Disciplining Expectations: Using Survey Data in Learning Models

Arturo Ormeño*

Job Market Paper

8th November, 2009

Abstract

Do learning models of expectations formation capture actual expectations? In this study we address this question by using data from the Survey of Professional Forecasters in the estimation of a dynamic stochastic general equilibrium model under learning. Exploiting the moments of this data allows us to identify the parameters of the model better, particularly those related with the formation of the expectations. We find that, without the use of survey data, inflation expectations estimated under the assumption of learning capture some features of the data not related with actual expectations, such as changes in the trend of inflation. Once surveys are included, learning not only matches actual expectations but also emerges as a key determinant of inflation persistence, explaining around 30 percent, while exogenous shocks only account for approximately 15 percent.

JEL classification: C11, D84, E30, E52

Keywords: Survey data; Learning models; Inflation persistence; Monetary policy analysis; Bayesian econometrics.

---

*Department of Economics and Business, Universitat Pompeu Fabra. I thank Fabio Canova, Klaus Adam, Anton Braun, Marco del Negro, Davide Debortori, Jose Dorich, Christina Felfe, Jordi Gali, Albert Marcet, Kristoffer Nimark, Franck Portier, Thijs van Rens, Sergey Slobodyan, Robert Zymek and the participants of the Society for Economic Dynamics (SED) 2009 Annual Meeting, the XIV Workshop on Dynamic Macroeconomics organized by Universidad de Vigo, the 8th Macroeconomic Policy Research Workshop on DSGE model organized by Magyar Nemzeti Bank and the Centre for Economic Policy Research (CEPR), and the Macroeconomic student seminar at UPF for useful comments and suggestions. I am grateful to Sergey Slobodyan for sharing his MATLAB codes. Any comments and/or suggestions are welcome at arturo.ormeno@upf.edu

1. Introduction

Expectations are a key element in economic analysis because they affect the decisions of households and firms. One way to model expectations that has gained popularity in recent years is learning. Under this assumption, agents' expectations about future variables are generated by forecasting models that reflect their understanding about how the economy works. These models are obtained by analyzing historical data and are updated whenever new data becomes available (see Evans and Honkapohja 2001). Learning is particularly attractive because, where present, it increases the propagation mechanisms of the model and renders the addition of ad hoc elements like persistent exogenous shocks unnecessary. However, learning has also been subjected to criticism as it relies heavily on the researcher's (arbitrary) assumptions about the forecasting model used by the agents to generate their expectations. To address this criticism, this study uses information contained in surveys on inflation expectations in order to determine the forecasting model that agents most likely use to predict inflation. Additionally, by exploiting the moments derived from survey information, we better identify the parameters of the model, particularly those related with the formation of the expectations.

The empirical question we address in this study relates to the identification of the sources of inflation persistence. Policy recommendations aimed at reducing inflation persistence – which would permit monetary authorities to keep inflation under control without affecting real activity too much – hinge crucially upon the sources of the observed persistence. Yet there is no consensus about whether the latter mainly arises from features of goods and labor markets (such as indexation to past inflation), from persistent shocks that affect the pricing decision of firms, or from people's ways of forming their expectations about future inflation. Estimations under learning indicate that the process of expectations formation plays an important role as a source of inflation persistence (Slobodyan and Wouters 2009). We evaluate this evidences using survey information.

The results of our study provide evidence that inflation expectations implied by a learning model capture mostly low frequency movements of inflation when survey data are ignored. This becomes evident as soon as the estimation is implemented for a period of stable inflation levels. In that case, learning seems to be irrelevant for the generation of inflation persistence while exogenous shocks play the most prominent role. However, the incorporation of survey data not only allows the model to match actual expectations but also allows learning to emerge as a key determinant of inflation persistence. Thus, even in a period of stable inflation, learning explains around 30 percent of inflation persistence while exogenous shocks only account for approximately 15 percent.

The estimations implemented in this study are derived from the medium-size New Keynesian DSGE model developed by Smets and Wouters (2007), one of the benchmark models for empirical analysis. In this sense, this study is related with the work of Slobodyan and Wouters (2007 and 2009), who are the first to estimate learning in the context of a medium-size DSGE model. Working with this type of model reduces the risk that some omitted variable could distort the contribution of learning to the estimated dynamics.

Only Del Negro and Eusepi (2009) have incorporated survey data into the estimation of a DSGE model so far. As in the present study, the work of Del Negro and Eusepi is based on a New Keynesian model. However, the type of imperfect information considered by these authors is related to time-varying inflation targets of policy-makers, as in Erceg and Levin (2003). Here, by contrast, the representative agent is not aware of the law of motion of inflation, and survey expectations are employed to determine the model which agents most likely use.

The remainder of the paper is organized as follows. In the next section we describe the model used for the estimation. Section 3 discusses the setup of the learning process, while Section 4 presents the series of macroeconomic indicators used in the estimation as well as their relationship with the variables of the model, and the forecasting model for inflation used in the estimation under learning. Section 5 contains the results of estimating the model under RE and learning with and without the use of survey data on inflation expectations. There, we show the sources of inflation persistence for each of the cases and how survey data improve their identification. Finally, Section 6 concludes and outlines possible avenues for future research.

2. The Model

Our estimation is based on a New Keynesian model similar to Smets and Wouters (2007).

This model incorporates several frictions affecting both nominal as well as real decisions of households and firms. Households maximize their utility over an infinite life-time horizon. Their utility function depends on the consumption of goods, which is considered relative to a time-varying external habit variable, and on the amount of labor supplied. They also own the stock of capital in the economy, which they can either rent to firms or accumulate, subject to an adjustment cost. Differentiation in the household's labor provides them of monopolistic power over wages. Firms, on the other hand, produce differentiated goods, decide on the amount of labor and capital services and finally set their prices. Both, prices and wages are affected by nominal rigidities *à la* Calvo and additionally incorporate partial indexation with respect to past inflation. Finally, the model features a deterministic growth rate driven by labor-augmenting technological progress.

The version used in this study departs from the original specification by Smets and Wouters (2007) in only two respects. First, monetary policy rule does not adjust to the output gap (i.e. the difference between the output obtained under nominal rigidities and under flexible prices). Instead, monetary policy reacts to changes in the level of output (produced by the economy with rigidities) from one period to the next. This modification allows us to avoid the estimation of a parallel economy under flexible prices, which reduces the number of variables in the model considerably (as in Slobodyan and Wouters, 2009)[1]. Second, the stochastic shocks that affect wages and prices directly, namely price and wage mark-up shocks, are assumed to be autoregressive processes that do not incorporate past perturbations (in other words they are AR(1) processes, not ARMA(1,1)).

The model contains the following thirteen endogenous variables: output, y; consumption, c; investment, i; the value of the capital stock, Q^k; the installed stock of capital, $\overline{k}$; stock of capital, k; inflation, π; the capital utilization rate, u; the real rental rate on capital, r^k; the real marginal cost, mc; real wages, w; hours worked, L; and the interest rate, R. In addition, the stochastic part of the model is characterized by seven exogenous autoregressive processes, each of them including an *iid*-normally distributed error term. After detrending the model with respect to the deterministic growth rate of the labor-augmenting technological progress and linearizing it around the steady-state of the detrended variables, the model can be summarized in the following set of equations (where ^ represents detrended variables and * their steady state values)[2, 3]:

[1] This modification, however, does not affect the results obtained by Smets and Wouters (2007).

[2] The optimization problem of the households, firms and government as well as the equilibrium conditions are shown in Appendix 1.

[3] The exact representation of the exogenous stochastic process is presented in Appendix 2.

(1) $\hat{y}_t = \hat{g}_t + \frac{c_*}{y_*}\hat{c}_t + \frac{i_*}{y_*}\hat{i}_t + \frac{r_*^k k_*}{y_*}\hat{u}_t$

(2) $\hat{c}_t = \frac{1}{(1+h)}\left(E_t\hat{c}_{t+1} + h\hat{c}_{t-1}\right) - c_1(E_t\hat{L}_{t+1} - \hat{L}_t) - c_2(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \hat{b}_t$

(3) $\hat{i}_t = \frac{1}{1+\overline{\beta}\gamma}(\hat{i}_{t-1} + \overline{\beta}\gamma E_t\hat{i}_{t+1} + \frac{1}{\gamma^2 S''}\hat{Q}_t^k) + \hat{q}_t$

(4) $\hat{Q}_t^k = -(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{r_*^k}{r_*^k + (1-\delta)}E_t\hat{r}_{t+1}^k + \frac{(1-\delta)}{r_*^k + (1-\delta)}E_t\hat{Q}_{t+1}^k + \tilde{b}_t$

(5) $\hat{y}_t = \Phi(\alpha\hat{k}_t + (1-\alpha)\hat{L}_t + \hat{A}_t)$

(6) $\hat{k}_t = \hat{u}_t + \hat{\hat{k}}_{t-1}$

(7) $\hat{u}_t = \frac{1-\psi}{\psi}\hat{r}_t^k$

(8) $\hat{\hat{k}}_t = (1 - i_*/\overline{k}_*)\hat{\hat{k}}_{t-1} + \frac{i_*}{\overline{k}_*}\hat{i}_t + \frac{i_*}{\overline{k}_*}\left(1+\overline{\beta}\gamma\right)\gamma^2 S''\hat{q}_t$

(9) $\hat{\pi}_t = \frac{1}{\left(1+\overline{\beta}\gamma l_p\right)}(l_p\hat{\pi}_{t-1} + \overline{\beta}\gamma E_t\hat{\pi}_{t+1} + c_{mc}\widehat{mc}_t) + \hat{\lambda}_{p,t}$

(10) $\widehat{mc}_t = \left(1-\alpha\right)\hat{w}_t + \alpha\hat{r}_t^k - \hat{A}_t$

(11) $\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t$

(12) $\hat{w}_t = \frac{1}{(1+\overline{\beta}\gamma)}(\hat{w}_{t-1} + \overline{\beta}\gamma(E_t\hat{w}_{t+1} + E_t\hat{\pi}_{t+1}) - (1+\overline{\beta}\gamma l_w)\hat{\pi}_t + l_w\hat{\pi}_{t-1} +$

$$+c_w(\frac{1}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) + \sigma_l\hat{L}_t - \hat{w}_t)) + \hat{\lambda}_{w,t}$$

(13) $\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(r_\pi\hat{\pi}_t + r_{\Delta y}(\hat{y}_t - \hat{y}_{t-1})) + \hat{r}_t$

