Which inflation to target? A small open economy with sticky wages indexed to past inflation*

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Abstract

There is common agreement on price inflation stabilization being one of the objectives of monetary policy. But, in an open economy, two alternative measures of inflation coexist: domestic inflation (DI) and consumer price inflation (CPI). Which one of the two should be the target variable? Most of the literature suggests that the monetary authority should try to stabilize DI. This is in sharp contrast with the practice of many inflation-targeting central banks which are using CPI as target variable. I use a small open economy model to show that CPI targeting can be rationalized by the presence of sticky wages indexed to past CPI. The latter assumption is highly plausible, as documented by the empirical evidence reported in, e.g., Smets and Wouters (2003).

After deriving the welfare function from a second order approximation of the utility function, I compute the fully optimal monetary policy under commitment and use it as a benchmark to compare the performance of different monetary policy rules. The rule performing best is the one targeting wage inflation and CPI. Moreover, this rule delivers results very close to those obtained under the fully optimal monetary policy with commitment.

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1 Introduction

The purpose of the present paper is to analyse which measure of inflation should be chosen as target variable in an open economy framework. In a closed economy context there is common agreement on price inflation stabilization being one of the main objectives of monetary policy. From the ad-hoc interest rate rule proposed by Taylor (1993), to the more recent New Keynesian literature deriving optimal monetary policy rules from the minimization of a microfounded loss function, the monetary instrument has to be chosen in order to match a given inflation target (among with other targets). However, in an open economy context two alternative measures of inflation coexist: Domestic Inflation (DI) and Consumer Price Inflation (CPI). Which one of these two should be the target variable? This is the question addressed in the paper.

With this purpose in mind, I develop a small open economy model similar to the one used by Galí and Monacelli (2005). The main difference with respect to the existing open economy literature is that, in addition to the standard hypothesis of sticky prices, I assume sticky wages. I also allow for a partial indexation to past CPI. In each period only a fraction of workers reoptimize while the others partially index their nominal wages to past CPI. Under those assumptions, the volatility of CPI and the impossibility for some workers to adjust their wages in order to keep their mark-ups constant make the stabilization of CPI relevant in this context. In particular, the assumption on wages has two main consequences: first, given the presence of wage rigidities, strict inflation targeting will no longer be optimal (as Erceg, Henderson and Levin (2000) show in a closed economy setup); second, fluctuations in CPI will induce undesired fluctuations in wage mark-ups and, therefore, in firms' marginal costs and DI. The link between CPI and DI through firm's marginal cost is further increased when there is a positive degree of wage indexation.

The main result of the paper is that, reacting to changes in CPI instead of focusing on targeting DI, the monetary authority will obtain better results not only in the stabilization of CPI but also in that of wage inflation, DI, and output gap. This makes it desirable to stabilize CPI rather than DI. The importance of this result is that, differently from the existing open economy literature, it is in line with the practice of inflation-targeting central banks. Indeed, from an operational point of view, there seems to be an unanimous consensus among central banks on CPI being the correct target. In particular, as stressed by Bernanke and Mishkin (1997), starting from 1990 the following countries have adopted an explicit target to CPI: Australia, Canada, Finland, Israel, New Zealand, Spain, Sweden, UK. In the EMU, the European Central Bank has the object to stabilize the Harmonized Index of Consumer Prices (HCPI) below 2%. In contrast, from a theoretical point of view, most of the literature suggests that the monetary authority should choose DI as target variable for inflation[1]. Hence, the contribution of the paper is to show that the introduction of sticky wages indexed to past inflation reconciles the workhorse model for monetary policy analysis in open economy with the practice of many monetary authorities.

[1] A detailed review of the related literature is provided in the next section.

Regarding the assumptions on which the results of the paper are built, there is strong empirical evidence of wage rigidity in the economy[2], as underlined by Christiano, Eichenbaum and Evans (2005) and by Smets and Wouters (2003). Moreover, Smets and Wouters (2003) estimate the degree of wage indexation to past inflation for the EURO area to be around 0.65. The main conclusion of both Christiano et al. (2005) and Smets and Wouters (2003) is that the introduction of wage rigidity is a crucial assumption in order to improve the ability of the New-Keynesian models to match the data. Consequently, there is empirical evidence in favour of the importance of modelling also wage rigidity in order to obtain more reliable dynamics.

Solving the model under the assumption of sticky wages and looking at the Phillips Curve and the wage inflation equation there emerges a link between DI, CPI and wage inflation. Given this link, it is clearly difficult to stabilize DI without stabilizing also CPI and wage inflation. In order to obtain a more precise analysis of what a central bank should do, I derive the welfare function as a second order approximation of the utility function and I compute the fully optimal monetary policy under commitment. Using the optimal monetary policy as a benchmark, I then compare different, implementable, monetary policy rules. In the choice of possible targets for monetary policy I disregard the output gap because it cannot be considered a feasible target since it is not clear how to estimate the natural level of output. Therefore, I concentrate on the other three variables that appear in the loss function i.e., DI, CPI and wage inflation. I focus on interest rate rules targeting either just one or two of the three variables at the same time. I simulate the model under these monetary policy rules and for different degrees of wage indexation in order to analyse how this feature of the model affects the results. If we consider rules targeting just one variable per time, the rule performing best is the one targeting CPI, even when there is no wage indexation. The rule targeting DI performs much worse in terms of welfare. The reason is that, targeting CPI instead of DI, improves substantially the stabilization of all the main variables. Looking at rules targeting two variables at the same time, the first thing that emerges is that central banks should use wage inflation as their second target variable. In the case of no indexation, a rule targeting DI and wage inflation is almost undistinguishable from one targeting CPI and wage inflation in terms of welfare. But, as soon as a positive degree of indexation is introduced, the policy rule that gives the best results is the one targeting both CPI and wage inflation. Increasing the level of indexation reinforces the results. Simulating the model under the optimal monetary policy rule and under the interest rate rules and looking at the correlations among the series simulated in the different scenarios, it is clear that the rule targeting at CPI and wage inflation delivers a behaviour of the economy that is very close to the one obtained under the fully optimal rule.

These results therefore confirm the original hypothesis that the introduction of wage rigidity would have affected the ranking among policy rules giving more importance to the stabilization of CPI, therefore rationalizing the observed behaviour of many central banks.

[2] For a review of the micro evidence of wage stickiness and of the importance of modelling wage rigidities together with price rigidities see Taylor (1998).

The structure of the paper is the following: section 2 presents the related literature, section 3 introduces the open economy model, section 4 presents the analysis of the welfare function, section 5 computes the optimal monetary policy under commitment, section 6 shows how different, implementable, monetary policy rules perform under different degrees of indexation and section 7 concludes.

2 Related literature

Clarida, Galí and Gertler (2001) analyse a small open economy model with price rigidities and frictions in the labour market. They find that, as long as there is perfect exchange rate pass-through, the target of the central bank should be DI. This is what they call "*the isomorphic result*" meaning that the form of the optimal interest rate rule is not affected by the consideration of being in an open economy. Openness only affects the aggressiveness with which the central bank should react to shocks. Therefore, the central bank should target DI and not CPI. However, in their paper they do not explicitly model frictions in the labour market. They just assume an exogenous stochastic process for the wage mark-up. This is an important difference with respect to the model I develop because, even if assuming an exogenous process for the wage mark-up makes price stability no more optimal (like here), the link between fluctuations in the wage mark-up and fluctuations in CPI is missing. A similar result is obtained in Galí and Monacelli (2005)[3] where strict DI targeting turns out to be the optimal monetary policy, consequently outperforming a CPI targeting rule. Aoki (2001) shows that in a two-sector closed economy with different price rigidities, more weight should be attributed to the inflation of the stickier sector[4]. The extension of this result to a small open economy context implies that the monetary authority should target the DI. Clarida, Galí and Gertler (2002) show, in a two-country model with sticky prices that, in the case of no coordination, the two monetary authorities should adjust the interest rate in response to DI. Benigno (2004) studies optimal monetary policy in a currency area using a two-country model with monopolistic competition and sticky prices in both regions. There are two independent fiscal authorities while there is only one monetary authority. The result is a generalization of the one obtained by Aoki (2001) in the closed economy, two-sector model. In the special case where prices are rigid only in one country, the central bank should stabilize DI in the country with sticky prices. In a more general case, where prices are rigid in both countries and the degree of price stickiness differs across the two regions, in the class of inflation targeting rules

[3]Under the assumptions of log utility in consumption and unit elasticity of substitution among foreign goods.

