How important is Intra-household Risk Sharing for Savings and Labor Supply?

Salvador Ortigueira*
Universidad Carlos III de Madrid

Nawid Siassi†
European University Institute

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Abstract

While it is recognized that the family is primarily an institution for risk sharing, little is known about the quantitative effects of this informal source of insurance on savings and labor supply. In this paper, we present a model where workers (females and males) are subject to idiosyncratic employment risk and where capital markets are incomplete. A household is formed by a female and a male who make collective decisions on consumption, savings and labor supplies. In a calibrated version of our model, we find that precautionary savings are only 55% of those generated by a similar economy but lacking access to insurance from the family. We also find that intra-household risk sharing has its largest impact among wealth-poor households. While wealth-rich households use mainly savings to smooth consumption across unemployment spells, wealth-poor households rely on spousal labor supply. For instance, in the group of households with wealth less than two months worth of income, average hours worked by wives of unemployed husbands are 8% higher than those worked by wives of employed husbands. This response in wives' hours makes up 9% of lost family income. We also find crowding out effects of public unemployment insurance that are comparable to those estimated from the data.

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*Address: Economics Department, Calle Madrid 126, 28903 Getafe (Madrid), Spain. E-Mail: Salvador.Ortigueira@uc3m.es.

†Address: Economics Department, Via della Piazzuola 43, 50133 Florence, Italy. E-Mail: Nawid.Siassi@eui.eu

1 Introduction

The lack of a formal, private insurance market against employment risk makes this type of risk different from most of others faced by individuals. Even though public, compulsory unemployment insurance schemes are present in many countries, they typically fall short of providing full insurance and workers must rely on self-insurance and on informal insurance mechanisms in order to smooth consumption across unemployment spells. Precautionary savings and labor supply are the two instruments individuals can use as self-insurance against employment risk. The family, on the other hand, is the main informal insurance mechanism available to individuals. The standard argument being that information and payment enforceability are better within than between households.[1]

In this paper, we present an incomplete markets economy with idiosyncratic employment risk and assess quantitatively the role of the family as provider of insurance. Intra-household risk sharing, more than any other informal insurance mechanism, has important behavioral implications that affect no only the demand of self insurance, but also how this is crowded out by public insurance programs. Indeed, recent empirical evidence on patterns of insurance against employment risk found in a large panel of U.S. households sheds light on these crowding out effects. More specifically, Cullen and Gruber (2000) and Engen and Gruber (2001) estimate the response in two forms of insurance —accumulation of financial assets and spousal labor supply— to changes in the level of unemployment benefits and find significant crowding out effects on both. The extent to which public insurance crowds out other forms of (private) insurance is of paramount importance for public policy assessment [see, e.g., Attanasio and Rios-Rull (2000), Di Tella and MacCulloch (2002), Golosov and Tsyvinski (2007) and Chetty and Saez (2010) for analyses on the optimal level of social insurance when other forms of private insurance are also available.]

The model economy we present in this paper consists of a large number of two-person households, each pooling risks and making *collective* decisions on individual consumptions, labor supplies and joint savings in a risk-free asset, subject to a borrowing constraint. The two persons forming a household, a female and a male, are assumed to have different individual preferences for risk and different elasticities of labor supply. Individual weights in the household's utility function are determined, among other variables, by their relative earning ability. There is a firms sector producing an homogeneous good with capital and labor services, and a government providing public unemployment insurance. In order to assess the consequences of within-household risk sharing, the equilibrium in this economy is then compared to that arising in an economy where individuals lack access to this form of insurance and are left with self-insurance and public benefits

[1] Blundell, Pistaferri and Preston (2008) estimate the degree of consumption insurance from U.S. data and find evidence that the family plays an important insurance role.

as their only instruments to cope with employment risk. This latter framework corresponds to a standard Aiyagari-Huggett economy augmented with a labor-leisure choice, which has been studied by, e.g., Flodén and Lindé (2001), Marcet, Obiols-Homs and Weil (2007) and Pijoan-Mas (2006).

Since the equilibrium of our model economy contains a distribution of households over financial assets and spouses' employment status, we can assess not only the average effects of intra-household risk sharing but also its effects for different groups of households. Thus, in a calibrated version of our model we find that precautionary savings are only 55% of those generated by a similar economy that lacks access to insurance from the family. This is a large drop in precautionary savings that should be taken into account when assessing the ability of general equilibrium models with idiosyncratic income risk to generate large volumes of precautionary savings (see, e.g., Díaz, Pijoan-Mas and Ríos-Rull 2003 for a discussion on the extent of precautionary savings in models with uninsurable income risk).

We also find that intra-household risk sharing has its largest impact among wealth-poor households. While wealth-rich households use savings to smooth consumption across unemployment spells, wealth-poor households rely on spousal labor supply. For instance, in the group of households with wealth less than two months worth of income, average hours worked by wives of unemployed husbands are 8% higher than those worked by wives of employed husbands. Moreover, this response in wives' hours makes up 9% of lost family income.

The crowding out effects of public unemployment insurance in our calibrated economy are comparable to those found in the data. On the contrary, the standard Aiyagari-Huggett model of self insurance over-predicts the response in savings to changes in public insurance by a large margin. For example, this model predicts an elasticity of asset holdings with respect to unemployment benefits that is almost four times the elasticity estimated by Engen and Gruber (2001).

There is a vast literature, both empirical and theoretical, assessing the effects of idiosyncratic income risk on consumption, labor supply and savings. With only few exceptions, this literature adopts the bachelor household formulation in order to measure individual responses to income shocks and the degree of endogenous self-insurance. A recent example of this type of exercise is the paper by Low, Meghir and Pistaferri (2008). These authors assume that individuals (they focus only on males) are subject to a rich array of idiosyncratic shocks, including productivity and employment shocks. These shocks are assumed to differ in their available insurance opportunities (employment shocks are partially insured by the public unemployment insurance system while productivity shocks are not). The authors then use a bachelor household model to measure the effects of these shocks and the individual willingness to pay to avoid them. Since they consider endogenous mobility choices, their paper extends previous results in the literature by adding a new channel from shocks to individual responses to shocks.

Kotlikoff and Spivak (1981) is one of the first papers in economics to study the family as a provider of insurance to its members. In particular, they present a model where the only risk is that of unexpected longevity. Their model abstracts from labor earnings and assumes that an initial level of wealth is the only source of resources available to consumers. They show that efficient risk-sharing within the family closes much of the utility gap between no annuities and complete annuities. For example, the utility gain of marriage at age 30 is about 50% of the utility gain of an annuities market. In a model with these ingredients, Kotlikoff, Shoven and Spivak (1986) study precautionary savings arising from longevity risk. They compare savings under perfect insurance markets with savings under intra-household risk sharing. They find significant differences in savings.

A more recent exception to the use of the bachelor household formulation is the work of Attanasio, Low and Sánchez-Marcos (2005), who present a partial equilibrium model with a two-person *unitary* household to assess the response of female labor market participation (extensive margin) to idiosyncratic earnings risk within the family. In their model, male participation is exogenous. An important feature of this model is the process of female human capital formation, which is assumed to depend on labor market participation. The authors find that the higher the uncertainty the higher female participation. They also find that the welfare cost of uncertainty is lower when households can adjust female labor market participation.

Heathcote, Storesletten and Violante (2008) also use a two-person, unitary household model to study the welfare implications of the observed changes in the U.S. wage structure. In particular, they present an incomplete-markets, life-cycle model to quantify the effects of the rising college premium, the narrowing wage gender gap and the increasing wage volatility. Their model allows for an endogenous education choice and for a matching process of females and males into households. Even though the welfare consequences of the above-mentioned changes in wages are highly heterogenous across different types of households, they find that, on average, recent cohorts of households enjoy welfare gains, as the new structure of wages translates into higher educational attainment.

Unitary models of the household, however, assume a utility function for the household and are thus silent about the decision process between its members. The collective model (see Browning, Chiappori and Lechene 2006 for a formal definition of this model) establishes instead that this decision process leads to within-household Pareto optimality and that Pareto weights on individuals' private utility functions depend on prices, policy variables and distribution factors. Thus, in this latter model, changes in the wage gender gap, in public unemployment benefits and/or in tax rates imply within-household distributional effects that unitary models fail to capture. Moreover, in economies with idiosyncratic risks and incomplete asset markets, these effects, along with heterogeneity in individuals' risk preferences, have sizable implications for precautionary savings and labor supplies. Consequently, the two models of the household predict different crowding out effects of public unemployment insurance. Tests of these two competing models of the household have been carried out by, e.g., Fortin and Lacroix (1997), who find evidence against the unitary model. In particular, they reject the income pooling restrictions and the symmetry of cross-wage effects which are embodied in this model.

The remaining of the paper is organized as follows. Section 2 describes the economic environment and presents the problems solved by the bachelor and the collective household. Section 3 defines a stationary equilibrium with incomplete markets in the collective household economy. It also presents the parameterization and calibration of this economy; it shows the steady-state equilibrium and discusses some features of the policy functions. Section 4 presents the main results on the aggregate and individual consequences of intra-household risk sharing. Section 5 concludes. The paper contains four appendices.

2 The Economic Environment

Consumers The economy is populated by a continuum of measure two of infinitely-lived workers/consumers. Half of this population of workers/consumers will be referred to as females, and the other half as males. All enjoy the consumption of an aggregate good and of leisure time (with possibly different utility functions). Agents supply time to work in the production sector and face idiosyncratic labor market risk in the form of employment shocks.

Employment shocks, s, take on values in $S \equiv \{0, 1\}$ and follow a Markov chain with transition matrix Π^i, where superscript i denotes the gender: females (f) and males (m). Thus, $\pi^i_{s'|s}$ is the probability for an agent of gender i to receive employment shock s' tomorrow conditional on employment shock s today, for $i = f, m$. These probabilities satisfy $\sum_{s'} \pi^i_{s'|s} = 1$, $\pi^i_{s'|s} > 0$, and $\pi^i_{1|1} \geq \pi^i_{1|0}$ for $i = f, m$. The long-run probabilities of the two employment shocks in S are denoted by q^i_0 and q^i_1. There are no others shocks in the economy.

Markets are incomplete. The only asset in the economy is physical capital, which pays out the risk-free interest rate r. Moreover, there is a minimum level of asset holdings, $\underline{a}$, which is a borrowing or liquidity constraint.

Lifetime preferences for an agent of gender i over stochastic consumption and leisure streams are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t, l_t), \quad \text{for} \quad i = f, m. \tag{2.1}$$

where c_t denotes consumption and l_t is leisure. We make the following assumptions on U^i:

A1. Utility $U^i(c, l) : R_+ \times [0, 1] \rightarrow R$ is bounded, continuous and twice continuously differentiable in the interior of its domain.

A2. Utility is separable in consumption and leisure.

A3. Utility U^i is strictly increasing and strictly concave in each of its arguments. Moreover, $\lim_{c\to 0} U^i_c(c,l) = +\infty$, and $\lim_{l\to 0} U^i_l(c,l) = +\infty$.

Firms Production of the aggregate good is conducted by competitive firms. Production technology is represented by the neoclassical production function $F(K,L)$, where K is the aggregate stock of capital and L is aggregate labor. The depreciation rate of capital is denoted by $\delta > 0$. Throughout the paper, we will assume the standard Cobb-Douglas production function, $F(K,L) = K^{\alpha}L^{1-\alpha}$, where $0 < \alpha < 1$ is the capital's share of income and $L \equiv \lambda L^m + (1-\lambda)L^f$. That is, female and male labor are perfect substitutes and parameter $0 < \lambda < 1$ pins down relative, gross-of-taxes wages. The firm's maximization problem is static: given a rental price of capital r and gross wages for females and males $\bar{w}^f$ and $\bar{w}^m$, respectively, first-order conditions are:

$$F_K(K,L) = r + \delta \tag{2.2}$$

$$\lambda F_L(K,L) = \bar{w}^m \tag{2.3}$$

$$(1-\lambda)F_L(K,L) = \bar{w}^f. \tag{2.4}$$

Government There is a government that provides public insurance against unemployment shocks. The government pays out benefits b^i to unemployed workers of gender $i = f, m$. Only workers who receive an unemployment shock are entitled to benefit payments. The government finances its expenditures by levying linear taxes on labor income: given tax rates τ^i, we will denote after-tax wage rates by $w^i = (1-\tau^i)\bar{w}^i$. The government is required to balance its budget on a period-by-period basis.

2.1 The Bachelor versus the Collective Household Model

We now consider two different risk-sharing arrangements and study their implications for labor supply (of both females and males) and for precautionary savings. Each arrangement defines in turn a different type of household. We start out by presenting the problem of the bachelor household. This is the definition of household that has dominated not only the literature on precautionary savings, but also most of the macroeconomic literature. The defining feature of this type of household is that a single breadwinner chooses sequences of consumption, leisure and asset holdings in order to maximize his/her own lifetime utility. In most studies adopting this framework, the income process is estimated using data on males. The second type of household we study is a dynamic version of the collective household model pioneered by Chiappori (1988). In this latter case, we assume that two adult individuals, with possibly different preferences, wages and processes for employment shocks, form a household and then make collective decisions on consumptions, labor supplies and savings.

2.1.1 Bachelor Households

A household formed by a single agent of gender i solves

$$v^i(s,a;w^i,r) = \max_{c,l,a'} \left\{ U^i(c,l) + \beta \sum_{s'} \pi^i_{s'|s} v^i(s',a';w^i,r) \right\} \tag{2.5}$$

$$c + a' = w^i(1-l)s + (1-s)b^i + (1+r)a \tag{2.6}$$

$$c \geq 0,\ 0 \leq l \leq 1, \text{ and } a' \in [\underline{a}^i, \bar{a}], \tag{2.7}$$

where $\pi^i_{s'|s}$ are the elements of Π^i. The minimum level of asset holdings, $\underline{a}^i$, imposes a borrowing constraint, which can be either a solvency or a liquidity constraint. A version of this model where there is a measure one of same-gender workers is the workhorse model in the literature of uninsurable idiosyncratic risk, precautionary savings and labor supply [see, e.g., Marcet, Obiols-Homs and Weil (2007). Flodén and Lindé (2001) and Pijon-Mas (2006) also study a model with a measure one of same-gender, bachelor households but assume a richer labor income process. Instead of facing an employment/unemployment shock, workers in their models receive idiosyncratic shocks to the efficiency units of labor supply].

By construction, the bachelor household does not engage in informal insurance arrangements with other workers. The only sources of insurance available to this type of household are the public unemployment insurance system, own savings and own labor supply.