Equations (1) to (4) represent the demand side of the economy. Equation (1) is the aggregate resource constraint of the economy and indicates that output is spent on consumption, investment or absorbed via capital-utilization costs that are a function of the capital utilization rate, and exogenous spending $\hat{g}_t$. Equation (2) represents the Euler equation for consumption where $h = \eta/\gamma$, $c_1 = \frac{(\sigma_c - 1)w_*^h L_*/c_*}{\sigma_c(1+h)}$, $c_2 = \frac{1-h}{\sigma_c(1+h)}$. Note that η captures the external habit formation, γ is the growth rate of the labor-augmenting technological process, σ_c is the inverse of the elasticity of intertemporal substitution and w_*^h is the nominal wage received by households in the steady state. This equation implies that current consumption depends on a weighted average of past and expected future consumption, on the expected growth in hours worked, the ex-ante real interest rate $(\hat{R}_t - E_t\hat{\pi}_{t+1})$, and a disturbance term $(\hat{b}_t)$.The Euler equation for investment is represented by Equation (3), where $\overline{\beta} = \beta/\gamma^{\sigma_c}$. β represents the discount factor applied by households and S'' stands for the steady-state elasticity of the capital adjustment cost function. The impact of the real value of existing capital stock ($\hat{Q}^k$) on investment depends on this elasticity. $\hat{q}_t$ is a disturbance to the investment-specific technology process. Equation (4) represents the arbitrage equation for the value of capital. It states that the current value of the capital stock depends positively on its expected future value and the expected real rental rate on capital, but negatively on the ex ante real interest rate and the risk premium disturbance, $\tilde{b}_t = c_2\hat{b}_t$. δ represents the depreciation rate.

The supply side of the economy is characterized by Equations (5) to (12). The aggregate production function, Equation (5), indicates that output is produced using capital and labor services as inputs and is affected by the total factor productivity $\hat{A}_t$. α captures the share of capital in production and Φ equals one plus the share of fixed costs in production. Current capital used in production is assumed to be a linear function of the degree of capital utilization $\hat{u}_t$ and the installed capital in the previous period, $\hat{\bar{k}}_{t-1}$, Equation (6). The latter argument reflects the assumption that new capital becomes effective only with a one-quarter lag. The positive relationship between the degree of capital utilization and the rental rate of capital is represented by Equation (7). In this equation ψ is a positive function of the elasticity of the capital utilization adjustment cost function and is normalized to a value between zero and one. Equation (8) represents the accumulation of installed capital as a function of the flow of investment and the relative efficiency of the investment expenditures captured by the investment-specific technology disturbance. The New-Keynesian Phillips curve is represented by Equation (9) and incorporates partial indexation of lagged inflation, where l_p represents the degree of indexation to past inflation, ε_p the curvature of the Kimball (1995) goods market aggregator, $\phi_p - 1$ the share of the fixed cost in production, ξ_p the degree of price stickiness and c_{mc} the slope related to marginal cost, where

$$c_{mc} = \frac{(1-\xi_p\bar{\beta}\gamma)(1-\xi_p)}{\xi_p((\phi_p - 1)\varepsilon_p + 1)}.$$

Finally, $\hat{\lambda}_{p,t}$ stands for the price mark-up disturbance and follows an AR(1) process. The marginal cost $\widehat{mc}_t$ is defined by Equation (10). Equation (11) signifies that the rental rate of capital is positively related with the capital-labor ratio, but negatively with the real wage. In the same way that nominal rigidities affect the price level determination, real wages can only adjust gradually to their optimal level. Equation (12) shows how the real wage is determined, where l_w represents wage indexation, ε_w the curvature of the Kimball (1995) aggregator of labor, $\phi_w - 1$ the steady state labor market mark-up; and c_w represents

$$c_w = \frac{\left(1-\xi_w \overline{\beta}\gamma\right)\left(1-\xi_w\right)}{\xi_w\left(\left(\phi_w - 1\right)\varepsilon_w + 1\right)}.$$

Analogously to above, $\hat{\lambda}_{w,t}$ stands for the wage mark-up disturbance and it follows an AR(1) process.

Finally, Equation (13) represents the monetary policy rule where ρ_R captures the degree of smoothing over the policy instrument and r_π and $r_{\Delta y}$ represent the responses of this instrument to deviations of inflation and output growth from their targets. $\hat{r}_t$ represents the non-systematic component of the interest rate and is assumed to follow an AR(1) process.

3. Learning mechanism of expectations formation

The model of the previous section incorporates expectations of several future variables. When dealing with expectations, researchers have traditionally adopted the rational expectations (RE) assumption. This assumption implies that agents have perfect knowledge of the true stochastic process of the economy. Arguably such high level of cognitive abilities and computational skills seem to be implausible in practice, researchers have developed models of imperfect knowledge and associated learning processes. One of the most popular learning mechanisms used in macroeconomics is a form of adaptive learning. Under this approach agents are assumed to use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data becomes available (see Evans and Honkapohja 2001).

It is common in the literature on learning to assume that agents update the coefficients of their forecasting models using constant gain least squares (CG-LS). Under CG-LS, the most recent observations receive higher weights in the least square estimation. More precisely, the weight decreases geometrically depending on the distance in time to the most recent observation. This learning mechanism implies that agents are concerned about changes in the structural parameters of the economy.

In the remainder of this section we provide details of the algorithm followed by the representative agent to update her expectations and characterize the resulting dynamics of the economy. We also specify the forecasting models used in this study and the initial conditions of the recursive CG-LS.

3.1 Ordinary Least Squared with constant gain

The forecasting model agents use to generate one-period-ahead expectations of the set of variable Y^f can be represented as:

(14) $$Y_t^f = \beta' X_{t-1},$$

the recursive expression for the estimate of β under CG-LS is:

(15a) $$\hat{\beta}_t = \hat{\beta}_{t-1} + g \ \left(R_t\right)^{-1} X_t \left(Y_t^f - \hat{\beta}'_{t-1} X_t\right)'$$

(15b) $$R_t = R_{t-1} + g \ \left(X_t X_t' - R_{t-1}\right)$$

where g represents the *constant gain parameter* and R_t the variance-covariance matrix of the regressors included in the forecasting model. The “gain” refers to the relative weight of the most recent observation and 1 – g is the discount factor over less recent observations (in ordinary least squares, the gain is equal to *1/t*, where *t* is the position of the observation since the beginning of the sample).

Using β, we can generate the forecast of the variables included in Y_t^f :

(16) $$\hat{E}_t Y_{t+1}^f = \hat{\beta}'_{t-1}[X_t]$$

Employing $\hat{\beta}_{t-1}$ instead of $\hat{\beta}_t$ in equation (3.3) is a standard procedure in learning estimation in order to avoid the simultaneous determination of $\hat{\beta}_t$ and the variables included in the solution of the model.

3.2 Model expectations augmented by learning

Using equations (1) to (13), described in Section 2, we can derive the equilibrium conditions describing the dynamics and the interactions of all endogenous variables under RE. For ease of representation, it is useful to use a generic form of the solution under RE:

$$\text{(17)} \qquad \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA^{re} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB^{re} \left[\vartheta_t \right]$$

where Y_t contains all the endogenous variables of the model, Z_t contains all the exogenous variables and ϑ_t contains all their iid-normal perturbations. The matrices AA^{re} and BB^{re} are nonlinear functions of the structural parameters of the DSGE model.

When estimated under learning, the set of equations (1) – (13) is augmented by equation (16), by equations (15a) and (15b) which describe the estimation procedure for the β, and by some initial conditions for the CG-LS algorithm (which we describe later). The only parameter that is added to the set of structural parameters is the gain parameter.

Under learning, we can rewrite the system containing all endogenous variables of the model and replace those in expectations in the following compact way:

$$\text{(18)} \qquad \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA_{t-1}^{learning} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB_{t-1}^{learning} \left[\vartheta_t \right]$$

Matrices $AA_{t-1}^{learning}$ and $BB_{t-1}^{learning}$ vary over time, as they contain not only the parameters of the structural model, but also the time-varying coefficients of the forecasting models ($\hat{\beta}_{t-1}$). The time variations of these coefficients depend on the value of the gain parameters *g*. If these parameters are equal to zero, the matrices $AA^{learning}$ and $BB^{learning}$ are constant. However, even in this situation they might not be equal to the corresponding matrices under RE, because $AA^{learning}$ and $BB^{learning}$ depend on the selection of the forecasting model and the initial conditions of the CG-LS.

3.3 Learning setting used in this study

We use survey data on inflation expectations to determine the forecasting model for inflation used by agents. Due to lack of availability for the complete sample, surveys can unfortunately not be applied in the selection of the forecasting model for all other variables that appear in expectations in the model. We therefore use as forecasting models specifications with the same set of regressors that are included in the RE case.

In our estimations, Equation (14) can be rewritten in the following way:

(19a) $$Y_t^{\pi} = \beta^{\pi}{}' X_{t-1}^{\pi}$$

(19b) $$Y_t^{non-\pi} = \beta^{non-\pi}{}' \left[X_{t-1}^{non-\pi} \right]$$

where $Y_t^{\pi} = \left[dlP\right]_t$, $Y_t^{non-\pi} = [\hat{c}_t \quad \hat{i}_t \quad \hat{q}_t \quad \hat{r}_t^k \quad \hat{w}_t \quad \hat{L}_t]$, $X_{t-1}^{\pi} = \left[1 \quad dlP_{t-1}\right]$, and $X_{t-1}^{non-\pi} = [1 \quad Y_{t-1} \quad Z_{t-1}]$, where *dlP* refers to the actual series of inflation used in the estimation of the DSGE model (see the next section). $\beta^{non-\pi}$ contains rows of matrices AA^{re} and BB^{re} corresponding to the variables in $Y_t^{non-\pi}$. X_{t-1}^{π} and $X_{t-1}^{non-\pi}$ includes an intercept to allows for temporal deviations from the estimated steady state of the model with respect to the pure RE specification.

The previous equations indicate that we are dealing with two learning blocks: one related to the inflation process and the other to the processes of the remaining variables which appear with expectations in the model. Therefore, we have not one but two gain parameters and we need to define two sets of initial conditions for the CG-LS.