[4]Another closed economy model dealing with which inflation variable to target is the one by Huang and Liu (2005). In their model there are two sectors, one for the production of intermediate goods and one for the production of final goods. Intermediate goods are produced using labour as the only input while to produce final goods labour is combined with the intermediate goods. Prices are rigid in both sectors and there are sector specific shocks. The main conclusion is that an interest rate rule targeting both CPI and PPI (producer price inflation) would attain better results than one seeking to stabilize CPI. Anyway, as stressed by the authors in the paper, "the PPI [...] does not have a clear counterpart in an open economy setup" making a comparison with an open economy model difficult.

where the target is a weighted average of the DI in the two countries, higher weight needs to be attributed to the DI of the country with relatively more rigid prices. Still, as in the previous papers, the target variable is DI and not CPI.

Differently from the aforementioned papers, Corsetti and Pesenti (2005) and DePaoli (2004) find that DI is not always the optimal target. But, the focus in those papers is not on which inflation to target but more on the general question of whether the policy should be inward-looking or outward-looking. Corsetti and Pesenti (2005) use a two-country model with firms' prices set one period in advance and incomplete pass-through to show that "inward-looking policy of domestic price stabilization is not optimal when firms' markups are exposed to currency fluctuations". DePaoli (2004) extends the welfare analysis for the small open economy of Galí and Monacelli (2005) allowing for a more general specification of the utility function and of the elasticity of substitution among domestically produced and foreign goods and finds that the monetary authority should target also the exchange rate, therefore supporting an outward-looking monetary policy. A paper dealing directly with the question of whether the monetary authority should target DI or CPI is the one by Svensson (2000). He uses a small open economy framework to analyse inflation targeting monetary policies and he underlines that "all inflation-targeting countries have chosen to target CPI...None of them has chosen to target domestic inflation". He assumes an ad-hoc loss function that includes both CPI and DI in addition to other variables. The result of the model (that is not fully microfounded) is that flexible CPI targeting is better than flexible DI targeting. Also in Monacelli (2005), the monetary authority is assumed to target CPI instead of DI, in order to behave like many central banks do in practice, but the welfare function is not derived.

Summarizing, with the exception of the paper by Svensson (2000), that is not fully microfounded, the papers claiming for an outward-looking monetary policy do not deal with the question of which measure of inflation should be chosen by the monetary authority. Here is where the contribution of the paper lies.

3 The model

Like in Galí and Monacelli (2005), there is a continuum $[0, 1]$ of small, identical, countries. Differently from the original model, I introduce the assumption of monopolistic competition on the supply side of the labour market. I also assume the presence of wage rigidities. It is worthy to note that, since I assume complete markets and separable utility, households differ in the amount of labour supplied (consequence of the presence of sticky wages) but share the same consumption. I also keep the simplifying assumption that the law of one price holds for individual goods at all times. From now on I will use "h" as index for a particular household, "i" to refer to a particular country and "j" as sector index. When no index is specified the variables refer to the home country.

3.1 Households

Household "h" maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + V(N_t(h))\right] \tag{1}$$

where $N_t(h)$ is the labour supply and C_t is a consumption index which aggregate bundles of domestic and imported goods:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where α represents the degree of openness, and $C_{H,t}$ and $C_{F,t}$ are two aggregate consumption indices, respectively for domestic and imported goods:

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\theta_p-1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p-1}} \tag{3}$$

$$C_{F,t} \equiv \left[\int_0^1 C_{i,t}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

$$C_{i,t} \equiv \left[\int_0^1 C_{i,t}(j)^{\frac{\theta_p-1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p-1}} \tag{5}$$

The parameter $\theta_p > 1$ represents the elasticity of substitution between two varieties of goods produced in the same country, while the parameter $\eta > 0$ represents the elasticity of substitution between home produced goods and goods produced abroad. Each household h maximizes (1) subject to a sequence of budget constraints. The results regarding the optimal allocation of expenditure across goods are not affected by the introduction of monopolistic competition in the labour market, so using the results of Galí and Monacelli (2005), I can directly write the budget constraint after having aggregated over goods:

$$P_t C_t + E_t\left[Q_{t,t+1} D_{t+1}\right] \leq D_t + (1+\tau_w) W_t(h) N_t(h) + T_t \tag{6}$$

where $Q_{t,t+1}$ is the stochastic discount factor, D_t is the payoff in t of the portfolio held at the end of $t-1$, T_t is a lump-sum transfer (or tax) which also includes profits resulting from ownership of firms, τ_w is a subsidy to labour income and P_t is the aggregate price index:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{7}$$

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}} \tag{8}$$

$$P_{F,t} \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di\right]^{\frac{1}{1-\eta}} \tag{9}$$

$$P_{i,t} \equiv \left[\int_0^1 P_{i,t}(j)^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}} \tag{10}$$

Each household supplies a differentiated labour service in each sector j, so that the total labour supplied by household h is given by $N_t(h) = \int_0^1 N_{t,h}(j)dj$. Consequently, he will maximize (1) w.r.t. $W_t(h)$ subject to the labour demand and the budget constraint. Given that the production function in each sector j is given by $Y_t(j) = A_t N_t(j)$ with $N_t(j) \equiv \left[\int_0^1 N_{t,j}(h)^{\frac{\theta_w - 1}{\theta_w}} dh\right]^{\frac{\theta_w}{\theta_w - 1}}$, the cost minimization problem of firms yields to the following demand for labour faced by individual h:

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\theta_w} N_t \tag{11}$$

where $\theta_w > 1$ represents the elasticity of substitutions between workers, and the aggregate wage index is given by $W_t \equiv \left[\int_0^1 W_t(h)^{1-\theta_w} dh\right]^{\frac{1}{1-\theta_w}}$.

3.1.1 Wage decisions

In each period only a fraction $(1-\xi_w)$ of households can reset wages optimally. For the fraction ξ_w of households that cannot optimize I allow for a partial indexation to past CPI. It is worth noticing that this mechanism not only accounts for the presence of written rules among firms and workers providing wage indexation. A more interesting way to think about it is that, in each period, there is a fraction of workers that find it easier, instead of fully reoptimize, just to follow a simple rule (like assumed in Christiano et al. (2005) for firms) trying to preserve their real wages. That is why the indexation is to CPI and not to DI. Like Smets and Wouters (2003), I have introduced the parameter γ_w so that it will be possible to study, later on, how certain results may be affected by different degrees of indexation. Therefore, the wage of the fraction ξ_w of households that can not reoptimize in t is given by:

$$W_t(h) = \Pi_{t-1}^{\gamma_w} W_{t-1}(h) \tag{12}$$

where Π_t is the CPI. Each household that can reoptimise in t will choose $W_t(h)$ considering the possibility that, with some probability, he will not be able to reoptimise any more in the future. Consequently, he will maximize (1) under (6) and (11) taking into account the probability of not being allowed to reoptimise in the future. The FOC of this optimisation problem with respect to $W_t(h)$ is:

$$E_t \sum_{T=0}^{\infty} (\beta\xi_w)^T \left[U_C[C_{t+T}] \frac{W_t(h)\Pi_{tT}^{\gamma_w}}{P_{t+T}} (1+\tau_w) \frac{\theta_w - 1}{\theta_w} + V_N[N_{t+T}(h)]\right] N_{t+T}(h) = 0 \tag{13}$$

with $\Pi_{tT} = \Pi_t \Pi_{t+1} .... \Pi_{T-1} = \frac{P_{t+T-1}}{P_{t-1}}$. From (13) it is clear that the solution $\widetilde{W}_t(h)$ will be the same for all households that are allowed to reoptimise in t. To solve for the optimal wage we need first to log linearize (13) around the steady state:

$$E_t \sum_{T=0}^{\infty} (\beta \xi_w)^T \left[\widehat{\Psi}_{t+T} - \widehat{MRS}_{t+T}(h) \right] = 0 \tag{14}$$

where $\Psi_{t+T} = \frac{\widetilde{W}_t \Pi_{tT}^{\gamma_w}}{P_{t+T}}$ is the real wage, $MRS_t = -\frac{V_{N,t}}{U_{C,t}}$ and $\widehat{\Psi}_{t+T}$ and $\widehat{MRS}_{t+T}(h)$ are the log deviations from their levels with flexible prices. Rearranging terms I get the following equation for the optimal wage:

$$\log \widetilde{W}_t = -log(1-\Phi_w)+(1-\beta\xi_w) E_t \sum_{T=0}^{\infty} (\beta \xi_w)^T \left[\log MRS_{t+T}(h) + \log P_{t+T} - \gamma_w \log \Pi_{tT} \right] \tag{15}$$

