The Implications of Sectoral Heterogeneity for Monetary Policy and Welfare in a Small Open Economy: A Linear Quadratic Framework*

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Abstract

Modern economies exhibit various structural and dynamic characteristics. At the same time, many central banks have implemented the similar strategy, i.e. inflation targeting, as an operational framework. Controversial normative issue - is such stabilization objective welfare maximizing for more complex models with heterogeneous elements across sectors? This article analyzes optimal monetary strategy and policy trade-offs in a DSGE model of an open economy with traded and non-traded sectors. We approximate the utility of the representative consumer to obtain a micro-founded quadratic loss function of the form extensively used for monetary policy assessment. The central bank's optimal strategy is computed and optimal and simple policy rules compared according to the derived welfare measure. We assess the role of openness, structural characteristics, and relative prices for monetary policy design. The model is calibrated to match the moments of main macroeconomics variables of Canadian economy. The findings suggest that social welfare objectives display sector-specific features thus generating important implications for optimal policy and welfare. The analysis of the performance of simple rules indicates that flexible CPI targeting regime that includes a certain degree of internal relative prices management is able to closely replicate the optimal solution. Finally, we assess the implications of sectoral heterogeneity in price stickiness for benefits of targeting the core versus broader price indices.

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1 Introduction

In recent decades the approach to monetary policy conduct has shifted to a more systematic one. Many central banks have formulated their policy objectives explicitly and, more specifically, have announced their commitment to price stabilization as the overriding policy goal. As a result, a new operational framework, inflation targeting, has been introduced by the most advanced central banks. At the same time, important features of modern economies, such as the social and economic consequences of unemployment, uncertainties of various types, asymmetric economic structure, and interrelations with the rest of the world, have brought about efforts to widen the range of policy objectives beyond inflation (price) stability alone. Therefore, over the past several years, the attention of economists has turned to the issue of whether strict inflation targeting indeed represents the best strategy from the welfare viewpoint.

The important attribute of real economies is that they represent the complex systems with various structural and dynamic characteristics. Should policymakers account for structural heterogeneity across economic elements when implementing the monetary strategy, or should they assume that welfare can be maximized under the uniform specification of the policy objectives? This paper aims to contribute to the discussion of this crucial issue of monetary policy design and practical implementation.

The analysis of optimal monetary strategies has been performed in a number of studies. One thread in the literature computes optimal policy under *assumed* welfare objectives. In particular, the loss function of the central bank usually takes the quadratic form with terms such as inflation (CPI or domestic) and the output gap, with the weights in front of each target chosen ad hoc. This approach is very popular in applied research because it greatly simplifies the derivations and brings the model dynamics closer to the real data. At the same time, such an approach assumes certain policy objectives a priori. An alternative methodology analyzes optimal monetary policy on the basis of the objective function of the central bank which is derived from micro-foundations. This paper contributes to the second class of literature and adds to the analysis of optimal policy in *open* economies, where the formulation of policy targets appears to be more controversial compared to a closed economy setting. It has been shown that welfare-maximizing monetary policy in a closed economy should aim to completely stabilize CPI inflation and the output gap (Woodford, 2003). In the literature on open economies, the critical questions are whether the central bank should also target open economy variables, i.e. the exchange rate, and how the targeting of domestic variables changes under the exposure of the economy to external factors. Another topic which has attracted a great deal of attention from both researchers and practitioners is related to the determination of the appropriate inflation measure that has to be stabilized. This issue gains particular relevance for the studies of models with heterogeneous economic structure, which implies the different response of domestic elements to disturbances of the same type.

A surprising conclusion drawn by several authors who have performed explicit welfare derivation for models of open economies is that exchange rate fluctuations have no direct impact on welfare. Specifically, Clarida, Gali, and Gertler (2001) find that under perfect exchange rate pass-through, the qualitative results for the closed economy carry over to the open economy. Gali and Monacelli (2005), who characterize the welfare of a small open economy for a special case of parameter values and under the balanced trade assumption, support the previous result and conclude that the small open economy problem is identical to that of a closed economy. The above results taken at face value imply optimality of complete exchange rate flexibility.

However, a number of recent studies have challenged this finding. Specifically, Corsetti and Pesenti (2005), Sutherland (2002), and Monacelli (2003) show that under incomplete pass-through, optimal policy is not purely inward looking. Benigno and Benigno (2006) analyze the gains from international monetary policy cooperation. They study the conditions under which individual countries have incentives to influence the terms of trade and thus to deviate from the socially optimal point. De Paoli (2006) finds that the simple violation of purchasing power parity (PPP), which arises from home bias in consumption, brings in a role for targeting the real exchange rate in a one-sector small open economy model. Liu and Pappa (2005) consider a two-sector, open economy model in a two-country framework. Their study provides interesting insights into the impact of an asymmetric structure between sectors on the gains from cooperation. Their results suggest that in an economy with multiple sectors, and thus multiple sources of nominal rigidities, optimal monetary policy cannot replicate a flexible price allocation creating the scope for coordination. The important limitation of their work for the analysis of optimal monetary policy is the assumption of unitary elasticity of substitution across goods and a logarithmic utility function. As a result, under this very special case, important welfare effects vanish and general conclusions concerning the optimal monetary policy cannot be derived.

In this work, we analyze the stabilization objectives of optimal monetary policy and the trade-offs facing the central bank in a two-sector, small open economy model obtained as a limiting case of a two-country Dynamic Stochastic General Equilibrium framework. We assess the role of structural asymmetries, general preferences, and multiple relative prices for monetary policy design and welfare evaluation. We contribute to the normative analysis of open economies by introducing a more complicated economic structure, namely, multiple domestic sectors combined with a variety of sector-specific and foreign shocks. In addition, we consider a general specification of preferences (the elasticity of substitution is non-unitary). These features of the model differentiate our work from the previous studies, which derived their results for the special cases of unitary elasticity of substitution across goods or, alternatively, relied on the ad hoc objective functions. By abstracting from those simplifying assumptions we are able to uncover additional welfare effects specific to the open multisectoral economy and make a methodological contribution by deriving the utility-based welfare measure and the optimal reaction function of the central bank under more generalized preferences. For this purpose we employ the linear-quadratic solution methods discussed in Benigno and Benigno (2006) and Benigno and Woodford (2005), which involve computation of a second-order approximation of the model structural equations. This approach enables us to analyze the determinants of optimal monetary policy and rank alternative monetary policy regimes on the basis of a rigorous welfare measure derived from micro foundations and approximated by a tractable quadratic form. In addition, we study how the optimal price index that has to be stabilized is affected by structural asymmetries. In particular, we evaluate the welfare benefits from targeting sector-specific versus aggregate price indices (domestic or CPI inflation) for various degrees of relative price stickiness. We calibrate the model to match the moments of variables of Canadian economy.

The results of our study suggest that the loss function of the central bank, which describes the welfare maximizing stabilization objectives, displays the features of both an open economy and multisectoral economic structure. Specifically, it is shown that social welfare is affected by variations in domestic inflation rates and output gaps (with sector-specific weights) as well as in the relative prices (including the exchange rate). We derive the optimal targeting rule, which determines the variables (targets) to which the central bank should respond in order to achieve efficient allocation of resources as well as the magnitude of such a response. Furthermore, we experiment with alternative simple rules and analyze their ability to replicate the optimal solution. Our results suggest that targeting domestic inflation is not always the best approximation for the optimal policy, and social welfare can be improved by accounting for other policy objectives, namely, the output gap and the relative prices. We present a ranking of alternative simple rules, which indicates the costs of implementing alternative monetary strategies and can provide useful information for managing the conflicting policy objectives. The rules with sector-specific and aggregate terms are compared. We show that the simple rules with aggregate variables which incorporate a response to relative price changes achieve better stabilization of sector-specific volatilities and thus improve welfare. Such a result is important because a strategy which differentiates the response between domestic sectors is difficult to design and implement in practice. Generally, the simple rules perform quite well in terms of macroeconomic stabilization (relative to the optimal rule) and can deliver reasonable welfare results. We perform a sensitivity analysis in order to study the impact of sectoral heterogeneity in the degree of price stickiness on the relative performance of policy rules with sector-specific and aggregate variables (inflation rates). We find that the implications of asymmetric nominal rigidities differ for closed and open economies.

The paper is organized as follows. Section 2 presents the model and section 3 describes the equilibrium dynamics. Section 4 analyzes the monetary policy problem and welfare. Section 5 describes the results of the numerical simulation. Section 6 illustrates the welfare implications of alternative simple rules. The sensitivity analysis is presented in section 7. Finally, the results of the paper are summarized in section 8.

2 A Two-Sector, Small Open Economy Model

The framework is represented by a two-country dynamic general equilibrium model where both sides, Home (the open economy – H) and Foreign (the rest of the world, the relatively closed economy – F), are explicitly modeled. The small open economy problem is derived as a limiting case of such a framework (as in De Paoli, 2006). Each country has two domestic sectors, which produce traded and non-traded goods; the share of non-traded goods may vary in the consumption basket of each country. A continuum of infinitively lived households consumes the final consumption good, which includes goods produced in both domestic sectors as well as imported goods. Households produce differentiated intermediate goods and receive disutility from production. We introduce monopolistic distortion and sticky prices in both sectors. These assumptions represent the standard way of introducing the role for monetary policy into such class of models. Households as consumers maximize their utility and solve the optimal price-setting problem as producers.

The model specification allows us to consider the closed economy, the open one-sector economy, and the economy with unitary elasticity of substitution as special cases of our more general analysis. We assume sector-specific productivity, fiscal, and mark-up shocks; the degree of nominal rigidities may also differ across sectors. Furthermore, we assume production subsidies in order to offset the monopolistic distortions in both sectors. The international and domestic asset markets are complete.

2.1 Representative Households

In our two-country framework a continuum of domestic households belong to the interval $[0, n)$, while foreign agents belong to the segment $(n, 1]$. The utility function of a representative consumer in country H or F is given by:

$$U_t^j = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_{s,T}(j), A_{s,T}^i) - V(y_{s,N}(j), A_{s,N}^i)] \right\},$$

where j is the index specific to the household, and i is the country index; E_t denotes the expectation operator conditional on the information set at time t, and β is the intertemporal discount factor. U(.) represents the flows of utility from consumption of a composite good and $V(.)$ stands for the flows of disutility from production of differentiated goods. Each household produces two types of differentiated goods – traded and non-traded. The home economy produces a continuum of differentiated traded goods indexed on the interval $[0, n]$, whereas the foreign economy's traded goods belong to the interval $(n, 1]$. In addition, a continuum of differentiated non-traded goods are indexed on the interval $[0, n]$ and $(n, 1]$ for the home and foreign country, respectively. A denotes a productivity shock that can be country and sector specific. The subscript T stands for the traded sector, whereas N denotes the non-traded sector.

In our analysis we assume that preferences have isoelastic functional form:

$$U(C_s^j) = \frac{(C_s^j)^{1-\rho}}{1-\rho}, \qquad V(y_{s,L}(j), A_{s,L}^i) = (A_{s,L}^i)^{-\eta}\frac{(y_{s,L}(j))^{1+\eta}}{1+\eta},$$

where $L = H, N$; $\rho > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $\eta \geq 0$ is equivalent to the inverse of the elasticity of goods production. The composite consumption good C is a Dixit-Stiglitz aggregator of traded and non-traded goods defined as:

$$C^j = [\gamma^{\frac{1}{\omega}}(C_N^j)^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}}(C_T^j)^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}},$$

where C_N and C_T are the consumption sub-indexes that refer to the consumption of non-traded and traded goods, respectively, $\omega > 0$ is the intratemporal elasticity of substitution, and γ is a preference parameter that measures the relative weight that individuals put on non-traded goods.

Preferences for the rest of the world are specified in a similar fashion:

$$C^{j*} = [(\gamma^*)^{\frac{1}{\omega}}(C_N^{*j})^{\frac{\omega-1}{\omega}} + (1-\gamma^*)^{\frac{1}{\omega}}(C_T^{*j})^{\frac{\omega-1}{\omega}}]^{\frac{\omega}{\omega-1}},$$

where the asterisk denotes a foreign country variable.

Traded consumption goods are the aggregators of goods produced at home and abroad and defined as:

$$\begin{aligned} C_T^j &= [\upsilon^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \\ C_T^{j*} &= [(\upsilon^*)^{\frac{1}{\theta}}(C_H^*)^{\frac{\theta-1}{\theta}} + (1-\upsilon^*)^{\frac{1}{\theta}}(C_F^*)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \end{aligned}$$

where υ and υ^* are the parameters that determine the preferences of agents in countries H and F, respectively, for the consumption of goods produced at Home.

As in Sutherland (2002) and De Paoli (2006) we assume that υ^*, the share of imported goods from country H in the consumption basket of country F, increases proportionally to the relative size of the home economy n and the degree of openness $\widetilde{\upsilon}^*$. Thus we assume that $\upsilon^* = n \cdot \widetilde{\upsilon}^*$. Similarly, $(1-\upsilon) = (1-n) \cdot \widetilde{\upsilon}^*$. Such a specification allows modeling of home bias in consumption as a consequence of different country size and degree of openness.

The consumption sub-indices of non-traded, home-produced, and foreign-produced differentiated goods are defined as follows:

$$C_N = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_N(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_N^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_N^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}},$$

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}},$$

$$C_{H^*} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated goods.

The corresponding consumption-based price indexes for countries H and F take the form:

$$P = [\gamma P_N^{1-\omega} + (1-\gamma) P_T^{1-\omega}]^{\frac{1}{1-\omega}} \tag{1}$$

$$P_T = [\upsilon P_H^{1-\theta} + (1-\upsilon) P_F^{1-\theta}]^{\frac{1}{1-\theta}} \tag{1a}$$

$$P^* = [(\gamma^*)(P_N^*)^{1-\omega} + (1-\gamma^*)(P_T^*)^{1-\omega}]^{\frac{1}{1-\omega}} \tag{2}$$

$$P_T^* = [(\upsilon^*)(P_H^*)^{1-\theta} + (1-\upsilon^*)(P_F^*)^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{2a}$$

The price sub-indices for home, foreign, and non-traded goods in the two economies are:

$$P_N = \left[\left(\frac{1}{n}\right) \int_0^n p_N(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}} \qquad P_N^* = \left[\left(\frac{1}{1-n}\right) \int_n^1 p_N^*(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}},$$

$$P_H = \left[\left(\frac{1}{n}\right) \int_0^n p_h(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}} \qquad P_F = \left[\left(\frac{1}{1-n}\right) \int_n^1 p_f(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}},$$

$$P_H^* = \left[\left(\frac{1}{n}\right) \int_0^n p_h^*(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}} \qquad P_F^* = \left[\left(\frac{1}{1-n}\right) \int_n^1 p_f^*(z)^{1-\sigma} d(z) \right]^{\frac{1}{1-\sigma}},$$

where $p_N(z), p_H(z)$, and $p_F(z)$ are prices in units of the domestic currency of the home-produced non-traded and traded goods, and foreign-produced goods. The law of one price holds for differentiated goods, i.e., $p_h(z) = S \cdot p_h^*(z)$ and $p_f(z) = S \cdot p_f^*(z)$, where S is the nominal exchange rate, defined as the price of the foreign currency in terms of the domestic currency. This in turn implies that $P_H = S \cdot P_H^*$ and $P_F = S \cdot P_F^*$. However, equations (1) and (2) demonstrate that the presence of non-traded goods and the home bias in consumption result in a violation of the Purchasing Power Parity (PPP), i.e., $P \neq S \cdot P^*$. Thus, the real exchange rate is not equal to one and is defined as $ER = \frac{S \cdot P^*}{P}$. The real exchange rate determinants will be more explicitly analyzed in subsection 2.5.

2.2 Aggregate Demand

By solving the consumer's cost minimization problem, we derive the total demand for the differentiated goods produced in countries H and F as well as the demand for the non-traded goods in both countries. The resulting demand equations for country H take the following form:

$$y_h^d(z) = \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_H}{P_T}\right)^{-\theta} \times \\ \left\{ \begin{array}{l} \upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega} \times \\ \left[\begin{array}{c} \left(\frac{\upsilon^*}{\upsilon+(1-\upsilon)(P_{FH})^{1-\theta}}\right) + \\ \left(\frac{1-\upsilon^*}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right) \end{array} \right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)\upsilon^* C^* \frac{1-n}{n} \end{array} \right\} + G_H \end{array} \right] \tag{3}$$

$$y_N^d(z) = \left(\frac{p_N(z)}{P_N}\right)^{-\sigma} \left[\left(\frac{P_N}{P}\right)^{-\omega} \gamma C + G_N \right], \tag{4}$$

and for goods produced in country F:

$$y_f^d(z) = \left(\frac{p_f(z)}{P_F}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_F}{P_T}\right)^{-\theta} \times \\ \left\{ \begin{array}{l} (1-\upsilon)(1-\gamma)C\frac{n}{1-n} + \left(\frac{1}{ER}\right)^{-\omega} \times \\ \left[\begin{array}{c} \left(\frac{\upsilon^*}{\upsilon+(1-\upsilon)(P_{FH})^{1-\theta}}\right) + \\ \left(\frac{1-\upsilon^*}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right) \end{array} \right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)(1-\upsilon^*)C^* \end{array} \right\} + G_F^* \end{array} \right] \tag{5}$$

$$y_N^d(z) = \left(\frac{p_N^*(z)}{P_N^*}\right)^{-\sigma} \left[\left(\frac{P_N^*}{P^*}\right)^{-\omega} \gamma^* C^* + G_N^* \right], \tag{6}$$

where G and G^* are country and sector-specific government purchase shocks, $P_{FH} = \frac{P_F}{P_H}$ is the relative price of foreign to home-produced goods, i.e., the terms of trade, and ER is the real exchange rate.

In order to obtain the small open economy version of our general two-country framework, we apply the assumptions $\upsilon^* = n \cdot \widetilde{\upsilon}^*$ and $(1-\upsilon) = (1-n) \cdot \widetilde{\upsilon}^*$ and take the limit $n \to 0$ similar to De Paoli (2006). As a result, the demand equations can be simplified to:

$$y_h^d(z) = \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_H}{P_T}\right)^{-\theta} \times \\ \left\{\upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega} \left[\left(\frac{1}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right)\right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)\widetilde{\upsilon}^* C^*\right\} + G_H \end{array} \right] \tag{7}$$

$$y_f^d(z) = \left(\frac{p_f(z)}{P_F}\right)^{-\sigma} \left[\begin{array}{l} \left(\frac{P_T}{P}\right)^{-\omega} \left(\frac{P_F}{P_T}\right)^{-\theta} \times \\ \left\{\left(\frac{1}{ER}\right)^{-\omega} \left[\left(\frac{1}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right)\right]^{\frac{\theta-\omega}{1-\theta}} (1-\gamma^*)C^*\right\} + G_F^* \end{array} \right]. \tag{8}$$

Therefore, the demand side for our two-sector, small open economy model is represented by equations (4), (6), (7), and (8).