With respect to the forecasting model of inflation, the initial values β_0^{π} and R_0^{π} are obtained using actual data of inflation for a pre-sample. The initial values $\beta_0^{non-\pi}$ and $R_0^{non-\pi}$ for the forecasting model of the remaining variables are taken from the solution under rational expectations. The value of $\beta_0^{non-\pi}$ comes from the expression (17), while the value of $R_0^{non-\pi}$ can be derived from the expression of the unconditional variance matrix of Y_t obtained from the solution of RE.

Finally, it is important to mention that the equilibrium achieved by the model is not longer compatible with the equilibrium under RE because we use a forecasting model (for inflation) that is misspecified in terms of the solution under RE. The relevant equilibrium concept in this case is the *restricted perceptions equilibrium* (RPE)[4] and is motivated by agents that, because they are unaware of the "true" structure of the economy, find it optimal to use small

[4] The name of Restricted Perceptions Equilibrium (RPE) was given by Evans and Honkapohja (2001). Branch (2004) discusses the generality of RPE as it encompasses many forms of misspecified equilibria such as the Self-Confirming Equilibrium in Sargent (1999) and the Consistent Expectations Equilibrium in Hommes and Sorger (1998).

(misspecified) forecasting models. Two conditions are required for this equilibrium to exist: first, the selected forecasting model should generate a lower mean-squared error than the one produced by other potential models and second, the equilibrium achieved has to be expectationally stable. Both conditions hold in this study.

4. Data, measurement equations and priors

The model is estimated using the same quarterly macroeconomic indicators for the US as in Smets and Wouters (2007), but in addition we also use survey data on inflation expectations provided by the Survey of Professional Forecasters (SPF). In particular, for each quarter, we calculate the median value of the reported one-period-ahead forecast of the percentage increase of the GDP deflator. The resulting series is henceforth referred to as "*exp_dlP*". As this data is available only from 1968Q4 onward, this date marks the starting point for our sample. The sample covers all quarters until 2008Q2. The other macroeconomic indicators are the log difference of real GDP ("*dlGDP*"), of real consumption ("*dlCons*"), of real investment ("*dlInv*") and of the real wage ("*dlWage*"), as well as the log of hours worked ("*lHours*"), the log difference of the GDP deflator ("*dlP*") and the federal funds rate ("*FedFunds*"). Appendix 3 contains a description of the data.

The way in which these macroeconomic indicators are related with the variables of the model under RE and learning when survey data on inflation expectations are not included is summarized by the following measurement equations:

$$\begin{bmatrix} dlGDP_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFunds_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}$$

where $\overline{\gamma} = 100(\gamma - 1)$ represents the common quarterly trend growth rate, $\overline{l}$ the steady state hours worked, $\overline{\pi} = 100(\Pi_* - 1)$ the quarterly steady state inflation rate and $\overline{r} = 100(\gamma^{\sigma_c}\Pi_* / \beta - 1)$ the quarterly steady state nominal interest rate.

When survey data is incorporated in the estimation under RE and under learning, the measurement equations are:

$$\begin{bmatrix} dlGDP_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFunds_t \\ \exp\_dlP_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ E_t\hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \zeta_t \end{bmatrix}$$

where ζ_t represents the measurement error (*iid*) related to the surveys on inflation expectations. Hence, survey data is taken as a noisy measurement of actual expectations.

Besides being used for estimating the DSGE model, the macroeconomic series are used to select the forecasting model for inflation employed in the learning estimation. In particular, we compare the models that can be obtained using as regressors (besides an intercept) all possible combinations of the lagged series of dlGDP, dlCons, dlInv, dlWage, dlP, FedFunds and lHours. Then, we rank these models (127 in total) according to their forecasting performance with respect to the survey data on inflation expectations, measured by the Mean Squared Error (MSE). Table 1 shows the five best-performing forecasting models for the periods 1968Q4 – 2008Q2 and 1984Q1 – 2008Q2 (the latter is used for robustness check of some of the results presented in the next section)[5]. In both cases, the best model is the one that considers as regressors only lagged inflation and the intercept.

[5] Let us outline the manner in which this ranking is constructed. First, each model is estimated using a recursive CG-LS, which allows for compatibility with the algorithm behind the inflation expectations formation under learning. Second, recursive CG-LS requires the definition of the initial values for the coefficients to be estimated and of the variance-covariance matrix of the regressors. These values are obtained using Ordinary Least Squares (OLS) over a pre-sample. Third, different values of the constant gain are employed to produce forecasts for each of the models (these values are taken from a grid of points that goes from 0 to 0.30, with steps of 0.0125). The ranking is then established taking into account for each of the models the value of the constant gain that results in the best MSE. Finally, given that the ordering could vary depending on the choice of the pre-sample, different pre-samples are considered and we select the one with the lowest MSE among the top models.

Table 1
Ranking of forecasting models by MSE

	Period: 1968Q4 - 2008Q2			Period: 1984Q1 - 2008Q2		
Rank	Model	Gain	MSE	Model	Gain	MSE
1	dlP	0.125	0.0294	dlP	0.225	0.0238
2	dlP lHours	0.113	0.0300	dlP dlCons	0.238	0.0250
3	dlP dlCons	0.100	0.0302	dlP dlCons FedFunds	0.150	0.0264
4	dlP dlCons lHours	0.125	0.0303	dlP FedFunds	0.150	0.0266
5	dlP dlGDP	0.125	0.0315	dlP dlCons dlInv FedFunds	0.138	0.0277

Note: the models are estimated by recursive CG-LS. The initial conditions for the periods 1968Q4-2008Q2 and 1984Q1-2008Q2 are obtained from the periods 1950Q1-1968Q3 and 1974Q1-1983Q4, respectively. Regression: dlP_t = intercept + $model_{t-1}$

The model contains 38 structural parameters. 33 of them are estimated while the remaining 5 are fixed to the same values as in Smets and Wouters (2009)[6]. The learning estimation adds another two parameters (the gains for inflation and for the other variables that appear in expectations). When estimating the model with survey data, we consider one extra parameter, namely the standard deviation of the measurement error of the surveys (ζ_t). The prior distributions of the structural parameters are as in Smets and Wouters; for the gains, uniform distributions over the [0,0.30] domain; and for the standard error of ζ_t, an inverse gamma distribution with zero mean and standard deviation of 2. The prior distributions for all the parameters are presented in Appendix 4.

The estimation of the DSGE model is performed using Bayesian estimation methods. Employing the random walk Metropolis-Hastings algorithm I obtain 500 000 draws from each model's posterior distribution. The first half of these draws is discarded and 1 out of every 10 is selected in order to estimate the moments of the posterior distributions.

5. Results

The main objective of this study is to identify the sources of inflation persistence under learning when survey data on inflation expectations are incorporated in the estimation. As we show in this section, the estimation of learning models is sensitive to the presence of low frequency movements (such as changes in trend) of the series under analysis. For this reason,

[6] These parameters are the depreciation rate (fixed to 0.025), the exogenous spending-GDP ratio (0.18), the steady state mark-up in the labor market (1.5) and the curvature parameters of the Kimball (1995) aggregators in the goods and labor market (both set at 10).

as a first step, we compare the predictions for inflation persistence yielded by the models under learning in a period of stable inflation (post-1984 sample) when survey data are included in the estimation and when they are not. In a second step, we repeat the previous analysis but considering a sample that includes a period of important changes in the inflation trend. Finally, we discuss the robustness of our findings when different forecasting models for inflation are used under learning.

5.1 Learning estimation in a period of stable inflation

As first step in our analysis we estimate the DSGE model under learning for the post-1984 sample (1984Q1-2008Q2) and discuss the results obtained in terms of the posterior statistics of the parameters which are closely related to the dynamics of inflation (Table 2). The posterior statistics of the complete set of parameters can be found in Appendix 5.

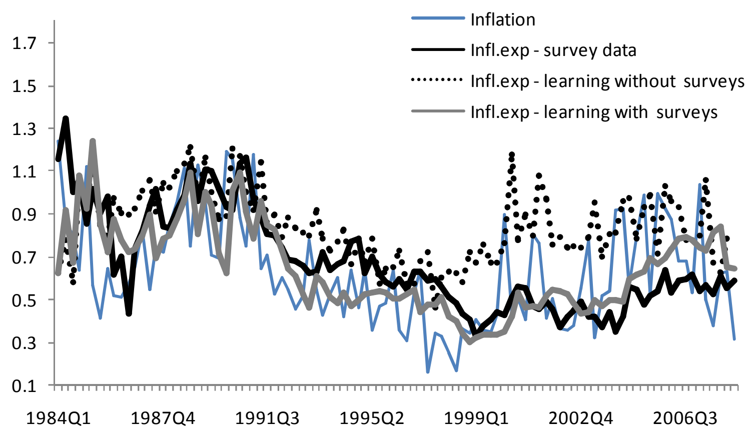
Table 2
Posterior distribution statistics under learning, Post-1984 sample

		(1) Without surveys		(2) With surveys	
	Symbol	Median	Std	Median	Std
Wage stickiness	ξ_w	0.464	0.070	0.461	0.067
Price stickiness	ξ_p	0.619	0.025	0.681	0.034
Wage indexation	ι_w	0.392	0.135	0.368	0.125
Price indexation	ι_p	0.463	0.118	0.665	0.102
TR: inflation	r_π	1.650	0.170	1.526	0.169
TR: lag interest rate	ρ_R	0.806	0.024	0.793	0.034
TR: change in output	$r_{\Delta y}$	0.183	0.048	0.181	0.044
aut. Price Mk up shock	ρ_p	0.075	0.053	0.113	0.067
std. Price mkup shock	σ_p	0.111	0.012	0.178	0.016
gain - inflation	g^{π}	0.006	0.004	0.201	0.006
gain - others	$g^{non\pi}$	0.047	0.037	0.005	0.009
Measurement exp error	σ_{exp}			0.151	0.010

As shown by Table 2, the inclusion of survey data in the estimation of the DSGE model under learning leads to higher levels of price stickiness, a higher degree of price indexation and a higher autoregressive component of the price mark-up shocks. However, the most significant change in terms of the median value of the posterior distributions is related to the gain parameter of inflation learning process (its posterior median increases from zero to 0.20). As we show bellow, the change in this parameter generates a reduction in inflation persistence that is partially compensated by the increases in price stickiness, price indexation and the autoregressive component of the price mark-up shocks. As a whole, these changes have an important impact on the way in which the learning model matches the survey data on inflation expectations, on the evolution of the coefficients of the forecasting model of inflation and on the identification of the sources of inflation persistence.

