Eyno Rots

Learning and the Market for Housing

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(Tanulás a lakáspiacon)

Written by Eyno Rots

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Abstract

House prices have inertia, which may be because housing-market participants need time to recognize long booms and recessions. Within a dynamic stochastic general-equilibrium model with markets for housing and defaultable mortgages, I consider the case of imperfect knowledge and learning about the persistence of exogenous shocks. I evaluate the performance of the model against the last 40 years of key U.S. macroeconomic data. Bayesian comparison strongly favors the model with learning over the baseline case with perfect knowledge, although additional assumptions about the learning process may be necessary for an adequate account of house-price dynamics.

JEL: E32, E37, R31.

Keywords: housing market, DSGE, signal extraction, Bayesian estimation.

Összefoglaló

A lakásárak ragadósságának egy lehetséges magyarázata, hogy a piac résztvevőinek időre van szüksége a hosszabb recessziós és konjunkturális időszakok felismeréséhez. Egy lakás- és jelzálogpiacot is tartalmazó dinamikus és sztochasztikus általános egyensúlyi modell segítségével azt vizsgálom, hogy egy részleges információs környezetben a piaci résztvevők hogyan szereznek tudást a gazdaságot érő sokkok tartósságáról. A modellt az USA 40 évet átfogó makroadatain értékelem ki. A bayes-i illeszkedésvizsgálat egyértelműen a tanulási folyamatot is tartalmazó modellt támogatja a teljes információs alapmodellel szemben, bár a tanulási folyamatról alkotott feltevések nagyban befolyásolják a modell sikerességét.

1 Introduction

Before the financial crisis of 2007–2009, the U.S. housing market had been booming for over a decade; rising house prices fueled the lenders' desire to create new, risky types of mortgages and offer them to the widest set of households. By the end of 2006, household debt was worth 80 percent of annual GDP, of which mortgage balances accounted for 60 percent of GDP.[1] The end of the housing boom triggered the most severe financial crisis and the longest recession in decades. Events like these require that economic theory provides a thorough understanding of the housing market. What is particularly puzzling about the housing market is how slow it is to adjust. As Figure 1 shows, the house price index declined steadily for five years upon the onset of the recent crisis.[2] However, within the context of frictionless models with rational expectations, the general prediction is that market prices should quickly reflect all the news about the state of the economy. For comparison, the S&P500 Index shows that the corresponding downward price adjustment in the market for capital lasted only six quarters.

A potential explanation of this dynamic feature of the housing market is that its participants lack knowledge about the nature of business-cycle fluctuations. In case of the Great Recession, economic agents observed the deteriorating economy, but did not expect the downturn to be so persistent. That is, they were initially over-optimistic. Households were betting on a shorter recession; they were willing to keep their houses and mortgages. As the economic downturn continued, they eventually understood its length. Such gradual recognition of a persistent recession could potentially account for the slow adjustment of house prices, as well as other variables. For example, as long as households kept updating their beliefs towards a longer recession, they saw unexpected house-price declines, which fueled mortgage foreclosures.[3] Figure 1 shows that during the recent crisis, the rate of mortgage-foreclosure starts hovered for over three years above the one-percent mark, more than double its average value for 2002–2006.

The goal of this paper is to study learning about the persistence of exogenous shocks to economy as an explanation for inertia in house prices. I take three steps towards this goal. First, I build a dynamic stochastic general-equilibrium model with an endogenous market for housing and mortgages that is driven by exogenous shock processes with two components: a transitory component, which is a white noise; and a persistent component, which is an AR(1) process. Second, I consider the situation of imperfect knowledge when economic agents cannot observe the individual components of each shock process. The agents would observe the process through time and rely on Kalman-type signal-extraction to gradually learn about its components. And third, I evaluate the ability of the models with perfect and imperfect knowledge to explain the 40 years of key U.S. data on housing and the aggregate economy. To that end, Bayesian methods help compare the two models. Numerical simulations reveal posterior odds that are strongly in favor of the model with imperfect knowledge and learning; however, the ability of the latter to simulate inertia in house prices seems to have space for improvement. I show that additional assumptions about the learning mechanism can make the model with imperfect knowledge a likelier data-generating process that can mimic the sluggish house-price dynamics. In particular, a parameterization of the learning process that is less restrictive than the classical Kalman filter seems promising.

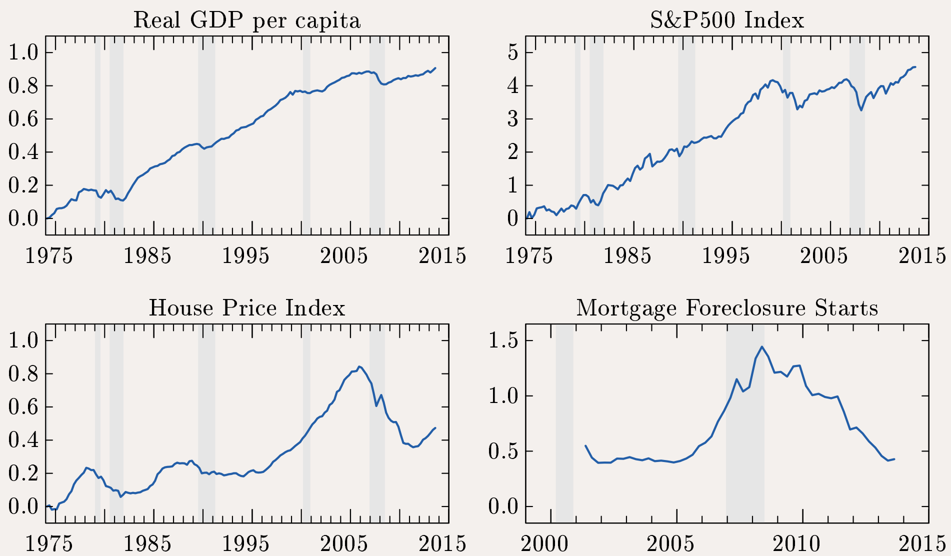
Several arguments can motivate the assumption of imperfect knowledge as a way to account for house-price dynamics. First, in models with rational expectations and complete information, market prices immediately reflect exogenous shocks. Without additional assumptions, gradual price adjustments in such models would have to come from sequences of small shocks, which are incompatible with the assumption that the shocks are i.i.d. Second, the assumption of learning finds its support in the literature. For example, an empirical study by Foote et al. (2012) finds that at the beginning of the 2007–2009 financial crisis, mortgage-market participants did not know the state of economy; they had beliefs that were ex-post over-optimistic and acted rationally subject to them. Finally, the assumption of imperfect knowledge is particularly sensible in case of the housing market. This market encompasses millions of households that make only a few purchases and sales during their lifetime. As for

[1] FRB of New York, Consumer Credit Panel

[2] House prices briefly increased around the end of 2008 after the Federal Reserve announced its large-scale MBS purchase program. See Fuster and Willen (2010) for a discussion of the effects of the program.

[3] Gerardi et al. (2008) point out that unexpected changes in house prices drive mortgage foreclosures.

Figure 1
U.S. economy and housing market, 1975–2015.



All the series are adjusted by inflation and seasonally, and cast in logs with base 2. Foreclosure starts are yearly percentage rates. Shades indicate NBER-dated recessions. *Source*: BEA, BLS, S&P Dow Jones Indices, FHFA, MBA, FRED, NBER.

capital market, its participants are professionals who closely monitor the state of the economy. It is reasonable to believe that participants in the housing market have a less detailed knowledge about the state of economy.

The fact that house prices evolve slowly is well-known in the literature. Case and Shiller (1989) have noted that house prices possess inertia, or high auto-correlation in their growth. A number of more recent studies confirm for the U.S. and other countries that house prices exhibit long duration of upturns and downturns, and generally estimate each of them between 4.5 and 6.5 years on average: Ceron and Suarez (2006); Cunningham and Kolet (2011); Agnello and Schuknecht (2011); Bracke (2013).

The novelty of my work is that it addresses the dynamic features of house prices within the context of a DSGE model. The benefit of the DSGE approach is its straightforward application to the data for the purposes of estimation and forecasting and a study of linkages between the housing market and the aggregate economy. The crisis of 2007–2009 and the preceding housing boom have brought the housing market to many economists' attention, and has motivated the development of general-equilibrium models with housing that allow for endogenous treatment of the house price, foreclosure rate, mortgage risk premium, etc. Examples include Iacoviello (2005), Monacelli (2009), Chatterjee and Eyigungor (2011). I draw from three key sources to construct the model. Iacoviello and Neri (2010) study the impact of the housing market on aggregate economy. They introduce a rich technology structure that accounts for long-run growth and a portion of short-run fluctuations in the house price and other variables. I use a similar structure but expand it to incorporate persistent and transitory components. To introduce an endogenous market for mortgages with defaults that happen in equilibrium, I follow Forlati and Lambertini (2011) who develop a dynamic new-Keynesian model to study the impact of mortgage-market financial shocks on the aggregate economy. Finally, to implement imperfect knowledge, I follow Gilchrist and Saito (2006) who augment the financial-frictions model of Bernanke et al. (1999) to study the implications of imperfect knowledge for monetary policy.

A considerably successful approach to add momentum to the housing market that has been taken in the existing literature is through frictions within the context of a searching model. These frictions would typically affect the way houses are traded, as in Guren (2015); Head et al. (2014). I view my approach as complementary to this explanation of the house-price momentum. Informational frictions have also been studied within searching models. Piazzesi and Schneider (2009) and Glaeser and Nathanson (2015) consider departures from rational expectations and generate momentum in house prices. My work shares the idea about rationality under incomplete information with Anenberg (2014), who suggests that traders in the housing market face a noisy signal about the fundamental value of housing stock and rely on learning to gradually uncover it. His model features quick learning and, as a result, house-price persistence that is low compared to the data. I depart from the searching framework and construct a general-equilibrium model which can be used in tandem with standard likelihood-based estimation methods. In addition, I address the problem that learning may generate limited persistence.

Burnside et al. (2011) develop a model with heterogeneous beliefs to study the implications of incomplete knowledge about fundamentals for housing-market dynamics. The authors argue that it is difficult to generate protracted house-price dynamics in case of homogeneous beliefs because a change in beliefs quickly translates into prices. They model an infection-like mechanism that gradually affects the beliefs of the population and creates protracted house-price dynamics. On the contrary, I attempt to account for dynamics of the house price in a model with *homogeneous* beliefs. In my model, the idea is that learning makes changes in homogeneous beliefs gradual and creates sluggishness.

The paper proceeds as follows: section 2 defines the model and discusses its key assumptions; section 3 describes the estimation procedure and section 4 summarizes the results; section 5 discusses the extensions; section 6 concludes.

2 Model

Time is discreet and one period lasts one quarter. There are two perfectly competitive sectors that produce consumption good and housing stock. There are N households that supply a fixed amount of labor and enjoy the consumption good and the stream of services flowing from housing stock. A fraction Ψ of total population are impatient households with a low discount factor $\check{\beta}$, and a fraction $1-\Psi$ are patient households with a high discount factor $\hat{\beta}$: $0<\check{\beta}<\hat{\beta}<1$. In equilibrium, the prices will be such that impatient households find it optimal to borrow in order to front-load their purchases, which can only be done by means of mortgages. Patient households will save in equilibrium; they provide the borrowers with mortgage loans and own all the physical assets used for production.[4] Henceforth, I refer to the impatient households as borrowers, and to the patient ones as savers; all the variables marked with a hat represent savers, and the variables marked with an inverted hat represent borrowers.

Idiosyncratic shocks make a fraction of borrowers default on mortgages in equilibrium. Yet, idiosyncratic payoffs are pooled within every household, so that each borrower and saver is a representative agent within their group. This set-up allows for an endogenous treatment of defaults, and yet it keeps the model highly tractable. I start the description of the model with the market for mortgages in order to clarify this issue first.

2.1 MORTGAGES

For the sake of tractability, mortgage market is simplified along two dimensions. One simplification is that there are only one-period mortgages assumed available in the economy. In reality, mortgages may have different terms (up to 30 years), amortization schedules, down-payments, etc. These loan parameters matter for the performance of the mortgage market and the whole economy.[5] An account for these parameters would require a model that tracks an endogenous distribution of households across stages of mortgage amortization and mortgage types. In order to keep the model tractable, I rather choose to stick to the representative-agent framework and focus on aggregate behavior of the variables such as house price and mortgage premium.

