Debt Maturity without Commitment*

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Abstract

This paper analyzes how sovereign risk paired with social costs of default shape the government debt maturity structure. Governments balance benefits of default induced redistribution and costs due to income losses in the wake of a default. Their choice of short- versus long-term debt issuance affects default and rollover decisions by subsequent policy makers whose price impact gives rise to revenue effects on inframarginal units of debt. When considering whether to issue additional debt of a particular maturity, the government weighs the benefits of smoothing disposable income and the costs due to these revenue effects. Consistent with the evidence, closed-form solutions of the model predict an interior maturity structure with positive gross positions and a shortening of the maturity structure when debt issuance is high or output low. In simulations, the model replicates additional features of the data.

Keywords: Debt; maturity structure; no commitment; default.
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1 Introduction

Sovereign borrowers exert considerable effort to structure their debt maturities optimally. This is difficult to reconcile with a frictionless benchmark model in which the equilibrium

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allocation concurs with net financial positions while gross financial positions and the maturity structure are indeterminate (Modigliani and Miller, 1958; Barro, 1974). Existing theories (discussed below) point to a role of government debt maturity in avoiding "bad" equilibria with rollover crises or in improving insurance possibilities for the government. However, the predictions of these theories are not robust or not in line with the empirical evidence, leading Faraglia, Marcet and Scott (2008, p. 28) to conclude that "[w]e remain in search of a plausible theory of debt management."

In this paper, we pursue an alternative explanation for borrowers' scrupulous choice of maturity, arguing that lack of commitment paired with social costs in the wake of a government default undermines the neutrality of the maturity structure. Focusing on these two factors is natural given that a large literature concerned with sovereign borrowing emphasizes the pervasiveness of limited contract enforceability and the significant social costs in the aftermath of defaults.[1] The implications of this alternative explanation turn out to be consistent with the evidence.

We consider a government issuing real non-contingent debt of various maturities to investors on the international financial market. Successive governments (or selves of the government) decide whether, and to what extent, to honor maturing debt. They also choose the level of taxation and debt issuance to finance debt repayment and government purchases. The desire to redistribute from foreign bondholders to domestic taxpayers creates an incentive for the government to default.[2] The wish to avoid the costs of a default which take the form of income losses for taxpayers creates a counteracting incentive to repay maturing debt. Both bondholders and the government form rational expectations. The price of a debt maturity therefore reflects its expected repayment rate, and government policy is subgame perfect.

In equilibrium, the risk-adjusted returns on short- and long-term funding are identical and the maturity structure is determined on the demand side. In particular, it is critically shaped by revenue effects on inframarginal units of debt. A direct consequence of lack of commitment, these revenue effects arise because debt issuance affects the default and rollover choices of subsequent governments and thus, the prices of maturities currently issued. When considering whether to sell additional debt of a certain maturity, a government weighs the benefits from smoothing taxpayers' disposable income and consumption across periods and states—which depend on the state contingent equilibrium repayment rates—and the costs due to the associated revenue effects.

To understand the implications of this tradeoff for the equilibrium maturity structure, we consider first a version of the model that can be solved in closed form. In this version, the revenue effects on inframarginal units of debt relative to the consumption smoothing benefits from the marginal unit of any specific maturity are convex. As a consequence,

[1] See Eaton and Fernandez (1995) for an overview over the literature and Reinhart and Rogoff (2004), Sturzenegger and Zettelmeyer (2006, pp. 49–52) or Panizza, Sturzenegger and Zettelmeyer (2009), among many others, for a discussion of the costs of sovereign defaults.

[2] The incentive to default might alternatively derive from the government's desire to transfer funds from the private to the public sector, in order to avoid tax distortions. Focusing on the redistributive motive is attractive for two reasons. On the one hand, it appears empirically relevant, see the discussion later in the text. On the other hand, abstracting from tax distortions allows to disregard a second source of time inconsistency, related to the optimal timing of taxes (Lucas and Stokey, 1983).

the equilibrium maturity structure is interior and smoothes cost-benefit ratios across maturities, in parallel with the familiar "tax (distortion) smoothing" prescription (Barro, 1979; Lucas and Stokey, 1983). While the latter specifies a Ramsey tax sequence and associated net government debt sequence that minimizes the detrimental effects of tax distortions, the "maturity smoothing prescription" defines the gross positions of each maturity (and thus, net debt positions and taxes) that maximize welfare when lack of commitment is binding.

The exact shape of the equilibrium maturity structure depends on the distribution function of income losses in the wake of a default. With an exponential distribution function, the cost-benefit ratios are symmetric across maturities and the maturity smoothing prescription therefore implies a fully balanced maturity structure. With any other distribution function satisfying a regularity condition, the equilibrium maturity structure generally is tilted towards the long end. Driving this result is the interaction between the two manifestations of lack of commitment. On the one hand, the ex-post choice of repayment rate which causes the revenue effects on inframarginal units and on the other hand, the ex-post choice of new debt issuance which affects the size of these revenue effects.

Due to the convexity of the cost-benefit ratio, a higher amount of inherited, outstanding debt leads a government to reduce its issuance of short-term debt (the second manifestation). Long-term debt issuance therefore increases the amount of debt maturing in the long term by less than one-to-one, in contrast with one-period debt issuance which results in a one-to-one increase in the amount of debt coming due in the subsequent period. Ceteris paribus, long-term debt issuance then has a smaller price impact than short-term debt issuance, due to the tight connection between the amount of debt coming due in a period and the default risk in that period (the first manifestation). This smaller price impact is reflected in smaller revenue effects on inframarginal units and thus, an advantageous cost-benefit ratio of long-term debt issuance. As a result, the equilibrium maturity structure is tilted towards the long end.

Higher quantities of debt reduce this cost advantage of long-term debt because they lessen the extent to which a successor government's debt issuance responds to the amount of outstanding debt. High debt-to-GDP ratios therefore go hand in hand with a more balanced maturity structure. This has implications for the government's portfolio over the cycle: In periods of high marginal utility, total debt issuance increases and the maturity structure shortens.

Output volatility tends to lengthen the equilibrium maturity structure as well. When output is low and marginal utility high, governments find it optimal to issue more debt. Since this increases the risk of default in the future, output is positively correlated with the price of newly-issued and outstanding debt. Ex ante, long-term debt therefore provides a useful hedge for the government since its return correlates positively with output. As a result, governments issue more long-term debt if the environment is becoming riskier.

Being unable to commit, a government cannot force its successors to pay a certain rate of return, including zero. Debt acceleration and cross-default on *outstanding* debt therefore is an equilibrium outcome—not a choice by the government defaulting on *maturing* debt. In the model, this equilibrium outcome occurs randomly, due to an exogenous shock that makes it costless for subsequent governments to default on currently outstanding debt if the current government defaults on maturing debt. Default choices when cross default is feasible depend on the quantities of maturing and outstanding debt. Ex ante, debt issuance therefore triggers revenue effects on inframarginal units across all maturities. In the special case where the probability of acceleration conditional on a default equals one and the environment is deterministic, the cost-benefit ratio of long-term debt always exceeds the corresponding ratio of short-term debt, implying that the maturity structure is concentrated on the short end.

The broad picture that emerges from the model's closed-form solutions is one of an interior maturity structure with positive gross positions, in line with the empirical evidence, but in contrast with predictions from models that stress the role of the maturity structure in completing markets or avoiding bad equilibria with rollover crises (see below). The model predicts a shortening of the maturity structure when debt issuance is high, in line with evidence summarized by Rodrik and Velasco (1999); around times of low output ("crises"), consistent with the evidence reported by Broner, Lorenzoni and Schmukler (2007); and in periods with low output volatility.[3]

Simulations of the general model that needs to be solved numerically corroborate the theoretical predictions and show that they are robust. Moreover, the simulations match stylized facts. If subjected to a cyclical output process with developing-country like volatility, the model generates equilibrium dynamics of the maturity structure and the spreads of different maturities that are qualitatively in line with the data. It also generates a realistic default frequency although this frequency is not targeted in the process of calibrating the model.

The model of this paper is silent about the choice of maturity structure in countries whose debt is perceived to be default-risk free.[4] In the aftermath of the recent turmoil on financial markets and the related deterioration of government budgets, the number of such countries is shrinking as indicated by sovereign bond ratings.[5] Credibility problems therefore are likely to bear on the maturity structure in a wide range of developing and developed economies.

As discussed in the paper, revenue effects on inframarginal units of debt constitute an inherent feature of these credibility problems. Closely related to these revenue effects, previous literature has emphasized the role of debt dilution. In particular, it has been pointed out that debt issuance reduces the value of outstanding debt and that this effect may increase governments' incentives to issue debt ex post.[6] In contrast, the revenue

[3]According to Rodrik and Velasco (1999), "the overall debt burden (debt/GDP ratio) is positively correlated with short-term borrowing in the time-series (but not in the cross-section)" (p. 21). According to Broner et al. (2007) "emerging economies issue relatively more short-term debt during periods of financial turmoil, and wait for tranquil times to issue longterm debt" (p. 3).

[4]Debt structure in those countries may be affected by liquidity concerns. The UK Debt Management Office "argues that cost is not the only factor. There is a virtue in being predictable, and in keeping all sections of the bond market supplied with debt to trade" (*The Economist*, "Losing interest," June 14th 2008).

[5]See, for example, *The Economist*, "Rate and see," December 12th 2009.

[6]Dilution may be present even if outstanding debt is prioritized. For example, Bizer and DeMarzo (1992) analyze a model where increased borrowing leads a borrower to take actions that lower the probability of repayment.

effects of interest in the present paper arise with respect to contemporaneously issued debt and are fully internalized by the government seeking funding. Moreover, incentives to dilute are absent. Because of the social losses in the wake of a default, the difference in the market value of outstanding debt due to debt issuance ex post is not transferred to new lenders.

Importantly, these results follow under entirely standard premises. For example, the assumption that debt contracts stipulate non-contingent payments and failure to make these payments triggers social losses is standard, presumably reflecting informational constraints that prevent sovereign borrowers from entering into more sophisticated financial arrangements. The present paper does not address the reasons for such constraints, nor does it question other central tenets in the sovereign debt literature, in particular lack of commitment. Instead, the paper maintains this standard set of assumptions and analyzes the determinants of sovereign debt maturity within their context.

Related Literature Lack of commitment and the associated difficulty to sustain borrowing take center stage in the sovereign debt literature.[7] Eaton and Gersovitz (1981) suggest that the threat of financial autarky discourages strategic default. Bulow and Rogoff (1989*b*), Grossman and Han (1999), Kletzer and Wright (2000) and Ljungqvist and Sargent (2004, ch. 19), among many others, discuss this hypothesis and the role that the set of available financial instruments plays in it. Cole and Kehoe (1998) and Sandleris (2006) argue that a sovereign default serves as a negative signal, inducing parties outside of the credit relationship to initiate actions that are costly for the government. Tabellini (1991), Dixit and Londregan (2000), Kremer and Mehta (2000), Gonzalez-Eiras (2003), Niepelt (2004) or Guembel and Sussman (2009) argue that distributive motives can counteract a sovereign's incentive to default. More direct default costs of the type considered here are present, for example, in the models of Bulow and Rogoff (1989*a*), Bulow and Rogoff (1989*b*), Cole and Kehoe (2000), Aguiar and Gopinath (2006) and Arellano (2008).[8]

To motivate an optimal maturity structure, some authors suggest that short-term debt renders a country vulnerable to rollover crises, and that long-term debt reduces such vulnerability (Calvo, 1988; Alesina, Prati and Tabellini, 1990; Giavazzi and Pagano, 1990; Rodrik and Velasco, 1999; Cole and Kehoe, 2000). However, Chamon (2007) shows that a simple mechanism is able to eliminate the coordination failure associated with rollover crises. Phelan (2004) draws a distinction between the maturity of debt and the sequencing of debt rollovers which matters for such crises. Broner et al. (2007) argue that supply side features induce emerging markets to borrow short-term in spite of the increased risk of a rollover crisis. In their three-period model, lenders are risk averse and heavily exposed to the intermediate-period price risk of long-term sovereign debt. Higher quantities of long-term debt therefore drive up term premia and thus, the costs of long-term funding.