2.1.2 Collective Households

We now consider two-person, collective households formed by an egotistical female and an egotistical male. We assume that the two members of the household share labor market risk in such a way that intra-household allocations are efficient.[2] Following the literature of collective households (see Chiappori and Donni 2010 for a recent survery), the utility of each individual in the household carries a weight, reflecting the relative power of that individual in the household. Individual weights are assumed to depend on variables such as premarital wealth, the population sex ratio, relative incomes and government policy. Under full commitment, that is, when households are not allowed to break up, individual weights are set when the household is formed and remain unchanged thereafter. Thus, transitory shocks, which are small relative to lifetime income, have

[2] It should be noted, however, that in two-person households the number of family members involved in risk pooling is too small to achieve full insurance against labor market risk.

no effect on individual weights. Only variables known or predicted at the time of household formation can affect those weights.[3] In our model there are four sources of income differences between females and males that affect relative Pareto weights: 1) They have different gross wages; 2) They may pay different tax rates; 3) They may receive different levels of unemployment benefits; and, 4) Finally, females and males may be subject to different employment and unemployment spells. We write the Pareto weight on female's utility as $\mu(x, \boldsymbol{z}) \in (0, 1)$, where variable x is a measure of the relative income-earning ability of the two spouses, which we write as,

$$x \equiv \frac{q_1^f (1 - \tau^f) \bar{w}^f + q_0^f b^f}{q_1^m (1 - \tau^m) \bar{w}^m + q_0^m b^m}, \tag{2.8}$$

where q_i^j for $j = f, m$ and $i = 0, 1$ is, as written above, the long-run probability of employment state i for an agent of gender j. Vector $\boldsymbol{z}$ includes variables such as the population sex ratio, the initial contribution to household wealth, etc., which we do not model explicitly in this paper. It must be noted that in our model the Pareto weight function, $\mu(x, \boldsymbol{z})$, is not obtained as the outcome of an explicit bargaining process between females and males. Instead, we will use estimates of the sharing rule provided by Browning, Bourguignon, Chiappori and Lechene (1994) to parameterize and solve our model.[4]

Household-level state variables for the two-person, collective household are the vector of employment shocks $\boldsymbol{s} = (s^f, s^m)$, which we assume to be uncorrelated within the household,[5] and the level of asset holdings, a. The state space of a household is $X = S \times S \times [\underline{a}, \bar{a}]$. We denote by $\mathcal{B}$ the Borel sigma algebra of X. The transition matrix for $\boldsymbol{s}$ is denoted by Π and obtained from the individual transition matrixes as $\Pi = \Pi^m \otimes \Pi^f$. The vector of after-tax wages for the household, (w^f, w^m), is denoted by $\boldsymbol{w}$.

The maximization problem of a collective household with Pareto weight $\mu(x, \boldsymbol{z})$ on female's utility

[3] For a test of intra-household commitment to future allocations, see Mazzocco (2007). Using data from the Consumer Expenditure Survey, this author rejects the hypothesis of commitment. Since our model abstracts from permanent shocks and assumes only transitory shocks to labor income, we will retain, for the sake of analytical tractability, the assumption of commitment.

[4] In a recent paper Heathcote, Storesletten and Violante (2009) endogenize the Pareto weight function as the solution to a symmetric Nash bargaining problem within the household.

[5] We will discuss further this independence assumption below.

is

$$V(\boldsymbol{s},a;x,\boldsymbol{z},r) = \max_{c^f,c^m,l^f,l^m,a'} \Bigg\{ \mu(x,\boldsymbol{z})U^f(c^f,l^f) + [1-\mu(x,\boldsymbol{z})]U^m(c^m,l^m) + \beta \sum_{\boldsymbol{s'}} \pi_{\boldsymbol{s'}|\boldsymbol{s}} V(\boldsymbol{s'},a';x,\boldsymbol{z},r) \Bigg\} \tag{2.9}$$

s.t.

$$c^f + c^m + a' = \sum_{i=f,m} w^i(1-l^i)s^i + \sum_{i=f,m}(1-s^i)b^i + (1+r)a \tag{2.10}$$

$$c^f, c^m \geq 0, \quad 0 \leq l^f, l^m \leq 1, \text{ and } a' \in [\underline{a}, \bar{a}], \tag{2.11}$$

where $\pi_{\boldsymbol{s'}|\boldsymbol{s}}$ are the elements of Π. Note that while we allow for different preferences over consumption and leisure for females and males, we assume that both spouses share a common discount factor β. In our model, $\boldsymbol{z}$ is the only source of variation in Pareto weights across households. We represent the distribution of these weights in the population of households by $G(\mu)$. The support of this distribution is denoted by $M \equiv (0,1)$.

Contrary to unitary models of the household, the utility function of the collective household depends, *via* the Pareto weight, on wages and policy variables, which leads to household demands that fail to meet the Slutsky conditions. This failure is the defining feature of the collective model. Also while in unitary models household decisions do not depend on who receives the income within the household, in our collective model decisions depend not only on total income, but on who receives the income (whether is the female or the male).

The dependency of the household's utility function on prices and policy must also be acknowledged when setting the Frisch elasticities of labor supply for females and males. In particular, these elasticities are functions of the derivative of the Pareto weight with respect to wages. Our assumption that both labor supplies can vary continuously in response to wages and non-labor income is common in the literature of collective labor supply [see, e.g., Chiappori (1988) and Chiappori, Fortin and Lacroix (2002)].[6] Likewise, the household's attitude towards risk in the collective model depends both on individual preferences and on the relative Pareto weight. Since we will assume individual preferences which are not of the ISHARA type (i.e., household members do not share a common coefficient of harmonic risk-aversion), the household does not behave as a single-decision maker, in the sense that an increase in risk aversion of one household member does not necessarily increase risk aversion of the household. [For an analysis of a two-period, collective model of the household with uncertainty see Mazzocco (2004)].

It should be noted that our assumption of egotistical preferences is not crucial. Actually, Browning,

[6] For a recent study of collective labor supply allowing for non continuity see Blundell, Chiappori, Magnac and Meghir (2007). These authors present a collective model of the household where the female makes a continuous labor supply choice but the male decides simply whether or not to participate.

Chiappori and Lechene (2006) show that under caring preferences of the form where female's instantaneous utility is $U^f(c^f,l^f)+\psi^f U^m(c^m,l^m)$ and male's utility is $U^m(c^m,l^m)+\psi^m U^f(c^f,l^f)$, with $0<\psi^f,\psi^m\leq 1$ denoting the caring parameters, the utility function of the household can be written down as for the case of egotistical preferences, after a re-definition of Pareto weights. The new relative weight on female's utility $U^f(c^f,l^f)$ would be $\hat{\mu}\equiv(\mu+(1-\mu)\psi^m)/(1+\mu\psi^f+(1-\mu)\psi^m)$. Note that this weight converges to 0.5 as ψ^f and ψ^m converge to 1, for all values of μ.

We now present the first-order conditions to the maximization problem (2.9)-(2.11). As explained above, the collective model of the household implies full risk-sharing within the household, i.e., the ratio of marginal utilities of consumption equals relative Pareto weights and is thus independent of the realized vector of employment shocks. That is,

$$\mu U_c^f=(1-\mu)U_c^m. \tag{2.12}$$

This equation defines the individual risk-sharing rules, which, for a given level of household consumption, specify how much is consumed by each of its members. It is straightforward to show that the derivative of the risk-sharing rules is positive and given by the product of the household's coefficient of absolute risk aversion and the individual's coefficient of absolute risk tolerance.[7] Therefore, the member of the household showing higher risk tolerance will be the one absorbing most of the variation in total household consumption. (In Appendix IV we present the derivatives of the risk-sharing rules for the case of CRRA utility functions.)

First-order conditions to female and male labor supply are, respectively,

$$\frac{U_l^f}{U_c^f}\geq w^f s^f \qquad \text{with inequality if } l^f=1 \tag{2.13}$$

$$\frac{U_l^m}{U_c^m}\geq w^m s^m \qquad \text{with inequality if } l^m=1. \tag{2.14}$$

Moreover, if the labor supply decision is interior for both household members then

$$\frac{U_l^f}{w^f s^f}=\frac{1-\mu}{\mu}\frac{U_l^m}{w^m s^m}. \tag{2.15}$$

The first-order condition to savings is,

$$U_c^f=\beta(1+r)\sum_{s'}\pi_{s'|s}\,U_c^{f\prime} \qquad \text{if } a'>\bar{a} \tag{2.16}$$

$$U_c^f\geq\beta(1+r)\sum_{s'}\pi_{s'|s}\,U_c^{f\prime} \qquad \text{if } a'=\underline{a}. \tag{2.17}$$

We can now characterize some properties of the value function and optimal decision rules for a household with Pareto weight $\mu\in M$:

Proposition 1. Assume $\mathbf{A1}-\mathbf{A3}$, $\boldsymbol{w}>0$, $(1+r)>0$, $\beta(1+r)\leq 1$, and $b^f=b^m=0$. Then:

[7] Risk tolerance is defined as the reciprocal of risk aversion.

(a) $V(\boldsymbol{s}, a, \mu)$ is strictly increasing and strictly concave in a. Decision rules $c^f(\boldsymbol{s}, a; \mu)$, $c^m(\boldsymbol{s}, a; \mu)$, $l^f(\boldsymbol{s}, a; \mu), l^m(\boldsymbol{s}, a; \mu)$ and $a'(\boldsymbol{s}, a; \mu)$ are continuous in a and strictly positive.

(b) Decision rules for consumption, $c^f(\boldsymbol{s}, a; \mu)$ and $c^m(\boldsymbol{s}, a; \mu)$, are strictly increasing in a. Decision rules for savings, $a'(\boldsymbol{s}, a; \mu)$, and leisure, $l^f(s^f = 1, s^m, a; \mu)$, $l^m(s^m = 1, s^f, a; \mu)$, are increasing in a.

(c) Decision rules for consumption are increasing in the own employment shock: $c^j(s^j = 1, s^i, a; \mu) \geq c^j(s^j = 0, s^i, a; \mu)$.

(d) Decision rules for leisure are increasing in the spouse's employment shock: $l^j(s^j = 1, s^i = 1, a) \geq l^j(s^j = 1, s^i = 0, a; \mu)$ for all a.

(e) If $\beta(1+r) \leq 1$, then for all $a \in [\underline{a}, \overline{a}]$, $a'(s^f = 0, s^m = 0, a; \mu) \leq a$ (with strict inequality if $\underline{a} < a < \overline{a}$ and $\beta(1 + r) < 1$).

Proof: See the Appendix.

We now present some results on the asymptotic properties of the consumption program, savings and labor supply of a household with Pareto weight μ, for different values of wages, (w^f, w^m), and of the interest rate, r. More specifically, we extend results by Marcet, Obiols-Homs and Weil (2007) for the bachelor household to our two-person, collective household model. We also extend the results to non-homogeneous utility functions. With this aim, let us denote by $\tilde{a}(\mu)$ the minimum level of asset holdings for which both spouses within a household with Pareto weight μ will stop supplying labor. The value $\tilde{a}(\mu)$ is pinned down as follows. First, since utility is separable in consumption and leisure, we can plug (2.12) into (2.14) and thus rewrite the first-order conditions to female and male labor supply as

$$U_c^f s^f \leq \frac{U_l^f}{w^f} \qquad \text{with inequality if } l^f = 1 \tag{2.18}$$

$$U_c^f s^m \leq \frac{U_l^m}{w^m}\frac{1-\mu}{\mu} \qquad \text{with inequality if } l^m = 1. \tag{2.19}$$

Define $\widetilde{U}_l^i$ as the marginal utility of leisure for individual $i = f, m$, at $l^i = 1$. Also, define

$$\widetilde{U}_c^f(\mu) \equiv \min\left\{\frac{\widetilde{U}_l^f}{w^f}\ ,\ \frac{\widetilde{U}_l^m}{w^m}\frac{1-\mu}{\mu}\right\} \tag{2.20}$$

and $\widetilde{U}_c^m(\mu) \equiv \frac{\mu}{1-\mu}\widetilde{U}_c^f(\mu)$. Let $\widetilde{c}^i(\mu)$ be the level of consumption for which the corresponding marginal utility of consumption equals $\widetilde{U}_c^i(\mu)$. Then the level of asset holding $\tilde{a}(\mu)$ mentioned above is defined as

$$\widetilde{a}(\mu) \equiv \frac{1}{r}\Big[\widetilde{c}^f(\mu) + \widetilde{c}^m(\mu)\Big]. \tag{2.21}$$

It can easily be checked that at $\widetilde{a}(\mu)$, equations (2.10) – (2.14) are satisfied for all possible realizations of s^f and s^m if consumption levels equal $\widetilde{c}^f(\mu)$ and $\widetilde{c}^m(\mu)$, hours worked equal zero and asset holdings remain constant. In the case that $\beta(1+r)=1$, equation (2.16) is satisfied, because consumption is constant. Hence, if $\beta(1+r)=1$, optimal decision rules are

$$c^i(\boldsymbol{s}, \widetilde{a}(\mu); \mu) = \widetilde{c}^i(\mu) \tag{2.22}$$

$$l^i(\boldsymbol{s}, \widetilde{a}(\mu); \mu) = 1 \tag{2.23}$$

$$a'(\boldsymbol{s}, \widetilde{a}(\mu); \mu) = \widetilde{a}(\mu), \tag{2.24}$$

for $i = f, m$ and for all $\boldsymbol{s} \in S \times S$. Thus, if the household ever reaches $\widetilde{a}(\mu)$, it will maintain a constant consumption stream without ever working. For lower interest rates, constant consumption does not satisfy the FOC for asset holdings, and the household never reaches $\widetilde{a}(\mu)$. The following proposition formalizes this result.

Proposition 2: Assume **A1 − A3**, $\overline{a} > \widetilde{a}(\mu)$, $\boldsymbol{w} > 0$ and $(1+r) > 0$. Then:

- **(a)** If $\beta(1+r) \leq 1$, for any $a \leq \widetilde{a}(\mu)$, $a'(\boldsymbol{s}, a; \mu) \leq \widetilde{a}(\mu)$.
- **(b)** If $\beta(1+r) = 1$, for any $a \geq \widetilde{a}(\mu)$ and any $\boldsymbol{s}$ we have $a'(\boldsymbol{s}, a; \mu) = a$, $l^f(\boldsymbol{s}, a; \mu) = 1$, $l^m(\boldsymbol{s}, a; \mu) = 1$ and $c^f(\boldsymbol{s}, a; \mu) + c^m(\boldsymbol{s}, a; \mu) = a\, r$ such that $\mu U_c^f = (1-\mu)\, U_c^m$.
- **(c)** If $\beta(1+r) = 1$ and $a \leq \widetilde{a}(\mu)$, then $a_t \xrightarrow{a.s.} \widetilde{a}(\mu), c_t^i \xrightarrow{a.s.} \widetilde{c}^i(\mu)$, $l_t^i \xrightarrow{a.s.} 1,\ i = f, m$.