where $\log(1 - \Phi_w) = \log(1 + \tau_w) - \log(\mu_w)$ and $\mu_w = \frac{\theta_w}{\theta_w - 1}$ is the wage markup. Whenever $\tau_w = \frac{1}{\theta_w - 1}$, then $\Phi_w = 0$ and the fiscal policy completely eliminates the distortion caused by the presence of monopolistic competition in the supply of labour. When instead $\tau_w < \frac{1}{\theta_w - 1}$, then $-\log(1 - \Phi_w) > 0$ and a distortion is present in the economy[5]. From now on the following specification for the utility function will be assumed:

$$U(C) + V(N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{16}$$

where σ represents the relative risk aversion coefficient while φ is the inverse of the labour supply elasticity. Given this specification, and with some algebra, it is possible to derive the following expression:

$$\begin{aligned} \log \widetilde{W}_t = & \\ & -\frac{(1-\beta\xi_w)}{1+\varphi\theta_w} \sum_{T=0}^{\infty} (\beta\xi_w)^T E_t[\widehat{\mu}_{w,t+T}] + \log(W_t) + \\ & + \sum_{T=1}^{\infty} (\beta\xi_w)^T E_t \log \Pi_{w,t+T} + \\ & -\gamma_w (1-\beta\xi_w) \sum_{T=0}^{\infty} (\beta\xi_w)^T E_t \log \Pi_{tT} \end{aligned} \tag{17}$$

where $\widehat{\mu}_{w,t} = \log(W_t) - \log(P_t) - \log(MRS_t) + \log(1 - \Phi_w)$ represents the fluctuation in the wage markup. The optimal wage today will be higher the higher the expectations

[5] Note that if $\frac{\theta_w}{\theta_w - 1} = 1 + \tau_w$ the fiscal policy is able to completely eliminate the distortion arising from labour markets. Following Woodford (2003) I define $1 - \Phi_w = (1 + \tau_w) \frac{\theta_w - 1}{\theta_w}$, where Φ_w represents the distortion in the economy. Whenever $\Phi_w > 0$ the level of employment in the flexible price equilibrium will be lower than the one that we would have without distortions. When doing welfare analysis I will assume for simplicity $\Phi_w = 0$ but now I can consider the more general case.

about future wages. Future CPI has instead a negative impact because of indexation. In particular, the higher the level of indexation and the higher the expected future CPI, the lower will be the optimal wage today. This is because agents know that even if they will not be allowed to reoptimise in the near future, their wages will increase anyway because of indexation. This effect would disappear with $\gamma = 0$. Note that, with the labour subsidy in place, the distortion in the labour market is smaller than the one that we would have without subsidy, indeed $-\log(\mu_w) < log(1-\Phi_w) \leq 0$. Still, $\widehat{\mu}_{w,t} = 0$ means that the wage charged is higher then the one that would be charged with perfect competition on the labour market. So, even if the monetary authority manages to eliminate the distortions arising from the nominal rigidities, the level of employment will be lower then the natural one, unless $\Phi_w = 0$.

The next step is to analyse the wage inflation equation. Given that the fraction $(1 - \xi_w)$ of households that is allowed to reoptimise will choose the same wage, while the others will follow the indexation rule, the aggregate wage index is:

$$W_t = \left[(1 - \xi_w)\widetilde{W}_t^{1-\theta_w} + \xi_w (W_{t-1}\Pi_{t-1}^{\gamma_w})^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}} \tag{18}$$

The log linearized version of this equation is given by:

$$logW_t = (1 - \xi_w)\log \widetilde{W}_t + \xi_w \log W_{t-1} + \gamma_w \xi_w \log \Pi_{t-1} \tag{19}$$

It is useful to rewrite (17) in the following way:

$$log\widetilde{W}_t - \beta\xi_w E_t \log \widetilde{W}_{t+1} = -\frac{1 - \beta\xi_w}{1 + \varphi\theta_w}\widehat{\mu}_{w,t} + (1 - \beta\xi_w)\log W_t \tag{20}$$

From now on all the lower case letters denote the log of the variables. Combining (20) with (19) gives:

$$\pi_{w,t} = -\lambda_w \widehat{\mu}_{w,t} + \beta E_t[\pi_{w,t+1}] - \xi_w \gamma_w \beta \pi_t + \gamma_w \pi_{t-1} \tag{21}$$

where $\lambda_w = \frac{1-\xi_w}{\xi_w}\frac{1-\beta\xi_w}{1+\varphi\theta_w}$. As in the case of no indexation, current wage inflation depends positively on the expected future wage inflation and negatively on the deviation of the markup from its frictionless level. In particular when $\widehat{\mu}_{w,t} > 0$ the markup charged is higher than its optimal level and that is way wages respond negatively to a positive $\widehat{\mu}_{w,t}$. This result is consistent with the one obtained in Galí (2003) in the closed economy case with no indexation. The presence of indexation introduces two new elements: a negative impact of current CPI and a positive impact of past CPI. For what concern present inflation, because of indexation households know that, even if they will not be able to change wages in the next period, their wages will increase because of the link with current inflation, so there is no need to increase them today. Past inflation, instead, has a positive impact on current wage inflation because agents that are not allowed to reoptimize in t will see their wages increase because of indexation. In case of no indexation, fluctuations in CPI will induce fluctuations in wage inflation only through their impact on the wage mark-up.

Having discussed the wage decisions, I move to the consumption choice which is standard.

3.1.2 Consumption Decisions

Maximizing (1) with respect to consumption and asset holdings subject to (6), leads to the standard Euler Equation:

$$\beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] = 1 \tag{22}$$

with $R_t = \frac{1}{E_t[Q_{t,t+1}]}$.

3.2 Firms

The production function of a domestic firm in sector j is given by:

$$Y_t(j) = A_t N_t(j) \tag{23}$$

with $a_t \equiv \log(A_t)$ and

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \tag{24}$$

where $\varepsilon_{A,t}$ is an i.i.d shock with zero mean. The aggregate domestic output is given by:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{25}$$

Up to a first order approximation Galí and Monacelli (2005) demonstrate that:

$$y_t = a_t + n_t \tag{26}$$

In each period only a fraction $(1 - \xi_p)$ of firms can reset prices optimally.

Given that the elasticity of substitution between varieties of final goods is $\theta_p > 1$, the markup that each firm would like to charge is $\mu_p = \frac{\theta_p}{\theta_p - 1}$. Assuming the presence of a subsidy τ_p to the firm's output, optimal price-setting of a home firm j must satisfy the following FOC:

$$E_t \sum_{T=0}^{\infty} \xi_p^T Q_{t,t+T} Y_{t+T}(j) \left[(1 + \tau_p) \frac{\theta_p - 1}{\theta_p} P_{H,t}(j) - MC_{t+T} \right] = 0 \tag{27}$$

where MC_t represents the nominal marginal cost. Like for wages, it is useful to define $1 - \Phi_p \equiv (1 + \tau_p) \frac{\theta_p - 1}{\theta_p}$, where Φ_p indicates the distortion due to monopoly power on the firm side that is still present in the economy after the intervention of the fiscal authority. If the fiscal authority optimally chooses τ_p in order to exactly offset the monopoly distortion then $\Phi_p = 0$. If $\Phi_w > 0$ and/or $\Phi_p > 0$ then the flexible price allocation will deliver an output and an employment level lower then the natural ones.

From the log-linear approximation of (27) around the steady state it is possible to derive the standard log-linear optimal price-setting rule:

$$\widetilde{p}_{H,t} = -\log(1-\Phi_p) + (1-\beta\xi_p)E_t \sum_{T=0}^{\infty} (\beta\xi_p)^T [mc_{t+T} + p_{H,t}] \tag{28}$$

where $\widetilde{p}_{H,t}$ represents the (log) price chosen by the firms that are allowed to reoptimise in t, and mc_t represents the (log) real marginal cost.

3.3 Equilibrium Conditions

To close the model some relations between home and foreign variables are needed. A "star" will be used to denote world variables. The derivation of the following equations[6] can be found in Galí and Monacelli (2005):

$$C_t^* = Y_t^* \tag{29}$$

$$c_t = c_t^* + \frac{1-\alpha}{\sigma} s_t \tag{30}$$

where $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ are the effective terms of trade and (30) represents the international risk sharing condition. The market clearing condition is given by:

$$Y_t = C_t S_t^{\alpha} \tag{31}$$

The world output is assumed to follow an exogenous law of motion:

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{y,t}. \tag{32}$$

with $\varepsilon_{y,t}$ i.i.d. shock with zero mean. The terms of trade can be expressed also in function of the aggregate and the home price indexes:

$$\alpha s_t = p_t - p_{H,t} \tag{33}$$

The relation between the home output and the world output is given by:

$$s_t = \sigma_\alpha (y_t - y_t^*) \tag{34}$$

with $\sigma_\alpha \equiv \frac{\sigma}{1-\alpha+\alpha\omega} > 0$ and $\omega \equiv \sigma\eta + (1-\alpha)(\sigma\eta - 1)$.

