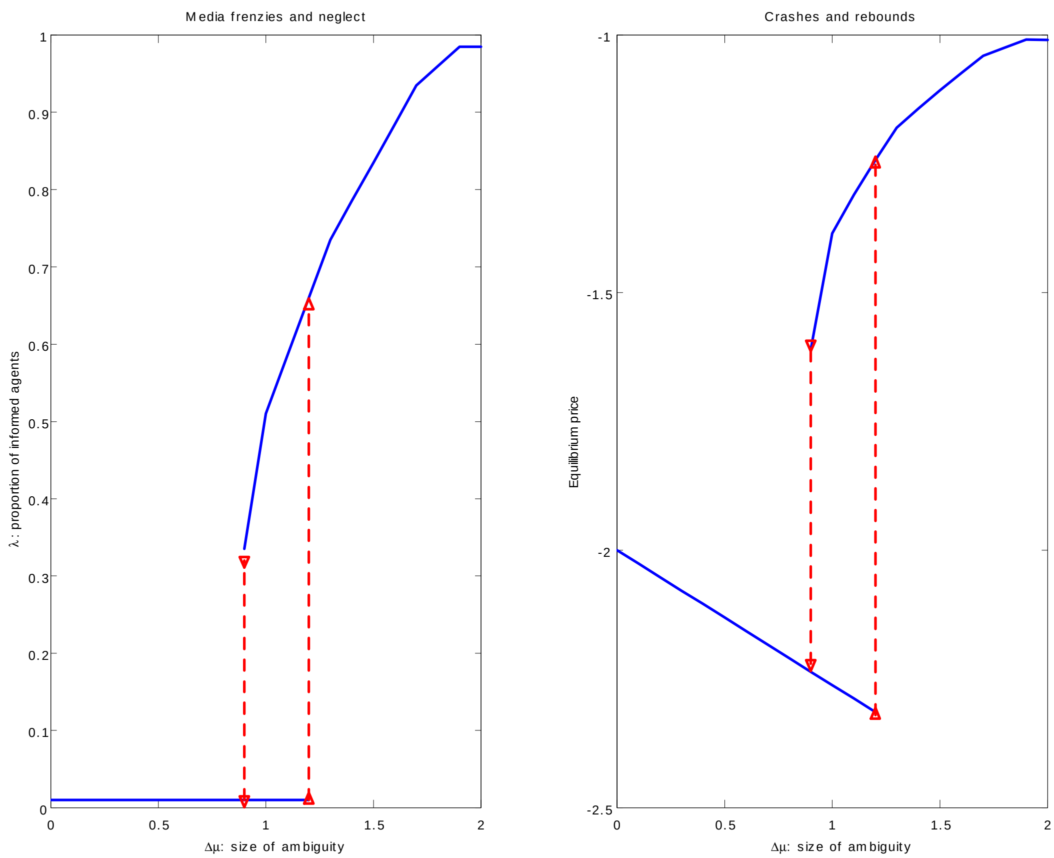

FIGURE 4. Media frenzies, media neglects, and market crashes and rebounds. Parameters values are $\omega_\theta = \omega_\epsilon = \omega_z = \tau = \mu_z = 1$, and $c = 0.5$.

5. Multi-period market

5.1. Information and equilibrium

We consider a sequence of two-period markets, in which the asset payoff as of time t is $f_t = \theta_t + \epsilon_t$, where θ_t denotes its "persistent" component,

$$\theta_{t+1} = (1 - \rho_\theta)\,\mu_t + \rho_\theta \theta_t + \sigma_\theta \theta_t \eta_{t+1}, \tag{12}$$

and ϵ_t and η_t are both independent and identically distributed with $\epsilon_t \sim N(0, \omega_\epsilon)$ and $\eta_t \sim N(0, 1)$, ρ_θ is the persistence parameter, and σ_θ is the instantaneous volatility of θ_{t+1}/θ_t. Finally, μ_t is independent and identically drawn from some distribution with support $[\bar{\mu}, \underline{\mu}]$. As in the static case we denote $\Delta\mu = \bar{\mu} - \underline{\mu}$.

In Appendix D, we show that the equilibrium price is the same as that in Eq. (6) of Proposition I, with some time-varying coefficients and with $var\left(\theta_{t+1} \middle| \theta_t\right) = \sigma_\theta^2 \theta_t^2$ replacing ω_θ, and $var\left(s_t \middle| \theta_t\right) = \frac{\lambda_t^2}{\tau^2 \omega_\epsilon} \omega_\epsilon \sigma_\theta^2 + \sigma_z^2$ replacing ω_s. Finally, in the presence of multiple equilibria, we rely on the following selection criterion: if λ_{t-1}^* is a stable equilibrium at time t, then $\lambda_t^* = \lambda_{t-1}^*$; otherwise λ_t^* is selected to be the stable equilibrium closest to the λ_{t-1}^*.

5.2. Predictions

5.2.1. Price dynamics

We compare the price dynamics to those predicted by the Grossman and Stiglitz (multi-period) market, arising when $\Delta\mu = 0$. To make the comparison meaningful, we fix the information cost c for our model and, then, search for the information cost in the Grossman-Stiglitz market such that the average value of information equals the corresponding value in the market with ambiguity, over all the simulations. Moreover, to avoid additional volatility in the price, we center the ambiguity size, $\Delta\mu$, around μ_t, and set $\mu_t = 1$.

Figure 5 shows that the prices in the market with ambiguity are more volatile than those without. The rationale underlying the price swings in this figure relate to the information complementarities our model generates. An increase in the value of the fundamentals, θ_t, leads to an increased conditional variance, $var\left(\theta_{t+1} \middle| \theta_t\right) = \sigma_\theta^2 \theta_t^2$, which ultimately results in a higher value of information, an effect similar to that in Veldkamp (2006). In our model, information complementarities arise when the increase in θ_t reaches a critical value such that the agents coordinate to switch from an equilibrium with information neglects to one with information frenzies, similarly as for the comparative statics of Figure 3 relating to changes in $\Delta\mu$.

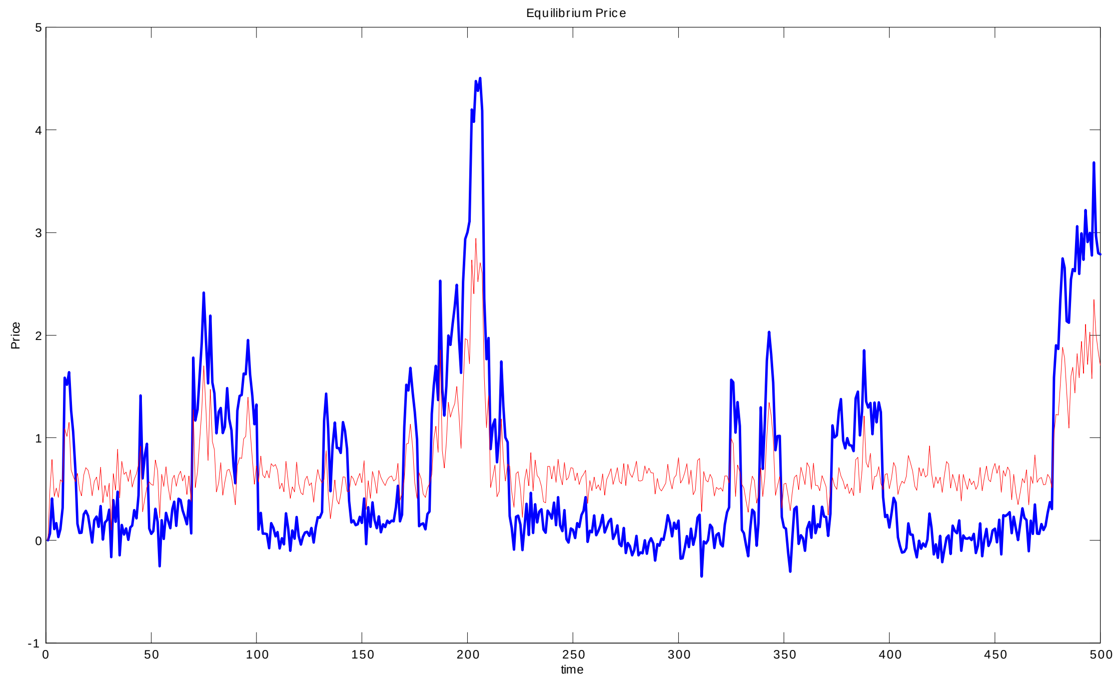

FIGURE 5. Bold solid line: the price in a market with ambiguity. Solid line: the price in market without ambiguity. Parameter values are $\mu_z = 1$, $\tau = 2.5$, $\omega_z = 0.25^2$, $\omega_\epsilon = 0.30^2$, $\rho_\theta = 0.95$, and $\sigma_\theta = 0.10$, $\Delta\mu = 0.5/(1-\rho_\theta)$. The cost of information in the market with ambiguity is $c = 0.164$, and in the market without ambiguity is $c = 0.155$. These values guarantee the average value of information is the same in the two markets.