As Figure 1 shows, without the use of survey data, the model-implied inflation expectations resulting from learning fail to match survey inflation expectations. In particular, this series overestimates the value of actual expectations during most of the sample and also reflects high frequency movements in the underlying behavior of inflation. As would be expected, the introduction of survey data in the estimation of the learning model results in a better match between the model-implied inflation expectations and the survey data. In particular, the high frequency movements of the model-implied inflation expectations are significantly reduced. The better fit of the estimated model with respect to the survey data can be summarized by the correlation coefficient between model-implied inflation expectations and the inflation expectations provided by surveys which increases from 0.43 when surveys are not included in the estimation to 0.75 when surveys are included.

Figure 1
Inflation and inflation expectations, Post-1984 sample



It is important to note that the use of surveys improves the fit of the model estimates with respect to most of the series incorporated in the estimation, not only with respect to inflation expectations. For instance, in terms of the root mean squared errors (RMSE), the model explains the evolution of the series of consumption, output and real wage growths; the interest rate and the number of hours worked better (see Table 3). The only series that display higher RMSE when the estimation includes surveys are the series of investment growth and inflation. The slight decrease in the fit of the latter series is a result of the fact that the estimated model now has to match the data on inflation as well as the data on inflation expectations. Thus, the previous results should be understood as the interaction of two effects. On the one hand, the incorporation into the estimation of one extra moment to match negatively affects the fit of some of the variables incorporated previously. On the other hand, the larger amount of information helps to identify some parameters better which were defined by their prior distribution without the additional information, as is the case for the gain parameter for inflation expectations.

Table 3
In sample RMSE - estimation under learning, Post-1984 sample

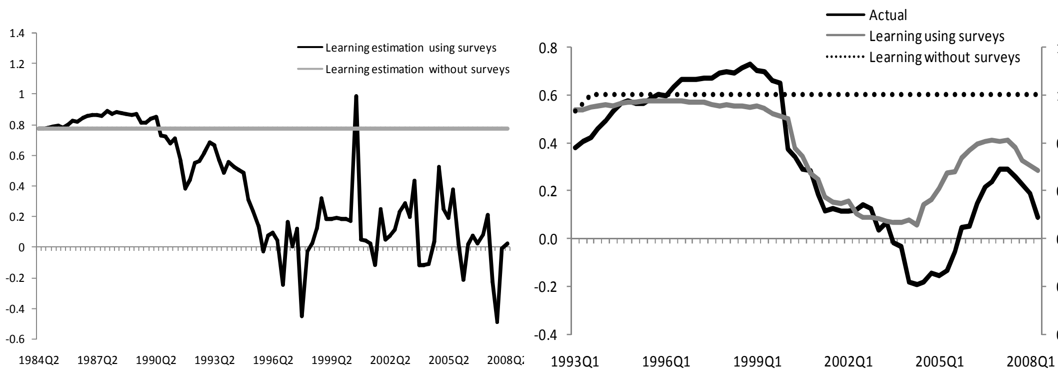
	(1)	(2)	(1)-(2)
Variable	w/o survey	w survey	
Consumption growth	0.542	0.521	0.021
Investment growth	1.486	1.497	-0.010
Wages growth	0.704	0.697	0.007
Inflation	0.227	0.228	-0.001
Output growth	0.559	0.552	0.007
Interest rate	0.107	0.101	0.006
Hours worked	0.390	0.382	0.008
Inflation expectations		0.139	

Another implication of the use of survey data in the estimation of learning models is more closely related with the estimates of the gain parameter of the learning process of inflation. The high posterior median value obtained for this parameter implies that the coefficients of the forecasting model for inflation are time-varying. For instance, Figure 2a shows the evolution of the coefficient of lagged inflation, also denoted “perceived inflation persistence”. The initial value of this coefficient is 0.77 but over the sample period it drops significantly to around 0.1. The time-variability of this coefficient makes it possible for the learning model to match the evolution of the correlation between inflation and expected inflation provided by survey data (see Figure 2b). When surveys are not incorporated in the estimation, the coefficient of lagged inflation remains equal to its initial value (the gain parameter is equal to zero in this case, which implies that new information does not affect the coefficient estimates). As a result, the learning model fails to match the correlation between inflation and expected inflation (this correlation is equal to one when the coefficients of the forecasting model are not time-varying).

Figure 2

Evolution of lagged inflation coefficient and the correlation between inflation and inflation expectations

(a) (b)*



*The dates in the X-axis indicate the end of a rolling windows that includes 36 quarterly observations

Finally, changes in the parameter estimates resulting from the use of surveys in the estimation of the learning model have a significant impact on the relative importance of the sources of inflation dynamics. Thus, when estimating the learning model without using survey data (Table 4, Column 1[7]), the percentage of inflation persistence associated with the autoregressive components of the exogenous shocks is 38%. This derives mostly from highly autoregressive shocks such as the wage mark-up shock, and to a lesser extent from the price mark-up shock whose autoregressive coefficient displays a much lower value. However, once surveys are included in the estimation, the learning process associated with inflation explains 34% of inflation persistence while shocks only explain 13%[8] (Column 2). This important change in the composition of the sources of inflation persistence springs from the significant reduction of the

[7] The marginal contribution of the different sources of inflation persistence is calculated as follows. First, we simulate the macroeconomic indicators using the model with the parameters equal to: (i) their posterior median values, (ii) their posterior median values except for the autoregressive coefficients of the shocks which are equal to zero, (iii) their posterior median values except for the gain parameter of the learning process of inflation which is equal to zero, and (iv) their posterior median values except for the gain parameter of the learning process not related with inflation which is equal to zero. Second, we calculate for each of these cases the simulated inflation persistence. The marginal contribution of the autoregressive coefficients over the total inflation persistence corresponds to the percentage difference of the inflation persistence estimates obtained between (i) and (ii); the marginal contribution of learning associated with inflation can be measured as the percentage difference of the inflation persistence estimates obtained between (i) and (iii); and so on. Last, we repeat the previous steps for 10 sets of innovations per each draw of parameter sets taken from the posterior distributions. As a total, we use 500 draws of parameters sets to build the distributions of the marginal contributions per sources of inflation persistence.

[8] The calculation of the marginal contributions per sources changes in this case because when the gain parameter of the learning process of inflation is equal to zero the simulated inflation persistence increases. The latter situation constitutes the reference level of persistence to consider at the time of calculating the marginal contribution of the autoregressive coefficients of the shocks or the gain parameter of the learning processes over variables other than inflation.

coefficient on lagged inflation in the forecasting model of inflation. This coefficient not only affects the correlation between inflation and expected inflation but also the correlation between inflation and its own lag, or inflation persistence. The lower the value of this coefficient, the lower is the persistence of inflation. For this reason, the simulated inflation persistence is 0.641 when surveys are used and 0.745 when surveys are not included and the coefficient of lagged inflation remains at its initial value. The reduction of inflation persistence generated by the learning process also explains the increases in price stickiness, the degree of price indexation and the autoregressive component of the price mark-up shocks, which go some way towards compensating for this reduction. This is the case despite the fact that the simulated model overestimates the actual level of inflation persistence (the actual inflation persistence for this period is 0.4829 while the learning model with surveys generates a persistence of 0.641, and when surveys are not included a persistence of 0.745). The overestimation is a result of the fact that the DSGE model used in this estimation lacks ways to replicate the high frequency movements of inflation.

Table 4
Simulated inflation persistence under learning, Post-84 sample

	(1)			(2)		
	Without surveys			With surveys		
Actual inflation persistence: **0.4829**	16%	Median	84%	16%	Median	84%
Complete model	0.648	0.745	0.832	0.566	0.641	0.727
% explained by autocorr shocks	27%	38%	51%	5%	13%	22%
% explained by inflation learning	0%	0%	0%	27%	34%	39%
% explained by non infl. learning	0%	2%	3%	0%	1%	1%

Note: This table shows the median and the 16th and 84th percentiles of the inflation persistence coefficient obtained from simulated series. The marginal contributions associated with the autoregressive part of the shocks and the learning processes are measured in percentages.

To sum up, we find that the addition of survey data into the estimation of the learning model results not only in a better match of the survey inflation expectations but also of most of the variables included in the estimation. Moreover, it allows the model to reproduce changes in the correlation between inflation and expected inflation. Importantly, the learning process related with inflation plays a key role in explaining inflation persistence, a role that is completely omitted when survey data are not included in the estimation.

5.2 Learning estimation and the effects of changes in inflation trend

In this subsection we estimate the learning model for the complete sample (1968Q4-2008Q2). This sample incorporates some years in which inflation experienced significant fluctuations that are more likely to be associated with changes in trend than with business cycle movements (see Cogley and Sbordone 2005). The presence of this type of movements could affect the estimates under learning because under this procedure a model consistence condition over expectations, as under rational expectations, is absent.

The posterior distribution statistics for those parameters closely related to the dynamics of inflation are presented in Table 5 while the statistics for the rest of parameters can be found in Appendix 5.

Table 5
Posterior distribution statistics under learning, complete sample

		(1) Without surveys		(2) With surveys	
	Symbol	Median	Std	Median	Std
Wage stickiness	ξ_w	0.565	0.066	0.552	0.046
Price stickiness	ξ_p	0.489	0.038	0.470	0.032
Wage indexation	ι_w	0.335	0.101	0.334	0.115
Price indexation	ι_p	0.506	0.111	0.518	0.115
TR: inflation	r_π	1.390	0.120	1.409	0.113
TR: lag interest rate	ρ_R	0.778	0.025	0.773	0.029
TR: change in output	$r_{\Delta y}$	0.210	0.045	0.204	0.045
aut. Price Mk up shock	ρ_p	0.138	0.072	0.168	0.089
std. Price mkup shock	σ_p	0.211	0.013	0.203	0.014
gain - inflation	g^{π}	0.187	0.012	0.140	0.008
gain - others	$g^{non\pi}$	0.105	0.043	0.017	0.024
Measurement exp error	σ_{exp}			0.176	0.011

The posterior distribution statistics obtained when surveys are included in the estimation are very similar to those obtained when surveys are not. In comparison with the results obtained for the period post-1984, the main difference is that the gain parameters of the learning process are now very high when surveys are not included. In fact, in the previous subsection we show that these parameters are basically zero while here both are significantly higher, even higher than the estimates obtained using survey data.