[6] All these relations, with the only exception of (29) that is an exact relation, hold exactly only under the assumption that $\sigma = \eta = 1$. Otherwise they hold up to a first order approximation.

3.4 The New Keynesian Phillips Curve (NKPC)

The relation between DI and real marginal cost is not affected by the presence of sticky wages:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t \tag{35}$$

with $\lambda \equiv \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$ and with $\widehat{mc}_t$ denoting log deviations of the real marginal cost from its level in the absence of nominal rigidities (i.e. $\widehat{mc}_t = mc_t - mc$ with $mc = \log(1 - \Phi_p)$). The presence of sticky wages leads to an additional term in the standard equation relating the marginal cost with the output gap (the derivation can be found in the appendix):

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)(y_t - \overline{y}_t) + \widehat{\mu}_{w,t} \tag{36}$$

When wages are fully flexible $\widehat{\mu}_{w,t} = 0$. When wages are sticky this is no longer true and in particular, when $\widehat{\mu}_{w,t} > 0$, the markup charged by workers is higher than the optimal one and firms bear a higher real marginal cost. Consequently the NKPC for a small open economy with both price and wage rigidities is:

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda(\sigma_\alpha + \varphi)(y_t - \overline{y}_t) + \lambda\widehat{\mu}_{w,t} \tag{37}$$

Even assuming that the only distortions left in the economy are the ones generated by the presence of nominal rigidities, clearly, as in Erceg et al. (2000), since it is not possible to stabilize at the same time DI, wage inflation and output gap, the flexible price allocation is no longer a feasible target. Is it still true then, that a Taylor rule targeting DI is the one that performs best? It is interesting to analyse the impact of an increase in p_t on $\pi_{H,t}$. To keep the wage markup constant wages should increase to offset the change in prices but, because of stickiness, this is not possible for all households, so some of them will charge a wage that is lower than the desired one and $\widehat{\mu}_{w,t}$ will become negative. This will have a negative impact on DI. On the other hand, because of indexation to past inflation, in $t+1$ the aggregate wage index will increase and so will $\widehat{\mu}_{w,t+1}$. This will lead to an increase of $E_t\pi_{H,t+1}$. So, other things equal, an increase in p_t will cause an increase of $\pi_{H,t+1}$, whereas the impact on current DI is not clear. Given this link between DI, CPI and wage inflation, it seems reasonable to postulate that targeting only one of these variables may not be optimal because, if CPI and wage inflation are very volatile, it will be hard to stabilize only DI.

To prove this conjecture, in the next section, I derive the welfare function from a second order approximation of the utility of the representative household. I then use the welfare function to study the behavior of the economy under optimal monetary policy. Finally, using the results under optimal monetary policy as benchmark, I compare different welfare losses obtained using different, implementable, policy rules.

4 Welfare function

Before starting with the welfare analysis it is important to underline that in the open economy model there are 5 distortions: monopolistic power in both goods and labour markets; nominal rigidities in both wages and prices; incentives to generate an exchange rate appreciation. In a closed economy framework it is enough to require $\Phi_w = \Phi_p = 0$ to ensure that the flexible price allocation will coincide with the optimal one, but this is no more true in an open economy. As emphasised by Corsetti and Pesenti (2001), a monetary expansion has two consequences in this context: it increases the demand for domestically produced goods and it deteriorates the terms of trade of domestic consumers. So in some cases the monetary authority may have the incentive to generate an exchange rate appreciation, even at the cost of a level of output (employment) lower than the optimal one. From now on I will assume $\sigma = \eta = 1$ (i.e. log utility in consumption and unit elasticity of substitution between home produced goods and goods produced abroad). In this case the equilibrium conditions derived in 3.3 hold exactly and maximizing (1) under the production function $Y_t = A_t N_t$, (30) and (31) leads to the following FOC:

$$-\frac{U_N}{U_C} = (1-\alpha)A^{1-\alpha}N^{-\alpha}(Y^*)^{\alpha} \tag{38}$$

The solution is a constant, optimal, level of employment $N = (1-\alpha)^{\frac{1}{1+\varphi}}$. Let us now analyse under which conditions the flexible price equilibrium delivers the optimal allocation. Under flexible prices, in every period $\widehat{\mu}_{w,t} = \widehat{mc}_t = 0$. Combining these two conditions together with the equilibrium conditions, it is possible to derive:

$$N_t^{1+\varphi}\frac{\mu_w}{1+\tau_w} = \frac{1+\tau_p}{\mu_p} \tag{39}$$

Once having substituted for the optimal level of N, (39) tells us how the two subsidies should be set in order to attain the optimal allocation in the flexible prices equilibrium. From now on I will assume that the subsidies are set such that the flexible price equilibrium coincides with the Pareto optimum[7].

All households have the same level of consumption but different levels of labour. For this reason, when computing the welfare function, we need to average the disutility of labour across agents:

$$W_t = U(C_t) + \int_0^1 V(N_t(h))dh \tag{40}$$

The details of the derivation of the welfare function as a second order approximation of the utility of the representative consumer can be found in Appendix B. The expected welfare loss in a small open economy with both price and wage rigidities and wage indexation to past CPI is given by:

$$L = -\frac{1-\alpha}{2}\left[(1+\varphi)Var(x_t) + \frac{\theta_p}{\lambda}Var(\pi_{H,t}) + \frac{\theta_w}{\lambda_w}Var(\pi_{w,t}) + \beta\gamma_w^2\frac{\theta_w}{\lambda_w}Var(\pi_t)\right] \tag{41}$$

[7]In the simulation I set $\Phi_w = 0$ and consequently, $1 - \Phi p = 1 - \alpha$.

From the comparison between this equation and the one obtained by Galí and Monacelli (2005) it emerges that the loss function is affected by two extra terms: the variance of wage inflation and the variance of CPI.

The next step is to analyse the behaviour of the economy under the fully optimal monetary policy with commitment. Then, using the results under optimal monetary policy as a benchmark, I simulate the model under different, ad-hoc, policy rules, to make a ranking among them (section 6).

5 Optimal monetary policy with commitment

In this section, the fully optimal monetary policy under commitment is computed following Clarida, Galí and Gertler (1999) and Giannoni and Woodford (2002).

The first step, in order to make optimal monetary policy easier to compute, is to reduce the original system of equations fully characterizing the model (see Appendix C) as much as possible. The system can be reduced to the following equations[8]:

$$\alpha(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t - y_t^*) = \alpha(x_{t-1} + \frac{\log(1-\alpha)}{1+\varphi} + a_{t-1} - y_{t-1}^*) + \pi_t - \pi_{H,t} \quad (42)$$

$$\pi_{w,t} = w_t + \pi_t - w_{t-1} \quad (43)$$

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \lambda_w \left[w_t - \alpha y_t^* + \varphi a_t - (1+\varphi-\alpha)(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t) \right] - \xi_w \gamma_w \beta \pi_t + \gamma_w \pi_{t-1} \quad (44)$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda(1+\varphi)x_t + \lambda \left[w_t - \alpha y_t^* + \varphi a_t - (1+\varphi-\alpha)(x_t + \frac{\log(1-\alpha)}{1+\varphi} + a_t) \right] \quad (45)$$

$$y_{t+1}^* = \rho_y y_t^* + \varepsilon_{y,t}. \quad (46)$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \quad (47)$$

With the inclusion of a monetary policy rule, equations (42), (43), (44) and (45) define the variables x_t, $\pi_{H,t}$, $\pi_{w,t}$, π_t and w_t, while the last two equations define the law of motion of the two exogenous shocks.

[8] The variable $x_t \equiv y_t - \bar{y}_t$ represents the output gap.

To compute the optimal monetary policy under commitment the central bank has to choose $\{x_t, \pi_{H,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^{\infty}$ in order to maximize[9]:

$$W = -\frac{1-\alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\varphi) x_t^2 + \frac{\theta_p}{\lambda} \pi_{H,t}^2 + \frac{\theta_w}{\lambda_w} \pi_{w,t}^2 + \beta \gamma_w^2 \frac{\theta_w}{\lambda_w} \pi_t^2 \right] \tag{48}$$

subject to the sequence of constraints defined by equations (42), (43), (44) and (45).