Our model predicts that the price overshoots the benchmark without ambiguity. These overshoots arise as during information neglects, the price falls substantially, due to ambiguity discounts, while during information frenzies, the price increases for the opposite reason. The downward overshoots are quite peculiar to our model: no model is known to display information complementarities that produce such downward overshoots.

5.2.2. The impact of an uncertainty shock

What is the price impact of an uncertainty shock? We address this issue by assuming uncertainty is a persistent process subject to jumps: once an uncertainty shock hits the market, it is absorbed gradually, such that uncertainty reverts to its long-run value $\overline{\Delta\mu}$,

$$\Delta\mu_{t+1} = \left(1 - \rho_{\Delta\mu}\right) \overline{\Delta\mu} + \rho_{\Delta\mu}\Delta\mu_t + \sigma_{\Delta\mu} J_{t+1}, \tag{13}$$

where $\rho_{\Delta\mu}$ is the persistence parameter, and J_t is the uncertainty shock, which is independent and identically binomially distributed with a "small" frequency p. The parameter $\sigma_{\Delta\mu}$ defines the size of the uncertainty shock. We assume $\Delta\mu_t$ is observed at time t prior to the trading stage but after information choices are made.

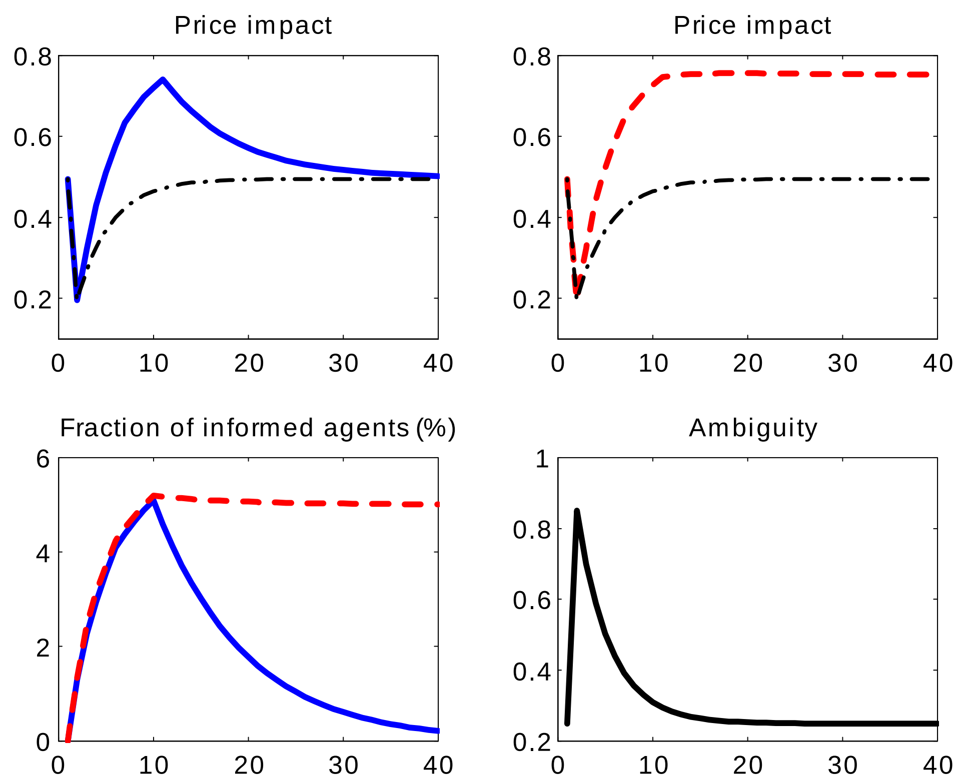

FIGURE 6. Equilibrium price and information response to an uncertainty shock. The top panels depict the price response to an uncertainty shock; the bottom panels depict the endogenous information response to an uncertainty shock (bottom left) and the uncertainty shock (bottom right). Dotted-dashed lines: price response in a market without information (top panels). Solid (dashed) lines: price and information response in a market with endogenous information acquisition and information stickiness for $c = 0.192$ ($c = 0.191$). Other parameter values are as in Figure 5 except

$\tau = 2$, $\alpha = 90\%$, and the parameters governing the dynamics of ambiguity in Eq. (13) fixed at $\overline{\Delta\mu} = 5$, $\rho_{\Delta\mu} = 0.75$, $\sigma_{\Delta\mu} = 12$, and $p = 1\%$.

We consider a market where successive generations of traders work in long-lived financial institutions. Traders have a one-period investment horizon, and trade a short-lived asset, a claim to the dividend process with the persistent component in Eq. (12). We assume that financial institutions entertain long-term information contracts, in that in each period, these contracts can only be dissolved with some probability $1 - \alpha$. Moreover, we assume the market of information is sticky on the upside as well, in that it takes time for this market to entirely absorb new demand for information and that as a result, new contracts can only be purchased with some probability $1-\alpha$. To summarize, every trader in his generation initially inherits the information choice (whether to purchase costly information or not) of the preceding trader he replaces. However, every new trader might successfully be satisfied with his information choice. In each period, then, only a fraction $(1 - \alpha)$ of the new (positive or negative) demand for information is satisfied:

$$\lambda_t - \lambda_{t-1} = (1 - \alpha) \left(\lambda_t^* - \lambda_{t-1} \right), \tag{14}$$

where λ_t^* denotes the fraction of agents who would become informed in the absence of any friction.

Figure 6 displays shows the model's predictions about the impact of an uncertainty shock. In the top panels, we compare the price response in the market with ambiguity but with uninformed agents, with the response in the market with endogenous information acquisition, assuming the market for information is sticky, with λ_t as in Eq. (14), and $\alpha = 90\%$. An uncertainty shock has an immediate negative price impact on both markets: the market crashes, as the agents are caught by surprise by the uncertainty shock and, then, demand, suddenly, a sizeable ambiguity premium. In a market where agents cannot acquire information, the uncertainty shock is absorbed, but only gradually as uncertainty itself (bottom-right panel) exogenously reverts to its long-run value. In a market with endogenous information acquisition, instead, the agents' incentives to purchase information lead them to remove the ambiguity premium occurred after the uncertainty shock. As a consequence, the asset price recovers quickly. In fact, the price overshoots, as it becomes more informative than it was prior to the uncertainty shock and uncertainty is (endogenously) removed faster than in the no information case. In the top-right panel, with a lower cost of information, the market is "trapped" in a new equilibrium regime in which information demand remains positive even after uncertainty resolves. In such a new regime, more efficient prices sustain persistently higher asset valuations.

Would these model's implications help explain the actual market response to an uncertainty shock? Figure 7 depicts the impulse-response function of the aggregate stock market to an uncertainty shock, estimated through a VAR model including monthly stock returns and a series of uncertainty shocks, from January 1957 to December 2008. Stock returns are those based upon the Fama and French (1993) market benchmark, and uncertainty shocks are those identified by Bloom (2009), which we use to define a time series which takes a value equal to one during the month when the shock took place, and zero otherwise. We control these estimates for the economic conditions under which the uncertainty shocks occurred, by feeding the VAR with the corporate spread and the term spread (defined as in Fama and French, 1989) and the dividend yield, as well as the recession indicator calculated by the National Bureau of Economic Research. In Appendix D, we provide details about the VAR methodology we employ, and a disaggregated description of the uncertainty shocks and subsequent market developments.

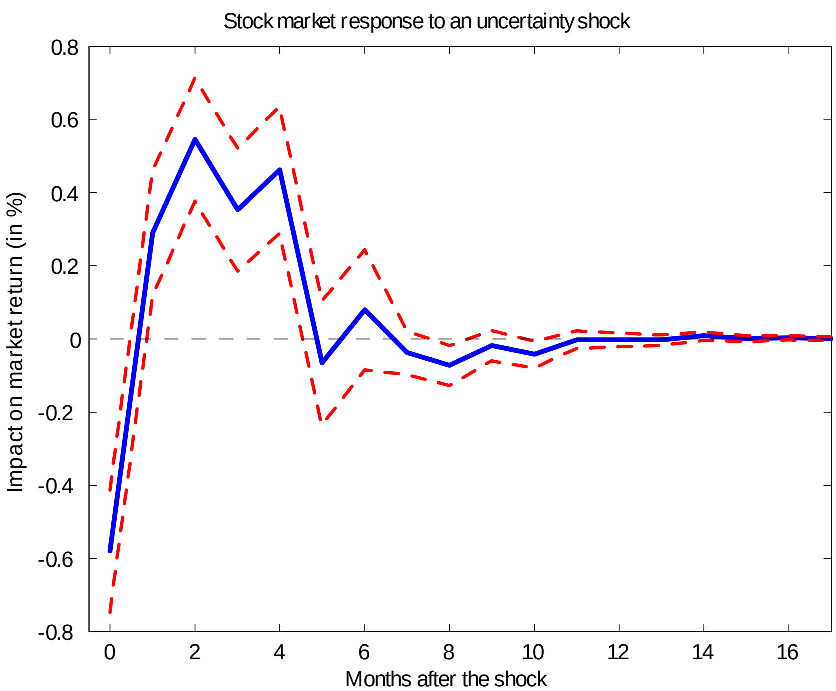

FIGURE 7. This picture depicts the response of the aggregate stock market to an uncertainty shock. The solid line is the first moment of the posterior distribution of the orthogonalized impulse-response function of the market to an uncertainty shock. Dashed lines are one standard-error bands around the response, constructed through the second moments of the posterior impulse-response

function.