The same reasons justify the second simplification, which is the treatment of default. Let each household be a unit mass of household members that are ex-ante identical; each member i conducts the policy that is optimal for the aggregate household. Then, if a variable Z_t is a part of the household's optimal policy, it is true that $Z_t=\int_0^1 Z_{i,t}di$, and $Z_{i,t}=Z_t$. Suppose a member i of a borrowing household wants to buy a house $H_{i,t-1}=H_{t-1}$ in period $t-1$ at price P_{t-1} (measured in consumption good). To finance this purchase, the member can use the house as collateral and obtain a one-period loan B_{t-1} with a pre-determined interest rate $\bar{r}_{m,t-1}$. Next period, the outstanding debt is $B_{t-1}(1+\bar{r}_{m,t-1})$, and the value of the house is $H_{t-1}P_t(1-\delta_h)\omega_{i,t}$, where δ_h is the rate of housing-stock depreciation and $\omega_{i,t}\sim F(\omega,\varsigma_t)$ is the idiosyncratic shock to member i's housing stock that is i.i.d. across household members. The shock $\omega_{i,t}$ can represent the effect of neighborhood externalities as in Hilber (2005), or stochastic depreciation as in Jeske and Krueger (2005). The cumulative distribution $F(\omega,\varsigma_t)$ is time-dependent:

$$\omega_{i,t}\sim F(\omega,\varsigma_t),\ \text{such that}\ \ln\omega_{i,t}\sim N\left(-\frac{\varsigma_t^2}{2},\ \varsigma_t^2\right).$$

The variance ς_t^2 itself is an exogenous shock process, which means that idiosyncratic risk associated with housing varies with time; yet, the distribution $F(\omega,\varsigma_t)$ is chosen so that $E_t[\omega_{i,t}]=1$ for all t.

Mortgages are non-recourse in the model: the borrower's only cost of default is that the lender seizes the foreclosed property. Therefore, the household member will repay only if the value of the house exceeds the outstanding debt:

$$H_{t-1}P_t(1-\delta_h)\omega_{i,t}\geq B_{t-1}(1+\bar{r}_{m,t-1}).$$

[4] Monacelli (2009) shows that different discount factors create savers and borrowers in equilibrium.

[5] For example, Garriga and Schlagenhauf (2010), Corbae and Quintin (2015) address this issue. It suffices to say that a household with recently acquired mortgage is highly leveraged and likelier to default; especially if the mortgage comes with delayed amortization.

Equivalently, the household member will repay the debt if the the idiosyncratic shock is realized above the threshold: $\omega_{i,t} \geq \bar{\omega}_t$, where

$$\bar{\omega}_t = \frac{B_{t-1}(1+\bar{r}_{m,t-1})}{H_{t-1}P_t(1-\delta_h)}. \tag{1}$$

Given realizations $\{\omega_{i,t}\}_{i\in[0,1]}$, only a fraction of household members will default, but the household pools the ex-post payoffs from all its members' mortgage arrangements and is not subject to idiosyncratic risk. The model with such set-up neglects household wealth distribution but remains tractable and retains the essential interplay between housing-market variables and the borrowing constraint. Henceforth, every borrower and saver represents a collection of household members.

In period $t-1$, the borrowing household buys a house H_{t-1} and gets a loan $\{B_{t-1}, \bar{r}_{m,t-1}\}$; in period t, it pools the following payoff:

$$H_{t-1}P_t(1-\delta_h)\int_{\bar{\omega}_t}^{\infty}\omega dF(\omega,\varsigma_t) - B_{t-1}(1+\bar{r}_{m,t-1})\int_{\bar{\omega}_t}^{\infty}dF(\omega,\varsigma_t).$$

That is, the borrower only keeps the houses and repays the loans of the members who do not default. Every defaulting household member repudiates the loan and loses the house to the saver and thus brings zero payoff. The saver collects the foreclosed property and the payments for mortgage loans that are not repudiated:

$$H_{t-1}P_t(1-\delta_h)\ (1-\mu)\int_0^{\bar{\omega}_t}\omega dF(\omega,\varsigma_t) + B_{t-1}(1+\bar{r}_{m,t-1})\int_{\bar{\omega}_t}^{\infty}dF(\omega,\varsigma_t).$$

Fraction μ captures the cost of default paid by the lender. Foreclosed property is usually sold at a significant discount; there may be legal fees, debt collector's commission, etc. Such costs are presumably proportionate to the size of the house. Parameter μ can also represent the cost of state verification paid by the lender in order to collect the whole value of the foreclosed property. Thus, μ captures financial frictions in the market for mortgages. For brevity of notation, define

$$\Gamma(\bar{\omega}_t,\varsigma_t) = \int_0^{\bar{\omega}_t}\omega dF(\omega,\varsigma_t) + \bar{\omega}_t\int_{\bar{\omega}_t}^{\infty}dF(\omega,\varsigma_t), \tag{2}$$

$$G(\bar{\omega}_t,\varsigma_t) = \int_0^{\bar{\omega}_t}\omega dF(\omega,\varsigma_t), \tag{3}$$

where $\Gamma(\bar{\omega}_t,\varsigma_t)$ is the debt repaid to the lender expressed as a share of housing collateral, and $G(\bar{\omega}_t,\varsigma_t)$ is the average idiosyncratic shock associated with repudiated mortgages. Using equations (1)–(3) and the fact that $\int_0^{\infty}\omega dF(\omega,\varsigma_t) = 1$, the borrower's payoff becomes

$$H_{t-1}P_t(1-\delta_h)\big(1-\Gamma(\bar{\omega}_t,\varsigma_t)\big), \tag{4}$$

and the saver's payoff becomes

$$H_{t-1}P_t(1-\delta_h)\big(\Gamma(\bar{\omega}_t,\varsigma_t)-\mu G(\bar{\omega}_t,\varsigma_t)\big).$$

In effect, a mortgage contract involves two parties co-funding the purchase of a house and splitting its value upon the settlement: the saver claims share $\Gamma(\cdot)$, the borrower retains $1-\Gamma(\cdot)$, and $\mu G(\cdot)$ is lost due to default. To further the intuition, let $r_{m,t}$ be the saver's ex-post return on mortgage:

$$B_{t-1}(1+r_{m,t}) = H_{t-1}P_t(1-\delta_h)\big(\Gamma(\bar{\omega}_t,\varsigma_t)-\mu G(\bar{\omega}_t,\varsigma_t)\big). \tag{5}$$

Using (5), the borrower's payoff (4) can be written to say that the mortgage loan $\{B_{t-1}, \bar{r}_{m,t-1}\}$ is designed so that the borrower repays the interest $r_{m,t}$ and bears the cost of default $\mu G(\cdot)$:

$$H_{t-1}P_t(1-\delta_h)\big(1-\mu G(\bar{\omega}_t,\varsigma_t)\big) - B_{t-1}(1+r_{m,t}).$$

2.2 HOUSEHOLDS

Households have lifetime utility $\sum_{t=0}^{\infty}\beta^t E_0\big[U(C_t,H_t)\big]$, where instant utility comes from consumption C_t and the stream of housing services derived from housing stock H_t that belongs to the household:

$$U(C_t,H_t) = \nu_t(\ln C_t + \psi_t \ln H_t).$$

Shocks ν_t and ψ_t are mean-reverting exogenous processes that affect household behavior. A positive innovation to ν_t makes households temporarily less thrifty, since they start valuing current utility relatively more. A positive innovation to ψ_t produces a temporary increase in relative preference for housing.

Each period, savers maximize the expected lifetime utility by choosing the amount of consumption $\hat{C}_t$, housing stock $\hat{H}_t$, mortgage lending $\hat{S}_t$ extended to borrowers, land $\hat{l}_t$ used for housing construction, and capital $\hat{K}_{y,t}$ used for the production of consumption good and $\hat{K}_{x,t}$ used for housing construction, subject to the budget constraint:

$$\hat{C}_t + \hat{H}_t P_t + \hat{S}_t + \hat{l}_t P_{l,t} + \frac{\hat{K}_{y,t}}{A_{k,t}} + \hat{K}_{x,t} = W_t + \hat{H}_{t-1} P_t (1-\delta_h) + (1+r_{m,t})\hat{S}_{t-1} + \\ + \left(R_{l,t} + P_{l,t}\right)\hat{l}_{t-1} + \left(R_{y,t} + \frac{1-\delta_y}{A_{k,t}}\right)\hat{K}_{y,t-1} + \left(r_{x,t} + 1 - \delta_x\right)\hat{K}_{x,t-1}. \quad (6)$$

A purchase of one unit of housing stock in period $t-1$ yields $(1-\delta_h)$ units in period t.[6] Land (priced at $P_{l,t}$) yields a rent $R_{l,t}$. Mortgage lending yields an ex-post return $r_{m,t}$; capital rent and depreciation in the two sectors are denoted by $R_{y,t}$, $r_{x,t}$, δ_y, and δ_x. Each household sells one unit of labor to both sectors, and perfect mobility and substitutability of labor implies that both sectors pay the same wage W_t. Note that $A_{k,t}$ is the marginal cost of one unit of capital $K_{y,t}$; it is a technology shock specific to capital creation.[7] The saver's maximization problem produces the set of standard Euler equations outlined below: for every available asset, the utility cost is compared with the expected utility gain from the next period's payoff. In case of housing stock, the additional benefit is that it directly increases current period's utility, as shown by equation (7):

$$U'_{\hat{C},t} P_t = U'_{\hat{H},t} + \hat{\beta} E_t \left[U'_{\hat{C},t+1} (1-\delta_h) P_{t+1} \right]; \quad (7)$$

$$\frac{U'_{\hat{C},t}}{A_{k,t}} = \hat{\beta} E_t \left[U'_{\hat{C},t+1} \left(R_{y,t+1} + \frac{1-\delta_y}{A_{k,t+1}} \right) \right]; \quad (8)$$

$$U'_{\hat{C},t} = \hat{\beta} E_t \left[U'_{\hat{C},t+1} \left(r_{x,t+1} + 1 - \delta_x \right) \right]; \quad (9)$$

$$U'_{\hat{C},t} = \hat{\beta} E_t \left[U'_{\hat{C},t+1} \left(r_{m,t+1} + 1 \right) \right]; \quad (10)$$

$$U'_{\hat{C},t} P_{l,t} = \hat{\beta} E_t \left[U'_{\hat{C},t+1} \left(R_{l,t+1} + P_{l,t+1} \right) \right]. \quad (11)$$

The borrowers choose consumption $\check{C}_t$, housing $\check{H}_t$, and a mortgage contract $\{\check{B}_t, \bar{r}_{m,t}\}$ to maximize the expected lifetime utility subject to three constraints:

$$\check{C}_t + \check{H}_t P_t - \check{B}_t = W_t + \check{H}_{t-1} P_t (1-\delta_h)\left(1 - \Gamma(\bar{\omega}_t, \varsigma_t)\right); \quad (12)$$

$$\check{B}_t = E_t \left[\hat{\beta} \frac{U'_{\hat{C},t+1}}{U'_{\hat{C},t}} \check{H}_t P_{t+1} (1-\delta_h) \left(\Gamma(\bar{\omega}_{t+1}, \varsigma_{t+1}) - \mu G(\bar{\omega}_{t+1}, \varsigma_{t+1}) \right) \right]; \quad (13)$$

$$\bar{\omega}_t = \frac{\check{B}_{t-1}(1+\bar{r}_{m,t-1})}{\check{H}_{t-1} P_t (1-\delta_h)}. \quad (14)$$

In the budget constraint (12), the last term on the right-hand side is the payoff from the previous mortgage arrangement, as given by expression (4). The participation constraint (13) is a combination of equations (10) and (5): it tells that the mortgage arrangement must yield a satisfactory expected payoff to the saver. Finally, constraint (14) simply reproduces equation (1) and tells that the borrower understands the chance of default implied by the mortgage contract. The optimality conditions are the

[6] Note that idiosyncratic risk does not affect the saver's payoff: the saver keeps all the houses previously purchased by the household members, for whom $\int_0^\infty \omega F(\omega, \varsigma_t) d\omega = 1$.

[7] A more intuitive formulation is $\hat{K}_{y,t} = A_{k,t}\hat{C}_{k,t}$, where $\hat{C}_{k,t}$ is the amount of consumption good spent on capital. I follow Iacoviello and Neri (2010) and note that technology $A_{k,t}$ mostly refers to IT and is therefore not as applicable to construction-sector capital.

following:

$$U'_{\check{C},t}P_t = U'_{\check{H},t} + \check{\beta}E_t\left[U'_{\check{C},t+1}P_{t+1}(1-\delta_h)\big(1-\Gamma(\bar{\omega}_{t+1},\varsigma_{t+1})\big)\right] + \tag{15}$$

$$U'_{\check{C},t}E_t\left[\hat{\beta}\frac{U'_{\hat{C},t+1}}{U'_{\hat{C},t}}P_{t+1}(1-\delta_h)\big(\Gamma(\bar{\omega}_{t+1},\varsigma_{t+1})-\mu G(\bar{\omega}_{t+1},\varsigma_{t+1})\big)\right];$$

$$E_t\left[\check{\beta}\frac{U'_{\check{C},t+1}}{U'_{\check{C},t}}\Gamma'_{\bar{\omega}}(\bar{\omega}_{t+1},\varsigma_{t+1})\right] = \tag{16}$$

$$= E_t\left[\hat{\beta}\frac{U'_{\hat{C},t+1}}{U'_{\hat{C},t}}\big(\Gamma'_{\bar{\omega}}(\bar{\omega}_{t+1},\varsigma_{t+1})-\mu G'_{\bar{\omega}}(\bar{\omega}_{t+1},\varsigma_{t+1})\big)\right].$$

The first-order condition with respect to housing stock (15) is similar to that of the saver, except that, apart from direct impact on utility and an increase in next period's wealth, one more marginal benefit of housing stock for the borrower is that it serves as collateral and increases access to borrowing (the last term on the right-hand side). Equation (16) is the first-order condition with respect to mortgage interest rate $\bar{r}_{m,t}$: on the left hand, a higher mortgage rate increases the chance of default and decreases the borrower's payoff from mortgage; on the right hand, the saver will claim a larger fraction of the housing stock,[8] which expands borrower's current access to borrowing.