The FOCs of this problem are ($\Phi_{i,t}$ is the Lagrange multiplier associated to the constraint i):

- x_t :

$$-(1-\alpha)(1+\varphi)x_t - \alpha \Phi_{1,t} + \beta \alpha E_t \Phi_{1,t+1} + \alpha \lambda \Phi_{4,t} + \lambda_w (1+\varphi-\alpha) \Phi_{3,t} = 0 \tag{49}$$

- $\pi_{H,t}$:

$$-(1-\alpha) \frac{\theta_p}{\lambda} \pi_{H,t} - \Phi_{1,t} - \Phi_{4,t} + \Phi_{4,t-1} = 0 \tag{50}$$

- $\pi_{w,t}$:

$$-(1-\alpha) \frac{\theta_w}{\lambda_w} \pi_{w,t} - \Phi_{2,t} - \Phi_{3,t} + \Phi_{3,t-1} = 0 \tag{51}$$

- π_t :

$$-(1-\alpha) \beta \gamma_w^2 \frac{\theta_w}{\lambda_w} \pi_t + \Phi_{1,t} + \Phi_{2,t} - \xi_w \gamma_w \beta \Phi_{3,t} + \gamma_w \beta E_t \Phi_{3,t+1} = 0 \tag{52}$$

- w_t :

$$\Phi_{2,t} - \beta E_t \Phi_{2,t+1} - \Phi_{3,t} \lambda_w + \lambda \Phi_{4,t} = 0 \tag{53}$$

Equations (49)-(53) plus the constraints (42)-(45) fully characterize the behaviour of the economy under optimal monetary policy. Using Uhlig's toolkit[10] it is possible to solve the system of equations and to study the behavior of the variables under optimal monetary policy. In the next section several, implementable, policy rules are considered. Their performance is evaluated using the optimal monetary policy as the benchmark case.

6 Evaluation of different policy rules

Now we can go back to the original question i.e., once wage rigidity is introduced in a small open economy, is it better to choose DI as target variable, or is it preferable to target at CPI? To answer this question I will compare the performance of several rules.

[9] Giannoni and Woodford (2002) do the optimization including also the IS equation among the constraints and maximizing also with respect to the interest rate. Following Clarida et al. (1999) it is possible to divide the problem in two steps. The first is to maximize the welfare with respect to $\{x_t, \pi_{H,t}, \pi_{w,t}, \pi_t, w_t\}_{t=0}^{\infty}$ without considering the IS. The second step, once obtained the optimal responses of those variables to the exogenous shocks, is to use the IS in order to see how the interest rate has to be set under optimal monetary policy.

[10] To simulate the model I used the Matlab program developed by Harald Uhlig. See Uhlig (1995).

6.1 Implementable policy rules

The welfare loss is function of π, π_H, π_w and the output gap. In the choice of possible targets for monetary policy, I disregard the output gap, that cannot be considered a feasible target since it is not clear how to estimate the natural level of output. I therefore concentrate on the other three variables. I consider the following interest rate rules:

$$\begin{aligned} r_t &= \rho + \phi_p \pi_t, \qquad & r_t &= \rho + \phi_p \pi_t + \phi_{p,H} \pi_{H,t} \\ r_t &= \rho + \phi_{p,H} \pi_{H,t}, \qquad & r_t &= \rho + \phi_p \pi_t + \phi_w \pi_{w,t} \\ r_t &= \rho + \phi_w \pi_{w,t}, \qquad & r_t &= \rho + \phi_{p,H} \pi_{H,t} + \phi_w \pi_{w,t} \end{aligned} \tag{54}$$

Instead of imposing *a priori* given coefficients for ϕ_p, $\phi_{p,H}$ and ϕ_w, I chose the values minimizing the welfare loss for a given grid of parameters[11]. I did this exercise for different degrees of wage indexation in order to analyse how this feature of the model affects the results.

6.2 Calibration of the parameters

Most of the parameters have been calibrated like in Erceg et al. (2000). The average contract duration is four quarters, i.e. $\xi_p = \xi_w = 0.75$. The elasticities of substitution between workers and between goods are $\theta_p = \theta_w = 4$. The discount factor is $\beta = 0.99$. The productivity shock follows an AR(1) process with $\rho_a = 0.95$. The exogenous shock to productivity is an i.i.d with zero mean and standard deviation $\sigma_a = 0.0071$. The parameters related to the open economy are calibrated following Galí and Monacelli (2005): $\alpha = 0.4$ and the world output follows an AR(1) process with $\rho_y = 0.86$. The exogenous shock to world output is i.i.d with zero mean and with standard deviation $\sigma_y = 0.0078$. The correlation between the two exogenous shocks is $corr_{a,y} = 0.3$. Since the loss function has been derived under the assumption $\sigma = \eta = 1$, I keep this assumption in the simulation. Finally $\varphi = 3$, i.e. the labour supply elasticity is set equal to $\frac{1}{3}$. For what concern the level of wage indexation, the model is simulated under different parameter values for γ_w in order to be able to evaluate the impact of different degrees of indexation on the results.

6.3 Performance of different monetary policy rules

The purpose of this section is twofold: first, to make a ranking among the interest rate rules; second, to quantitatively evaluate how close they are to the optimal monetary policy. To this end, the first step is to rank the policy rules using the welfare losses associated to each of them (table 1). In general, two rules could deliver exactly the same loss and, nonetheless, be different i.e., they could generate very different impulse responses to the exogenous shocks. Therefore, to have more conclusive results, it is important to look at: the standard deviations of the variables of interest (table 2); the

[11] I used a grid from 1 to 10 with intervals of 0.25.

correlations between the simulated series obtained under optimal monetary policy and the ones obtained under the different interest rate rules (table 3). This last measure is particularly interesting because tells us how close the rule is to the optimal one.

Table 1 reports the welfare losses associated to the interest rate rules[12]. If we consider rules targeting just one variable it emerges clearly that responding to movements in DI instead of reacting to changes in CPI generates much bigger welfare losses. Even in the case of zero indexation, targeting DI implies a welfare loss of 0.22%. With a positive degree of indexation the loss increases and reaches 0.66% when $\gamma_w = 0.65$. Those losses, especially considering the ones usually obtained in this kind of literature[13], are substantial. In this class of simple rules, targeting CPI outperforms the other two targets, the only exception being when $\gamma_w = 0.25$, in which case we can obtain better results targeting wage inflation. The intuition for this result is that, when there is a relatively small degree of wage indexation, reacting to movements in wage inflation implies also responding indirectly to movements in lagged CPI and this improves the performance of the rule[14]. If we consider the possibility for the central bank to target two variables at the same time, it emerges clearly that the second target should be wage inflation. The rule with the best performance is the one targeting CPI and wage inflation and this is true for all levels of wage indexation. However, with zero indexation, the loss associated with the rule targeting DI and wage inflation is almost undistinguishable from that of the rule targeting DI and wage inflation. As the level of indexation increases it becomes more costly to target DI instead of CPI. The rule targeting CPI and wage inflation is the best among the six considered and it delivers losses very close to those under optimal monetary policy. Therefore, the first result is that the presence of wage rigidity is enough to justify the choice of CPI as the target variable for inflation rather then DI. The best would be to introduce also wage inflation as second target. A positive degree of wage indexation reinforces that result.

Table 2 reports the (percentage) standard deviations of output gap, DI, CPI and wage inflation under different rules[15], for different degrees of wage indexation. Recall that the mechanism presented in the paper is such that, because of sticky wages, fluctuations in CPI generate undesired fluctuations in the wage mark-up and, therefore, in firms' marginal costs. For this reason, the intuition for targeting CPI instead of DI is that it make it easier to stabilize wage inflation and DI. This intuition is confirmed by the volatilities presented in table 2. Indeed, when the monetary authority targets CPI instead of DI, we observe a reduction in the volatility of all the four variables. This is true even in the benchmark case of zero indexation. A positive degree of indexation strengthen the result. Comparing the volatilities with the one under the optimal monetary policy, we can see that the rule targeting CPI reduces the volatility

[12]The welfare losses are measured as percentage units of steady state consumption and are expressed in deviation from the loss under optimal monetary policy.

[13]See, for example, Galí and Monacelli (2005).

[14]We could say that, for a rule targeting wage inflation, a level of indexation of 0.25 constitutes an optimal degree of indexation.

[15]In this exercise, when I allow for two target variables, I disregard the rule targeting at CPI and DI because, in terms of welfare losses, it performs always worse than the other two making it clear that, if the central bank has two targets, the second one should be wage inflation.

of CPI and DI too much, at the cost of a higher volatility in wage inflation and output gap. If the central bank targets at the same time also wage inflation this problem is considerably reduced. Therefore, we can conclude that the rule targeting at CPI and wage inflation delivers a welfare loss lower than the others because it reduces the overall variance of the main variables.