The clear pattern emerging from Figure 7 is that the aggregate stock market plummets after an uncertainty shock, although, then, it rebounds rapidly and overshoots for several months. A simple explanation for these rallies is that uncertainty resolves quickly. However, anecdotal evidence suggests uncertainty shocks persist for more than just two or three months. Moreover, a mere exogenous reduction in uncertainty by itself seems unlikely to generate price overshoots. Our model offers an alternative explanation, based on a sustained information acquisition process, resulting from an higher value of information due to the uncertainty shock. Our model actually predicts two possible outcomes, as explained: one, where prices return to their level before the shock, as in the top-left bottom of Figure 6, and a second, where information acquisition leads to a change in information regime, with a boosted stock market. These two outcomes might account for uncertainty episodes exerting different market impacts, which are documented in Appendix D, on top of the average impact as summarized by the estimated impulse-response function in Figure 7.

6. Conclusion

References

Anderson, E., E. Ghysels and J. Juergens (2009): "The Impact of Risk and Uncertainty on Expected Returns." *Journal of Financial Economics* 94, 233-263.

Barlevy, G., and P. Veronesi (2000): "Information Acquisition in Financial Markets." *Review of Economic Studies* 67, 79-90. [Corrigendum, 2008, *Review of Economic Studies*]

Bloom, N. (2009): "The Impact of Uncertainty Shocks." *Econometrica* 77, 623-685.

Bossaert, P., P. Ghirardato, S. Guarneschelli and W. Zame (2010): "Ambiguity in Asset Markets: Theory and Experiment." *Review of Financial Studies* 23, 1325-1359.

Caballero, R.J., and A. Krishnamurthy (2008): "Collective Risk Management in a Flight to Quality Episode." *Journal of Finance* 63, 2195-2230.

Cao, H., T. Wang, and H. Zhang (2005): "Model Uncertainty, Limited Market Participation and Asset Prices." *Review of Financial Studies* 18, 1219-51.

Caskey, J. (2009): "Information in Equity Markets with Ambiguity-Averse Investors." *Review of Financial Studies* 22, 3595-3627.

Chamley, C. (2008): "On Acquisition of Information in Financial Markets." *Review of Economic Studies* 75, 1081-1084.

Dow, J. and S. Werlang (1992): "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio." *Econometrica* 60, 197-204.

Easley, D., and M. O'Hara (2009): "Ambiguity and Non-Participation: the Role of Regulation." *Review of Financial Studies* 22, 1817-1843.

Ellsberg, D. (1961): "Risk, Ambiguity and the Savage Axioms." *Quarterly Journal of Economics* 75, 643-69.

Epstein, L., and M. Schneider (2008): "Ambiguity, Information Quality and Asset Pricing." *Journal of Finance* 63, 197-228.

Epstein, L., and T. Wang (1994): "Intertemporal Asset Pricing under Knightian Uncertainty." *Econometrica* 62, 83-322.

Epstein, L., and T. Wang (1995): "Uncertainty, Risk-Neutral Measures and Security Price Booms and Crashes." *Journal of Economic Theory* 67, 40-82.

Epstein, L.G. and M. Schneider (2010): "Ambiguity and Asset Markets." Working paper, Boston University and Stanford University.

Fama, E. F., and French, K.R. (1989): "Business Conditions and Expected Returns on Stock and Bonds." *Journal of Financial Economics* 25, 23-49.

Fama, E.F. and K.R. French (1993): "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics* 33, 3-56.

Froot, K.A., D.S. Scharfstein, and J.C. Stein (1992): "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation." *Journal of Finance* 47, 1461-1484.

Gagliardini, P., P. Porchia and F. Trojani (2009): "Ambiguity Aversion and the Term Structure of Interest Rates." *Review of Financial Studies* 22, 4157-4188.

García D. and G. Strobl (2008): "Relative Wealth Concerns and Complementarities in Information Acquisition." Working paper, University of North Carolina at Chapel Hill.

Gilboa, I. and D. Schmeidler (1989): "Maxmin Expected Utility with a Non-Unique Prior." *Journal of Mathematical Economics* 18, 141-153.

Gollier, C. (2005): "Does Ambiguity Aversion Reinforce Risk Aversion? Applications to Portfolio Choices and Asset Prices." Working Paper, University of Toulouse.

Grossman, S.J. and J.E. Stiglitz (1980): "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70, 393-408.

Hansen, L.P. and T.J. Sargent (2008): *Robustness.* Princeton NJ: Princeton University Press.

Hellwig, C. and L. Veldkamp (2009): "Knowing What Others Know: Coordination Motives in Information Acquisition." *Review of Economic Studies* 76, 223-251.

Ozsoylev, H. and J. Werner (2008): "Liquidity and asset prices in rational expectations equilibrium with ambiguous information." Working paper, University of Oxford.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005): "A Smooth Model of Decision Making under Ambiguity." *Econometrica* 73, 1849-92.

Knight, F. (1921): *Risk, Uncertainty and Profit.* Boston, MA: Houghton Mifflin.

Leippold, M., F. Trojani, and P. Vanini (2008): "Learning and Asset Pricing under Ambiguous Information." *Review of Financial Studies* 21, 2565-2597.

Maenhout, P. (2004): "Robust Portfolio Rules and Asset Pricing." *Review of Financial Studies* 17, 951-83.

Uppal, R. and T. Wang (2003): "Model Misspecification and Underdiversification." *Journal of Finance* 58, 2465-2486.

Veldkamp, L. (2006): "Media Frenzies in Markets for Financial Information." *American Economic Review* 96, 577-601.

Appendices

Appendix A: Proofs for Section 2

Proof of Proposition I. By the market clearing condition, Eq. (5), the equilibrium price arising when the uninformed agents do not participate is:

$$P(s) = -\frac{\tau\omega_\epsilon}{\lambda}\mu_z + \frac{\tau\omega_\epsilon}{\lambda}s,$$

which is the second line in Eqs. (6). Next, we compute the uninformed agents' expectation of the asset payoff, in the states of nature in which these agents participate. Using $\omega_s = \left(\frac{\lambda}{\omega_\epsilon\tau}\right)^2\omega_\theta + \omega_z$, straight forward computations leave:

$$E^{\mathrm{buy}}(f|\, S = s) = \frac{\tau^2\omega_\epsilon^2\omega_z}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}\underline{\mu} + \frac{\lambda\tau\omega_\epsilon\omega_\theta}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}s \tag{A1}$$

$$E^{\mathrm{sell}}(f|\, S = s) = \frac{\tau^2\omega_\epsilon^2\omega_z}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}\bar{\mu} + \frac{\lambda\tau\omega_\epsilon\omega_\theta}{\lambda^2\omega_\theta + \tau^2\omega_\epsilon^2\omega_z}s \tag{A2}$$

Next, we plug Eqs. (A1)-(A2) into the demand schedule, Eq. (2), replace the result into the market clearing condition, Eq. (5), conjecture the piece-wise linear price function in Eqs. (6), and solve for undetermined coefficients, obtaining,

$$\begin{aligned}
\underline{a} &= \frac{-\lambda^2\mu_z\tau\omega_\epsilon\omega_\theta + \left(\underline{\mu}\,(1-\lambda)\,\omega_\epsilon - \mu_z\tau\omega_\epsilon\,(\omega_\epsilon + \omega_\theta)\right)\tau^2\omega_z\omega_\epsilon}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2} \\
\bar{a} &= \underline{a} + \frac{\Delta\mu\,(1-\lambda)\,\tau^2\omega_z\omega_\epsilon^2}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2} \\
a &= -\frac{\tau\omega_\epsilon}{\lambda}\mu_z \\
b &= \frac{\left(\lambda\omega_\theta + \omega_z\tau^2\omega_\epsilon\,(\omega_\theta + \omega_\epsilon)\right)\tau\omega_\epsilon}{\lambda^2\omega_\theta + \lambda\tau^2\omega_\theta\omega_z\omega_\epsilon + \tau^2\omega_z\omega_\epsilon^2}
\end{aligned}$$

Finally, we determine the threshold for the compound signal, $\underline{s}$ and $\bar{s}$. We use the cutoff conditions in Eq. (3). As for $\underline{s}$, consider the first equation, $E^{\mathrm{buy}}\left(f|\, P(\cdot,\cdot) = \underline{p}\right) = \underline{p}$. For $s \leq \underline{s}$, the conjectured price function is linear in s. Therefore, we solve for $\underline{p}$ by equivalently solving for $\underline{s}$ in the following condition,

$$E^{\mathrm{buy}}(f|\, S = \underline{s}) = \underline{p} = \underline{a} + b\underline{s},$$

where $E^{\mathrm{buy}}(f|\, S = \underline{s})$ is given by Eq. (A1), and the second equality holds by the first line of the conjectured price function in Eqs. (6). We do the same to determine $\bar{s}$, by solving,

$$E^{\mathrm{sell}}(f|\, S = \bar{s}) = \bar{p} = \bar{a} + b\bar{s},$$

where $E^{\text{sell}}\left(f \middle| S = \bar{s} \right)$ is given by Eq. (A2). The expressions for $\underline{s}$ and $\bar{s}$ given in Proposition I then follow by simple computations. Finally, we need to compute the threshold prices $\bar{p}$ and $\underline{p}$. We plug Eqs. (A1)-(A2) into Eq. (3), use the price function in Eqs. (6), and obtain,

$$\underline{p} = \underline{\mu} + \frac{\lambda \omega_\theta}{\tau \omega_\epsilon \omega_z} \mu_z, \quad \bar{p} = \bar{\mu} + \frac{\lambda \omega_\theta}{\tau \omega_\epsilon \omega_z} \mu_z.$$

The previous expressions confirm that $\underline{p} < \bar{p}$. ■

Proof of Eq. (7). Follows by Eqs. (A1)-(A2). ■

Appendix B: Proofs for Section 3

Remark on notation. To alleviate the notation, we fix $\Phi(\cdot) \equiv \Phi(\cdot; 0, 1)$.