The demand equations illustrate the small open economy implications, the impact of the economic structure, and a more general specification of preferences. In particular, the demand for goods produced at Home depends on both domestic and foreign consumption, whereas the demand for foreign-produced goods is not affected by changes in Home consumption. Moreover, the two-sector model specification brings in the differentiated impact of the terms of trade and the real exchange rate on the total demand for tradable goods. This happens under the general assumption that $\theta \neq \omega$. The literature on open economies usually assumes that $\theta > \omega$, $\theta > 1$, and ω is small. This implies that non-traded and traded goods are complements in the consumption basket. At the same time, home and foreign-produced goods are considered as substitutes.

2.3 International Risk Sharing

Foreign and domestic households have access to the international financial market, where state-contingent nominal bonds are traded. Households at home and abroad make their optimal consumption-saving decisions. They maximize their utility subject to the sequence of budget constraints for $t = 0, 1, ..$:

$$P_t C_t + E_t D_{t,t+1} B_{t+1} \leq B_t + \Pi_t + T_t,$$

where B_{t+1} is the holding of a nominal state-contingent bond that pays one unit of home currency in period $t+1$, $D_{t,t+1}$ is the period t price of the bond, Π_t is the profit income from goods production, and T_t is the transfer from the government. The complete-market assumption implies that the marginal rate of substitution between consumption in the two countries is equalized:

$$\frac{U_C(C^*_{t+1})}{U_C(C^*_t)}\frac{P^*_t}{P^*_{t+1}}\frac{S_t}{S_{t+1}}=\frac{U_C(C_{t+1})}{U_C(C_t)}\frac{P_t}{P_{t+1}}. \tag{9}$$

The international risk-sharing equation presented above illustrates the equality of nominal wealth in both countries in all states and time periods. The violation of PPP implies that fluctuations in the real exchange rate may result in a divergence in consumption across countries even under optimal risk sharing.

Consumers' optimization problem implies the following Euler equation:

$$U_C(C_t)=\beta\left[U_C(C_{t+1})R_t\frac{P_t}{P_{t+1}}\right],$$

where R_t is the nominal interest rate. Log-linearization of this condition leads to the following expression:

$$\widehat{r}_t=\rho\left(\widehat{C}_{t+1}-\widehat{C}_t\right)+E\pi_{t+1}.$$

2.4 Optimal Pricing Decisions

Each household is a monopolistic producer of one differentiated traded and one non-traded good. The domestic household sets the price $p_N(z)$ and $p_h(z)$ and takes as given P, P_N, P_H, P_F, and C. The price-setting behavior is modeled according to Calvo (1983). In countries H and F in each time period a fraction $\alpha_L\in[0,1)$ of randomly picked producers in each sector $(L=N,H)$ are not allowed to change their prices. Thus the parameter α_L reflects the level of price stickiness. The remaining fraction $(1-\alpha_L)$ can choose the optimal sector-specific price by maximizing the expected discounted value of profits:

$$E_t\sum_{S=t}^{\infty}(\alpha_L\beta)^{S-t}\left[\frac{U_C(C_S)}{P_S}(1-\tau_S)\widetilde{p}_{t,L}(z)\widetilde{y}_{t,S,L}(z)-V(\widetilde{y}_{t,S,L}(z),A_{S,L})\right],$$

where after-tax revenues in each sector are evaluated using the marginal utility of nominal income, $\frac{U_C(C_S)}{P_S}$, which is identical for all households in the country under the assumption of complete markets; τ_S is the tax rate; $\widetilde{p}_{t,L}(z)$ is the price of the differentiated good z, which is produced in sector L, chosen at time t , and $\widetilde{y}_{t,S,L}(z)$ is the total demand for good z, produced in sector L, at time S, conditional on the fact that the price $\widetilde{p}_{t,L}(z)$ has not been changed. All producers who belong to the fraction $(1-\alpha_L)$ choose the same price.

The optimal price $\widetilde{p}_{t,L}(z)$, which is derived from the first-order conditions, takes the following form:

$$\widetilde{p}_{t,L}(z) = \frac{E_t \sum\limits_{S=t}^{\infty} (\alpha_L \beta)^{S-t} V(\widetilde{y}_{t,S,L}(z), A_{S,L}) \widetilde{y}_{t,S,L}(z)}{E_t \sum\limits_{S=t}^{\infty} (\alpha_L \beta)^{S-t} \frac{U_C(C_S)}{P_S} \frac{1}{\mu_S} \widetilde{y}_{t,S,L}(z)}, \tag{10}$$

where $\mu_{S,L} = \frac{\sigma}{(1-\tau_{S,L})(\sigma-1)}$ represents the overall degree of monopolistic distortion and leads to an inefficient gap between the marginal utility of consumption and the marginal disutility of production. Benigno and Benigno (2006) and De Paoli (2006) refer to this gap as the mark-up shock. A Calvo-type setting implies the following law of motion for the sectoral price indices:

$$P_{L,t} = [\alpha_L (P_{L,t-1})^{1-\sigma} + (1-\alpha_L) \widetilde{p}_{t,L}(z)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \tag{11}$$

Similar conditions can be derived for the producers in country F.

2.5 Real Exchange Rate Decomposition and PPP Violation

In order to explore the structural economic factors that result in PPP violation, we consider the real exchange rate decomposition. The real exchange rate is defined as $ER = \frac{S \cdot P^*}{P}$. We use the price indexes (1), (1a), (2), and (2a) to express the real exchange rate as a function of relative prices and preference parameters. We also use the fact that the law of one price holds for tradable goods, i.e., $P_H = S \cdot P_H^*$ and $P_F = S \cdot P_F^*$. The real exchange rate can be presented as:

$$ER = \left(\frac{\upsilon^* + (1-\upsilon^*)(P_{FH})^{1-\theta}}{\upsilon + (1-\upsilon)(P_{FH})^{1-\theta}} \right)^{\frac{1}{1-\theta}} \left(\frac{\gamma^* (P_{NT}^*)^{1-\omega} + (1-\gamma^*)}{\gamma (P_{NT})^{1-\omega} + (1-\gamma)} \right)^{\frac{1}{1-\omega}}, \tag{12}$$

where P_{FH} is the terms of trade defined in the previous sections, and $P_{NT} = \frac{P_N}{P_T}$ and $P_{NT}^* = \frac{P_N^*}{P_T^*}$ are the relative prices of non-traded goods in the two countries. Such a decomposition enables us to analyze the different channels of PPP violation. First of all, we note that under $\upsilon \neq \upsilon^*$,the ER is affected by the terms of trade. For our small open economy model specification, given the assumptions on υ and υ^*, the difference in country size necessarily results in different shares of consumption of home-produced goods in countries H and F. This so-called home bias channel has also been analyzed by De Paoli (2006) and Sutherland (2002).

Another important component that explains the deviation of the ER from PPP is determined by the multisectoral economic structure. Specifically, different preferences for consumption of non-traded goods across countries, i.e., $\gamma \neq \gamma^*$, as well as changes in the relative price of non-traded goods determine the fluctuation in the ER. The divergence in relative prices may occur as a result of country or sector-specific productivity shocks. Moreover, the law of one price holds for traded goods only. Nothing can ensure that the same equality will hold for the goods produced in the non-traded sector. Therefore, the exchange rate in our model is a composite term of two types of relative prices. As far as the policy issues are concerned, such a distinction implies a more difficult task of exchange rate management.

3 Equilibrium Dynamics

The *equilibrium* is described by the allocations of $C_{H,t}$, $C_{F,t}$, $C_{N,t}$, B_{t+1} and $C^*_{H,t}$, $C^*_{F,t}$, $C^*_{N,t}$, B^*_{t+1} for domestic and foreign households, respectively; the allocations of $y_{t,N}(z)$ and price $\widetilde{p}_{t,N}(z)$ for non-traded goods produced in country H and $y^*_{t,N}(z)$, $\widetilde{p^*}_{t,N}(z)$ for the intermediate goods produced in country F; the allocations $y^d_{t,H}(z)$ and price $\widetilde{p}_{t,H}(z)$ for traded goods produced in the domestic economy, and $y^d_{t,F}(z)$, $\widetilde{p}_{t,N}(z)$ for traded goods produced abroad; and prices $D_{t,t+1}$, S_t, ER_t, $P_{t,}P_{N,t}$, $P_{T,t}$, $P_{H,t}$, $P^*_{t,}P^*_{N,t}$, $P^*_{T,t}$, $P^*_{F,t}$ that satisfy the following equilibrium conditions:

1. taking prices as given, the household's allocation in each country solves the consumer's utility maximization problem;
2. taking aggregate prices as given, the demand allocations and the price of each non-traded differentiated good solve the producer's profit maximization problem;
3. taking aggregate prices as given, the demand allocations and the price of each traded differentiated good solve the producer's profit maximization problem;
4. the world bond market `clears`.

3.1 Sticky Price Equilibrium

The equilibrium dynamics under sticky prices are characterized by the optimality conditions derived in section 2. Here, we present a log-linearized version of the model. We define $\widehat{x}_t \equiv \ln \frac{x_t}{\overline{x}}$ as the log deviation of the equilibrium variable x_t under sticky prices from its steady state value. $\widehat{x}_t{}^{flex} \equiv \ln \frac{x_t^{flex}}{\overline{x}}$ represents the log deviation of the equilibrium variable x_t under flexible prices from its steady state value. Under the assumption of flexible prices, producers can re-optimize every period so that their pricing decisions are synchronized. As a result the price dispersion among the differentiated goods is zero. Therefore, the price index in each sector is equal to the price set by each producer in this sector, and the main source of domestic distortion is eliminated. We will refer to $\widehat{x}_t - \widehat{x}_t^{flex}$ as the deviation of the variable $\widehat{x}_t$ from its natural level, i.e., the gap. At the same time, Benigno and Woodford (2005) and De Paoli (2006) demonstrate that under certain conditions, the flexible price equilibrium does not represent the most efficient allocation of resources, and the desired levels of variables which the policymaker wishes to achieve in order to eliminate the loss may differ from the flexible price allocation. Specifically, in the presence of mark-up and fiscal shocks as well as the condition $\rho\theta \neq 1$, the flexible price allocation diverges from the desired targets. Therefore, in general, the optimal policy aims to stabilize of the variables relative to their *target* level. Thus, we define the welfare relevant gap as $\widehat{x}_t - \widehat{x}_t^T$, where $\widehat{x}_t^T$ is the target level of the variable $\widehat{x}_t$. Both the flexible price equilibrium and the target variables are functions of shocks that affect the economy.

Moreover, we define the price change in the traded sector as $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}}$ and that in the non-traded sector as $-\Pi_N = \frac{P_{N,t}}{P_{N,t-1}}$; consequently, the producer price inflation rates in the traded and non-traded sectors are $\pi_{H,t} \equiv \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ and $\pi_{N,t} \equiv \ln\left(\frac{P_{N,t}}{P_{N,t-1}}\right)$, respectively. We approximate the model around the steady state, in which producer prices do not change, i.e., $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}} = 1$ and $\Pi_N = \frac{P_{N,t}}{P_{N,t-1}} = 1$ at all times. A more detailed description of the steady state is presented in the Appendix.

3.1.1 Log-Linearization of the Optimality Conditions

We log-linearize the equilibrium conditions (4), (6)–(10), and (12) and obtain the following set of log-linear equations describing the dynamics of the multisectoral small open economy:

$$\pi_{H,t} = k_H\left(\eta\widehat{Y}_{H,t} + \rho\widehat{C}_t + (1-\upsilon)\widehat{P}_{FH,t} + \gamma\widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta\widehat{A}_{H,t}\right) + \beta E_t\pi_{H,t+1}, \quad (13)$$

$$\pi_{N,t} = k_N\left(\eta\widehat{Y}_{N,t} + \rho\widehat{C}_t - (1-\gamma)\widehat{P}_{NT,t} + \widehat{\mu}_{N,t} - \eta\widehat{A}_{N,t}\right) + \beta E_t\pi_{N,t+1}, \quad (14)$$

$$\widehat{Y}_{H,t} = -[\theta + (\theta-\omega)\upsilon]\widehat{P}_{HT,t} + \omega\gamma\widehat{P}_{NT,t} + \upsilon\widehat{C}_t + w(1-\upsilon)\widehat{ER}_t + (1-\upsilon)\widehat{C}_t^* + \widehat{g}_{H,t}, \quad (15)$$

$$\widehat{Y}_{N,t} = \widehat{C}_t - w(1-\gamma)\widehat{P}_{NT,t} + \widehat{g}_{N,t}, \quad (16)$$

$$\widehat{C}_t = \frac{1}{\rho}\widehat{ER}_t + \widehat{C}_t^*, \quad (17)$$

$$\widehat{ER}_t = \upsilon\widehat{P}_{FH,t} - \gamma\widehat{P}_{NT,t} + \gamma^*\widehat{P}_{NT,t}^*, \quad (18)$$

$$\Delta\widehat{P}_{NT,t} = \pi_{N,t} - \pi_{H,t} - (1-\upsilon)\Delta\widehat{P}_{FH,t}. \quad (19)$$

Moreover, from the price index relation (1a) we note that:

$$\widehat{P}_{HT,t} = -(1-\upsilon)\widehat{P}_{FH,t}. \quad (19a)$$

The Phillips curve relations in the two sectors are presented by equations (13) and (14), where $k_L = \frac{(1-\alpha_L\beta)(1-\alpha_L)}{\alpha_L(1+\sigma\eta)}$ is the constant that measures the response of the sectoral inflation rates to variations in real marginal costs. The characterization of real marginal costs in the open economy setting differs from that of the closed economy due to the gap between production and consumption as well as to the impact of relative prices, which reflect the distinction between domestic and consumer prices. An improvement in the terms of trade (a decrease in $\widehat{P}_{FH}$) or a positive productivity shock results in a fall in marginal costs in the traded sector. The marginal costs in the non-traded sector are independent of direct changes in the terms of trade. However, the sectoral marginal costs are linked through the relative prices of non-traded goods. This impact is opposite in sign and symmetric in magnitude. Producers' pricing decisions are forward-looking due to price stickiness. As a result, the Phillips curve takes the expectation-augmented form. Equations (15) and (16) describe the aggregate demand for domestic goods in the two sectors. We consider $\widehat{C}_t^*$ as a term that cannot be affected by dynamics in the home country. This variable is exogenous from the small open economy perspective. Relation (17) is the log-linearized optimal risk-sharing condition. It describes variations in domestic consumption depending on fluctuations in the real exchange rate and consumption abroad. Equation (18), which is derived from (12), summarizes the determinants of the real exchange rate. Again, the relative price of non-traded goods in the foreign country is treated as exogenous. This equation illustrates the implication of the multisectoral economic structure. In particular, changes in the terms of trade do not necessarily imply a corresponding adjustment of the exchange rate, due to the impact of the relative prices of non-traded goods at home and abroad. Finally, expression (19), which is in fact an identity, is obtained from the definitions of non-traded and traded goods inflation and describes the evolution of the price indexes for both sectors. The equation that characterizes traded goods inflation is presented in the next sub-section.

3.1.2 Domestic Inflation, CPI Inflation, and Some Aggregation Results

In this sub-section, we present several useful definitions and identities, which will be used in the subsequent analysis. Log-linearization of price indexes (1) and (1a) yields :

$$\widehat{P}_t = \gamma \widehat{P}_{N,t} + (1-\gamma)\widehat{P}_{T,t} \tag{20}$$

$$\widehat{P}_{T,t} = \upsilon \widehat{P}_{H,t} + (1-\upsilon)\widehat{P}_{F,t}. \tag{21}$$

Applying the definition of inflation $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \widehat{P}_t - \widehat{P}_{t-1}$, we obtain the expressions for CPI inflation and traded inflation:

$$\pi_t = \gamma\pi_{N,t} + (1-\gamma)\pi_{T,t} \tag{22}$$

$$\pi_{T,t} = \upsilon\pi_{H,t} + (1-\upsilon)\pi_{F,t}. \tag{23}$$

Moreover, the definition of the terms of trade implies that $\pi_{F,t} = \Delta\widehat{P}_{FH,t} + \pi_{H,t}$. The combination of the equations presented above results in the following relationship between CPI and domestic inflation:

$$\pi_t = \pi_t^D + (1-\gamma)(1-\upsilon)\Delta\widehat{P}_{FH,t}, \tag{24}$$

where domestic inflation equals:

$$\pi_t^D = \gamma\pi_{N,t} + (1-\gamma)\pi_{H,t}. \tag{25}$$

Total output is given by:

$$P_tY_t = P_{N,t}Y_{N,t} + P_{H,t}Y_{H,t}. \tag{26}$$

Log-linearization of equation (26) yields:

$$\widehat{Y}_t = \gamma\widehat{Y}_{N,t} + (1-\gamma)\widehat{Y}_{H,t} - (1-\gamma)(1-\upsilon)\widehat{P}_{FH,t}. \tag{26a}$$

This relation implies that in an open multi-sectoral economy, aggregate output is not only the weighted average of the sectoral outputs, but also a function of relative prices.

Moreover, the evolution of the nominal exchange rate is derived from the definition of the real exchange rate and takes the form:

$$\widehat{ER}_t - \widehat{ER}_{t-1} = \widehat{S}_t - \widehat{S}_{t-1} + \pi_t^* - \pi_t, \tag{27}$$

where $\widehat{S}_t$ is the nominal exchange rate, and π_t^* is CPI inflation for the foreign country. We assume that the monetary authority abroad is implementing an inflation-targeting policy, and thus, $\pi_t^* = 0$. Such an assumption is common in the small open economy literature (Gali and Monacelli, 2005).

4 The Monetary Policy Problem and Welfare

This section will present the formulation of the monetary policy strategy and an analysis of the competing objectives of the central bank. We will see that the model specification implies deviations of the optimal monetary policy from complete price stabilization. Specifically, we present a formal welfare analysis and derive the objective function of the central bank based on a second-order approximation of both the household's utility and the structural equilibrium conditions (13)–(19). Optimal monetary strategy involves the maximization of the quadratic social welfare function (a minimization of the loss function) subject to linear constraints. Monetary policy is able to achieve the best outcome from the welfare perspective by implementing the optimal plan. In this analysis, we focus on optimal targeting rules, which are strongly advocated by Svensson and Woodford.

