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Abstract

This paper tests whether the exchange rate of the Czech koruna, the Hungarian forint, and the Polish zloty were anchored by the market expectations concerning the euro-locking rate in the period of 15.Dec.2004.–3.Aug.2006. First, I derive the process of the exchange rate as a function of the processes of the factors, namely the latent exchange rate and the market expectation concerning both the euro-locking rate and the time of locking. Then I filter the expected final conversion rate. The time-varying volatilities of the state variables are estimated from cross-sectional data on option prices.

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The views expressed are those of the authors and do not necessarily reflect the official view of the Magyar Nemzeti Bank. This disclaimer is particularly important in the case of the future euro-locking rate of the Hungarian forint. The filtered market expectations concerning the euro-locking rate do not necessarily coincide with the preferred euro-locking rate of the Magyar Nemzeti Bank.

Are The Exchange Rates Of The EMU Accession Countries Anchored By Their Expected Euro-Locking Rates?

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1 Introduction

This paper investigates the stabilizing feature of the market expectations concerning the euro-locking rate or in other words of the final conversion rate. I apply the analysis to three EMU candidate countries, Czech Republic, Hungary, and Poland. First, I construct an economic model where the exchange rate is a function of three factors, namely the latent exchange rate and the market expectation concerning both the final conversion rate and the time of locking. Then, in the empirical part of the paper, I decompose the historical changes of the exchange rate into changes of each of the factors. By investigating the filtered market expectation concerning the euro-locking rate I make inference on the stabilizing effect of the locking on the exchange rate.

The nominal exchange rate can be viewed as an asset price. Thus the exchange rate not only includes information about current conditions, but also expectations on future events. The main difference between exchange rate without future locking and with future locking is the different time horizon. The time horizon of events affecting the exchange rate without locking or many other assets is infinite making their pricing difficult. Luckily, the case I analyze, the exchange rate with future locking, rules out many of these difficulties due to the finite horizon and the irrevocable feature of the final locking.

An alternative source of information on the expectations of the market is the Reuters poll which surveys the expectations of the analysts concerning the times of EMU and ERM2 entry of the accession countries and also concerning the central parities in the ERM2. The reported expected central parity could be considered as the market expectation concerning the final conversion rate. However, I opt to filter the market expectation for the following reason. Extracting the market expectation concerning the final conversion rate from daily historical exchange rate data has many advantages over the Reuters polls data. The filtered expectations may embody more accurate and more up-to-date information than the monthly or quarterly Reuters polls. Moreover, the higher frequency of the filtered expectations enables us to investigate the stabilizing effect of the locking on the exchange rate.

There is an expanding literature investigating how financial markets assess the outlooks of EU members of adapting the euro in the future. Market expectation concerning the probability of a country's adopting the euro at a certain time is usually estimated from interest rate differentials, currency option prices, or Arrow-Debreu contracts. Bates (1999) provides a review of this literature and he highlights the novelty and the potential weaknesses of the usually applied methods.

Csajbók-Rezessy (2005) estimate the expected Euro-zone entry date of Hungary from the forint and euro yield curves and find the estimates to be relatively close to the reported expectations of the analysts provided by the Reuters polls. For this reason, the present paper does not aim at filtering the market expectation concerning the time of locking. Instead, I use the reported expectations of the analysts provided by the Reuters polls.

The accession countries of the Euro-zone aim at choosing the final irrevocable conversion rates to be equal to the equilibrium exchange rates. Therefore, once we had reliable estimates on the equilibrium exchange rate, then we might consider it as an estimate on the market expectations concerning the final conversion rate. There are at least three problems with this concept. First, there are different concepts and estimating methods [1] of equilibrium real exchange rate. Second, these are estimates on the real and not on the nominal exchange rate. Third, the market expectation might be different even from an estimated nominal equilibrium exchange rate, especially if the choice of the final conversion rate is based not only on economic, but also political considerations. The novelty of this paper is that it filters the subjective market expectation concerning the final conversion rate, which mirrors not only the economic considerations, but also the possible political considerations. Then, by comparing the time series of the filtered subjective market expectation and that of the historical exchange rate we can make inferences about the stabilizing effect of the locking on the exchange rate.

Our model is similar to Krugman (1991) target-zone model in many aspects. The Krugman paper investigates the stabilizing feature of the target zone taking the floating regime as a benchmark. Here, we explore the stabilizing effect of the future locking on the exchange rate and we take the regime with no locking as the benchmark regime. In our model the exchange rate with future locking is derived from the fundamental, just like the target-zone exchange rate in the Krugman model.

The market expectation concerning the final conversion rate has a similar role in this model like the medium term target exchange rate of the central bank in Karádi (2005). In our model, the monetary authority can influence the exchange rate by altering the market expectation concerning the final conversion rate, whereas in Karadi's model, by modifying the target exchange rate. Since the market expectation concerning the final conversion rate and the target exchange rate in Karádi's paper are likely to be less volatile than the exchange rate, the market expectation in this model and the target exchange rate in Karádi's paper can smooth the exchange rate.

The main difference between this model and a model with exchange rate targeting is that the target exchange rate of the central bank is a short or medium term target in the latter, whereas the market expectation concerning the locking is a long term anchor

[1] Williamson (1994) gives an overview on the widely used FEER, BEER, NATREX methods.

in this model. Consequently, the relative importance of these anchors are different in the two models. Moreover, the time to reach a medium term target exchange rate is constant over time, whereas the time until the locking is changing over time. Hence, the exchange rate elasticity with respect to the target exchange rate is time-invariant, whereas the corresponding elasticity with respect to the market expectation concerning the final conversion rate is time-varying.

I take into account the uncertainty concerning the time of locking and the final conversion rate by assuming that the expectations both on the time of locking and the locking rate follow stochastic processes. This uncertainty is due to the fact that market participants update their expectations concerning the final conversion rate and the time of locking whenever new information are released, for instance, about the preferred final conversion rate by the competent authorities or about the chances for fulfilling the Maastricht criteria.

I find that the log exchange rate is the weighted average of the expected log final conversion rate and another term called the log latent exchange rate. The weights are changing over time which is in line with our intuition: if the time until locking is infinite, or, in other words, locking will never be achieved, then the exchange rate equals to the latent exchange rate. As the time until the locking decreases the weight of the expected final conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected final conversion rate approaches one.

The dynamics of the exchange rate is such that it tends towards the actual market expectation concerning the final conversion rate in expected term. The closer the time of locking or the expected time of locking is, the higher the speed of convergence is.

I apply the Kalman Filter technique to extract the time series of two factors out of the three. I treat the third one, the expected time of locking as being exogenous.

The roadmap of the paper is as follows. The paper consists in a theoretical model for the exchange rate with future locking. This model provides a functional relationship between the exchange rate and the factors. The model tells us what determines the stabilizing feature of the locking and how the locking alters the process of an exchange rate relative to the case of no future locking. By applying the Kalman Filter I filter the market expectation concerning the locking rate and the latent exchange rate. In order to filter the factors some parameters need to be estimated or calibrated. The time-varying volatilities of the filtered factors are estimated from cross-sectional data on option prices. The estimation of the volatilities are based on a theoretical option pricing model derived in the paper. This option pricing model is such, that the options with longer maturities depend more on the volatility of one of the filtered factors, than the options with shorter maturities do. The option pricing model and our data on options with different maturities ensure that the volatilities of the filtered factors are identified. By comparing the process of the filtered market expectation with that of the historical exchange rate, I make inferences on the stabilizing effect of the locking.

The paper is structured as follows. Section 2 presents the economic model. Section 3 derives an option pricing formula, which is utilized for parameter estimation in the empirical part of the paper. In section 4 first I define the filtering problem, then I show how the parameters are set, finally I present the results of the Kalman Filtering. Section 5 concludes.

2 Economic model

Our model is similar to Krugman's target-zone model. I introduce our model by pointing out the analogy between the two models. In the Krugman paper, the target-zone exchange rate is derived from the fundamental. Similar to Krugman's approach, our starting point is that the exchange rate with locking s_t is a function of the fundamental v_t. In Krugman's model, the logarithm of the target-zone exchange rate is equal to a fundamental plus a term proportional to the conditional expected change of the logarithm of the exchange rate. Moreover, the exchange rate would be equal to the fundamental if there would be no target zone. In our model, the log exchange rate with future locking is equal to the fundamental plus a term proportional to the conditional expected instantaneous change of the log exchange rate. If there would be no locking at all, than the term proportional to the conditional expected instantaneous change of the exchange rate would be zero, consequently the exchange rate would be equal to the fundamental. Motivated by the fact, that the fundamental is identical to the exchange rate without future locking, I refere to the fundamental v as the log latent exchange rate. The latent exchange rate would be the exchange rate in case of no future locking.