Derivation of the utilities for the would-be uninformed and informed agents.

Would-be uninformed agents. By Eqs. (A1)-(A2), we have,

$$x_U(s) = \begin{cases} \dfrac{E^{\text{buy}}\left(f \middle| s \right) - P(s)}{\tau \omega_{f|s}} = \dfrac{\delta}{\tau \omega_{f|s}} \left(\underline{s} - s \right), & \text{for } s < \underline{s} \\[2ex] 0, & \text{for } s \in [\underline{s}, \bar{s}] \\[2ex] \dfrac{E^{\text{sell}}\left(f \middle| s \right) - P(s)}{\tau \omega_{f|s}} = \dfrac{\delta}{\tau \omega_{f|s}} \left(\bar{s} - s \right), & \text{for } s > \bar{s} \end{cases}$$

where $\omega_{f|s}$ is the variance of f conditional on s, $P(s)$ is the equilibrium price in Eqs. (6) of Proposition I, and:

$$\omega_{f|s} = \omega_\epsilon + \frac{\omega_z \omega_\theta}{\omega_s}, \quad \delta = \frac{\tau^3 \omega_z \omega_\epsilon^3 \left(\lambda^2 \omega_\theta + \tau^2 \omega_z \omega_\epsilon^2 + \tau^2 \omega_z \omega_\theta \omega_\epsilon \right)}{\left(\lambda^2 \omega_\theta + \lambda \tau^2 \omega_z \omega_\theta \omega_\epsilon + \tau^2 \omega_z \omega_\epsilon^2 \right) \left(\lambda^2 \omega_\theta + \tau^2 \omega_z \omega_\epsilon^2 \right)}. \tag{A3}$$

Accordingly, the interim utility is,

$$v_U(s) = -e^{-\tau \mathcal{C}_U(s)} = \begin{cases} -\exp\left(-\dfrac{1}{2} \dfrac{\delta^2}{\omega_{f|s}} \left(s - \underline{s} \right)^2 \right), & \text{for } s < \underline{s} \\[2ex] -1, & \text{for } s \in [\underline{s}, \bar{s}] \\[2ex] -\exp\left(-\dfrac{1}{2} \dfrac{\delta^2}{\omega_{f|s}} \left(s - \bar{s} \right)^2 \right), & \text{for } s > \bar{s} \end{cases} \tag{A4}$$

Integrating over the distribution of the compound signal, s, leaves

$$E_{\mu}\left[v_U\left(s\right)\right] = -\int_{-\infty}^{\infty} e^{-\tau \mathcal{C}_U(t)} d\Phi\left(t;\mu_s\left(\mu\right),\omega_s\right) \equiv \sum_{\ell \in \{\text{buy},\text{np},\text{sell}\}} J_{\mu}^{\ell},$$

where

$$\begin{aligned}
J_{\mu}^{\text{buy}} &= -\int_{-\infty}^{\underline{s}} e^{-\tau \mathcal{C}_U(t)} d\Phi\left(t;\mu_s\left(\mu\right),\omega_s\right), \\
J_{\mu}^{\text{np}} &= -\int_{\underline{s}}^{\bar{s}} d\Phi\left(t;\mu_s\left(\mu\right),\omega_s\right), \\
J_{\mu}^{\text{sell}} &= -\int_{\bar{s}}^{\infty} e^{-\tau \mathcal{C}_U(t)} d\Phi\left(t;\mu_s\left(\mu\right),\omega_s\right).
\end{aligned}$$

A direct computation of these integrals yields,

$$\begin{aligned}
J_{\mu}^{\text{buy}} &= -\kappa \exp\left(-\frac{\delta^2\left(\underline{s}-\mu_s\left(\mu\right)\right)^2}{2\left(\omega_{f|s}+\delta^2\omega_s\right)}\right)\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\underline{s}-\mu_s\left(\mu\right)\right)\right), \\
J_{\mu}^{\text{np}} &= -\left[\Phi\left(\frac{\bar{s}-\mu_s\left(\mu\right)}{\sqrt{\omega_s}}\right)-\Phi\left(\frac{\underline{s}-\mu_s\left(\mu\right)}{\sqrt{\omega_s}}\right)\right], \\
J_{\mu}^{\text{sell}} &= -\kappa \exp\left(-\frac{\delta^2\left(\bar{s}-\mu_s\left(\mu\right)\right)^2}{2\left(\omega_{f|s}+\delta^2\omega_s\right)}\right)\left[1-\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\bar{s}-\mu_s\left(\mu\right)\right)\right)\right],
\end{aligned}$$

where,

$$\kappa = \sqrt{\frac{\omega_{f|s}}{\omega_{f|s}+\delta^2\omega_s}}. \quad \blacksquare$$

Would-be informed agents. Let $\mu_{\theta|s}\left(\mu\right)$ and $\omega_{\theta|s}$ denote the conditional expectation and variance of θ given s, which are easily shown to be:

$$\mu_{\theta|s}\left(s;\mu\right) = \frac{\tau^2\omega_{\epsilon}^2\omega_z}{\lambda^2\omega_{\theta}+\tau^2\omega_{\epsilon}^2\omega_z}\mu + \frac{\lambda\tau\omega_{\epsilon}\omega_{\theta}}{\lambda^2\omega_{\theta}+\tau^2\omega_{\epsilon}^2\omega_z}s, \qquad \omega_{\theta|s} = \frac{\omega_z\omega_{\theta}}{\omega_s}. \tag{A5}$$

We have,

$$E_{\mu}\left[v_I\left(\theta,s\left(\theta,z\right)\right)\right] = e^{\tau c}\int_{-\infty}^{\infty} E_{\mu}\left[\left.v_I\left(\theta,s\right)\right|s\right] d\Phi\left(s;\mu_s\left(\mu\right),\omega_s\right) \tag{A6}$$

where,

$$E_{\mu}\left[\left.v_I\left(\theta,s\right)\right|s\right] = e^{\tau c}\int_{-\infty}^{\infty} v_I\left(\theta,s\right) d\Phi(\theta;\mu_{\theta|s}\left(s;\mu\right),\omega_{\theta|s}).$$

Computing the integrals yields,

$$E_\mu\left[v_I(\theta,s)|\,s\right] = -e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\exp\left(-\frac{1}{2}\frac{\left(\mu_{\theta|s}(s;\mu)-P(s)\right)^2}{\omega_{f|s}}\right), \tag{A7}$$

where $P(s)$ is the equilibrium price in Eqs. (6) of Proposition I. Replacing $P(s)$ and the expression for $\mu_{\theta|s}(s;\mu)$ in Eq. (A5) into Eq. (A7), leaves:

$$E_\mu\left[v_I(\theta,s)|\,s\right] = e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}\bar{v}_I(s;\mu), \tag{A8}$$

where, for $\hat{s}=\frac{1}{2}(\underline{s}+\bar{s})$ and δ defined as in Eq. (A3),

$$\bar{v}_I(s;\mu) = \begin{cases} -\exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s-\underline{s}-\frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\mu-\underline{\mu})\right)^2\right), & \text{for } s<\underline{s} \\ -\exp\left(-\frac{1}{2}\frac{\hat{\delta}^2}{\omega_{f|s}}\left(s-\hat{s}-\frac{\lambda}{\tau\omega_\epsilon}\mu\right)^2\right), & \text{for } s\in[\underline{s},\bar{s}] \\ -\exp\left(-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s-\bar{s}+\frac{\hat{\delta}}{\delta}\frac{\lambda}{\tau\omega_\epsilon}(\bar{\mu}-\mu)\right)^2\right), & \text{for } s>\bar{s} \end{cases} \tag{A9}$$

and

$$\hat{\delta} = \frac{\tau\omega_\epsilon\omega_z}{\lambda\omega_s}.$$

Finally, substituting Eq. (A8) into Eq. (A6), and integrating, leaves Eq. (10) in the main text, with

$$E_\mu\left[\bar{v}_I(s;\mu)\right] = \sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} I_\mu^\ell,$$