Lucas and Stokey (1983) characterize the Ramsey tax policy in a closed economy where the government has access to state contingent debt. They show that, due to general

[7]Kydland and Prescott (1977) and Fischer (1980) discuss the government's ex-post incentive to default when taxes are distorting.

[8]See also Tirole (2006, p. 180) where a default might trigger a costly loss of social capital.

equilibrium effects, a complete set of maturities allows to implement the Ramsey policy even if the government can only commit to debt repayment and not to taxes. Policy deviations ex post would affect interest rates, devaluing some debt maturity positions and appreciating others. An appropriate choice of maturity structure ex ante then allows the government to balance the benefits and costs of policy changes ex post and thus, to render the Ramsey policy sustainable. Abstracting from time-consistency issues, Bohn (1990) emphasizes the general equilibrium insurance benefits of non-state contingent long-term debt and Angeletos (2002) shows that a sufficiently rich maturity structure of non-contingent bonds may serve as substitute for state-contingent debt (see also Gale, 1990). However, as documented by Faraglia et al. (2008), the quantitative implications of this "complete market approach" are at odds with the data. Nosbusch (2008) shows that a tax smoothing policy very similar to the one under complete markets can be sustained with only few maturities. Similar to Faraglia et al. (2008), the basic prescription for the government in Nosbusch's (2008) model is to borrow long and invest short, in contrast with the positive short- and long-term debt positions observed in the data.

Closer in spirit to the present paper, Calvo and Guidotti (1990) and Missale and Blanchard (1994) discuss the role of the maturity structure of nominal debt for the government's incentive to engineer surprise inflation. Hatchondo and Martinez (2009) analyze numerically how the duration of government debt affects debt issuance, default choices and risk premia (see also Chatterjee and Eyigungor, 2010). Arellano and Ramanarayanan (2010) numerically solve a model with two bonds of unequal duration and other elements to match empirical bond spreads and portfolio choices.[9] Finally, a large literature in corporate finance analyzes the role of commitment problems for the financial structure of firms, see Tirole (2006) for an overview and Jeanne (2004) for an application in the sovereign debt context.

Outline The remainder of the paper is structured as follows. Section 2 presents the model and Section 3 shows how lack of commitment paired with default induced social losses introduces a role for the debt maturity structure. Sections 4 and 5 contain the analytical and numerical results, respectively. Section 6 concludes.

2 Model

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The small open economy is inhabited by a representative taxpayer and a government that interacts with foreign investors. The government levies taxes, τ_t, chooses the repayment rate on maturing debt, r_t, and issues

[9] Arellano and Ramanarayanan (2010) assume that default triggers cross-default, income losses proportional to the realization of output as well as temporary exclusion from capital markets. They also posit an asset pricing kernel of international investors that is stochastic (this assumption on its own can generate a determinate maturity structure) and correlated with output in the borrowing country, and they assume that the recovery rate after defaults depends on outstanding debt. In an example with three periods, risk-neutral investors, permanent and complete loss of output after a default and without several of the features listed above, the maximal deficit is attained by issuing both short- and long-term debt. No general results are derived.

zero-coupon debt of maturities 1 to M, $\{b_{t,s}\}_{s=t+1}^{t+M}$, where the first and second subscript denotes the issuance and maturity date, respectively. Vector ι_t summarizes the government's debt issuance in period t, $\iota_t \equiv (b_{t,t+1}, \ldots, b_{t,t+M})$. Without loss of generality, government spending other than debt repayment is normalized to zero.

2.1 Private Sector

Taxpayers do not save nor borrow.[10] They have time- and state-additive preferences over consumption with strictly increasing and concave felicity function $u(\cdot)$ and discount factor $\delta \in (0, 1)$. Welfare of taxpayers in period t is given by

$$\mathbb{E}\left[\sum_{s \geq t} \delta^{s-t} u(y_s^p - \tau_s)|s_t, r_t, \iota_t\right],$$

where y_t^p denotes pre-tax income and s_t denotes the state, to be specified below.

Foreign investors are competitive, risk neutral and require a riskfree gross interest rate $\beta^{-1} > 1$. Since taxpayers do not save, all government debt is held by foreign investors. To guarantee positive debt positions, we assume $\delta \ll \beta$.

The assumption that the sets of taxpayers and investors do not "overlap" is unimportant for the central results but simplifies the analysis; modeling a mixed rather than concentrated ownership structure of debt would require a theory of how this ownership structure is determined in equilibrium.[11]

2.2 Government

The government maximizes the welfare of taxpayers.[12] Crucially, it cannot commit its successors (or future selves). In each period, the government therefore chooses debt issuance as well as the uniform (pari passu) repayment rate on all maturing debt, $b_{x,t} \equiv \sum_{s=t-M}^{t-1} b_{s,t}$. Taxes follow residually from the government's dynamic budget constraint.

2.3 Default Costs

A government default—a situation where the repayment rate falls short of unity—triggers temporary income losses for taxpayers (cf. Eaton and Gersovitz, 1981; Cole and Kehoe,

[10] Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with "savers" and "spenders."

[11] The government's default decision depends on the ownership structure of debt relative to the distribution of tax burdens across the population, see below. Changes in the ownership structure therefore affect the default decision ex post and thus, investment decisions ex ante.

Tabellini (1991) and Dixit and Londregan (2000) provide theories of the ownership structure of debt. They assume that households can only save in government debt (Tabellini, 1991), or that the return on the only alternative asset is household specific (Dixit and Londregan, 2000). Both assumptions are not applicable in the current context. See also Niepelt (2004).

[12] If the government maximized a weighted average of taxpayers' and investors' welfare and attached a sufficiently large weight to the welfare of investors, interior repayment rates might result, in contrast to what follows. If the government attached a strictly positive weight to the welfare of investors and if investors were risk averse, investor wealth would constitute a state variable, in contrast to what follows.

2000; Aguiar and Gopinath, 2006; Arellano, 2008). More specifically, a default in period t triggers an income loss $L_t \geq 0$ where L_t is the realization of an i.i.d. random variable with cumulative distribution function $F(\cdot)$ and associated density function $f(\cdot)$, $f(L) > 0$ for all $L \geq 0$. The government learns about the realization of L_t at the beginning of the period, before choosing its policy instruments. Pre-tax income of taxpayers is given by $y_t^p = y_t - \mathbf{1}_{[r_t<1]} L_t$ where y_t denotes a realization of the exogenous stochastic output process in period t and $\mathbf{1}_{[x]}$ denotes the indicator function for event x.

The assumption of temporary rather than persistent income losses is motivated by two considerations. First, temporary default costs constitute a natural benchmark.[13] Second, and more importantly, the assumption of temporary losses is more plausible. In particular, while permanent exclusion from trade or credit markets and other forms of long-term punishment may serve as threat points they are unlikely to materialize in equilibrium if the parties renegotiate.[14] Empirical evidence supports the notion of temporary rather than permanent default costs as well as the notion that these costs arise in the form of output losses (cf., for example, Panizza et al., 2009).[15]

2.4 Cross Default and Debt Accumulation

Being unable to commit, a government cannot force its successors to pay a certain rate of return, including zero. This implies that a government may not *directly* default on outstanding debt. Indirectly, however, such a cross default may arise. In particular, the random variable cd_t takes the value 1 with probability $\pi \in [0, 1]$ and the value 0 with probability $1 - \pi$. If $\mathrm{cd}_t = 1$, then a default on debt maturing in period t (carrying income losses L_t) reduces to zero the costs for subsequent governments of defaulting on debt outstanding in period t. If $\mathrm{cd}_t = 1$, a default on maturing debt therefore triggers a devaluation of outstanding debt as well since investors know that future governments will find it in their interest to default on the latter.[16] Letting $b_{x,t,t+s}$ denote the amount of debt outstanding in period t and maturing in period $t + s, 0 \leq s \leq M - 1$ (with $b_{x,t,t} = b_{x,t}$), the law of motion for the debt maturities is given by

$$b_{x,t+1,t+s} = b_{x,t,t+s}(1 - \mathbf{1}_{[r_t<1\ \&\ \mathrm{cd}_t=1]}) + b_{t,t+s},\ 1 \leq s \leq M. \tag{1}$$

Equation (1) states that the stock of debt outstanding in period $t + 1$ and maturing in period $t + s$ is given by the debt outstanding in period t and maturing in period $t + s$

[13]The results of this paper remain valid under the assumption of permanent default costs if these costs do not interact with future debt issuance and repayment rate decisions. This is the case, for example, if the utility function is linear.

[14]Suppose, for example, that upon defaulting the sovereign enters into negotiations with creditors. These negotiations last one period, generating income losses L_n, and result in a settlement where lenders secure a repayment rate $\bar{r}_t$. The analysis in this paper is consistent with this interpretation; for simplicity, it sets $\bar{r}_t = 0$.

[15]According to Panizza et al. (2009, p. 692), "[c]apital exclusion periods [in the wake of a default] are brief; effects on the cost of borrowing are temporary and small ... defaulting debtors have been able to issue new debt domestically (including to foreign investors) at relatively low cost. If anything, defaults appear to be deterred by the domestic collateral damage that tends to accompany debt crises".

[16]Alternatively, the cross default can be interpreted as a debt buyback at very low prices that reflect equilibrium expectations of subsequent governments' default decisions.

as well as the s-period-maturity debt issued in period t. However, in case of default the former component vanishes if $\mathrm{cd}_t = 1$. The random variables y_t, cd_t and L_t are pairwise independent.

While the "cross-default shock" cd_t allows to reconcile the assumption of no commitment with the equilibrium occurrence of cross default, its stochastic specification allows to capture the different extent of cross default and restructuring across default episodes, due to varying incentives of the sovereign and certain creditors to delay or prevent acceleration.[17] The latter include lenders that seek to avoid an immediate deterioration of their balance sheet as otherwise implied by mark-to-market regulation, or the government itself if it purchased the country's debt on the secondary market.[18] In addition to such institutional factors, the structure of government debt securities if likely to affect the extent of cross default too. For example, zero coupon bonds might be expected to be less exposed to cross-default risk than the coupon payments of a single console.

2.5 Equilibrium

Apart from time (in the finite-horizon case), the state in this economy is given by the realizations $(y_t, \mathrm{cd}_t, L_t)$ as well as the quantities of maturing and outstanding debt:

$$s_t = (y_t, \mathrm{cd}_t, L_t, b_{x,t}, \{b_{x,t,t+s}\}_{s=1}^{M-1}).$$

Throughout the paper, we exclude non-fundamental state variables of the type sustaining trigger strategies.

Denote by $q_{t,s}(s_t, r_t, \iota_t)$ the price of debt issued in period t state s_t and maturing in period s if the government implements the policy (r_t, ι_t). All governments in period t and earlier take the price functions $\{q_{t,s}(\cdot)\}_{s=t+1}^{t+M}$ as given when choosing their policies. Define the deficit in period t as the market value of debt issued in period t,

$$d_t(s_t, r_t, \iota_t) \equiv \sum_{s=t+1}^{t+M} b_{t,s}\ q_{t,s}(s_t, r_t, \iota_t).$$

The dynamic budget constraint of the government, $\tau_t = b_{x,t} r_t - d_t(s_t, r_t, \iota_t)$, implies that period-$t$ consumption of taxpayers, c_t, is given by $c_t = y_t - \mathbf{1}_{[r_t<1]} L_t - b_{x,t} r_t + d_t(s_t, r_t, \iota_t)$.