2.3 PRODUCTION

The production of consumption good Y_t takes place in a perfectly competitive sector that requires labor $n_{y,t}$ and capital stock $K_{y,t-1}$:

$$Y_t = (A_{y,t}n_{y,t})^{1-\alpha_y}K_{y,t-1}^{\alpha_y}. \tag{17}$$

Technology $A_{y,t}$ is specific to consumption-good production. Standard expressions for wage and capital rent describe profit-maximization in the sector:

$$W_{y,t} = (1-\alpha_y)Y_t/n_{y,t}, \tag{18}$$

$$R_{y,t} = \alpha_y Y_t/K_{y,t-1}. \tag{19}$$

Households consume the consumption good, convert it into capital, and allocate an amount X_t as an intermediary input for housing construction. In addition, as the following resource constraint suggests, a small part of output is lost due to mortgage defaults:

$$Y_t - \mu\Psi\check{H}_{t-1}P_t(1-\delta_h)G(\bar{\omega}_t,\varsigma_t) = \Psi\check{C}_t + (1-\Psi)\hat{C}_t + \frac{K_{y,t}}{A_{y,t}} - (1-\delta_y)\frac{K_{y,t-1}}{A_{y,t-1}} + K_{x,t} - (1-\delta_x)K_{x,t-1} + X_t. \tag{20}$$

Housing construction employs labor $n_{x,t}$, capital stock $K_{x,t-1}$, consumption good X_t, and land l_{t-1} to produce new housing stock:

$$IH_t = (A_{x,t}n_{x,t})^{1-\alpha_{xk}-\alpha_{xx}-\alpha_{xl}}K_{x,t-1}^{\alpha_{xk}}X_t^{\alpha_{xx}}l_{t-1}^{\alpha_{xl}}. \tag{21}$$

Technology $A_{x,t}$ is specific to housing construction. The resulting output must equal net purchases of new housing stock by both types of households:

$$IH_t = \Psi\check{H}_t + (1-\Psi)\hat{H}_t - \big(\Psi\check{H}_{t-1} + (1-\Psi)\hat{H}_{t-1}\big)(1-\delta_h). \tag{22}$$

To justify the inclusion of consumption good into housing production function, note that housing construction involves household appliances and furnishing.[9] It makes it easier to solve the model and control for the elasticity of housing supply. The inclusion of land (whose stock is fixed) stabilizes the quantity of produced housing and therefore adds volatility to house prices. As Iacoviello and Neri (2010) point out, it is a useful feature of the model if one of its goals is to explain the dynamics of the house price. Construction sector is perfectly competitive, and the following optimality conditions summarize profit-maximization:

$$X_t = \alpha_{xx}IH_tP_t; \tag{23}$$

$$W_{x,t} = (1-\alpha_{xk}-\alpha_{xx}-\alpha_{xl})IH_tP_t/n_{x,t}; \tag{24}$$

$$r_{x,t} = \alpha_{xk}IH_tP_t/K_{x,t-1}; \tag{25}$$

$$R_{l,t} = \alpha_{xl}IH_tP_t/l_{t-1}. \tag{26}$$

[8] $\Gamma(\cdot) - \mu G(\cdot)$ is increasing in $\bar{\omega}$ around the steady state; see Bernanke et al. (1999).

[9] Y_t can represent intermediate good that is used to produce housing, capital, or consumption good.

2.4 SHOCKS

Technologies $A_{y,t}$, $A_{x,t}$, and $A_{k,t}$ are growth-stationary stochastic processes:[10] for $i \in \{y, x, k\}$,

$$\ln A_{i,t} - \ln A_{i,t-1} = \gamma_{i,t} + u_{i,t}, \qquad u_{i,t} \sim N(0, \sigma^2_{u,i}), \text{ i.i.d.}; \tag{27}$$

$$\gamma_{i,t} - \gamma_i = \rho_i(\gamma_{i,t-1} - \gamma_i) + v_{i,t}, \qquad v_{i,t} \sim N(0, \sigma^2_{v,i}), \text{ i.i.d.} \tag{28}$$

The first component of technology growth is an AR(1) process $\gamma_{i,t}$ which is stationary around γ_i. The shock $v_{i,t}$ is persistent: it has a lasting impact on technological growth. The second component is a white-noise process $u_{i,t}$ labeled transitory shock. In addition, there are three non-technological exogenous processes with persistent and transitory components: for $i \in \{\psi, \nu, \varsigma\}$,

$$\ln i_t = \gamma_{i,t} + u_{i,t}, \qquad u_{i,t} \sim N(0, \sigma^2_{u,i}), \text{ i.i.d.}; \tag{29}$$

$$\gamma_{i,t} - \gamma_i = \rho_i(\gamma_{i,t-1} - \gamma_i) + v_{i,t}, \qquad v_{i,t} \sim N(0, \sigma^2_{v,i}), \text{ i.i.d.} \tag{30}$$

First, ψ_t is the share of housing stock in household utility. It is a housing demand shock that affects household preferences toward housing services. Second, ν_t is a shock to inter-temporal preferences; it makes households value current period's instant utility differently. Finally, ς_t is a financial shock that affects the level of risk in the market for mortgages and affects the flow of credit from savers to borrowers.

The equilibrium is defined dynamically by equations (6)–(30) and a set of market-clearing conditions: $(1-\Psi)\hat{S}_t = \Psi\check{B}_t$; $W_{y,t} = W_{x,t} = W_t$; $(1-\Psi)\hat{K}_{y,t} = K_{y,t}$; $(1-\Psi)\hat{K}_{x,t} = K_{x,t}$; $(1-\Psi)\hat{l}_t = l_t = L$; $n_{y,t} + n_{x,t} = N$.

2.5 STATE-SPACE FORM

Cobb-Douglas specification of production and utility functions guarantees the existence of a balanced-growth path and facilitates de-trending. The three separate growth-stationary technology processes allow for the existence of different growth rates in observable variables. Let $A_{c,t}$ denote the measure of total productivity in consumption-good sector:

$$A_{c,t} = A_{y,t} A_{k,t}^{\alpha_y/(1-\alpha_y)}.$$

Note that it depends on the efficiency of capital creation in this sector. Consumption and most other variables are de-trended using $A_{c,t}$. Let $A_{h,t}$ be the measure of total housing-sector productivity:

$$A_{h,t} = A_{c,t}^{\alpha_{xk}+\alpha_{xx}} A_{x,t}^{1-\alpha_{xk}-\alpha_{xx}-\alpha_{xl}}.$$

It depends on $A_{c,t}$ because housing sector employs inputs that originate from the consumption-good sector. Housing variables $\check{H}_t$, $\hat{H}_t$, and IH_t are de-trended using $A_{h,t}$. The house price is de-trended using productivity in both sectors: $p_t = P_t A_{h,t}/A_{c,t}$. That is, it grows together with the supply of consumption good relative to housing stock. Note that all three technology processes ultimately affect the long-run growth of the house price.

Upon de-trending, the model can be log-linearized and cast in the following linear state-space form:[11]

$$\mathbf{z}_t = \Phi_0 + \Phi_1 \mathbf{s}_{1,t} + \Phi_2 \mathbf{s}_{2,t} + \Phi_3 t; \tag{31}$$

$$\mathbf{s}_{1,t} = \mathbf{T}_1 \mathbf{s}_{1,t-1} + \mathbf{H}_1 \mathbf{e}_t; \tag{32}$$

$$\mathbf{s}_{2,t} = \mathbf{s}_{2,t-1} + \mathbf{H}_2 \mathbf{e}_t. \tag{33}$$

Equation (31) decomposes the vector of logged observable variables $\mathbf{z}_t$. The first two terms on the right-hand side capture the appropriately de-trended and log-linearized model: Φ_0 contains logged steady-state values, and $\mathbf{s}_{1,t}$ summarizes log-deviations from the steady state. The last two terms capture non-stationary components of the model, which come from technology processes. Note that the precision of the local approximation does not suffer from the fact that the model is non-stationary,

[10] A trend-stationary version of the model produces a posterior distribution that is centered very close to 1 for some AR coefficients. Stationarity in growth seems to be a more appropriate specification.

[11] (31)–(33) is a Beveridge-Nelson decomposition, where $\Phi_2 \mathbf{s}_{2,t}$ defines cointegrating relationships.

because log-linearization applies only to the stationary component of the system, which is the de-trended model; for example, see Chang et al. (2007).

To illustrate how the model fits into system (31)–(33), consider house price $P_t \in \mathbf{z}_t$. It can be expressed as

$$\ln P_t = \ln p + \tilde{p}_t + \ln A_{c,t} - \ln A_{h,t},$$

where $\ln p \in \Phi_0$ is the logged steady-state value of the de-trended price and $\tilde{p}_t \in \mathbf{s}_{1,t}$ is the log-deviation from the steady state approximated by the linear model (32). Furthermore, equations (27) and (28) let express any technology i as

$$\ln A_{i,t} = a_{i,t} + \gamma_i t - \tfrac{\rho_i}{1-\rho_i}\tilde{\gamma}_{i,t}; \tag{34}$$

$$a_{i,t} = a_{i,t-1} + \tfrac{1}{1-\rho_i} v_{i,t} + u_{i,t}. \tag{35}$$

Stationary component $\tilde{\gamma}_{i,t} = \gamma_{i,t} - \gamma_i$ defined by equation (28) is a part of the state vector $\mathbf{s}_{1,t}$. Non-stationary components $a_{i,t} \in \mathbf{s}_{2,t}$ and $\gamma_i t$; are captured by the last two terms of the system (31). All the other observable variables have a similar decomposition.

2.6 IMPERFECT KNOWLEDGE AND LEARNING

In case of perfect knowledge, economic agents can observe both persistent and transitory components of exogenous processes defined by equations (27)–(30). In case of imperfect knowledge, the agents only observe the total value $\gamma_{i,t} + u_{i,t}$, $i \in \{y, k, x, \psi, \nu, \varsigma\}$, but not the individual components $\gamma_{i,t}$ and $u_{i,t}$. For example, if the agents observe an unusually large increase in technology $A_{i,t}$, they cannot immediately tell if they should expect higher growth in the future as well, since the increase can be due to a persistent shock $v_{i,t}$, a transitory shock $u_{i,t}$, or both. To resolve this uncertainty, the agents can observe the growth rate and use a linear steady-state Kalman filter to gradually learn about the sources of growth. The motivation behind the steady-state filter is that the agents have established the features of each exogenous process through a long history of observations. The agents are assumed to know the values σ_{vi}, σ_{ui}, ρ_i, and γ_i for all $i \in \{y, k, x, \psi, \nu, \varsigma\}$. An interesting question that I address in section 5 is whether the values that the agents "know" correspond to the actual values.

Let $\bar{\tilde{\gamma}}_{i,t} = E(\tilde{\gamma}_{i,t} | \ \tilde{\gamma}_{i,0} + u_{i,0} \ , \dots, \tilde{\gamma}_{i,t} + u_{i,t})$ be the persistent component of process i that agents infer at time t based on all past observations. The standard Kalman-filtering equation follows from equations (27)–(30) and summarizes the way agents estimate $\bar{\tilde{\gamma}}_{i,t}$:

$$\bar{\tilde{\gamma}}_{i,t} = \lambda_i(\tilde{\gamma}_{i,t} + u_{i,t}) + (1-\lambda_i)\rho_i \bar{\tilde{\gamma}}_{i,t-1},$$

where parameter $\lambda_i \in [0, 1]$ is the steady-state Kalman gain:

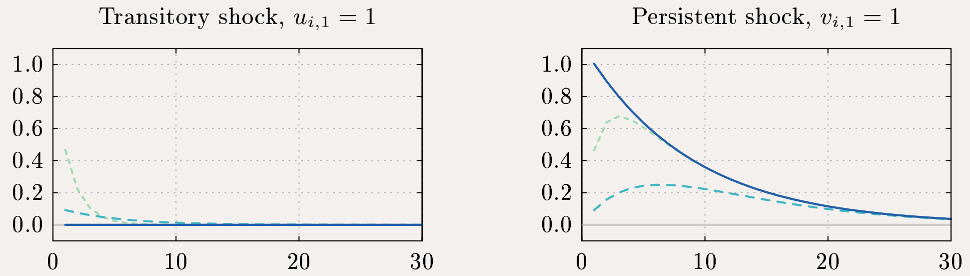
$$\lambda_i = \frac{d_i - (1-\rho_i^2) + \sqrt{(1-\rho_i^2)^2 + d_i^2 + 2(1+\rho_i^2)d_i}}{2 + d_i - (1-\rho_i^2) + \sqrt{(1-\rho_i^2)^2 + d_i^2 + 2(1+\rho_i^2)d_i}}. \tag{36}$$

Kalman gain λ_i measures how quickly the agents attribute a given change to the persistent component; it positively depends on $d_i = \sigma_{v,i}^2/\sigma_{u,i}^2$ and ρ_i. Intuitively, the persistent shock that is volatile is likelier to create an observed change. In addition, if the change persists, the agents will quickly attribute it to the persistent component rather than a sequence of transitory shocks.