The implicit relationship between the target zone exchange rate and the fundamental in the Krugman model is the same as the one between the exchange rate with future locking and the latent exchange rate. I can formulate the relationship between the exchange rate and the latent exchange rate as follows [2]:

$$s_t = v_t + c\frac{E_t(ds_t)}{dt} \quad . \tag{1}$$

Here, s_t is the log exchange rate, and v_t is the log latent exchange rate. The constant c is the time scale. The term $\frac{E_t(ds_t)}{dt}$ is the expected [3] instantaneous change of the exchange rate. As I will derive it, the expected instantaneous change of the exchange rate depends on the log latent exchange rate v_t, the market expectation concerning the log final conversion rate x_t and concerning the time of locking T_t.

From this point on I discuss exclusively the model on the exchange rate with locking and I do not explain all the potential analogies to the Krugman target zone model.

The latent exchange rate is defined as in footnote (2):

$$v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^* \quad . \tag{2}$$

Where y denotes the domestic real output, q is the real log exchange rate, ψ is the risk premium, p^* is the foreign log price, m denotes the domestic nominal money supply, i^*

[2] Svensson (1991) presents one possible structural model for the reduced form (1):

(1f) $m_t - p_t = \alpha y_t - ci_t \quad \alpha > 0 \quad c > 0$ money market equilibrium

(2f) $q_t = s_t + p_t^* - p_t$ real exchange rate

(3f) $\psi_t = i_t - i_t^* - \frac{E(ds_t)}{dt}$ risk premium

(4f) $v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^*$ fundamental/latent exchange rate

In this model the parameter c can be interpreted as the interest rate elasticity of the money demand.

[3] I consider two different types of expectations in the paper. One is the subjective market expectation, and the other is the mathematical expected value of a random variable. Here, I refer to the latter one. In order to distinguish between the two, I refer to the first type of expectation as the market expectation. However, under rational expectation the two are the same.

denotes the foreign interest rate. For the sake of simplicity, I assume that p^*, m and i^* are constant, moreover, normed to zero.

The exchange rate is aimed to be fixed at its equilibrium level. Among the various concepts of equilibrium exchange rate I use the behavior equilibrium exchange rate (BEER). The strong law of purchasing power parity (PPP) should hold under this equilibrium concept. Consequently, the log nominal exchange rate at the time of locking T is equal to the difference between the domestic and foreign log prices: $s_T = p_T - p_T^*$. Under rational expectation the market expects the final conversion rate at time t to be $x_t = E_t(s_T)$, which gives

$$x_t = p_t + \int_t^{T_t} E_t(\pi_\tau) d\tau \quad . \tag{3}$$

Where π denotes the inflation rate.

Fulfilling the Maastricht criteria is a prerequisite for countries aiming to join the EMU. Consequently, the market expectation concerning the time of locking T_t depends both on the inflationary and fiscal shocks. Later I specify how T_t depends on x_t and v_t.

2.1 Dynamics

First, I specify the processes of the factors. Then, I use Ito's stochastic change-of-variable formula to obtain an expression for the expected change of the log exchange rate and to derive the process of the exchange rate. Moreover, I derive the functional relationship $s_t = f(t, v_t, x_t, T_t)$ between the log exchange rate and the factors, namely the log latent exchange rate v_t, the market expectation concerning the log final conversion rate x_t and the market expectation concerning the time of locking T_t.

I assume that all three factors T_t, v_t and x_t follow Brownian motions. This assumtion can be decomposed into an assumtion on the martingale property of the processes and into the Gaussian distribution of the innovations. The Gaussian distribution of the innovations are assumed for technical reason. The martingale property of these processes can be easily explained:

- Under rational expectation, the *expectation of the market participants concerning the log final conversion rate* is the expected value of the log final conversion rate given all the information available at the time the expectation is formed by the market ($x_t = E_t(s_T)$). And also the *market expectation concerning the time of locking* is the expected value of the true time of locking T given all the information available at the time the expectation is formed ($T_t = E_t(T)$). The *law of iterated expectations* implies that the process of both T_t and x_t are martingales, since $E_t(E_{t+1}(s_T)) = E_t(s_T)$ and $E_t(E_{t+1}(T)) = E_t(T)$.

- The assumption on the martingale property of the process of the *log latent exchange rate* can be derived from the economic model under the assumption that the right hand side variables of equation (2) have martingale processes. The martingale property of v_t allows us to focus entirely on the dynamics caused by the future final locking, as opposed to the effects of predictable future changes in the latent exchange rate.

The process of the market expectation concerning the log euro-locking rate x_t can be derived from equation (3). In a discrete time framework it is $\Delta x_t = [\pi_{t+1} - E_t(\pi_{t+1})] + \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)]$. If we assume that both the expectation errors and the change of expectations are independent and normally distributed with zero mean then this process of x_t can be rewritten in a continuous time framework as

$$dx_t = \begin{cases} \sigma_{x,t} dz_{x,t} & \text{, if } t < T_t \\ 0 & \text{, otherwise} \end{cases} \tag{4}$$

where $dz_{x,t}$ is a Wiener process.

The discrete time process of the log latent exchange rate v_t can be derived from (2), (4) and from two additional equations of the model [4].

By defining χ_t by its discrete corresponding process as $\Delta\chi_t = (\alpha+\gamma)\beta \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)] + c\Delta\psi_t$ the process of the latent exchange rate is $dv_t = -(\alpha+\gamma)\beta\sigma_{x,t}dz_{x,t} + d\chi_t$. If χ_t is assumed to follow Brownian motion then the continuous time process of the log latent exchange rate is

$$dv_t = \sigma_{v,t} dz_{v,t} \quad . \tag{5}$$

where $dz_{v,t}$ is a Wiener process. By assuming that the expectation error $(\pi_{t+1} - E_t(\pi_{t+1}))$ is orthogonal to the sum of changes of expectations $(\sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)])$, moreover, the risk premium ψ_t is othogonal to both the expectation error and the sum of changes of expectations, we get that the correlation between $dz_{v,t}$ and $dz_{x,t}$ is

$$\rho\left(dz_{v,t}, dz_{x,t}\right) = -(\alpha+\gamma)\beta \frac{\sigma_{x,t}}{\sigma_{v,t}} \quad . \tag{6}$$

The assumed process of the market expectation concerning the time of locking is the following martingale,

$$dT_t = \begin{cases} (T_t - t)\sigma_{T,t} dz_{T,t} & \text{, if } t < T_t \\ 0 & \text{, otherwise} \end{cases} \quad . \tag{7}$$

Where $dz_{T,t}$ is a Wiener process.

One can see that the market expectation concerning the log final conversion rate x_t reacts mainly to the inflationary shocks, whereas the log latent exchange rate v_t is more related to the real output and hence to the fiscal shocks. The accession country may join EMU just after the Maastricht criteria are fulfilled. Consequently, the market expectation concerning the time of locking should be closely related to both the inflationary and fiscal shocks and also to x_t and v_t. I pose the following intuitive restrictions on the interdependence of T_t and x_t and also of T_t and v_t. First, higher uncertainty relating x_t and v_t makes T_t more volatile. Second, the higher is c, the interest rate elasticity of money demand, the more efficient can be the monetary policy by influencing inflation and output. In that case the expected time of locking is less dependent on x_t and v_t, hence the smaller are $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$. Third, T_t is less dependent on x_t and v_t if the corresponding Maastricht criteria is already fulfilled, consequently, $\rho(dz_{T,t}, dz_{x,t})$ is a positive function of the expected time until locking $T_t - t$ and x_t. And $\rho(dz_{T,t}, dz_{v,t})$ is also a positive function of $T_t - t$ and v_t.

Along these lines one can make restrictions on the process of the expected time of locking. However, the restrictions to be posed are not uniquely determined by the above

[4] I extend the model with a supply curve and an equation capturing the Balassa-Samuelson effect:

(5f) $y_t - y_{t-1} = \beta(\pi_t - E_{t-1}(\pi_t)) \quad \beta > 0$ supply curve

(6f) $dq_t = -\gamma dy_t \quad \gamma > 0$ Balassa-Samuelson effect (real appreciation) .

intuitive requirements. I chose restrictions (8) and (9) for technical reason. My choice on the restrictions is motivated by the demand for a nice analytical solution to the function $s_t = f(t, v_t, x_t, T_t)$. The analytical solution enables me to apply the Kalman Filter. These restrictions on the processes imply the solution in (10), which has some attractive properties apart from being a closed form solution. I discuss these properties at a later point.