Let $G_t(s_t)$ denote the value of the government's program conditional on the state s_t and let $\mathcal{I}$ denote a bounded set such that equilibrium choices of ι_t lie in the interior of this set in all periods. An equilibrium is given by price functions $\{q_{t,s}(\cdot)\}_t$, value functions $\{G_t(\cdot)\}_t$, and policy functions $\{r_t(\cdot), \iota_t(\cdot)\}_t$ (of s_t) such that

i. conditional on the price functions, the value and policy functions solve

$$\begin{aligned} G_t(s_t) \quad = \quad & \max_{r_t \in [0,1], \iota_t \in \mathcal{I}} u(y_t - b_{x,t} r_t - \mathbf{1}_{[r_t<1]} L_t + d_t(s_t, r_t, \iota_t)) + \delta \mathbb{E}\left[G_{t+1}(s_{t+1}) | s_t, r_t, \iota_t\right] \\ & \text{s.t. (1) for all } s_t, t; \end{aligned}$$

[17] Acceleration of bonds often requires support by creditors representing a significant share (typically 25 percent) of the outstanding bonds.

[18] See Buchheit (2009) for a discussion in the context of Ecuador's sovereign bond default.

ii. the price functions reflect rational expectations by investors,

$$q_{t,t+s}(s_t, r_t, \iota_t) = \beta^s \, \mathbb{E}\left[\prod_{i=t+1}^{t+s-1}(1-\mathbf{1}_{[r_i(s_i)<1 \text{ \& } \mathrm{cd}_i=1]})r_{t+s}(s_{t+s})|s_t, r_t, \iota_t\right] \quad (2)$$
$$\text{s.t. (1) for all } s_t, r_t \in [0,1], \iota_t \in \mathcal{I}, t, 1 \leq s \leq M.$$

The second condition states that in equilibrium, investors earn the required rate of return. This insulation from the effects of government policy contrasts with the exposure of domestic taxpayers whose disposable income depends on taxes and income losses in the wake of defaults. According to the first condition, the benevolent government chooses the repayment rate and the issuance of new debt in order to minimize the detrimental effects of these taxes and income losses, taking into account how subsequent governments respond ex-post optimally to these choices.

3 Policy Choices

We focus on the case of two maturities, $M = 2$. Short-term debt matures after one period, long-term debt after two. Accordingly, the state is given by $s_t = (y_t, \mathrm{cd}_t, L_t, b_{x,t}, b_{x,t,t+1})$.

3.1 Debt Repayment

Consider first the government's choice of repayment rate, r_t. Since the marginal cost of reducing r_t equals zero for $r_t < 1$, the optimal repayment rate equals either zero or unity. The threshold value of L_t at which the repayment rate changes depends on cd_t. In particular,

$$r_t(s_t) = \begin{cases} 1 & \text{if } L_t - b_{x,t} \geq \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) \cdot \mathrm{cd}_t \\ 0 & \text{if } L_t - b_{x,t} < \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) \cdot \mathrm{cd}_t \end{cases} . \quad (3)$$

If $\mathrm{cd}_t = 0$, the government's repayment choice maximizes $y_t - b_{x,t}r_t - \mathbf{1}_{[r_t<1]}L_t + d_t(s_t, r_t, \iota_t)$ and the right-hand side of the inequalities in (3) equals zero. If $\mathrm{cd}_t = 1$, in contrast, then the choice of repayment rate affects the evolution of outstanding debt in equation (1) and the right-hand side of the inequalities in (3) is given by $\alpha_t(y_t, b_{x,t}, b_{x,t,t+1})$ where the function $\alpha_t(\cdot)$ is defined by the indifference condition

$$u(y_t - b_{x,t} + d_t(s_t, 1, \iota_t(s_t))) + \delta\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 1, \iota_t(s_t)] \equiv$$
$$u(y_t - b_{x,t} - \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) + d_t(s_t, 0, \tilde{\iota}_t(s_t))) + \delta\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 0, \tilde{\iota}_t(s_t)] \text{ if } \mathrm{cd}_t = 1 \quad (4)$$

and where $\alpha_t(\cdot)$ is positive for $b_{x,t,t+1} \geq 0$.[19]

[19] Equilibrium debt issuance may differ depending on whether the government defaults or not, thus the distinction between $\iota_t(s_t)$ and $\tilde{\iota}_t(s_t)$. To see that $\alpha(\cdot)$ is positive for $b_{x,t,t+1} \geq 0$, note that

$$\begin{aligned} & u(y_t - b_{x,t} + d_t(s_t, 1, \iota_t(s_t))) + \delta\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 1, \iota_t(s_t)] \\ \equiv\ & u(y_t - b_{x,t} - \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) + d_t(s_t, 0, \tilde{\iota}_t(s_t))) + \delta\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 0, \tilde{\iota}_t(s_t)] \\ \geq\ & u(y_t - b_{x,t} - \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) + d_t(s_t, 0, \iota_t(s_t))) + \delta\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 0, \iota_t(s_t)] \text{ if } \mathrm{cd}_t = 1. \end{aligned}$$

Condition (3) states that a government defaults when the income losses L_t are relatively small. This is consistent with the notion that governments tend to default when the political costs—specifically income losses of pivotal pressure groups—are low.[20] Governments also tend to default when economic activity is depressed (Borensztein, Levy Yegati and Panizza, 2006; Tomz and Wright, 2007). The model is consistent with this fact as well if it is slightly extended to include direct default costs for the government in addition to the income losses for taxpayers.[21] As discussed in Appendix A, corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those invoked here.[22]

Equation (3) pins down expected repayment rates and thus, equilibrium prices of newly-issued debt. To streamline notation, let $1 - F_t^{\text{cd}=0} \equiv 1 - F(b_{x,t})$, $1 - F_t^{\text{cd}=1} \equiv 1 - F(b_{x,t} + \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}))$, and let $1 - F_t$ denote either $1 - F_t^{\text{cd}=0}$ or $1 - F_t^{\text{cd}=1}$, depending on the realization of cd_t. Similarly, let $f_t^{\text{cd}=0} \equiv f(b_{x,t})$ etc. Moreover, let $S_t \equiv (s_t, r_t(s_t), \iota_t(s_t))$, $q_{t,t+1}([S_t]) \equiv q_{t,t+1}(s_t, r_t(s_t), \iota_t(s_t))$, $d_t([S_t]) \equiv d_t(s_t, r_t(s_t), \iota_t(s_t))$ and, for future reference, $u'([S_t]) \equiv u'(y_t - b_{x,t}r_t(s_t) - \mathbf{1}_{[r_t(s_t)<1]}L_t + d_t([S_t]))$. From (2), the price of short-term debt then equals

$$\begin{aligned} q_{t,t+1}(s_t, r_t, \iota_t) &= \beta\mathbb{E}[r_{t+1}(s_{t+1})|s_t, r_t, \iota_t] = \beta\mathbb{E}[1 - F_{t+1}|s_t, r_t, \iota_t] \\ &= \beta\left(\pi\mathbb{E}[1 - F_{t+1}^{\text{cd}=1}|s_t, r_t, \iota_t] + (1-\pi)(1 - F_{t+1}^{\text{cd}=0}|s_t, r_t, \iota_t)\right). \qquad (5) \end{aligned}$$

The price of long-term debt reflects "cross-default risk" in the subsequent period as well as default risk in the long term. Condition (2) and pari passu imply[23]

$$\begin{aligned} q_{t,t+2}(s_t, r_t, \iota_t) &= \beta^2\mathbb{E}[(1 - \mathbf{1}_{[r_{t+1}(s_{t+1})<1 \ \& \ \text{cd}_{t+1}=1]})r_{t+2}(s_{t+2})|s_t, r_t, \iota_t] \\ &= \beta\mathbb{E}[(1 - \mathbf{1}_{[r_{t+1}(s_{t+1})<1 \ \& \ \text{cd}_{t+1}=1]})q_{t+1,t+2}([S_{t+1}])|s_t, r_t, \iota_t] \\ &= \beta\pi\mathbb{E}\left[(1 - F_{t+1}^{\text{cd}=1})q_{t+1,t+2}([S_{t+1}])|\text{cd}_{t+1} = 1, L_{t+1} = \infty, s_t, r_t, \iota_t\right] \\ &\quad +\beta(1-\pi)\mathbb{E}\left[q_{t+1,t+2}([S_{t+1}])|\text{cd}_{t+1} = 0, s_t, r_t, \iota_t\right]. \qquad (6) \end{aligned}$$

Without cross-default risk, $\pi = 0$ and

$$\begin{aligned} q_{t,t+1}(s_t, r_t, \iota_t) &= \beta(1 - F(b_{x,t,t+1} + b_{t,t+1})), \\ q_{t,t+2}(s_t, r_t, \iota_t) &= \beta^2\mathbb{E}[1 - F(b_{t,t+2} + b_{t+1,t+2}(s_{t+1}))|s_t, r_t, \iota_t]. \end{aligned}$$

Moreover, $d_t(s_t, 1, \iota_t(s_t)) \leq d_t(s_t, 0, \iota_t(s_t))$ if $\text{cd}_t = 1$ since outstanding debt decreases the market price of newly issued debt, see below. It follows that $-b_{x,t} \geq -b_{x,t} - \alpha_t(y_t, b_{x,t}, b_{x,t,t+1})$.

[20] Tomz (2002) documents that domestic audiences opposed the government of Argentina to suspend debt payments in 1999 but supported such action two years later. Kohlscheen (2004) documents that parliamentary democracies rarely resort to rescheduling (despite shorter office terms of their executives), presumably because domestic constituencies opposed to default are more likely to be politically influential in representative democracies. MacDonald (2003) suggests that it is precisely in countries where a default does not generate clearly identifiable winners and losers among politically influential groups where sovereign defaults have been avoided.

[21] If default triggers costs K to the government in addition to the income losses for taxpayers, the default decision (in the case $\text{cd}_t = 0$) reduces to $r_t = 1$ iff $u(y_t - b_{x,t} + d_t) \geq u(y_t - L_t + \tilde{d}_t) - K$. Concavity of $u(\cdot)$ implies that low income levels render a default more likely.

[22] Interior repayment rates could arise if the government attached sufficiently strong weight to the welfare of foreign investors.

[23] If $\text{cd}_{t+1} = 1$ but L_{t+1} is sufficiently large for a default to be avoided then debt issuance is independent of the particular realization of L_{t+1} and expectations can be conditioned on $L_{t+1} = \infty$.

Intuitively, the price of each maturity is decreasing in its quantity because higher debt issuance reduces the probability of repayment. Similarly, higher outstanding debt reduces the price of short-term debt and higher expected short-term debt issuance by the subsequent government reduces the price of long-term debt. If cross-default risk is maximal, in contrast, then $\pi = 1$ and the prices of the two maturities satisfy

$$q_{t,t+2}(s_t, r_t, \iota_t) = q_{t,t+1}(s_t, r_t, \iota_t)\mathbb{E}\left[q_{t+1,t+2}([S_{t+1}])|\mathrm{cd}_{t+1} = 1, L_{t+1} = \infty, s_t, r_t, \iota_t\right] + \Phi_t$$

where Φ_t denotes a covariance term.