where,

$$I_\mu^{\text{buy}} = -\kappa\exp\left(-\frac{\delta^2\left(\frac{\omega_s}{\omega_z}\mu_z+\gamma_0(\mu-\underline{\mu})\right)^2}{2\left(\omega_{f|s}+\delta^2\omega_s\right)}\right)\Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z-\gamma_1(\mu-\underline{\mu})\right)\right)$$

$$I_\mu^{\text{np}} = -\hat{\kappa}\exp\left(-\frac{\hat{\delta}^2(\frac{\omega_s}{\omega_z}\mu_z)^2}{2(\omega_{f|s}+\hat{\delta}^2\omega_s)}\right)\left[\Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z+\gamma_2(\bar{\mu}-\mu)\right)\right)-\Phi\left(\frac{\hat{\kappa}}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z-\gamma_2(\mu-\underline{\mu})\right)\right)\right]$$

$$I_{\mu}^{\text{sell}} = -\kappa \exp\left(-\frac{\delta^2\left(\frac{\omega_s}{\omega_z}\mu_z - \gamma_0\left(\bar{\mu}-\mu\right)\right)^2}{2\left(\omega_{f|s} + \delta^2\omega_s\right)}\right)\left[1 - \Phi\left(\frac{\kappa}{\sqrt{\omega_s}}\left(\frac{\omega_s}{\omega_z}\mu_z + \gamma_1\left(\bar{\mu}-\mu\right)\right)\right)\right]$$

and:

$$\hat{\kappa} = \sqrt{\frac{\omega_{f|s}}{\omega_{f|s} + \hat{\delta}^2\omega_s}}, \quad \gamma_0 = \left(\frac{\hat{\delta}}{\delta} - 1\right)\frac{\lambda}{\tau\omega_\epsilon}, \quad \gamma_1 = \left(1 + \delta\frac{\hat{\delta}\omega_s}{\omega_{f|s}}\right)\frac{\lambda}{\tau\omega_\epsilon}, \quad \gamma_2 = \left(1 + \hat{\delta}\frac{\hat{\delta}\omega_s}{\omega_{f|s}}\right)\frac{\lambda}{\tau\omega_\epsilon}. \quad \blacksquare$$

Proof of Proposition II. We claim that $\mu_s = \hat{s} \equiv \frac{1}{2}\left(\underline{s} + \bar{s}\right)$, or equivalently, that for all $\epsilon > 0$,

$$-\Delta_\epsilon \mathcal{U}_U \equiv -\int_{-\infty}^{\infty} v_U\left(s\right)\Delta_\epsilon\varphi\left(s\right)ds > 0, \tag{A10}$$

where $\Delta_\epsilon\varphi\left(s\right) \equiv \varphi\left(s;\hat{s},\omega_s\right) - \varphi\left(s;\hat{s}+\epsilon,\omega_s\right)$, and $\varphi\left(\cdot,\mu,\sigma^2\right)$ denotes the Normal density function, with mean μ and variance σ^2. Note that the function $v_U\left(s\right)$ in Eq. (A4) is symmetric about $\hat{s}$, so that Proposition II follows once, we show that the inequality in (A10) holds true for all $\epsilon > 0$.

We have:

$$\Delta_\epsilon\varphi\left(s\right) = \begin{cases} f\left(s - \hat{s} - \frac{1}{2}\epsilon\right) \geq 0 & \text{for all } s \in \left(-\infty, \hat{s} + \frac{1}{2}\epsilon\right] \\ -f\left(\hat{s} + \frac{1}{2}\epsilon - s\right) \leq 0 & \text{for all } s \in \left[\hat{s} + \frac{1}{2}\epsilon, \infty\right) \end{cases}$$

where we have defined:

$$f\left(x\right) \equiv \frac{1}{\sqrt{2\pi\omega_s}}\left(e^{-\frac{1}{2\omega_s}\left(x+\frac{1}{2}\epsilon\right)^2} - e^{-\frac{1}{2\omega_s}\left(x-\frac{1}{2}\epsilon\right)^2}\right).$$

Next, define the two functions,

$$h_1\left(s\right) \equiv \begin{cases} e^{-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(\underline{s}-s\right)^2} & \text{for all } s \in \left(-\infty, \underline{s}\right] \\ 1 & \text{for all } s \in \left[\underline{s}, \hat{s} + \frac{1}{2}\epsilon\right) \end{cases}$$

and

$$h_2\left(s\right) \equiv \begin{cases} 1 & \text{for all } s \in \left[\hat{s} + \frac{1}{2}\epsilon, \bar{s}\right) \\ e^{-\frac{1}{2}\frac{\delta^2}{\omega_{f|s}}\left(s-\bar{s}\right)^2} & \text{for all } s \in \left(\bar{s}, \infty\right] \end{cases}$$

In terms of h_1 and h_2, we have, $-v_U\left(s\right) = h_1\left(s\right)\mathbb{I}_{\left\{s \leq \hat{s}+\frac{1}{2}\epsilon\right\}} + h_2\left(s\right)\mathbb{I}_{\left\{s \geq \hat{s}+\frac{1}{2}\epsilon\right\}}$, where $\mathbb{I}_{\{\cdot\}}$ denotes the indicator function, and the expression for $\Delta_\epsilon\mathcal{U}_U$ in (A10) is,

$$\begin{aligned} -\Delta_\epsilon\mathcal{U}_U &= \int_{-\infty}^{\hat{s}+\frac{1}{2}\epsilon} h_1\left(s\right) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right)ds - \int_{\hat{s}+\frac{1}{2}\epsilon}^{\infty} h_2\left(s\right) f\left(\hat{s} + \frac{1}{2}\epsilon - s\right)ds \\ &= \int_{-\infty}^{\hat{s}+\frac{1}{2}\epsilon} h_1\left(s\right) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right)ds - \int_{-\infty}^{\hat{s}-\frac{1}{2}\epsilon} h_1\left(s\right) f\left(s - \hat{s} - \frac{1}{2}\epsilon\right)ds \\ &> 0, \end{aligned}$$

where the second equality follows by the symmetry of $v_U\left(s\right)$ about $\hat{s}$. $\blacksquare$

Proof of Proposition III. Consider the indifference condition in Eq. (11). We wish to show that for $\Delta\mu > 0$,

$$\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)} < e^{\tau c}\sqrt{\frac{\omega_\epsilon}{\omega_{f|s}}}.$$

Because $E_{\mu_I}[\bar{v}_I(s;\mu)]$ and $E_{\mu_U}[v_U(s)]$ are both strictly negative, the previous inequality holds true if:

$$E_{\mu_I}[\bar{v}_I(s;\mu)] > E_{\mu_U}[v_U(s)], \tag{A11}$$

where we define, as in the main text:

$$\mu_I \in \arg\min_\mu E_\mu[\bar{v}_I(s;\mu)], \quad \mu_U \in \arg\min_\mu E_\mu[v_U(s)].$$

To show that (A11) is true, suppose the contrary, i.e. that:

$$E_{\mu_I}[\bar{v}_I(s;\mu)] \leq E_{\mu_U}[v_U(s)]. \tag{A12}$$

By direct comparison of Eq. (A4) and Eq. (A9), we have that $0 > \bar{v}_I(s,\mu) \geq v_U(s)$ for all $\mu \in [\underline{\mu},\bar{\mu}]$, and $s \in \mathbb{R}$, the second inequality being strict on some open set in $\mathbb{R}$. As a consequence, we must have the inequality, $E_{\mu_I}[\bar{v}_I(s;\mu)] > E_{\mu_I}[v_U(s)]$ which, combined with (A12), yields,

$$E_{\mu_I}[v_U(s)] < E_{\mu_I}[\bar{v}_I(s;\mu)] \leq E_{\mu_U}[v_U(s)],$$

contradicting that μ_U minimizes $E_\mu[v_U(s)]$. $\blacksquare$

Proof of Corollary 1. Let $\lambda^*(\Delta\mu)$ solve the indifference condition:

$$\frac{\mathcal{U}_I(c,\lambda)}{\mathcal{U}_U(\lambda)} = 1.$$

Assume now that $\lambda^*(0) \geq \lambda^*(\Delta\mu)$, for some $\Delta\mu > 0$. By Proposition III, this cannot be the case as we would have

$$\frac{\mathcal{U}_I(c,\lambda^*(\Delta\mu))}{\mathcal{U}_U(\lambda^*(\Delta\mu))} < 1. \quad \blacksquare$$

Proof of Proposition IV. We wish to show that

$$\frac{\mathcal{U}_I(c,0)}{\mathcal{U}_U(0)} > \frac{\mathcal{U}_I(c,1)}{\mathcal{U}_U(1)},$$

or

$$\frac{\mathcal{I}_0}{\mathcal{J}_0}\frac{\mathcal{J}_1}{\mathcal{I}_1} > \sqrt{\frac{\omega_{f|s,\lambda=0}}{\omega_{f|s,\lambda=1}}}, \tag{A13}$$

where $\omega_{f|s,\lambda=0} = \lim_{\lambda\to 0}\omega_{f|s}$, $\omega_{f|s,\lambda=1} = \lim_{\lambda\to 1}\omega_{f|s}$, and,

$$\mathcal{I}_{\lambda^*} = \lim_{\lambda\to\lambda^*}\sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} I^{\ell}_{\mu_I}, \quad \mathcal{J}_{\lambda^*} = \lim_{\lambda\to\lambda^*}\sum_{\ell\in\{\text{buy},\text{np},\text{sell}\}} J^{\ell}_{\mu_U}, \quad \lambda^*\in\{0,1\}\,.$$

We now proceed with determining $\mathcal{I}_{\lambda^*}$ and $\mathcal{J}_{\lambda^*}$ for $\lambda^*\in\{0,1\}$, and for μ_z sufficiently large. Then, we shall prove that (A13) holds true for μ_z sufficiently large. We shall need the results recorded in the next two lemmas.