Figure 2 demonstrates the process of learning. There is a shock at $t = 1$ that increases $\tilde{\gamma}_{i,t} + u_{i,t}$. The graph on the left shows the effect of a transitory shock $u_{i,1}$. The persistent component $\tilde{\gamma}_{i,1}$ does not change, but agents suspect that the shock may be persistent immediately upon the shock. The impact of the shock on $\tilde{\gamma}_{i,t} + u_{i,t}$ disappears in all subsequent periods $t > 1$, and agents eventually learn that the shock is transitory. The graph on the right shows the case of persistent shock $v_{i,1}$. Initially, the inferred value of the persistent component is lower than the actual value. As agents keep observing elevated values of $\tilde{\gamma}_{i,t} + u_{i,t}$ at $t > 1$, they gradually learn that the shock has been persistent, since the observations are far more likely to be the result of a persistent shock rather than a series of transitory shocks. The inferred and the actual values of the persistent component eventually converge. Importantly, a relatively volatile transitory component makes agents slow to learn about persistent shocks.

In order to impose the case of imperfect knowledge on the model, I follow Gilchrist and Saito (2006): I take the linear system (32) and replace the actual values of persistent components $\{\tilde{\gamma}_{i,t}\}^i \in \mathbf{s}_{1,t}$ and shocks $\{v_{i,t}, u_{i,t}\}^i \in \mathbf{e}_t$ with their estimates inferred

Figure 2
Changes in growth rate and learning.



Response of the persistent component of an exogenous shock process due to one-percent positive persistent and transitory innovation. The solid lines show the actual responses of the variable $\tilde{\gamma}_{i,t} = \rho_i \tilde{\gamma}_{i,t-1} + v_{i,t}$. The dashed lines show the values $\bar{\tilde{\gamma}}_t$ inferred by the agents. The dark dashed lines are for $\sigma^2_{v,i}/\sigma^2_{u,i} = 0.05$ and $\rho_i = 0.9$. The light dashed lines are for $\sigma^2_{v,i}/\sigma^2_{u,i} = 0.5$ and $\rho_i = 0.9$.

through Kalman filtering. Also, for each of the exogenous processes, I add a set of linear equations that relate the Kalman-filtered estimates to their actual values:

$$\bar{v}_{i,t} = \lambda_i(\tilde{\gamma}_{i,t} + u_{i,t}) - \lambda_i \rho_i \bar{\tilde{\gamma}}_{i,t-1};$$
$$\bar{u}_t = (1 - \lambda_i)(\tilde{\gamma}_{i,t} + u_{i,t}) - (1 - \lambda_i)\rho_i \bar{\tilde{\gamma}}_{i,t-1};$$
$$\tilde{\gamma}_{i,t} = \rho_i \tilde{\gamma}_{i,t-1} + v_t.$$

3 Estimation

3.1 RELATING THE MODEL AND THE DATA

The aggregate resource constraint combines constraints (20) and (22); it illustrates the model's counterpart to the observed variables:

$$
\begin{aligned}
Y_t + IH_tP_t - \mu\Psi\check{H}_{t-1}P_t(1-\delta_h)G(\bar{\omega}_t,\varsigma_t) = \\
= \frac{K_{y,t}}{A_{y,t}} - (1-\delta_y)\frac{K_{y,t-1}}{A_{y,t-1}} + K_{x,t} - (1-\delta_x)K_{x,t-1} + \Psi\check{C}_t + (1-\Psi)\hat{C}_t + \\
+ P_t\Big(\big(\Psi\check{H}_t + (1-\Psi)\hat{H}_t\big) - \big(\Psi\check{H}_{t-1} + (1-\Psi)\hat{H}_{t-1}\big)(1-\delta_h)\Big). \quad (37)
\end{aligned}
$$

The left-hand side of the equation represents GDP: it combines the value added in the two sectors of production and accounts for the cost of mortgage defaults. The second line of the constraint represents capital investment and consumption. The real aggregate consumption AC_t is defined as $\Psi\check{C}_t + (1-\Psi)\hat{C}_t$; the real non-residential investment IK_t is defined as $K_{y,t} - K_{y,t-1}(1-\delta_y)$.[12] The last term of the equation is the product of two observed variables, real house price P_t and real residential investment IH_t. The data set includes the four quarterly series $Z = \{AC_t, IK_t, IH_t, P_t\}_{t=1}^{T}$ and spans 1975–2014.

3.2 CALIBRATION

Likelihood-based estimation outlined below focuses on the features of exogenous processes: autoregressive coefficients and standard deviations of shocks. All the other parameters are calibrated, since they have proven to be either hard to identify in preliminary likelihood estimations, or particularly convenient to pin down with a set of standard steady-state targets. The chosen set of calibrated parameters defines the steady state that is equivalent for the two considered models. Table 1 summarizes the calibration.

I fix the steady-state technology growth rates using the average rates of growth of consumption, capital investment, and house price. The resulting values are in line with the posterior estimated by Iacoviello and Neri (2010). Residential investment is the most volatile series; its average growth rate is computed as a residual between the growth rates of consumption and the house price.[13]

The share of borrowers in population is set equal to two thirds, which corresponds to the share of homeowners with mortgages according to BLS.[14] Saver's discount factor is set at $\hat{\beta} = 0.997$; given the average growth rate of consumption around $\gamma_c = 0.44\%$, this sets the real quarterly interest rate equal to 0.74%. Given $\hat{\beta}$, the steady-state versions of equations (13), (14), and (16) allow to jointly determine parameters $\{\check{\beta}, \sigma_\omega, \mu\}$ so that the steady state matches three mortgage-market targets: mortgage premium, foreclosure rate, and loan-to-value ratio. The chosen values $\check{\beta} = 0.951$, $\mu = 0.1186$, and $\sigma_\omega = 0.1278$ seem generally in line with the literature.[15] Given these values, the steady-state quarterly mortgage premium equals 32 basis points, which matches the average value reported by the Federal Reserve for 1984–2014.[16] The delinquency rate on real-estate

[12] NIPA do not provide quarterly data to separate real private capital investment in construction and other sectors; however, given the relatively small scale of investment in construction-sector capital, it seems safe to approximate capital investment in both sectors with IK_t. See Iacoviello and Neri (2010).

[13] High volatility of residential investment is also the reason why the posterior is not estimated for the growth rates. In the steady state, de-trended residential investment equals $(\Psi\check{h} + (1-\Psi)\hat{h}) \times (\gamma_h + \delta_h)/(1+\gamma_h)$. Posterior estimations would put the housing growth rate γ_h close to $-\delta_h$, so that residential investment would become infinitesimal but volatile in the model. To avoid this result, I fix the growth rates with calibration. In short, long-run growth rates in a growth-stationary model turn out to be difficult to identify with likelihood estimation.

[14] BLS, Consumer Expenditure Survey, 2006–2010

[15] Forlati and Lambertini (2011) set $\sigma_\omega = 0.2$ and $\mu = 0.12$, where the latter is based on the data on foreclosure discounts. As for the borrowers' discount factor $\check{\beta}$, its value should be sufficiently low in order for the borrowers to accept risky mortgages with costly default in equilibrium. An overview of the existing estimates of discount factors by Iacoviello (2005) suggests that $\check{\beta} = 0.951$ is reasonable.

Table 1
Calibrated parameters

Parameter	Value	Meaning
Ψ	0.6667	Borrowers' share in population
$\hat{\beta}$	0.997	Saver's discount factor
$\check{\beta}$	0.951	Borrower's discount factor
σ_ω	0.1278	Standard error of idiosyncratic shock to house size
μ	0.1186	Cost of mortgage foreclosure
ψ	0.15	Share of housing in utility
α_y	0.25	Share of capital in consumption-good production
α_{xk}	0.1	Share of capital in housing construction
α_{xh}	0.1	Share of housing stock in housing construction
α_{xx}	0.1	Share of consumption good in housing construction
δ_y	0.02	Depreciation rate of consumption capital
δ_x	0.03	Depreciation rate of housing construction capital
δ_h	0.01	Depreciation rate of housing stock
γ_y	0.0034	Consumption technology growth
γ_x	0.0021	Housing technology growth
γ_k	0.0030	Capital technology growth

loans averaged 2.2% in 1991–2006, and peaked as high as 11% during the following recession.[17] Notice, however, that not every delinquent mortgage ends with a foreclosure. The steady-state value of the foreclosure rate in the model is 2%. The LTV ratio for mortgage loans has averaged at 76.5% in 1984–2014;[18] calibration implies 75% for the model.

The remaining parameters have to be determined jointly in order for the model to match a set of targets established in the literature. Capital share in consumption sector is $\alpha_y = 0.25$, which is a reasonable value for the model without a capital-intensive government sector.[19] I follow Iacoviello and Neri (2010) and Davis and Heathcote (2005) in using NIPA Input-Output tables to set housing-sector shares of capital, land, and consumption good at $\alpha_{xk} = \alpha_{xl} = \alpha_{xx} = 0.1$. The share of consumption good α_{xx} turns out to be difficult to identify. An informal sensitivity analysis has shown that the steady-state and dynamic properties of the considered models allow for some variation in this parameter. The chosen share of land α_{xl} makes land valuable in the model because it entitles its owners to the rent from housing production: its stock is priced at 50.8% of annual GDP. As for capital shares, they are chosen jointly with depreciation rates so that the steady state has realistic ratios of capital investment to GDP (21.1% in the model), residential investment to GDP (6.1%), and consumption- and housing-sector capital stock to GDP (1.94 and 0.04). BEA provides data on fixed assets that allows to estimate the average quarterly rates of depreciation for 1984–2014:[20] 3% for construction-sector capital, 2% for capital in other sectors, and 0.6% for residential property. The corresponding

[16] The average quarterly premium between the one-year adjustable-rate mortgages and the Treasury notes with the same maturity reported in Federal Reserve Economic Data for 1984–2014 is 36 basis points. I choose a slightly lower target in order to account for what I believe is an abnormally high premium observed in 2008–2014.

[17] FRB, Charge-Off and Delinquency Rates on Loans and Leases at Commercial Banks

[18] FHFA, Monthly Interest Rate Survey, Table 9

[19] See DeJong and Dave (2011), for example.

[20] BEA, Fixed Assets Accounts, Tables 3.1, 3.4, 4.1, 4.4, 5.1, 5.4

choices for the model are $\delta_x = 0.03$, $\delta_y = 0.02$, and $\delta_h = 0.01$. The share of housing in the utility is set at $\psi = 0.15$; the resulting steady-state value of housing stock relative to annual GDP is 1.23.

3.3 BAYESIAN ESTIMATION

Let $\mathcal{P}$ denote the model with perfect knowledge, and $\mathcal{I}$ denote the model with imperfect knowledge about shocks. Posterior odds in favor of the model $\mathcal{I}$ compare the empirical performance of the two models. Assuming no prior odds, the posterior odds are equal to the Bayes factor:

$$PO = \frac{p(Z|\mathcal{I})}{p(Z|\mathcal{P})}.$$

Therefore, the central goal of the outlined empirical exercise is to compute the marginal likelihoods

$$p(Z|\mathcal{M}) = \int L(Z|\theta,\mathcal{M})\pi(\theta|\mathcal{M})d\theta, \quad \mathcal{M} \in \{\mathcal{I},\mathcal{P}\}, \tag{38}$$

where $L(Z|\theta,\mathcal{M})$ is the likelihood function evaluated for a parameter vector θ, and $\pi(\theta|\mathcal{M})$ is its prior density.[21] Given parameters θ, I can cast the model $\mathcal{M}$ in linear state-space form (31)–(33) and find the likelihood $L(Z|\theta,\mathcal{M})$. I follow Iacoviello and Neri (2010) to specify the prior $\pi(\theta|\mathcal{M})$ loosely. As for the integral (38), I use Markov-Chain Monte Carlo integration with random-walk Metropolis algorithm to draw a sample from the posterior distribution $p(\theta|Z,\mathcal{M}) \propto L(Z|\theta,\mathcal{M})\pi(\theta|\mathcal{M})$,[22] and follow Chib and Jeliazkov (2001) to convert the output of the sampling procedure into an estimate of the marginal likelihood. The following section presents the results of the estimation.

[21] Note that parameters are grouped in two vectors, θ and θ_C. Parameters θ_C are calibrated. The Bayesian estimation treats θ_C as fixed and equal for the two models and integrates the expression (38) with respect to vector θ.