We proceed under the assumption that the price functions be differentiable in $(b_{x,t,t+1}, \iota_t)$ and the government's program well behaved such that the policy functions are smooth. Below, when considering special cases of the model, we verify that this is indeed the case.[24]

3.2 Debt Issuance

Issuing debt of a particular maturity has two effects on the deficit. On the one hand, it raises revenue from the marginal unit of debt, in proportion to its price. On the other hand, it affects the revenue raised from inframarginal units of debt, by changing the repayment probabilities and thus, prices of these units. This second effect is a direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent rollover and repayment decisions. Formally,

$$\begin{aligned}
\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}} &= q_{t,t+1}(s_t, r_t, \iota_t) \underbrace{+b_{t,t+1}\frac{\mathrm{d}q_{t,t+1}(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}}}_{\mathcal{R}_{t,ss}} \underbrace{+b_{t,t+2}\frac{\mathrm{d}q_{t,t+2}(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}}}_{\mathcal{R}_{t,sl}}, \\
\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+2}} &= q_{t,t+2}(s_t, r_t, \iota_t) \underbrace{+b_{t,t+1}\frac{\mathrm{d}q_{t,t+1}(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+2}}}_{\mathcal{R}_{t,ls}} \underbrace{+b_{t,t+2}\frac{\mathrm{d}q_{t,t+2}(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+2}}}_{\mathcal{R}_{t,ll}}
\end{aligned}$$

with revenue effects on inframarginal units denoted by $\mathcal{R}_{t,\cdot\cdot}$. For example, $\mathcal{R}_{t,sl}$ denotes the revenue effects on inframarginal long-term debt caused by a marginal increase of short-term debt.[25]

Consider the effect on the government's value of a marginal increase in the stock of maturing debt, given by

$$\frac{\partial G_t(s_t)}{\partial b_{x,t}} = \begin{cases} -u'([S_t]) & \text{if } L_t - b_{x,t} \geq \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) \cdot \mathrm{cd}_t \\ 0 & \text{if } L_t - b_{x,t} < \alpha_t(y_t, b_{x,t}, b_{x,t,t+1}) \cdot \mathrm{cd}_t \end{cases}$$

[24] In general, the objective function need not be concave in debt issuance because higher debt issuance reduces the probability of repayment in the future and because it implies increasingly smaller revenue effects on inframarginal units of debt if the price function is convex.

[25] Negative revenue effects on inframarginal units of debt imply that the funds a government can raise are limited. In particular, the deficit is maximized at the peak of the "debt-Laffer surface" which, in an interior maximum, is attained if the two marginal effects equal zero. If $\delta = 0$, each successive government aims at maximizing the deficit.

and implying

$$\frac{\partial \mathbb{E}[G_{t+1}(s_{t+1})|s_t, r_t, \iota_t]}{\partial b_{x,t+1}} = \mathbb{E}\left[-(1-F_{t+1})u'([S_{t+1}])|L_{t+1} = \infty, s_t, r_t, \iota_t\right].$$

In states where the government repays, higher maturing debt reduces the government's value proportionally to taxpayers' marginal utility because taxes need to be raised at the margin. Adjustments in debt issuance may also occur but they do not have first-order effects on the value, due to an envelope condition.

Consider next the effect on the government's value of a marginal increase in the stock of outstanding debt. In case of cross-default, this marginal effect equals zero since outstanding debt is defaulted upon. Otherwise, it is given by

$$\frac{\partial G_t(s_t)}{\partial b_{x,t,t+1}} = u'([S_t])\frac{\mathrm{d}d_t([S_t])}{\mathrm{d}b_{x,t,t+1}}|_{\text{direct}} + \delta \frac{\partial \mathbb{E}[G_{t+1}(s_{t+1})|S_t]}{\partial b_{x,t+1}} \text{ if } \mathrm{cd}_t = 0 \text{ or } r_t(s_t) = 1.$$

First-order effects of higher outstanding debt only result from induced price changes and thus, revenue effects on inframarginal units of newly-issued debt. (Since the repayment rate and new debt issuance are chosen optimally, indirect welfare effects caused by adjustments in these policy instruments are not of first order.) These revenue effects are identical to those triggered by the issuance of short-term debt,

$$\frac{\mathrm{d}d_t([S_t])}{\mathrm{d}b_{x,t,t+1}}|_{\text{direct}} = \mathcal{R}_{t,ss} + \mathcal{R}_{t,sl} \text{ if } \mathrm{cd}_t = 0 \text{ or } r_t(s_t) = 1.$$

Accordingly

$$\begin{aligned}
&\frac{\partial \mathbb{E}[G_{t+1}(s_{t+1})|s_t, r_t, \iota_t]}{\partial b_{x,t+1,t+2}} = \\
&\quad \pi \mathbb{E}\left[(1-F^{\mathrm{cd}=1}_{t+1})\left\{u'([S_{t+1}])(\mathcal{R}_{t+1,ss} + \mathcal{R}_{t+1,sl}) + \delta \frac{\partial \mathbb{E}[G_{t+2}(s_{t+2})|S_{t+1}]}{\partial b_{x,t+2}}\right\}\right. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad |\mathrm{cd}_{t+1} = 1, L_{t+1} = \infty, s_t, r_t, \iota_t\Big] \\
&+ \quad (1-\pi)\mathbb{E}\left[u'([S_{t+1}])(\mathcal{R}_{t+1,ss} + \mathcal{R}_{t+1,sl}) + \delta \frac{\partial \mathbb{E}[G_{t+2}(s_{t+2})|S_{t+1}]}{\partial b_{x,t+2}}|\mathrm{cd}_{t+1} = 0, s_t, r_t, \iota_t\right].
\end{aligned}$$

With these results at hand, we can derive the welfare effects of debt issuance. Consider first short-term debt. A marginal increase in $b_{t,t+1}$ raises the government's value by

$$u'(c_t)\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}} + \delta \frac{\partial \mathbb{E}[G_{t+1}(s_{t+1})|s_t, r_t, \iota_t]}{\partial b_{x,t+1}}$$

which can be expressed as

$$u'(c_t)(\mathcal{R}_{t,ss} + \mathcal{R}_{t,sl}) + \mathbb{E}[(1-F_{t+1})(\beta u'(c_t) - \delta u'([S_{t+1}]))|L_{t+1} = \infty, s_t, r_t, \iota_t]. \quad (7)$$

This marginal effect consists of two parts. On the one hand, a standard consumption smoothing term on the right-hand side reflecting the fact that debt issuance at price

$\beta\mathbb{E}[(1-F_{t+1})|s_t, r_t, \iota_t]$ allows to shift consumption across periods. On the other hand, a term on the left-hand side reflecting the consequences of revenue effects on inframarginal units of debt.

A direct consequence of lack of commitment, this term on the left-hand side arises because a government's choice of debt issuance alters the subsequent government's choice of repayment rate and debt rollover and thus, current prices and deficit.[26] Note that, in spite of the subsequent government's altered choice of repayment rate and debt rollover, there are no related first-order welfare effects operating through the continuation value. This is a consequence of an envelope condition—the subsequent government is indifferent at the margin between repaying or defaulting and between issuing slightly more or less debt—as well as the congruence of the subsequent government's objective function and the current government's continuation value function. Appendix B analyzes the role played by social rather than private losses in the wake of a default in shaping the welfare consequences of revenue effects on inframarginal units of debt.

Consider next long-term debt. A marginal increase in $b_{t,t+2}$ raises the government's objective by

$$u'(c_t)\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+2}} + \delta\frac{\partial\mathbb{E}[G_{t+1}(s_{t+1})|s_t, r_t, \iota_t]}{\partial b_{x,t+1,t+2}}$$

which can be expressed as

$$\begin{aligned}
& u'(c_t)(\mathcal{R}_{t,ls} + \mathcal{R}_{t,ll}) \\
+ \quad & \pi\mathbb{E}\left[(1-F^{\mathrm{cd}=1}_{t+1})\delta u'([S_{t+1}])(\mathcal{R}_{t+1,ss} + \mathcal{R}_{t+1,sl})|\mathrm{cd}_{t+1}=1, L_{t+1}=\infty, s_t, r_t, \iota_t\right] \\
+ \quad & (1-\pi)\mathbb{E}\left[\delta u'([S_{t+1}])(\mathcal{R}_{t+1,ss} + \mathcal{R}_{t+1,sl})|\mathrm{cd}_{t+1}=0, s_t, r_t, \iota_t\right] \\
+ \quad & \pi\mathbb{E}\left[(1-F^{\mathrm{cd}=1}_{t+1})\mathbb{E}\left[(1-F_{t+2})(\beta^2 u'(c_t) - \delta^2 u'([S_{t+2}]))|L_{t+2}=\infty, S_{t+1}\right]\right. \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left.|\mathrm{cd}_{t+1}=1, L_{t+1}=\infty, s_t, r_t, \iota_t\right] \\
+ \quad & (1-\pi)\mathbb{E}\left[(1-F_{t+2})(\beta^2 u'(c_t) - \delta^2 u'([S_{t+2}]))|L_{t+2}=\infty, \mathrm{cd}_{t+1}=0, s_t, r_t, \iota_t\right].
\end{aligned} \tag{8}$$

Parallel to (7), the marginal effect in (8) consists of consumption-smoothing terms (the last two terms) and terms reflecting the consequences of revenue effects on inframarginal units (the first three terms). In contrast to (7), these revenue effects also arise with respect to debt issued in the subsequent period because its price is affected by the state variables in that period.

If short-term debt issuance in the subsequent period is interior then the government is indifferent between redeeming long-term debt after one period or holding it to maturity. In this case, the marginal effect in (8) can be re-expressed in terms of the return characteristics and revenue effects of long-term debt that is redeemed after one period at price $\beta\mathbb{E}[1-F_{t+2}|s_{t+1}, r_{t+1}, \iota_{t+1}]$. Formally, combining (7) as of period $t+1$ and (8) as of

[26] The interests of the current and the subsequent government may not only be misaligned even debt is issued but also if long-term debt is prematurely redeemed ($b_{t,t+1} < 0$). Premature redemption reduces the likelihood of a default by the subsequent government and thereby raises the price at which the current government buys back its bonds.

period t yields

$$
\begin{aligned}
& u'(c_t)(\mathcal{R}_{t,ls} + \mathcal{R}_{t,ll}) \\
+ \quad & \pi\mathbb{E}\left[(1 - F^{\mathrm{cd}=1}_{t+1})\beta(1 - F_{t+2})(\beta u'(c_t) - \delta u'([S_{t+1}]))|\mathrm{cd}_{t+1} = 1, L_{t+1} = \infty, s_t, r_t, \iota_t\right] \\
+ \quad & (1 - \pi)\mathbb{E}\left[\beta(1 - F_{t+2})(\beta u'(c_t) - \delta u'([S_{t+1}]))|\mathrm{cd}_{t+1} = 0, s_t, r_t, \iota_t\right]
\end{aligned}
$$

which can be expressed as

$$
\begin{aligned}
& u'(c_t)(\mathcal{R}_{t,ls} + \mathcal{R}_{t,ll}) \\
+ \quad & \mathbb{E}\left[(1 - F_{t+1})q_{t+1,t+2}([S_{t+1}])(\beta u'(c_t) - \delta u'([S_{t+1}]))|L_{t+1} = \infty, s_t, r_t, \iota_t\right] \qquad (9) \\
+ \quad & (1 - \pi)\mathbb{E}\left[\int_0^{b_{x,t+1}} q_{t+1,t+2}([S_{t+1}])(\beta u'(c_t) - \delta u'([S_{t+1}]))dF(L_{t+1})|\mathrm{cd}_{t+1} = 0, s_t, r_t, \iota_t\right].
\end{aligned}
$$

The marginal effect of long-term debt issuance in (9) differs twofold from the effect of short-term debt issuance in (7). First, long-term debt issuance generates different revenue effects than short-term debt issuance, represented by $\mathcal{R}_{t,ls} + \mathcal{R}_{t,ll}$ versus $\mathcal{R}_{t,ss} + \mathcal{R}_{t,sl}$, respectively. Second, the consumption-smoothing terms differ across maturities: While short-term debt shifts consumption between the current period and the *repayment states* in the subsequent period, long-term debt shifts consumption between the current period and *states without cross-default* in the subsequent period. Moreover, the *return* per unit of long-term debt in such states without cross-default is given by the price of new short-term debt while the return on a unit of short-term debt equals one.