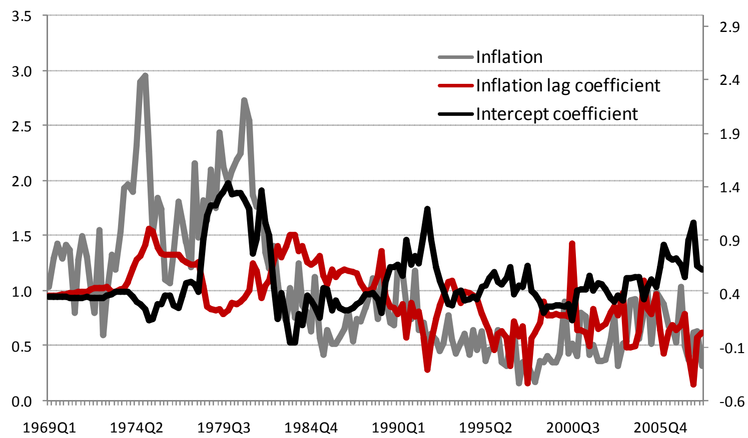
As indicated previously, the higher these parameters are, the more time-varying are the coefficients of the forecasting models. In fact, Figure 3a shows that both coefficients of the forecasting model for inflation change significantly over time[9]. However, this figure also reveals that these coefficients are capturing part of the low frequency movements of inflation observed during this period. In particular, the intercept seems to reflect precisely the magnitude and the duration of the significant increase of inflation which occurred between the

[9] The learning processes not related with inflation do not affect the persistence of inflation. For this reason we do not extent our analysis over the evolution of the coefficients of the forecasting models associated with these variables.

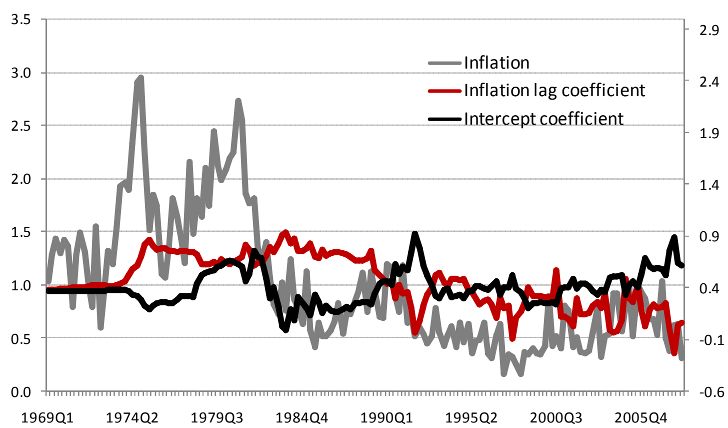
last quarter of 1977 and the beginning of 1982. The sharp and relatively short increase in inflation which resulted from the 1973 oil crisis is mainly captured by an increase in the coefficient of lagged inflation, and by a decrease in the intercept of the forecasting model. For the remaining sample, even though both coefficients keep varying, the intercept remains in the vicinity of 0.4 while the coefficient of the lagged inflation index fluctuates around 0.15.

Figure 3
Evolution of the coefficients of the forecasting model for inflation

(a) Estimation without using survey data

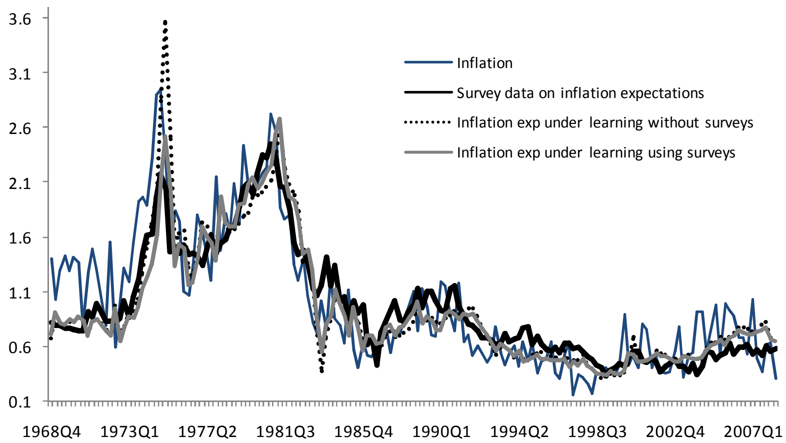


(b) Estimation using survey data



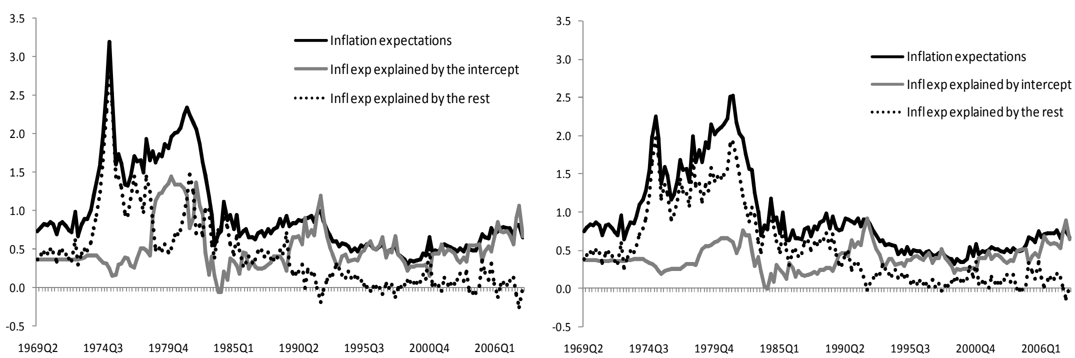
In contrast, the evolution of both coefficients obtained from the estimation that includes survey data are much less affected by the changes in the inflation trend. Figure 3b shows, on one hand, that the coefficient of the lagged inflation or “perceived inflation persistence” is around 0.9 during the years of high inflation (end of the 70s and beginning of the 80s), and after that starts to decline over the remainder of the sample period to around 0.2. On the other hand, the intercept does not deviate considerably from the sample mean of 0.4.

Figure 4
Inflation and inflation expectations, complete sample



The discrepancy in the evolution of these coefficients does not significantly affect the series of expected inflation that can be obtained under both learning estimations (with and without the use of surveys) but it does affect the underlying process of expectations formation. As Figure 4 shows, the main difference between both series arises because the series obtained without using survey data captures the peak of inflation observed in the year 1974 but this is not translated into expectations in the same way.

Figure 5
Composition of Inflation expectations
(a) Without using survey data (b) Using survey data



With respect to the underlying process of expectations formation, Figure 5 indicates that when surveys are not incorporated in the estimation, an important part of the increase in inflation expectations during years of high inflation (end of the 70s and first years of the 80s) is driven by the changes in the intercept of the forecasting model (or "perceived mean") for inflation. This is not the case when surveys are added in the estimation of the learning model. In this case, expectations are driven by the coefficient of lagged inflation (or "perceived persistence") multiplied by the value of inflation. In other words, without survey data, inflation expectations are driven by changes in the perceived inflation mean while using survey data, inflation expectations are driven by changes in the perceived persistence. Again, the comparison of the RMSE for the series incorporated in the estimation shows that survey data results in a better fit (see Table 6).

Table 6
In sample RMSE - estimation under learning, complete sample

	(1)	(2)	(1)-(2)
Variable	w/o survey	w survey	
Consumption growth	0.581	0.578	0.003
Investment growth	1.889	1.886	0.003
Wages growth	0.602	0.597	0.005
Inflation	0.305	0.302	0.003
Output growth	0.715	0.716	-0.001
Interest rate	0.264	0.256	0.008
Hours worked	0.519	0.517	0.002
Inflation expectations		0.188	

To sum up, the evidence presented in this and the previous subsection suggests that without the use survey data, learning captures low frequency movements in inflation but not necessarily inflation persistence. Thus, once we estimate the model only for a period of stable inflation, learning seems to be irrelevant as a source of inflation persistence. However, estimates that incorporate the additional moments from survey data capture the persistence pattern of inflation during both periods of stable and volatile inflation.

5.3 Robustness check: adding different forecasting models

In order to examine the robustness of the results presented above, we use different forecasting models for inflation in the estimation of the DSGE model under learning. Table 7 reports the posterior distribution statistics of the gain parameter of the learning process with respect to inflation, the log marginal likelihood and the in-sample root of the mean of squared inflation innovations (R*MSE inflation*) for different forecasting models[10].

[10] The in-sample root of the mean of squared innovations represents a measurement for the individual fit of each of the series included in the estimation.

Table 7
Comparison across forecasting models, learning estimation Post-84 sample[11]

	WITHOUT survey data				WITH survey data			
	Gain		Log Mg.	RMSE	Gain		Log Mg.	RMSE
Forecasting model	Median	Std	likelihood	inflation	Median	Std	likelihood	inflation
dlP	0.006	0.004	62.5	0.227	0.201	0.006	184.2	0.228
dlP dlCons	0.206	0.047	49.5	0.235	0.199	0.007	185.4	0.234
dlP dlCons FedFunds	0.005	0.003	62.1	0.228	0.150	0.008	170.0	0.245
dlP FedFunds	0.165	0.012	53.9	0.246	0.160	0.009	171.8	0.236
dlP dlCons dlInv FedFunds	0.005	0.005	58.0	0.231	0.150	0.015	167.9	0.238

Without considering survey data, the median of the posterior distributions of the gain parameter varies depending on the selected forecasting model. However, the cases where this parameter is zero also display the highest log marginal likelihoods (measurement of global fit of the model) as well as the lowest RMSEs for inflation (individual measurement fit). In other words, models with the best fit are the ones that report zero gain parameter. As indicated previously, a gain parameter close to zero indicates that learning plays only a small role in explaining inflation persistence.

When survey data is included in the estimation, the posterior median of the gain parameters lies between 0.15 and 0.20. These magnitudes imply an important time-variability in the structure of the economy and therefore a potential role for learning in explaining persistence. In fact, in terms of persistence the results are very similar to those reported in the previous subsections. Hence, survey data seem to be essential for clarifying the role of learning when explaining inflation persistence. Ignoring information from survey data can result in false conclusions being drawn. In particular, without survey data the models that fit the data best are those where learning does not introduce time-variation in parameters (gain parameter equal to zero).

In conclusion, our findings suggest that survey data on inflation expectations is important for an adequate identification of the role of learning in explaining the persistence of inflation. Ignoring survey data, different results can be obtained depending on the exact forecasting model for inflation which is selected. As soon as survey data is taken into account the message is unambiguous: learning plays a key role in explaining inflation persistence.