Among the possible restrictions I choose the followings:

$$\rho(dz_{T,t}, dz_{x,t})\sigma_{x,t} = \frac{1}{c}(T_t - t)x_t\sigma_{T,t} \quad . \tag{8}$$

$$\rho(dz_{T,t}, dz_{v,t})\sigma_{v,t} = \frac{1}{c}(T_t - t)v_t\sigma_{T,t} \quad . \tag{9}$$

2.2 Functional relationship between the exchange rate and the factors

Here, I derive the functional relationship between the exchange rate and the latent exchange rate, the market expectations concerning the locking time and locking rate. First, I derive the process of the log exchange rate s_t from the processes of the factors by using Ito's stochastic change-of-variable formula. Then, we will obtain that the only function satisfying the derived process and the terminal condition $s_T = x_T$ and (1),(8),(9) is given by

$$s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{T_t - t}{c}}\right) v_t + e^{-\frac{T_t - t}{c}} x_t \quad . \tag{10}$$

According to Ito's formula, the function $f(t, v_t, x_t, T_t)$ satisfies (11).

$$\begin{aligned} df = \Bigg[& \frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{\partial f}{\partial T_t}\mu_{T,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial T_t^2}\sigma_{T,t}^2(T_t - t)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t \partial x_t}\rho\left(dz_{T,t}, dz_{x,t}\right)(T_t - t)\sigma_{T,t}\sigma_{x,t} + \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial T_t \partial v_t}\rho\left(dz_{T,t}, dz_{v,t}\right)(T_t - t)\sigma_{T,t}\sigma_{v,t} + \frac{1}{2}\frac{\partial^2 f}{\partial x_t \partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\Bigg] dt + \\ & + \frac{\partial f}{\partial v_t}\sigma_{v,t}dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t}dz_{x,t} + \frac{\partial f}{\partial T_t}(T_t - t)\sigma_{T,t}dz_{T,t}. \end{aligned} \tag{11}$$

The different μ's denote the drift terms, whose values are zero in our model. The ρ's denote correlations.

At time T the exchange rate s_T is equal to the market expectation concerning the final conversion rate x_T, because at that time the market already knows the final conversion rate. Consequently, the function $f(t, v_t, x_t, T_t)$ should satisfy the terminal condition

$$f(T, v_T, x_T, T) = x_T \quad . \tag{12}$$

The solution is given by (10) and the proof can be found in the Appendix. Equation (10) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights are changing over time;

if the time until locking is infinite, or in other words, there will be no locking at any time, then the weight of the latent exchange rate is one, and the weight of the expected final conversion rate is zero. As the time until the locking decreases, the weight of the expected final conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected final conversion rate approaches one.

In order to examine the dynamics of the exchange rate, I rewrite equation (11) in the following way. By substituting (10),(8),(9) and $v_t = \frac{1}{1-e^{-\frac{T_t-t}{c}}} s_t - \frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}} x_t$ into equation (11) we obtain

$$ds_t = \frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}(x_t - s_t)\,dt + \left(1 - e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}dz_{v,t} + \tag{13}$$

$$+e^{-\frac{T_t-t}{c}}\sigma_{x,t}dz_{x,t} - \frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}(x_t - s_t)(T_t - t)\sigma_{T,t}dz_{T,t} \quad .$$

Equation (13) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation concerning the final conversion rate. Moreover, the closer the time of locking, the faster the convergence is.

Equations (4), (5), (7) and (10) define a three-factor model. One factor is the market expectation concerning the final conversion rate; another factor is the market expectation concerning the time of locking; the third factor is the latent exchange rate. This model is linear in two of the factors, but not in T_t.

3 Option pricing

In this section I show a pricing formula for European type options what fits our model. This option pricing formula is used to estimate the time varying volatilities of the filtered factors. The historical option prices are given in terms of implied volatility, consequently, I derive the option prices in terms of volatility as well.

In the theoretical model the uncertainty is present due to the stochastic innovations ($dz_{v,t}$, $dz_{x,t}$, $dz_{T,t}$) of the factors, consequently the price of an option is a function of the variances and covariances of these normally distributed innovations. From equation (13), we can derive, that the instantaneous variance of the log changes of the exchange rate at time t is

$$\begin{aligned}\sigma_{s,t}^2 = {\sigma^*}_{s,t}^2 &+ \left(\frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}\right)^2 (x_t - s_t)^2 (T_t - t)^2\sigma_{T,t}^2 + \\ &- 2\frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}(x_t - s_t)(T_t - t)\sigma_{T,t}\left(1 - e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}\rho\left(dz_{T,t}, dz_{v,t}\right) + \\ &- 2\frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}(x_t - s_t)(T_t - t)\sigma_{T,t}e^{-\frac{T_t-t}{c}}\sigma_{x,t}\rho\left(dz_{T,t}, dz_{x,t}\right) \quad .\end{aligned} \tag{14}$$

Where ${\sigma^*}_{s,t}^2$ is

$${\sigma^*}_{s,t}^2 = \left(1 - e^{-\frac{T_t-t}{c}}\right)^2 \sigma_{v,t}^2 + \left(e^{-\frac{T_t-t}{c}}\right)^2 \sigma_{x,t}^2 + \tag{15}$$

$$+2\left(1-e^{-\frac{T_t-t}{c}}\right)\left(e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}\sigma_{x,t}\rho\left(dz_{v,t},dz_{x,t}\right) \quad .$$

The magnitude of the terms of (14) other than ${\sigma^*}^2_{s,t}$ are negligible compared to the magnitude of ${\sigma^*}^2_{s,t}$, because their common component, $\sigma_{T,t}$, is likely to be relatively small. Consequently, I will disregard these terms in the theoretical option pricing formula and approximate $\sigma^2_{s,t}$ by ${\sigma^*}^2_{s,t}$. Moreover, I make the following simplification. Until now, I aloud $\sigma_{v,t}$, $\sigma_{x,t}$ and $\sigma_{T,t}$ to change over time. I do not rule out this possibility. However, I think that the option prices are not much influenced by the changes of the volatilities. The pricing formula for the stochastically changing volatility case is different from the one I derive, however the derived one is a good approximation for the theoretical value in case of ATM options with a maximum of one year maturity [5] . The price of a European option in terms of volatility is approximated by

$$g(t,m,\sigma_{x,t},\sigma_{v,t},\rho\left(dz_{v,t},dz_{x,t}\right)) = \left[\int_t^{t+m} {\sigma^*}^2_{s,\tau} d\tau\right]^{\frac{1}{2}} =$$

$$= \left[\int_t^{t+m} \left(1-e^{-\frac{T_\tau-\tau}{c}}\right)^2 \sigma^2_{v,\tau} + \left(e^{-\frac{T_\tau-\tau}{c}}\right)^2 \sigma^2_{x,\tau} + \right.$$

$$\left. + 2\left(1-e^{-\frac{T_\tau-\tau}{c}}\right)\left(e^{-\frac{T_\tau-\tau}{c}}\right)\sigma_{v,\tau}\sigma_{x,\tau}\rho\left(dz_{v,\tau},dz_{x,\tau}\right) d\tau\right]^{\frac{1}{2}} \quad . \quad (16)$$

Where the option is sold at time t. The time until maturity is denoted by m.

In this formula T_τ ($\tau > t$) is stochastic and unknown at time t. In order to avoid complication coming from the stochastic nature of T_τ I approximate [6] T_τ by T_t. By applying this final approximation and by calculating the integrals we obtain the option pricing formula

[5] As it is pointed out by Hull (1997) page 620: "For options that last less then a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases. The pricing impact in percentage terms can be quite large for deep-out-of-the-money options."

[6] An alternative approximation can also be applied, where the function $h(T_\tau)$ is approximated by its second order Taylor series expansion around T_t: $h(T_\tau) = h(T_t) + \frac{1}{2}\frac{\partial^2 h}{\partial T_t^2}(T_t - t)^2\sigma^2_{T,t}(\tau - t)$. This approximation is more precise, than the applied one. The value added of applying this approximation depends highly on the magnitude of $\sigma_{T,t}$. In our case it proved to be relatively minor.

$$
\begin{aligned}
&g^2(t,m,\sigma_{x,t},\sigma_{v,t},\rho\left(dz_{v,t},dz_{x,t}\right)) = \\
&\qquad \sigma_{v,t}^2 \left\{ m - 2ce^{-\frac{1}{c}(T_t-t-m)} + 2ce^{-\frac{1}{c}(T_t-t)} + \frac{c}{2}e^{-\frac{2}{c}(T_t-t-m)} - \frac{c}{2}e^{-\frac{2}{c}(T_t-t)} \right\} + \\
&\qquad + \sigma_{x,t}^2 \left\{ \frac{c}{2}e^{-\frac{2}{c}(T_t-t-m)} - \frac{c}{2}e^{-\frac{2}{c}(T_t-t)} + 2ce^{-\frac{1}{c}(T_t-t-m)}\rho\left(dz_{v,t},dz_{x,t}\right)\frac{\sigma_{v,t}}{\sigma_{x,t}} + \right. \\
&\qquad\qquad - 2ce^{-\frac{1}{c}(T_t-t)}\rho\left(dz_{v,t},dz_{x,t}\right)\frac{\sigma_{v,t}}{\sigma_{x,t}} + \\
&\qquad\qquad \left. - ce^{-\frac{2}{c}(T_t-t-m)}\rho\left(dz_{v,t},dz_{x,t}\right)\frac{\sigma_{v,t}}{\sigma_{x,t}} + ce^{-\frac{2}{c}(T_t-t)}\rho\left(dz_{v,t},dz_{x,t}\right)\frac{\sigma_{v,t}}{\sigma_{x,t}} \right\} \qquad (17)
\end{aligned}
$$

This option pricing formula (17) is used to estimate the time varying volatilities $\sigma_{v,t}$, $\sigma_{x,t}$ of the filtered factors. By using formula (17) and cross-sectional data on options with different maturities but with the same issuing date t, the volatilities $\sigma_{v,t}$, $\sigma_{x,t}$ can be estimated for each time t. The intuition behind the identification is that longer options are more exposed to shocks occurring in the far future than options with shorter maturities. Or in other words, $\sigma_{x,t}$ has higher relative weight in a longer option, then in a shorter one. And the opposite holds for $\sigma_{v,t}$.