Lemma 1. *There exists a $\hat{\mu}_z > 0$ such that for all $\mu_z \geq \hat{\mu}_z$, we have that* $\arg\min_\mu \left(I^{\text{buy}}_\mu + I^{\text{sell}}_\mu\right) = \underline{\mu}$.

Proof. We have,

$$\begin{aligned}
I^{\text{buy}}_\mu &= -c_0 \exp\left(-\frac{\left(\tau\omega_f\mu_z + \left(\mu-\underline{\mu}\right)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z - \tau\omega_z\left(\mu-\underline{\mu}\right)\right)\right)\\
I^{\text{sell}}_\mu &= -c_0 \exp\left(-\frac{\left(\tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\left[1-\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z + \tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)\right]
\end{aligned}$$

where $c_0 = \left(1+\tau^2\omega_f\omega_s\right)^{-\frac{1}{2}}$. It is easy to show that $\mu \mapsto I^{\text{buy}}_\mu$ is increasing. We are left to show that with μ_z sufficiently large, we have that $\mu\mapsto I^{\text{sell}}_\mu$ is increasing as well. We have,

$$\begin{aligned}
c_0^{-1}\frac{\partial}{\partial\mu}I^{\text{sell}}_\mu &= -\frac{\partial}{\partial\mu}\exp\left(-\frac{\left(\tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\left[1-\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z + \tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)\right]\\
&\quad + \exp\left(-\frac{\left(\tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\frac{\partial}{\partial\mu}\left[1-\Phi\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z + \tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)\right]\\
&= \exp\left(-\frac{\left(\tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right)\right)^2}{2\left(\omega_f+\tau^2\omega_f^2\omega_s\right)}\right)\\
&\quad\times\left(\frac{\tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right)}{\omega_f+\tau^2\omega_f^2\omega_s} + \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{c_0}{\sqrt{\omega_z}}\left(\mu_z+\tau\omega_z\left(\bar{\mu}-\mu\right)\right)\right)^2\right)\frac{c_0}{\sqrt{\omega_z}}\tau\omega_z\right).
\end{aligned}$$

This expression is positive for all $\mu_z : \tau\omega_f\mu_z - \left(\bar{\mu}-\mu\right) \geq 0$, which it does when $\mu_z \geq \frac{\Delta\mu}{\tau\omega_f}$. ■

Lemma 2. *There exists a $\check{\mu}_z > 0$ such that for all $\mu_z \geq \check{\mu}_z$ we have that* $\arg\min_\mu \left(J^{\text{buy}}_\mu + J^{\text{np}}_\mu + J^{\text{sell}}_\mu\right) = \bar{\mu}$.

Proof. Follows directly by Proposition II, once we set $\check{\mu}_z = \frac{\lambda\omega_z}{\tau\omega_\epsilon\omega_s}\bar{\mu}$. ■

We are now ready to compute $\mathcal{I}_0$, $\mathcal{J}_0$, $\mathcal{I}_1$ and $\mathcal{J}_1$.

- As for $\mathcal{I}_0$, note that, clearly, $\mathcal{I}_0 = \min_\mu (I_\mu^{\text{buy}} + I_\mu^{\text{sell}})$. Therefore, by Lemma 1, and a simple computation, we have that for all $\mu_z \geq \hat{\mu}_z$, and with $\hat{\mu}_z$ as in the proof of Lemma 1,

$$\begin{aligned}\mathcal{I}_0 &= I_{\underline{\mu}}^{\text{buy}} + I_{\underline{\mu}}^{\text{sell}} \\ &= -c_0 \left[\exp\left(-\frac{\mu_z^2 \tau^2 \omega_f c_0^2}{2}\right) \Phi\left(\frac{\mu_z}{\sqrt{\omega_z}} c_0\right) + \exp\left(-\frac{\left(\Delta\mu - \mu_z \tau \omega_f\right)^2 c_0^2}{2\omega_f}\right) \Phi\left(-\frac{\mu_z + \Delta\mu\tau\omega_z}{\sqrt{\omega_z}} c_0\right)\right],\end{aligned}$$

where the second equality follows by a simple computation.

- As for $\mathcal{J}_0$ and $\mathcal{I}_1$, it is easily seen that $\mathcal{J}_0$ and $\mathcal{I}_1$ are independent of μ. They are,

$$\mathcal{J}_0 = -c_0 \exp\left(-\frac{\mu_z^2 \tau^2 \omega_f c_0^2}{2}\right), \quad \mathcal{I}_1 = -c_1 \sqrt{\frac{\omega_\theta + \tau^2 \omega_z \omega_\epsilon \omega_f}{c_2}} \exp\left(-\frac{1}{2} \tau^2 \omega_\epsilon c_1^2 \mu_z^2\right),$$

where $c_1 = \left(1 + \tau^2 \omega_z \omega_\epsilon\right)^{-\frac{1}{2}}$ and $c_2 = \tau^2 \omega_z \omega_\epsilon^2 + \omega_\theta$.

- As for $\mathcal{J}_1$ we have, by Lemma 2 and a direct computation, that for all $\mu_z \geq \check{\mu}_z$, where $\check{\mu}_z$ is as in Lemma 2,

$$\begin{aligned}\mathcal{J}_1 = &-c_1 \sqrt{\frac{\omega_\theta + \tau^2 \omega_z \omega_\epsilon \omega_f}{c_2}} \\ &\times \left[\exp\left(-\frac{\tau^2 \omega_\epsilon c_1^2}{2} \frac{\left(\Delta\mu\tau\omega_z\omega_\epsilon - \mu_z c_2\right)^2}{c_2^2}\right) \Phi\left(\rho_0 \mu_z + \rho_1\right) + \exp\left(-\frac{\tau^2 \omega_\epsilon c_1^2}{2} \mu_z^2\right) \Phi\left(-\rho_0 \mu_z\right)\right] \\ &- \left[\Phi\left(\rho_2 \mu_z + \rho_3\right) - \Phi\left(\rho_2 \mu_z + \rho_4\right)\right],\end{aligned} \tag{A14}$$

for some constants $\rho_0 > 0$, $\rho_2 > 0$, ρ_1, ρ_3, ρ_4 independent of μ_z.

We are now ready to show that the inequality in (A13) holds true. Consider the following ratios:

$$\begin{aligned}\frac{\mathcal{I}_0}{\mathcal{J}_0} &= \Phi\left(\frac{\mu_z}{\sqrt{\omega_z}} c_0\right) + \exp\left(\frac{c_0^2}{2\omega_f} \left(2\mu_z \tau \omega_f - \Delta\mu\right) \Delta\mu\right) \Phi\left(-c_0 \frac{\mu_z + \Delta\mu\tau\omega_z}{\sqrt{\omega_z}}\right) \\ \frac{\mathcal{I}_1}{\mathcal{J}_1} &= -c_1 \sqrt{\frac{\omega_\theta + \tau^2 \omega_z \omega_\epsilon \omega_f}{\tau^2 \omega_z \omega_\epsilon^2 + \omega_\theta}} \left[\exp\left(\frac{\tau^2 \omega_\epsilon c_1^2}{2} \mu_z^2\right) \mathcal{J}_1\right]^{-1}\end{aligned}$$

where $\mathcal{J}_1$ is as in Eq. (A14). We claim that,

$$\lim_{\mu_z \to \infty} \frac{\mathcal{I}_0}{\mathcal{J}_0} = 1, \quad \lim_{\mu_z \to \infty} \frac{\mathcal{I}_1}{\mathcal{J}_1} = 0.$$

The first limit holds by the property of the cumulative distribution function, and by the L'Hôpital's rule. To show that the second limit holds, note that by the expression for $\mathcal{J}_1$ in Eq. (A14), we only need to show that

$$\lim_{\mu_z \to \infty} \exp\left(\frac{\tau^2 \omega_\epsilon c_1^2}{2c_2^2} \left(c_2^2 \mu_z^2 - \left(\Delta\mu\tau\omega_z\omega_\epsilon - c_2 \mu_z\right)^2\right)\right) = \infty,$$

which is easily seen to be true. ■

Appendix C: Partial resolution of Knightian uncertainty

We consider a model where a mass $(1-\lambda)$ of uninformed agents correctly believe that $\mu_0 \in [\underline{\mu}, \bar{\mu}]$, as in the main model. The informed agents, however, by paying a cost $c > 0$, resolve only a fraction $(1-\xi)$ of their uncertainty: upon paying c, they would correctly believe that $\mu_0 \in [\xi\underline{\mu}, \xi\bar{\mu}]$, with $\xi \in (0,1)$. We assume, as usual, that $\mu_0 = 0$. By arguments similar to that we used to solve for the main model, we are looking for an equilibrium piecewise linear price function. Let $s = -(z - \mu_z)$. We conjecture, and verify, that there are three threshold values of the signal, say $\underline{s}$, s^* and $\bar{s}$, which lead to identify four regions of the equilibrium price schedule, as in Figure A.1: two regions where everyone trades, which occur whenever the signal is low, $s \leq \underline{s}$ (in which case everyone is buyer) or high $s \geq \bar{s}$ (in which case everyone is seller); and two regions where only the informed agents trade, either as buyers, when $s \in (\underline{s}, s^*)$, or as sellers, when $s \in (s^*, \bar{s})$.