Proof: See the Appendix.

It follows that in the case $\beta(1+r) < 1$ the household can reach any value of asset holdings from any initial capital stock in finite time, and a stationary distribution arises in the long run. Moreover, in the case $\beta(1+r) = 1$ capital accumulation in the long run is bounded and it converges asymptotically to $\widetilde{a}(\mu)$. This is in contrast to the case of inelastic labor supply where savings asymptotically grow to infinity if $\beta(1+r) = 1$. As it should be apparent from the results above, the endogenous labor-leisure decision changes the asymptotic behavior of consumption and assets with respect to the inelastic labor case by removing income uncertainty. When household wealth is high enough, labor supply equals zero and thus employment shocks no longer affect household income. Thus, under non-stochastic income, unbounded assets accumulation is no longer optimal under $\beta(1+r) = 1$.

Finally, note that if we set $\overline{a} > \max_{\mu \in M} \widetilde{a}(\mu)$ and choose initial capital holdings for all households with relative Pareto weight μ such that $a_0(\mu) \leq \widetilde{a}(\mu)$, then the upper limit on capital is never binding. In other words, under these conditions the upper bound on asset holdings, which was imposed to guarantee existence and uniqueness of the value function, does not bind.

3 Stationary Equilibrium with Incomplete Markets

We define now a stationary equilibrium with incomplete markets in the collective household economy. Let $\psi(B;\mu)$ be a probability measure describing the mass of households with fixed Pareto weight μ at each point in the state space X, where $\psi(B;\mu)$ is defined on the Borel sigma algebra $\mathcal{B}$. Denote by $P(\boldsymbol{s}, a, B; \mu)$ the probability that a household with Pareto weight μ at state $(\boldsymbol{s}, a)$ will transit to a state that lies in $B \in \mathcal{B}$ in the next period. The transition function P can be constructed as

$$P(\boldsymbol{s}, a, B; \mu) = \sum_{\boldsymbol{s}' \in B_{\boldsymbol{s}}} \Pi_{\boldsymbol{s}'|\boldsymbol{s}} \, \mathcal{I}_{a'(\boldsymbol{s},a;\mu) \in B_a},$$

where $\mathcal{I}$ is an indicator function taking on a value of 1 if its argument is true and 0 otherwise, and $B_{\boldsymbol{s}}$ and B_a are the projections of $\mathcal{B}$ on $S \times S$ and $[\underline{a}, \overline{a}]$ respectively. Note that these transition functions will in general differ across households with different Pareto weights μ. We are now ready to define the equilibrium concept for our model.

Definition: A stationary recursive competitive equilibrium with incomplete markets in the economy with collective households is a list of functions $\left\{V, c^f, c^m, l^f, l^m, a', K, L^f, L^m\right\}$, a measure of households ψ and a set of prices $\left\{r, \bar{w}^f, \bar{w}^m\right\}$, taxes $\left\{\tau^f, \tau^m\right\}$ and benefits $\left\{b^f, b^m\right\}$ such that:

1) For given prices, taxes and benefits, V is the solution to (2.9) – (2.11), and $c^f(\boldsymbol{s}, a; \mu)$, $c^m(\boldsymbol{s}, a; \mu)$, $l^f(\boldsymbol{s}, a; mu)$, $l^m(\boldsymbol{s}, a; \mu)$ and $a'(\boldsymbol{s}, a; \mu)$ are the associated optimal policy functions.

2) For given prices, K, L^f and L^m satisfy the firm's first-order conditions (2.2) – (2.4).

3) Aggregate factor inputs are generated by the policy functions of the agents:

$$K = \int_M \int_X a(\boldsymbol{s}, a; \mu) d\psi dG, \tag{3.1}$$

$$L^f = \int_M \int_X s^f [1 - l^f(\boldsymbol{s}, a; \mu)] d\psi dG, \tag{3.2}$$

$$L^m = \int_M \int_X s^m [1 - l^m(\boldsymbol{s}, a; \mu)] d\psi dG. \tag{3.3}$$

4) The time-invariant stationary distribution ψ is determined by the transition function P as

$$\psi(B;\mu) = \int_X P(\boldsymbol{s}, a, B; \mu) d\psi \quad \text{for all} \quad B \in \mathcal{B}. \tag{3.4}$$

5) The government budget is balanced: $q_0^f b^f + q_0^m b^m = \tau^f \bar{w}^f L^f + \tau^m \bar{w}^m L^m$.

Under assumptions **A1 – A3** the interest rate in the stationary equilibrium under incomplete markets must be such that $\beta(1+r) < 1$. This implies that the equilibrium capital-labor ratio under incomplete markets is higher than under complete markets.

3.1 Stationary Equilibrium with Complete Markets

In the complete markets economy households can trade a set of Arrow securities which pay contingent on the realization of the idiosyncratic shocks of both spouses.[8] It is then straightforward to show that in a stationary equilibrium the interest rate must be such that $\beta(1+r) = 1$. In addition, marginal utilities of consumption are equalized across states and periods, which in conjunction with assumption **A2** implies that female and male consumption levels are independent of the vector of the household's employment shocks $\boldsymbol{s} = (s^f, s^m)$ and constant over time. In a stationary equilibrium with complete markets the capital-labor ratio K/L and optimal household decision rules are uniquely determined, whereas the absolute values of K and L are not pinned down: in fact, there are infinitely many different distributions of households that generate pairs of aggregate capital K and aggregate labor L which are all consistent with the equilibrium capital-labor ratio. Hence, when comparing the stationary complete markets equilibrium with the incomplete markets economy, we must choose an equilibrium selection mechanism in the complete markets economy. An obvious candidate is the steady state equilibrium that arises after the transition from the incomplete markets economy. That is, when markets are completed, we compute the long-run equilibrium using the stationary equilibrium of the incomplete markets economy as initial conditions.[9]

3.2 Parameterization and Calibration

3.2.1 Parameterization

Preferences Instantaneous utility functions for females and males are parameterized as follows,

$$U^i(c,l) = \varphi_c^i \, \frac{c^{1-\sigma^i} - 1}{1-\sigma^i} + \varphi_l^i \, \frac{l^{1-\gamma^i} - 1}{1-\gamma^i} \quad \text{for} \quad i = f, m, \tag{3.5}$$

where φ_c^i and φ_l^i are parameters (φ_c^f is normalized to one) and σ^i is the coefficient of relative risk aversion of an individual of gender i. It must be noted that in the model with collective households —and contrary to the model with bachelor households— the Frisch elasticity of labor supply of an individual of gender i depends not only on parameter γ^i, but is also a function of variables and parameters that affect the expected, intra-household earnings differential through the Pareto weight (see Appendix III for a derivation of Frisch elasticities in the collective household economy). Also, as anticipated above, household's risk aversion is determined by individual preferences for risk and by the household sharing rule μ. It is only when the two household members share the

[8] See Appendix II for a complete characterization of this economy.

[9] Pijoan-Mas (2006) contains a detailed description of the computational algorithm for an Aiyagari-Hugget economy without public insurance and with ex-ante identical, same-gender individuals.

same preferences for risk, i.e., $\sigma^f = \sigma^m$, that the household's coefficient of relative risk aversion becomes independent of Pareto weights (see Appendix IV for a derivation of the household's coefficient of risk aversion).

Technology As written above, the production takes place according to the standard Cobb-Douglas technology, $F(K, L) = K^\alpha L^{1-\alpha}$, where labor is $L \equiv \lambda L^m + (1 - \lambda)L^f$. Parameter α is the capital share of income and λ pins down relative gross wages, since $\overline{w}^f/\overline{w}^m = (1 - \lambda)/\lambda$.

Pareto weights We will make the following simplifying assumption on the distribution of Pareto weights over the population, G. In our benchmark economy we assume that all households are ex-ante identical and have a relative Pareto weight equal to 0.5. This amounts to assuming a degenerate distribution over the vector $\boldsymbol{z}$ so that females and males have a Pareto weight exactly equal to 0.5 in all households. It should be noted, however, that a Pareto weight of 0.5 would endogenously arise under caring preferences of the form discussed above for high enough caring parameters ψ^f and ψ^m.

We also need the derivative of the Pareto weight function with respect to x, $\mu_1(x, \boldsymbol{z})$, in order to pin down the Frisch elasticities of labor supply. We will set the value of this derivative using empirical estimates of the sharing rule. We detail this empirical evidence and our procedure below.

3.2.2 Parameter Values

Our model contains eight preference parameters: β, φ_c^m, φ_l^f, φ_l^m, σ^f, σ^m, γ^f and γ^m. There are three technology parameters: α, λ and δ. The two transition matrices Π^f and Π^m contain four parameters. The parameter $\underline{a}$ defines the minimum level of asset holdings for any household, i.e. the borrowing limit. Fiscal policy is described by labor income tax rates and the level of unemployment benefits: τ^f, τ^m, b^f and b^m. Finally, we have to pin down the derivative of the Pareto weight function with respect to x, μ_1.

The length of a period in the model is set to one quarter. We will normalize φ_c^f to 1, which is equivalent to dividing both instantaneous utility functions by this parameter. The borrowing limit is set to zero, i.e. households are restricted to hold non-negative asset holdings at all times. In order to calibrate the remaining parameters we choose a set of statistics from aggregate and household survey data for the U.S economy, such that the incomplete markets equilibrium of our collective household economy matches these targets. Using estimates for the quarterly capital depreciation rate and the capital share of income, we set $\delta = 0.025$ and $\alpha = 0.36$, which are both standard values in the macro literature.

In our benchmark economy, we impose equal labor income tax rates for females and males, $\tau^f = \tau^m$. Consequently, the value for λ can be pinned down using *a priori* information on the gender wage gap. We set this parameter equal to 0.575, which implies a ratio of female to male wages of 0.74. This corresponds to the gender wage gap in 2004 as reported by Heathcote, Storesletten and Violante (2008) for the U.S. economy.

Transition probabilities for idiosyncratic employment shocks are assumed to be identical for females and males. While the female unemployment rate averaged 1.5% percentage points higher than the male rate during the period 1960-1980, the female-male gap disappeared after the early 1980's. Even though male unemployment rates generally increase more than female rates during recessions —mainly due to the fact that men dominate industries like manufacturing and construction— the average difference between female and male unemployment rates over the period 1980-2009 is practically zero. Explanations for the narrowing gap in unemployment rates point to the relative increase of service-oriented industries which employ a large proportion of women. We use the following transition probabilities which match an average employment rate of 93% after normalizing with the participation rate,[10]

$$\Pi^i = \begin{pmatrix} 0.09 & 0.91 \\ 0.06 & 0.94 \end{pmatrix} \quad \text{for } i = f, m. \tag{3.6}$$

Our assumption that within-household unemployment shocks are uncorrelated can be supported from SIPP data. Indeed, from the April 1996 panel of the Survey of Income and Program Participation, which covers 48 months between April 1996 and March 2000, it is possible to compute the within-household unemployment correlation. Since information on occupation is available in these data, unemployment correlations can be computed both for households where husband and wife report different occupation and for households reporting same occupation. Within-household unemployment correlation in the first group is 0.05, and 0.23 in the second. It should be noted, however, that the fraction of households reporting same occupation for husband and wife is only 3.2% of the total. (For a detailed explanation on the calculation of these correlations, see Shore and Sinai 2010.)

The remaining twelve parameters are set such that our model matches the following targets:

1. Married females' average hours of work if working represent 28% of their discretionary time. Married males' average hours of work if working represent 40% of their discretionary time.[11]

2. Estimates for males' Frisch elasticity of labor supply in the presence of potentially binding borrowing constraints range from 0.2 to 0.6 (see Domeij and Flodén 2006). Blundell and

[10] These transition probabilities are similar to the ones used in the previous literature, see e.g. Imrohoroglu (1989), Krusell and Smith (1998) and Marcet, Obiols-Homs and Weil (2007).

[11] Mazzocco, Ruiz and Yamaguchi (2008) use PSID data from 1968 to 1996 to compute mean annual hours worked if working for married females and males; he finds values of 1660 and 2312 respectively. We make the assumption that the disposable daily time endowment is 16 hours.

MacCurdy (1999) find that for females this elasticity is 3-4 times larger than for males. We will target values of 0.37 and 1.2 for males and females, respectively.

3. Non-gender-based estimates of the average coefficient of relative risk aversion have yielded values ranging from 1 to 10. When gender is taken into account, females are found to be more risk-averse than males.[12] We set individual preferences for risk at $\sigma^f = 2$ and $\sigma^m = 1.5$, which yield an average coefficient of relative risk aversion for the collective household of

4. The capital-to-output ratio is around 10.

5. The ratio of annual hours worked by single working women to annual hours worked by single working men is $1861/2095 = 89$ percent.[13] We will match this value using the equilibrium of the bachelor economy.

6. The average net unemployment benefit replacement rate in the United States is roughly 30 percent (see OECD 2010). We will set b^f and b^m to match this target as fractions of the average wage income both for females and males. Labor income tax rates are set to balance the budget constraint of the government.

7. The derivative of the Pareto weight function with respect to the expected income differential, μ_1, is set to match the sharing rule estimates presented in Browning, Bourguignon, Chiappori and Lechene (1994).

As for the last target, Browning et al. (1994) use data on couples with no children to estimate the parameters of the sharing rule: they find that the wife's share in total expenditure increases modestly with her share in household income. Specifically, increasing the wife's contribution to household income from 25% to 75% (holding total expenditure constant) raises her share in total expenditure by about 2.3%. In addition, the impact of total expenditure on the wife's share is positive and sizable. For instance, an increase in total expenditures (holding her relative contribution to household income fixed) by 60% raises the wife's share by about 12%. We use these empirical estimates to ascertain the value of $\mu_1|_{\mu=0.5}$ as follows. Starting from the benchmark equilibrium and $\mu = 0.5$, we increase the value of x — e.g. by raising w^f/w^m — and then compute the new Pareto weight, say $\tilde{\mu}$, such that the implied increase in the wife's relative contribution to household income yields an increase in the wife's share of total expenditure, $c^f/(c^f + c^m)$, that matches the one implied by the sharing rule as estimated in Browning, Bourguignon, Chiappori and Lechene (1994).[14] Given the imputed value $\tilde{\mu}$, we then use a linear approximation to obtain $\mu_1|_{\mu=0.5}$.