It is clear from (7) and (9) that a government's preferred maturity structure generally is determinate. This contrasts with the situation in a model with commitment. If the government could commit its successors to honor maturing debt at face value, all revenue effects on inframarginal units in the above expressions would be absent and (7) and (9) would reduce to the consumption smoothing benefits

$$
\begin{aligned}
& q_{t,t+1}u'(c_t) - \delta\mathbb{E}[u'(c_{t+1})|s_t, \iota_t], \\
& q_{t,t+2}u'(c_t) - \delta\mathbb{E}[u'(c_{t+1})q_{t+1,t+2}|s_t, \iota_t],
\end{aligned}
$$

respectively. Under the maintained assumption of risk neutrality on the part of investors, the absence of default risk would imply $q_{t,t+1} = q_{t+1,t+2} = \beta$ and $q_{t,t+2} = \beta^2$ and thus, equality of the two marginal effects and indeterminacy of the portfolio choice. To restore determinacy in a setting with commitment, the price of default-free outstanding debt, $q_{t+1,t+2}$, would need to be state contingent, for example due to an endogenous asset pricing kernel of investors (see Angeletos, 2002; Nosbusch, 2008), and taxpayers would need to be risk averse.

In the model of this paper, the equilibrium maturity structure is determinate even if the asset pricing kernel of investors is not stochastic and even if taxpayers are risk neutral.

4 Equilibrium: Analytical Results

To understand the equilibrium implications of lack of commitment for the maturity structure, we first consider several special cases of the model. In all these cases, marginal utility $u'(c_t)$ is assumed to be exogenous, potentially dependent on the (exogenous) level of output: $u'(c_t) = \mu(y_t)$ with $\mu(y^h) < \mu(y^l)$ for $y^l < y^h$.

The assumption about the marginal utility function is motivated by tractability considerations. If marginal utility is independent of the stock of maturing debt or the income loss in case of default, then so is equilibrium debt issuance, $b_{t,t+1}(y_t, \mathrm{cd}_t, b_{x,t,t+1})$ and $b_{t,t+2}(y_t, \mathrm{cd}_t, b_{x,t,t+1})$. This allows to characterize the equilibrium maturity structure in closed form. The assumption is satisfied if taxpayers are risk neutral and it is satisfied approximately if variations in output have a much stronger effect on marginal utility than policy does. Numerical simulations in Section 5 will show that the results derived under the assumption about the marginal utility function are robust to relaxing this assumption.

4.1 No Cross-Default

In a first part, we abstract from the possibility of cross-default, $\pi = 0$, and focus on the effect of the distribution function $F(\cdot)$ as well as of cyclicality and risk on the equilibrium maturity structure. Throughout, we assume that a regularity condition is satisfied: Let $H(L) \equiv f(L)/(1 - F(L))$ denote the hazard function.

(C) For all $L \geq 0$, (i) $H'(L) \geq 0$ and (ii) $2H'(L)^2 - H(L)H''(L) \geq 0$, for example because the hazard function is concave.

Many distribution functions typically used in economic applications satisfy condition (C).[27] The condition implies that the marginal effects of debt issuance on the government's value are monotone such that equilibrium analysis can be based on first-order conditions.

Absent cross-default, the equilibrium price functions satisfy

$$\begin{aligned} q_{t,t+1}(s_t, r_t, \iota_t) &= \beta(1 - F(b_{x,t,t+1} + b_{t,t+1})), \\ q_{t,t+2}(s_t, r_t, \iota_t) &= \beta^2 \mathbb{E}[1 - F(b_{t,t+2} + b_{t+1,t+2}(y_{t+1}, b_{t,t+2}))|s_t, \iota_t], \end{aligned}$$

implying that no revenue effects across maturities are present, $\mathcal{R}_{t,sl} = \mathcal{R}_{t,ls} = 0$. The effect on the government's value of a marginal increase in $b_{t,t+1}$ and $b_{t,t+2}$, respectively, therefore reduces to

$$\begin{aligned} &-\mu(y_t) b_{t,t+1} \beta f(b_{x,t+1}) + (1 - F(b_{x,t+1})) \mathbb{E}\left[\beta\mu(y_t) - \delta\mu(y_{t+1})|s_t\right], \\ &-\mu(y_t) b_{t,t+2} \beta^2 \mathbb{E}\left[f(b_{x,t+2}) \left(1 + \frac{\partial b_{t+1,t+2}(y_{t+1}, b_{t,t+2})}{\partial b_{x,t+1,t+2}}\right) |s_t\right] \\ &\qquad\qquad + \mathbb{E}\left[\beta(1 - F(b_{x,t+2}))(\beta\mu(y_t) - \delta\mu(y_{t+1}))|s_t\right]. \end{aligned}$$

[27] Examples of distribution functions with increasing hazard functions include uniform, normal, exponential, logistic, extreme value, Laplace, power, Weibull, gamma, chi-squared, chi, or beta distributions (see, e.g., Bagnoli and Bergstrom, 2005). If L_t is distributed according to an exponential distribution, $F(L) = 1 - \exp(-\lambda L)$, then the hazard function is constant, $H(L) = \lambda$. If L_t is distributed according to a Weibull distribution, $F(L) = 1 - \exp(-L^\lambda), \lambda > 1$, then the hazard function is strictly increasing, $H(L) = \lambda L^{\lambda - 1}$; moreover, for $1 \leq \lambda \leq 2$, the hazard function is concave, and for all $\lambda > 1$, $H'(L)^2 - H(L)H''(L) > 0$.

The terms on the left-hand side of each expression represent welfare consequences of revenue effects on inframarginal units, the terms on the right-hand side consumption smoothing benefits. While the revenue effects on inframarginal short-term debt reflect the endogeneity of the subsequent repayment rate, the revenue effects on inframarginal long-term debt reflect the endogeneity of the subsequent repayment rate and debt issuance. In this sense, the revenue effects on short-term debt reflect one, those on long-term debt two channels through which lack of commitment operates.

Under condition (C), the two marginal effects equal zero in equilibrium and the equilibrium maturity structure balances for each maturity the consumption-smoothing benefits from the marginal unit of debt and the costs due to revenue effects on inframarginal units:

$$b_{t,t+1} = \frac{1-F(b_{x,t+1})}{f(b_{x,t+1})}\mathbb{E}\left[1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}|s_t\right] \geq 0, \tag{10}$$

$$b_{t,t+2} = \frac{\mathbb{E}\left[(1-F(b_{x,t+2}))\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)|s_t\right]}{\mathbb{E}\left[f(b_{x,t+2})\left(1+\frac{\partial b_{t+1,t+2}(y_{t+1},b_{t,t+2})}{\partial b_{x,t+1,t+2}}\right)|s_t\right]} > 0. \tag{11}$$

Equation (10) defines a policy function $b_{t,t+1}(y_t, b_{x,t,t+1})$ that is positive, decreasing and convex (the latter due to part (ii) of condition (C)): $b_{t,t+1}(y_t, b_{x,t,t+1}) > 0$, $-1 < \partial b_{t,t+1}(y_t, b_{x,t,t+1})/\partial b_{x,t,t+1} <$ 0 and $\partial^2 b_{t,t+1}(y_t, b_{x,t,t+1})/(\partial b_{x,t,t+1})^2 \geq 0$. Higher outstanding debt reduces the equilibrium level of short-term debt issuance because it increases the negative revenue effects on inframarginal units. This effect only is absent if the hazard function is constant as is the case with exponentially distributed income losses and it is otherwise weakened as the quantity of outstanding debt increases.[28] Subject to the policy function for short-term debt issuance (in the subsequent period), equation (11) defines a policy function for short-term debt issuance, $b_{t,t+2}(y_t) > 0$. Summarizing, we have the following preliminary result:

Lemma 1. If $\pi = 0$ and condition (C) holds, then there exists an equilibrium in which the policy functions $b_{t,t+1}(s_t)$ and $b_{t,t+2}(s_t)$ do not depend on $b_{x,t}$ or L_t. The maturity structure in this equilibrium is unique with $b_{t,t+1}(s_t), b_{t,t+2}(s_t) > 0$.

The equilibrium characterized in the Lemma is the only equilibrium that arises in a finite horizon economy (with the number of periods potentially approaching infinity). This follows from a straightforward backward induction argument. In the discussion, we focus on this type of equilibrium.

[28]With Weibull distributed income losses, short-term debt issuance depends negatively on the quantity of outstanding debt but the parametric assumption $\lambda = 2$ renders the dependence analytically tractable,

$$b_{t,t+1}(y_t, b_{x,t,t+1}) = -\frac{b_{x,t,t+1}}{2} + \frac{1}{2}\sqrt{b_{x,t,t+1}^2 + 2\mathbb{E}_{y_{t+1}}\left[1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}|s_t\right]}.$$

Consider the case of a constant hazard function, $H(L) = H$, and suppose that output follows a deterministic process. The equilibrium conditions (10) and (11) then reduce to

$$\begin{aligned} b_{t,t+1}H &= 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}, \\ b_{t,t+2}H &= 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}, \end{aligned}$$

implying that the equilibrium maturity structure is fully balanced at all times. Intuitively, with a constant hazard function, the revenue effects on inframarginal units of debt relative to the revenue gain on the marginal unit are symmetric across maturities as well as convex. Equilibrium policy therefore "smoothes maturities" or more specifically, the convex losses associated with them for parallel reasons as those driving Barro's (1979) "tax-smoothing" prescription:

Proposition 1. If $\pi = 0$, the hazard function is constant (such that condition (C) holds) and y_t is deterministic, then the equilibrium maturity structure is fully balanced. Debt issuance is high when marginal utility relative to marginal utility in the subsequent period is high.

The result of a fully balanced maturity structure hinges on the feature that the revenue effects on inframarginal units relative to the revenue on the marginal unit are symmetric across maturities. With a strictly increasing hazard function, this symmetry disappears because long-term debt issuance affects short-term debt issuance in the subsequent period. Consider the equilibrium in an environment where the consumption smoothing term $\mathcal{C}_t \equiv 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}$ is constant over at least three periods, $t-1, t, t+1$, such that $b_{t-1,t+1} = b_{t,t+2} \equiv b_{\text{long}}$ and $b_{t,t+1} = b_{t+1,t+2} \equiv b_{\text{shrt}}$.[29] The equilibrium conditions then read

$$\begin{aligned} b_{\text{shrt}}H(b_{\text{shrt}} + b_{\text{long}}) &= 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}, \\ b_{\text{long}}H(b_{\text{shrt}} + b_{\text{long}})\left(1 + {b_{\text{shrt}}}'(b_{\text{long}})\right) &= 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}. \end{aligned}$$

Since short-term debt issuance responds negatively to the quantity of outstanding debt (from the first equation), long-term debt issuance increases the debt amount at maturity by less than one-to-one (from the second equation), in contrast to short-term debt issuance. Ceteris paribus, long-term debt issuance therefore has a smaller price impact, rendering it "cheaper" from the government's perspective. As a consequence, the equilibrium maturity structure is tilted towards the long end, $b_{\text{shrt}} < b_{\text{long}}$. Moreover, due to the convexity of $b_{\text{shrt}}(b_{\text{long}})$, the tilt towards long-term debt becomes smaller as the total amount of debt increases. Higher debt-to-GDP ratios therefore go hand in hand with a shortening of the maturity structure, in line with the evidence (Rodrik and Velasco, 1999):

[29] Short-term debt issuance in period t is a function of $b_{t-1,t+1}$ and $\mathcal{C}_t$. A parallel statement holds for period $t+1$. If $\mathcal{C}_t = \mathcal{C}_{t+1}$, then governments issuing long-term debt in periods $t-1$ and t anticipate the same type of policy response as far as short-term debt issuance by their respective successor governments is concerned. If, moreover, $\mathcal{C}_{t-1} = \mathcal{C}_t$ then long-term debt issuance in periods $t-1$ and t coincides. As a consequence, short-term debt issuance in periods t and $t+1$ coincides as well.