4.1 The Objective Function of the Central Bank for an Open Economy with Multiple Domestic Sectors

In order to obtain the analytical expression for welfare in a purely quadratic form, we apply the linear-quadratic solution methods described in Woodford (2003) and Benigno and Woodford (2005). This approach is based on the idea presented in Sutherland (2002) to explore the dynamic characteristics of the model and thus to account for the impact of the second moments of the variables on their levels. The derivation of the objective function of the central bank is presented in the Mathematical Appendix. We show that the utility function of the representative household can be approximated by the following expression:

$$W_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{28}$$
$$\left[\begin{array}{c} \widehat{C}_t - (\overline{\mu}_N)^{-1}\gamma \widehat{Y}_{N,t} - (\overline{\mu}_H)^{-1}(1-\gamma)\widehat{Y}_{H,t} + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ -\frac{1}{2}(\overline{\mu}_N)^{-1}\gamma(1+\eta)\widehat{Y}_{N,t}^2 - \frac{1}{2}(\overline{\mu}_H)^{-1}(1-\gamma)(1+\eta)\widehat{Y}_{H,t}^2 \\ +(\overline{\mu}_N)^{-1}\gamma\eta\widehat{A}_{N,t}\widehat{Y}_{N,t} + (\overline{\mu}_H)^{-1}(1-\gamma)\eta\widehat{A}_{H,t}\widehat{Y}_{H,t} \\ -\frac{1}{2}\gamma\frac{\sigma}{\overline{\mu}_N k_N}\pi_{N,t}^2 - \frac{1}{2}(1-\gamma)\frac{\sigma}{\overline{\mu}_H k_H}\pi_{H,t}^2. + t.i.p + (\left\|\xi^3\right\|) \end{array} \right].$$

We eliminate the linear terms in the objective function by using a second-order approximation of the equilibrium structural equations (13–19). As a result, we obtain an objective function that is purely quadratic. The expression takes the following form:

$$L_{to} = U_C \overline{C} E_{to} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{29}$$
$$\left[\begin{array}{c} \frac{1}{2}W_{Y_N}(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^T)^2 + \frac{1}{2}W_{Y_H}(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^T)^2 + \frac{1}{2}W_{ER}(\widehat{ER}_t - \widehat{ER}_t^T)^2 \\ +\frac{1}{2}W_{P_{NT}}(\widehat{P}_{NT,t} - (\widehat{P}_{NT,t}^T)^2 + W_{Y_N Y_H}(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^T)(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^T) \\ +W_{ER,P_{NT}}(\widehat{ER}_t - \widehat{ER}_t^T)(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^T) + \frac{1}{2}W_{\pi_N}(\pi_{N,t})^2 + \frac{1}{2}W_{\pi_H}(\pi_{H,t})^2 \end{array} \right] + t.i.p,$$

where $\widehat{Y}_{N,t}^{T}$, $\widehat{Y}_{H,t}^{T}$, $\widehat{ER}_{t}^{T}$, and $\widehat{P}_{NT,t}^{T}$ are welfare-relevant target variables, which are functions of stochastic shocks and, in general, may not be identical to the flexible price allocations.

Equation (29) implies that the social welfare of the two-sector, small open economy is affected by deviations in the sectoral inflation rates, output gaps, and relative prices from their target values.

In fact, the objective function reflects the impact of various economic distortions on social welfare and illustrates their relative contributions to the loss. First of all, price rigidities and monopolistic distortions in both sectors, which may not be fully offset by production subsidies, result in economic inefficiencies and introduce a role for inflation and output gap stabilization. The cross-output variable $(\widehat{Y}_{N,t}-\widehat{Y}_{N,t}^{T})(\widehat{Y}_{H,t}-\widehat{Y}_{H,t}^{T})$ describes the impact of co-movement in the sectoral output gaps on social welfare. When the weight in the objective function associated with the interaction term is positive, the sectoral asymmetries might be welfare improving. When this weight is negative, a co-movement of the sectoral outputs reduces welfare losses. In general, the weights next to each of the quadratic terms are represented by complicated functions of the structural parameters of the model (details are presented in the Appendix).

Furthermore, when price rigidities are present in both sectors and domestic shocks are imperfectly correlated, price changes are not synchronized following a shock. This results in inefficient output dispersion *between* sectors and introduces a role for relative prices into the monetary policy design problem. In this case, not only do the levels of inflation in both sectors matter for welfare, but so does the deviation of the relative price from its target level. The open economy formulation brings an additional, cross-country, dimension into the problem described above. Specifically, nominal rigidities may prevent prices in both countries from adjusting efficiently after exchange rate movements. In other words, the so-called relative price channel can fail to function accurately; this may result in welfare gains from exchange rate stabilization. On the other hand, in an open economy the policymaker can manipulate the terms of trade in order to increase expected consumption and decrease the expected disutility of production, i.e., to improve welfare. Those incentives are called the terms of trade externality and were analyzed by Benigno and Benigno (2006). Therefore, the weight next to the exchange rate term in the loss function balances the stability objective determined by the economic distortions (nominal rigidities) with the incentive of creating additional volatility in excess of the fundamental shocks. The cross factor $(\widehat{ER}_{t}-\widehat{ER}_{t}^{T})(\widehat{P}_{NT,t}-\widehat{P}_{NT,t}^{T})$ represents another "international dimension" term, which appears due to the fact that the relative price of non-traded to traded goods partially drives the evolution of the real exchange rate. This term, therefore, describes the additional welfare effects that originate from the correlation between the two relative prices.

Equation (29) indicates that the loss function derived for our model specification is not identical to the one of the closed economy or to the loss function obtained under the assumption $\rho = \theta = \omega = 1$. The general welfare representation, however, embodies these two special cases, which coincide in terms of policy objectives and imply that $W_{Y_N Y_H} = 0$ and $W_{ER} = W_{P_{NT}} = W_{ER,P_{NT}} = 0$.

The presence of open economy terms is not the only implication of the exposure to external factors that can be observed in the objective function. The relative weights on the sectoral inflation rates and output gaps are not only affected by the structural asymmetries, like in the case of the closed economy, but also display the incentives that arise under openness to trade of one of the domestic sectors. Specifically, in an open economy, the weights in the objective function imply relatively higher stabilization of the non-traded sector compared to the traded sector variables. Figures 1 and 2 present the weights on inflation rates and output gaps as functions of the non-traded sector size derived for the closed and open economies, respectively. The weights are computed under the baseline parameterization, for illustration purpose the nominal price rigidities are assumed to be equal across sectors and are set to the value 0.66.

Figure 1: Sector-Specific Weights for the Closed Economy Model

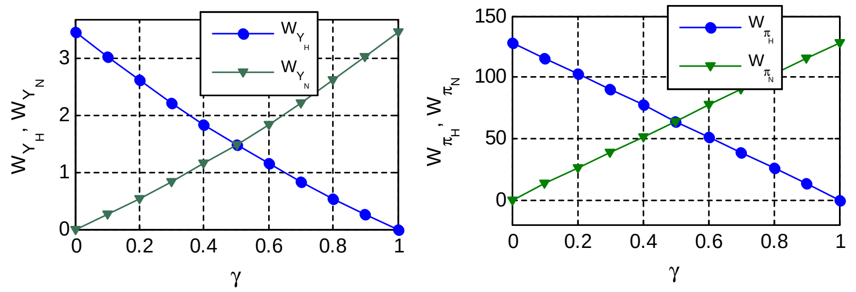
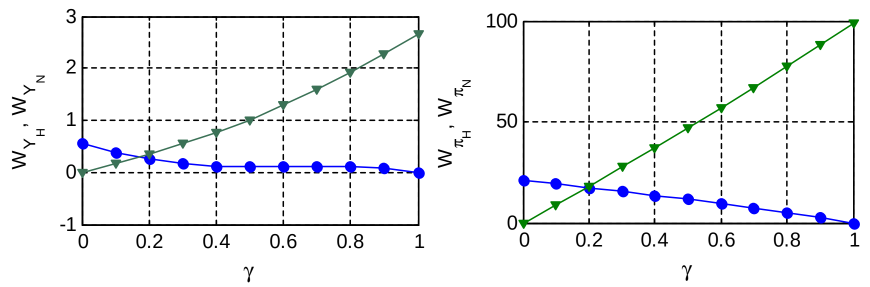


Figure 2: Sector-Specific Weights for the Open Economy Model



Two important results can be highlighted when analyzing Figures 1 and 2. First, these graphs indicate that both sectors are more volatile under the optimal policy when the economy is open (the weights are lower for all values of γ). Secondly, the decomposition of weights between sectors changes depending on whether the economy is subject to external factors. In particular, Figure 1 indicates that the weights derived for the closed economy model are symmetric and determined mainly by the parameter γ (under the equal stickiness of prices). The equal size of both sectors ($\gamma = 0.5$) implies their equal contribution to the loss function. In contrast, Figure 2 demonstrates that in the open economy, the stabilization "bias" is shifted toward the non-traded sector. In other words, the sector that is open to trade is allowed to adjust more at the optimum compared to the sector that produces goods only for internal consumption. Such a result is driven by various sensitivity of sectors to shocks and incentives to explore the terms of trade externality in a welfare-improving manner. Specifically, domestic households can benefit from volatility in the traded sector by varying the consumption and output of home goods. The possibility to substitute for foreign goods in the consumption basket enables households to "divert" a part of production abroad and thus to lower the costs of the home-goods inflation and reduce the economic inefficiencies. Therefore, the terms of trade externality influences the weights of both the relative price terms and the domestic variables in the loss function. This effect is increasing in the elasticity of substitution between home and foreign traded goods θ.

4.2 The Optimal Monetary Policy Rules

In order to obtain the optimal targeting policy rules, we minimize the objective function (29) subject to the set of constraints, which are given by:

$$\pi_{H,t} = k_H \left[\eta(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) + \frac{1}{\upsilon}(\widehat{ER}_t - \widehat{ER}^T_t) + \frac{\gamma}{\upsilon}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + u^H_t \right] + \beta E_t \pi_{H,t+1}, \tag{30}$$

$$\pi_{N,t} = k_N \left[\eta(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) + (\widehat{ER}_t - \widehat{ER}^T_t) - (1-\gamma)(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + u^N_t \right] + \beta E_t \pi_{N,t+1}, \tag{31}$$

$$(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) = \frac{l+1}{\rho\upsilon}(\widehat{ER}_t - \widehat{ER}^T_t) + \gamma \left[\frac{(l+1) + \upsilon^2(\rho\omega - 1)}{\rho\upsilon} \right] (\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \chi^H_t, \tag{32}$$

$$(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) = \frac{1}{\rho}(\widehat{ER}_t - \widehat{ER}^T_t) - \omega(1-\gamma)(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \chi^N_t, \tag{33}$$

$$(1-\upsilon)\Delta(\widehat{ER}_t - \widehat{ER}^T_t) = \upsilon(\pi_{N,t} - \pi_{H,t}) - (\upsilon + \gamma(1-\upsilon))\Delta(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + \varepsilon_t, \tag{34}$$

where $l = (\rho\theta - 1)(1-\upsilon)(1+\upsilon)$, and the terms $u^H_t, u^N_t, \chi^H_t, \chi^N_t,\ \varepsilon_t$ are functions of exogenous shocks and arise when the target levels of variables and flexible price allocations diverge. The conditions (30)–(34) are obtained by combining the log-linearized equilibrium conditions (13)–(19) and expressing the relations in terms of gap variables. We assume that the central bank can commit to the policy that maximizes welfare and consider the timeless perspective approach described in Woodford (2003). The timeless perspective optimal policy assigns the particular value to the commitment to expectations prior to period 0. The constraints on the initial conditions result in the time-invariant first-order conditions and thus optimal policy. Therefore, the time inconsistency problem is eliminated. Following such a strategy, the policymaker chooses the path for endogenous variables $\pi_{H,t}$, $\pi_{N,t}$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{ER}_t$, $\widehat{P}_{NT,t}$ subject to constraints (30)–(34) and given the initial conditions on π_{Ho}, π_{No}, $\widehat{Y}_{Ho}$, $\widehat{Y}_{No.}$ The Lagrange multipliers associated with the set of constraints are $\lambda_{1,t} - \lambda_{5,t}$ respectively. In addition before the optimization, we divided equation (30) by k_H, equation (31) by k_N, and equation (34) by υ. The first-order conditions to the problem are given by:

$$W_{\pi_H} k_H \pi_{H,t} = \lambda_{1,t} - \lambda_{1,t-1} + \lambda_{5,t} k_H, \tag{35}$$

$$W_{\pi_N} k_N \pi_{N,t} = \lambda_{2,t} - \lambda_{2,t-1} - \lambda_{5,t} k_N, \tag{36}$$

$$W_{Y_H}(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) + W_{Y_N Y_H}(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) = \lambda_{3,t} - \eta\lambda_{1,t}, \tag{37}$$

$$W_{Y_N}(\widehat{Y}_{N,t} - \widehat{Y}^T_{N,t}) + W_{Y_N Y_H}(\widehat{Y}_{H,t} - \widehat{Y}^T_{H,t}) = \lambda_{4,t} - \eta\lambda_{2,t}, \tag{38}$$

$$W_{ER}(\widehat{ER}_t - \widehat{ER}^T_t) + W_{ER,P_{NT}}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) = \tag{39}$$
$$-\frac{1}{\upsilon}\lambda_{1,t} - \lambda_{2,t} - \frac{(l+1)}{\rho\upsilon}\lambda_{3,t} - \frac{1}{\rho}\lambda_{4,t} + \frac{1-\upsilon}{\upsilon}(\lambda_{5,t} - \beta\lambda_{5,t+1})$$

$$W_{P_{NT}}(\widehat{P}_{NT,t} - \widehat{P}^T_{NT,t}) + W_{ER,P_{NT}}(\widehat{ER}_t - \widehat{ER}^T_t) = -\frac{\gamma}{\upsilon}\lambda_{1,t} + (1-\gamma)\lambda_{2,t} - \tag{40}$$
$$-\frac{\gamma(l+1+\upsilon^2(\rho\omega-1))}{\rho\upsilon}\lambda_{3,t} + \omega(1-\gamma)\lambda_{4,t} + \left(1 + \frac{(1-\upsilon)\gamma}{\upsilon}\right)(\lambda_{5,t} - \beta\lambda_{5,t+1}).$$

Combining equations (35)–(40), we can eliminate the Lagrange multipliers and express the optimal policy rule in the following general form:

$$A^0\Delta\widetilde{X}_t + A^1\Delta\widetilde{X}_{t-1} + A^2\Delta\widetilde{X}_{t+1} = 0, \tag{41}$$

where A^0, A^1, A^2 are the matrices of parameters, $\Delta\widetilde{X}_t = \widetilde{X}_t - \widetilde{X}_{t-1}$, and $\widetilde{X}_t = \widehat{X}_t - \widehat{X}^T_t$, i.e., $\widetilde{X}_t$ denotes the vector of the endogenous variables ($\pi_{H,t}$, $\pi_{N,t}$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{ER}_t$, $\widehat{P}_{NT,t}$) in deviations from their target values. Therefore, the optimal policy rule is represented by a fairly complicated expression that prescribes the response to deviations in the sectoral inflation rates and output gaps as well as to fluctuations in relative prices. The reaction function (41) includes both backward and forward-looking endogenous variables. The matrices of the parameters A, which describe the optimal magnitude of the response, depend on the optimal weights and the structural parameters of the model.

For comparison, the optimal policy rule derived with the use of the similar methodology for the one-sector, open economy model takes the general form: $A^0 \Delta \widetilde{X}_t = 0$. Therefore, the multi-sectoral model specification brings in more complex dynamics of variables under the optimal policy. Specifically, rule (41) is more persistent, i.e., it prescribes the response to the first *and* the second lag of the endogenous variables. Moreover, the rule contains forward-looking components since $A^2 \neq 0$. The characteristics of the policy rule mentioned above are determined by the persistent structure of one of the model equations (34), which describes the evolution of the sector-specific inflation rates and the two types of relative prices.

4.2.1 Policy Trade-Offs

The welfare function (29) indicates that the monetary authority is confronted with several policy objectives. In particular, the central bank has to control the sector-specific inflation rates and output gaps, as well as relative prices. In order to study the optimal plan, it is important to investigate whether the policy goals can be simultaneously attained or the central bank has to decide how to balance them appropriately. Where the objectives do not conflict with each other, the central bank can achieve the first best allocation and completely eliminate the loss. In this section, we describe the policy trade-offs that arise in a generalized model of a two-sector, small open economy.

We analyze the combination of equations (18) and (19) expressed in terms of the welfare-relevant gap variables:

$$(1-\upsilon)\Delta(\widehat{ER}_t - \widehat{ER}_t^T) = \upsilon(\pi_{N,t} - \pi_{H,t}) - (\upsilon + \gamma(1-\upsilon))\Delta(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^T) + \varepsilon_t. \quad (42)$$

The gaps depend on the target levels of the variables, which in turn are functions of the shocks and parameters and vary over time. Equation (42) indicates that it is not possible to stabilize inflation rates in each sector and to eliminate the gaps between relative prices and their target values at the same time. In fact, relative prices act as endogenous shocks that do not allow the same policy to attain zero inflation in both sectors. For example, under a productivity shock in the non-traded goods sector (Figure 4), the optimal policy implies depreciation of the nominal exchange rate. Complete stability of non-traded inflation would require an even larger increase in the exchange rate. This, however, would result in a further worsening of the terms of trade and a greater rise in home-goods inflation. A similar trade-off exists under fiscal and mark-up shocks. Moreover, the impulse-responses indicate that the magnitude of the response differs across sector-specific variables. The different sensitivity of the domestic sectors to shocks is determined not only by structural asymmetries such as sector size, elasticity of substitution, and the level of nominal rigidities, but also by the openness to trade of one of the domestic sectors. Therefore, the optimal policy cannot comply with all the sector-specific stabilization objectives simultaneously. Woodford (2003) illustrates that a corresponding trade-off also exists in the closed economy model (υ=1) if the target rate of the relative price (the natural rate) is not constant.

Furthermore, we address the question of whether complete stability of the *aggregate* variables is attainable under the given economic structure. We present the Phillips curve relations in terms of gap variables and use the definition of domestic inflation. Moreover, in this analysis we assume for simplicity that the target variables and flexible price allocations coincide and the degree of nominal rigidities is equal across sectors. We combine the constraints (30)–(33) and apply the definition of domestic inflation (25). As a result, the following relationship arises:

$$\pi_t^D = k \left[\begin{array}{c} (\eta+\rho)\left(\gamma(\widehat{Y}_{N,t} - \widehat{Y}_{N,t}^{flex}) + (1-\gamma)(\widehat{Y}_{H,t} - \widehat{Y}_{H,t}^{flex})\right) - \\ \frac{(1-\gamma)}{\upsilon} l(\widehat{ER}_t - \widehat{ER}_t^{flex}) - \frac{\gamma(1-\gamma)}{\upsilon}\widetilde{l}(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^{flex}) \end{array} \right] + \beta E_t \pi_{t+1}^D, \qquad (43)$$

where $l = (\rho\theta - 1)(1-\upsilon)(1+\upsilon)$, $\widetilde{l} = l - (\rho\omega - 1)(1-\upsilon)\upsilon$, and the flexible price allocations of the variables are functions of the exogenous shocks $\widehat{A}_{H,t}$, $\widehat{A}_{N,t}$, $\widehat{P}^*_{NT,t}$, C_t^*. Moreover, we make use of equation (26a) and provide the alternative domestic Phillips curve relation in order to analyze the impact of the aggregate output gap instead of the differentiation between the sectoral variables:

$$\pi_t^D = k \left[\begin{array}{c} (\eta+\rho)\left((\widehat{Y}_t - \widehat{Y}_t^{flex}) + (1-\gamma)(1-\upsilon)(\widehat{P}_{FH,t} - \widehat{P}_{FH,t}^{flex})\right) - \\ \frac{(1-\gamma)}{\upsilon} l(\widehat{ER}_t - \widehat{ER}_t^{flex}) - \frac{\gamma(1-\gamma)}{\upsilon}\widetilde{l}(\widehat{P}_{NT,t} - \widehat{P}_{NT,t}^{flex}) \end{array} \right] + \beta E_t \pi_{t+1}^D. \qquad (44)$$

We present two special cases of our more general analysis in order to describe the role of relative prices in generating the policy trade-offs. First, we consider a two-sector, closed-economy setting, i.e., $\upsilon = 1, \gamma > 0$. In such a situation $l = \widetilde{l} = 0$. Equations (43) and (44) illustrate that the sectoral Phillips curves reduce to the classical aggregate relation, which, at the same time, describes the dynamics for the one-sector, closed economy. Therefore, there is no conflict between inflation and output gap stabilization, and optimal monetary policy is able to implement the first best, i.e., flexible price allocation. This result has been shown by Woodford (2003).