[12]For a recent study of risk aversion and gender see Maestripieri, Sapienza and Zingales (2009), who find a negative relation between testosterone levels and risk aversion.

[13]See Mazzocco, Ruiz and Yamaguchi (2008).

[14]When computing the wife's shares of income and expenditures, we take the average over all households.

Table 1 presents parameter values for our benchmark economy.

Table 1. Baseline Parameters

Description	Parameter	Value	Description	Parameter	Value
Female risk aversion	σ^f	2	Utility weight	φ_c^f	1
Male risk aversion	σ^m	1.5	Utility weight	φ_c^m	2.15
Regulates Frisch elasticity (f)	γ^f	2	Utility weight	φ_l^f	2.662
Regulates Frisch elasticity (m)	γ^m	3.75	Utility weight	φ_l^m	0.911
Pareto weight	μ	0.5	Discount factor	β	0.989
Derivative Pareto weight	μ_1	0.038	Unemployment benefits (f)	b^f	0.083
Elasticity of output to capital	α	0.36	Unemployment benefits (m)	b^m	0.161
Depreciation rate of capital	δ	0.025	Relative wages	λ	0.575

3.3 Steady-state Equilibrium

Aggregate variables in the steady-state equilibrium with incomplete markets are presented in Table 2 below, both for the collective and the bachelor household economies. Note that the two economies differ only in the insurance opportunities available to individuals, and, therefore, differences in aggregates variables reflect the equilibrium effects of intra-household risk sharing. Aggregate capital is higher in the bachelor economy, as the lack of insurance from the family in this economy leads individuals to rely more on savings. Aggregate work effort by females and males rank differently in the two economies. While male labor is higher in the collective economy, females work more in the bachelor economy. In this latter economy, females are relatively poorer and, since they lack the consumption insurance provided by the family, must supply more hours of work. On the contrary, males finance part of female consumption in the collective economy (even with equal Pareto weights) and must therefore work longer hours. Total labor is higher in the bachelor household economy. We will elaborate further on this below. The capital-labor ratio is lower in the economy with intra-household risk sharing, yielding a higher interest rate as compared to the economy with bachelor households. Finally, production is higher in the economy with bachelor households, which results from larger aggregate capital and labor.

Table 2. Steady-state equilibrium: Aggregate Variables

	Y	K	L	K/L	L^f	L^m	$1+r$
Collective household economy	1.2723	12.6820	0.3490	36.3351	0.2799	0.4001	1.1115
Bachelor household economy	1.3081	13.0410	0.3588	36.3452	0.3363	0.3754	1.1109

3.3.1 Policy Functions

The relative contribution of households across the wealth and employment distribution to the differences in economic aggregates shown in Table 2 are now explored. Labor supply and saving policy functions of collective households are presented in Figures 1 and 3, respectively. The top panel of Figure 1 plots hours worked by females and males in households where the two spouses are employed. Hours decrease with household wealth, and the rate of decline is higher for females, implying that they work relatively less in asset-rich households. As asset holdings approach the borrowing limit, policy functions for hours bend upwards, capturing the fact that asset-poor households use labor supply to smooth consumption more intensively. Hours worked by females and males when the spouse is unemployed are plotted in the bottom panel of Figure 1 (for convenience, we plot them along with those emerging when the two spouses are employed). First, hours supplied increase if spouse is unemployed, both for females and males, and the increase is especially marked for females in asset-poor households. For example, a female in a household with no assets will supply almost half of her available time to work if the spouse is unemployed, as opposed to 0.37 when the spouse is employed, which represents a decline of more than 25%.

We now display the effects of intra-household risk sharing on hours worked at different levels of asset holdings and employment shocks. Figure 2 (top panel) plots excess hours worked by two bachelors (each with wealth $a/2$) over hours worked by a two-person collective household (with wealth a). For all households where only the male is employed, intra-household risk sharing increases household hours. For households where the female is employed, with the exception of low-wealth households with the male unemployed, intra-household risk sharing decreases household hours. The bottom panel of the Figure shows the average of these excess hours across households along the employment distribution. As it is apparent, the effects of intra-household risk sharing on hours are highest among wealth-poor households.

Savings policy functions in the collective model are presented in Figure 3 (for convenience we plot the net change in asset holdings $a' - a$). Households where the two spouses are employed choose positive net savings at the borrowing limit and at all values in the support of the equilibrium distribution of assets. For households with at least one of the spouses unemployed, net savings are zero at the borrowing limit and negative for a large set of asset holdings. Negative net savings are larger in households where the male is unemployed. The saving effects of intra-household risk sharing at different levels of asset holdings are shown in Figure 4. The top panel of Figure 4 plots excess savings of two bachelors (each with wealth $a/2$) over a two-person collective household (with wealth a). The bottom panel plots the average of excess savings across employment shocks. Clearly, although risk sharing affects the saving decisions of all households across the wealth distribution, its effects are highest among wealth-poor households.

3.4 Aggregate Precautionary Savings and Precautionary Labor Supply

We now move to assessing the consequences of completing markets and how these depend on the ability to share risks within the family. As already noted, aggregate precautionary savings in our framework are small, regardless of whether intra-household risk sharing is available or not. This is a consequence of our specification of the income process —employment/unemployment shocks coupled with unemployment benefits— which lacks the necessary persistence to generate large incentives to save for precautionary reasons. However, a comparison of precautionary savings and work effort across the economies with collective and bachelor households will help us assess the implications of intra-household risk sharing.

In Table 3 we present aggregate precautionary savings and precautionary labor supply in the collective model relative to those in the bachelor model. That is, we report $\Delta^{col.}/\Delta^{bach.}$, where Δ^i for $i = col., bach.$ denotes precautionary aggregates (savings and work effort) under households of type i. For our baseline parameter values, precautionary savings —measured as the fraction of capital held for precautionary motives— in the economy with collective households represent 55% of those in the economy with bachelor households. That is, access to insurance from the family reduces aggregate precautionary savings by 45%.

Aggregate precautionary work effort is equally measured by the fraction of hours worked for precautionary motives, i.e., $(L_{IM} - L_{CM})/L_{IM}$. Both for females and males, aggregate work effort is higher in the complete markets economy, implying negative aggregate precautionary labor under both households arrangements. This is a consequence of an ex-post wealth effect operating in the incomplete markets economy. That is, conditional on being employed, individuals work relatively less hours in the incomplete markets economy because the inability to buy employment insurance makes them ex-post richer. Marcet, Obiols-Homs and Weil (2007) were the first to uncover the implications of this ex-post wealth effect for aggregate precautionary labor in the Aiyagari-Hugget model. In Table 3 we report precautionary labor in the collective household economy relative to that in the bachelor economy. The percentage increase in aggregate female labor resulting from completing markets in the collective household economy is only 37% of the increase under bachelor households. The increase in aggregate male labor represents 75% of the increase under bachelor households. That is, the ex-post wealth effect is weaker in the collective economy.

Table 3. Relative Precautionary Savings and Precautionary Work Effort

	K	L	L^f	L^m
$\Delta^{col.}/\Delta^{bach.}$	0.5552	0.5586*	0.3769*	0.7502*

Notes: $\Delta^i \equiv 1 - CM^i/IM^i$ for $i = col., bach.$, represent the fraction of capital held and hours worked for precautionary motives in an economy with households of type i. That is, CM^i and

IM^i refer to aggregates under complete and incomplete markets, respectively. * For the case of aggregate labor, both $\Delta^{col.}$ and $\Delta^{bach.}$ are negative. I.e., both in the collective and the bachelor economies aggregate work effort is higher under complete markets than under incomplete markets.

4 Intra-household Risk Sharing and the Crowding out Effects of Unemployment Benefits

In our model economy there are two insurance mechanisms —in addition to public unemployment benefits— households can use to smooth consumption across unemployment spells: savings and labor supply. In the economy with intra-household risk sharing, spousal labor supply is a potentially important instrument to smooth consumption upon a spousal's unemployment spell. Changes in the level of public insurance call forth adjustments in the demand for other forms of insurance. The extent to which the ability to share risks within the household shapes the crowding out effects of public unemployment insurance is explored in this section.

4.1 Household Financial Assets and the Generosity of Unemployment Benefits

An implication of our model, as of any model with uninsurable income risk, is that household asset holdings increase with income uncertainty. Engen and Gruber (2001) exploit the variation in generosity in the unemployment insurance schedules across U.S. states to test this implication and to estimate the extent of the precautionary savings motive. Since the level of unemployment benefits is directly correlated with household income risk, this variation can be used to measure the extent to which benefits crowd out household financial assets. These authors use data from the Survey of Income and Program Participation (SIPP), which follows a cross section of individuals over a period of 2.5 years and find that the elasticity of the average household's financial assets-to-income ratio with respect to unemployment benefits is -0.28. That is, reducing the replacement rate of unemployment benefits by 50% would rise the average household's assets-to-income ratio by 14%.

In this subsection, we use our model economy to compute the elasticity of the average assets-to-income ratio with respect to unemployment benefits. The purpose of this exercise is twofold. On the one hand, we use it as a test for our model with collective households to match this estimated measure of the precautionary savings motive. On the other hand, we also compute this elasticity using the bachelor household model and assess by how much it overestimates the precautionary motive. In this latter model there is no intra-household risk sharing and, therefore, variation in unemployment benefits amounts to larger changes in household income risk and, consequently, to larger effects on savings.

Because the empirical test conducted by Engen and Gruber (2001) relies on the variation in unemployment benefits for workers living in different states in the U.S., we conduct our quantitative exercise keeping the interest rate constant at the equilibrium level of our benchmark economy when we vary unemployment benefits. That is, we interpret unemployment benefits in our benchmark economy as an average across all states. Then, we vary these benefits and solve the model keeping the interest rate unchanged, an strategy which is in accordance with the existence of a unique financial market. The results of this exercise are presented in Table 4. As shown there, our collective household model does much better at matching the empirical elasticity estimated by Engen and Gruber (2001) than the bachelor household model. The economy with intra-household risk sharing yields an elasticity of the asset-income ratio of -0.39, against an elasticity of -1.05 in the bachelor household economy. Intra-household risk sharing reduces this elasticity by more than 60%, which indicates the importance of this informal source of insurance to assessing the crowding out effects of public unemployment insurance.

It should be noted, however, that our collective household economy does not match exactly the empirical elasticity, -0.39 in the model against -0.28 in the data. A possible interpretation of this result is that the two-person household falls short of embedding all informal insurance arrangements available to individuals. Indeed, some authors have emphasized the important insurance role of the extended family, friends and other social networks.

Table 4. Unemployment Benefits and Financial Assets

	Elasticity of average assets-to-income ratio w.r.t. replacement rate
Data (Engen and Gruber 2001)	-0.28
Collective Household Economy	-0.39
Bachelor Household Economy	-1.05

Notes: This table shows how household asset holdings respond to the generosity of unemployment benefits.

4.2 Spousal Labor Supply as Insurance

In the face of unemployment risk and capital market imperfections, spousal labor supply becomes a potential source of household self-insurance. The change in a household member's labor supply induced by unemployment spells of another household member —the added worker effect— has been largely studied in the empirical literature. Most of this literature has focused on the labor supply response of married women to their husband's unemployment spells. The main argument in favor of restricting the attention to labor supply of women is that they are the secondary wage earners in most households (according to Cullen and Gruber 2000, in 87% of married couples in the U.S. the husband earns more and in 73% the husband works more hours).

Early literature on the added worker effect (see Cullen and Gruber 2000 for a short review) has singled out liquidity constraints as one of the main reasons married women increase hours worked during their husband's unemployment spells. Empirical estimates have however produced mixed results, failing to find strong support for this effect.[15] Cullen and Gruber (2000), using data from the 1984-88 and 1990-92 panels of the Survey of Income Program Participation for married couples aged between 25 and 54 years old, report means for wives' monthly hours worked during husbands' spells of employment and unemployment, respectively. Conditional on working women, these authors find that the average amount of work per month of wives of unemployed husbands is 149 hours, as opposed to 132.4 hours worked by wives of employed husbands. When non working wives are included, i.e. those who work 0 hours, the change in average hours is small: 98.2 hours for wives with an unemployed husband, against 97.9 hours for those with an employed husband.

In this section we use our model economy with collective households to study the response of female labor supply to male's unemployment spells in two groups of households. In order to highlight the role of liquidity constraints on wives' labor supply responses, we follow Zeldes (1989) in defining a household as liquidity constrained if its non-housing wealth is less than two months of average income. Table 5 below reports the added worker effect in our model economy. For the group of liquidity-constrained households, average hours worked by wives of unemployed husbands are 8% higher than those worked by wives of employed husbands, an increase comparable to that found by Cullen and Gruber (2000) in their sample of working women. When all households are taken into account the increase in hours is only 0.06%. That is, spousal labor supply is an important insurance mechanism for wealth-poor households but not for the wealth rich. *Notes:* This table

Table 5. Female Labor Supply and Male Employment Status

	Households with wealth less than two months worth of income	All households
Employed Husband	173.1	145.1
Unemployed Husband	187.8	145.9

[15] Stephens (2002) estimates the added worker effect taking into account not only the current period of the husband's job loss but also the periods before and after a job loss. This author finds small pre-displacement effects but large, persistent post-displacement effects.

shows average monthly hours of work by working females in households with employed and unemployed males in our baseline economy with collective households.

How effective is wives' labor supply as insurance against income fluctuations due to husbands' unemployment? In other words, what is the fraction of lost family income that is made up by the wife's response to the husband's unemployment spell? To answer this question we compute, for each level of asset holdings a, the following fraction,

$$\frac{[h^f(0,1,a) - h^f(1,1,a)]w^f}{h^m(1,1,a)w^m - b^m},$$

where $h^f(0,1,a)$ denotes hours worked by a female with an unemployed husband and $h^f(1,1,a)$ denotes female hours worked if the husband is employed. The denominator represents lost income due to husband's unemployment. The numerator is the increase in income due to the wife's response in hours. We then average out across asset holdings. For the group of liquidity-constraint households (i.e., with asset holdings less than two months worth of income) we obtain that wives' response makes up about 9% of lost family income, while this number is only 1% when we consider all households. Households with high levels of asset holdings use savings to smooth consumption upon husband's unemployment rather than using spouse labor supply. Liquidity-constrained households must rely, however, on spousal labor supply.