[11] The forecasting models behind the results of this table are the top-five forecasting models obtained using the same steps presented in subsection 5.1 but for the simple 1984Q1-2008Q2. The pre-sample used to initialize the CG-LS is the period 1974Q1-1983Q4.

6. Conclusions

In this paper we evaluate the role of learning in explaining the persistence of inflation when survey data on inflation expectations are incorporated in the estimation of this type of models.

Employing surveys in the context of a DSGE model under learning is a novel strategy and we use it in the following two ways. First, we exploit the information provided by surveys to determine the forecasting model used by agents. Second, we add surveys to the estimation of a DSGE model which allows us exploit the moments derived from this information, and thereby to identify better the parameters of the model.

When ignoring survey data, we find that inflation expectations generated by a model under learning are influenced by low frequency movements of inflation. This result becomes evident when the estimation is implemented during a period of stable inflation levels. In this case, learning does not seem to play a role as a source of inflation persistence while exogenous shocks are the most prominent source. However, this is no longer true once survey data are added into the estimation. The use of this information allows the model to capture other properties of the data, particularly inflation persistence. Incorporating survey expectations, learning plays a key role in explaining inflation persistence even in a period of stable inflation, while exogenous shocks lose their predominant importance.

There are some important issues not addressed in this study. First, in order to show that learning without surveys captures mainly low frequency movements of inflation, we use a sub-sample during which inflation was more stable. One alternative to this procedure is to model the low frequency movements of inflation explicitly, as in Sbordone (2007), and then to evaluate whether learning still adds persistence to inflation and whether survey data helps to identify this role correctly. In fact, considering all potential sources of inflation persistence (trend inflation, learning, exogenous shocks and frictions such as indexation), and using survey data to incorporate a compatibility condition over the expectation mechanism, constitutes a completely novel framework of analysis. Second, we only use the median value of inflation forecasting across the reports of all forecasters included in the SPF at each point in time. However, information about other moments – such as the dispersion – can be exploited to evaluate issues like the credibility of the central bank or how periods of high disagreement in expectations affect the conduct of monetary policy. Finally, survey data are also available for a variety of other macroeconomic indicators, and the use of this information may change existing results regarding the identification of the sources of the business cycle, among other things. It is true that surveys about inflation expectations have been a more common subject of academic studies and have attracted less criticism concerning their quality than other variables. Nevertheless, this should not imply that survey data of other indicators do not contain any useful information at all.

To conclude, this study is one of the first to show that information about expectations formation contained in survey data can have significant macroeconomic implications. So far, however, the information collected by surveys such as the Survey of Professional Forecasters (SPF), but also the Livingstone and Michigan surveys or the Greenbook, have been largely neglected by empirical macroeconomic studies. The use of this information could improve our understanding of the workings of the economy and might change some pre-established ideas.

Bibliography

Boivin, Jean and Marc P. Giannoni, "Has Monetary Policy Become More Effective?", The Review of Economics and Statistics, volume 88, No3, August 2006, pages 445–462.

Canova, Fabio, "What Explains the Great Moderation in US? A Structural Analysis", Journal of European Economic Association, volume 7, No4, June 2009, pages 697-721.

Canova, Fabio and Luca Gambetti, "Do Expectations Matter? The Great Moderation Revisited", working paper, 2009.

Del Negro, Marco and Stefano Eusepi, "Modeling Observed Inflation Expectations", mimeo 2009.

Evans, George and Seppo Honkapohja, Learning and Expectations in Macroeconomics, Princeton University Press, 2001.

Evans, George and Seppo Honkapohja, "Expectations, Learning and Monetary Policy: An Overview of Recent Research", Centre for Dynamic Macroeconomic Analysis, Working Paper Series, January 2008.

Gali, Jordi and Luca Gambetti, "On the Sources of the Great Moderation", American Economic Journal: Macroeconomics, volume 1, 2009, pages 26-57.

Kimball, Miles, "The Quantitative Analytics of the Basic Neomonetarist Model", Journal of Money, Credit, and Banking, volume 27, issue 4, pages 1241-1277.

Lansing, Kevin, "Time-varying U.S. inflation dynamics and the New Keynesian Phillips curve", Review of Economic Dynamics, volume 12, 2009, pages 304-326.

Leduc, Sylvain, Keith Sill and Tom Stark, "Self-fulfilling Expectations and the Inflation of the 1970s: Evidence from the Livingston Survey", Journal of Monetary Economics, volume 54, 2007, pages 433-459.

Milani, Fabio, "Learning, Monetary Policy Rules, and Macroeconomic Stability", Journal of Economic Dynamics and Control, volume 32, issue 10, October 2008, pages 3148-3165.

Milani, Fabio, "Expectations, Learning and Macroeconomic Persistence", Journal of Monetary Economics, volume 54, 2007, pages 2065–2082.

Orphanides, Athanasios, "Monetary Policy Rules, Macroeconomic Stability an Inflation: A view from the Trenches", Journal of Money, Credit, and Banking, volume 36, No 2, April 2004, pages 151-175.

Orphanides, Athanasios and John C. Williams, "Imperfect Knowledge, Inflation Expectations, and Monetary Policy", NBER Chapters, in: The Inflation-Targeting Debate, 2004, pages 201-246.

Orphanides, Athanasios and John C. Williams, “Inflation-scares and Forecast-based Monetary Policy”, Review of Economic Dynamics, volume 8, No 2, 2005, pages 498-527.

Orphanides, Athanasios and John C. Williams, “Robust Monetary Policy with Imperfect Knowledge”, Journal of Monetary Economics, volume 54, 2007, pages 1406-1436.

Rabanal, Pau and Juan F. Rubio-Ramírez, “Comparing New Keynesian models of the Business Cycle: A Bayesian approach”, Journal of Monetary Economics, volume 52, 2005, pages 1151–1166.

Roberts, John M., “Inflation Expectations and the Transmission of Monetary Policy”, Finance and Economics Discussion Series 1998-43, Board of Governors of the Federal Reserve System.

Sbordone, Argia, “Inflation persistence: Alternative interpretations and policy implications”, Journal of Monetary Economics, volume 54, June 2007, pages 1311–1339.

Slobodyan, Sergey and Raf Wouters, “Learning in an estimated medium-scale DSGE model”, working paper, 2007.

Slobodyan, Sergey and Raf Wouters, “Estimating a medium-scale DSGE model with expectations based on small forecasting models”, working paper, 2009.

Smets, Frank and Rafael Wouters, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach”, The American Economic Review, volume 97, No3, June 2007, pages 586-606.

Appendix 1: Optimization problem of the agents and equilibrium conditions

1. Final good producers

As in Kimball (1995), the final good Y_t is a composite made of a continuum of intermediate goods $Y_t(i)$. The final good producers buy intermediate goods, package Y_t, and sell it to consumers, investors and the government in a perfectly competitive market. Their maximization problem is as follows:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

$$s.t. \left[\int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \varepsilon_t^p\right) di \right] = 1$$

where P_t and $P_t(i)$ are the prices of the final and intermediate goods, respectively. G is a strictly concave and increasing function characterized by *G(1)=1 and* ε_t^p is a stochastic parameter that determines the time-varying markup in the goods market. Combining the first order conditions (FOCs) of the above outlined maximization process yields the following expression:

$$Y(i) = Y_t G'^{-1} \left[\frac{P_t(i)}{P_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}\right) \frac{Y_t(i)}{Y_t} di \right]$$

Hence, the assumptions on *G()*, as defined in Kimball (1995), imply a demand for intermediate goods that is decreasing in its relative price, while the elasticity demand is increasing in the relative price.

2. Intermediate goods producers

The technology used by the intermediate good producer *i* is defined like:

$$Y_t(i) = \varepsilon_t^a K_t(i)^\alpha \left[\gamma^t L_t(i) \right]^{1-\alpha} - \gamma^t \Phi$$

where $K_t(i)$ is the capital services used in production, $L_t(i)$ is a composite labor input and Φ is a fixed cost. γ^t represents the labor-augmenting deterministic growth in the economy and ε_t^a is the total factor productivity.

Considering that profits are defined as:

$$P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t(i)$$

where W_t is the aggregate nominal wage rate and R_t^k is the rental rate of capital. The resulting cost minimization conditions are:

$$(\partial L_t(i)) : \Theta_t(i)\gamma^{(1-\alpha)t}(1-\alpha)\varepsilon_t^{\alpha} K_t(i)^{\alpha} L_t(i)^{-\alpha} = W_t$$

$$(\partial K_t(i)) : \Theta_t(i)\gamma^{(1-\alpha)t}\alpha\varepsilon_t^{\alpha} K_t(i)^{\alpha-1} L_t(i)^{1-\alpha} = R_t^k$$

where $\Theta_t(i)$ is the Lagrange multiplier associated with the production function and equals the marginal cost MC_t.

Combining the previous optimization conditions and considering that the capital-labor ratio is equal across the firms implies that:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k} L_t$$

The marginal cost is the same across all the firms and equals to:

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \gamma^{-(1-\alpha)t} (\varepsilon_t^{\alpha})^{-1}$$

The optimal price set by the firm is determined considering a Calvo pricing setup with partial indexation in order to pass inflation. The optimization problem that the intermediate firm faces is as follows:

$$\max_{\widetilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\widetilde{P}_t(i) \left(\prod_{l=1}^{s} \pi_{t+l-1}^{l_p} \pi_*^{1-l_p} \right) - MC_{t+s} \right] Y_{t+s}(i)$$

$$s.t. Y_{t+s}(i) = Y_{t+s} G'^{-1}\left(\frac{P_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s} \right)$$

where $\widetilde{P}_t(i)$ is the newly set price, 1-ξ^p is the Calvo probability of being allowed to optimize, π_t is the gross inflation, where $\pi_t = \frac{P_t}{P_{t-1}}$, $\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$ is the nominal discount factor for firms, which equals the discount factor of the households who are the final owner of the firms. Lastly, $\tau_t = \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}\right) \frac{Y_t(i)}{Y_t} di$ and $X_{t,s} = \left\{ \begin{matrix} 1 \; for s=0 \\ \left(\prod_{l=1}^{s} \pi_{t+l-1}^{l_p} \pi_*^{1-l_p} \right) for s=1,\ldots,\infty \end{matrix} \right\}$.