Secondly, we assume the special case of unitary elasticity of substitution and a unitary coefficient of relative risk aversion, i.e., the balanced trade model specification as in Liu and Pappa (2005). Again, we have $l = \widetilde{l} = 0$. Thus, the exchange rate and relative prices vanish from the Phillips curve relations (43) and (44). Moreover, the assumption $\rho = \theta = \omega = 1$ implies that the exchange rate does not characterize a welfare-relevant policy objective. In this situation, the terms of trade act as an endogenous "cost-push shock," which generates tension between domestic inflation and the output gap. In fact, such a trade-off can be generated in closed economy models in the presence of mark-up shocks or adjustment costs (Benigno and Woodford, 2005; Erceg and Levin, 2006).

Finally, we consider a two-sector model under general preferences. The Phillips curve (43) illustrates that the stabilization of domestic inflation and outputs in both sectors does not involve equivalent policies due to the presence of relative prices. Moreover, equation (44) indicates that there is tension between domestic inflation and relative price (internal and external,i.e. the exchange rate) stability in addition to the trade-off between domestic inflation and the aggregate output gap variability. Therefore, unless preferences are specified in the general form, the conflict between managing domestic inflation and the relative prices ceases to exist.

The fairly complex economic structure and general model specification determine the non-trivial task facing policymakers, i.e., the search for the second-best optimal policy given that the flexible price efficient allocation of resources cannot be replicated. The optimal reaction function (41), in fact, represents such a second-best solution. A similar result is obtained in the one-sector, open-economy model analyzed by De Paoli (2006). In our case, however, the definition of the real exchange rate implies a distinction between the two types of relative prices and enables us to characterize the dynamics and impact of each variable separately. Moreover, the multiple sectors imply an additional policy challenge, i.e., the proper management of the "between-sector" terms.

5 Impulse-Response Functions

In this section we examine the impulse-responses of key macroeconomic variables to exogenous shocks. Specifically, we compare the numerical results under the optimal plan with the outcomes achieved under the basic simple rules common in the literature, such as domestic inflation targeting (DIT), consumer price index inflation targeting (CPIT), and an exchange rate peg (PEG). We consider four types of shocks, i.e., productivity, foreign, fiscal, and mark-up shocks. For the numerical exercise we calibrate the model parameters to match the moments of Canadian data (Table 2). We assume the coefficient of relative risk aversion $\rho = 3$ and the elasticity of substitution between differentiated goods $\sigma = 6$ as in Benigno and Benigno (2006). Following Rotenberg and Woodford (1997) we set $\beta = 0.99$ and $\eta = 0.47$. The elasticity of substitution between traded home and foreign goods θ is assumed to be equal to 1.5 and the parameter that measures the substitution between non-traded and traded goods ω is set to 0.5. These assumptions are common in the open economy literature. The level of price rigidities in tradable sector is set to

$\alpha_H = 0.55$ and in non-tradable sector the Calvo parameter is assumed to be somewhat higher and equal to $\alpha_N = 0.6$. The share of non-traded goods in the consumption basket γ is set to 0.5. The corresponding parameter for the foreign country $\gamma^* = 0.6$. The degree of openness $\upsilon = 0.6$, implying a 40% import share. Finally, the steady state mark-up in the traded sector $\overline{\mu}_H$ is set to the value $1/\upsilon$ as in Liu and Pappa (2005) and De Paoli (2006) in order to guarantee the optimal subsidy policy. In addition, the equal size of both domestic sectors implies that $\overline{\mu}_H = \overline{\mu}_N$. The calibration of the parameters of stochastic processes and the policy rule are based on Dib (2008) and Ortega and Rebei (2006), who performed the Bayesian estimation of multi-sectoral DSGE models of Canadian economy. The calibrated parameters are summarized in the Table 1.

Figure 3 represents the impulse-responses to a productivity shock in the traded sector, $\widehat{A}_H$. All regimes (except PEG) imply a reaction of the monetary authority that induces a depreciation of the nominal exchange rate. Such dynamics, together with a fall in the price of home goods, worsen the terms of trade and thus result in a real depreciation. The increase in the exchange rate is the largest under DIT, because in this case the central bank stabilizes inflation more aggressively. In fact, higher home-goods inflation stability is traded for some additional exchange rate volatility. CPI inflation rises under DIT and the optimal plan. Under PEG, the nominal exchange rate is stable and the effect of the productivity shock on CPI inflation is determined by the fall in inflation in the home-goods sector. Domestic output increases due to the real exchange rate depreciation. Domestic goods become relatively cheaper than foreign goods. However, the increase in output is not large enough to boost production above its target level and the total impact on the output gap is negative. The expenditure switching effect is the most pronounced under the DIT regime, which implies no control over the exchange rate and thus allows for greater real depreciation. As a result, the output response is the largest. On the contrary, under PEG and CPIT the expenditure switching effect is minimized and the output gap falls by more compared to the other regimes. The negative response of home-goods inflation under all the regimes is determined by the direct impact of the productivity shock, which lowers the marginal costs in this sector. However, the marginal costs in the non-tradable sector increase. Non-traded output increases and the relative price of non-traded to traded goods $\widehat{P}_{NT}$ falls under DIT and the optimal plan, due to nominal depreciation. As a result, non-traded inflation rises.

Figure 4 presents the impulse-response to a productivity shock in the non-traded sector, $\widehat{A}_N$. The dynamics of the variables can be described in a similar fashion. It is important to note that non-traded inflation is stabilized to a greater extent under the optimal plan compared to the alternative simple rules. The reason for such a policy reaction is that the optimal welfare function assigns the greatest weight to stabilization of non-traded inflation. At the same time, the productivity shock $\widehat{A}_N$ directly affects the price change in this sector and, hence, induces greater dynamics of this variable. In order to prevent large swings in non-traded inflation, the central bank allows greater adjustments in relative prices and output. In addition, the response of relative prices ($\widehat{P}_{NT}$ and $\widehat{ER}$) is almost two times stronger than the responses of these variables following the productivity shock $\widehat{A}_H$. Again, the reason is that instability of non-traded inflation has larger negative welfare consequences than changes in home-goods inflation. The output reaction is positive in both sectors due to the large expenditure switching effect under DIT and the optimal plan. Unlike the negative response of the output gap following the productivity shock in the home-goods sector, the $\widehat{A}_N$ shock results in an increase of output above its target level due to the more expansionary policy.

Figure 5 presents the responses of domestic variables to the innovation in foreign consumption, $\widehat{C}^*$. We can observe that the DIT regime is very similar to the optimal plan in terms of the direction and magnitude of the response. The foreign consumption shock raises domestic consumption through the risk-sharing condition. This, in turn, may induce an increase in domestic output. At the same time, the nominal and real exchange rates appreciate and the terms of trade fall. Domestic goods become relatively less competitive and demand shifts to foreign goods. The net effect on home output is negative under DIT and the optimal plan. The impact of the shock on the macro-variables is qualitatively different under the CPIT and PEG regimes. Specifically, the monetary authority stabilizes relative prices and the real appreciation is small. The expenditure switching effect is dominated by the positive impact of the shock on domestic consumption and demand. As a result, the outputs in both sectors as well as the output gap show a significant increase. Such a boost in production increases marginal costs, and inflation in both sectors rises.

Figure 6 presents the impulse-responses to a shock to foreign relative prices, $\widehat{P}^*_{NT}$. The DIT regime almost perfectly replicates the optimal response. The policy reaction following the $\widehat{P}^*_{NT}$ shock displays a sharp contrast between the responses under the CPIT and PEG regimes, whereas under the other types of shocks these two regimes induce very similar changes in economic activity. Specifically, under the CPIT regime the central bank prevents large movements in the terms of trade at the expense of additional domestic inflation volatility. The policy implies a large nominal depreciation so as to mitigate the negative impact of foreign prices on the terms of trade. The nominal depreciation under the stabilized CPI inflation results in real depreciation. This, in turn, increases domestic production and inflation in both sectors. On the contrary, the PEG regime induces a policy that is closer to the optimal plan and DIT. When foreign and home goods are substitutes, the optimal response implies a greater nominal exchange rate stabilization in order to improve the terms of trade and divert production abroad by switching to consumption of foreign goods. Such a policy is welfare improving because it enables one to take advantage of the foreign productivity shock by reducing domestic marginal costs and the inefficient output dispersion associated with price rigidities.

Figure 7 shows the impulse responses to a mark-up shock in the home-goods sector, $\widehat{\mu}_H$. The optimal policy diverges from complete domestic inflation stabilization and the other alternative simple rules. The positive shock leads to a rise in home-goods inflation, which returns to its initial level after several periods of deflation, and a temporary fall in the output gap. The extent to which the shock affects output versus inflation depends on the weight that the central bank places on output gap variability. Specifically, the optimal policy, unlike the alternative simple rules, implies a certain degree of output gap stability. Therefore, inflation is allowed to increase more and the output gap to fall less under the optimal plan. The response of the monetary authority to a mark-up shock implies fall in the nominal interest rate, depreciation of the exchange rate, an increase in the terms of trade, and a fall in the relative price of non-traded to traded goods. Outputs in both domestic sectors and consumption rise in response to a shock. The output gap, however, falls due to the fall in home-goods output below its target value.

The responses to a mark-up shock in the non-traded sector, $\widehat{\mu}_N$, are presented in figure 8. Again, the central bank has to balance conflicting policy objectives – to absorb the upward pressure on inflation in the non-traded sector by a fall in the output gap. The exchange rate appreciates and consumption and output decrease under the optimal plan. The DIT regime implies a greater economic contraction and thus the largest fall in output and consumption. CPIT and PEG represent strongly suboptimal regimes because they induce excessive stabilization of relative prices and a higher response of non-traded inflation. The comparative analysis of impulse-responses under the $\widehat{\mu}_H$ and $\widehat{\mu}_N$ shocks suggests that the optimal policy reacts more aggressively under the disturbance to a non-traded mark-up.

Figures 9 and 10 illustrate the responses to fiscal shocks in the traded and non-traded sectors, respectively. Again, the optimal policy differs significantly from the simple policy rules. The rise in government spending $\widehat{g}_H$ increases home-goods output. The central bank, which aims at domestic inflation stabilization, offsets the upward pressure on home-goods inflation by a corresponding decrease in non-traded inflation. The response induces an initial appreciation of the exchange rate, a fall in the terms of trade, and a rise in the relative price of non-traded to traded goods. As a result, consumption and non-traded output decrease. The optimal plan, on the contrary, implies an expansionary policy. The exchange rate depreciates, implying an additional stimulus to output in both domestic sectors. Such a policy prevents the initial drop in consumption. The CPI and PEG regimes imply greater stability of relative prices.

The government spending shock $\widehat{g}_N$ increases non-traded output and creates upward pressure on non-traded inflation. Therefore, unlike in the previous case, the optimal policy implies the economic contraction. The response of the central bank is the most aggressive compared to the alternative policy rules. As a result, greater non-traded inflation stability is achieved at the expense of additional volatility of inflation and output in the traded sector, as well as a larger adjustment of relative prices.

The analysis of the numerical results suggests that the type of shock and the economic structure are important determinants of the comparative performance of optimal versus simple policy rules. Specifically, the responses under the optimal policy differ the most from the simple rules under fiscal and mark-up shocks. On the contrary, the DIT regime better approximates the optimal plan under foreign and productivity shocks. In addition, the optimal and PEG regimes come closer under a foreign relative price shock. Shocks of the same type but affecting different domestic sectors may induce qualitatively distinct economic responses. This happens due to the different sensitivity of welfare-relevant economic variables to sector-specific shocks and greater stabilization of the non-traded sector under the optimal policy. In particular, the optimal policy is expansionary with respect to fiscal and mark-up shocks in the traded sector, whereas identical shocks in the non-traded sector call for an economic contraction. The DIT regime induces a more expansionary policy under a traded-sector productivity shock, whereas the policy is less active following foreign shocks. Fiscal and mark-up disturbances result in an economic contraction under DIT. Under the CPI and PEG regimes, the policy is less aggressive in response to domestic productivity shocks and it becomes more expansionary under foreign shocks.

6 Welfare Implications of the Alternative Simple Rules

The study of the optimal policy problem presented in the previous sections provides a useful theoretical foundation for the design of monetary strategy and offers a rigorous benchmark for comparing the performance of alternative monetary regimes. At the same time, the prescriptions of the optimal policy given by expression (41) might be too difficult for the general public to interpret and too difficult to put into practice. Therefore, the analysis of the alternative policy rules, which deliver reasonable welfare results and at the same time are simple and transparent, and the optimal rule, which has normative implications, should interact in a complementary way in order to provide beneficial economic conclusions. In this section we enhance the analysis of the optimal policy with a discussion of the alternative simple rules and present their comparative performance. Specifically, we use Dynare software in order to compute *optimal* simple rules (OSRs) of the form: $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + \psi \widehat{X}_t + \varepsilon_r$, where $\widehat{X}_t$ is a vector of endogenous variables, ψ is a vector of optimized parameters, and ε_r is a policy shock with standard deviation set to 0.003.We also set the value of the parameter ρ_r to 0.75. In fact, we compute the parameters of a policy rule which maximize a linear-quadratic loss function (29) subject to constraints (30-34). As a result, we are able to analyze the performance of rules with a simple structure but with optimal coefficients.

We address two important issues. First, we consider several types of alternative simple rules classified depending on the variables entering the rules and investigate the extent to which alternative monetary regimes are able to replicate the optimal solution. Secondly, we explore the implications of the alternative simple rules for macroeconomic volatility.

The welfare ranking is performed on the basis of the value of the loss, which is computed by taking the unconditional expectations of expression (29), i.e., the second-order approximation to the utility of the representative consumer, expressed as a fraction of the steady state consumption. As a result, we present the value of the loss in terms of the variances/covariances of the sector-specific inflation rates, output gaps, and relative prices:

$$V \equiv \frac{1}{2}\frac{\beta}{1-\beta} \times \left[\begin{array}{c} W_{Y_N} var(\widetilde{Y}_{N,t}) + W_{Y_H} var(\widetilde{Y}_{H,t}) + 2W_{Y_N Y_H} covar(\widetilde{Y}_{N,t}\widetilde{Y}_{H,t}) + \\ +W_{ER} var(\widetilde{ER}_t) + W_{P_{NT}} var(\widetilde{P}_{NT,t}) + 2W_{ER,P_{NT}} covar(\widetilde{ER}_t \widetilde{P}_{NT,t}) + \\ +W_{\pi_N} var(\pi_{N,t}) + W_{\pi_H} var(\pi_{H,t}) \end{array} \right]. \tag{45}$$

Table 3 reports the welfare losses associated with various types of OSRs. Specifically, we consider simple rules which include domestic variables and rules that prescribe the response to both closed and opened economy terms. In addition, we would like to evaluate the benefits of targeting sector-specific inflation rates and outputs versus aggregate variables. This issue is practically important since central banks do not usually differentiate their policy response depending on the economic sector and consider aggregate variables, due to the problem of policy implementation and a lack of information.

Table 3 indicates that the welfare losses under the OSRs that target domestic inflation are on average 15-30% larger compared to the optimal rule. The losses associated with strict CPI inflation targeting are somewhat larger than rules that completely stabilize the domestic inflation. At the same time, certain forms of flexible CPI targeting may outperform flexible DIT rules (compare rules 4,5 and 10,11). Targeting the non-tradable inflation provides better welfare results than DIT or CPI targeting (rules 13 and 4,10). In general, the DIT regime performs worse compared to the results obtained in the previous literature. In particular, in the special case of the open economy model presented in Gali and Monacelli (2005) and the framework with ad-hoc welfare objectives as in Soto (2004), the DIT regime represents or nearly replicates the first-best. In our case, the presence of mark-up and government spending shocks determines the deviation of optimal policy from the DIT. The ranking of alternative regimes suggests that strict inflation targeting (DIT or CPI) is suboptimal compared to policies that account for other objectives, namely the interest rate smoothing, the output gap, and/or the relative prices. The rules that target the sector-specific variations in outputs and inflation rates perform significantly better compared to rules targeting the aggregate variable. Thus stabilization of the appropriately weighted average of the sectoral inflation rates (rule 12) produces better results than DIT or CPI. At the same time, augmenting the rule that responds to the aggregate inflation (domestic or CPI) and total output gap with the relative price term allows one to better account for sector-specific features of the economy. For example, rule 11 indicates that flexible CPI targeting regime that includes a certain degree of the internal relative price management can achieve a welfare result that is close to the case of targeting the sector-specific inflation rates. Furthermore, across all types of rules (4 and 5, 10 and 11, 13 and 14), the internal relative prices do better job in capturing sector-specific characteristics than external relative prices (the exchange rate). Thus the inclusion of the relative price of non-traded goods in the policy rule brings higher welfare gains. The improvement in welfare coming from the response to the change in the exchange rate is higher for the CPI targeting rules because of the excess smoothness of relative prices which this regime entails.

The values of the optimized coefficients k1, k2, k3, and k4 displayed in table 3 provide information about the relative magnitude of the policy response to deviations in key macroeconomic variables. Specifically, the OSRs indicate that the policy should respond more aggressively to variations in the non-traded sector variables (output and inflation rates).

The important criterion for evaluating the performance of the simple rules is the level of macroeconomic stability which they induce. Alternative regimes may generate comparable welfare results but, at the same time, imply different volatility of the macroeconomic variables. This issue becomes particularly important prior to entering the Eurozone, when the monetary authority has to fulfill specific and sometimes conflicting stabilization objectives. Table 4 presents the standard deviations of the key variables under different OSRs relative to the standard deviations implied by the optimal policy.