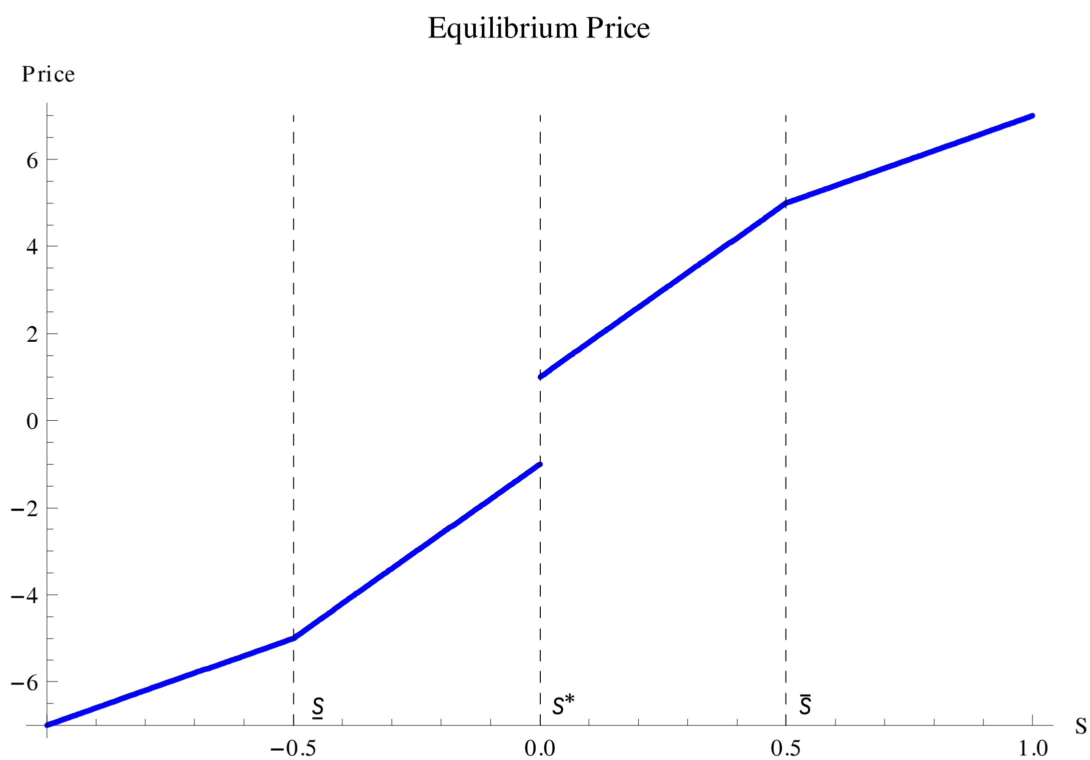

FIGURE A.1. The equilibrium price schedule in a market with partial resolution of Knightian uncertainty. Parameter values are $\lambda = 0.5$, $\tau = 2$, $\omega_f = \omega_z = 1$; $\xi = 0.2$; $\bar{\mu} = -\underline{\mu} = 5$ and $\mu_z = 0$.

The price schedule is:

$$P(s) = \begin{cases} \underline{a}_1 + \dfrac{1}{\rho} s, & \text{for } s \leq \underline{s} \\ \underline{a}_2 + \dfrac{1}{\lambda\rho} s, & \text{for } s \in [\underline{s}, s^*) \\ \bar{a}_2 + \dfrac{1}{\lambda\rho} s, & \text{for } s \in (s^*, \bar{s}] \\ \bar{a}_1 + \dfrac{1}{\rho} s, & \text{for } s \geq \bar{s} \end{cases}$$

where $\rho = \frac{1}{\tau(\omega_\theta + \omega_\epsilon)}$, $\underline{a}_1 = \underline{\mu}\left(1 - \lambda\left(1 - \xi\right)\right) - \frac{1}{\rho}\mu_z$, $\bar{a}_1 = \bar{\mu}\left(1 - \lambda\left(1 - \xi\right)\right) - \frac{1}{\rho}\mu_z$, $\underline{a}_2 = \xi\underline{\mu} - \frac{1}{\lambda\rho}\mu_z$, $\bar{a}_2 = \xi\bar{\mu} - \frac{1}{\lambda\rho}\mu_z$, and the signal thresholds are given by:

$$\underline{s} = \rho\lambda\left(1 - \xi\right)\underline{\mu} + \mu_z, \quad s^* = \mu_z, \quad \bar{s} = \rho\lambda\left(1 - \xi\right)\bar{\mu} + \mu_z.$$

Lengthy computations lead to the following expressions for the ex-ante utilities. The ex-ante utilities of the uninformed agents are:

$$E\left[v_U\left(s\right)\right] = \sum_{\ell \in \{\text{buy,np,sell}\}} J^\ell,$$

where

$$\begin{aligned} J^{\text{buy}} &= -\frac{1}{\sqrt{1 + \tau^2\omega_f\omega_z}} \exp\left(-\frac{\underline{s}^2\omega_f\tau^2}{2\left(1 + \tau^2\omega_f\omega_z\right)}\right) \Phi\left(\frac{\underline{s}}{\sqrt{\omega_z}} \frac{1}{\sqrt{1 + \tau^2\omega_f\omega_z}}\right), \\ J^{\text{np}} &= -\left[\Phi\left(\frac{\bar{s}}{\sqrt{\omega_z}}\right) - \Phi\left(\frac{\underline{s}}{\sqrt{\omega_z}}\right)\right], \\ J^{\text{sell}} &= -\frac{1}{\sqrt{1 + \tau^2\omega_f\omega_z}} \exp\left(-\frac{\bar{s}^2\omega_f\tau^2}{2\left(1 + \tau^2\omega_f\omega_z\right)}\right) \left[1 - \Phi\left(\frac{\bar{s}}{\sqrt{\omega_z}} \frac{1}{\sqrt{1 + \tau^2\omega_f\omega_z}}\right)\right]. \end{aligned}$$

and $\zeta = \tau^2\omega_f$. The ex-ante utilities of the informed agents are, instead,

$$E\left[v_I\left(s\right)\right] = \sum_{\ell \in \{\text{buy,np,sell}\}} I^\ell,$$

where,

$$\begin{aligned} I^{\text{buy}} &= -\frac{1}{\sqrt{1 + \tau^2\omega_f\omega_z}} \exp\left(-\frac{\left(\underline{s}\tau\omega_f - \underline{\mu}\left(1 - \xi\right)\right)^2}{2\omega_f\left(1 + \tau^2\omega_f\omega_z\right)}\right) \Phi\left(\frac{1}{\sqrt{\omega_z}} \frac{\underline{s} + \underline{\mu}\left(1 - \xi\right)\tau\omega_z}{\sqrt{1 + \tau^2\omega_f\omega_z}}\right), \\ I^{\text{np}} &= -\sqrt{\frac{\lambda^2}{\lambda^2 + \tau^2\omega_f\omega_z}} \exp\left(-\frac{\left(\mu_z\tau\right)^2\omega_f}{2\left(\lambda^2 + \tau^2\omega_f\omega_z\right)}\right) \times \end{aligned}$$

$$\left[\Phi\left(\frac{1}{\lambda\sqrt{\omega_z}}\frac{\bar{s}\lambda^2+(\bar{s}-\mu_z)\,\tau^2\omega_z\omega_f}{\sqrt{\lambda^2+\tau^2\omega_f\omega_z}}\right)-\Phi\left(\frac{1}{\lambda\sqrt{\omega_z}}\frac{\underline{s}\lambda^2+(\underline{s}-\mu_z)\,\tau^2\omega_z\omega_f}{\sqrt{\lambda^2+\tau^2\omega_f\omega_z}}\right)\right],$$

$$I^{\text{sell}} = -\frac{1}{\sqrt{1+\tau^2\omega_f\omega_z}}\exp\left(-\frac{\left(\bar{s}\tau\omega_f-\bar{\mu}\,(1-\xi)\right)^2}{2\omega_f\,(1+\tau^2\omega_f\omega_z)}\right)\left[1-\Phi\left(\frac{1}{\sqrt{\omega_z}}\frac{\bar{s}+\bar{\mu}\,(1-\xi)\,\tau\omega_z}{\sqrt{1+\tau^2\omega_f\omega_z}}\right)\right].$$

Figure A.2 illustrates how strategic complementarities and multiple equilibria might arise in this model of partial resolution of uncertainty.