The analysis of the variances is useful in understanding where the losses come from. Still, it could be the case that two rules deliver exactly the same variances but generate very different responses to the exogenous shocks. Therefore, the last step is the study of the correlations among the series simulated using the fully optimal monetary policy rule and the ones simulated using the interest rate rules (table 3). When there is no wage indexation the correlations for all the rules considered are relatively small. With a positive degree of wage indexation instead, the interest rate rule with CPI and wage inflation delivers very high correlations. Under that rule the behaviour of the variables is very close to what we would observed under the fully optimal monetary policy with commitment.

7 Conclusions

The starting point of this paper was to analyse whether the introduction of wage rigidities in a small open economy model is enough to rationalize the observed behaviour of many central banks that are targeting CPI. As in the closed economy case, once both price and wage rigidities are present, it is no longer possible to reach the flexible price allocation because the central bank cannot simultaneously stabilize price inflation, wage inflation and the output gap. Given this, an interesting question was if it were still true that targeting DI is the best that a central bank can do and, if not, how the new results are affected by the presence of wage indexation. To this purpose I derived the loss function from a second order approximation of the utility of the representative consumer. Compared with the one obtained by Galí and Monacelli (2005), the presence of sticky wages makes the loss function depending also on the variance of wage inflation while the presence of indexation introduces the volatility of CPI. After deriving the optimal monetary policy under commitment, I simulated the model under different, implementable, monetary policy rules, in order to make a ranking among them, using the optimal monetary policy as a benchmark. The main result is that, even with zero indexation, a rule targeting only DI delivers considerably higher welfare losses than a rule targeting at CPI. The performance of a rule focusing exclusively on DI further deteriorates as the level of wage indexation increases. If a central bank implements a rule targeting CPI and wage inflation, she will obtain welfare losses very close to the ones delivered by the optimal monetary policy under commitment.

Concluding, the introduction of wage rigidity is enough to justify CPI targeting instead of DI. The difference in terms of welfare loss is quantitatively relevant when the central bank is targeting only one variable. In order to obtain welfare losses very close to those under optimal monetary policy, the central bank should target also wage inflation. Increasing the level of indexation strengthens all the previous results.

A Derivation of $\widehat{mc}_t$

Making use of some of the equilibrium conditions defined in (3.3), the real marginal cost can be written as:

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= mrs_t + log(\mu_{w,t}) + p_t - p_{H,t} - a_t \\ &= \sigma * y_t^* + (1-\alpha)s_t + \varphi(y_t - a_t) + \alpha * s_t - a_t + log(\mu_{w,t}) \\ &= (\sigma - \sigma_\alpha)y_t^* + (\sigma_\alpha + \varphi)y_t - (1+\varphi)a_t + log(\mu_{w,t}) \end{aligned} \tag{55}$$

where $\mu_{w,t}$ represents the actual markup charged in each period[16]. From equation (55) we can express the level of output as:

$$y_t = \frac{mc_t}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1+\varphi}{\sigma_\alpha + \varphi} a_t - \frac{log(\mu_{w,t})}{\sigma_\alpha + \varphi} \tag{56}$$

Let's define $\bar{y}_t$ the natural level of output, i.e. the level of output in absence of nominal rigidities:

$$\bar{y}_t = \frac{mc}{\sigma_\alpha + \varphi} - \frac{\sigma - \sigma_\alpha}{\sigma_\alpha + \varphi} y_t^* + \frac{1+\varphi}{\sigma_\alpha + \varphi} a_t + \frac{log(1-\Phi_w)}{\sigma_\alpha + \varphi} \tag{57}$$

Then,

$$y_t - \bar{y}_t = \frac{\widehat{mc}_t}{\sigma_\alpha + \varphi} - \frac{\widehat{\mu}_{w,t}}{\sigma_\alpha + \varphi} \tag{58}$$

that is exactly equation (36).

B Derivation of the welfare function

B.1 Step 1: $W_t - \overline{W}$

All the results in this section are derived under the assumption $\sigma = \eta = 1$. Under this assumption the relations defined in (3.3) hold exactly and it is possible to derive a second order approximation of the utility function using first order approximation of the structural equations.

From now on all the variables of the type $\widehat{a}_t$ represent log deviations from the steady state.

We will substitute the following expression of the second order derivative: $V_{NN} = \varphi * V_N N^{-1}$. We will also use the fact that:

$$\frac{X_t - \overline{X}}{\overline{X}} = \widehat{x}_t + \frac{1}{2}\widehat{x}_t^2 + o(\|a\|^3) \tag{59}$$

[16] Note that with the presence of taxes that exactly offset the monopoly distortions, the wedge between the real wage and the mrs_t is do only to the presence of stickiness, whereas when $\Phi_w > 0$ then $\mu_{w,t}$ reflects both the presence of stickiness and the presence of monopoly power.

The first step is to compute a second order approximation around the steady state of 40.
Up to a second order approximation it is true that:

$$\begin{aligned} U(C_t) = \\ &+U(\overline{C}) + \overline{U}_C(C_t - \overline{C}) + \frac{1}{2}\overline{U}_{CC}(C_t - \overline{C})^2 + o(\|a\|^3) \end{aligned} \tag{60}$$

Using (59) and the relations between consumption and output defined in (3.3)the previous equation becomes:

$$\begin{aligned} U(C_t) - U(\overline{C}) &= \widehat{c}_t + o(\|a\|^3) \\ &= (1-\alpha)\widehat{y}_t + o(\|a\|^3) \end{aligned} \tag{61}$$

In an analogous way it's true that:

$$\begin{aligned} E_h V(N_t(h)) = \\ &V(\overline{N}) + E_h[\overline{V}_N(N_t - \overline{N})] + \frac{1}{2}E_h[\overline{V}_{NN}(N_t - \overline{N})^2] + o(\|a\|^3) \end{aligned} \tag{62}$$

that using (59) and the relation between first order and second order derivatives leads to:

$$\begin{aligned} E_h[V(N_t(h)] = \\ &V(\overline{N}) + \overline{V}_N \overline{N} E_h\left[\widehat{n}_t(h) + \frac{1+\varphi}{2}\widehat{n}_t^2(h)\right] + o(\|a\|^3) \end{aligned} \tag{63}$$

Combining (61) and (63) leads to:

$$\begin{aligned} W_t - \overline{W} = \\ &(1-\alpha)\widehat{y}_t + \overline{V}_N \overline{N} E_h[\widehat{n}_t(h) + \frac{1+\varphi}{2}\widehat{n}_t^2(h)] + o(\|a\|^3) \end{aligned} \tag{64}$$

The second step is to compute the approximation of the two expected values.

B.2 Step 2: Derivation of $E_h[\widehat{n}_t(h)]$ and $E_h[\widehat{n}_t^2(h)]$

Since in general, for $A = \left[\int_0^1 A(i)^\phi di\right]^{\frac{1}{\phi}}$, it's true that[17] $\widehat{a}_t = E_i[\widehat{a}(i)] + \frac{1}{2}\phi * Var_i[\widehat{a}(i)] + o(\|a\|^3)$ then, given the way in which aggregate labour has been defined, it is possible to write:

$$\widehat{n}_t = E_h[\widehat{n}_t(h)] + \frac{1}{2}\frac{\theta_w - 1}{\theta_w} Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \tag{65}$$

[17] The reference for the results in this section is Erceg et al. (2000).