Comparing the volatility under the alternative regimes we note that the rules that strictly target aggregate variables naturally perform the worst in terms of stabilization of the particular economic sectors. Thus, under the DIT, CPI, and PEG regimes, the volatility of the sector-specific variables diverges the most from the deviations implied by the optimal rule. In particular, sectoral inflation rates are 50% over (for home inflation) and about 2 times under-stabilized (for non-traded inflation) under the strict DIT regime. At the same time, the output gaps in the home-goods and non-traded sectors are 10% and 17% respectively more volatile compared to their standard deviations under the optimal policy. In all cases of strict inflation targeting (rules 1,2,6,7,12), the fulfillment of the inflation objectives comes at the expense of somewhat higher volatility of the output gap, at sector-specific and/or aggregate levels. The comparison of DIT, CPI and the rule that targets the properly weighted domestic inflation index (rule 12) indicates that under the latter, the volatility of sector-specific inflation rates is closer to the optimal values and thus non-traded inflation is less volatile. At the same time relative prices and especially traded inflation display higher volatility. Greater stability of non-traded inflation is achieved due to the different magnitude of the optimal policy response with respect to the sectoral inflation rates expressed by the values of the parameters k1 and k2. The rules that do not differentiate the response across sectoral variables but instead incorporate the reaction to changes in the relative prices (rules 5,10,11,13,14) allow the standard deviations of the sector-specific inflation rates to be brought closer to the optimal values. Such an improvement can be achieved at the expense of increased domestic and/or CPI inflation volatility as well as the standard deviations of some of the relative prices. Moreover, regimes, which display the features of an open economy, i.e., prescribes a certain degree of exchange rate management (rules 4,16,17) bring higher stability of the CPI inflation, but may imply somewhat higher variation in output and domestic and non-tradable inflation.

The results of this section demonstrate the tension between the sector-specific inflation objectives, inflation and relative price stabilization as well as the inflation-output gap policy trade-off common in the literature. We also numerically assess the welfare benefits of differentiating the policy response depending on economic sectors compared to stabilizing aggregate variables. Moreover, we show that the welfare results achieved under the "sector-specific" targeting rules can be closely replicated by a rules with an appropriate combination of aggregate variables, namely, the CPI inflation, total output gap and the internal relative price change. Responding to the relative prices may facilitate targeting the sector-specific variables and contribute to welfare improvements when the central bank does not have enough information about domestic sectors.

The exercise presented in this section has important practical implications. In particular, it could provide policymakers with a tool for analyzing the relative importance (in terms of welfare consequences) of various monetary policy objectives and facilitate the design of strategies aimed at achieving several competing goals.

7 Sensitivity analysis

In the previous sections, we analyzed the performance of various policy rules under the assumption that prices in the non-traded sector are more rigid compared to the level of nominal rigidities in the traded sector. In this section, we would like to provide a more general analysis of the impact of sectoral heterogeneity in the degree of price stickiness on the relative performance of policy rules with sector-specific and aggregate variables (inflation rates). In other words, we would like to check how sectoral asymmetries affect the optimal inflation index being stabilized. Moreover, we will compare the implications of asymmetric nominal rigidities for closed and open economies. Specifically, we compare our results with conclusions derived by Aoki (2001) who studied the optimal policy in a two-sector closed economy model where prices are fully flexible in one sector but sticky in the other. His main result implies that the central bank should target the core inflation rather than changes of a broader price index.

For the sensitivity analysis we evaluate 5 types of rules: optimal policy, DIT and CPI targeting with interest rate smoothing (rules 2 and 7), policy rule with sector-specific inflation rates (rule 12), and the rule 11 which incorporates the response to the CPI inflation and the relative price change. We construct a measure of the sectoral asymmetries in relative price rigidities $0 \leq \chi \leq 1; \chi = \frac{\alpha_i}{\alpha_i + \alpha_j}$. It measures the level of price stickiness in a sector i relative to the overall level of nominal rigidities. In the case that $\chi = 0.5$ is chosen, $\alpha_i = \alpha_j$ i.e. sectoral prices are equally sticky. This measure allows us to vary the assumed relative stickiness of prices in the two sectors between the two extremes of complete flexibility in sector i (non-tradable, $\alpha_i = 0, \chi = 0$) and complete flexibility in sector j (tradable, $\alpha_j = 0, \chi = 1$). We compute welfare losses for values of χ from 0 to 1, for each point we consider all possible combinations of α_i and α_j and aggregate the results across all options.

The results are presented on the Figures 11 and 12. We plot the welfare losses under alternative policy regimes for various degrees of relative price rigidities. We compute optimal and DIT policy for the closed economy as a special case of an open economy, i.e. we assume that the share of imports is equal to zero (degree of openness) and open economy shocks are shut off. Figures 11 and 12 indicate that implications of equal degree of nominal rigidities across sectors ($\chi = 0.5$) differ for closed and open economies. In particular, in the closed economy model where the sectoral prices are equally sticky, the DIT policy is nearly optimal. At the same time, for values $\chi < 0.5$ or $\chi > 0.5$, targeting of aggregate price index is suboptimal. The central bank should weigh the sectoral inflation rates according to their price stickiness; the sector with higher rigidities should be more stabilized at the optimum. This result corresponds to the one shown by Aoki. The distance between the DIT and the optimal policy is higher on the interval where nominal rigidities in sector j are greater than in sector i ($\chi < 0.5$). In this case, inflation in sector j is under-stabilized under the DIT. Under the assumption of asymmetric disturbances, which greater affect the sector j, welfare losses decline under the DIT regime as sector j becomes more flexible. In case of identical sector-specific shocks the welfare losses following the DIT policy would be symmetric for the intervals $\chi < 0.5$ and $\chi > 0.5$ because, in the closed economy, domestic sectors are equally sensitive to shocks.

The results obtained for the open economy model indicate that equality of sectoral price rigidities does not imply the optimality of targeting the aggregate inflation indices (CPI or domestic). At the same time, the regimes that stabilize the measures of sector-specific and aggregate inflations produce similar welfare results on the interval $0.1 \leq \chi \leq 0.4$. In particular, for $\chi = 0.1 - 0.2$ there are almost no gains from targeting sector-specific versus domestic inflation rates. This implies that the optimal policy in the open economy may prescribe the equivalent response to changes in sectoral price indices even under the diverse values of nominal rigidities (prices in the non-tradable sector are more flexible). Such a result is obtained because the non-tradable inflation has to be more stabilized under the optimal policy relative to the tradable inflation (the sensitivity to shocks differs across sectors). The gain from targeting sector-specific versus aggregate inflation rates is increasing on the interval $\chi > 0.4$, where prices in sector i (non-tradable) become stickier and the discrepancy between optimal weights assigned to sectoral inflations is increasing. For all values of relative price stickiness, the policy rule that includes CPI and relative price targeting is able to closely replicate the policy that differentiates the response across sectoral inflations. In addition, Figure 11 indicates that nominal rigidities in non-tradable sector are more costly comparing to the case when prices in the tradable sector are stickier (the point where $\chi = 0$ implies lower welfare losses compared to the point $\chi = 1$). This result indicates that greater stability of non-tradable inflation comes at the expense of higher volatility of other welfare relevant variables. The stabilization of tradable inflation generates less severe volatility trade-offs.

8 Conclusions

In this paper we analyzed the stabilization objectives of optimal monetary policy in a two-sector small open economy model obtained as a limiting case of a two-country DSGE framework. We assessed the role of sectoral heterogeneity, general preferences, and multiple relative prices in monetary policy design and welfare evaluation. The stabilization objectives derived for our model specification and represented by the loss function display the features of an open economy and reflect a multisectoral economic structure. Specifically, it is shown that social welfare is affected by deviations in inflation rates and output gaps (with sector-specific weights) as well as in relative prices from their target values. Therefore, the micro-founded welfare objective function differs from the ad hoc forms widely assumed in the applied literature. The exposure of one of the domestic sectors to the external environment not only determines the presence of open economy terms in the loss function, but also affects the decomposition of weights between domestic variables. In particular, the sector that is open to trade is allowed to adjust more at the optimum compared to the sector that produces goods for internal consumption only. Such a result implies a qualitatively different magnitude of the response to deviations in sector-specific variables compared to the closed economy setting and determines the asymmetric response of the domestic sectors to various shocks. We characterized the optimal policy by the optimal targeting rule, which is a rather complex expression.

Furthermore, we experimented with alternative simple rules and analyzed their ability to replicate the optimal solution. The numerical results suggest that the type of shock is an important determinant of the comparative performance of optimal versus simple policy rules. Specifically, the optimal responses differ the most from the simple rules under fiscal and mark-up shocks. On the contrary, the DIT regime better approximates the optimal plan under foreign and productivity shocks.

An analysis of the welfare implications of alternative simple rules suggests that strict targeting of domestic and CPI inflation does not yield the best approximations for the optimal policy, and social welfare can be improved by accounting for other policy objectives, namely, the output gap and the relative prices. We presented a ranking of alternative simple rules and evaluated the welfare benefits of targeting the core versus broader inflation indices. In addition, we showed that the simple rules which incorporate a response to the relative price changes achieve more efficient stabilization of sector-specific variables. Finally, the sensitivity analysis demonstrates that implications of equal degree of price stickiness across sectors differ for closed and open economies. Unlike the policy implemented in closed economy, the optimal strategy in the open economy may prescribe the equivalent response to changes in sector-specific price indices (and thus targeting the aggregate price index) even under the diverse values of sectoral nominal rigidities.

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9 Appendix

9.1 The Steady State

We approximate the model around the steady state, in which $\overline{A}_N = \overline{A}_H = 1$, $\overline{G}_H = \overline{G}_N = 0$, $\overline{\mu}_H \geq 1$, $\overline{\mu}_N \geq 1$. We assume that producer prices do not change in the steady state, i.e., $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}} = 1$ and $\Pi_N = \frac{P_{N,t}}{P_{N,t-1}} = 1$ at all times. The optimal risk-sharing condition implies that $ER_t = \frac{U_C(C_t^*)}{U_C(C_t)} k_o$. Under the given functional forms, we obtain the condition for the steady state: $\overline{ER} = \left(\frac{\overline{C}}{\overline{C}^*}\right)^{\rho} k_o$. By choosing $k_o = \left(\frac{\overline{C}}{\overline{C}^*}\right)^{-\rho}$ we obtain the steady state real exchange rate equal to unity, i.e., $\overline{ER} = 1$. We normalize the price indexes of traded goods at home and abroad so that $\overline{P}_H = \overline{P}_F$, as usually assumed in the literature, i.e., in the steady state the terms of trade P_{FH} are equal to unity. Moreover, from the price index equation (1a) it follows that $\overline{P}_H = \overline{P}_T$. We can write the general price index (1) as: $1 = [\gamma \overline{p}_N^{1-\omega} + (1-\gamma)\overline{p}_T^{1-\omega}]^{\frac{1}{1-\omega}}$ where $\overline{p}_N = \frac{\overline{P_N}}{\overline{P}}$, $\overline{p}_T = \frac{\overline{P_T}}{\overline{P}}$. From this relation we obtain $\overline{P}_N = \overline{P}_T = \overline{P}$. The price index equations and the fact that $\overline{ER} = 1$ imply that in the steady state prices at home and abroad are equalized. Furthermore, the price setting equations imply the following relationships in the steady state:

$$U_C(\overline{C})\frac{\overline{P_H}}{\overline{P}} = \overline{\mu}_H V_y(\overline{Y}_H), \tag{1}$$

$$U_C(\overline{C})\frac{\overline{P_N}}{\overline{P}} = \overline{\mu}_N V_y(\overline{Y}_N)\ . \tag{2}$$

From the aggregate demand equations (7) and (4) (main text) we obtain:

$$\overline{Y}_H = \left[(1-\gamma)\upsilon\overline{C} + (1-\gamma^*)\widetilde{\upsilon}^*\overline{C}^*\right], \tag{3}$$

$$\overline{Y}_N = \gamma\overline{C}. \tag{4}$$

The world aggregate resource constraint is given by: $\overline{Y} + \overline{Y}^* = \overline{C} + \overline{C}^*$. Combining this condition with (3) and (4) we obtain:

$$\frac{\overline{C}}{\overline{C}^*} = \frac{(1-\gamma^*)\widetilde{\upsilon}^*}{(1-\gamma)(1-\upsilon)}. \tag{5}$$

This relation demonstrates that even under the complete market assumption, the structural asymmetries result in a wedge between consumption in the two countries. Finally, $k_o = \left(\frac{\overline{C}}{\overline{C}^*}\right)^{-\rho} = \left(\frac{(1-\gamma^*)\widetilde{\upsilon}^*}{(1-\gamma)(1-\upsilon)}.\right)^{-\rho}$.

9.2 Second-Order Approximation to the Utility Function and Equilibrium Conditions

We apply the methodology described in Woodford (2003) and Benigno and Woodford (2005) in order to obtain the second-order approximation to the utility function of the form:

$$U_t^j = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_{s,T}(j), A_{s,T}^i) - V(y_{s,N}(j), A_{s,N}^i)] \right\}. \tag{6}$$

We assume that preferences have isoelastic functional form and we arrive at the following expression:

$$W_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \times \tag{7}$$

$$\begin{bmatrix} \widehat{C}_t - (\overline{\mu}_N)^{-1}\gamma \widehat{Y}_{N,t} - (\overline{\mu}_H)^{-1}(1-\gamma)\widehat{Y}_{H,t} + \frac{1}{2}(1-\rho)\widehat{C}_t^2 \\ -\frac{1}{2}(\overline{\mu}_N)^{-1}\gamma(1+\eta)\widehat{Y}_{N,t}^2 - \frac{1}{2}(\overline{\mu}_H)^{-1}(1-\gamma)(1+\eta)\widehat{Y}_{H,t}^2 \\ +(\overline{\mu}_N)^{-1}\gamma\eta \widehat{A}_{N,t}\widehat{Y}_{N,t} + (\overline{\mu}_H)^{-1}(1-\gamma)\eta \widehat{A}_{H,t}\widehat{Y}_{H,t} \\ -\frac{1}{2}\gamma \frac{\sigma}{\overline{\mu}_N k_N}\pi_{N,t}^2 - \frac{1}{2}(1-\gamma)\frac{\sigma}{\overline{\mu}_H k_H}\pi_{H,t}^2. + t.i.p + (\left\|\xi^3\right\|) \end{bmatrix},$$

where $t.i.p.$ denotes terms that are independent of policy and $(\left\|\xi^3\right\|)$ denotes terms that are of third order and higher. We can write (7) in a vector-matrix form as:

$$W_{to} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[z_x' x_t - \frac{1}{2} x_t' Z_x x_t - x_t' Z_\xi \xi_t - \frac{1}{2} z_{\pi_H} \pi_{H,t}^2 - \frac{1}{2} z_{\pi_N} \pi_{N,t}^2 \right] + t.i.p + (\left\|\xi^3\right\|), \tag{8}$$

where

$$x_t' \equiv \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{C}_t & \widehat{P}_{HT,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix},$$

$$\xi_t' \equiv \begin{bmatrix} \widehat{A}_{H,t} & \widehat{A}_{N,t} & \widehat{\mu}_{H,t} & \widehat{\mu}_{N,t} & \widehat{g}_{H,t} & \widehat{g}_{N,t} & \widehat{C}_t^* & \widehat{P}_{NT,t}^* \end{bmatrix},$$

$$z_x' \equiv \begin{bmatrix} (-(\overline{\mu}_H)^{-1}(1-\gamma)) & (-(\overline{\mu}_N)^{-1}\gamma) & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$Z_x \equiv \begin{bmatrix} (\overline{\mu}_H)^{-1}(1-\gamma)(1+\eta) & 0 & 0 & 0 & 0 & 0 \\ 0 & (\overline{\mu}_N)^{-1}\gamma(1+\eta) & 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Z_\xi \equiv \begin{bmatrix} -(\overline{\mu}_H)^{-1}(1-\gamma)\eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\overline{\mu}_N)^{-1}\gamma\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$z_{\pi_H} \equiv (1-\gamma)\frac{\sigma}{\overline{\mu}_H k_H} \qquad z_{\pi_N} \equiv \gamma\frac{\sigma}{\overline{\mu}_N k_N},$$

where $k_L = \frac{(1-\alpha_L\beta)(1-\alpha_L)}{\alpha_L(1+\sigma\eta)}$, for $L = H, N$.