4.2.1 Spousal Labor Supply and the Generosity of Unemployment Benefits

Some authors have argued that the finding of a moderate to nil added worker effect may be partially explained by the presence of public unemployment insurance schemes. That is, unemployment payments during the husband's unemployment spell crowd out wife's labor supply. To quantify this effect, Cullen and Gruber (2000) estimate the response in wives' hours of work during their husbands' spells of unemployment to changes in unemployment benefits. They find evidence of a crowd out effect, i.e., increasing the benefits received by unemployed husbands reduces their wives' hours of work. Moreover, they also find a differentially larger response of wives' labor supply among those households that are less able to smooth consumption through own savings.

We use our model economy to compute the crowding out of unemployment benefits on wives' labor supply. Table 6 below presents the results of this exercise. A 50% reduction in unemployment benefits received by the husband increases wife's hours by almost 5% for the group of liquidity-constrained households. This increase is only 0.71% when all households are considered. The relatively higher sensitivity of spousal labor supply to unemployment benefits among liquidity-constrained households found in our model in is line with the finding of Cullen and Gruber (2000).[16]

[16] In order to compare the relative responsiveness of couples with differing levels of assets these authors split their

Table 6. Unemployment Benefits and Female Labor Supply During Male's Unemployment Spells

	Households with wealth less than two months worth of income	All households
10% reduction in b^m	+0.95%	+0.14%
50% reduction in b^m	+4.97%	+0.71%

Notes: This table shows the percentage increase in female labor supply upon a male's unemployment spell yielded by 10% and 50% reductions in unemployment benefits in our baseline economy with collective households.

Even though a direct comparison of our results with those estimated by Cullen and Gruber (2000) is not straightforward, it seems that our model under predicts the crowding out effect of unemployment benefits on spouse labor supply. According to their estimates, a 50% reduction in potential unemployment benefits of the husband (75 USD per week) would imply an increase in monthly hours worked by the wife (conditional on working) of 13.42 hours, which amounts to an increase of about 9%. Our model predicts that a 50% reduction in benefits receipt increases spouse labor supply, in the group of liquidity-constrained households, by 5%. It should be noted however that the estimate in Cullen and Gruber is not statistically significant, thus hindering the assessment of our model's predictions.

4.3 Consumption Loss Upon Unemployment

In economies with imperfect capital markets, the loss of the job implies a reduction in the level of individual consumption. In the case of complete markets, the consumption loss upon unemployment is equal to zero. In the opposite extreme case of bachelor individuals with no assets, unable to borrow and without entitlement to unemployment benefits, the consumption loss is one hundred percent. In intermediate cases with partial consumption insurance, the degree of transmission of unemployment shocks to consumption depends on factors such as the generosity of unemployment benefits, on the level of accumulated wealth and on whether risks are shared within the household.

In this section we use our benchmark economy to assess the contribution of intra-household risk sharing to individual consumption insurance, as measured by the degree of transmission of unemployment shocks to consumption. We do so by comparing individual consumption losses upon unemployment in the collective household model to those in the bachelor model. We compute the

sample of unemployment spells according to the age of the couple. Then, they interpret that households where the two spouses are under 40 years of age are liquidity constrained.

percentage change in consumption upon unemployment, $\triangle c/c$ for all asset holdings in the support of the corresponding equilibrium distribution. For the collective economy, individual consumption losses for females and males, both with an employed spouse and with an unemployed spouse, are computed as,

$$\frac{c^j(s^j = 1, s^i, a) - c^j(s^j = 0, s^i, a)}{c^j(s^j = 1, s^i, a)}$$

for $j = f, m$, $i = f, m$ and $i \neq j$, both for $s^i = 1$ and $s^i = 0$. For the bachelor economy, individual consumption losses upon unemployment are simply computed as, $(c^j(1, a) - c^j(0, a))/c^j(1, a)$ for $j = f, m$.

In Table 7 we report average individual consumption losses, both for the group of liquidity-constraint individuals and for all individuals. We use the respective equilibrium asset and employment distributions to average out individual consumption losses. The results show that intra-household risk sharing provides important consumption smoothing opportunities, especially for liquidity-constraint individuals. Thus, the average consumption loss for a liquidity-constraint female in the bachelor economy is -21.34%, against only -2.72% in the collective economy, which is ten times smaller. For a liquidity-constraint male, intra-household risk sharing reduces the consumption loss from -30.68% to -6.55%. These numbers imply that the family is an important provider of consumption insurance for a significant fraction of individuals.

It should be noted that, even in the collective household economy, liquidity-constraint individuals suffer a significant loss in consumption when faced with an unemployment shock. Despite public unemployment benefits and the insurance provided by the family, the average consumption loss for a male in this group is 6.55% and 2.27% for a female. This result is qualitatively consistent with the empirical finding of Blundell, Pistaferri and Preston (2008) about the degree of insurability of transitory shocks. These authors find that whereas the impact of these shocks on consumption is small when estimated from all households in their sample, the impact is significant when estimated for the group of wealth-poor households (these authors define a household as wealth poor if its wealth in the first year they are observed is in the bottom 20 percent of the distribution of initial wealth.

that transitory income shocks have a significant impact on consumption among low-wealth households.

4.3.1 Consumption Loss and the Generosity of Unemployment Benifits

We now turn to the sensitivity of household consumption losses upon unemployment with respect to the generosity of unemployment benefits, and assess the extent to which our model economy with collective households matches the empirical findings of Browning and Crossley (2001). These authors use a Canadian panel data set to estimate how changes in household consumption following

Table 7. Individual consumption loss upon unemployment

	Collective Model		Bachelor Model	
	Liquidity-constrained individuals	All individuals	Liquidity-constrained individuals	All individuals
Females, $\triangle c^f/c^f$	-2.72%	-0.12%	-21.34%	-0.35%
Males, $\triangle c^m/c^m$	-6.55%	-0.32%	-30.68%	-0.57%

a job loss vary with the generosity of unemployment benefits. They obtain two main results. First, the level of unemployment benefits has small average effects on household consumption loss upon unemployment. In particular, a 10 percentage-point reduction in benefits leads to an average fall in consumption of 0.8%.[17] Second, the consumption effects of unemployment benefits are not homogeneous across households. For instance, for the sub-sample of liquidity-constrained households at the time of job separation these effects are substantially larger. (These authors also follow Zeldes (1989) in defining a household as liquidity-constrained if its non-housing wealth is less than two months of average disposal income.) These results show the importance of unemployment benefits as a consumption smoothing instrument for a large number of households. They also highlight the importance of carrying out analyse which go beyond a representative agent and thus beyond estimating mean effects.

Table 8 below presents the elasticities of consumption loss with respect to unemployment benefits in our model economy with and without intra-household risk sharing, and compares the results to the estimates in Browning and Crossley (2001).

It is important to note that estimates by Browning and Crossley (2001) of the elasticity of household consumption loss upon unemployment with respect to unemployment benefits use Canadian data, while our baseline parameter values have been chosen to match some U.S. stylized facts. Since it is likely that this elasticity differs when evaluated at U.S. equilibrium values, our exercise in this section should not be taken as an attempt at matching the estimated Canadian elasticity. It serves, however, to shed further light on the role of intra-household risk sharing. The elasticity predicted by the bachelor economy, 0.2, is more than two times the elasticity under collective households.

[17] Gruber (1997) uses U.S. data on food consumption from the Panel Study of Income Dynamics (PSID) and finds a larger mean effect of unemployment benefits on consumption losses upon unemployment. This author estimates that a 10 percentage-point increase in benefits reduces the fall in consumption by 2.65%.

Table 8. Elasticity of Household Consumption Loss to Unemployment Benefits

	Households with wealth less than two months worth of income	All households
Data (Browning and Crossley 2001)	0.0922	0.05
Collective Household Economy	0.0875	0.001
Bachelor Household Economy	0.2	0.002

5 Concluding Remarks

To be written

6 Appendix I: Proofs

Proof of Proposition 1:

(a) The proof of this part follows from the Contraction Mapping Theorem and Theorem 3 and Corollary 2 in Denardo (1967).

(b) Case 1: We consider first values of a such that $a'(\boldsymbol{s}, a) > \underline{a}$ (interior solution).

(i) $c^f(\boldsymbol{s}, a)$, $c^m(\boldsymbol{s}, a)$ are strictly increasing in a. Take the envelope condition (using **A2**):

$$V_a(\boldsymbol{s}, a; \mu) = \mu U_c^f(c^f(\boldsymbol{s}, a), \cdot)(1 + r) = (1 - \mu)U_c^m(c^m(\boldsymbol{s}, a), \cdot)(1 + r). \tag{6.1}$$

Since $V(\boldsymbol{s}, a, \mu)$ is strictly concave, $V_a(\boldsymbol{s}, a; \mu)$ is strictly decreasing in a. It follows that $U_c^i(c^i(\boldsymbol{s}, a; \mu), \cdot)$, $i = f, m$, must be strictly decreasing in a as well. Since U^i is strictly concave in c^i, the result follows.

(ii) $a'(\boldsymbol{s}, a)$ increasing in a. By contradiction: suppose there were values a_1, a_2 such that $a_2 > a_1$ and $a'(\boldsymbol{s}, a_2) < a'(\boldsymbol{s}, a_1)$. Then since $c^f(\boldsymbol{s}, a)$ is strictly increasing in a (as shown before), it has to be that $c^f(\boldsymbol{s}, a'(\boldsymbol{s}, a_2)) < c^f(\boldsymbol{s}, a'(\boldsymbol{s}, a_1))$. As utility is separable and the marginal utility of consumption does not depend on the level of leisure, the following holds:

$$\beta(1 + r)E\left[U_c^f(c^f(\boldsymbol{s}', a'(\boldsymbol{s}, a_2)), \cdot)\right] > \beta(1 + r)E\left[U_c^f(c^f(\boldsymbol{s}', a'(\boldsymbol{s}, a_1)), \cdot)\right].$$

However, the Euler equation then implies $U_c^f(c^f(\boldsymbol{s}, a_2), \cdot) > U_c^f(c^f(\boldsymbol{s}, a_1), \cdot)$, which is a contradiction because $c^f(\boldsymbol{s}, a_2) > c^f(\boldsymbol{s}, a_1)$.

(iii) $l^f(s^f=1,s^m,a)$ and $l^m(s^m=1,s^f,a)$ increasing in a. Intratemporal optimality requires:

$$\frac{U_l^i}{U_c^i} \geq w^i s^i, \quad \text{for } i=f,m, \tag{6.2}$$

with inequality if $l^i=1$. Since $c^i(\boldsymbol{s},a)$ is strictly increasing in a, $U_c^i(c^i(\boldsymbol{s},a),\cdot)$ is strictly decreasing in a. Hence, $U_l^i(\cdot,l^i(s^i=1,s^j,a))$ has to be decreasing in a, too. This implies that $l^i(s^i=1,s^j,a)$ is increasing in a.

Case 2: Consider now values of a such that $a'(\boldsymbol{s},a)=\underline{a}$ (non-interior solution).

In this case the budget constraint reads

$$c^f(\boldsymbol{s},a)+c^m(\boldsymbol{s},a)=w^f(1-l^f(\boldsymbol{s},a))s^f+w^m(1-l^m(\boldsymbol{s},a))s^m+(1+r)a-\underline{a}. \tag{6.3}$$

The proof is by contradiction:

(i) Suppose that $l^f(\boldsymbol{s},a)$ is decreasing in a and $l^m(\boldsymbol{s},a)$ is increasing in a. From intratemporal optimality (6.2) it follows that $c^f(\boldsymbol{s},a)$ must be decreasing in a and that $c^m(\boldsymbol{s},a)$ must be increasing in a. This is a contradiction with (2.12).

(ii) Suppose that $l^f(\boldsymbol{s},a)$ is increasing in a and $l^m(\boldsymbol{s},a)$ is decreasing in a. From intratemporal optimality (6.2) it follows that $c^f(\boldsymbol{s},a)$ must be increasing in a and that $c^m(\boldsymbol{s},a)$ must be decreasing in a. This is a contradiction with (2.12).

(iii) Suppose that $l^f(\boldsymbol{s},a)$ and $l^m(\boldsymbol{s},a)$ are decreasing in a. From intratemporal optimality (6.2) it follows that $c^f(\boldsymbol{s},a)$ and $c^m(\boldsymbol{s},a)$ must be decreasing in a. This is a contradiction with (6.3).

Hence, $l^f(\boldsymbol{s},a)$ and $l^m(\boldsymbol{s},a)$ are increasing in a, and (6.3) implies that c^f and c^m are strictly increasing in a.

(c) Case 1: Consider values of a such that $a'(\boldsymbol{s},a)>\underline{a}$ (interior solution).

As in the proof of Lemma 1 in Huggett (1993), it can be shown by induction that $V_a(s^j=1,s^i,a)\leq V_a(s^j=0,s^i,a)$, $\forall s^i$, using the assumption that $\pi^i_{1|1}\geq\pi^i_{1|0}$. The result then follows immediately from the envelope condition (6.1).

Case 2: We consider now values of a such that $a'(\boldsymbol{s},a)=\underline{a}$ (non-interior solution).

First we show that $c^j(s^j=1,s^i=0,a)\geq c^j(s^j=0,s^i=0,a)$. Evaluating the budget constraint at these two household's employment shocks we obtain,

$$\begin{aligned}
&c^j(s^j=1,s^i=0,a)+c^i(s^j=1,s^i=0,a)+\underline{a}-(1+r)a-w^j(1-l^j(s^j=1,s^i=0,a))=0\\
&c^j(s^j=0,s^i=0,a)+c^i(s^j=0,s^i=0,a)+\underline{a}-(1+r)a=0.
\end{aligned} \tag{6.4}$$

This implies that $c^j(s^j = 1, s^i = 0, a) + c^i(s^j = 1, s^i = 0, a) \geq c^j(s^j = 0, s^i = 0, a) + c^i(s^j = 0, s^i = 0, a)$. The result follows from the first-order condition for consumption, (2.12).

We now show that $c^j(s^j = 1, s^i = 1, a) \geq c^j(s^j = 0, s^i = 1, a)$. Using the budget constraint and eliminating terms we get,

$$\begin{aligned} & c^j(s^j = s^i = 1, a) + c^i(s^j = s^i = 1, a) - w^i(1 - l^i(s^j = s^i = 1, a)) - w^j(1 - l^j(s^j = s^i = 1, a)) \\ = \;& c^j(s^j = 0, s^i = 1, a) + c^i(s^j = 0, s^i = 1, a) - w^i(1 - l^i(s^j = 0, s^i = 1, a)). \end{aligned} \tag{6.5}$$

Suppose, towards a contradiction, that $c^i(s^j = 1, s^i = 1, a) < c^i(s^j = 0, s^i = 1, a)$. Intratemporal optimality (6.2) then requires $l^i(s^j = 0, s^i = 1, a) > l^i(s^j = 1, s^i = 1, a)$, and (2.12) implies $c^j(s^j = 1, s^i = 1, a) < c^j(s^j = 0, s^i = 1, a)$. Hence, the right hand side of equation (6.5) is strictly larger than the first three terms on the left hand side, which immediately leads to a contradiction.