4 Filtering Factors

I apply the Kalman Filter technique to extract the time series of the factors from the time series of the observable exchange rate. Filtering all three factors from only one series would be overambitious. It is likely that such an exercise would not provide robust results. Luckily, I have alternative source of information on the market expectation concerning the time of locking. This source of information is the Reuters poll which seems to be reliable concerning the Euro entry date. So, I treat the time of locking T_t as being exogenously given. As T_t is not independent of the other two factors I use the conditional distributions of x_t and v_t, where I condition on the realization of T_t.

The Kalman Filter technique can be applied to filter factors only if the model is linear [7] in all the factors to be filtered. The log exchange rate s_t is linear in the remaining two factors, namely the latent exchange rate v_t and the market expectation concerning the final conversion rate x_t.

In this section, I filter the market expectation concerning the final conversion rate of the Czech koruna, Hungarian forint and Polish zloty. I use historical daily exchange rate data from the period of 15.Dec.2004.–3.Aug.2006. The sample size is 421 in case of Hungary and it is somewhat shorter, 391 in case of Czech Republic and Poland due to missing observations. First, I define the filtering problem, and then I show how the parameters are set, finally I present the results.

[7] To filter all three factors one should apply a different technique then the Kalman Filter, because the model is not linear in T_t. The Extended Kalman Filter and the Particle Filter are possible candidates.

4.1 Filtering problem

In our filtering problem one of the factors T_t is exogenous. As T_t is not independent of the other two factors I have to use the conditional distributions of x_t and v_t, where I condition on the realisation of T_t. The conditional expected innovations of x_t and v_t are $\rho(dz_{T,t}, dz_{x,t})dz_{T,t}$ and $\rho(dz_{T,t}, dz_{v,t})dz_{T,t}$ respectively, where the ρ's denote correlations. These expected changes of $dz_{x,t}$ and $dz_{v,t}$ are taken into account in the model by having a contsant as a third state variable. The system covariance matrix $Q(t)$ is also conditional on T_t.

The filtering problem can be written in the usual form:

$$\Lambda(t+1) = A(t)\Lambda(t) + w_1(t+1) \tag{18}$$

$$\Omega(t) = C(t)\Lambda(t) + w_2(t) \tag{19}$$

$$E\left[\begin{pmatrix} w_1(t+1) \\ w_2(t) \end{pmatrix} \begin{pmatrix} w_1(t+1) & w_2(t) \end{pmatrix}\right] = \begin{pmatrix} Q(t) & 0 \\ 0 & R \end{pmatrix} \tag{20}$$

In our problem, the vector of states is $\Lambda(t) = \begin{pmatrix} v_t \\ x_t \\ 1 \end{pmatrix}$. The system matrix is $A(t) = \begin{pmatrix} 1 & 0 & \sigma_{v,t}\rho(dz_{T,t}, dz_{v,t})\frac{dT_t}{\sigma_{T,t}(T_t-t)} \\ 0 & 1 & \sigma_{x,t}\rho(dz_{T,t}, dz_{x,t})\frac{dT_t}{\sigma_{T,t}(T_t-t)} \\ 0 & 0 & 1 \end{pmatrix}$. The vector $w_1(t)$ is assumed to be a Gaussian vector white noise. The observable variable is the log exchange rate $\Omega(t) = s_t$. Equation (10) implies that the observation matrix is $C(t) = \begin{pmatrix} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{T_t - t}{c}} & 0 \end{pmatrix}$.

The system covariance matrix can be written as

$$Q(t) = \begin{pmatrix} Q_{1,1}(t) & Q_{1,2}(t) & 0 \\ Q_{1,2}(t) & Q_{2,2}(t) & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

where the covariance is conditional on the observed T_t, therefore

$$Q_{1,1}(t) = \sigma_{v,t}^2 \left[1 - \rho^2(dz_{T,t}, dz_{v,t})\right],$$

$$Q_{1,2}(t) = \sigma_{v,t}\sigma_{x,t}\left[\rho(dz_{x,t}, dz_{v,t}) - \rho(dz_{T,t}, dz_{v,t})\rho(dz_{T,t}, dz_{x,t})\right],$$

$$Q_{2,2}(t) = \sigma_{x,t}^2 \left[1 - \rho^2(dz_{T,t}, dz_{x,t})\right].$$

I assume that the error term $w_2(t)$ is zero. In other words, I assume that we observe the exchange rate without error and the model (10) perfectly describes the relationship between the factors and the exchange rate. Hence, the variance of the observation error term R is set to zero. The Kalman Filter remains valid even in this case [8] .

In our problem, the observation matrix $C(t)$, the system matrix $A(t)$ and the system covariance $Q(t)$ are changing over time.

The parameters of the observation matrix c, T_t and the parameters $\sigma_{v,t}$, $\sigma_{x,t}$, $\sigma_{T,t}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$ of the system covariance $Q(t)$ and of the system matrix $A(t)$ need to be either calibrated or estimated. Moreover, the initial values x_{t_0} and v_{t_0} of the factors belonging to the beginning of the sample period, $t_0 = 15.Dec.2004.$, need to be set as well. I describe in the next section how these parameters are estimated and calibrated.

[8] See Harvey (1990) page 108 for a detailed discussion.

4.2 Parameters

First, I describe how T_t, is set based on the Reuters poll. Then, I show how the parameters x_{t_0}, v_{t_0}, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{v,t})$ and $\sigma_{T,t}$ are calibrated. Finally, I describe how the parameters $\sigma_{v,t}$, $\sigma_{x,t}$ and c are estimated from historical option prices and exchange rate data.

For calibrating the *expected time of locking* T_t, I take into consideration that the exchange rates of the countries newly entered the ERM2 system are almost fixed: the volatility of the Estonian kroon, the Lithuanian lita, the Slovenian tolar, the Cyprus pound and the Maltese lira dropped below 1% after entering the ERM2 regime [9] .This finding makes the assumption plausible that the locking does not take place at the time the country is entering the Monetary Union, but the time it enters the ERM2 regime. The monthly and quarterly Reuters polls survey the expectation of market analysts concerning the time of EMU and ERM2 entries of the accession countries. I model the time of locking as the time of ERM2 entry and set the parameter of the time of locking equal to the average of the reported expectations of the individual analysts concerning the time of ERM2 entry of each of the three Visegrad countries [10] .

The Reuters poll queries the analysts opinion on the expected date of ERM2 entry of Hungary on every month, whereas the expectations on the date of ERM2 entry of Poland and Czech Republic are queried only quarterly. In case of Hungary the analysts are queried by the Reuters poll in the middle of each month, usually after all the new monthly macro indexes become public. If the expectations of the analysts are mainly based on these new releases of macro data then one has no reason to assume that the expectations are changing between two monthly Reuters polls. I assume that the expected time of ERM2 entry reported on one specific day of a month is formed exactly on that day. Along these lines, I can simply interpolate the monthly observations on T of Hungary by a constant to have daily data. The same interpolation is applied to the quarterly Reuters poll data of Poland and Czech Republic.

Figure 1 shows the average reported expected time of ERM2 entry of Czech Republic, Hungary and Poland in the period of 15.Dec.2004.–3.Aug.2006. From the figure, we can see that the market expectations were relatively stable until autumn of 2005. The expectations have changed between the quarterly polls of August and November in case of Czech Republic and Poland. Whereas in case of Hungary, it can be better detected from the monthly polls when the expectation changed. The shift in expectations were between September and October of 2005. Until autumn of 2005 it was expected that the three Visegrad countries would enter the ERM2 sometime during the year of 2007. Thereafter, the expectations changed dramatically, as it is reported by the monthly and quarterly Reuters polls. The expected time of ERM2 entry postponed to 2008 for Czech Republic, to 2009 for Poland and to 2010 for Hungary.

[9] The Estonian kroon, the Lithuanian lita and the Slovenian tolar joined ERM2 on 27 June 2004. On 2 May 2005 three other Member States joined ERM2: Cyprus, Latvia and Malta.

[10] In order to check the robustness of the results, in an alternative specification I model the time of locking as the time of EMU entry. In the alternative specification the parameter of the time of locking is set equal to the average of the reported expectations of the individual analysts concerning the time of EMU entry. Since the results of the ERM2 entry date specification do not differ qualitatively from those of the EMU entry date specification, I only present the former ones.