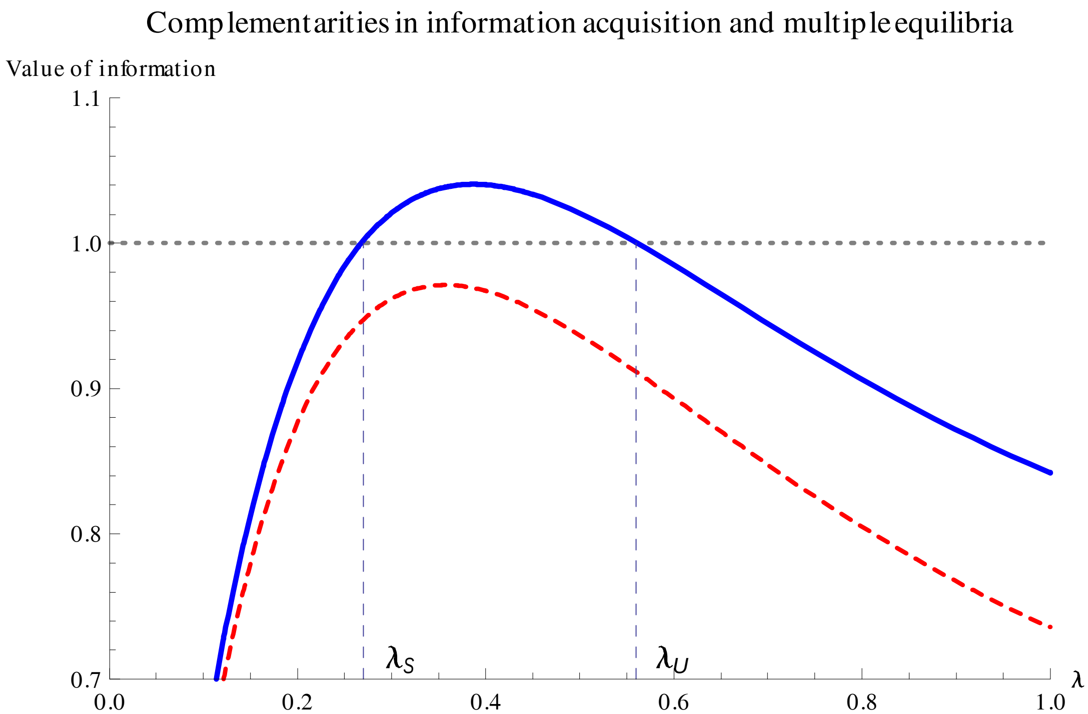

FIGURE A.2. The value of information in a market with partial resolution of Knightian uncertainty, as a function of the fraction of the informed agents, λ. Parameter values are as in Figure A.1, with the exception of the average asset supply, μ_z, which equals 4. The solid (dashed) line depicts the value of information when $\Delta\mu = 10$ ($\Delta\mu = 10.5$).

The value of information, arising for $\Delta\mu = 10$ (the solid line), leads to three equilibria: two of these are interior, $\lambda^* = \lambda_S$ (stable) and $\lambda^* = \lambda_U$ (unstable); the third one is that with $\lambda^* = 1$ (stable). Increasing $\Delta\mu$ to 10.5 leads to a unique equilibrium, that with $\lambda^* = 1$.

Appendix D: Details for the multi-period market and VAR estimates of Section 5

Multi-period market. The equilibrium price in this market is the same as that in Eq. (6) of Proposition I, with $(\underline{a}_t, \bar{a}_t, b_t, \lambda_t, s_t, \underline{\mu}_t^{\mathrm{p}}, \bar{\mu}_t^{\mathrm{p}})$ replacing $(\underline{a}, \bar{a}, b, \lambda, s, \underline{\mu}, \bar{\mu})$, where:

$$s_t = \frac{\lambda_t \theta_{t+1}}{\tau \omega_\epsilon} - (z_t - \mu_z), \quad \underline{\mu}_t^{\mathrm{p}} = (1-\rho_\theta)\left(\mu_0 - \frac{1}{2}\Delta\mu_t\right) + \rho_\theta \theta_t, \quad \bar{\mu}_t^{\mathrm{p}} - \underline{\mu}_t^{\mathrm{p}} = (1-\rho_\theta)\,\Delta\mu_t.$$

Description of data and VAR methodology. The impulse-response function in Figure 7, and the confidence bands around it, relate to a VAR(6) model with two endogenous variables: (i) the series of aggregate stock market returns of Fama and French (1993), and (ii) a series of uncertainty shocks, defined to be always zero, except during the month when an uncertainty event takes place, in which case the shock series equals one. All data are monthly, and span the period from January 1957 to December 2008. The uncertainty shock series is the "first volatility" event series identified by Bloom (2009) (Table A.1, p. 676). It equals one on the first months a stock market volatility index (defined below) is higher than two standard deviations above the Hodrick-Prescott detrended (with parameter 129,600) mean of the same volatility series. As regards the sampling period 1986-2008, the stock market volatility series is the CBOE VXO index of implied volatility for one month at-the-money options on the S&P100. As for the 1957-1985 period, when the CBOE index is not available, stock market volatility is defined as the monthly standard deviation of the daily returns on the S&P500 index, normalized to have the same mean and standard deviation of the VXO index for the period 1986-2008. Table A.1 contains a description of the uncertainty events, as well as the market returns during and after the events.

Table A.1. Stock market developments after an uncertainty shock.

		months after the event						
Date	Event	0	+1	+2	+3	+4	+5	+6
October 1962	Cuban missile crisis	−0.04	10.81	0.96	4.93	−2.42	3.06	4.49
November 1963	Assassination of JFK	−0.82	1.88	2.28	1.46	1.45	0.17	1.48
August 1966	Vietnam buildup	−7.95	−1.12	3.78	1.35	0.22	8.12	0.73
May 1970	Cambodia and Kent State	−6.96	−5.69	6.9	4.47	4.21	−2.28	4.58
December 1973	OPEC I, Arab-Israel War	0.52	−0.19	−0.35	−2.9	−5.35	−4.95	−2.89
September 1974	Franklin National	−11.78	16.05	−4.64	−3.4	13.58	5.41	2.61
November 1978	OPEC II	2.68	0.99	4.18	−3.41	5.75	0.05	−2.18
March 1980	Afghanistan, Iran hostages	−13.23	3.97	5.2	3.16	6.41	1.72	2.21
August 1982	Monetary cycle turning point	11.14	1.17	11.27	4.56	0.78	3.51	2.41
October 1987	Black Monday	−23.14	−7.58	6.64	4.2	4.71	−2.1	0.64
September 1990	Gulf War I	−5.98	−1.93	6.00	2.35	4.39	7.10	2.45
November 1997	Asian Crisis	2.66	1.31	0.02	6.93	4.74	0.66	−2.98
September 1998	Russian, LTCM default	5.92	7.12	5.89	5.93	3.48	−4.16	3.36
September 2001	9/11 terrorist attack	−9.43	2.58	7.71	1.63	−1.75	−2.31	4.34
July 2002	Worldcom and Enron	−8.26	0.66	−10.14	7.36	6.01	−5.44	−2.44
February 2003	Gulf War II	−1.63	0.93	8.18	6.26	1.53	2.24	2.42
August 2007	Credit Crunch	0.74	3.77	2.26	−5.27	−0.71	−6.44	−2.33

Each of the two equations of the VAR(6) is also fed by the current values of (i) the corporate spread, defined as the difference between the baa industrial bond yield and the ten year Government bond yield;

(ii) the term spread, defined as the difference between the ten year government bond yield and the yield on three month Treasury Bills; (iii) the price-dividend ratio on the S&P500 index; (iv) the National Bureau of Economic Research recession indicator. All data are collected from the Global Financial Data database, with the exception of the Fama and French market returns, which are taken from Kenneth French webpage.

We compute the first two moments of the posterior distribution of the impulse-response function in the VAR, by simulations. First, we simulate the VAR coefficients, drawing them from a normal distribution centered at the point coefficient estimates, and variance-covariance matrix equal to $\boldsymbol{S} \otimes (\boldsymbol{X}^\intercal \boldsymbol{X})^{-1}$, where $\boldsymbol{X}$ is the matrix containing the series of exogenous variables and the lagged endogenous variables, and $\boldsymbol{S}$ is the variance-covariance matrix of the VAR residuals, assumed to have a Normal-inverse Wishart posterior distribution, $\boldsymbol{S}^{-1} \sim \text{Wishart}\left((T\hat{\boldsymbol{S}})^{-1}, T-P\right)$, and $\hat{\boldsymbol{S}}$ is the point estimate of $\boldsymbol{S}$, T is the sample size and P is the dimension of each series in $\boldsymbol{X}$. For each simulation, we compute the impulse-response function of stock market returns to a one standard deviation change in the uncertainty series, and aggregate across 5000 simulations to calculate the cross-sectional average and standard errors used to produce the market responses in Figure 7.