[22] For each model, I obtain 2.8 million draws (upon the burn-in run), and set the thinning step of 2,000 observations, which gives a sample of 1,400 draws. For a detailed coverage of the approach, see An and Schorfheide (2007), Chib and Greenberg (1995), Guerrón-Quintana and Nason (2012), Geweke (1999).

4 Results

The key numerical finding is that the log of posterior odds in favor of model $\mathcal{I}$ against model $\mathcal{P}$ is estimated to be 60, which is most decisively in favor of the assumption of imperfect knowledge. This section explains the reason behind this strong result.

4.1 POSTERIOR DISTRIBUTION

The prior distribution and the results of posterior estimations are summarized in Table 2. Compared to Iacoviello and Neri (2010), given that the set of observables included into the data and the model specification is different, the posterior of model $\mathcal{P}$ is quite similar. One notable distinction is a much less volatile capital-technology growth. As for model $\mathcal{I}$, the main result is that learning mostly affects the dynamics of the model through technology and housing-preference shocks, as these are the only processes that have both transitory and persistent components volatile in the model's posterior.[23]

Simulated posteriors indicate that the transitory component of consumption technology is important for both specifications. In addition, the persistent component generally shows a higher auto-regressive coefficient and variance in the posterior of the model with learning. Because of this, it is only important for the dynamics of model $\mathcal{I}$. Capital technology only matters under imperfect knowledge, where its persistent and transitory components are both prominent. The growth of construction technology behaves similarly in the posterior of both models: both components are volatile, and the transitory component is more so. Housing-preference shock can drive housing-market variables. Its persistent component is prominent in both models; moreover, in the model with learning, there is a large transitory component as well. Persistent shock to inter-temporal preferences, an important factor for the consumption-saving decision, is sizable in both models $\mathcal{P}$ and $\mathcal{I}$. Other shocks do not seem important.

4.2 IMPULSE RESPONSES

For a clear account of the effect of imperfect knowledge on the dynamic features of the model, I construct impulse responses for the mean parameter vector of model $\mathcal{I}$'s posterior, but under both assumptions of imperfect and perfect knowledge. I limit the discussion to shocks that are significantly affected by learning. The key conclusion is that learning seems to work well to combine persistent and transitory shocks in order to create sluggish responses whenever the immediate responses to the two types of shocks are opposite. In other words, learning washes out the opposite responses immediately upon shocks and therefore protracts them. On the contrary, when both persistent and transitory components of an exogenous process move the variables in the same direction under perfect knowledge, the dynamic effect of imperfect knowledge on these variables is ambiguous.

4.2.1 CAPITAL TECHNOLOGY

Figure 3 shows responses to negative one-standard-deviation shocks to capital technology. Under perfect knowledge, the immediate reaction of the savers is that they substitute away or towards capital investment.[24] A negative transitory shock to capital technology makes consumption-sector capital costlier; savers delay capital investment and shift towards consumption and housing sector. Rising house prices decrease the rate of mortgage defaults and the cost of mortgages. If the shock is persistent, the cost of capital is supposed to keep growing faster in the future. In effect, savers anticipate a higher capital gain from holding capital stock and buy more of it at the cost of other purchases: capital investment initially rises, while consumption, residential investment, and house prices fall. Upon the initial increase in capital stock, consumption-sector output rises, together

[23] My judgment about the size of the shocks is based on their contribution to the dynamics of the observable variables as summarized by variance decomposition below.

[24] Note that the growth of capital technology is much less volatile in the posterior of model $\mathcal{P}$, so the impulse-responses under perfect knowledge are much smaller than shown in Figure 3.

Table 2
Prior and posterior distributions for the estimated models

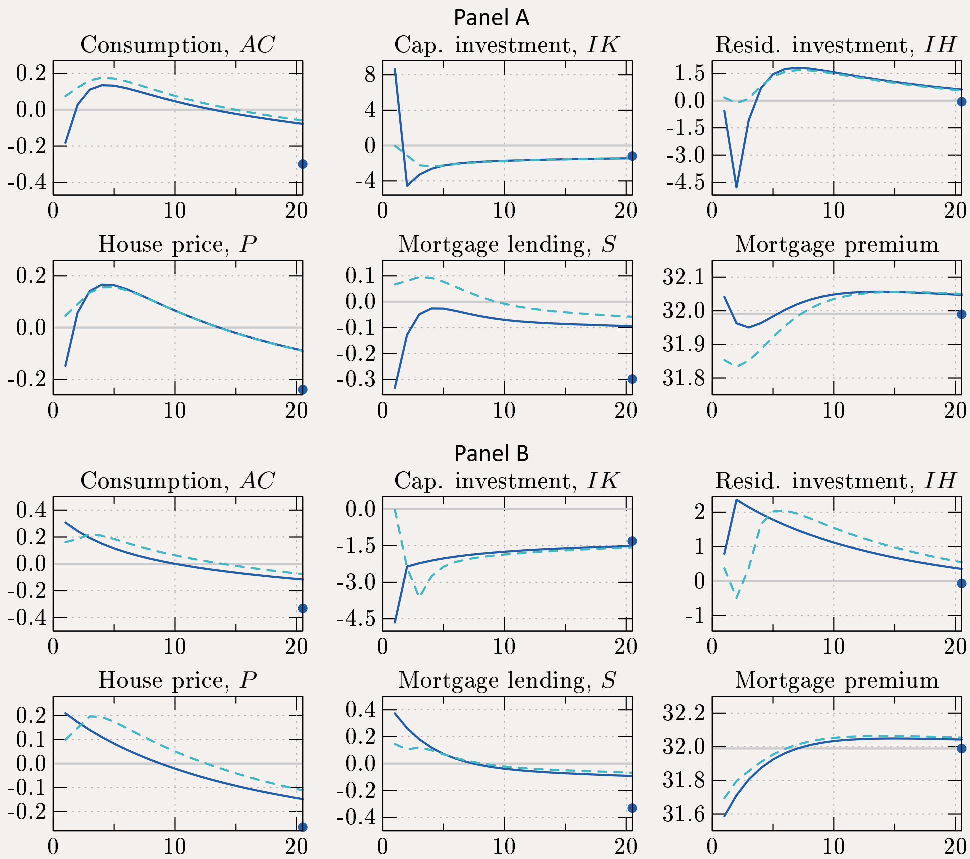
Parameter	Prior, all models: Type	Mean	S. D.	Posterior mean (95-percent confidence interval): Model $\mathcal{P}$		Model $\mathcal{I}$		Model $\mathcal{F}$	
AR coefficients									
ρ_y	$\mathcal{B}$	0.8	0.1	0.72	(0.47, 0.90)	0.86	(0.66, 0.97)	0.82	(0.74, 0.89)
ρ_k	$\mathcal{B}$	0.8	0.1	0.62	(0.37, 0.84)	0.48	(0.41, 0.56)	0.45	(0.40, 0.50)
ρ_x	$\mathcal{B}$	0.8	0.1	0.43	(0.35, 0.53)	0.86	(0.66, 0.96)	0.54	(0.45, 0.63)
ρ_ψ	$\mathcal{B}$	0.8	0.1	0.98	(0.95, 0.99)	0.97	(0.94, 0.99)	0.97	(0.95, 0.99)
ρ_ν	$\mathcal{B}$	0.8	0.1	0.97	(0.94, 0.99)	0.96	(0.91, 0.99)	0.96	(0.92, 0.99)
ρ_ς	$\mathcal{B}$	0.8	0.1	0.80	(0.57, 0.95)	0.79	(0.56, 0.94)	0.80	(0.56, 0.95)
Persistent component's st. dev.									
$100 \times \sigma_{v,y}$	$\mathcal{G}^{-1}$	0.1	1.0	0.09	(0.02, 0.51)	0.16	(0.05, 0.36)	0.56	(0.45, 0.69)
$100 \times \sigma_{v,k}$	$\mathcal{G}^{-1}$	0.1	1.0	0.06	(0.02, 0.13)	0.48	(0.33, 0.63)	0.07	(0.02, 0.19)
$100 \times \sigma_{v,x}$	$\mathcal{G}^{-1}$	0.1	1.0	1.01	(0.65, 1.34)	0.12	(0.03, 0.29)	1.47	(1.27, 1.67)
$100 \times \sigma_{v,\psi}$	$\mathcal{G}^{-1}$	0.1	1.0	2.42	(1.82, 3.38)	1.83	(1.15, 2.85)	1.95	(1.40, 2.63)
$100 \times \sigma_{v,\nu}$	$\mathcal{G}^{-1}$	0.1	1.0	2.60	(1.93, 3.92)	1.97	(1.12, 3.41)	2.28	(1.47, 3.47)
$100 \times \sigma_{v,\varsigma}$	$\mathcal{G}^{-1}$	0.1	1.0	0.10	(0.02, 0.47)	0.10	(0.02, 0.38)	0.07	(0.02, 0.19)
Transitory component's st. dev.									
$100 \times \sigma_{u,y}$	$\mathcal{G}^{-1}$	0.1	1.0	1.28	(0.96, 1.49)	1.09	(0.90, 1.28)	0.06	(0.02, 0.18)
$100 \times \sigma_{u,k}$	$\mathcal{G}^{-1}$	0.1	1.0	0.07	(0.02, 0.18)	1.04	(0.71, 1.35)	1.21	(0.99, 1.45)
$100 \times \sigma_{u,x}$	$\mathcal{G}^{-1}$	0.1	1.0	1.79	(1.48, 2.13)	2.04	(1.78, 2.34)	0.10	(0.02, 0.54)
$100 \times \sigma_{u,\psi}$	$\mathcal{G}^{-1}$	0.1	1.0	0.09	(0.02, 0.40)	9.16	(5.26, 13.35)	8.50	(5.66, 11.79)
$100 \times \sigma_{u,\nu}$	$\mathcal{G}^{-1}$	0.1	1.0	0.05	(0.02, 0.13)	0.06	(0.02, 0.18)	0.06	(0.02, 0.18)
$100 \times \sigma_{u,\varsigma}$	$\mathcal{G}^{-1}$	0.1	1.0	0.09	(0.02, 0.39)	0.11	(0.02, 0.65)	0.07	(0.02, 0.23)
Kalman gains									
λ_y	$\mathcal{B}$	0.5	0.2					0.10	(0.04, 0.19)
λ_k	$\mathcal{B}$	0.5	0.2					0.22	(0.21, 0.23)
λ_x	$\mathcal{B}$	0.5	0.2					0.10	(0.03, 0.21)

$\mathcal{P}$ denotes the model with perfect knowledge; $\mathcal{I}$ denotes the model with imperfect knowledge; $\mathcal{F}$ denotes the model with learning and Kalman gains independent of the parameters of technological processes.

with household income, aggregate consumption, and housing demand. In the long run, regardless of the persistence of the shock, higher cost of consumption-sector capital results in its smaller stock, as well as lower output, consumption, investment, and house prices.

Under imperfect knowledge, savers are not completely sure what to do immediately upon shock. Because persistent and transitory shocks create opposite immediate responses, imperfect knowledge washes them out for *both* types of shock. Since capital-technology shocks have the largest impact on capital and residential investment, I expect that learning works through these shocks to add inertia to investment and not so much to consumption or house prices. In addition, note that capital-technology shocks generate negative correlation between capital investment and the other observable variables under perfect knowledge. Imperfect knowledge seems to wash out this counter-cyclicality of capital investment. Huang et al. (2009) and Edge et al. (2007) discuss similar findings with respect to learning, washing-out, and correlation. Under perfect knowledge,

Figure 3
Impulse-responses to capital-technology shocks



Persistent shock $v_{k,1} = -\sigma_{v,k}$ in Panel A and transitory shock $u_{k,1} = -\sigma_{u,k}$ in Panel B. Solid lines (perfect knowledge, $\mathcal{P}$) and dashed lines (imperfect knowledge, $\mathcal{I}$) are percentage-deviations from the initial balanced-growth path (solid grey lines). Absolute values in basis points are given for mortgage premium. Dots on the right axes show where the responses stabilize after 400 quarters.

capital technology shocks are estimated to be small, probably because they create excessive counter-cyclical volatility in capital investment.[25]