The first order condition is given by:

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{t+s}(i) \left[X_{t,s} \widetilde{P}_t(i) + \left(\widetilde{P}_t(i) X_{t,s} - MC_{t+s} \right) \frac{1}{G'^{-1}(z_{t+s})} \frac{G'(x_{t+s})}{G''(x_{t+s})} \right]$$

where $x_t = G'^{-1}(z_t)$ and $z_t = \frac{P_t(i)}{P_t} \tau_t$.

The aggregate price index is in this case given by:

$$P_t = (1-\xi_p) P_t(i) G'^{-1} \left[\frac{P_t(i)\tau_t}{P_t} \right] + \xi_p \pi_{t-1}^{l_p} \pi_*^{1-l_p} P_{t-1} G'^{-1} \left[\frac{\pi_{t-1}^{l_p} \pi_*^{1-l_p} P_{t-1} \tau_t}{P_t} \right]$$

3. Households

Household *j* chooses consumption *C*$_t$*(j)*, hours worked *L*$_t$*(j)*, bonds *B*$_t$*(j)*, investment *I*$_t$*(j)* and capital utilization *Z*$_t$*(j)* in order to maximize the following objective function:

$$E_t \sum_{s=0}^{\infty} \left[\frac{1}{1-\sigma_c} (C_{t+s}(j) - \eta C_{t+s-1})^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}(j)^{1+\sigma_l}\right)$$

subject to the budget constraint:

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_t^b R_{t+s} P_{t+s}} - T_{t+s}$$

$$\leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{t+s}^h(j) L_{t+s}(j)}{P_{t+s}} + \frac{R_{t+s}^k Z_{t+s}(j) \bar{K}_{t+s-1}(j)}{P_{t+s}} - a(Z_{t+s}(j)) \bar{K}_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}}$$

and the capital accumulation equation:

$$\bar{K}_t(j) = (1-\delta) \bar{K}_{t-1}(j) + \varepsilon_t^q \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j)$$

The degree of external habit formation is captured by η while σ_c and σ_l denote the inverse of the elasticity of intertemporal substitution (for constant labor) and the inverse of the elasticity of labor supply, respectively. The one-period bond is expressed on a discount basis. The term ε_t^b represents an exogenous premium on the return to bonds and should be interpreted as a reflection of inefficiencies in the financial sector that generates a premium on the deposit rates with respect to the risk free rate set by the central bank or a risk premium that households require in order to hold one-period bond. T_{t+s} are lump sum taxes or subsidies. In nominal terms, the income from labor effort is $W_{t+s}^{h}(j)L_{t+s}(j)$ and from renting capital services $R_{t+s}^{k}Z_{t+s}(j)\bar{K}_{t+s-1}(j)$, while the cost of changing the capital utilization is $P_{t+s}a(Z_{t+s}(j))\bar{K}_{t+s-1}(j)$. In period t, the amount of effective capital that households can rent to the firms is denoted as:

$$K_t(j)=Z_t(j)\bar{K}_{t-1}(j)$$

With respect to the capital accumulation equation, δ represents the depreciation rate, $S(\cdot)$ the adjustment cost, with $S(\gamma)=0$, $S'(\gamma)=0$ and $S''(\cdot)>0$. ε_t^q is a stochastic shock to the price of investment relative to consumption goods.

In equilibrium the decisions about consumption, hours worked, bonds, investment and capital utilization are the same across all the households. The first order conditions with respect to each of the previously mentioned variables are as follows:

$$(\partial C_t):\Xi_t=\exp(\frac{\sigma_c-1}{1+\sigma_l}L_t^{1+\sigma_l})(C_t-\eta C_{t-1})^{-\sigma_c}$$

$$(\partial L_t):\left[\frac{1}{1-\sigma_c}(C_t-\eta C_{t-1})^{1-\sigma_c}\right]\exp(\frac{\sigma_c-1}{1+\sigma_l}L_t^{1+\sigma_l})(\sigma_c-1)L_t^{\sigma_l}=-\Xi_t\frac{W_t^h}{P_t}$$

$$(\partial B_t):\Xi_t=\beta\varepsilon_t^b R_t E_t\left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right]$$

$$(\partial I_t):\Xi_t=\Xi_t^k\varepsilon_t^q\left(1-S(\frac{I_t}{I_{t-1}})-S'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}\right)+\beta E_t\left[\Xi_{t+1}^k\varepsilon_{t+1}^q S'(\frac{I_{t+1}}{I_t})\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

$$(\partial\bar{K}_t):\Xi_t^k=\beta E_t\left[\Xi_{t+1}\left(\frac{R_{t+1}^k}{P_{t+1}}Z_{t+1}-a(Z_{t+1})\right)+\Xi_{t+1}^k(1-\delta)\right]$$

$$(\partial u_t):\frac{R_t^k}{P_t}=a'(Z_t)$$

where Ξ_t and Ξ_t^k are the Lagrange multipliers associated with the budget and capital accumulation constraint respectively. Tobin's q is $Q_t=\frac{\Xi_t^k}{\Xi_t}$ and equals one in the absence of adjustment costs.

4. Intermediate labor union sector

As mentioned above there is a labor union in the economy that differentiates the labor services provided by the households and sets wages subject to a Calvo probability scheme with the labor packers. Labor packers take labor services from the union $L_t(l)$, package L_t and resell it to the intermediate goods producers. L_t is a composite "product" that aggregates $L_t(l)$ using the aggregator proposed by Kimball (1995).

Labor packers maximize profit in a perfectly competitive environment:

$$\max_{L_t, L_t(I)} W_t L_t - \int_0^1 W_t(I) L_t(I) dI$$

$$s.t. \left[\int_0^1 H\left(\frac{L_t(I)}{L_t}; \varepsilon_t^w\right) dI \right] = 1$$

where W_t and $W_t(I)$ represent the prices of the composite and intermediate labor services respectively, and H is a strictly concave and increasing function characterized by $H(1)=1$. ε_t^w is an exogenous process that reflects shocks to the aggregator function that result in changes in the elasticity of demand and therefore in the mark up. We will constrain $\varepsilon_t^w \in (0,\infty)$. Combining FOCs yields the following result:

$$L_t(I) = L_t H'^{-1}\left[\frac{W_t(I)}{W_t} \int_0^1 H'\left(\frac{L_t(I)}{L_t}\right) \frac{L_t(I)}{L_t} dI \right]$$

The labor unions represent the intermediates between households and the labor packers. In their negotiations with the labor packers they consider the marginal rate of substitution between consumption and labor of the households. Given that unions possess of market power they can generate some markups that are distributed among the households. However, the choice of the wage level is subject to nominal rigidities *à la* Calvo. Unions can adjust wages with a probability $1-\xi_w$ in each period. For those unions that cannot re-optimize within one period, $W_t(l)$ will increase at the deterministic growth rate of γ and weighted average rate of the steady-state inflation π_* and of the last period's inflation (π_{t-1}). The maximization problem faced by the unions allows for the possibility of getting stuck with the determined wage level for the following infinite periods having the previously mentioned indexation mechanism as the only way to adjust nominal wages. Thus, optimal wage level $\widetilde{W}_t(l)$ maximize the value of the following object:

$$\max_{\widetilde{W}_t(I)} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\widetilde{W}_t(I) \left(\prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_w} \pi_*^{1-l_w} - W_{t+s}^h \right) \right] L_{t+s}(I)$$

$$s.t. L_{t+s}(I) = L_{t+s} H'^{-1}\left(\frac{W_t(I) X_{t,s}^w}{W_{t+s}} \tau_{t+s}^w \right)$$

where $\tau_t^w = \int_0^1 H'\left(\frac{L_t(I)}{L_t} \right) \frac{L_t(I)}{L_t} dI$ and $X_{t,s} = \left\{ \begin{matrix} 1 \; for \, s=0 \\ \left(\prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_w} \pi_*^{1-l_w} \right) for \, s=1,\ldots,\infty \end{matrix} \right\}$.

The FOC is given by

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} L_{t+s}(I) \left[X_{t,s}^w \widetilde{W}_t(I) + \left(\widetilde{W}_t(I) X_{t,s}^w - W_{t+s}^h \right) \right] \frac{1}{H'^{-1}(z_{t,s}^w)} \frac{H'(x_{t,s}^w)}{H''(z_{t,s}^w)} = 0$$

where $x_{t,s}^w = H'^{-1}(z_t^w)$ and $z_t^w = \frac{W_t(I)}{W_t} \tau_t^w$

After some algebraic manipulation one achieves the following expression for aggregate wages:

$$W_t = (1-\xi_w) \widetilde{W}_t H'^{-1}\left[\frac{\widetilde{W}_t \tau_t^w}{W_t} \right] + \xi_w \gamma \pi_{t-1}^{l_w} \pi_*^{1-l_w} W_{t-1} H'^{-1}\left[\frac{\gamma \pi_{t-1}^{l_w} \pi_*^{1-l_w} W_{t-1} \tau_t^w}{W_t} \right]$$

5. Government policies

The central bank adjusts the nominal interest rate in response to deviations of the inflation and the output growth to their respective target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho} \left[\left(\frac{\pi_t}{\pi *} \right)^{r_\pi} \left(\frac{Y_t}{Y_{t-1}} \Big/ \frac{Y_t^*}{Y_{t-1}^*} \right)^{r_{\Delta y}} \right]^{1-\rho} \varepsilon_t^r$$

where R^* is the steady state nominal gross interest rate and Y_t^* is defined as the potential output taking into account only the exogenous process for total factor productivity and the trend growth of the economy:

$$Y_t^* = \varepsilon_t^a \overline{K}^{\alpha} \left[\gamma^t \overline{L} \right]^{1-\alpha} - \gamma^t \Phi$$

The government budget constraint has the following form:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

Government spending (G_t) is exogenous and expressed relative to the steady-state output path as $\varepsilon_t^g = \frac{G_t}{Y\gamma^t}$.

6. Resource constraints

The market clearing condition for the final goods market can be obtained by integrate the households' budget constraint across all of them and combine with the government budget constraint. The resulting resource constraint is:

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t$$

Appendix 2: Stochastic part of the model

The stochastic part of the model is characterized by seven exogenous processes: two which affect the intertemporal margin, such as the risk premium shocks, $\widehat{b_t}$, and the investment-specific technology shocks, $\widehat{q_t}$; further two that affect the intratemporal margin, such as the wage mark-up shocks, $\widehat{\lambda_{w,t}}$, and the price mark-up shocks, $\widehat{\lambda_{p,t}}$; another two policy shocks, the exogenous spending, $\widehat{g_t}$, and the monetary policy shocks, $\widehat{r_t}$; and last the total factor productivity shocks, $\widehat{A_t}$.