We now derive the second-order approximation to the structural equilibrium conditions. Following Benigno and Woodford (2005), we approximate the optimal price-setting equation (expression (10) in the main text) for both domestic sectors as well as the law of motion for the sectoral price indices (11). We combine the corresponding expressions and, after integrating forward, obtain the following relations:

$$V_0^H = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\times \tag{9}$$
$$\left\{ \begin{array}{l} \left[\eta\widehat{Y}_{H,t} + \rho\widehat{C}_t - \widehat{P}_{HT,t} + \gamma\widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta\widehat{A}_{H,t}\right] + \frac{1}{2}\gamma(1-\omega)(1-\gamma)\widehat{P}^2_{NT,t} \\ +\frac{1}{2}\left[\begin{array}{c} \eta\widehat{Y}_{H,t} + \rho\widehat{C}_t - \widehat{P}_{HT,t} + \\ \gamma\widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta\widehat{A}_{H,t} \end{array}\right] \times \left[\begin{array}{c} (2+\eta)\widehat{Y}_{H,t} - \rho\widehat{C}_t + \widehat{P}_{HT,t} - \\ \gamma\widehat{P}_{NT,t} + \widehat{\mu}_{H,t} - \eta\widehat{A}_{H,t} \end{array}\right] \\ +\frac{1}{2}\frac{\sigma(1+\eta)}{k_H}\pi^2_{H,t} + s.o.t.i.p. + (\left\|\xi^3\right\|) \end{array} \right\},$$

$$V_0^N = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\times \tag{10}$$
$$\left\{ \begin{array}{l} \left[\eta\widehat{Y}_{N,t} + \rho\widehat{C}_t - (1-\gamma)\widehat{P}_{NT,t} + \widehat{\mu}_{N,t} - \eta\widehat{A}_{N,t}\right] + +\frac{1}{2}\gamma(1-\omega)(1-\gamma)\widehat{P}^2_{NT,t} \\ +\frac{1}{2}\left[\begin{array}{c} \eta\widehat{Y}_{N,t} + \rho\widehat{C}_t - (1-\gamma)\widehat{P}_{NT,t} + \\ \widehat{\mu}_{N,t} - \eta\widehat{A}_{N,t} \end{array}\right] \times \left[\begin{array}{c} (2+\eta)\widehat{Y}_{N,t} - \rho\widehat{C}_t + (1-\gamma)\widehat{P}_{NT,t} + \\ +\widehat{\mu}_{N,t} - \eta\widehat{A}_{N,t} \end{array}\right] \\ +\frac{1}{2}\frac{\sigma(1+\eta)}{k_N}\pi^2_{N,t} + s.o.t.i.p. + (\left\|\xi^3\right\|) \end{array} \right\},$$

where $s.o.t.i.p.$ denotes second-order terms independent of policy. We can present equations (9) and (10) in a vector-matrix form as :

$$V_0^H = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[a_x^{'}x_t + a_\xi^{'}\xi_t + \frac{1}{2}x_t^{'}A_x x_t + x_t^{'}A_\xi\xi_t + \frac{1}{2}a_{\pi_H}\pi^2_{H,t}\right] + s.o.t.i.p. + (\left\|\xi^3\right\|), \tag{11}$$

$$V_0^N = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[b_x^{'} x_t + b_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} B_x x_t + x_t^{'} B_\xi \xi_t + \frac{1}{2} b_{\pi_N} \pi_{N,t}^2 \right] + s.o.t.i.p. + (\left\|\xi^3\right\|), \tag{12}$$

where

$$a_x^{'} \equiv \begin{bmatrix} \eta & 0 & \rho & -1 & \gamma & 0 \end{bmatrix},$$

$$a_\xi^{'} \equiv \begin{bmatrix} -\eta & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$A_x \equiv \begin{bmatrix} \eta(2+\eta) & 0 & \rho & -1 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \rho & 0 & -\rho^2 & \rho & \rho\gamma & 0 \\ -1 & 0 & \rho & -1 & \gamma & 0 \\ \gamma & 0 & \rho\gamma & \gamma & -\gamma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_\xi \equiv \begin{bmatrix} -\eta(1+\eta) & 0 & (1+\eta) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$a_{\pi_H} \equiv \frac{\sigma(1+\eta)}{k_H}.$$

$$b_x^{'} \equiv \begin{bmatrix} 0 & \eta & \rho & 0 & -(1-\gamma) & 0 \end{bmatrix},$$

$$b_\xi^{'} \equiv \begin{bmatrix} 0 & -\eta & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$B_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta(2+\eta) & \rho & 0 & -(1-\gamma) & 0 \\ 0 & \rho & -\rho^2 & 0 & \rho(1-\gamma) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(1-\gamma) & \rho(1-\gamma) & 0 & (1-\gamma)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$
B_{\xi} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\eta(1+\eta) & 0 & (1+\eta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
$$

$$
b_{\pi_N} \equiv \frac{\sigma(1+\eta)}{k_N}.
$$

The traded-goods demand equation is of the form:

$$
\begin{aligned}
Y_H = & \left(\frac{P_T}{P}\right)^{-\omega}\left(\frac{P_H}{P_T}\right)^{-\theta} \times \\
& \left\{ \upsilon(1-\gamma)C + \left(\frac{1}{ER}\right)^{-\omega}\left[\left(\frac{1}{\upsilon(P_{FH})^{\theta-1}+(1-\upsilon)}\right)\right]^{\frac{\theta-\omega}{1-\theta}}(1-\gamma^*)\widetilde{\upsilon}^* C^* \right\}.
\end{aligned} \tag{13}
$$

We take the second-order expansion of (13) and obtain the following relation:

$$
\begin{aligned}
\widehat{Y}_{H,t} \; = \; & -\left[\theta+(\theta-\omega)\upsilon\right]\widehat{P}_{HT,t} + \omega\gamma\widehat{P}_{NT,t} + \upsilon\widehat{C}_t + \omega(1-\upsilon)\widehat{ER}_t + (1-\upsilon)\widehat{C}_t^* + \\
& + \widehat{g}_{H,t} + \frac{1}{2}\upsilon(1-\upsilon)\widehat{C}_t^2 + \frac{1}{2}\omega^2\upsilon(1-\upsilon)\widehat{ER}_t^2 + \frac{1}{2}\omega(1-\omega)\gamma(1-\gamma)\widehat{P_{NT,t}}^2 - \\
& - \frac{1}{2}\frac{\upsilon}{(1-\upsilon)}\left[(1-\theta)(\theta-\omega)-(\theta-\omega)^2\upsilon^2\right]\widehat{P}_{HT,t}^2 - (\theta-\omega)\omega\upsilon^2\widehat{ER}_t\widehat{P}_{HT,t} - \\
& - \omega\upsilon(1-\upsilon)\widehat{ER}_t\widehat{C}_t + (\theta-\omega)\upsilon^2\widehat{C}_t\widehat{P}_{HT,t} + \omega\upsilon(1-\upsilon)\widehat{ER}_t\widehat{C}_t^* - (\theta-\omega)\upsilon^2\widehat{P}_{HT,t}\widehat{C}_t^* - \\
& - \upsilon(1-\upsilon)\widehat{C}_t\widehat{C}_t^* - \omega\gamma\widehat{P}_{NT,t}\widehat{g}_{H,t} + [\theta+\upsilon(\theta-\omega)]\widehat{P}_{HT,t}\widehat{g}_{H,t} - \upsilon\widehat{C}_t\widehat{g}_{H,t} - \\
& - \omega(1-\upsilon)\widehat{ER}_t\widehat{g}_{H,t} + s.o.t.i.p. + (\left\|\xi^3\right\|).
\end{aligned} \tag{14}
$$

In a vector-matrix form the expression above takes the following form:

$$
\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[c_x^{'}x_t + c_{\xi}^{'}\xi_t + \frac{1}{2}x_t^{'}C_x x_t + x_t^{'}C_{\xi}\xi_t\right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0, \tag{15}
$$

where

$$
c_x^{'} \equiv \begin{bmatrix} -1 & 0 & \upsilon & -\left[\theta+(\theta-\omega)\upsilon\right] & \omega\gamma & \omega(1-\upsilon) \end{bmatrix},
$$

$$
c_{\xi}^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & (1-\upsilon) & 0 \end{bmatrix}.
$$

$$C_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \upsilon(1-\upsilon) & (\theta-\omega)\upsilon^2 & 0 & -\omega\upsilon(1-\upsilon) \\ 0 & 0 & (\theta-\omega)\upsilon^2 & \frac{\upsilon}{(1-\upsilon)}\begin{bmatrix} (1-\theta)(\theta-\omega)- \\ (\theta-\omega)^2\upsilon^2 \end{bmatrix} & 0 & -(\theta-\omega)\omega\upsilon^2 \\ 0 & 0 & 0 & 0 & \omega(1-\omega)\gamma(1-\gamma) & 0 \\ 0 & 0 & -\omega\upsilon(1-\upsilon) & -(\theta-\omega)\omega\upsilon^2 & 0 & \omega^2\upsilon(1-\upsilon) \end{bmatrix},$$

$$C_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\upsilon & 0 & -\upsilon(1-\upsilon) & 0 \\ 0 & 0 & 0 & 0 & [\theta+\upsilon(\theta-\omega)] & 0 & -(\theta-\omega)\upsilon^2 & 0 \\ 0 & 0 & 0 & 0 & -\omega\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega(1-\upsilon) & 0 & \omega\upsilon(1-\upsilon) & 0 \end{bmatrix}.$$

Similarly, the demand equation for non-traded goods takes the following form:

$$Y_N = \left(\frac{P_N}{P}\right)^{-\omega} \gamma C. \tag{16}$$

The second-order approximation of this equation yields the following expressions:

$$\begin{aligned} \widehat{Y}_{N,t} = \widehat{C}_t - w(1-\gamma)\widehat{P}_{NT,t} + \widehat{g}_{N,t} + \frac{1}{2}(1-\gamma)\gamma\omega(1-\omega)\widehat{P}_{NT,t}^2 - \\ \widehat{C}_t\widehat{g}_{N,t} + \omega(1-\gamma)\widehat{P}_{NT,t}\widehat{g}_{N,t} + (\left\|\xi^3\right\|), \end{aligned} \tag{17}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[d_x^{'} x_t + d_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} D_x x_t + x_t^{'} D_\xi \xi_t \right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0. \tag{18}$$

$$d_x^{'} \equiv \begin{bmatrix} 0 & -1 & 1 & 0 & -w(1-\gamma) & 0 \end{bmatrix},$$

$$d_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\gamma)\gamma\omega(1-\omega) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega(1-\gamma) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

The second-order approximation of the risk-sharing equation (9) in the main text takes the form:

$$\widehat{C}_t = \frac{1}{\rho}\widehat{ER}_t + \widehat{C}_t^*. \tag{19}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[e_x^{'} x_t + e_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} E_x x_t + x_t^{'} E_\xi \xi_t \right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0. \tag{20}$$

$$e_x^{'} \equiv \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & \frac{1}{\rho} \end{bmatrix},$$

$$e_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

$$E_x = 0; \qquad E_\xi = 0.$$

Finally, the real exchange rate equation (12) approximated up to the second order takes the form:

$$\begin{aligned} \upsilon \widehat{P}_{HT,t} &= -(1-\upsilon)\widehat{ER}_t - \gamma(1-\upsilon)\widehat{P}_{NT,t} + \gamma^*(1-\upsilon)\widehat{P}^*_{NT,t} - \frac{1}{2}\frac{(1-\upsilon)}{\upsilon}(1-\theta)\widehat{ER}_t^2 - \\ &\quad -\frac{1}{2}\gamma(1-\upsilon)\left[\frac{\gamma(1-\theta)}{\upsilon} + (1-\omega)(1-\gamma)\right]\widehat{P}^2_{NT,t} - \frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta)\widehat{ER}_t\widehat{P}_{NT,t} + \\ &\quad + \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma^*\widehat{ER}_t\widehat{P}^*_{NT,t} + \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma\gamma^*\widehat{P}_{NT,t}\widehat{P}^*_{NT,t} + s.o.t.i.p. + (\left\|\xi^3\right\|). \end{aligned} \tag{21}$$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[f_x^{'} x_t + f_\xi^{'} \xi_t + \frac{1}{2} x_t^{'} F_x x_t + x_t^{'} F_\xi \xi_t \right] + s.o.t.i.p. + (\left\|\xi^3\right\|) = 0. \tag{22}$$

$$f_x^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & -\upsilon & -\gamma(1-\upsilon) & -(1-\upsilon) \end{bmatrix},$$

$$f_\xi^{'} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma^*(1-\upsilon) \end{bmatrix},$$

$$F_x \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma(1-\upsilon)\left[\frac{\gamma(1-\theta)}{\upsilon} + (1-\omega)(1-\gamma)\right] & -\frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta) \\ 0 & 0 & 0 & 0 & -\frac{\gamma(1-\upsilon)}{\upsilon}(1-\theta) & -\frac{(1-\upsilon)}{\upsilon}(1-\theta) \end{bmatrix},$$

$$F_\xi \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma\gamma^* \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1-\upsilon)}{\upsilon}(1-\theta)\gamma^* \end{bmatrix}.$$

We combine constraints (11), (12), (15), (18), (20), and (22) in order to get rid of the linear terms in the objective function (8). We collect vectors that contain the linear components of the above constraints and derive the vector λ, such that:

$$\begin{bmatrix} a_x & b_x & c_x & d_x & e_x & f_x \end{bmatrix} \times \lambda = z_x.$$

We solve the system of linear equations using the symbolic Matlab toolbox and derive values $\lambda_1 - \lambda_6$ associated with each of the constraints. After the linear terms cancel, we obtain the following expression for the loss function:

$$L_{to} = U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}x_t^{'}\widetilde{Z}_x x_t + x_t^{'}\widetilde{Z}_\xi\xi_t + \frac{1}{2}\widetilde{Z}_{\pi_H}\pi_{H,t}^2 + \frac{1}{2}\widetilde{Z}_{\pi_N}\pi_{N,t}^2\right] + K_0 + t.i.p + (\left\|\xi^3\right\|), \tag{23}$$

where

$$\widetilde{Z}_x = Z_x + \lambda_1 A_x + \lambda_2 B_x + \lambda_3 C_x + \lambda_4 D_x + \lambda_5 E_x + \lambda_6 F_x,$$

$$\widetilde{Z}_\xi = Z_\xi + \lambda_1 A_\xi + \lambda_2 B_\xi + \lambda_3 C_\xi + \lambda_4 D_\xi + \lambda_5 E_\xi + \lambda_6 F_\xi,$$

$$\widetilde{Z}_{\pi_H} = z_{\pi_H} + \lambda_1 a_{\pi_H},$$

$$\widetilde{Z}_{\pi_N} = z_{\pi_N} + \lambda_2 b_{\pi_N},$$

$$K_0 \equiv U_C\overline{C}\left[\lambda_1 V_0^H + \lambda_2 V_0^N\right].$$

Vectors $\widetilde{Z}_x$, $\widetilde{Z}_{\pi_H}, \widetilde{Z}_{\pi_N}$ represent the weights next to the endogenous variables in the objective function.

Furthermore, we would like to present the loss function (23) in terms of the variables $\widehat{Y}_{N,t}$, $\widehat{Y}_{H,t}$, $\widehat{ER}_t$, $\widehat{P}_{NT,t}$. Thus, we map the vector of all endogenous variables $x_t^{'} \equiv \begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{C}_t & \widehat{P}_{HT,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix}$ into the variables of interest with the use of matrices Q and Q_ξ such that:

$$x_t = Q\begin{bmatrix} \widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{P}_{NT,t} & \widehat{ER}_t \end{bmatrix}^{'} + Q_\xi\xi_t, \tag{24}$$

and

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (1-\gamma) & \gamma & \frac{-\gamma(1-\gamma)(\widetilde{l}+1-\upsilon)}{\rho\upsilon} & \frac{-(1-\gamma)(l+1-\upsilon)}{\rho\upsilon} \\ 0 & 0 & \frac{-\gamma(1-\upsilon)}{\upsilon} & \frac{-(1-\upsilon)}{\upsilon} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_\xi=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(1-\gamma) & -\gamma & 0 & \frac{\gamma^*(1-\gamma)(\widetilde{l}+1-\upsilon)}{\rho\upsilon} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\gamma^*(1-\upsilon)}{\upsilon} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $l=(\rho\theta-1)(1-\upsilon)(1+\upsilon)$ and $\widetilde{l}=l-(\rho\omega-1)(1-\upsilon)\upsilon$. Therefore, the loss function (23) can be expressed as follows:

$$L_{to}=U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}X_t'W_xX_t+X_t'W_\xi\xi_t+\frac{1}{2}W_{\pi_H}\pi_{H,t}^2+\frac{1}{2}W_{\pi_N}\pi_{N,t}^2\right]+K_0+t.i.p+(\left\|\xi^3\right\|), \tag{25}$$

where $X_t'=\begin{bmatrix}\widehat{Y}_{H,t} & \widehat{Y}_{N,t} & \widehat{P}_{NT,t} & \widehat{ER}_t\end{bmatrix}$, $W_x=Q'\widetilde{Z}_xQ$, $W_\xi=Q'\widetilde{Z}_xQ_\xi+Q'\widetilde{Z}_\xi$, $W_{\pi_H}=\widetilde{Z}_{\pi_H}$, $W_{\pi_N}=\widetilde{Z}_{\pi_N}$.

Finally, we present the variables in the objective function in terms of the deviations from their target values. Thus, we denote the gap as $\widetilde{X}_t=(X_t-X_t^T)$. The target values are functions of the exogenous shocks and take the following general form: $X_t^T=\left(-\frac{W_\xi}{W_x}\xi_t\right)$. As a result, we are able to present the objective function in the following quadratic form:

$$\begin{aligned} L_{to} &= U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}(X_t-X_t^T)'W_x(X_t-X_t^T)+\frac{1}{2}W_{\pi_H}\pi_{H,t}^2+\frac{1}{2}W_{\pi_N}\pi_{N,t}^2\right]+ \\ &\quad +K_0+t.i.p+(\left\|\xi^3\right\|). \end{aligned} \tag{26}$$

Expression (26) corresponds to formula (29) in the main text.

9.3 Tables and Figures

Table 1. Calibration of the parameters

Parameters	Definition	Value
Structural parameters		
β	discount factor	0.99
ρ	inverse of intertemporal elasticity of substitution	3
σ	intermediate goods elasticity of substitution	6
θ	elasticity of substitution between home and foreign goods	1.5
ω	elasticity of substitution between tradable and non-tradable goods	0.5
η	inverse of elasticity of goods production	0.47
γ	share of non-tradables	0.5
υ	degree of openness	0.6
γ^*	share of non-tradables in foreign country	0.6
α_H	Calvo price parameter, tradable sector	0.55
α_N	Calvo price parameter, non-tradable sector	0.6
Parameters of stochastic shocks		
ρ_{Ah}	technology autoregressive coefficient, tradable sector	0.8
σ_{Ah}	technology standard deviations, tradable sector	0.024
ρ_{An}	technology autoregressive coefficient, non-tradable sector	0.9
σ_{An}	technology standard deviations, non-tradable sector	0.008
ρ_{Gh}	autoregressive coefficient government spending shock, tradable sector	0.7
σ_{Gh}	standard deviations of government spending shock, tradable sector	0.01
ρ_{Gn}	autoregressive coefficient government spending shock, non-tradable sector	0.8
σ_{Gn}	standard deviations of government spending shock, non-tradable sector	0.008
$\rho_{\mu h}$	autoregressive coefficient mark-up shock, tradable sector	0.7
$\sigma_{\mu h}$	standard deviations of mark-up shock, tradable sector	0.02
$\rho_{\mu n}$	autoregressive coefficient mark-up shock, non-tradable sector	0.8
$\sigma_{\mu n}$	standard deviations of mark-up shock, non-tradable sector	0.014
ρ_{C^*}	autoregressive coefficient of foreign consumption shock	0.9
σ_{C^*}	standard deviations of foreign consumption shock	0.007
ρ_{pnt^*}	autoregressive coefficient of foreign relative price shock	0.7
σ_{pnt^*}	standard deviations of foreign relative price shock	0.012
Parameters of calibrated monetary policy rule		
ρ_r	smoothing coefficient	0.75
ψ_π	inflation coefficient	0.49
ψ_Y	output gap coefficient	0.038
ε_r	standard deviations of monetary policy shock	0.003

Table 2. Matching the moments: Data and the baseline model
(Canadian data, HP-filtered series, sample 1981Q1-2007Q4)

Series, std. in %	Data	Model
Total output	1.45	1.33
Output, tradable sector	3.14	3.77
Output, non-tradable sector	1.22	1.47
CPI inflation	0.47	0.48
Inflation, tradable sector	0.73	0.73
Inflation,non-tradable sector	0.44	0.40
Real exchange rate	3.48	3.60
Nominal interest rate	0.36	0.41

Definitions of used data series (data source - Statistics Canada):

- Real exchange rate: computed as nominal CDN$-US$ exchange rate deflated by Canadian and US CPI data.
- Nominal interest rate: Canadian 3-month T-bill interest rate.
- CPI inflation rate: the percentage change in the consumer price index.
- CPI inflation, tradable sector: the percentage change in the consumer price index for goods.
- CPI inflation, non-tradable sector: the percentage change in the consumer price index for services.
- Total output: GDP at 1997 constant dollars, s.a.
- Output, tradable sector: commodities and manufactured goods 1997 constant dollars, s.a.
- Output, non-tradable sector: services 1997 constant dollars, s.a.: utilities, construction, wholesale and retail trade, transportation and warehousing, information and cultural industries, finance and insurance, real estate and renting and leasing and management of companies and enterprises, professional scientific and technical services, administrative and support, waste management and remediation services, educational services, health care and social assistance, arts entertainment and recreation, accommodation and food services, other services, public administration.