For a given value of c, one can calibrate the initial values of the factors and the correlations. Later, I will discuss in detail how the parameter c is estimated. For now, let us assume that we know the parameter c and want to calibrate the parameters x_{t_0}, v_{t_0}, $\rho(dz_{T,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{v,t})$, and $\rho(dz_{x,t}, dz_{v,t})$. What makes this calibration somewhat difficult is that I have no direct information on the latent exchange rate. For the calibration of the *initial states*, x_{t_0} and v_{t_0} and some time invariant parameters, I used the Reuters polls. Although, I do not find the reported expectation on the central parity in the ERM2 system reliable, this is the only source of information that could be used for certain calibrations. Due to the limited reliability of these data, I have limited confidence in the calibrated parameters. In order to gain some confidence of our results, some sensitivity analysis is necessary relating the calibrated parameters. The initial values, x_{t_0} and v_{t_0}, are set as follows. I assume that x_{t_0} is equal to the log of averaged expectations on the central parity reported by the last Reuters polls of 2004. The initial value of v_{t_0} is calculated by plugging s_{t_0}, x_{t_0}, T_{t_0}, and c into equations (10).

One possible way to calibrate the *correlations* $\rho(dz_{T,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{v,t})$ and $\rho(dz_{x,t}, dz_{v,t})$ is to use not only the last Reuters poll data for the year 2004, but all the averaged expected central parities reported by the polls. By following this strategy of calibration, first the latent exchange rates corresponding to each of the monthly and quarterly observations are need to be calculated by using again equations (10) and the corresponding s_t, x_t, T_t, and c. Then the calibrated correlations can be calculated from these monthly and quarterly data on x, v and T. This strategy of calibration has the major drawback of only a few observations[11] can be used to calculate the correlations. Moreover, by following this strategy of calibration one might obtain correlations with a sign that is not in line with the theoretical considerations[12] presented in Subsection 2.1. Unfortunately, six of the nine correlation parameters of the three countries have the wrong sign if their calibration is based on the above method. Consequently, I opt to simply set all the nine correlations to zero.

The estimated c maximizes the filtering likelihood and the filtering likelihood is obviously a function of the calibrated initial state parameters. Consequently, the sequence of estimation and calibration should be the following. First, one should calibrate these parameters for every candidates of c. Then the filtering likelihood can be calculated for the set of calibrated parameters and the candidate for c. Finally, by searching for the optimal c, the estimated c parameter and the calibrated parameters depending on c are determined simultaneously.

It is difficult to estimate the *volatility of the market expectation concerning the time of locking* for the following reasons. First, this volatility is likely to fluctuate substantially over time; second, I have only a few observations on T to estimate the time varying $\sigma_{T,t}$. Consequently, I have to rely more on intuition, than on the data. The instantaneous volatility $\sigma_{T,t}$ is assumed to be very large, whenever the market expectation concerning the time of locking jumps. However, $\sigma_{T,t}$ is assumed to be negligible [13] , whenever the

[11]The number of observations is seven in case of Czech Republic and Poland and it is twenty in case of Hungary.

[12]Based on the theoretical considerations the correlations have to meet the following sign restrictions: $\rho(dz_{T,t}, dz_{x,t}) \geq 0$, $\rho(dz_{T,t}, dz_{v,t}) \geq 0$ and $\rho(dz_{x,t}, dz_{v,t}) \leq 0$.

[13]Whenever $\sigma_{T,t}$ is negligible, the option pricing formula (17) is valid, because all the applied approximations of the derivation of (17) are justified.

market expectation concerning the time of locking is unchanged. This assumption makes the system matrix A to be independent of the jumps in T [14] .

For a given value of time invariant parameters c and $\rho(dz_{x,t}, dz_{v,t})$ one can estimate the time varying *volatilities of the factors* v and x. Parameters $\sigma_{v,t}$ and $\sigma_{x,t}$ are estimated from 6 implied volatilities $\sigma_{t,i}^{imp}$ for each time t by OLS. The basic idea of the estimation is to minimize the distance between the theoretical option prices given by the option pricing formula (17) and the historical option prices. The 6 currency options have different maturities $m(i)$. In case of Czech koruna and Polish zloty the maturities are one-month $m(1)$, two-months $m(2)$, three-months $m(3)$, six-months $m(4)$, nine-months $m(5)$ and one-year $m(6)$. Whereas in case of the Hungarian forint the currency options have one-week $m(1)$, one-month $m(2)$, two-months $m(3)$, three-months $m(4)$, six-months $m(5)$ and one-year $m(6)$ maturities. The OLS estimates of $\sigma_{v,t}$ and $\sigma_{x,t}$ satisfy

$$\min_{\sigma_{v,t},\sigma_{x,t}} \sum_{i=1}^{6} \left[g(t, m(i), \sigma_{x,t}, \sigma_{v,t}, \rho\left(dz_{v,t}, dz_{x,t}\right)) - \sigma_{t,i}^{*,imp} \right]^2 \quad . \tag{21}$$

The term $\sigma_{t,i}^{*,imp}$ of equation (21) is either the historical implied volatility $\sigma_{t,i}^{imp}$ or a transformation of it. The possible need for a transformation of the historical implied volatilities can be explained along the following lines. Obviously, if the option pricing model of Section 3 would perfectly capture the relationship between the volatility of the factors and the implied volatilities, then there would be no need for any transformation. Since the filtered factors are heavily dependent on their estimated volatilities, it is crucial to investigate what else can effect the implied volatilities other than the volatilities of the factors. Moreover, if these other possible effects are not happen to be orthogonal to the volatilities of the factors in the option pricing formula, then we face the omitted variable problem. Hence, the estimated volatilities of the factors will be biased.

One possible omitted variable is the one that captures the effect of an implicit or explicit fluctuation band. Until this point, I have not taken into account, that the fluctuation of the exchange rate of the Hungarian forint versus euro is limited by an exchange rate band. Moreover, the other two countries, Czech Republic and Poland, might also apply an implicit fluctuation band what can have significant but different effect on the historical option prices with different maturities. The closer is the exchange rate to the edges of the band the limited its volatility is [15] . Moreover the diminishing effect on the volatility is higher in case of longer horizons. Consequently, the option prices with longer maturities should be more effected by the relative position of the exchange rate in the fluctuation band then the option prices with shorter maturity.

First, I transform the implied volatilities in order to purge the possible effect of an explicit or implicit band. Then, in case of finding empirical evidence of significant effect of the band on the implied volatilities I use the transformed data to estimate $\sigma_{v,t}$ and $\sigma_{x,t}$ by (21), whereas in case of lacking evidence for the effect of a possible target zone on the

[14] An alternative assumption is that the A matrix is effected by the changes of T. The time varying parameter $\sigma_{T,t}$ could be chosen so as the process of x is pulled back to its reported value in each months or quarters. This specification would be interesting only, if the Reuters poll data on the expected central parity would be more reliable and one would aim to filter x between every two Reuters polls.

[15] This finding is supported by the theoretical models on target zones by Krugman (1991) and Naszódi (2004) for instance.

	CZ	HU	PL
$R^2_{i=2}$	14.24%	13.89%	15.95%
$R^2_{i=3}$	13.70%	**29.83**%	12.81%
$R^2_{i=4}$	18.15%	**36.77**%	12.73%
$R^2_{i=5}$	23.55%	**41.45**%	11.54%
$R^2_{i=6}$	21.08%	**43.60**%	9.91%

Table 1: The portion of variations of the volatility wedges explained by a constant, the exchange rate and the square of exchange rate

volatilities I use the untransformed historical implied volatility data to estimate $\sigma_{v,t}$ and $\sigma_{x,t}$.

The applied transformation is such that it does not effect the implied volatility of the option with the shortest maturity $\sigma^{imp}_{t,1}$. All the other implied volatilities are transformed to $\sigma^{*,imp}_{t,i} = \sigma^{imp}_{t,i} - \hat{\beta}_{i,0} - \hat{\beta}_{i,1}S_t - \hat{\beta}_{i,2}S_t^2$, where the $\hat{\beta}_{i,.}$'s are the estimated parameters of the following regression.

$$\sigma^{imp}_{t,i} - \sigma^{imp}_{t,1} = \beta_{i,0} + \beta_{i,1}S_t + \beta_{i,2}S_t^2 + \epsilon^{imp}_{t,i} \qquad i \in \{2,3,4,5,6\} \quad . \tag{22}$$

In this regression the volatility wedge, defined as $\sigma^{imp}_{t,i} - \sigma^{imp}_{t,1}$, is regressed on a constant and on the exchange rate and on the square of the exchange rate.

Table 1 shows the common explanatory power of the constant, the exchange rate and the square of exchange rate for the five volatility wedges and for the three countries. As we can see, the R^2s are high only in case of Hungary. This can be interpreted as finding evidence for the effect of the target zone on the volatility wedges, on the differences between the option prices with different maturities. Whereas in case of Czech Republic and Poland the exchange rate does not explain much of the variance of the volatility wedges. The different findings in case of Hungary and the other two countries can be explained by the fact that Hungary maintained a target zone, but not the other two countries. The transformation of the implied volatility data is necessary in case of Hungary before estimating $\sigma_{v,t}$ and $\sigma_{x,t}$. Whereas, the option prices do not need to be transformed in case of Czech Republic and Poland.