Each of these processes are characterized as first-order autoregressive (AR(1)) process with and *iid*-normal distributed error term. Their representations are the following:

- $\hat{g}_t = \rho_g \hat{g}_{t-1} + \rho_{ga}\varepsilon_t^a + \varepsilon_t^g$
- $\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_t^b$
- $\hat{q}_t = \rho_q \hat{q}_{t-1} + \varepsilon_t^\mu$
- $\hat{A}_t = \rho_a \hat{A}_{t-1} + \varepsilon_t^a$
- $\widehat{\lambda_{p,t}} = \rho_p \widehat{\lambda_{p,t-1}} + \varepsilon_t^p$
- $\widehat{\lambda_{w,t}} = \rho_w \widehat{\lambda_{w,t-1}} + \varepsilon_t^w$
- $\hat{r}_t = \rho_R \widehat{r_{t-1}} + \varepsilon_t^r$

Appendix 3: Definition of the dataset[12]

Definition of data variables

- consumption = LN((PCEC / GDPDEF) / LNSindex) * 100
- investment = LN((FPI / GDPDEF) / LNSindex) * 100
- output = LN(GDPC96 / LNSindex) * 100
- hours = LN((PRS85006023 * CE16OV / 100) / LNSindex) * 100
- inflation = LN(GDPDEF / GDPDEF(-1)) * 100
- real wage = LN(PRS85006103 / GDPDEF) * 100
- interest rate = Federal Funds Rate / 4

Source of the original data:

GDPC96 : Real Gross Domestic Product - Billions of Chained 1996 Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

GDPDEF : Gross Domestic Product - Implicit Price Deflator - 1996=100, Seasonally Adjusted Source: U.S. Department of Commerce, Bureau of Economic Analysis

PCEC : Personal Consumption Expenditures - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

FPI : Fixed Private Investment - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

CE16OV : Civilian Employment: Sixteen Years & Over, Thousands, Seasonally Adjusted. Source: U.S. Department of Labor: Bureau of Labor Statistics

CE16OV index : CE16OV (1992:3)=1

Federal Funds Rate : Averages of Daily Figures – Percent. Source: Board of Governors of the Federal Reserve System (Before 1954: 3-Month Treasury Bill Rate, Secondary Market Averages of Business Days, Discount Basis)

LFU800000000 : Population level - 16 Years and Older - Not Seasonally Adjusted. Source: U.S. Bureau of Labor Statistics

LNS10000000 : Labor Force Status : Civilian noninstitutional population - Age : 16 years and over . Seasonally Adjusted - Number in thousands. Source: U.S. Bureau of Labor Statistics (before 1976: LFU800000000 : Population level - 16 Years and Older)

[12] Taken from the data documentation of Smets and Wouters (2007).

LNSindex : LNS10000000(1992:3)=1

PRS85006023 - Nonfarm Business, All Persons, Average Weekly Hours Duration : index, 1992 = 100, Seasonally Adjusted. Source : U.S. Department of Labor

PRS85006103 - Nonfarm Business, All Persons, Hourly Compensation Duration : index, 1992 = 100, Seasonally Adjusted. Source : U.S. Department of Labor

Appendix 4: Prior distributions of structural parameters

	Symbol	Distribution	Mean	Std.
Share of capital in production	α	Normal	0.30	0.05
Inv. Elasticity of Intertemporal substitution	σ_c	Normal	1.50	0.38
Fix cost in production	Φ	Normal	1.25	0.13
Adjust cost of investment	S''	Normal	4.00	1.50
Habits in consumption	η	Beta	0.70	0.10
Wage stickiness	ξ_w	Beta	0.50	0.10
inv. Elast. labor supply	σ_l	Normal	2.00	0.75
Price stickiness	ξ_p	Beta	0.50	0.10
Wage indexation	ι_w	Beta	0.50	0.15
Price indexation	ι_p	Beta	0.50	0.15
Capital utilization elasticity	ψ	Beta	0.50	0.15
Taylor rule: response to inflation	r_π	Normal	1.50	0.25
Taylor rule: response to lagged interest rate	ρ_R	Beta	0.75	0.10
Taylor rule: response to changes in output	$r_{\Delta y}$	Normal	0.13	0.05
Trend growth rate	γ	Normal	0.40	0.10
Steady state of inflation	π _bar	Gamma	0.63	0.10
Steady state of hours worked	l _bar	Normal	0.00	2.00
Steady state of nominal int rate	r _bar	Gamma	1.15	0.30
Autocorrelation coef. Price Mk up shock	ρ_p	Beta	0.50	0.20
Autocorrelation coef. Wage Mk up shock	ρ_w	Beta	0.50	0.20
Autocorrelation coef. Product. Shock	ρ_a	Beta	0.50	0.20
Autocorrelation coef. Risk premium shock	ρ_b	Beta	0.50	0.20
Autocorrelation coef. Government shock	ρ_g	Beta	0.50	0.20
Autocorrelation coef. Investment-Specific shock	ρ_q	Beta	0.50	0.20
Autocorrelation coef. Monet policy shock	ρ_r	Beta	0.50	0.20
Correlation Government and productivity shocks	ρ_{ga}	Normal	0.50	0.25
Std Price Mk up innovation	σ_p	Inv. Gamma	0.10	2.00
Std. Wage Mk up innovation	σ_w	Inv. Gamma	0.10	2.00
Std. Product. Innovation	σ_a	Inv. Gamma	0.10	2.00
Std. Risk premium innovation	σ_b	Inv. Gamma	0.10	2.00
Std. Government innovation	σ_g	Inv. Gamma	0.10	2.00
Std. Inv. Specific innovation	σ_q	Inv. Gamma	0.10	2.00
Std. Monet policy innovation	σ_r	Inv. Gamma	0.10	2.00
Gain - no inflation	g^{π}	Uniform	0.00	0.30
Gain - inflation	$g^{non\pi}$	Uniform	0.00	0.30
Std. measurement error on expectations	σ_{exp}	Inv. Gamma	0.10	2.00

Note: for uniform distributions the values assigned as mean and standard deviation correspond to the range of the domain.

Appendix 5: Posterior distribution statistics of learning estimations

		Post-1984 sample				Complete sample			
		Without surveys		With surveys		Without surveys		With surveys	
	Symbol	Median	Std.	Median	Std.	Median	Std.	Median	Std.
Share of K in production	α	0.173	0.023	0.185	0.023	0.184	0.018	0.183	0.019
Inv. Elast. Intertp. Sust.	σ_c	1.531	0.222	1.477	0.207	1.242	0.105	1.309	0.140
Fix cost product.	Φ	1.581	0.085	1.632	0.090	1.640	0.080	1.653	0.079
Adj.cost inv.	S''	7.121	1.231	7.147	1.170	7.115	1.242	7.059	1.017
Habits	η	0.741	0.076	0.751	0.051	0.822	0.029	0.805	0.031
Wage stickiness	ξ_w	0.464	0.070	0.461	0.067	0.565	0.066	0.552	0.046
Elast. labor supply	σ_l	2.603	0.666	2.382	0.571	2.426	0.612	2.491	0.627
Price stickiness	ξ_p	0.619	0.025	0.681	0.034	0.489	0.038	0.470	0.032
Wage indexation	ι_w	0.392	0.135	0.368	0.125	0.335	0.101	0.334	0.115
Price indexation	ι_p	0.463	0.118	0.665	0.102	0.506	0.111	0.518	0.115
Cap. Utiliz. Elast.	ψ	0.578	0.116	0.632	0.118	0.640	0.105	0.649	0.108
TR: inflation	r_π	1.650	0.170	1.526	0.169	1.390	0.120	1.409	0.113
TR: lag interest rate	ρ_R	0.806	0.024	0.793	0.034	0.778	0.025	0.773	0.029
TR: change in output	$r_{\Delta y}$	0.183	0.048	0.181	0.044	0.210	0.045	0.204	0.045
aut. Price Mk up shock	ρ_p	0.075	0.053	0.113	0.067	0.138	0.072	0.168	0.089
aut. Wage Mk up shock	ρ_w	0.977	0.010	0.969	0.018	0.946	0.029	0.948	0.018
aut. Product. Shock	ρ_a	0.995	0.005	0.996	0.003	0.962	0.012	0.971	0.008
aut. Risk premium	ρ_b	0.332	0.199	0.264	0.194	0.155	0.071	0.156	0.073
aut. Government shock	ρ_g	0.975	0.011	0.972	0.012	0.993	0.004	0.993	0.004
aut. Inv. Specific shock	ρ_q	0.909	0.048	0.929	0.030	0.842	0.035	0.837	0.033
aut. Monet policy shock	ρ_r	0.392	0.067	0.438	0.063	0.195	0.063	0.195	0.064
Corr. Gov & product sks	ρ_{ga}	0.461	0.102	0.435	0.094	0.572	0.098	0.592	0.093
std. Price Mk up shock	σ_p	0.111	0.012	0.178	0.016	0.211	0.013	0.203	0.014
std. Wage Mk up shock	σ_w	0.259	0.052	0.260	0.058	0.211	0.032	0.218	0.030
std. Product. Shock	σ_a	0.363	0.029	0.360	0.029	0.440	0.025	0.448	0.032
std. Risk premium	σ_b	0.161	0.040	0.175	0.038	0.247	0.022	0.239	0.023
std. Government shock	σ_g	0.391	0.027	0.387	0.032	0.497	0.027	0.495	0.029
std. Inv. Specific shock	σ_q	0.276	0.036	0.262	0.032	0.327	0.028	0.340	0.031
std. Monet policy shock	σ_r	0.125	0.012	0.125	0.011	0.257	0.014	0.259	0.016
Gain - others	$g^{non\pi}$	0.047	0.037	0.005	0.009	0.105	0.043	0.017	0.024
Gain - inflation	g^{π}	0.006	0.004	0.201	0.006	0.187	0.012	0.140	0.008
Measurement exp error	σ_{exp}			0.151	0.010			0.176	0.011

Total number of draws is 500 thousands. After discarding the first half, 1 out of every 10 is selected to estimate the moments of the posterior distribution.