Table 3. Welfare Ranking of Optimal Simple Rules

Policy Rule	Optimized coefficients				Loss to optimal
	k1	k2	k3	k4	$\frac{V^{OSR}}{V^{OPT}}$
Domestic Inflation Stabilization					
1.$\widehat{r} = 0.75\widehat{r}_{-1} + \infty\pi^D$	-	-	-	-	1.284
2. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D$	1.644	-	-	-	1.184
Flexible Domestic Inflation Targeting					
3. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y}$	1.68	0.017	-	-	1.18
4. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y} + k_3\Delta\widetilde{ER}$	1.81	0.018	0.056	-	1.178
5. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y} + k_3\Delta\widetilde{P}_{NT}$	1.656	0.018	0.288	-	1.137
6. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y}_H + k_3\widetilde{Y}_N$	3.44	0.044	0.601		1.144
CPI Inflation Stabilization					
7. $\widehat{r} = 0.75\widehat{r}_{-1} + \infty\pi^{CPI}$	-	-	-	-	1.294
8. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI}$	1.606	-	-	-	1.198
Flexible CPI Inflation Targeting					
9. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y}$	1.664	0.021	-	-	1.191
10. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y} + k_3\Delta\widetilde{ER}$	1.771	0.0178	-0.258	-	1.148
11. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y} + k_3\Delta\widetilde{P}_{NT}$	5.317	0.07	1.659	-	1.060
Sector-specific inflation targeting:					
12. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N$	1.345	5.721	-	-	1.055
13. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_N + k_2\Delta\widetilde{ER}$	3.935	0.183	-	-	1.123
14. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_N + k_2\Delta\widetilde{P}_{NT}$	4.789	-0.685	-	-	1.072
15. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\widetilde{Y}$	1.532	6.858	0.11	-	1.047
16. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\Delta\widetilde{ER}$	1.918	8.418	0.264	-	1.075
17. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\widetilde{Y} + k_4\Delta\widetilde{ER}$	2.292	10.586	0.174	0.33	1.04
18. PEG	-	-	-	-	3.09

Table 4. Macroeconomic Volatility under Alternative Policy Rules
(standard deviation in % under various OSRs relative to the optimal rule, $\frac{st.dev^{OSR}}{st.dev^{OPT}}$)

Policy Rule	Variables									
	π_H	π_N	π^D	π^{CPI}	$\widetilde{Y}_H$	$\widetilde{Y}_N$	$\widetilde{Y}$	$\widetilde{P}_{NT}$	$\widetilde{ER}$	$\widehat{r}$
Optimal, std %	0.66	0.17	0.28	0.52	4.25	1.57	1.6	3.36	3.96	0.65
Domestic Inflation Stabilization										
1.$\widehat{r} = 0.75\widehat{r}_{-1} + \infty\pi^D$	0.53	2.06	0	1.08	1.10	1.17	1.09	0.94	1.04	1.02
2. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D$	0.70	1.82	0.53	0.94	1.08	1.01	1.07	0.95	1.01	0.74
Flexible Domestic Inflation Targeting										
3. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y}$	0.70	1.88	0.54	0.94	1.06	1.02	1.05	0.95	1.01	0.74
4. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y} + k_3\Delta\widetilde{ER}$	0.70	1.90	0.56	0.86	1.06	1.03	1.06	0.94	0.99	0.69
5. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y} + k_3\Delta\widetilde{P}_{NT}$	0.83	1.53	0.75	1.23	1.07	0.96	1.05	1.01	1.05	0.91
6. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^D + k_2\widetilde{Y}_H + k_3\widetilde{Y}_N$	0.77	1.71	0.64	0.94	1.06	0.94	1.06	0.96	1.01	0.75
CPI Inflation Stabilization										
6. $\widehat{r} = 0.75\widehat{r}_{-1} + \infty\pi^{CPI}$	0.86	2	1	0	1.08	1.17	1.11	0.88	0.86	1.08
7. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI}$	0.86	1.82	0.93	0.44	1.07	1.01	1.09	0.92	0.90	0.71
Flexible CPI Inflation Targeting										
9. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y}$	0.86	1.82	0.93	0.46	1.06	1.02	1.07	0.93	0.91	0.72
10. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y} + k_3\Delta\widetilde{ER}$	0.82	1.65	0.75	0.71	1.06	1	1.06	0.95	1	0.85
11. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi^{CPI} + k_2\widetilde{Y} + k_3\Delta\widetilde{P}_{NT}$	1.03	0.94	1.07	0.83	1.07	0.88	1.07	0.99	0.99	0.65
Sector-specific inflation targeting:										
12. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N$	0.97	0.88	0.89	1.02	1.09	0.86	1.08	1.01	1.02	0.60
13. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_N + k_2\Delta\widetilde{ER}$	1.23	0.47	1.43	1.10	1.09	0.84	1.10	1.09	0.98	0.66
14. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_N + k_2\Delta\widetilde{P}_{NT}$	1.07	0.76	1.14	0.79	1.08	0.89	1.09	0.99	0.97	0.69
15. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\widetilde{Y}$	0.98	0.88	0.93	1.02	1.07	0.89	1.06	1.01	1.02	0.6
16. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\Delta\widetilde{ER}$	0.98	0.88	0.93	0.90	1.09	0.88	1.09	1.01	0.99	0.62
17. $\widehat{r} = 0.75\widehat{r}_{-1} + k_1\pi_H + k_2\pi_N + k_3\widetilde{Y} + k_4\Delta\widetilde{ER}$	0.98	0.94	0.93	0.90	1.07	0.90	1.07	1	0.99	0.63
18. PEG	1.5	4.52	2.86	1.38	1.10	1.56	1.17	0.83	0.77	0.91