By estimating (21) I obtain the time-varying volatilities of the factors. Figure 2 shows the time series of the shortest and longest implied volatilities (transfored implied volatilities for Hungary) and their fitted values. Figure 3 shows the time series of the estimated volatilities of the factors. Figure 3 shows that the estimated volatility of x is often zero. However, during turbulent times it can have extremely large values, between 10% and 30%. An extreme example for the turbulent times is July of 2006 in Hungary, when the estimated volatility of x was around 70%. The high estimated volatilities of x can be associated with those times, when the long implied volatility substantially exceeds the short implied volatility.

The parameters c is estimated by maximum likelihood (ML), the estimated c maximizes the likelihood function of the filtering problem. The estimated value of c is around two for all three countries; 1.80, 2.05, 2.365 for Czech Republic, Hungary, and Poland

	CZ	**HU**	**PL**
c	1.80	2.05	2.365
$(tstat_c)$	(8.92)	(8.90)	(8.08)

Table 2: Estimated c parameter and its t-statistics[16]

respectively. Table 2 shows, that these parameter estimates are highly significant. One can interpret a parameter value of c equal to two as follows. If a country will lock its exchange rate in four years, then the elasticity of the exchange rate with respect to the market expectation concerning the final conversion rate ($e^{-\frac{T-t}{c}} = e^{-\frac{4}{2}}$) is almost 14%. If the locking of the exchange rate takes place for instance in two years, then this elasticity is more than 40%.

Figure 4 shows the relative weights of the two components of the log exchange rate in the investigated period. The positive shocks in T decrease the relative weight of x whereas the negative shocks increase it. The largest change in the relative weights took place after September 2005, when the market expectation concerning the time of ERM2 entry shifted substantially in case of all three countries. However, the relative weight of x remained significant in case of all three countries. Even when the relative weight of x was the smallest, it exceeded 10% in case of Czech Republic and Poland and it exceeded 7% in case of Hungary.

4.3 Filtered expectation of the market

Figure 5 shows the historical exchange rates of the koruna, the forint and the zloty against the euro, the filtered states and the average expectations concerning the central parity of the analysts queried by the Reuters polls. The expectation of the market concerning the final conversion rate may be thought to be close to the expected central parity of the ERM2 regime. In that case, the expected central parity is a good reference for the filtered expected final conversion rate to be compared with. Here, we compare the filtered market expectation with the average expectations reported by the Reuters polls, although we think that the polls have only limited information content with respect to the central parity as it is shown by Figure 6. The views of the queried analysts on the central parity in the ERM2 varies a lot in each poll. There is at least 6% difference between the two extreme views of the analysts, however even a more than 20% difference is not rare. These differences indicate that the uncertainty around the reported expectations are likely to be big and one have to be careful by referring to the average reported expectations as the general view of the market on the central parity.

It can be seen on Figure 5, that the filtered expected final conversion rate has a similar pattern to the reported expected central parity in case of all three countries. Moreover, each pattern is similar to that of the corresponding historical exchange rate. However, the reported expectations of the market and the filtered x_t are significantly different in most of the time. The reported expectations of the market are usually outside the 90%

[16]The t-statistics are calculated from the asymptotic covariance matrix estimated by the BHHH algorithm.

confidence interval of the filtered x in case of all three countries.

We have important findings on both the level of the expected final conversion rate and on its volatility. If our previous view on the role of locking was based purely on the Reuters poll data on the averaged market expectations concerning the central parity, then these new findings may modify our view in some aspects.

The filtered market expectations concerning the euro-locking rate is bellow the averaged market expectations concerning the central parity for a long period in case of all three countries. In case of Czech Republic the filtered expected locking rate is almost always less than the reported averaged expectations concerning the central parity. The only exception is the most recent observation of August 2006. The same holds for Hungary and Poland for their first sub periods. The filtered expected locking rate is smaller than the reported averaged expectations concerning the central parity in Hungary until March of 2006 and for Poland before October of 2005. If one considers the filtered data to be more reliable than the Reuters poll data, then this paper contributes to our knowledge on the market expectations substantially: in the first part of the investigated period the market expected the koruna, the forint and the zloty to be locked at a stronger final conversion rate than what was suggested by the Reuters poll data. In case of the koruna and the zloty the difference between the filtered market expectations and the reported averaged expectations concerning the central parity decreased substantially for the second part of the sample. Whereas in case of the forint the market expected an even higher final conversion rate than what was suggested by the Reuters poll data in the second part of the sample. Finally, at the end of the sample the two seem to coincide in case of the forint as well.

The level of volatility of the market expectation concerning the final conversion rate is important, because a relatively stable market expectation can stabilize the exchange rate. The locking rate is often referred to as the nominal anchor of the exchange rate due to this stabilizing feature of the market expectation concerning the locking rate. Regarding the volatilities, one can see, that the filtered x is more volatile than the reported averaged expectations concerning the central parity in case of all three countries. This finding might adversely modify our previous view based purely on the Reuters polls on the stabilizing feature of the locking. Still, if the volatility of x is lower than that of s, then the market expectation concerning the final conversion rate might have a stabilizing effect on the exchange rate. What can be seen on Figure 5 is that most of the time the volatility of s exceeds the volatility of x in case of all three countries. In case of Czech Republic and Poland the locking seems to had a stabilizing effect on the exchange rate from March 2006 until August 2006. Moreover, in case of Poland there seems to be another stable period between March and October of 2005. In those periods the volatilities of the market expectation concerning the locking rates were almost always zero as it is shown by Figure 3, and the filtered xs were more stable than the exchange rates. Moreover, the weights of xs in the koruna and zloty were relative large, around 20%. In case of Hungary we can detect by visual inspection two periods characterized by the stabilizing effect of the locking. One of the periods is between October 2005 and January 2006, the other period is between March 2006 and June 2006. What might make the stabilizing feature of the locking smaller in case of Hungary relative to the other two countries is that the relative weight of x in s is around only 10% in these periods.

The big picture on the stabilizing feature of the locking in the entire sample period is provided by Table 3. The stabilizing effect of the locking is calculated either as the

	CZ	**HU**	**PL**
σ_S	4.70%	7.67%	9.26%
σ_V	5.02%	8.26%	11.98%
$\sigma_S - \sigma_V$	-0.32%	-0.59%	-2.72%
$\frac{\sigma_S - \sigma_V}{\sigma_V}$	-6.36%	-7.15%	-22.67%

Table 3: The volatiliy of the exchange rate (S) and of the filtered latent exchange rate (V)

absolute or as the relative difference between the volatilities of the historical exchange rate and the latent exchange rate. Based on the investigation of the entire sample period the stabilizing effect of the locking is the highest in Poland. The second highest in Hungary. The stabilizing effect is the least important in Czech Republic, however the volatility of the koruna would be the smallest among the three countries even if the Czech Republic would not aim at joining the Euro-zone.

5 Conclusion

This paper has investigated the expectation of the market concerning the final conversion rate. The paper has presented a theoretical model for the exchange rate with future locking. The dynamics of the exchange rate is such that it converges to the actual market expectation concerning the final conversion rate in expected term. The closer the time of locking, or the expected time of locking is, the higher the speed of convergence is. In the empirical part of the paper, we have filtered out the subjective expectation of the market participants concerning the final conversion rate from historical exchange rate data by Kalman Filter. I applied this analysis to three Visegrad countries, Czech Republic, Hungary, and Poland.

Our previous view on the role of locking, what was mainly based on Reuters poll data, has been modified in some aspects. First, the level of the filtered market expectation concerning the final conversion rate differ significantly from the averaged reported market expectations concerning the central parity in case of all three countries. Second, the stabilizing feature of the market expectation concerning the final conversion rate on the exchange rate proved to be smaller in case of filtered expectations than in case of the averaged reported market expectations. Still, we find empirical evidence on the exchange rate stabilizing effect of the locking even when the filtered expectation concerning the locking rate is considered to be the true expectation of the market. The magnitude of the stabilizing effect depends on two determinants. First, how stable are the market expectations concerning the locking rate. Second, how important are the expectation in determining the exchange rate. In case of an earlier entry to the Euro zone the stabilizing effect is likely to be more substantial because the market expectations concerning the locking rate are likely to be more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this intuitive argument the locking should contribute to the stabilization of the koruna the most and to that of the forint the least. The results somewhat contradict to this intuitive argument. Based on the investigation of the entire sample period the stabilizing effect of the locking is the highest in Poland. The second highest in Hungary. The stabilizing effect is the least important in Czech Republic, however the volatility of the koruna would be the smallest among the three countries even without any future locking.

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7 Appendix

Here, I prove, that the derived function $s_t = f(t, v_t, x_t, T_t)$ of (10) satisfies the dynamic condition given by (11), the terminal condition (12) and (1),(8),(9).