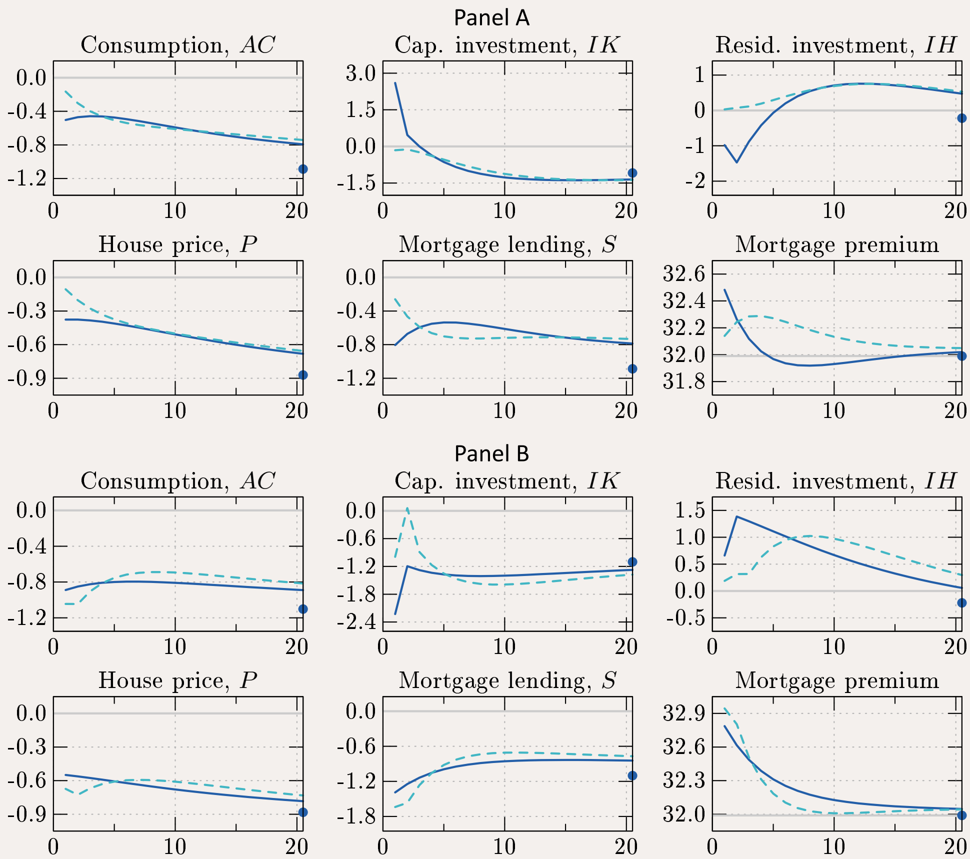
4.2.2 CONSUMPTION TECHNOLOGY

Figure 4 shows responses to a negative shock to growth in consumption technology. A fall in the output of consumption good drives household income down and decreases the demand for housing. House prices fall, which creates a spike in mortgage defaults and the cost of mortgages. The productivity of capital in consumption sector falls; whether savers shift towards or away from capital investment depends on the persistence of the shock, just like in the case of capital-technology shocks. Eventually, because consumption good is used to pay for housing stock and capital, lower productivity in consumption-good sector makes investment, house prices, and aggregate consumption stabilize below the initial balanced-growth path.

Qualitatively, imperfect knowledge has the following effect on the response to a persistent shock. Initially, there is a chance that the shock is transitory and that a quick recovery is about to follow, so consumption and house prices adjust by less; as households gradually learn about the persistent shock, the adjustment catches up. Learning keeps expectations deteriorating,

[25] I do not embed the model with such elements as capital-adjustment costs and variable capital utilization for clarity of the main argument. This adds volatility to capital investment in the model.

Figure 4
Impulse-responses to consumption-technology shocks



Persistent shock $v_{y,1} = -\sigma_{v,y}$ in Panel A and transitory shock $u_{y,1} = -\sigma_{u,y}$ in Panel B. Solid lines (perfect knowledge, $\mathcal{P}$) and dashed lines (imperfect knowledge, $\mathcal{I}$) are percentage-deviations from the initial balanced-growth path (solid grey lines). Absolute values in basis points are given for mortgage premium. Dots on the right axes show where the responses stabilize after 400 quarters.

the house price falling, and the rate of mortgage defaults and mortgage premium elevated over a longer period.[26] That is, imperfect knowledge delays the responses of the house price, consumption, and mortgage-market variables. Quantitatively, the effect is limited: although the initial adjustment of consumption and house prices is about 3 times smaller, agents quickly learn about the persistence of the shock, and the responses converge to the case of perfect knowledge within 4–6 quarters.[27] In case of a transitory shock, the effect of imperfect knowledge is essentially the opposite: it amplifies the initial responses of consumption and the house price, since economic agents suspect a long recession and over-react.

Learning does not seem to work well with consumption-technology shocks in order to add sluggishness to simulated house-price dynamics: it only moderately protracts the responses to persistent shocks but amplifies the responses to transitory shocks. Why does the effect of learning turn out to be limited? The answer comes from the fact that Kalman gains, which measure how quickly agents recognize a persistent shock, are defined by the parameters of technology processes according to equation (36). The features of consumption technology are such that agents quickly learn about the nature of the persistent shock. For a slower learning, the transitory component would have to be more volatile. However, it would not necessarily make model $\mathcal{I}$

[26] This link is quite in line with the finding by Gerardi et al. (2008) that unexpected house-price adjustments are a key driver of the mortgage foreclosure rate.

[27] Perhaps, this particular finding conforms with Burnside et al. (2011) who argue that it is hard to achieve protraction in house-price adjustments when household beliefs are homogeneous.

better at mimicking the sluggish house-price dynamics: volatile transitory shocks, coupled with the fact that consumption and house prices over-react to them in case of imperfect knowledge, would create excessive volatility in these variables.

Notice that this is not the case for investment variables. Under perfect knowledge, savers substitute away or towards capital investment immediately upon the shock, depending on its persistence. The immediate responses are opposite for the two types of shock, and learning washes them out and thus protracts them. Like in the case of capital-technology shocks, both types of investment look more likely to gain momentum due to learning than the house price or consumption.

4.2.3 HOUSING SHOCKS

A close look at shocks to housing preferences ψ_t and growth in housing-sector technology $A_{x,t}$ does not reveal any new insights. The responses are in line with the already provided conclusions and similar to the ones described by Iacoviello and Neri (2010).[28] Therefore, I only provide a brief summary of the effects of these shocks.

A negative shock to housing-sector technology makes housing scarce and expensive; through both wealth and substitution effect, it causes an increase in consumption. Low capital return in the housing sector eventually causes the residential investment to fall, but the immediate response of capital and residential investment depends on the persistence of the shock, like before. The shock matters the most for residential investment and the house price but causes a counter-factual negative correlation between them. A sizable housing-preference shock helps explain the joint dynamics of these variables. Quite naturally, a fall in the preference for housing causes residential investment, house price, and mortgage lending to fall. Consumption becomes preferred, but its initial response also depends on the wealth effect of the house-price decrease.

Just like in the case of consumption and capital technologies, the model with learning does not seem to be able to employ housing shocks to generate a lot of inertia in the house price. The reason is the same: both transitory and persistent shocks move this variable in the same direction, and volatile transitory shocks required for slow learning would cause excessive house-price volatility. This adds to the suspicion that learning has a limited contribution to house-price inertia.

4.3 VARIANCE DECOMPOSITION

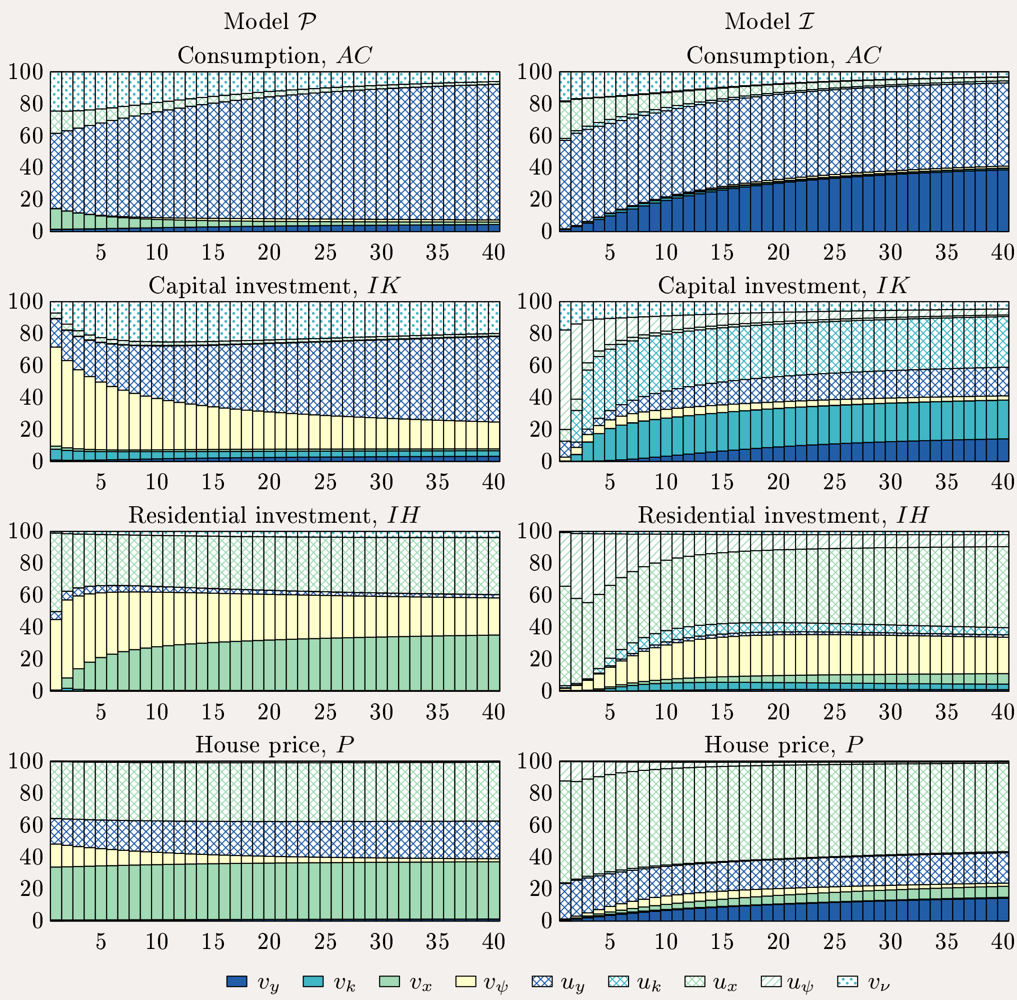
4.3.1 FORECAST VARIANCE

Figure 5 decomposes the forecast variance of growth in the observable variables for both models; it shows the relative importance of shocks. Consumption technology accounts for most of variability in consumption, virtually all of it in the long run (up to 80–90% at 40-quarter horizon for both models). In addition, consumption technology affects capital returns and therefore explains a lot of variability in capital investment. For both capital investment and consumption, inter-temporal preferences are also important because they affect the consumption-saving decision. In case of imperfect knowledge, over 50% of variability in capital investment is predicted due to capital technology that defines the cost of capital creation. In case of perfect knowledge, capital technology is unimportant. In both models, housing-preference shock matters for capital investment in the short run, due to substitutability between capital and residential investment as saving vehicles. Of course, housing preferences together with housing technology matter the most for residential investment; they account for over 90% of its variation in both models across the reported forecast range. As for the house price, its variation in both models is almost entirely due to housing- and consumption-sector technologies. The latter is important because the house price is measured in units of consumption good. In addition, up to 15% of house-price variation is expected to come in the short run from housing-preference shocks.

It is true for both models that technology shocks seem to be the most important drivers; and more so for longer forecast horizons, since they create permanent deviations from the balanced growth path. Under imperfect knowledge, persistent shocks to technologies matter despite their small estimated variances. These shocks accumulate their influence over longer forecast horizons. The conclusion is that, to a large degree, long-run evolution of variables simulated by the two models can be attributed to technology shocks, and in particular to their persistent components. Persistent shocks alone, especially in case

[28] To be precise, the transitory shock to housing-sector technology and the persistent shock to housing preferences produce the responses that are the closest to Iacoviello and Neri (2010), since the former process is stationary in growth and the latter one is stationary in levels in my models.

Figure 5
Forecast variance decomposition



Percentage-contribution of shocks to forecast variability in growth of every observable variable for model $\mathcal{P}$ with perfect knowledge and model $\mathcal{I}$ with learning: persistent technology shocks $\{v_y, v_k, v_x\}$, transitory technology shocks $\{u_y, u_k, u_x\}$, housing-preference shocks $\{v_\psi, u_\psi\}$, and persistent shock to inter-temporal preferences v_ν.