Figure 3: Impulse Response to a Productivity Shock in the Traded Sector

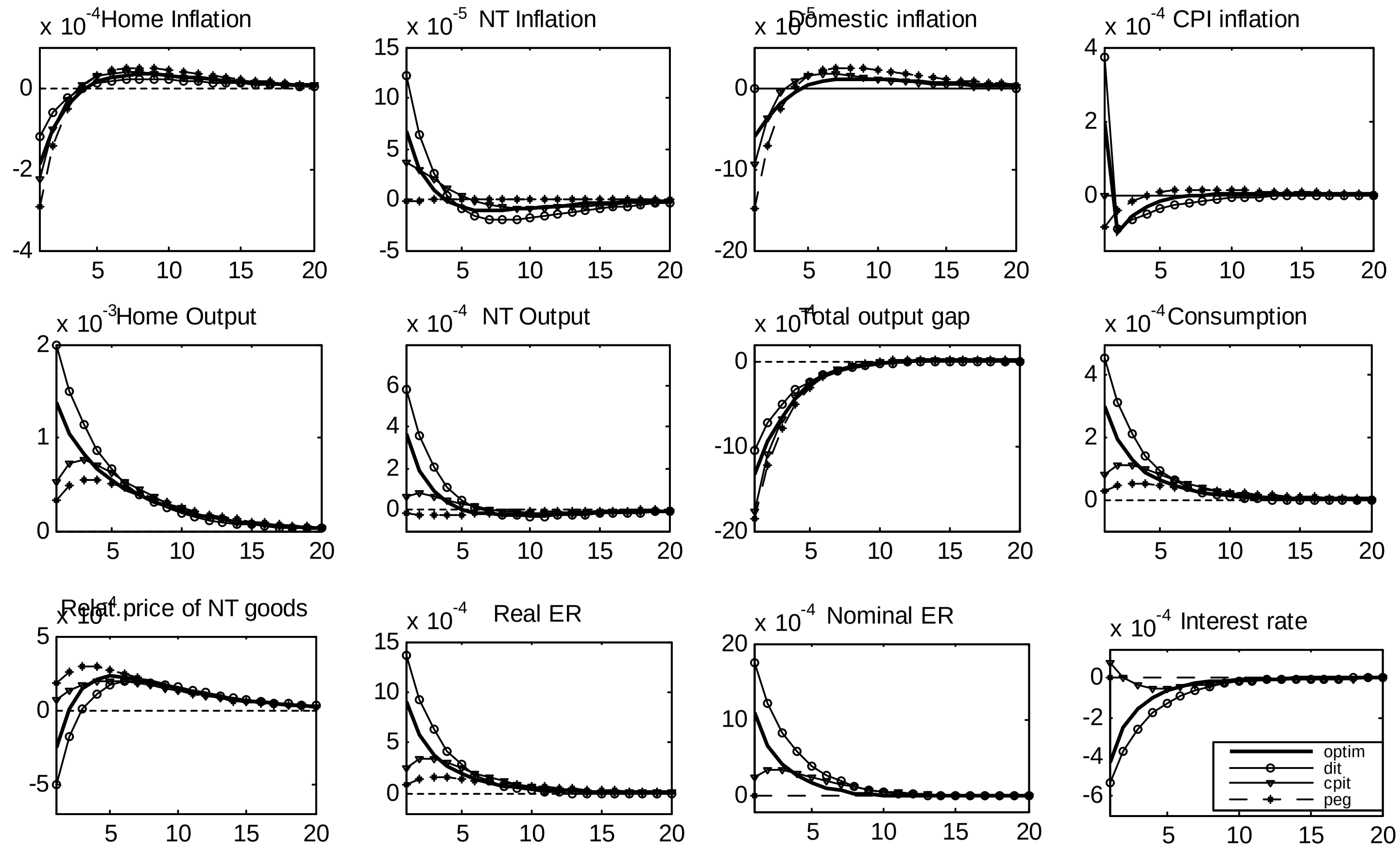

Figure 4: Impulse Response to a Productivity Shock in the Non-Traded Sector

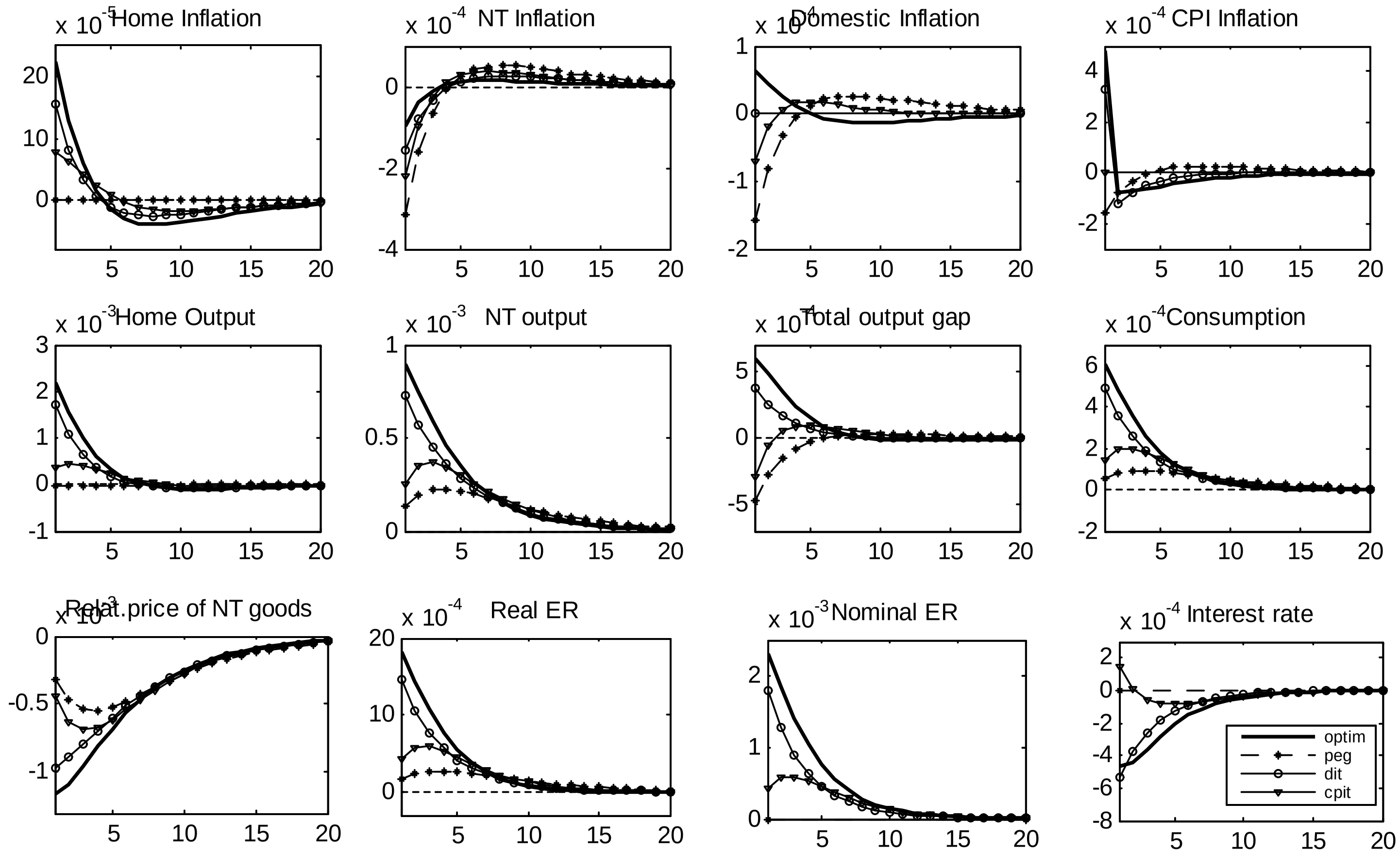

Figure 5: Impulse Response to a Foreign Consumption Shock

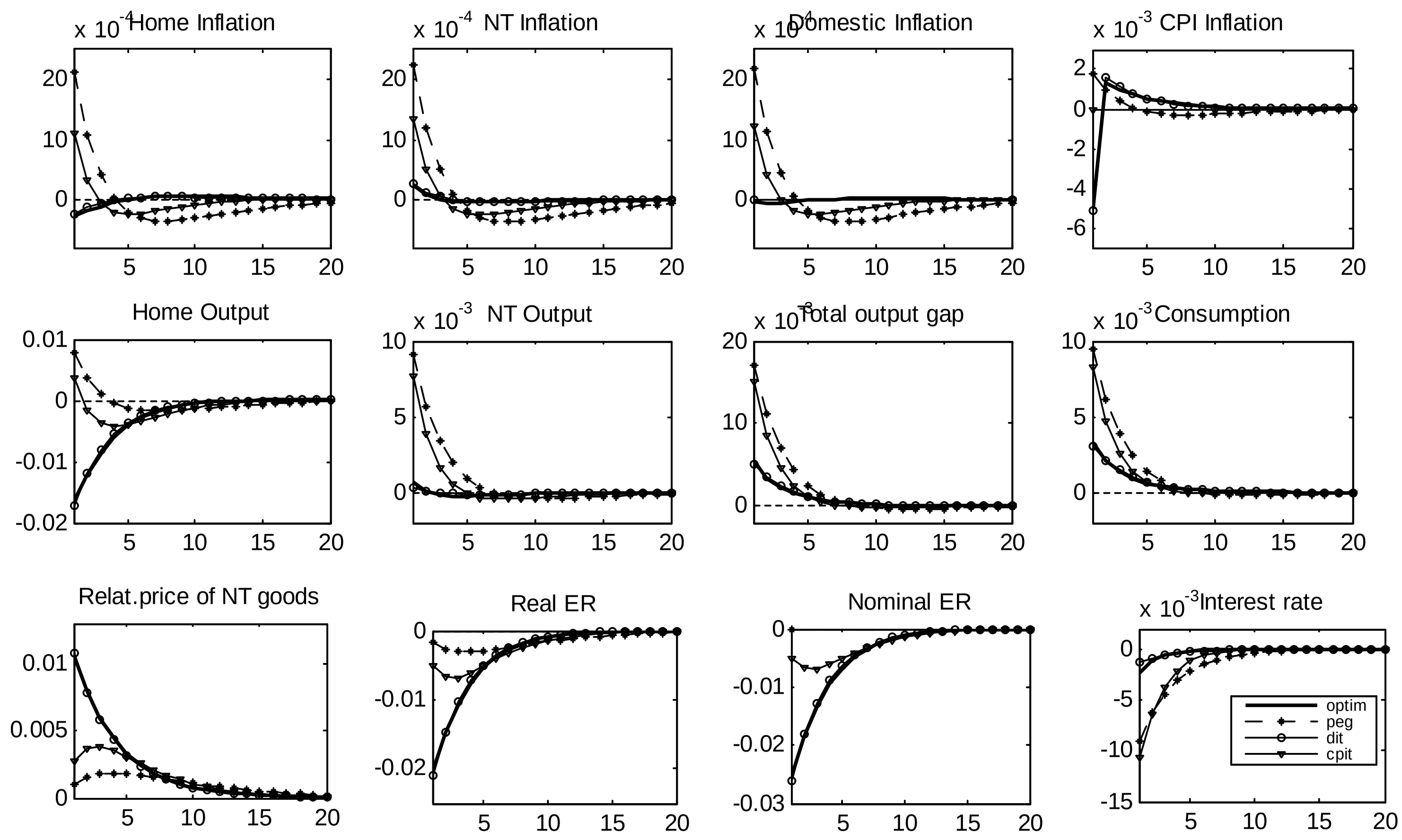

Figure 6: Impulse Response to a Foreign Relative Price Shock

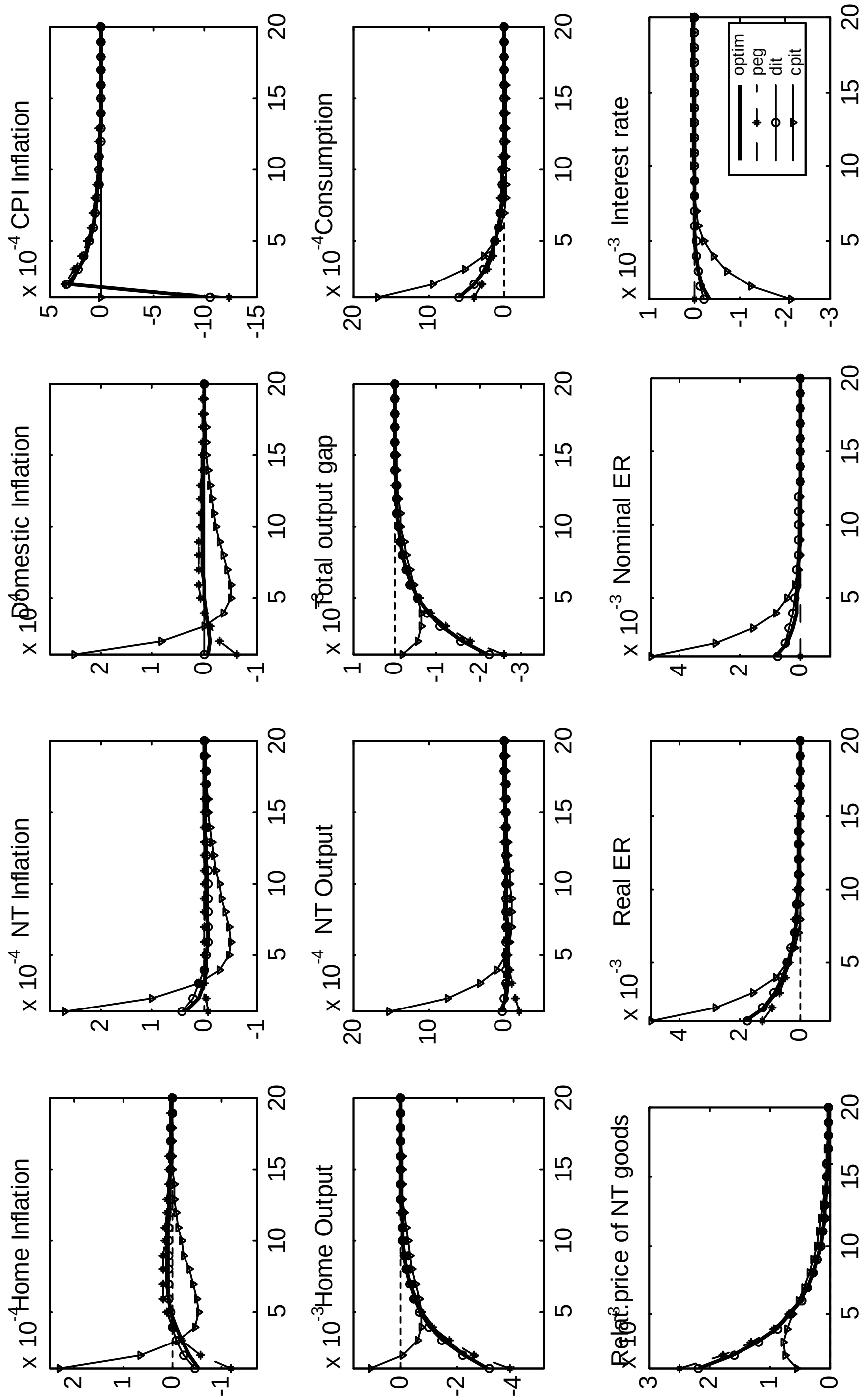

Figure 7: Impulse Response to a Mark-Up Shock in the Traded Sector

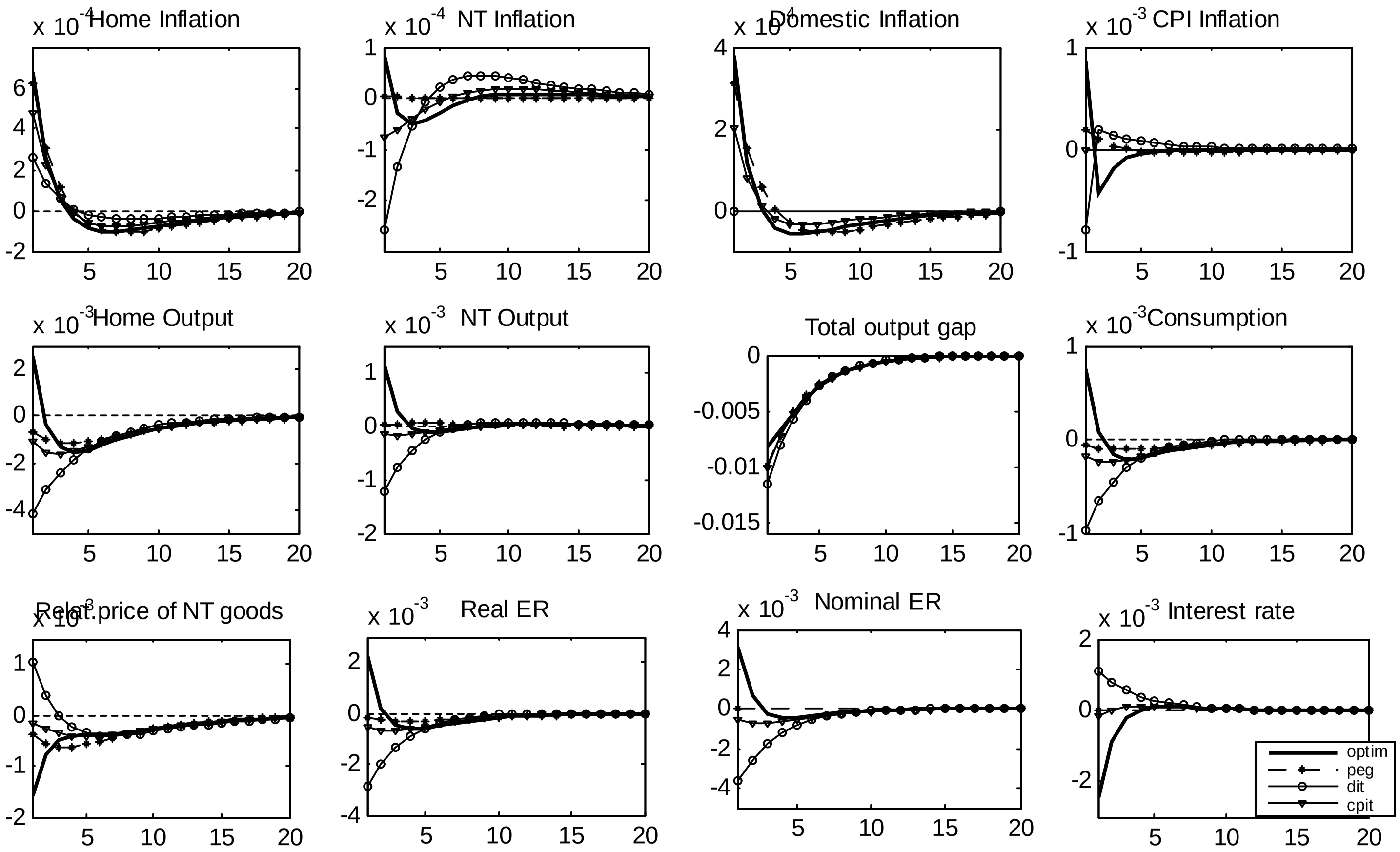

Figure 8: Impulse Response to a Mark-Up Shock in the Non-Traded Sector

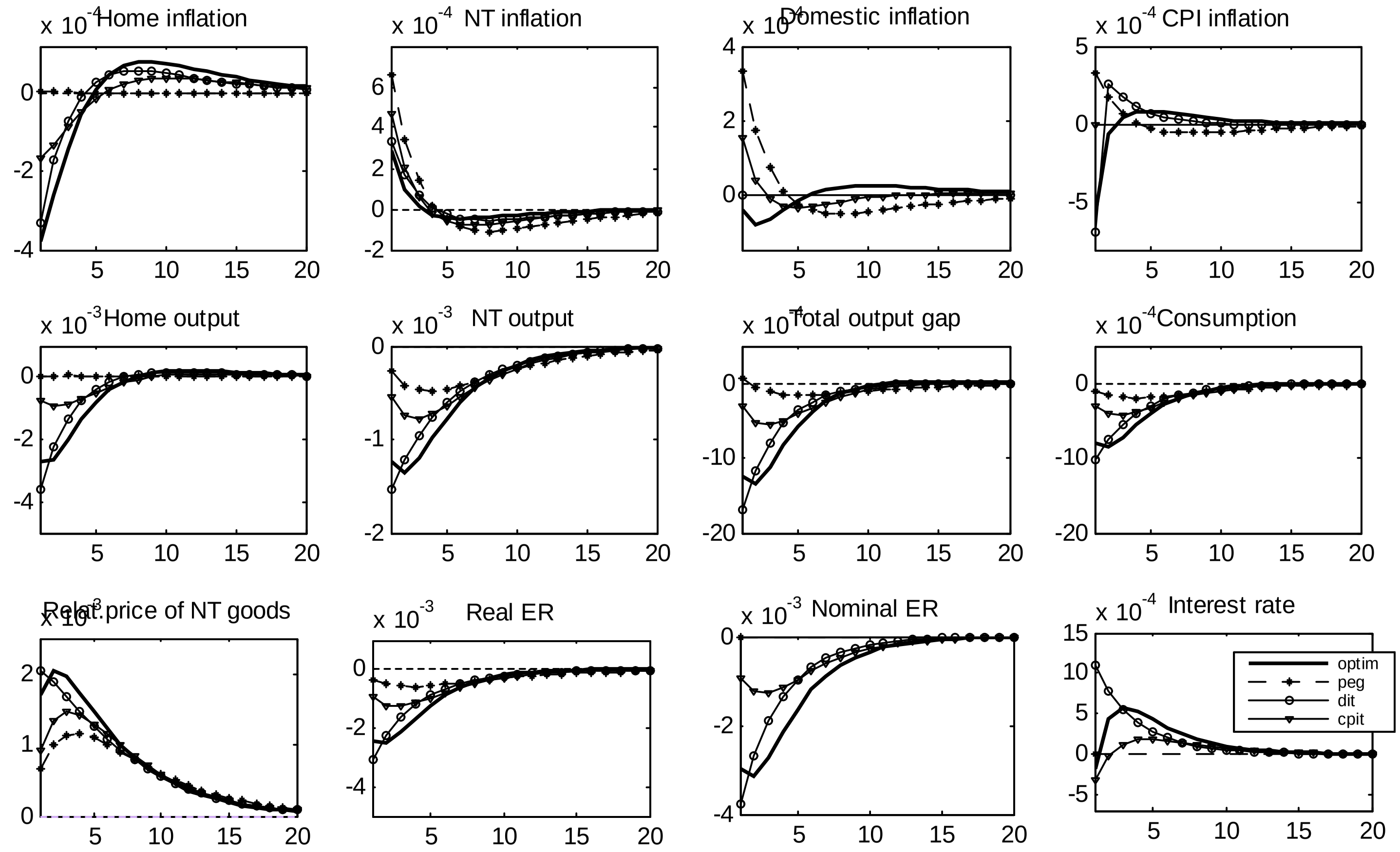

Figure 9: Impulse Response to a Fiscal Shock in the Traded Sector

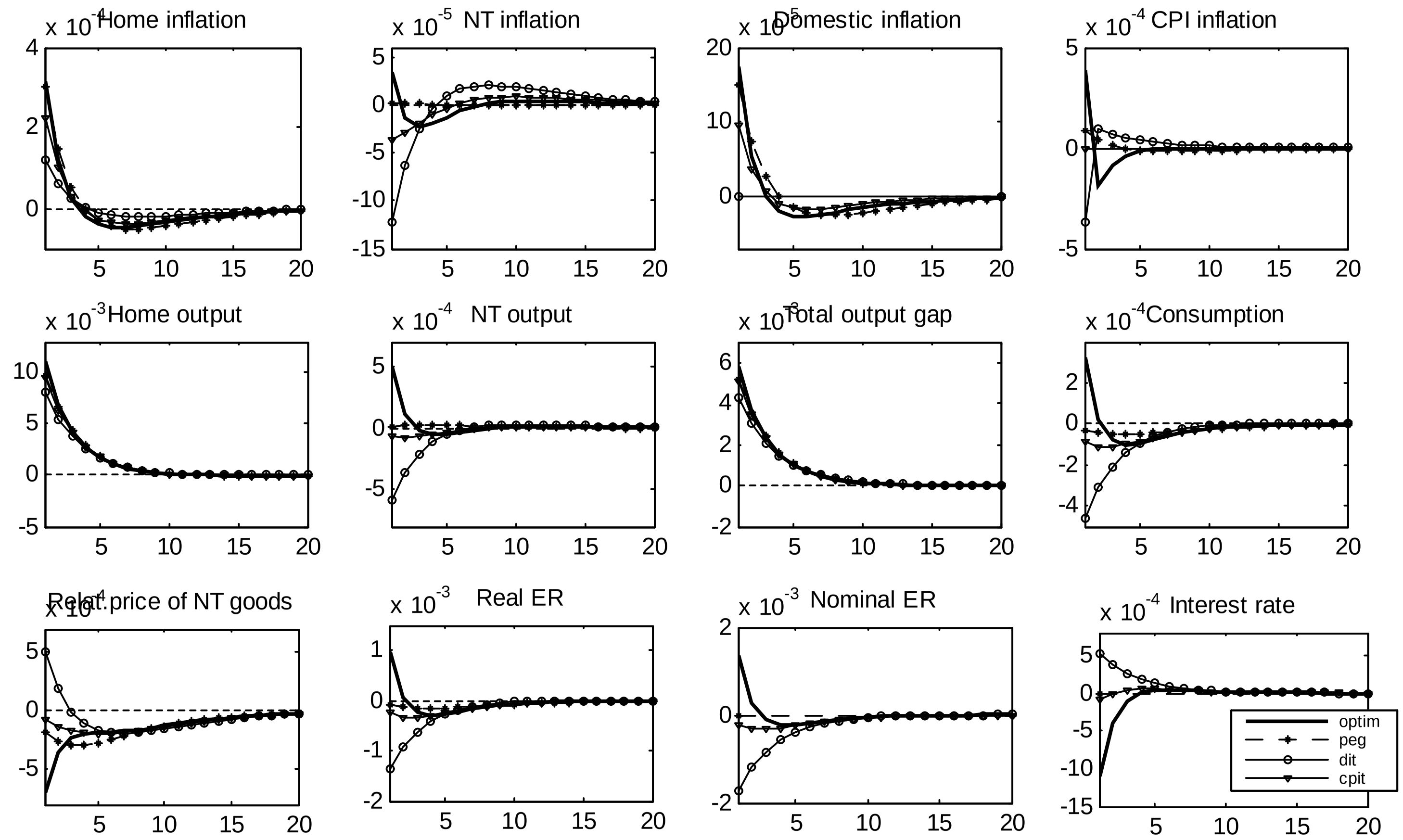

Figure 10: Impulse Response to a Fiscal Shock in the Non-Traded Sector

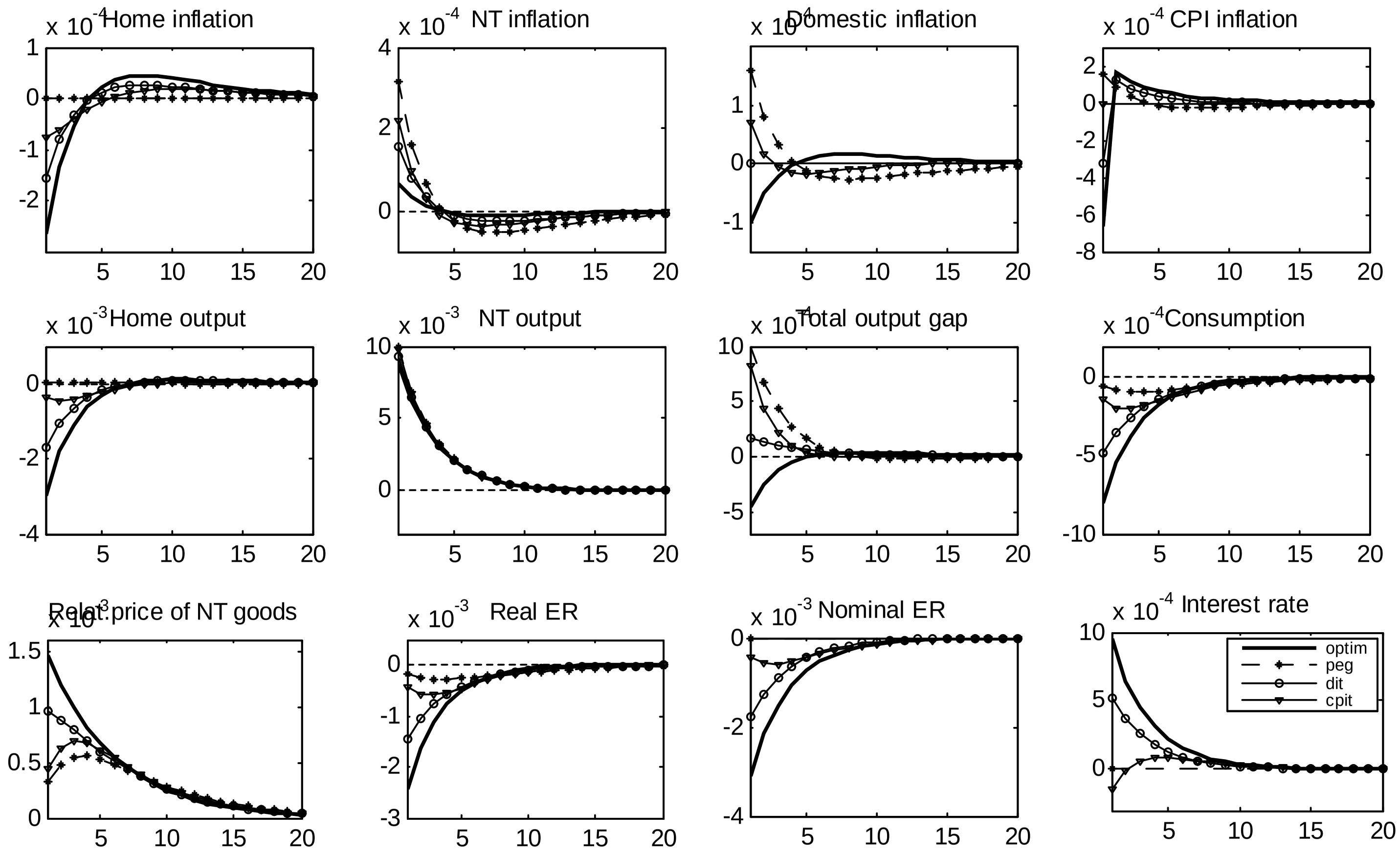

Figure 11: Welfare losses under alternative policies in the open economy

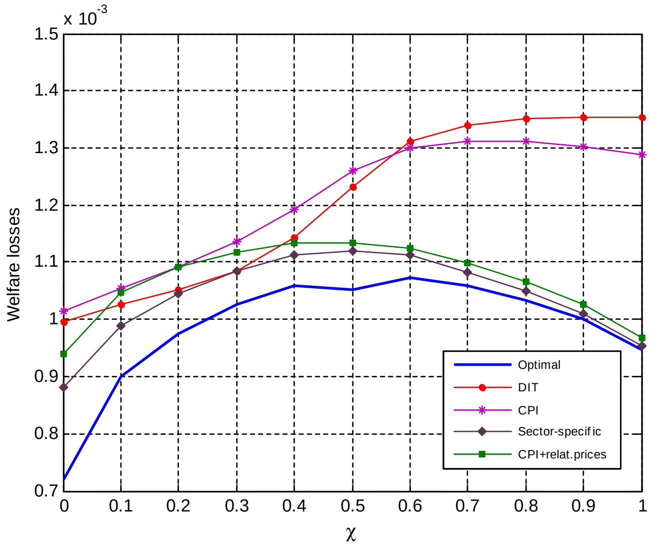

Figure 12: Welfare losses under alternative policies in the closed economy