I prove by substitution that (10) satisfies (11) as follows.

$$ds_t = \left[\frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t) + \frac{1}{2}\frac{1}{c^2}e^{-\frac{T_t-t}{c}}(x_t - v_t)\sigma_{T,t}^2(T_t - t)^2 + \right.$$
$$\left. -\frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{x,t}\right)(T_t - t)\sigma_{T,t}\sigma_{x,t} + \frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{v,t}\right)(T_t - t)\sigma_{T,t}\sigma_{v,t}\right]dt +$$
$$+\left(1 - e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}dz_{v,t} + e^{-\frac{T_t-t}{c}}\sigma_{x,t}dz_{x,t} - \frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t)(T_t - t)\sigma_{T,t}dz_{T,t}. \quad (23)$$

By substituting and (10) into (23), we get that the expected instantaneous change of the exchange rate is

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c}(s_t - v_t) \quad . \quad (24)$$

By substituting (24) into (1), we obtain an identity. This proves that (10) satisfies (1). Thus, (10) satisfies the terminal condition (12) as well. Hence, the function in (10) is the solution I was looking for.

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8 Figures

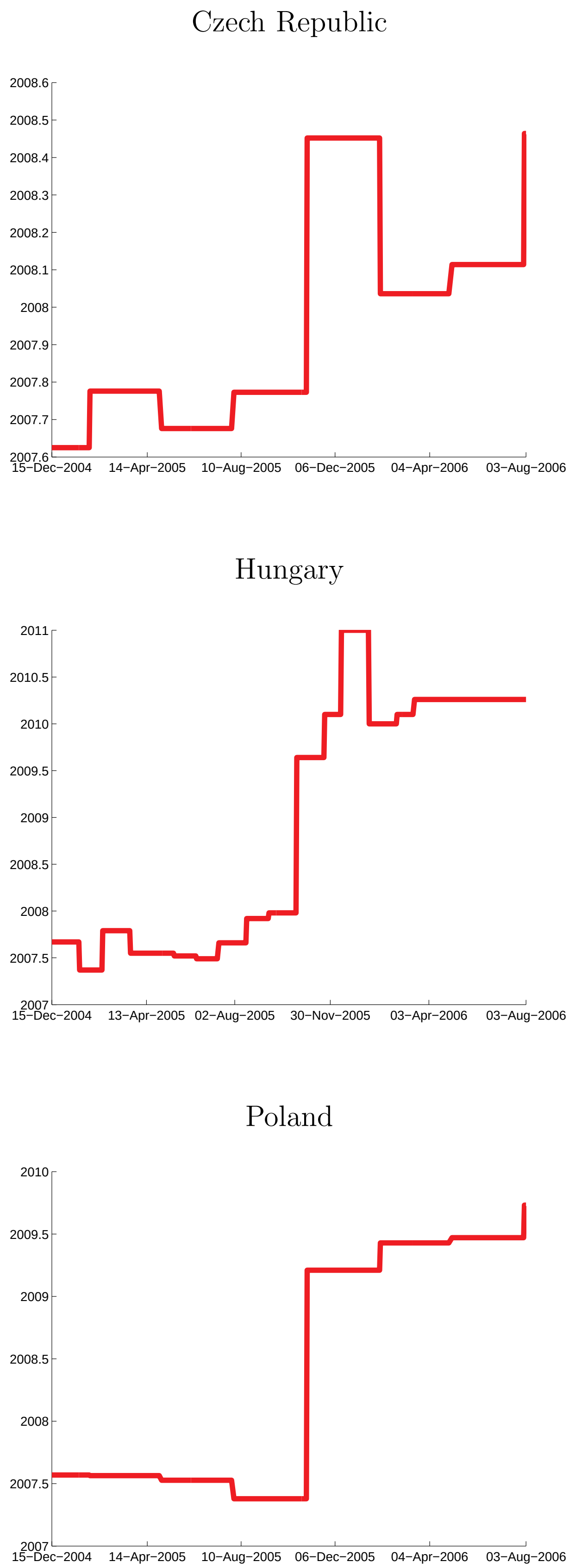


Figure 1: The average expectation of the analysts concerning the time of ERM2 entry

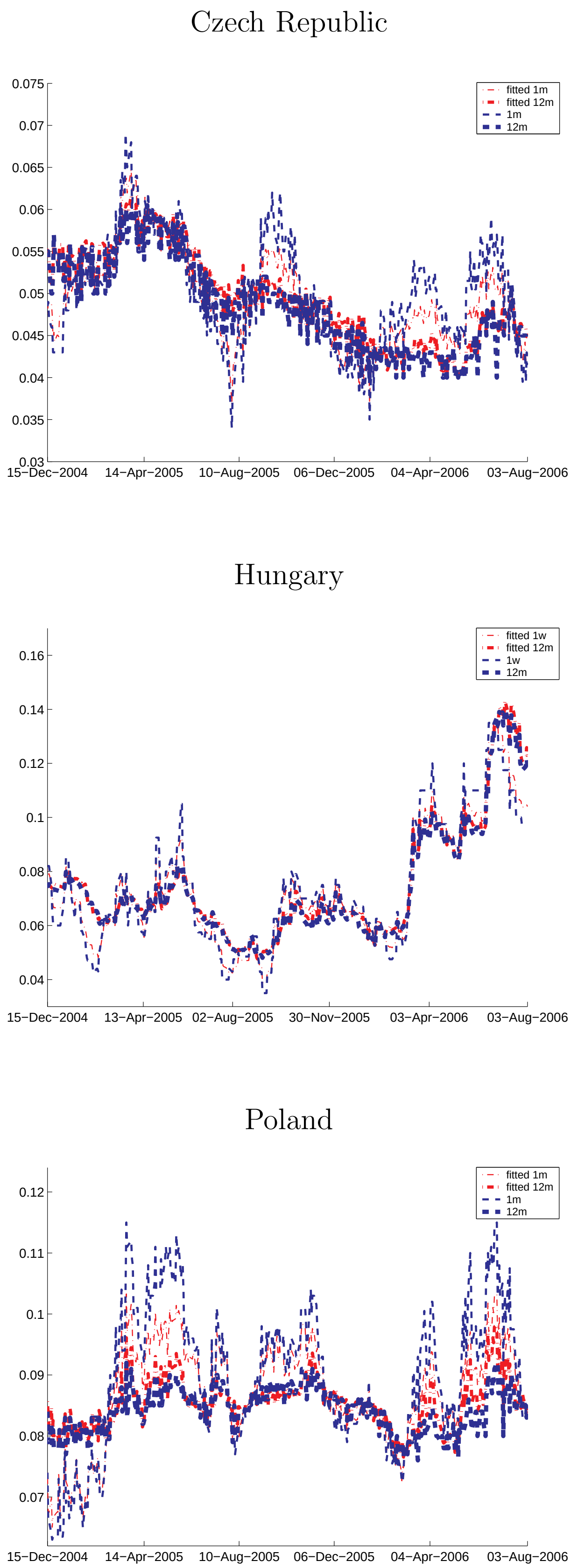
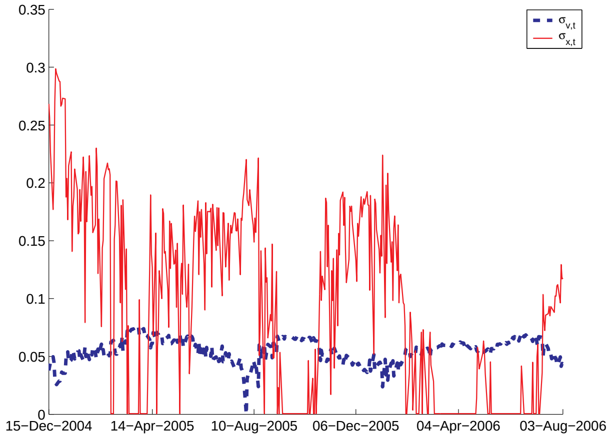
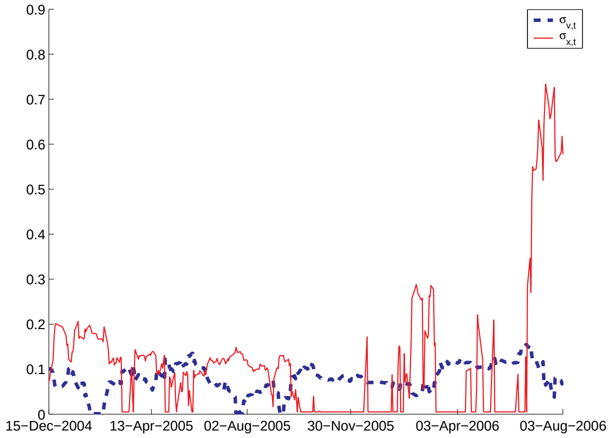


Figure 2: The option prices with the shortest and longest maturity and the fitted values in terms of volatility