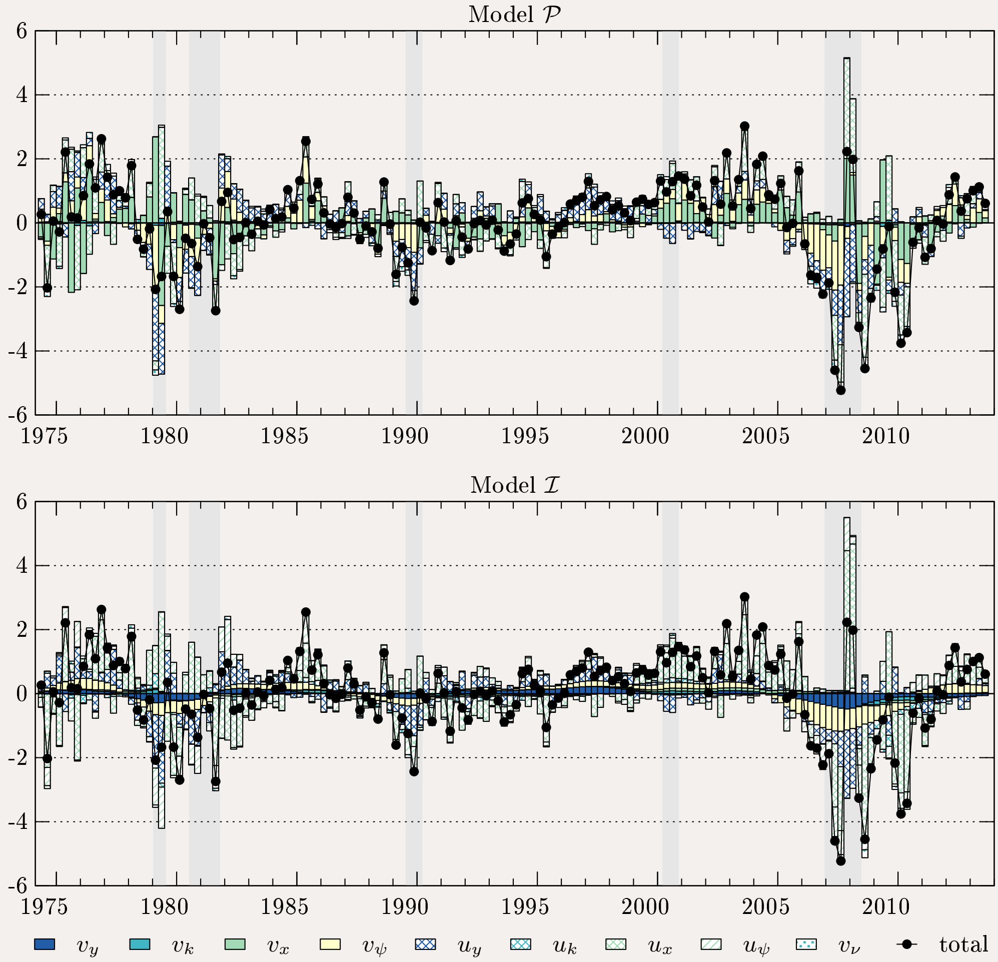
of learning (and especially if learning is slow), seem to matter over longer forecast horizons, or for low-frequency dynamics. However, learning mechanics depend on the presence of transitory components, and it is the interaction between the persistent and transitory components that is the most important for the model's ability to mimic the dynamic behavior of the house price and other variables.

4.3.2 HISTORICAL VARIANCE

Figure 6 decomposes the historical variance of house-price growth. It conforms with the decomposition of the forecast variance: most of variation in the house price is due to consumption- and housing-sector technologies. Moreover, forecast variance decomposition has indicated that persistent shocks gradually build up their influence over longer horizons, and the historical decomposition reflects this fact.

The figure shows that persistent shocks explain a good part of low-frequency dynamics; they gradually build up their impact over time, and they contribute the most during the periods of lasting, persistent upturns or downturns in the house price. It

Figure 6
Historical variance decomposition, house price



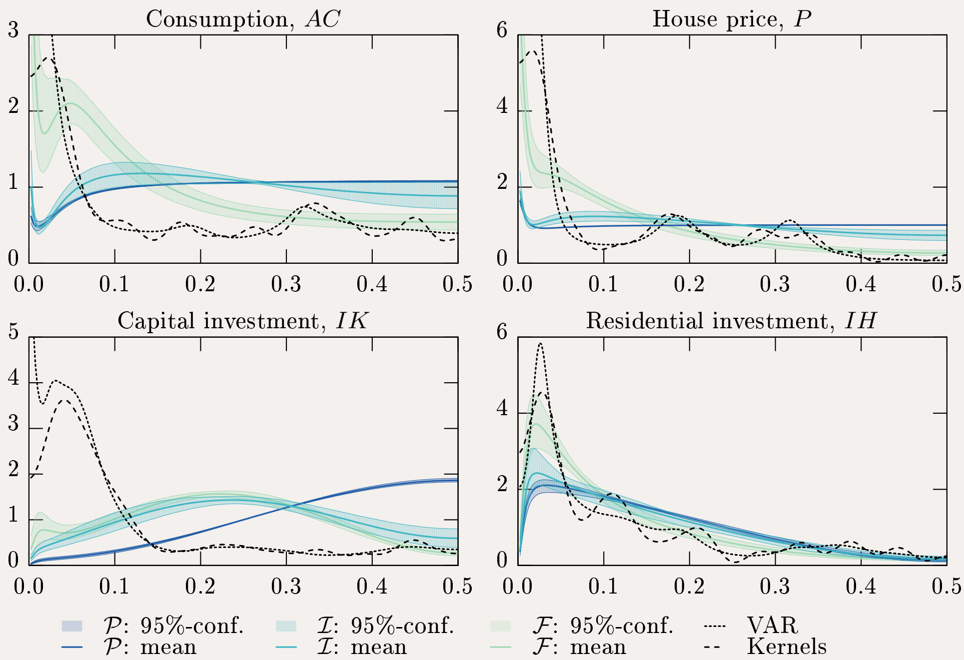
Decomposition of percentage house-price growth for models with perfect knowledge ($\mathcal{P}$) and imperfect knowledge ($\mathcal{I}$). The dotted line shows the observed growth in the variable. Shades indicate NBER-dated recessions.

is evident in the case of imperfect knowledge, where the response of house prices to persistent technological shocks is protracted due to learning, and where the historical decomposition reveals a more regular contribution of persistent components to house-price growth. This is an important point. An observable variable with inertia exhibits auto-correlated first differences. The house price is clearly the case. Learning may protract the responses to persistent shocks and, it seems, add auto-correlation to the contribution of such shocks to changes in the house price. If model $\mathcal{I}$ is the data-generating process under consideration, learning may make persistent shocks an efficient tool to simulate the house price with inertia. However, as already noted, the limitation is that slow learning requires volatile transitory components. Another problem is that the persistent components of the technologies show auto-correlated contributions largely because of high auto-correlation in their smoothed errors estimated for model $\mathcal{I}$. Spectral-density analysis provided below seems an appropriate test to determine whether model $\mathcal{I}$ is truly the data-generating process that is better capable of simulating inertia in the house price.

It is also interesting to note that the contribution of consumption technology is often consistently in the direction opposite to the observed house-price growth, as it is the case for the early 1990s or early 2000s. In other words, the prevailing direction of house-price growth is not always in line with consumption sector, the dominant sector of the economy. A booming aggregate economy and a growing aggregate consumption do not always correspond to growing house prices, and vice versa. This observation may point to the existence of house-price bubbles; alternatively, it may point to economic developments specific to housing and mortgage markets, such as introduction and spread of mortgage-backed securities.

4.4 SPECTRAL DENSITY

Figure 7
Spectral densities of the observed variables



Spectral densities of the first differences normalized by unconditional variances. The horizontal axes measure cycles per quarter. Spectral density is estimated over the posteriors of the models and compared against less parametric estimates: an estimate based on VAR, and a completely non-parametric estimate for individual series based on auto-covariances weighted with kernels. Line colors and adjacent shades, from darkest to lightest, indicate model $\mathcal{P}$ with perfect knowledge, model $\mathcal{I}$ with learning, and model $\mathcal{F}$ with learning and Kalman gains independent of the parameters of technological processes.

To construct Figure 7, I take the state-space forms estimated for the two models and convert them into spectral densities for first-differences of the four observable variables. These densities reflect the ability of the models to simulate cycles of various length in the variables. In addition, it is well known that the likelihood function can be cast over the frequency domain, and that a comparison of the models' spectra against the data can reveal the sources of higher likelihood.[29] For such comparison, I take the first differences of the data and construct two additional, non-parametric estimates of the spectra. First, I construct spectral densities from a VAR model with 6 lags estimated for the differenced series $\{\Delta AC, \Delta IK, \Delta IH, \Delta P\}$. Second, I construct a kernel-based estimate for each individual series. The two methods deliver similar results that I use as benchmarks for the performance of the two models.

Compared to model $\mathcal{P}$, the most visible impact of imperfect knowledge is on the spectrum of capital investment: there are large significant gains for frequencies less than 1/4, or for cycles longer than one year. In other words, model $\mathcal{I}$ looks better equipped to mimic the dynamics of capital investment, which exhibits non-trivial low-frequency cyclical component. The gains are also significant over cycles longer than 1 year for the house price. As for the rest of the variables, there are small and largely insignificant gains over lower frequencies.

[29] For example, see Christiano and Vigfusson (2003)

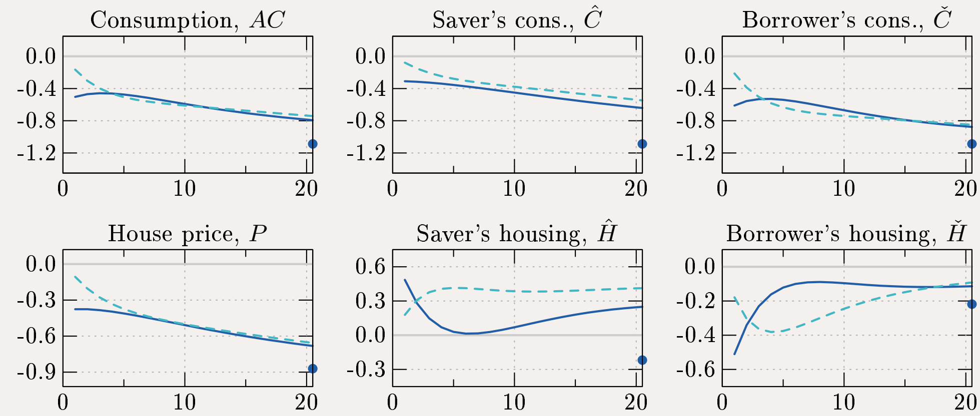
All of the described gains look desirable, as they bring the spectra closer to the estimates obtained more directly from the data. However, it would be hard to say precisely how spectral modifications brought by imperfect knowledge affect the posterior odds. It can be that the tilt of the odds in favor of model $\mathcal{I}$ is largely due to the ability of the latter to better mimic the dynamics of capital investment, not the house price or other variables.

As discussed before, for learning to add inertia to the house price, it must be slow in case of persistent shocks to exogenous processes that drive the house price. It is slow when the corresponding transitory components are volatile and Kalman gains are small. The problem is that imperfect knowledge amplifies the immediate responses of the house price to transitory shocks. Therefore, volatile transitory components create volatile house price and counter the dynamic effect of persistent components with slow learning. This is generally not the case for capital investment that has opposite initial responses to transitory and persistent shocks under perfect knowledge, which are washed out under imperfect knowledge. This explains the visibly strong impact of learning on the spectral density of capital investment and the absence of such for the house price and other variables.

5 Discussion

5.1 IMPLICATIONS: REDISTRIBUTION OF WELFARE

Figure 8
Impulse-responses to a negative persistent consumption-technology shock



Responses to a one-standard-deviation negative shock, $v_{y,1} = -\sigma_y$. Solid lines (perfect knowledge, $\mathcal{P}$) and dashed lines (imperfect knowledge, $\mathcal{I}$) are percentage-deviations from the initial balanced-growth path (solid grey lines). Dots on the right axes show where the responses stabilize after 400 quarters.

Consider a negative one-standard-deviation persistent shock to consumption technology, as shown in Panel A of Figure 4. It is a shock to productivity in the largest sector that causes a long economy-wide recession. Figure 8 augments Figure 4 with levels of consumption and housing stock chosen by both types of households. Note that these choices directly define household utility in the model. Compared to the baseline model $\mathcal{P}$, an additional effect of imperfect knowledge is that it redistributes consumption and housing stock between the two groups.

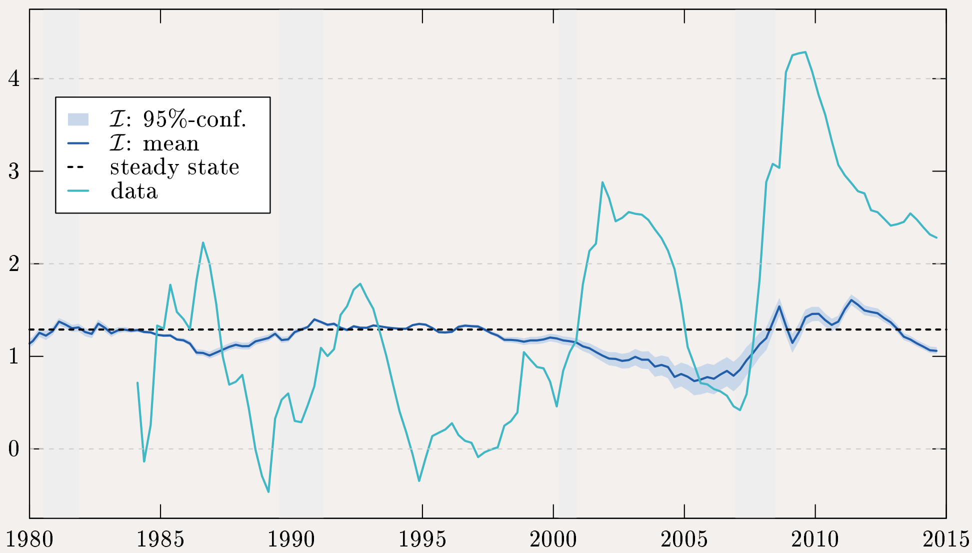
In case of imperfect knowledge and immediately upon the shock, households underestimate its persistence; they bet on a quick recovery and high house prices in the near future. Borrowers maintain relatively high levels of mortgage loans, housing stock, and consumption. Eventually, as households learn about the nature of the shock, their wealth declines due to the decreases in house prices and expected future earnings. In addition, the downward adjustment of the house price keeps the foreclosure rate high and mortgage loans expensive. Starting from three quarters upon the shock, the borrowers' choices of housing stock and consumption become lower than in the case of perfect knowledge. The mechanics of the savers' response is different: as households learn about the persistence of the shock and as the house price declines, savers are able to buy up more housing stock at a lower price, as well as afford a higher consumption. Once the initial bet on a quick recovery turns out to be wrong, savers win and borrowers lose.