Proposition 2. If $\pi = 0$, the hazard function is strictly increasing, condition (C) holds and the consumption smoothing term is time invariant, then the equilibrium maturity structure is tilted towards the long end. Higher debt-to-GDP ratios go hand in hand with a shortening of the maturity structure.

Consider next the equilibrium in an environment where the consumption smoothing term fluctuates in the sense that $\mathcal{C}_{t-1} + \varepsilon = \mathcal{C}_t = \mathcal{C}_{t+1}$ for $\varepsilon \geq 0$.[30] This scenario describes a "crisis" where the economy plunges from a peak in period $t-1$ into a trough in period t before returning to trend growth. From the previous results, one would expect the model to produce a decrease in total debt issuance in period $t-1$ (due to the weak consumption smoothing motive in that period) and a corresponding lengthening of the maturity structure (in light of Proposition 2). This should be followed by an increase in debt issuance at the trough in period t (due to the stronger consumption smoothing motive) and a shortening of the maturity structure, in line with the evidence (Broner et al., 2007). A general result along these lines can indeed be proved under the assumption that the hazard function is proportional (representing Weibull distributed income losses):[31]

Proposition 3. If $\pi = 0$, $H(L) = 2L$ (such that condition (C) holds), the consumption smoothing term fluctuates as defined above and ε is marginally increased around zero, then the equilibrium maturity structure lengthens in period $t-1$ and shortens in period t.

To understand the equilibrium implications of risk it is instructive to rewrite (11) as

$$b_{t,t+2} = \frac{\mathbb{E}\left[1 - F(b_{x,t+2})|s_t\right] \mathbb{E}\left[1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}|s_t\right] + \mathbb{C}\mathrm{ov}\left[1 - F(b_{x,t+2}), 1 - \frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}|s_t\right]}{\mathbb{E}\left[f(b_{x,t+2})\left(1 + \frac{\partial b_{t+1,t+2}(y_{t+1}, b_{t,t+2})}{\partial b_{x,t+1,t+2}}\right)|s_t\right]}.$$

The first term on the right-hand side of the equality represents the average consumption smoothing benefit from the marginal unit of long-term debt relative to the average cost due to revenue effects on inframarginal units. The second term represents an insurance benefit due to the covariance between the price of outstanding debt in the subsequent period and marginal utility. Short-term debt does not provide such insurance benefits since its repayment rate does not covary with output and marginal utility in the subsequent period.

The insurance benefit of long-term debt is positive if marginal utility in the subsequent period covaries negatively with the price of outstanding long-term debt and thus, if it covaries positively with short-term debt issuance. Consider the case of a constant hazard

[30] An alternative source of time variation of the maturity structure relates to changes of the hazard function over time. A priori, it is not clear how such changes should correlate with output and marginal utility.

[31] The proof is available on request.

function. Condition (11) then reduces to[32]

$$
\begin{aligned}
b_{t,t+2} &= \lambda^{-1}\frac{\mathbb{E}\left[\exp(-\lambda(b_{t+1,t+2}(y_{t+1})))\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)|s_t\right]}{\mathbb{E}\left[\exp(-\lambda(b_{t+1,t+2}(y_{t+1})))|s_t\right]} \\
&= b_{t,t+1}+\lambda^{-1}\frac{\mathbb{C}\mathrm{ov}\left[\exp(-\lambda(b_{t+1,t+2}(y_{t+1}))),\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)|s_t\right]}{\mathbb{E}\left[\exp(-\lambda(b_{t+1,t+2}(y_{t+1})))|s_t\right]},
\end{aligned}
$$

where the second equality follows from (10) with $H(L)=\lambda$. From (10), short-term debt issuance covaries positively with marginal utility if output is mean reverting in the sense that a realization of $\mu(y_t)$ above its mean implies that the consumption smoothing term $\mathcal{C}_t$ exceeds its mean as well. In this case, the covariance term in the expression for long-term debt issuance is positive and long-term debt issuance exceeds short-term debt issuance:

Proposition 4. If $\pi=0$, the hazard function is constant (such that condition (C) holds) and output is stochastic and mean reverting, then the equilibrium maturity structure is tilted towards the long end.

4.2 Cross-Default

The possibility of cross-default, $\pi>0$, introduces "cross-revenue effects," $\mathcal{R}_{t,sl},\mathcal{R}_{t,ls}\neq 0$. Intuitively, short-term debt issuance drives up the risk of default in the subsequent period and such a default does not only affect maturing debt but also, if $\mathrm{cd}_{t+1}=1$, outstanding debt. Similarly, long-term debt issuance may drive up the risk of default in the subsequent period as well if $\mathrm{cd}_{t+1}=1$.

From (5) and (6), the revenue effects on inframarginal units of debt are given by[33]

$$
\begin{aligned}
\mathcal{R}_{t,ss} &= -b_{t,t+1}\beta\left(\pi\mathbb{E}[f_{t+1}^{\mathrm{cd}=1}|s_t,r_t,\iota_t]+(1-\pi)(f_{t+1}^{\mathrm{cd}=0}|s_t,r_t,\iota_t)\right), \\
\mathcal{R}_{t,sl} &= -b_{t,t+2}\beta\pi\mathbb{E}[f_{t+1}^{\mathrm{cd}=1}q_{t+1,t+2}([S_{t+1}])|\mathrm{cd}_{t+1}=1,L_{t+1}=\infty,s_t,r_t,\iota_t], \\
\mathcal{R}_{t,ls} &= -b_{t,t+1}\beta\pi\mathbb{E}\left[f_{t+1}^{\mathrm{cd}=1}\frac{\partial\alpha_{t+1}(y_{t+1},b_{x,t+1,t+2})}{\partial b_{t,t+2}}|s_t,r_t,\iota_t\right], \\
\mathcal{R}_{t,ll} &= -b_{t,t+2}\beta\left(\pi\mathbb{E}\left[f_{t+1}^{\mathrm{cd}=1}\frac{\partial\alpha_{t+1}(y_{t+1},b_{x,t+1,t+2})}{\partial b_{t,t+2}}q_{t+1,t+2}([S_{t+1}])|\mathrm{cd}_{t+1}=1,L_{t+1}=\infty,s_t,r_t,\iota_t\right]\right. \\
&\qquad\left.-\pi\Delta_i-(1-\pi)\Delta_j\right),\quad \Delta_i,\Delta_j<0,
\end{aligned}
$$

while the consumption smoothing benefits from short- and long-term debt issuance are proportional to

$$
\begin{aligned}
&\mathbb{E}\left[(1-F_{t+1})\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)|L_{t+1}=\infty,s_t,r_t,\iota_t\right], \\
&\mathbb{E}\left[(1-F_{t+1})q_{t+1,t+2}([S_{t+1}])\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)|L_{t+1}=\infty,s_t,r_t,\iota_t\right]+(1-\pi)\Delta_c,\quad \Delta_c\geq 0,
\end{aligned}
$$

[32]If L is distributed exponentially with parameter λ, then $1-F(b_1+b_2)=(1-F(b_1))\exp(-\lambda b_2)$ and $f(b_1+b_2)=f(b_1)\exp(-\lambda b_2)=\lambda(1-F(b_1))\exp(-\lambda b_2)$.

[33]Since the stock of maturing debt does not affect marginal utility, it does not enter the function $\alpha_t(\cdot)$ either.

respectively.

Note that, if short-term debt issuance in the subsequent period is interior (as verified below), then $\partial\alpha_{t+1}(y_{t+1}, b_{x,t+1,t+2})/\partial b_{t,t+2} = q_{t+1,t+2}([S_{t+1}])|\mathrm{cd}_{t+1} = 1$.[34] A comparison of the above expressions therefore suggests that in the limiting case with $\pi = 1$ and without risk, the welfare implications of issuing one unit of short-term debt and $z \equiv 1/\frac{\partial\alpha_{t+1}(y_{t+1}, b_{x,t+1,t+2})}{\partial b_{t,t+2}}$ units of long-term debt are closely related. In particular, with $\pi = 1$ and without risk, one unit of short-term debt and z units of long-term debt raise the same revenue on the marginal unit of debt (see (5) and (6)). At the same time,

$$\begin{aligned} z\mathcal{R}_{t,ls} &= \mathcal{R}_{t,ss}, \\ z\mathcal{R}_{t,ll} &= \mathcal{R}_{t,sl} + zb_{t,t+2}\beta\Delta_i. \end{aligned}$$

Accordingly, the normalized revenue effects on inframarginal units of short-term debt due to short- or long-term debt issuance coincide while the normalized revenue effects on inframarginal units of long-term debt due to long-term debt issuance exceed those due to short-term debt issuance. This reflects the fact that long-term debt issuance increases the default likelihood in the long-term, conditional on no default occuring in the short term. Relative to the revenue raised on the marginal unit, long-term debt therefore generates larger adverse revenue effects. At the same time, the normalized consumption smoothing benefits of short- and long-term debt issuance coincide and short-term debt issuance therefore always dominates long-term debt issuance:

Proposition 5. If $\pi = 1$ and y_t is deterministic, then the maturity structure is concentrated on the short end.

5 Equilibrium: Numerical Results

To assess the robustness of the analytical results, we solve the general model numerically and simulate it. We first analyze how curvature in the utility function as well as the interaction between the individual effects highlighted by the analytical results affect equilibrium outcomes. Thereafter, we consider the predictions of the model subject to a benchmark calibration with risky output, risk averse taxpayers and cross-default risk.

We assume that $u(\cdot)$ is of the CIES form with σ, the inverse of the elasticity of intertemporal substitution, varying between 0.01 (approximately corresponding to the situation analyzed in Section 4) and 5; that income losses in the wake of a default are distributed according to a Weibull distribution with parameter $\lambda = 2$ (the simplest distribution to yield a time varying maturity structure in the benchmark cases); and that the annual

[34] Differentiating (4) with respect to $b_{x,t,t+1}$ yields

$$\mu(y_t)(\mathcal{R}_{t,ss} + \mathcal{R}_{t,sl}) + \delta\frac{\partial\mathbb{E}[G_{t+1}(s_{t+1})|s_t, 1, \iota_t(s_t)]}{\partial b_{x,t,t+1}} \equiv -\mu(y_t)\frac{\partial\alpha_t(y_t, b_{x,t,t+1})}{\partial b_{x,t,t+1}} \text{ if } \mathrm{cd}_t = 1.$$

From the first-order condition for $b_{t,t+1}$, the left-hand side of this equation equals $-\mu(y_t)\beta\mathbb{E}[1-F_{t+1}|\mathrm{cd}_t = 1, s_t, r_t, \iota_t]$. The result then follows.

riskfree interest rate equals 2 percent. One period in the model corresponds to 3 years and simulation statistics are based on a sequence of 20'000 observations.