Following Erceg et al. (2000), it is useful to write $\widehat{n}_t$ in function of the aggregate demand of labour by firms $N_t = \int_0^1 N_t(j)dj$:

$$\widehat{n}_t = E_j[\widehat{n}_t(j)] + \frac{1}{2}Var_j[\widehat{n}_t(j)] + o(\|a\|^3) \tag{66}$$

Clearly, since $\widehat{y}_t(j) = a_t + \widehat{n}_t(j)$ then, $Var_j[\widehat{n}_t(j)] = Var_j[\widehat{y}_t(j)]$ and $E_j[\widehat{n}_t(j)] = E_j[\widehat{y}_t(j)] - a_t$. Also, given the expression for aggregate output, $E_j[\widehat{y}_t(j)] = \widehat{y}_t - \frac{1}{2}\frac{\theta_p-1}{\theta_p}Var_j[\widehat{y}_t(j)] + o(\|a\|^3)$ therefore, we can write:

$$\begin{aligned} E_h[\widehat{n}_t(h)] &= \widehat{n}_t - \frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \\ &= E_j[\widehat{y}_t(j)] - a_t + \frac{1}{2}Var_j[\widehat{y}_t(j)] - \frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \\ &= \widehat{y}_t - a_t + \frac{1}{2\theta_p}Var_j[\widehat{y}_t(j)] - \frac{1}{2}\frac{\theta_w-1}{\theta_w}Var_h[\widehat{n}_t(h)] + o(\|a\|^3) \end{aligned} \tag{67}$$

For the other expected value:

$$E_h[\widehat{n}_t^2(h)] = Var_h[\widehat{n}_t(h)] + [E_h[\widehat{n}_t(h)]]^2 \tag{68}$$

B.3 Step 3: Derivation of $W_t - W_t^n$

Having chosen optimally τ_p and τ_w, the following holds $-\overline{V}_N\overline{N} = (1-\alpha)$. Then, using this relation and substituting (67) and (68) into (64), the second order approximation of the welfare function around the steady state becomes:

$$\begin{aligned} &W_t - \overline{W} = \\ &\quad (1-\alpha)a_t - \frac{(1-\alpha)}{2\theta_p}Var_j[\widehat{y}_t(j)] - \frac{(1-\alpha)(1+\varphi\theta_w)}{2\theta_w}Var_h[\widehat{n}_t(h)] + \\ &\quad -\frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t - a_t)^2 + o(\|a\|^3) \end{aligned} \tag{69}$$

Computing the approximation around the steady state of the welfare function in absence of nominal rigidities leads to[18]:

$$\begin{aligned} &W_t^n - \overline{W} = \\ &\quad (1-\alpha)a_t - \frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t^n - a_t)^2 + o(\|a\|^3) \end{aligned} \tag{70}$$

Consequently,

[18] With flexible prices and wages there are no differences across workers and firms so $Var_j = Var_h = 0$

$$
\begin{aligned}
W_t - W_t^n = \\
& -\frac{(1-\alpha)(1+\varphi)}{2}(\widehat{y}_t^2 - (\widehat{y}_t^n)^2) + (1-\alpha)(1+\varphi)(\widehat{y}_t - \widehat{y}_t^n)a_t + \\
& -\frac{(1-\alpha)}{2\theta_p}Var_j[\widehat{y}_t(j)] - \frac{(1-\alpha)(1+\varphi\theta_w)}{2\theta_w}Var_h[\widehat{n}_t(h)] + o(\|a\|^3)
\end{aligned}
\tag{71}
$$

From the log-linearization of equation (38), $a_t = \widehat{y}_t^n$.
From (71):

$$
\begin{aligned}
W \equiv \sum_{t=0}^{\infty}\beta^t(W_t - W_t^n) = \\
& -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left[(1+\varphi)x_t^2 + \frac{1}{\theta_p}Var_j\left[\widehat{y}_t(j)\right] + \frac{1+\varphi\theta_w}{\theta_w}Var_h\left[\widehat{n}_t(h)\right]\right]
\end{aligned}
\tag{72}
$$

where $x_t = \widehat{y}_t - \widehat{y}_t^n = y_t - y_t^n$. As proved by Woodford (2001),

$$
\sum_{t=0}^{\infty}\frac{\beta^t}{\theta_p}Var_j\left[\widehat{y}_t(j)\right] = \frac{\theta_p}{\lambda}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2 \tag{73}
$$

It remains to study $Var_h[\widehat{n}_t(h)]$. Let's first write the log linear labour demand faced by each household:

$$
\widehat{n}_t(h) = -\theta_w\log(W_t(h)) + \theta_w\log(W_t) + \widehat{n}_t + o(\|a\|^2) \tag{74}
$$

consequently:

$$
Var_h[\widehat{n}_t(h)] = \theta_w^2 Var_h[w_t(h)] \tag{75}
$$

with $w_t(h) = \log(W_t(h))$.
The next step is to compute $Var_h[w_t(h)]$.

B.4 Step 4: Derivation of $Var_h[w_t(h)]$

First it is useful to decompose the variance as[19]:

$$
\begin{aligned}
Var_h[w_t(h)] &= E_h[w_t(h) - E_h w_t(h)]^2 \\
&= \xi_w E_h[w_{t-1}(h) + \gamma_w\pi_{t-1} - E_h w_t(h)]^2 \\
&\quad +(1-\xi_w)[\widetilde{w}_t - E_h w_t(h)]
\end{aligned}
\tag{76}
$$

Using the log-linearized expression for the aggregate wage and the result by Erceg et al. (2000) that $w_t - E_h w_t(h) = o(\|a\|^2)$ then,

[19]In general, if X assumes value X_1 with probability α and X_2 with probability $(1-\alpha)$, then $E(X^2) = \alpha * X_1^2 + (1-\alpha)X_2^2$, but the fraction of workers that can not reoptimise in t will all have a different wage, that's why, like in Erceg et al. (2000), I need to take expectations again.

$$\begin{aligned} E_h[w_{t-1}(h)+\gamma_w\pi_{t-1}-E_hw_t(h)]^2 &= E_h[w_{t-1}(h)+\gamma_w\pi_{t-1}-\xi_wE_hw_{t-1}(h)-\xi_w\gamma_w\pi_{t-1}+ \\ &\quad -(1-\xi_w)\widetilde{w}_t]^2 \\ &= E_h[w_{t-1}(h)+\gamma_w\pi_{t-1}-w_t+o(\|a\|^2)]^2 \\ &= E_h[w_{t-1}(h)-E_hw_{t-1}(h)+\gamma_w\pi_{t-1}-\pi_{w,t}+o(\|a\|^2)]^2 \\ &= Var_hw_{t-1}+\pi_{w,t}^2+\gamma_w^2\pi_{t-1}^2+o(\|a\|^3) \qquad (77) \end{aligned}$$

With the same arguments I have:

$$\begin{aligned} [\widetilde{w}_t-E_hw_t(h)]^2 &= [\widetilde{w}_t-w_t]^2+o(\|a\|^3) \\ &= \left[\frac{\xi_w}{1-\xi_w}\pi_{w,t}-\frac{\xi_w}{1-\xi_w}\gamma_w\pi_{t-1}\right]^2+o(\|a\|^3) \qquad (78) \end{aligned}$$

Substituting (77) and (78) into (76) I can write:

$$Var_h[w_t(h)] = \xi_wVar_hw_{t-1}(h)+\frac{\xi_w}{1-\xi_w}\pi_{w,t}^2+\frac{\xi_w}{1-\xi_w}\gamma_w^2\pi_{t-1}^2 \qquad (79)$$

Like in Woodford (2001), let's define $\triangle_t^w = Var_h[w_t(h)]$. Consequently I can rewrite (79) as:

$$\triangle_t^w = \xi_w\triangle_{t-1}^w+\frac{\xi_w}{1-\xi_w}\pi_{w,t}^2+\frac{\xi_w}{1-\xi_w}\gamma_w^2\pi_{t-1}^2+o(\|a\|^3) \qquad (80)$$

Iterating backward the previous equation can be written has:

$$\triangle_t^w = \xi_w^{t+1}\triangle_{-1}^w+\sum_{s=0}^{t}\xi_w^s\frac{\xi_w}{1-\xi_w}\pi_{w,t-s}^2+\gamma_w^2\sum_{s=0}^{t}\xi_w^s\frac{\xi_w}{1-\xi_w}\pi_{t-1-s}^2+o(\|a\|^3) \qquad (81)$$

Following Woodford (2001):

$$\sum_{t=0}^{\infty}\beta^t\triangle_t^w = \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\sum_{t=0}^{\infty}\beta^t\pi_{w,t}^2+\gamma_w^2\frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\sum_{t=0}^{\infty}\beta^t\pi_{t-1}^2+t.i.p.+o(\|a\|^3) \qquad (82)$$

Now it's enough to note that we can rewrite the last sum as:

$$\gamma_w^2\frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\pi_{-1}^2+\gamma_w^2\frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\beta\sum_{t=0}^{\infty}\beta^t\pi_t^2 \qquad (83)$$

and π_{-1}^2 is a $t.i.p.$ like it was $\triangle_{-1}^w$. With this consideration, equation (82) became:

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^tVar_h[\widehat{w}_t(h)] &= \frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\sum_{t=0}^{\infty}\beta^t\pi_{w,t}^2+ \\ &\quad +\gamma_w^2\frac{\xi_w}{(1-\beta\xi_w)(1-\xi_w)}\beta\sum_{t=0}^{\infty}\beta^t\pi_t^2+t.i.p.+o(\|a\|^3) \qquad (84) \end{aligned}$$