This result motivates a welfare analysis of the redistribution between savers and borrowers due to imperfect knowledge. I refrain from doing so, however. First, it is not clear how to conduct such analysis. Ex post, one can trace the choices made by the two types of households and conclude whether it is better for the households to know about the persistent recession from the start. It may be not the case ex ante. In addition, the situation is different for transitory shocks. And second, if economic agents lack knowledge about the sources of changes in the economy and if perfect knowledge is a Pareto-improvement, it is not clear how to achieve it.

What is clear is that imperfect knowledge alters the welfare implications of house-price adjustments for the two types of households, especially through the cost of mortgages and the wealth effect. These implications matter for the discussion about policies to manage house prices, since the data are strongly in favor of imperfect knowledge. They certainly affect the choice of timing, length, and scale of the price adjustments under policymaker's consideration.

5.2 ACCOUNT FOR LENDING CONDITIONS IN THE MARKET FOR MORTGAGES

Figure 9
Mortgage premium: model against the data



Mortgage premium predicted by the model with imperfect knowledge ($\mathcal{I}$) and compared against the actual data. Annualized, percentage points. In the data, the mortgage premium is calculated as interest on one-year adjustable-rate mortgages net of the yield on one-year Treasury notes. Shades indicate NBER-dated recessions. *Source*: Freddie Mac; the Federal Reserve Board; FRED; NBER

Apart from its account for house prices, it is interesting to see whether the presented general-equilibrium model with endogenous markets for housing and mortgages is any good at explaining the state of the latter. Mortgage premium is a good indicator of financing conditions in the market for mortgages for which there is a long history of observations. In the U.S. data on mortgage rates, the shortest available maturity is one year. The model's counterpart for the annual premium is

$$MP_t^* = E_t\left[\prod_{i=0}^{3} \frac{1+\bar{r}_{m,t+i}}{1+r_{m,t+i+1}}\right] - 1,$$

where the quarterly mortgage premium $E_t[(1+\bar{r}_{m,t})/(1+r_{m,t+1})]$ is defined using equations (1) and (5). I use the data (available from 1984) to construct the premium MP_t in exactly the same way using the observations of the interest on one-year adjustable-rate mortgages and the yield on one-year Treasury notes. Figure 9 presents both the observed series and its counterpart fitted by model $\mathcal{I}$.

Model $\mathcal{I}$ is estimated using only the data on GDP components and house prices. As can be seen from the figure, the model's account of the tensions in the market for mortgages is rather poor. The standard deviation of the mortgage premium MP_t^* inferred from model $\mathcal{I}$ is five times smaller than that of the actual series, MP_t. The correlation between the two series is 0.31. Notably, the correlation between the fitted mortgage premium MP_t^* and the house-price growth is -0.59: model $\mathcal{I}$ mainly employs information about the house price to infer about the financing conditions in the mortgage market. The fact that this strategy provides a poor estimate of the actual mortgage premium points to the evolution of financing conditions that are not reflected by the house price. Interestingly, the model's predictions about the dynamics of the mortgage premium look better for the later part of the sample. The correlation between the predicted and the actual mortgage premium, $\rho(MP_t, MP_t^*)$, increases from −0.17 for 1984–1999 to +0.66 for 2000–2015. Following the same logic, it may imply that the recent house-price bubble has been largely due to the financing conditions in the mortgage market. Indeed, sub-prime mortgages and mortgage securitization were at the core of the housing boom that ended with the Great Recession. A more general implication is that the house price and the aggregate economy have shown a tighter link with the financing conditions in the mortgage market after 2000.

The inclusion of mortgage premium into the data set for estimations helps identify a financial shock. Preliminary posterior estimations have indicated that the idiosyncratic variance of the housing stock ς_t, which drives the risk of mortgage default and which is proposed by Forlati and Lambertini (2011), becomes a prominent shock that helps explain most of variation in the mortgage premium. In fact, it becomes extremely large: the standard deviation of the persistent component averages above 15 percent in the posterior of the model with imperfect knowledge. Interestingly, Forlati and Lambertini (2011) study the effect of a 40-percent increase in the idiosyncratic variance. This shock affects the tail of a distribution that guides the rate of default in the model. Because of its indirect effect on the default rate and the mortgage premium, it must be large in order to have an impact. For the same reason, I argue that such large estimated volatility of the shock does not diminish the precision of the first-order approximation to the solution of the model. A more serious concern is that the smoothed errors estimated for this shock are highly auto-correlated when the mortgage premium is included into the data. Details aside, it remains an imminent challenge to develop the financial component of the presented models in order to provide an adequate quantitative account of mortgage-market developments.

5.3 INDEPENDENT KALMAN GAINS

The idea that learning can help create low-frequency dynamics in the observable variables turns out to be flawed, because volatile transitory shocks are necessary for slow learning. The flaw is due to the assumption of model $\mathcal{I}$ that the speed of learning, as captured by Kalman gains $\lambda_i(\rho_i, \sigma_{v,i}, \sigma_{u,i})$, $i \in \{y, k, x, \psi, \nu, \varsigma\}$, is tied by equation (36) to the parameters of the exogenous processes. In case of capital investment, this assumption seems to pay off because transitory and persistent shocks create opposite responses in this variable, and learning helps wash them out. It does not pay off, however, for the house price, which over-reacts to transitory shocks in case of learning. It is interesting to see what can be done to simulate more house-price inertia in the model with imperfect knowledge.

An interesting experiment is to assume that Kalman gains are independent of the parameters that guide the exogenous processes. This means that economic agents, in addition to their inability to observe the individual components of the exogenous processes, have a wrong idea about the true features of these processes.[30] Let us consider model $\mathcal{F}$ with imperfect knowledge and Kalman gains that are free parameters. To be precise, only the Kalman gains specific to technologies are assumed to be free parameters; the rest of the processes are specified as before.[31] I run a separate MCMC routine to estimate the posterior $p(\theta|Z, \mathcal{F})$ of model $\mathcal{F}$ jointly for Kalman gains and the rest of the parameters and report the results in Table 2. Compared to models $\mathcal{P}$ and $\mathcal{I}$, what is different about the technologies in the posterior?

One major difference is that for consumption- and housing-sector technologies, the new posterior is tilted towards relatively large persistent components and small transitory components, and towards slower learning. To better illustrate the latter point, Table 3 reproduces from Table 2 the moments of the posterior $p(\theta|Z, \mathcal{F})$ for Kalman gains $\{\lambda_i\}^i \in \theta$ that are free parameters, along with the moments for Kalman gains $\{\lambda_i(\theta)\}^i$ that are constructed as functions of the parameters of technologies according to equation (36) and for the posterior of the same model $\mathcal{F}$. The table indicates that model $\mathcal{F}$ predicts Kalman gains for consumption and housing technologies significantly lower than they should be according to the assumptions of model $\mathcal{I}$. In other words, for consumption and housing technologies, model $\mathcal{F}$ features slow learning in case of persistent shocks and transitory shocks that are much less volatile than would have been necessary for such slow learning under the assumptions of model $\mathcal{I}$. Naturally, these features should contribute to sluggish house-price dynamics, because these technologies are key drivers of the house price (as shown by variance decomposition above).

As for capital technology, it becomes a process with a large transitory component and a very small persistent component. The posterior distribution of the corresponding Kalman gain indicates that households do not interpret the shocks to be transitory

[30] In terms of Evans and Honkapohja (1999), this is the case when agents learn based on mis-specified perceived law of motion.

[31] It suffices to consider free Kalman gains only for technologies for the sake of the argument. It is also easier to interpret the fact that agents do not know the true parameters of technologies than such processes as inter-temporal preferences or preferences for housing.

Table 3
Posterior of Kalman gains in a model with free Kalman gains

	$\lambda_i \in \theta$		$\lambda_i(\theta)$	
Parameter	**Mean**	**95%-conf. int.**	**Mean**	**95%-conf. int.**
λ_y	0.103	(0.042, 0.188)	0.983	(0.905, 0.999)
λ_k	0.217	(0.205, 0.230)	0.006	(0.001, 0.031)
λ_x	0.100	(0.026, 0.208)	0.989	(0.868, 0.999)

as quickly as they would had they known the true parameters of capital technology. This combination works to create inertia in capital investment because of the above-described washing-out that happens upon both persistent and transitory capital-technology shocks in case of learning.

Figure 7 shows that, compared to model $\mathcal{I}$, spectral density of the house price is significantly higher over low frequencies in the posterior of model $\mathcal{F}$. The gains are significant over cycles longer than about 8 quarters. In addition, slow learning about persistent shocks to consumption and housing technologies alters the low-frequency dynamics of aggregate consumption and residential investment. The spectrum of aggregate consumption is significantly higher over cycles longer than 2.5 years. The spectrum of residential investment is significantly higher over cycles longer than 3 years. The spectrum for capital investment of model $\mathcal{F}$ does not look significantly different from that of model $\mathcal{I}$.

Overall, the described modifications bring the estimated spectral densities closer to their non-parametric counterparts, which can explain why the posterior odds are in favor of the model with independent Kalman gains. The logged posterior odds are 36 in favor of model $\mathcal{F}$ against model $\mathcal{I}$. Like before, it is not clear how exactly these spectral modifications translate into higher marginal likelihood, although the modifications brought to aggregate consumption and house price seem responsible. In the end, the fact is that the kind of learning assumed in model $\mathcal{F}$ is capable of adding significant inertia to housing-market variables, as well as other observables, and make the model with endogenous housing market a likelier data-generating process.

As learning helps add inertia to the house price, it does so for the other observable variables as well. Presumably, even if the house price was not a part of the vector of observables, the outlined exercise would still reveal the posterior odds in favor of imperfect knowledge and learning. It is a well-known fact that consumption and investment are not as volatile as predicted by a general-equilibrium model without frictions, and learning performs well to slow down the simulated dynamics of these variables. Yet, there are more traditional ways to control for the dynamics of consumption and investment. For the sake of clear argument, the models presented above are limited in what Iacoviello and Neri (2010) summarize as real rigidities: consumption habits, capital utilization, capital-adjustment costs, etc. These elements could add to the model's ability to simulate low-frequency dynamics of consumption and investment in models with and without perfect knowledge. Hence, learning would have less space to improve the dynamic features of these variables in the model, and the focus of its impact would shift towards the housing-market variables. This conjecture deserves further investigation, although it seems that, real rigidities or not, learning will be able to improve the dynamic properties of the model with respect to observable variables other than the house price. Finally, the argument can be extended to say that the housing market can also be augmented with non-informational frictions, which would also limit the scope of impact of learning on the house price. This point does not impede the validity of learning as a mechanism to add inertia to the house price.

6 Conclusion

When shocks of different persistence are possible, it may take economic agents time to recognize and react to them, which may add momentum to market prices. This is the key motivation to impose imperfect information and learning about the state of the economy on a general-equilibrium model with the market for housing. Bayesian estimation against the last 40 years of key U.S. data on housing and the aggregate economy reveals decisive evidence in favor of the model with learning. The major impact of learning is on the dynamics of variables that respond oppositely to shocks of different persistence: learning protracts the responses mainly through washing out their initial part. Investment variables are definitely the case; they gain a significant low-frequency component from learning.

It seems, however, that the house price is *not* the case: under perfect information, it reacts similarly to shocks of different persistence. As a result, learning mutes and protracts the house-price response to persistent shocks, but amplifies the response to transitory shocks, which are necessary for slow learning to occur. As confirmed by spectral-density analysis, the combined effect of these alterations on simulated house-price dynamics is rather weak. An unrestricted parameterization of learning allows to have protracted responses to persistent shocks and virtually absent transitory shocks at the same time—a combination that lets the model generate low-frequency dynamics of the house price that match the data well. Estimations indicate that this combination is likely for shocks that drive the house price. In case of the Great Recession, this combination implies that households were stubbornly optimistic after the onset of the crisis, which was why it took house prices so long to adjust.

Of course, there is space for improvement of the presented model's ability to provide an adequate quantitative account of the markets for housing and mortgages. Yet, it is already clear that the households' lack of knowlegde about the nature of the unfolding cycles should affect the optimal policies to tame the housing market.

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