(d) Start from $c^j(s^j = 1, s^i, a) \geq c^j(s^j = 0, s^i, a), \forall a$. Then (2.12) implies that $c^i(s^j = 1, s^i, a) \geq c^i(s^j = 0, s^i, a)$. The result follows immediately from equations (2.13) and (2.14).

(e) By contradiction: suppose there is an $a \in [\underline{a}, \overline{a}]$ such that $a'(s^f = 0, s^m = 0, a) > a$ and

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) = \beta(1 + r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right], \quad i = f, m.$$

(The equality follows from $a'(s^f = 0, s^m = 0, a) > a \geq \underline{a}$.) Since (i) $\beta(1 + r) \leq 1$, (ii) $c^i(\boldsymbol{s}, a)$ strictly increasing in a and (iii) $c^i(\boldsymbol{s}, a)$ is time-invariant if factor prices are constant, it follows that:

$$\beta(1 + r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right] \leq E\left[U_c^i(c^i(\boldsymbol{s}', a), \cdot)\right].$$

Combining these two expressions implies that

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) \leq E\left[U_c^i(c^i(\boldsymbol{s}', a), \cdot)\right].$$

Using part (c) this can only hold if $c^i(\boldsymbol{s}, a)$ is the same for all $\boldsymbol{s} \in S \times S$ and, consequently, $a'(\boldsymbol{s}, a) > a$ for all $\boldsymbol{s}$. Since consumption is strictly increasing in a, this implies that future consumption will be strictly higher in any state $\boldsymbol{s}'$ and, hence,

$$U_c^i(c^i(\boldsymbol{s}, a), \cdot) > E\left[U_c^i(c^i(\boldsymbol{s}', a'(\boldsymbol{s}, a)), \cdot)\right].$$

The Euler equation, however, requires

$$U_c^i(c^i(\boldsymbol{s}, a), \cdot) = \beta(1 + r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s, a)), \cdot)\right],$$

which is impossible for $\beta(1 + r) \leq 1$.

Strict inequality: suppose there is an $a \in (\underline{a}, \overline{a})$ such that $a'(s^f = 0, s^m = 0, a) = a$. Using part (c) it follows that $a'(\boldsymbol{s}, a) \geq a$ for all $\boldsymbol{s}$. Since consumption is strictly increasing in a, this implies that future consumption will be at least as high as current consumption in any state s' and, hence,

$$U_c^i(c^i(s^f = 0, s^m = 0, a), \cdot) \geq E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, , a)), \cdot)\right].$$

The Euler equation, however, requires

$$U_c^i(c^i(s^f = 0, s^m =, a), \cdot) = \beta(1+r)E\left[U_c^i(c^i(\boldsymbol{s}', a'(s^f = 0, s^m = 0, a)), \cdot)\right],$$

(the equality follows from $a'(s^f = 0, s^m = 0, a) = a > \underline{a}$). This is impossible for $\beta(1+r) < 1$.

Proof of Proposition 2:

In order to compact notation, we will write $\tilde{a}(\mu)$ simply as $\tilde{a}$.

(a) Let us first assume $r > 0$. We prove that $a'(\boldsymbol{s}, \widetilde{a}) \leq \widetilde{a}$. The result then follows from the fact that $a'(\boldsymbol{s}, a)$ is increasing in a, as shown before. From part (c) of Proposition 1, $a'(s^f = 0, s^m = 0, \widetilde{a}) \leq \widetilde{a}$. Then using the budget constraint:

$$a'(s^f = 0, s^m = 0, \widetilde{a}) \leq \widetilde{a} \tag{6.6}$$

$$\begin{aligned} w^f \cdot \left(1 - l^f(s^f = 0, s^m = 0, \widetilde{a})\right) \cdot 0 + w^m \cdot \left(1 - l^m(s^f = 0, s^m = 0, \widetilde{a})\right) \cdot 0 & \\ +(1+r)\widetilde{a} - c^f(s^f = 0, s^m = 0, \widetilde{a}) - c^m(s^f = 0, s^m = 0, \widetilde{a}) &\leq \widetilde{a} \end{aligned} \tag{6.7}$$

$$c^f(s^f = 0, s^m = 0, \widetilde{a}) + c^m(s^f = 0, s^m = 0, \widetilde{a}) \geq r\widetilde{a}. \tag{6.8}$$

From before we know that decision rules for consumption are increasing in endowments; hence,

$$c^f(\boldsymbol{s}, \widetilde{a}) + c^m(\boldsymbol{s}, \widetilde{a}) \geq r\widetilde{a}, \qquad \forall \boldsymbol{s}.$$

Finally, use the definition of $\widetilde{a}$ from above and the FOC with respect to leisure to get

$$l^f(\boldsymbol{s}, \widetilde{a}) = l^m(\boldsymbol{s}, \widetilde{a}) = 1, \qquad \forall \boldsymbol{s}.$$

Hence, $a'(\boldsymbol{s}, \widetilde{a}) \leq \widetilde{a}$.
Case $r \leq 0$: Take $a_1 < a_2$ and thus $c^f(\boldsymbol{s}, a_1) + c^m(\boldsymbol{s}, a_1) < c^f(\boldsymbol{s}, a_2) + c^m(\boldsymbol{s}, a_2)$. Plug in the budget constraints:

$$\begin{aligned} w^f(1 - l^f(\boldsymbol{s}, a_1))s^f + w^m(1 - l^m(\boldsymbol{s}, a_1))s^m + (1+r)a_1 - a'(\boldsymbol{s}, a_1) &< \\ w^f(1 - l^f(\boldsymbol{s}, a_2))s^f + w^m(1 - l^m(\boldsymbol{s}, a_2))s^m + (1+r)a_2 - a'(\boldsymbol{s}, a_2) & \end{aligned} \tag{6.9}$$

and thus

$$a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1) < (1+r)(a_2 - a_1) + w^f(l^f(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^f + w^m(l^m(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^m.$$

Divide by $a_2 - a_1$:

$$\frac{a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1)}{a_2 - a_1} < (1+r) + \frac{1}{a_2 - a_1}\left[w^f(l^f(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^f + w^m(l^m(\boldsymbol{s}, a_1) - l^f(\boldsymbol{s}, a_2))s^m\right].$$

Since leisure is increasing in a, the last two terms are non-positive. Also, r is non-positive by assumption. Therefore,

$$\frac{a'(\boldsymbol{s}, a_2) - a'(\boldsymbol{s}, a_1)}{a_2 - a_1} < 1.$$

That is, the decision rule for capital accumulation has a slope that is strictly lower than 1 and strictly positive. This implies that for all s there is a level of asset holdings $\widetilde{a}(s)$ (this is not the same $\widetilde{a}$ as above!) such that $a'(\boldsymbol{s}, \widetilde{a}) \leq \widetilde{a}$, i.e. a' crosses the 45 degree line at most once.

(b) Take an arbitrary level of asset holdings $a_0 \geq \widetilde{a}$ and check whether the proposed allocation $\left\{\widehat{c}^f, \widehat{c}^m, \widehat{l}^f, \widehat{l}^m, \widehat{a}'\right\}$ satisfies first-order optimality:

- equation (2.12) is satisfied by definition
- $\widehat{c}^f + \widehat{c}^m = a\, r \geq \widetilde{a}\, r = \widetilde{c}^f + \widetilde{c}^m$; moreover, $\widehat{c}^i \geq \widetilde{c}^i \Longrightarrow \widehat{U}^i_c \leq \widetilde{U}^i_c$, $i = f, m$, which implies by (2.20) that equations (2.13) and (2.14) are satisfied
- the budget constraint (2.10) holds and
- the Euler equation (2.16) holds because consumption is constant.

Since the problem is concave, first-order optimality is sufficient for an optimum. Since the policy functions characterize the optimum, the proposed allocation is optimal.

(c) The proof exploits results in Chamberlain and Wilson (2000), which are also used in Marcet,Obiols-Homs and Weil (2007). Part (a) implies that $a_t \leq \widetilde{a}(\mu)$, $\forall t$, and part (b) of Proposition 1 together with part (b) of Proposition 2 imply that $c^i_t \leq \widetilde{c}^i(\mu)$, $i = f, m$, so that individual consumption levels are bounded almost surely. The first-order condition to savings (2.16) and (2.17) imply that $U^i_{c,t} \geq E_t(U^i_{c,t+1})$ almost surely, so that $U^i_{c,t}$ is a super-martingale. As $U^i_{c,t}$ is bounded from below by $U^i_c\left(\widetilde{c}^i(\mu)\right)$, we can apply the martingale convergence theorem, which implies that $U^i_{c,t}$ converges almost surely to a random variable. Suppose, by contradiction, that $U^i_{c,t}$ converged to a value strictly larger than $U^i_c\left(\widetilde{c}^i(\mu)\right)$, which would imply that consumption levels would converge to values $\widehat{c}^i < \widetilde{c}^i(\mu)$, so that the consumption-leisure choice would be interior for at least one of the two spouses when employed. In that case labor income would converge to $\iota \equiv w^f(1 - \widehat{l}^f)s^f + w^m(1 - \widehat{l}^m)s^m$, where $\widehat{l}^f$ and/or $\widehat{l}^m$ are strictly smaller than 1 and solve (2.13) and (2.14). ι is a non-degenerate random variable with positive variance, which implies that the lower or upper bounds on asset holdings would be violated with positive probability, a contradiction. This follows from the result of Chamberlain and Wilson (2000) that under $\beta(1 + r) = 1$ consumption and asset grow with no bound if income is suitably stochastic. Thus, $U^i_{c,t}$ cannot converge to a value strictly larger than $U^i_c\left(\widetilde{c}^i(\mu)\right)$ and it must converge to $U^i_c\left(\widetilde{c}^i(\mu)\right)$. Since U^i_c is invertible, consumption will converge to $\widetilde{c}^i(\mu)$. The budget constraint implies that a_t must converge to $\widetilde{a}(\mu)$.

Appendix II: The Complete Markets Economy

Let $\theta(\boldsymbol{s})$ denote the number of Arrow securities owned by the collective household. Then the household solves the following problem:

$$\begin{aligned} V\big(\boldsymbol{s}, \theta(\boldsymbol{s}); x, \boldsymbol{z}\big) &= \max_{c^f, c^m, l^f, l^m, \theta'(\boldsymbol{s}')} \Big\{ \mu(x, \boldsymbol{z}) U^f(c^f, l^f) + [1 - \mu(x, \boldsymbol{z})] U^m(c^m, l^m) \\ & \qquad + \beta \sum_{\boldsymbol{s}'} \pi_{\boldsymbol{s}'|\boldsymbol{s}} V\big(\boldsymbol{s}', \theta'(\boldsymbol{s}'); x, \boldsymbol{z}\big) \Big\} \qquad (6.10) \\ c^f + c^m &+ \sum_{\boldsymbol{s}'} p(\boldsymbol{s}, \boldsymbol{s}') \theta'(\boldsymbol{s}') = \sum_{i=f,m} w^i (1 - l^i) s^i + \sum_{i=f,m} w^i (1 - s^i) b^i + \theta(\boldsymbol{s}) \qquad (6.11) \\ c^f, c^m \geq 0, & \qquad 0 \leq l^f, l^m \leq 1. \qquad (6.12) \end{aligned}$$

where $p(\boldsymbol{s}, \boldsymbol{s}')$ denotes the price of an Arrow security that is purchased by a household in state s and pays one unit of the consumption good in the subsequent period if state $\boldsymbol{s}'$ is realized. For a household with relative Pareto weight μ in state $\boldsymbol{s}$, solving (6.10) yields the following system of optimality conditions:

$$\begin{aligned} \mu U_c^f &= (1 - \mu) U_c^m && && (6.13) \\ \frac{U_l^f}{U_c^f} &\geq w^f s^f && \text{with inequality if } l^f = 1 && (6.14) \\ \frac{U_l^m}{U_c^m} &\geq w^m s^m && \text{with inequality if } l^m = 1 && (6.15) \\ U_c^f &= \beta \frac{\pi_{\boldsymbol{s}'|\boldsymbol{s}}}{p(\boldsymbol{s}, \boldsymbol{s}')} U_c^{f\prime} && \forall \boldsymbol{s}' \in S \times S. && (6.16) \end{aligned}$$

Imposing the no-arbitrage condition, $1 + r = \frac{\pi_{\boldsymbol{s}'|\boldsymbol{s}}}{p(\boldsymbol{s}, \boldsymbol{s}')}$, one can rewrite the Euler equation as,

$$U_c^f = \beta(1 + r) U_c^{f\prime} \qquad \forall \boldsymbol{s}' \in S \times S. \qquad (6.17)$$

For a steady-state equilibrium to exist we will require $\beta(1 + r) = 1$, and the previous expression simplifies to

$$U_c^f = U_c^{f\prime} \qquad \forall \boldsymbol{s}' \in S \times S. \qquad (6.18)$$

That is, households choose $\theta(\boldsymbol{s}')$ such that the marginal utility of consumption is equalized across different states and different points in time. In the special case when utility is separable between consumption and leisure, consumption *levels* are independent of the individual state and constant over time.

Definition: A stationary recursive competitive equilibrium with complete markets in the economy with collective households is a list of functions $\{V, c^f, c^m, l^f, l^m, \theta, K, L^f, L^m\}$, a measure of households ψ, a set of prices $\{r, \bar{w}^f, \bar{w}^m\}$, taxes $\{\tau^f, \tau^m\}$ and benefits $\{b^f, b^m\}$, and a pricing function $p(s, s')$ such that:

(1) Given prices, taxes and benefits, V is the solution to (6.10) – (6.12), and $c^f(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)$, $c^m(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)$, $l^f(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)$, $l^m(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)$ and $\theta'(\boldsymbol{s}', \boldsymbol{s}, \theta(\boldsymbol{s}); \mu)$ are the associated optimal policy functions.

(2) For given prices, K, L^f and L^m satisfy the firm's first-order conditions:

   (i) $r = F_K(K, L) - \delta$

   (ii) $w^f = (1 - \lambda)F_L(K, L)$

   (iii) $w^m = \lambda F_L(K, L)$.