Czech Republic



Hungary



Poland

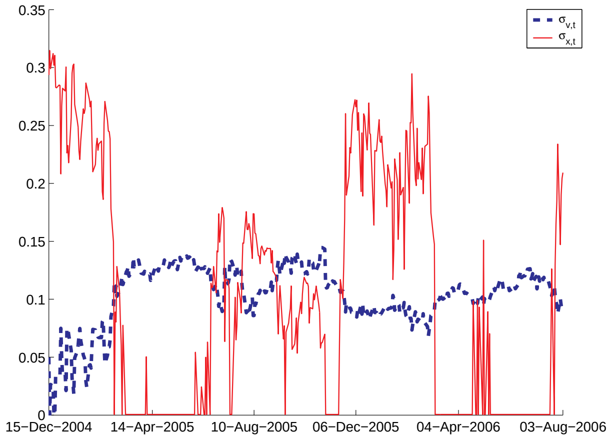
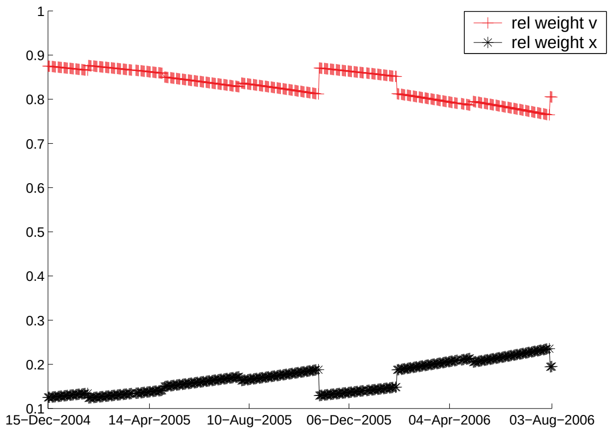


Figure 3: The estimated volatilities of x and v

Czech Republic



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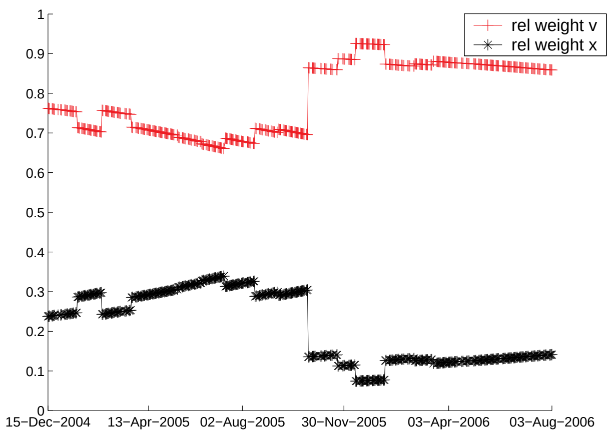

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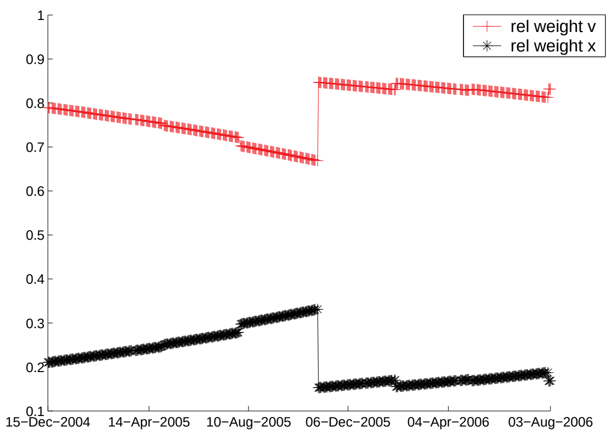
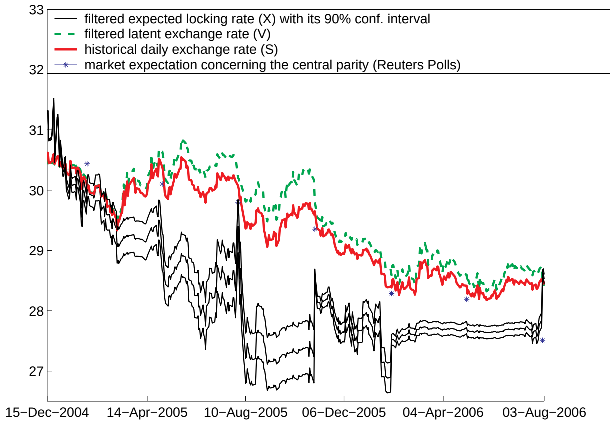
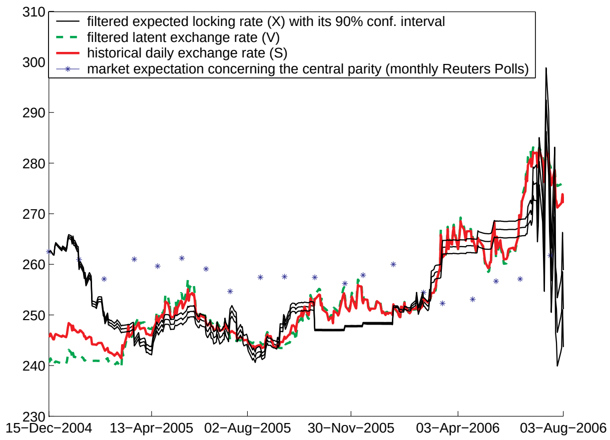


Figure 4: The relative weights of the expected log locking rate (x) and of the log latent exchange rate (v) in the log exchange rate (s)

Czech Republic



Hungary



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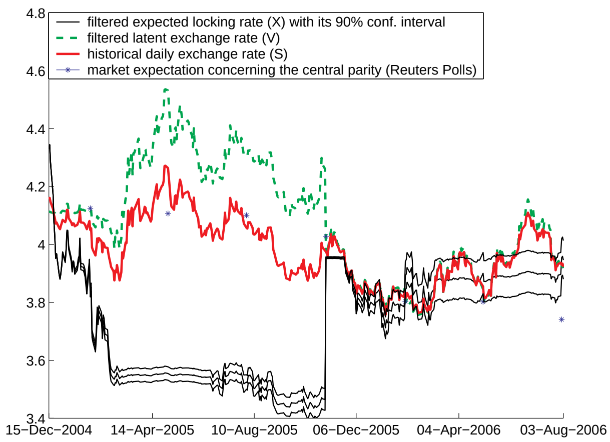

Figure 5: The filtered market expectation concerning the final conversion rate

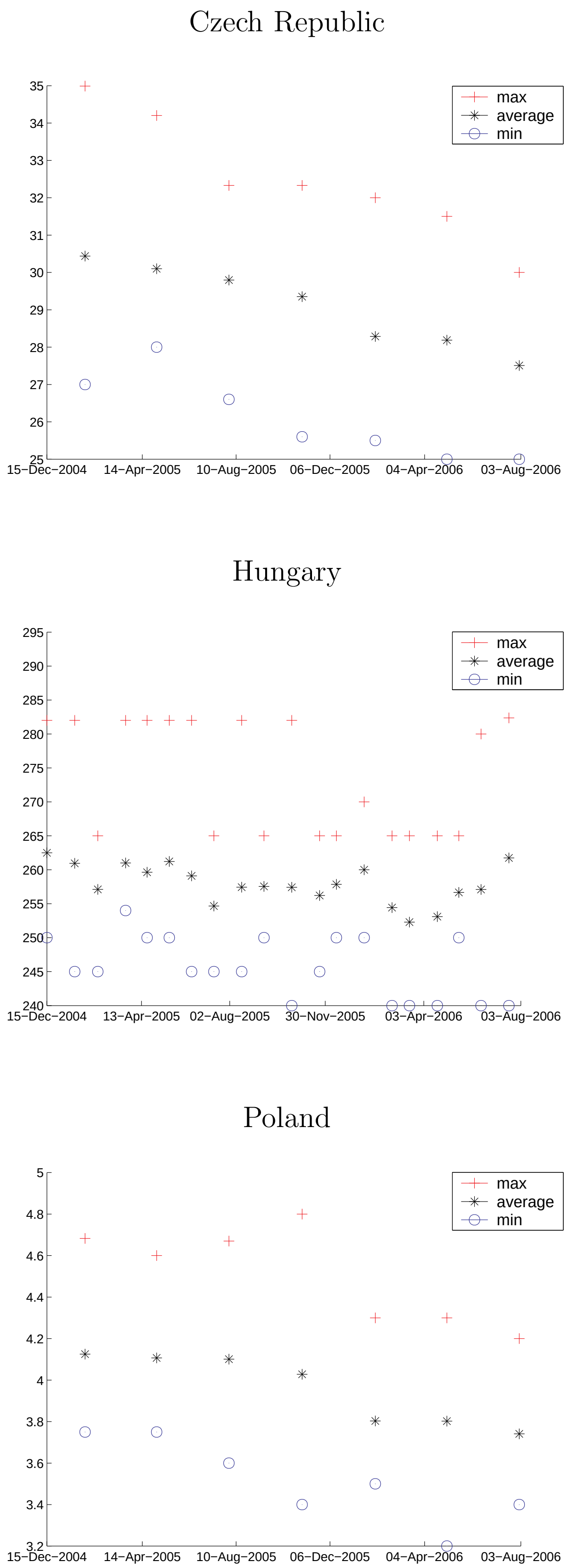

Figure 6: Minimum, maximum and average expectation of the market analysts concerning the central parity in the ERM2 (Reuters polls)