Exogenous output is assumed either to be constant or to fluctuate between a high, average and low state, y^h, $\bar{y}$ and y^l, respectively, with a standard deviation of 3.5 percent, corresponding to the standard deviation of detrended three-year Argentinian GDP. Mean exogenous output $\bar{y}$ as well as the parameter δ are calibrated by equalizing a debt-service-to-GDP ratio of 5.5 percent (Arellano, 2008) and debt-to-GDP ratio of 40 percent with their respective model counterparts under the assumptions of Proposition 2,

$$\begin{aligned} 0.055\bar{y} &= b_{\text{shrt}} + b_{\text{long}} - (\beta b_{\text{shrt}} + \beta^2 b_{\text{long}})(1 - F(b_{\text{shrt}} + b_{\text{long}})), \\ 0.400\bar{y} &= b_{\text{shrt}} + b_{\text{long}}. \end{aligned}$$

In the simulations with risk, the exogenous output process is generated by the transition matrix

$$\begin{pmatrix} 0.50 & 0.50 & 0.00 \\ 0.25 & 0.50 & 0.25 \\ 0.00 & 0.50 & 0.50 \end{pmatrix},$$

implying unconditional probabilities of high, average and low exogenous output of 25, 50 and 25 percent, respectively. In the simulations without risk, $y_t = \bar{y}$ in all periods. Summarizing, the assumed or implied parameter values are $\beta = 0.9412, \delta = 0.9005, y^h = 0.5998, \bar{y} = 0.5715, y^l = 0.5432$ as well as the transition matrix. To find the policy functions, we use an iterative algorithm that searches over the discretized state space. The shock sequence for income losses in the wake of a default is kept constant across all simulations. The shock sequences for exogenous output and the cross-default shock differ across those simulations whose transition matrix or value of π are unequal.

If exogenous output is constant and $\pi = 0$, the simulated default frequency lies between 3.8 and 5.4 percent, depending on the choice of σ. Higher curvature of the utility function tends to reduce the amount of debt coming due in a period and as a consequence, the default frequency. It also tends to reduce short-term debt issuance while having less clear-cut effects on long-term debt issuance. Long-term debt issuance tends to exceed short-term debt issuance, in line with the analytical results. Income losses L_t tend to be much smaller in periods where the government defaults than in periods where it does not.

If exogenous output is constant and $\pi = 1$, the simulated default frequency falls to roughly 2 percent, reflecting a reduction in debt positions, due to cross-default risk. In line with the analytical results, long-term debt issuance vanishes as σ approaches zero. However, for higher values of σ, the maturity structure does not display this extreme bias towards the short end. Indeed, for $\sigma = 1, 2, 5$, it tends to be tilted towards the long end on average.

If exogenous output is risky and $\pi = 0$, the simulated default frequency rises to between 4.3 and 6 percent. As σ approaches zero, the maturity structure tends to be tilted more strongly towards the long end than in the case without output risk, in line with the analytical result.

If exogenous output is risky and $\pi = 1$, the simulated default frequency is roughly 2.2 percent, again reflecting lower debt positions due to cross-default risk. Long-term debt issuance vanishes as σ approaches zero, in line with the analytical result for the deterministic case. However, as σ increases from this limiting value, the extreme bias in the maturity structure quickly disappears and long-term debt tends to exceed short-term debt on average.

In summary, this first set of simulation results confirms all the analytical results. Among those, only the extreme bias of the maturity structure towards its short end when $\pi = 1$ and $\sigma = 0$ (Proposition 5) turns out to be not robust. With risk aversion on the part of taxpayers, the prevalence of cross-default risk therefore does not have a strong effect on the *relative* size of short- and long-term debt positions while it does affect the absolute size of the two maturities.

We turn next to the model predictions subject to a benchmark calibration with risky output, $\pi = 0.8$ and $\sigma = 2$ (as, for example, in Arellano (2008) or Hatchondo and Martinez (2009)). We summarize these predictions by reporting the typical dynamics of the central exogenous and endogenous model variables in Figures 1–3. Among the endogenous variables, we also report the spreads on short- and long-term debt, respectively,

$$\begin{aligned} \rho_{t,t+1} &\equiv q_{t,t+1}^{-1} - \beta^{-1}, \\ \rho_{t,t+2} &\equiv q_{t,t+2}^{-1/2} - \beta^{-1}. \end{aligned}$$

Figure 1 reports the average path of the model variables during a window of eleven periods around a government default. The first two rows of the Figure show that exogenous output tends to be lower before a default period; a default tends to coincide with a cross-default shock and a low realization of default induced income losses; maturing and outstanding debt tend to fall after a default; and default periods tend to be isolated over time. The second two rows of the Figure indicate that debt issuance and, accordingly, spreads fall after a default; and the maturity structure of debt issuance shortens during the default episode.

Figures 2 and 3 shed light on the typical dynamics over the business cycle by conditioning on the realization of output. High output episodes (summarized in Figure 2) tend to go hand in hand with decreasing levels of maturing and outstanding debt, increasing repayment rates, low debt issuance and a longer than average maturity structure. Spreads tend to be low. In contrast, low output episodes (summarized in Figure 3) are characterized by buildups of maturing and outstanding debt, falling repayment rates, high debt issuance with a shorter than average maturity structure and increased spreads.

Importantly, the simulation-based results corroborate the analytical results derived earlier and extend them to the case of risk aversion. In particular, long-term debt issuance generally exceeds short-term debt issuance (cf. Proposition 2), the maturity structure shortens during periods with default or low output (cf. Proposition 3), and a more risky environment increases the portfolio share of long-term debt (see the simulation results reported earlier; cf. Proposition 4). This strong correspondence between the analytical and the simulation-based results suggests that the version of the model that can be solved in closed form incorporates the central aspects of the general framework.

Moreover, the simulation-based results correspond with the data along several dimensions. First, the model correctly predicts a shortening of the maturity structure during times of crisis and periods with low output. Second, the model generates an unconditional default probability of roughly 2.5 percent. Existing studies in the quantitative debt literature often target a probability of this size (for example, Arellano (2008) calibrates her model to generate a default probability of three percent); here, in contrast, the default probability did not serve as a target during the calibration. Third, the model predicts counter cyclical spreads, consistent with the data (Broner et al., 2007; Arellano and Ramanarayanan, 2010), and it does so without assuming any correlation between output and default induced income losses.[35] Finally, the model predicts that rising spreads go hand in hand with a stronger increase in the spread of short- relative to long-term spreads, again consistent with the data (Broner et al., 2007; Arellano and Ramanarayanan, 2010).

6 Conclusion

Lack of commitment paired with social losses in the wake of a default gives rise to a determinate maturity structure. Under regularity conditions, this maturity structure smoothes the revenue effects on inframarginal units of debt across the available maturities. Such smoothing of revenue effects has implications for unconditional and conditional moments of the maturity structure which are broadly consistent with the available evidence. It also has implications for equilibrium default frequencies and spreads that measure up to the data.

Enriching the standard sovereign debt model by introducing a choice of maturity opens a promising avenue for a better understanding of debt dynamics in countries with credibility problems.

[35] The predicted spreads are too low relative to the data. Arellano (2008) encounters a similar problem. She argues that correlation between output in the borrowing country and the lenders' asset pricing kernel increases the equilibrium risk premium in the model and that this effect might help explain high observed spreads.

A Alternative Specifications of Social Losses

Corner solutions for the optimal repayment rate follow under more general assumptions about the income losses in the wake of a default. Consider for example the case where income losses are proportional to L_t and the *default rate*,

$$\text{losses}_t = (1 - r_t)L_t.$$

The optimal repayment choice then is identical to the one given in the text.

Consider next the situation where income losses are proportional to L_t and the total *amount* defaulted upon,

$$\text{losses}_t = (1 - r_t)b_{x,t}L_t.$$

The optimal repayment rate then varies with L_t but does not depend on the amount of maturing debt, rendering such a specification unattractive.

Consider next the situation where income losses are a *concave* function of the amount defaulted upon, for example

$$\text{losses}_t = [(1 - r_t)b_{x,t}]^{1/2}L_t$$

or

$$\text{losses}_t = \mathbf{1}_{[r_t<1]}L_t + k(1 - r_t)b_{x,t},\ 0 < k < 1.$$

Again, the optimal repayment rate then equals either unity or zero since total costs from debt repayment and income losses are a concave function of the default rate.

If income losses are a *convex* function of the amount defaulted upon, for example

$$\text{losses}_t = [(1 - r_t)b_{x,t}]^2 L_t,$$

then the equilibrium repayment rate is no longer discrete. However, convexity of income losses appears less plausible than the previously discussed specifications, for at least two reasons. First, most notions of income losses are consistent with concave costs: The marginal cost of defaulting on the first 5 percent of debt exceeds the one from defaulting on the following 5 percent. Second, convex income losses would lead governments to *always* default at least partially, in contrast with the empirical evidence.

B Social Losses and the Incentive to Dilute

In this section, we analyze how the assumption of social losses in the wake of a default shapes the government's rollover decision. We focus on the case where the government issues short-term debt only and π equals zero. Recall from the text that, in this case,

$$\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}} = \beta \left(1 - F(b_{x,t+1}) \underbrace{- b_{t,t+1} f(b_{x,t+1})}_{\mathcal{R}_{t,ss}} \right)$$

while the marginal effect of short-term debt issuance on the government's objective is given by

$$u'(c_t)\beta\mathcal{R}_{t,ss} + (1 - F(b_{x,t+1}))\left(\beta u'(c_t) - \delta\mathbb{E}[u'([S_{t+1}])|s_t]\right).$$

Consider an alternative setup without social losses. Assume as before that the government either fully repays the maturing debt or suffers costs L_t. In contrast to the main model, however, suppose now that these costs correspond to a *transfer* to bondholders rather than a social loss. One can interpret this modified setting as a situation where the realization of L_t determines the bargaining power of bondholders vis-a-vis the government. According to this interpretation, bondholders can successfully press for full repayment if the realization of L_t is high. If the realization of L_t falls short of the maturing debt, however, bondholders must concede and settle for a reduced repayment equal to L_t.

In this modified setup, the repayment rate in period t is given by

$$r_t(s_t) = \begin{cases} 1 & \text{if } L_t \geq b_{x,t} \\ \frac{L_t}{b_{x,t}} & \text{if } L_t < b_{x,t} \end{cases}$$

and the expected repayment rate features a new component that accounts for payments in the partial default case:

$$\mathbb{E}\left[r_{t+1}(s_{t+1})|s_t\right] = 1 - F(b_{x,t+1}) + \underbrace{\frac{1}{b_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})}_{\text{new term}}.$$

Accordingly, the marginal effect of debt issuance in period t on the deficit in that period changes to

$$\begin{aligned}\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}b_{t,t+1}} &= \beta\left(1 - F(b_{x,t+1}) - b_{t,t+1}f(b_{x,t+1})\right) \\ &\quad \underbrace{+\beta\left(b_{t,t+1}f(b_{x,t+1}) + \frac{1}{b_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})\left(1 - \frac{b_{t,t+1}}{b_{x,t+1}}\right)\right)}_{\text{new terms}}.\end{aligned}$$

The presence of transfers rather than social losses introduces three marginal effects in addition to those present in the main model. First, the increase in $b_{t,t+1}$ raises more revenue because newly-issued debt is *partially* repaid in some states, as reflected in the term $\frac{1}{b_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})$. Second, as reflected in the term $b_{t,t+1}f(b_{x,t+1})$, an increase in $b_{t,t+1}$ raises the probability of *partial* repayment of the newly-issued debt at the critical income loss, $b_{x,t+1}$. Finally, the increase in $b_{t,t+1}$ causes revenue effects on newly-issued inframarginal debt, $-\frac{b_{t,t+1}}{b^2_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})$, because it reduces the repayment rate in case of *partial* default.

The second of these additional effects exactly balances the revenue effect on inframarginal units of debt that is present in the main model. Intuitively, the revenue gain due to more likely, partial repayment exactly compensates for the revenue loss due to less likely, full repayment. On net, the marginal effect on the deficit therefore amounts to $\beta(1-F(b_{x,t+1}))+\beta\frac{1}{b_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})\left(1-\frac{b_{t,t+1}}{b_{x,t+1}}\right)$. If $0<b_{x,t,t+1}<b_{x,t+1}$ such that debt is outstanding and the government issues additional debt, this marginal effect exceeds $\beta(1-F(b_{x,t+1}))$ because debt issuance effectively redistributes expected payments from owners of outstanding to owners of newly-issued debt, in contrast with the situation in the main model.