(3) Aggregate factor inputs are generated by the policy functions of the agents:

   (i) $K = \int_M \int_X p(\boldsymbol{s}, \boldsymbol{s}')\theta'(\boldsymbol{s}', \boldsymbol{s}, \theta(\boldsymbol{s}); \mu) d\psi dG$,

   (ii) $L^f = \int_M \int_X \boldsymbol{s}[1 - l^f(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)] d\psi dG$,

   (iii) $L^m = \int_M \int_X \boldsymbol{s}[1 - l^m(\boldsymbol{s}, \theta(\boldsymbol{s}); \mu)] d\psi dG$.

(3) The pricing function $p(s, s')$ satisfies a no-arbitrage condition: $1 + r = \frac{\pi_{s'|s}}{p(s,s')}$.

(4) The steady-state condition $\beta(1 + r) = 1$ holds.

(5) The government budget is balanced: $q_0^f b^f + q_0^m b^m = \tau^f \bar{w}^f L^f + \tau^m \bar{w}^m L^m$.

Appendix III: Frisch Elasticities of Labor Supply

Since the Pareto weight, $\mu(x, \boldsymbol{z})$, where

$$x \equiv \frac{q_1^f(1 - \tau^f)\bar{w}^f + q_0^f b^f}{q_1^m(1 - \tau^m)\bar{w}^m + q_0^m b^m}, \tag{6.19}$$

is a function of female and male wages, Frisch elasticities of labor supply depend both on the Pareto weight and its derivative with respect to wages. In this Appendix we derive the Frisch elasticity of labor supply for females and males. For convenience, we write again the first-order conditions with respect to leisure at an interior solution. If we use Λ to denote the marginal utility of wealth, these first-order conditions are

$$\mu(x, \boldsymbol{z})U_l^f = \Lambda w^f \tag{6.20}$$

$$(1-\mu(x, \boldsymbol{z}))U_l^m = \Lambda w^m. \tag{6.21}$$

The Frisch elasticity of labor supply, say η^i, of an agent of gender $i = f, m$ captures how her/his labor supply responds to an intertemporal reallocation of wages that leaves the marginal utility of wealth unchanged, i.e.

$$\eta^i \equiv \frac{d(1-l^i)}{dw^i}\frac{w^i}{1-l^i}\bigg|_\Lambda. \tag{6.22}$$

For females, the Frisch elasticity can be readily obtained after differentiating equation (6.20) with respect to w^f, which yields

$$\mu_1 \frac{q_1^f}{q_1^m w^m + q_0^m b^m} U_l^f + \mu U_{ll}^f \frac{dl^f}{dw^f} = \Lambda, \tag{6.23}$$

where μ_1 denotes the derivative of μ with respect to it first argument, x. After plugging the value for Λ and multiplying through by $w^f/(1-l^f)$ one obtains

$$\eta^f = -\frac{U_l^f}{(1-l^f)U_{ll}^f}\left(1 - \frac{\mu_1}{\mu}\frac{q_1^f w^f}{q_1^m w^m + q_0^m b^m}\right). \tag{6.24}$$

Equivalently, the Frisch elasticity for males can be derived by differentiating (6.21) with respect to w^m,

$$\mu_1 \frac{x q_1^m}{q_1^m w^m + q_0^m b^m} U_l^m + (1-\mu)U_{ll}^m \frac{dl^m}{dw^m} = \Lambda. \tag{6.25}$$

After rearranging terms, plugging in the value of Λ from the first-order condition and multiplying through by $w^m/(1-l^m)$ gives

$$\eta^m = -\frac{U_l^m}{(1-l^m)U_{ll}^m}\left(1 - \frac{\mu_1}{1-\mu}\frac{x q_1^m w^m}{q_1^m w^m + q_0^m b^m}\right). \tag{6.26}$$

Appendix IV: Household Risk Aversion with Risk Sharing

In this appendix we derive the coefficient of risk aversion of the two-person collective household as a function of individual preferences for risk and the relative Pareto weight. We also show that the derivative of the risk-sharing rule for a household member of gender $i = f, m$, is given by the product of the household's coefficient of risk aversion and the individual's coefficient of risk tolerance.

The coefficient of absolute risk aversion of a bachelor household with instantaneous utility function $U^i(c, l)$ is defined as

$$\rho^i = -\frac{U^i_{cc}}{U^i_c}, \text{ for } i = f, m.$$

For the utility function assumed in (3.5), this coefficient is σ^i/c.

When two individuals with different attitudes to risk form a household and share risks, the household's coefficient of risk aversion is obviously different from individual ones. Collective household's risk preferences will depend on individual preferences and Pareto weights.

Collective Household's Risk Aversion

Let us denote the utility function of the two-person, collective household over total household consumption, y, and individual leisures, l^f and l^m, by $u^\mu(y, l^f, l^m)$. Superscript μ refers to the household's Pareto weight. This utility function is defined as,

$$u^\mu(y, l^f, l^m) = \max_{c^f, c^m}\{\mu U^f(c^f, l^f) + (1-\mu)U^m(c^m, l^m)\} \tag{6.27}$$

$$s.t. = c^f + c^m = y. \tag{6.28}$$

With this utility function we can write the maximization problem solved by the collective household as,

$$\tilde{V}^\mu(\boldsymbol{s}, a) = \max_{l^f, l^m, a', \tilde{c}} \{u^\mu(\tilde{c}, l^f, l^m) + \beta \sum_{\boldsymbol{s'}} \pi_{\boldsymbol{s'}|\boldsymbol{s}} \tilde{V}^\mu(\boldsymbol{s'}, a')\} \tag{6.29}$$

$$s.t. \quad \tilde{c} + a' = \sum_{i=f,m} w^i(1-l^i)s^i + \sum_{i=f,m}(1-s^i)b^i + (1+r)a. \tag{6.30}$$

(6.31)

The coefficient of absolute risk aversion of a collective household with Pareto weight μ can then be defined as,

$$\rho_\mu = -\frac{u^\mu_{yy}}{u^\mu_y}.$$

To derive this coefficient of risk aversion let us consider the first-order condition to the static maximization problem embedded into the household problem,

$$\mu\varphi^f_c(c^f)^{-\sigma^f} = (1-\mu)\varphi^m_c(c^m)^{-\sigma^m}. \tag{6.32}$$

Taking logarithms on both sides of equation (6.32) and differentiating with respect to y yields,

$$\sigma^f \frac{dc^f}{dy}\frac{1}{c^f} = \sigma^m \frac{dc^m}{dy}\frac{1}{c^m}. \tag{6.33}$$

Using that $\frac{dc^f}{dy} + \frac{dc^m}{dy} = 1$, we can solve for for dc^f/dy as,

$$\frac{dc^f}{dy} = \left(1 + \frac{\sigma^f}{\sigma^m}\frac{c^m}{c^f}\right)^{-1}. \tag{6.34}$$

Now, if we take the derivative of u^μ with respect to y and use the first-order condition (6.32), it gives,

$$u^\mu_y = \mu\varphi^f_c(c^f)^{-\sigma^f}. \tag{6.35}$$

Differentiating (6.35) with respect to y again yields,

$$u^\mu_{yy} = -\sigma^f\mu\varphi^f_c(c^f)^{-\sigma^f-1}\frac{dc^f}{dy}. \tag{6.36}$$

Then, the coefficient of absolute risk aversion of a household with Pareto weight μ is,

$$\rho_\mu = \frac{\sigma^f\sigma^m}{\sigma^m c^f + \sigma^f c^m}, \tag{6.37}$$

and the coefficient of relative risk aversion is $\frac{\sigma^f\sigma^m(c^f+c^m)}{\sigma^m c^f+\sigma^f c^m}$.

Now, it is straightforward to show that the derivatives of the sharing rules, $\frac{dc^f}{dy}$ and $\frac{dc^m}{dy}$, are given by the household's coefficient of absolute risk aversion, ρ_μ, times the coefficient of absolute risk tolerance of each individual in the household. Simple algebra in equation (6.32) leads to

$$\frac{dc^f}{dy} = \left(\frac{\sigma^f\sigma^m}{\sigma^m c^f + \sigma^f c^m}\right)\frac{c^f}{\sigma^f}, \tag{6.38}$$

where the expression within brackets on the right-hand side is the household's coefficient of absolute risk aversion and the second term, c^f/σ^f, is the individual's coefficient of absolute risk tolerance. The same result can be shown for $\frac{dc^m}{dy}$.

References

[1] Aiyagari, S.R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics,* 109. pp. 659-684.

[2] Attanasio, O. and Rios-Rull, J.V. (2000), "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare? *European Economic Review,* 44, pp. 1225-1258.

[3] Attanasio, O., Low, H. and Sanchez-Marcos, V. (2005), "Female Labor Supply as Insurance Against Idiosyncratic Risk," *Journal of the European Economic Association,*

[4] Blundell, R., Chiappori, P.A., Magnac, T. and Meghir, C. (2007), "Collective Labor Supply: Heterogeneity and Non-participation," *Review of Economic Studies,* 74(2), pp. 417-445.

[5] Blundell, R. and MaCurdy, T.E. (1999), "Labor Supply: A Review of Alternative Approaches," in: Ashenfelter, Q., Card, D. (Eds.), Handbook of Labor Economics, vol. 3A. Elservier, Ambsterdam.

[6] Blundell, R., Pistaferri, L. and Preston, I. (2008), "Consumption Inequality and Partial Insurance," *American Economic Review,* 98(5), pp. 1887-1921.

[7] Browning, M., Bourguignon, F., Chiappori, P.A. and Lechene, V. (1994), "Income and Outcomes: A Structural Model of Intrahousehold Allocation," *Journal of Political Economy,* 102(6), pp. 1067-1096.

[8] Browning, M., Chiappori, P.A. and Lechene, V. (2006), "Collective and Unitary Models: A Clarification," *Review of Economics of the Household,* 4, pp. 5-14.

[9] Browning, M. and Crossley, T.F. (2001), "Unemployment Insurance Benefit Levels and Consumption Changes," *Journal of Public Economics,* 80, pp. 1-23.

[10] Chetty, R. and Saez, E. (2010), "Optimal Taxation and Social Insurance with Endogenous Private Insurance," *American Economic Journal: Economic Policy,* 2, pp. 85-114.

[11] Chiappori, P.-A. (1988), "Rational Household Labor Supply," *Econometrica,* 56, pp. 63-89.

[12] Chiappori, P.-A. and Donni, O. (2010), "Non-unitary Models of Household Behavior: A Survey of the Literature," in: Molina, A. (eds.) Household Economic Behaviors, Berlin: Springer.

[13] Chiappori, P.-A., Fortin, B. and Lacroix, G. (2002), "Marriage Market, Divorce Legislation and Household Labor Supply," *Journal of Political Economy,* 110(1), pp. 37-72.

[14] Cullen, J.B. and Gruber, J. (2000), "Does Unemployment Insurance Crowd out Spousal Labor Supply?," *Journal of Labor Economics,* 18, pp. 546-572.

[15] Di Tella, R. and MacCulloch, R. (2002), "Informal Family Insurance and the Design of the Welfare State," *Economic Journal,* 112, pp. 481-503.

[16] Díaz, A., Pijoan-Mas, J., and Ríos-Rull, J.V. (2003), "Precautionary Savings and Wealth Distribution under Habit Formation Preferences," *Journal of Monetary Economics,* 50(6), pp. 1257-1291.

[17] Domeij, D. and Floden, M. (2001), "The labor-supply elasticity and borrowing constraints: why estimates are biased," *Review of Economic Dynamics,* 9, pp. 242-262.

[18] Engen, E.M. and Gruber, J. (2001), "Unemployment Insurance and Precautionary Savings," *Journal of Monetary Economics,* 47, pp. 545-579.

[19] Flodén, M. and Lindé, J. (2001), "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics,* 4, pp. 406-437.

[20] Fortin, B. and Lacroix, G. (1997), "A Test of the Unitary and Collective Models of Household Labour Supply," *The Economic Journal,* 107, pp. 933-955.

[21] Golosov, M. and Tsyvinski, A. (2007), "Optimal Taxation with Endogenous Insurance Markets," *Quarterly Journal of Economics,* 122, pp. 487-534.

[22] Gruber, J. (1997) "The Consumption Smoothing Benefits of Unemployment Insurance," *American Economic Review,* 87, pp. 192-205.

[23] Gruber, J. and Cullen, J.B. (2000), "Does Unemployment Insurance Crowd Out Spousal Labor Supply?," *Journal of Labor Economics,* 18, pp. 546-572.

[24] Heathcote J., Storesletten, K. and Violante, G.L. (2008), "The Macroeconomic Implications of Rising Wage Inequality in the U.S.," mimeo.

[25] Heathcote J., Storesletten, K. and Violante, G.L. (2009), "The Macroeconomics of the 'Quiet Revolution'," mimeo.

[26] Huggett, M. (1993), "The Risk-free Rate in Heterogenous-agent Incomplete Insurance Economies," *Journal of Economic Dynamics and Control,* 17, pp. 953-969.

[27] Huggett, M. (1997), "The One-sector Growth Model with Idiosyncratic Shocks: Steady States and Dynamics," *Journal of Monetary Economics,* 39, pp. 385-403.

[28] Imrohoroglu, A. (1989), "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy,* 97(6), pp. 1364-1383.

[29] Kotlikoff, L.J., Shoven, J. and Spivak, A. (1986), "The Effect of Annuity Insurance on Savigns and Inequality," *Journal of Labor Economics,* 4(3), pp. 183-207.

[30] Kotlikoff, L.J. and Spivak, A. (1981), "The Family as an Incomplete Annuities Market," *Journal of Political Economy,* 89, pp. 372-391.

[31] Krusell, P. and Smith Jr., A. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy,* 106, pp. 867-896.

[32] Low, H., Meghir, C. and Pistaferri, L. (2008), "Wage Risk and Employment Risk over the Life Cycle," WP08/06 The Institute For Fiscal Studies.

[33] Maestripieri, D., Sapienza, P. and Zingales, L. (2009), "Gender Differences in Financial Risk Aversion and Career Choice are Affected by Testosterone," *Proceedings of the National Academy of Sciences,* 106(36), pp. 15268-72.

[34] Marcet, A., Obiols-Homs, F. and Weil, P. (2007), "Incomplete Markets, Labor Supply and Capital Accumulation", *Journal of Monetary Economics,*

[35] Mazzocco, M. (2004), "Savings, Risk Sharing and Preferences for Risk," *American Economic Review,* 94(4), pp. 1169-1182.

[36] Mazzocco, M. (2007), "Household Intertemporal Behavior: A Collective Characterization and a Test of Commitment," *Review of Economic Studies,* 74(3), pp. 857-895.