B.5 Final expression

Combining the results in previous sections:

$$W = -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left[(1+\varphi)x_t^2 + \frac{\theta_p}{\lambda}\pi_{H,t}^2 + \frac{\theta_w}{\lambda_w}\pi_{w,t}^2 + \beta\gamma_w^2\frac{\theta_w}{\lambda_w}\pi_t^2\right] \tag{85}$$

Taking unconditional expectation of (85) and letting $\beta \to 1$ the expected welfare loss is:

$$L = -\frac{1-\alpha}{2}\left[(1+\varphi)Var(x_t) + \frac{\theta_p}{\lambda}Var(\pi_{H,t}) + \frac{\theta_w}{\lambda_w}Var(\pi_{w,t}) + \beta\gamma_w^2\frac{\theta_w}{\lambda_w}Var(\pi_t)\right] \tag{86}$$

C System of equations fully characterizing the model

With the inclusion of a monetary policy rule the following system of equations fully characterize the model:

$$\alpha s_t = \alpha s_{t-1} + \pi_t - \pi_{H,t} \tag{87}$$

$$y_t = c_t + \alpha s_t \tag{88}$$

$$y_t^n = \frac{\log(1-\alpha)}{1+\varphi} + a_t \tag{89}$$

$$y_t = a_t + n_t \tag{90}$$

$$\begin{aligned} \pi_{w,t} &= w_t + \pi_t - w_{t-1} \\ w_t &= \log(W_t/P_t) \end{aligned} \tag{91}$$

$$s_t = y_t - y_t^* \tag{92}$$

$$x_t = y_t - \overline{y}_t \tag{93}$$

$$c_t = -\left[r_t - \rho - E_t\pi_{t+1}\right] + E_t c_{t+1} \tag{94}$$

$$\pi_{w,t} = \beta E_t\pi_{w,t+1} - \lambda_w\left[w_t - c_t - \varphi n_t\right] - \xi_w\gamma_w\beta\pi_t + \gamma_w\pi_{t-1} \tag{95}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda(1+\varphi)x_t + \lambda\left[w_t - c_t - \varphi n_t\right] \tag{96}$$

$$y^*_{t+1} = \rho_y y^*_t + \varepsilon_{y,t}. \tag{97}$$

$$a_{t+1} = \rho_a a_t + \varepsilon_{A,t}. \tag{98}$$

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Table 1: **Welfare cost of deviation from optimal policy.** *Welfare losses are in percentage units of steady state consumption. For the interest rate rules are also reported the coefficients of the policy rule minimizing the welfare losses. Moments have been computed as average over 200 simulations, each 100 periods long.*

γ_w	Interest Rate π	Interest Rate π_H	Interest Rate π_w
0	$\phi_p = 4.25$ **0.0344**	$\phi_{p,H} = 3.50$ 0.2214	$\phi_w = 9.25$ 0.0464
0.25	$\phi_p = 6.25$ 0.0295	$\phi_{p,H} = 2.25$ 0.1343	$\phi_w = 7$ **0.0068**
0.45	$\phi_p = 7.75$ **0.0195**	$\phi_{p,H} = 3.50$ 0.3834	$\phi_w = 4.50$ 0.0308
0.65	$\phi_p = 9.50$ **0.0153**	$\phi_{p,H} = 3.50$ 0.6593	$\phi_w = 3.50$ 0.0675

γ_w	Interest Rate $\pi_H - \pi_w$	Interest Rate $\pi - \pi_w$	Interest Rate $\pi - \pi_H$
0	$\phi_{p,H} = 3; \phi_w = 9.5$ 0.001	$\phi_p = 1.25; \phi_w = 9.5$ **0.0003**	$\phi_p = 9.5; \phi_{p,H} = 1.25$ 0.0413
0.25	$\phi_{p,H} = 1.25; \phi_w = 7$ 0.0093	$\phi_p = 2.25; \phi_w = 8.5$ **0.0013**	$\phi_p = 10; \phi_{p,H} = 1.25$ 0.0328
0.45	$\phi_{p,H} = 1.25; \phi_w = 4.5$ 0.0336	$\phi_p = 4; \phi_w = 7.5$ **0.0011**	$\phi_p = 10; \phi_{p,H} = 1.25$ 0.0268
0.65	$\phi_{p,H} = 1.25; \phi_w = 3.25$ 0.0771	$\phi_p = 8; \phi_w = 8.25$ **0.0022**	$\phi_p = 9.5; \phi_{p,H} = 1.25$ 0.0254

Table 2: **Standard deviations of several variables under the Optimal Monetary Policy Rule and under several Taylor's type rules.** *(%) Standard deviations have been computed as average over 200 simulations, each 100 periods long.*

γ_w	Rule	$\sigma(\pi)$	$\sigma(\pi_H)$	$\sigma(\pi_w)$	$\sigma(x)$
0	Optimal	0.3289	0.2076	0.0290	0.1741
	π	**0.0836**	**0.1610**	0.1175	**1.0647**
	π_H	0.2542	0.1834	0.1756	3.7474
	π_w	0.1999	0.2286	**0.0688**	1.7648
	$\pi_H - \pi_w$	0.3754	**0.1975**	0.0362	**0.2925**
	$\pi - \pi_w$	**0.2376**	0.2164	**0.0118**	0.3403
0.25	Optimal	0.1626	0.2109	0.0292	0.5995
	π	**0.0536**	**0.1536**	0.1151	1.1859
	π_H	0.5746	0.2649	0.2170	1.1724
	π_w	0.2544	0.2294	**0.0529**	**0.5166**
	$\pi_H - \pi_w$	0.2585	0.2126	0.0591	**0.5895**
	$\pi - \pi_w$	**0.1472**	**0.2088**	**0.0323**	0.7052
0.45	Optimal	0.1151	0.2165	0.0384	0.8428
	π	**0.0387**	**0.1603**	0.1124	1.1170
	π_H	0.6553	0.1846	0.3305	1.9289
	π_w	0.2410	0.2403	**0.0952**	**0.7536**
	$\pi_H - \pi_w$	0.2458	0.2019	0.1038	**0.8500**
	$\pi - \pi_w$	**0.1050**	**0.1994**	**0.0454**	0.9278
0.65	Optimal	0.0812	0.1931	0.0470	0.9832
	π	**0.0303**	**0.1531**	**0.1087**	1.1465
	π_H	0.6258	0.1573	0.4296	1.9493
	π_w	0.2309	0.2445	0.1331	**1.0156**
	$\pi_H - \pi_w$	0.2411	0.1928	0.1490	**1.0333**
	$\pi - \pi_w$	**0.0715**	**0.1920**	**0.0569**	1.0479

Table 3: **Correlations among the simulated series obtained under the Fully Optimal Monetary Policy Rule and the ones obtained under several Taylor's type rules.**

γ_w	Rule	$\rho(\pi)$	$\rho(x)$	$\rho(\pi_w)$	$\rho(\pi_H)$
0	π	-0.2809	**0.4328**	-0.0534	0.3954
	π_H	0.1385	-0.5719	-0.1432	0.3271
	π_w	**0.2415**	-0.6520	-0.2955	**0.4626**
	$\pi_H - \pi_w$	**0.3245**	**0.2210**	**0.2157**	**0.3994**
	$\pi - \pi_w$	0.2420	-0.2646	0.0147	0.3378
0.25	π	0.3465	0.8839	**0.7390**	0.9774
	π_H	0.3263	0.0794	-0.0281	0.5066
	π_w	**0.9770**	**0.9011**	-0.7207	**0.9872**
	$\pi_H - \pi_w$	0.9278	0.7310	0.2450	0.9915
	$\pi - \pi_w$	**0.9720**	**0.9942**	**0.3930**	**0.9919**
0.45	π	0.3853	0.9559	**0.8454**	**0.9778**
	π_H	0.1212	0.2826	0.2530	0.5067
	π_w	**0.9258**	**0.9570**	-0.6662	0.9605
	$\pi_H - \pi_w$	0.7839	0.8573	0.1804	0.9674
	$\pi - \pi_w$	**0.9706**	**0.9931**	**0.7604**	**0.9940**
0.65	π	0.4063	**0.9781**	**0.8946**	**0.9816**
	π_H	-0.0450	0.3737	0.3697	0.3845
	π_w	**0.8359**	**0.9781**	-0.5583	0.9072
	$\pi_H - \pi_w$	0.6100	0.9086	0.1990	0.9277
	$\pi - \pi_w$	**0.9783**	**0.9959**	**0.9376**	**0.9985**