The government's program in period t is unchanged relative to the original setup, except for the modified expression characterizing the deficit. (From the government's point of view, it is irrelevant whether income losses in period $t+1$ correspond to transfers to bond holders rather than social losses.) The effect of a marginal increase in $b_{t,t+1}$ therefore equals

$$u'(c_t)\beta\frac{1}{b_{x,t+1}}\int_0^{b_{x,t+1}} L_{t+1}\mathrm{d}F(L_{t+1})\frac{b_{x,t,t+1}}{b_{x,t+1}}+(1-F(b_{x,t+1}))\left(\beta u'(c_t)-\delta\mathbb{E}[u'([S_{t+1}])|s_t]\right),$$

reflecting the same consumption-smoothing effect as in the main model, but modified revenue effects on inframarginal units of debt. Without social losses in the wake of a default as they are present in the original setup, the government has an incentive to dilute outstanding debt.

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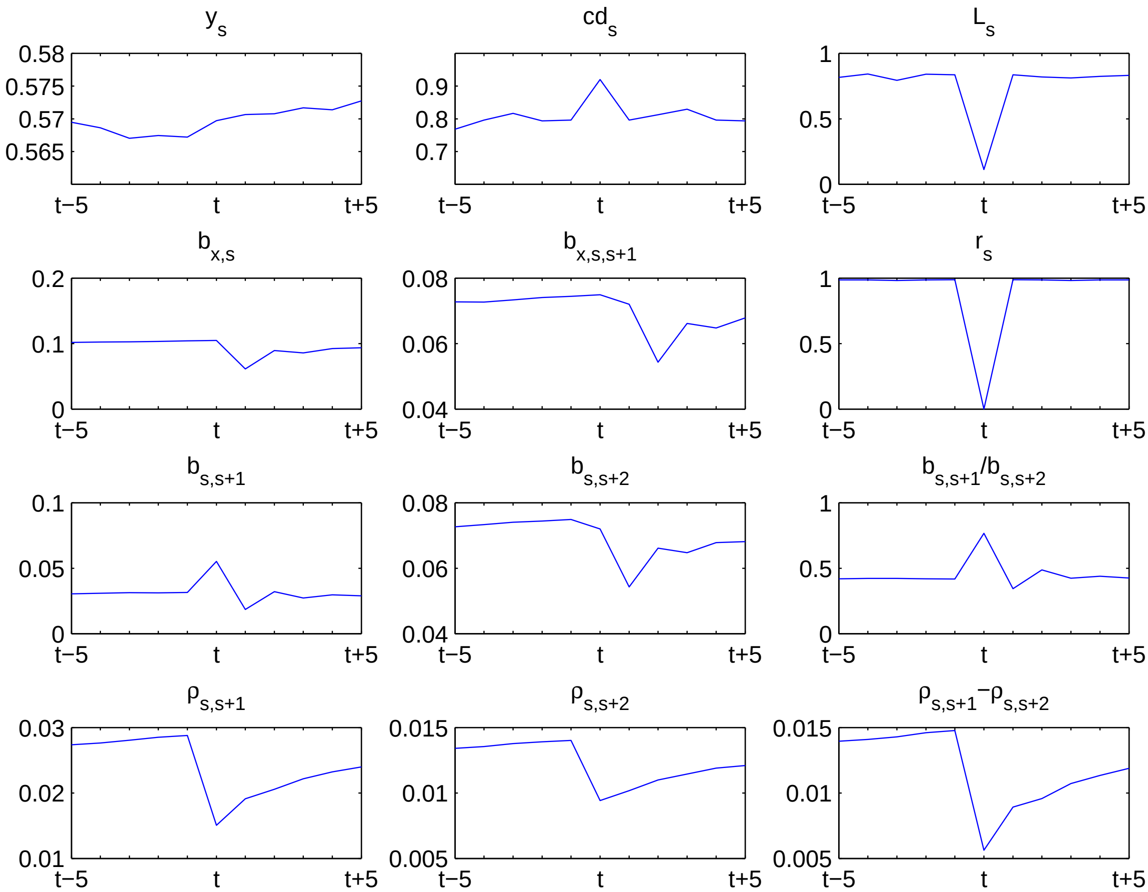

Figure 1: Typical dynamics around a default period (the panels display the sample averages of the respective variables in periods $s = t-5, \dots, t+5$ conditional on $r_t = 0$)

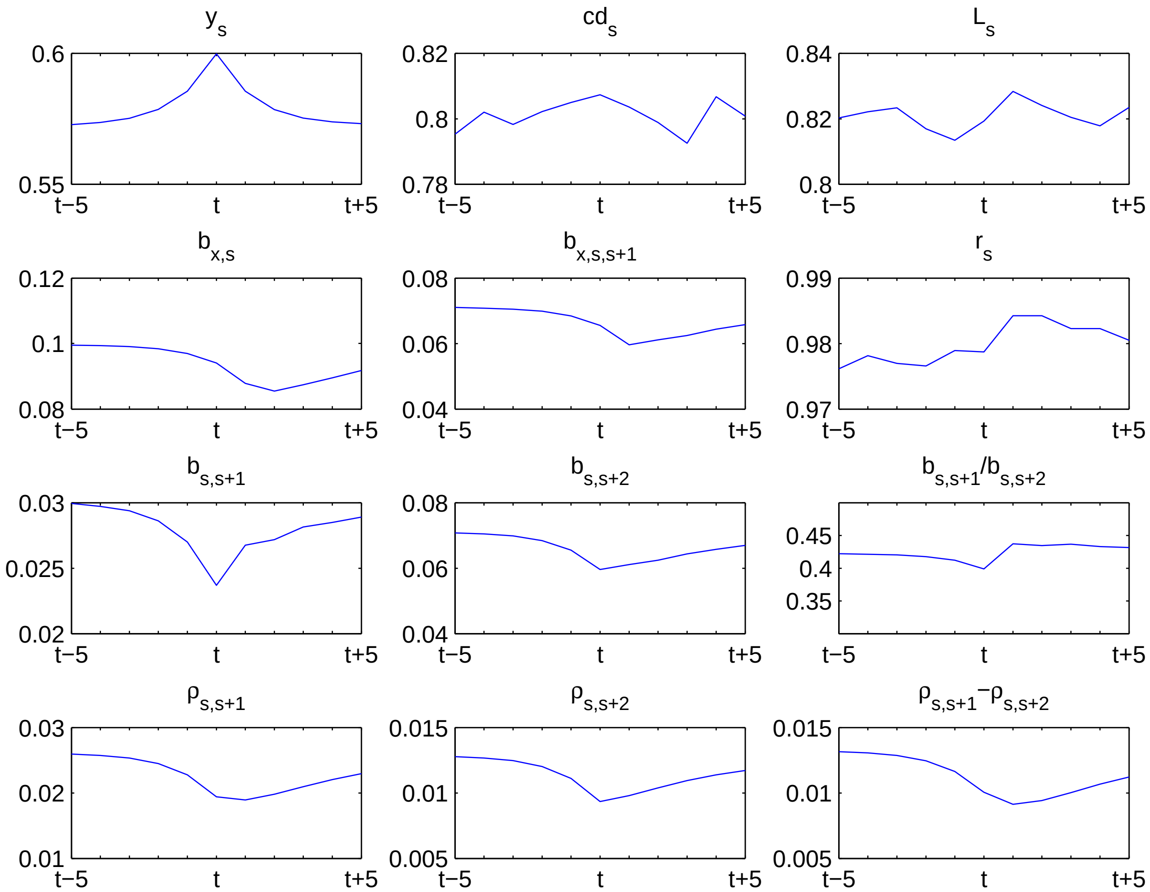

Figure 2: Typical dynamics around a high-income period (the panels display the sample averages of the respective variables in periods $s = t-5, \ldots, t+5$ conditional on $y_t = y^h$)

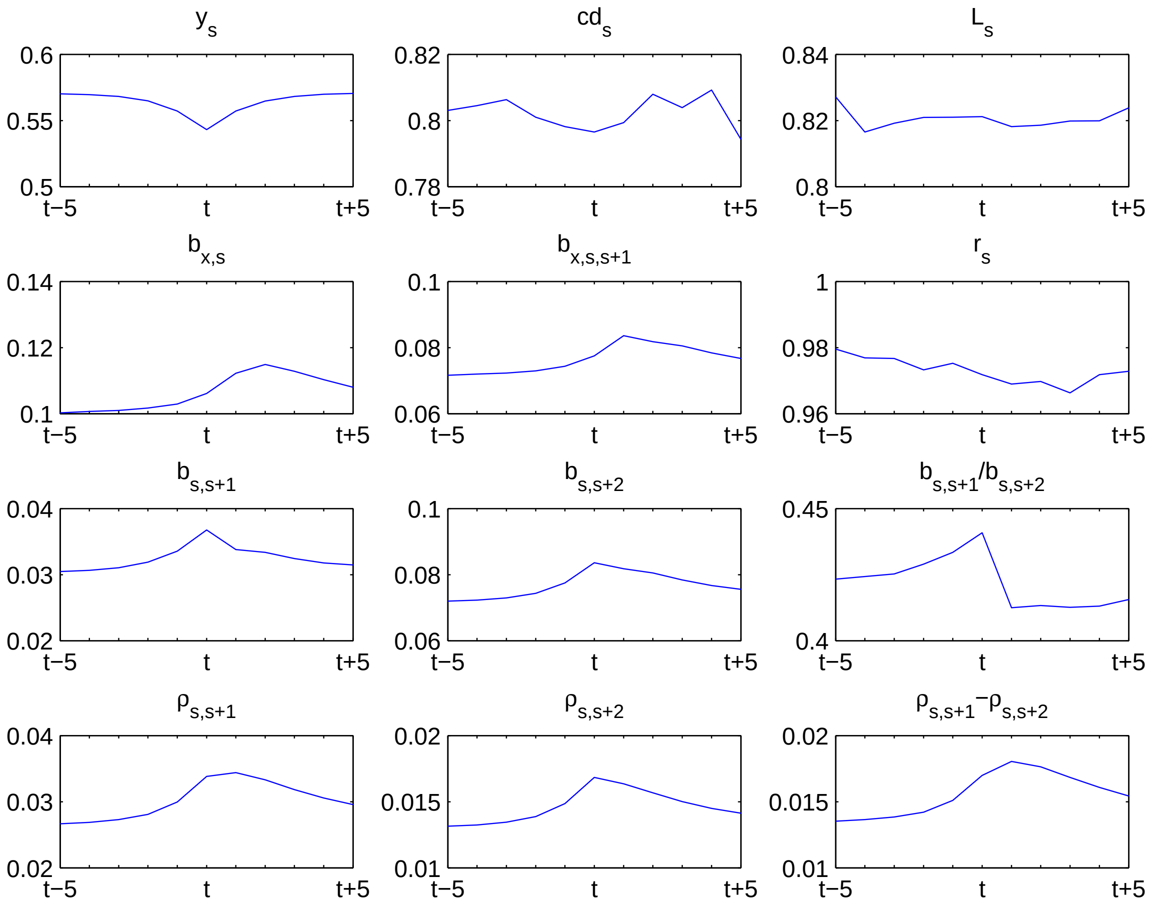

Figure 3: Typical dynamics around a low-income period (the panels display the sample averages of the respective variables in periods $s = t-5, \ldots, t+5$ conditional on $y_t = y^l$)