[37] Mazzocco, M., Ruiz, C. and Yamaguchi, S. (2008), "Labor Supply, Wealth Dynamics and Marriage Decisions," California Center for Population Research (CCPR).

[38] OECD Indicators (2010): "Net Replacement Rates (NRR) over a 5-Year Period following Unemployment 2001 – 2008", http://www.oecd.org/dataoecd/16/42/39720461.xls.

[39] Pijoan-Mas, J. (2006), "Precautionary Savings or Working Longer Hours?," *Review of Economic Dynamics,* 9, pp. 326-352.

[40] Shore, S.H. and Sinai, T. (2010), "Commitment, Risk, and Consumption: Do Birds of a Feather Have Bigger Nests?" *Review of Economics and Statistics* 92(2), pp. 408-424.

[41] Stephens Jr., M. (2002), "Worker Displacement and the Added Worker Effect," *Journal of Labor Economics,* 20(3), pp. 504-537.

[42] Zeldes, S. P. (1989), "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy,* 97(2), pp. 305-346.

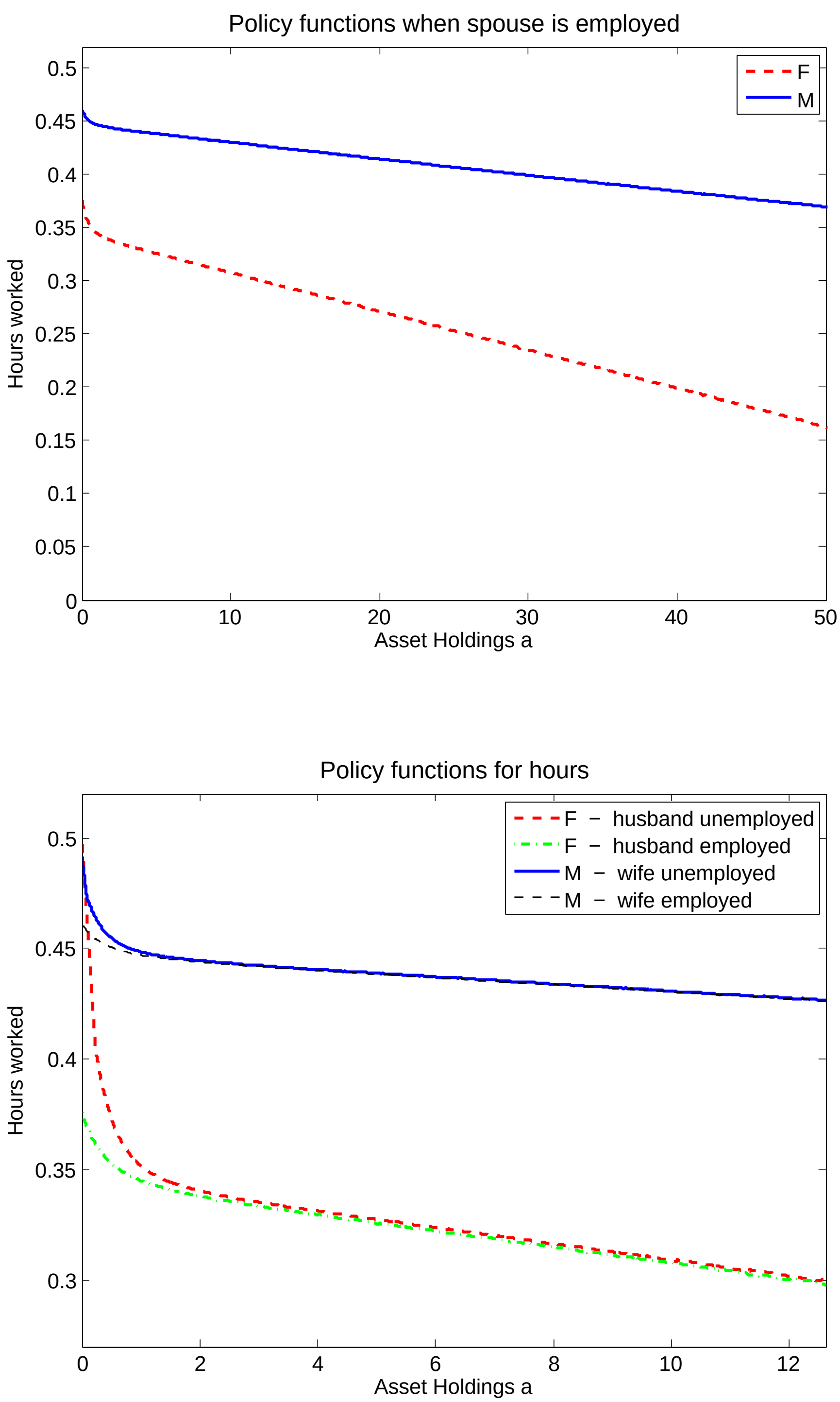

Figure 1: Policy functions for labor supply in the collective model

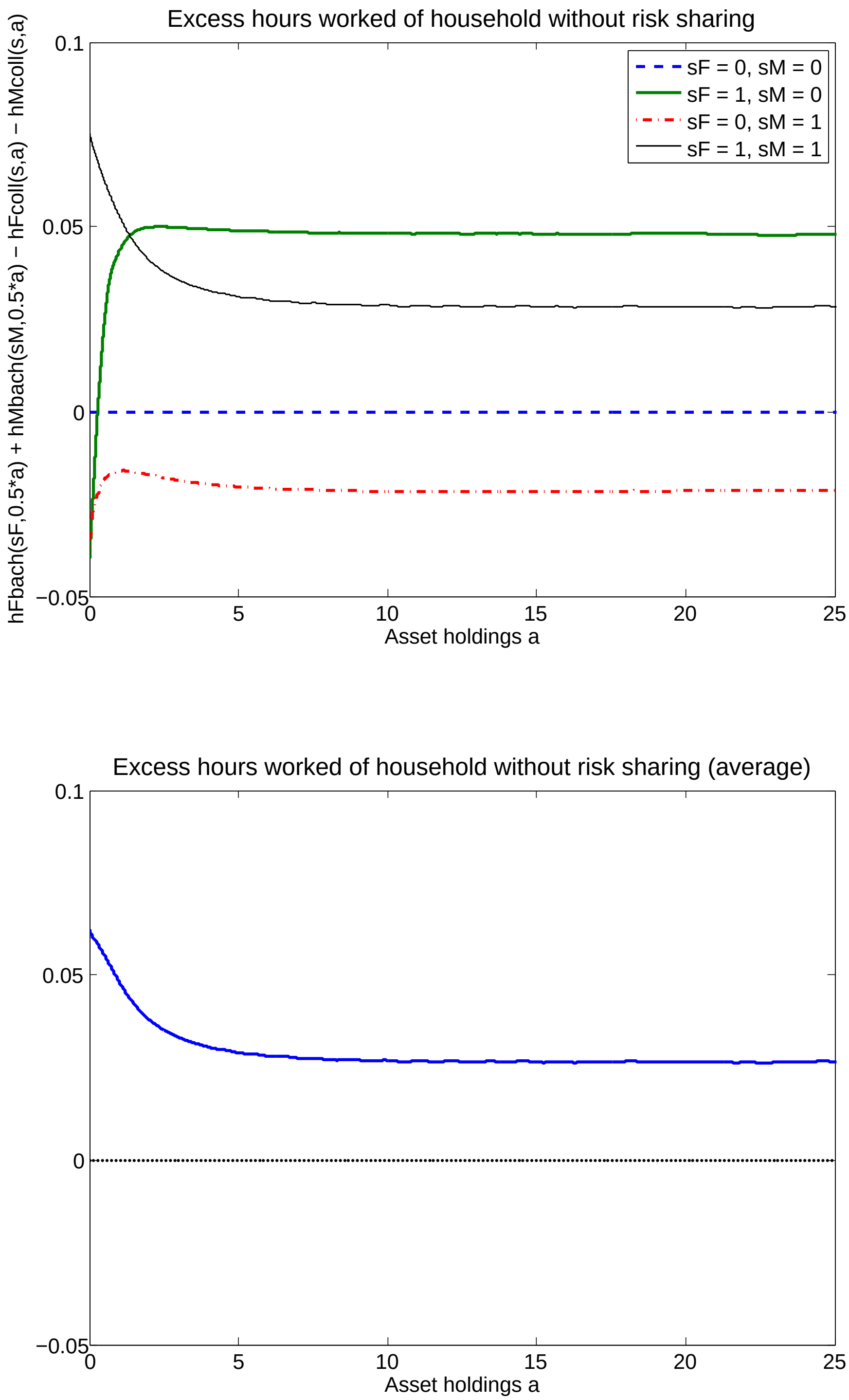

Figure 2: Excess hours worked of household without risk sharing

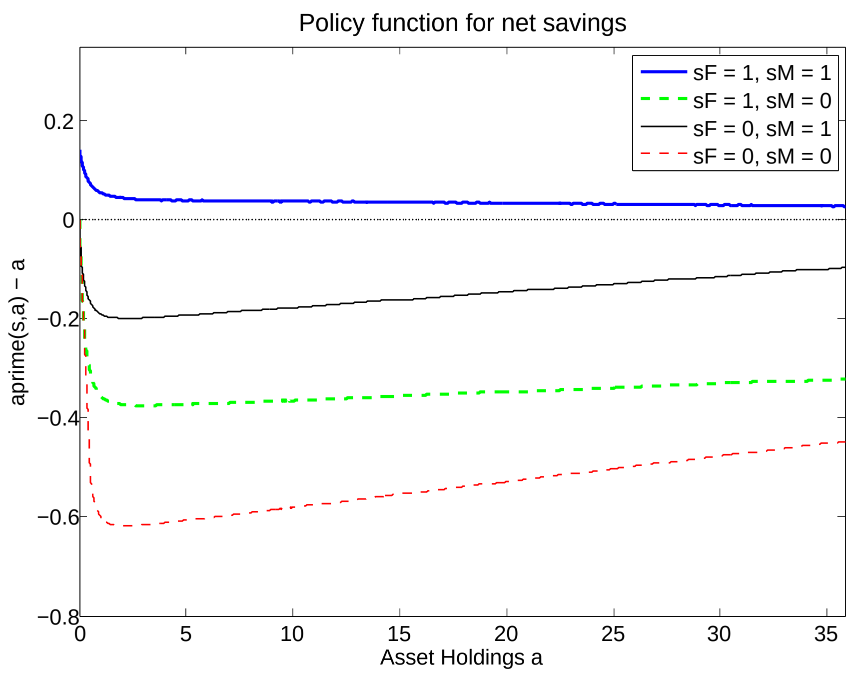

Figure 3: Policy function for net savings in the collective model

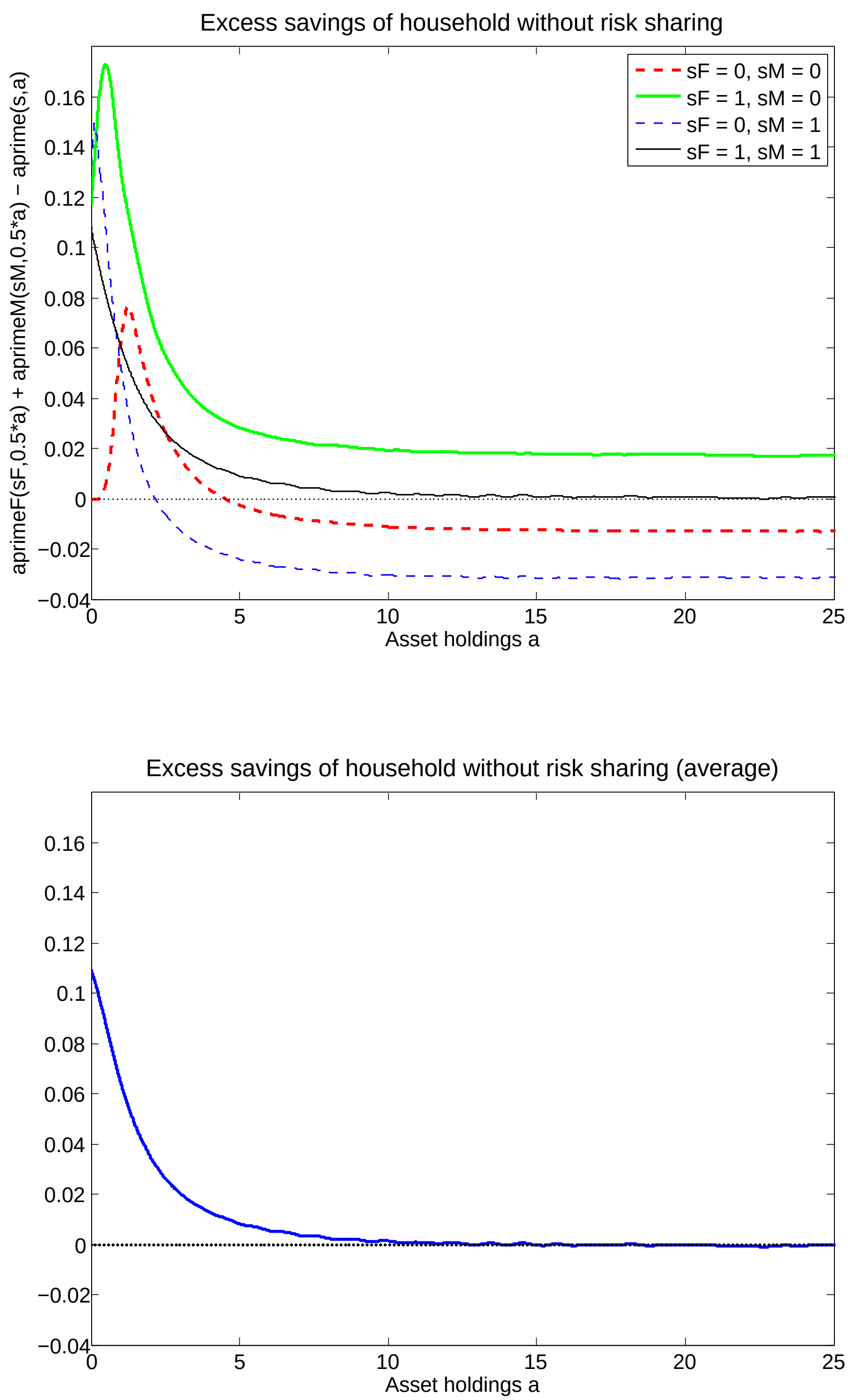

Figure 4: Excess savings of household without risk sharing