The Role of Financial Market Structure and the Trade Elasticity for Monetary Policy in Open Economies

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Abstract

This paper studies Ramsey optimal monetary policy in a two-country model with monopolistic competition and nominal rigidities under different assumptions on international financial markets: complete markets, financial autarky, incomplete markets-bond economy. I show that the optimality of (producer) price stability and inward-looking policies is only obtained in the special case in which risk sharing is complete and policymakers act coordinately. In all other cases, movements in international relative prices (the terms of trade or the real exchange rate) enter into the consideration of optimal monetary policy. I show how these optimal deviations from price stability depend on the degree of risk sharing a particular financial market structure provides and on whether domestic and foreign goods are substitutes or complements in consumption. Contrasting Nash and coordination policies I show that there are generally welfare gains from coordination to be achieved.

Keywords: Monetary Policy, Price Stability, Policy Coordination Financial Market Structure, Elasticity *JEL-Codes:* E52, E58, F42

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1 Introduction

This paper studies the role of international financial market structure and the trade elasticity for welfare maximizing monetary policy in a two-country imperfectly-competitive sticky-price model of the open economy. Several contributions in the recent literature have argued that the policy recommendations in an open economy can be isomorphic to the ones of a closed economy, in the sense that optimal policy should be only inward-looking and focus on (producer) price stability (see Clarida, Gali and Gertler (2001) and Gali and Monacelli (2005) for early contributions). However, often conclusions are drawn from a setup where risk sharing is perfect, because complete financial markets are assumed. Alternatively (or additionally), they are based on rather special preference specifications, such as the case of a unit inter- and intratemporal elasticity (Cobb-Douglas-logarithmic utility), which imply that relative wealth is always unaffected in response to country specific shocks and which provide automatic full risk sharing independent of the financial market structure assumed (see, Cole and Obstfeld (1991), Corsetti and Pesenti (2001)).

A number of imperfections characterize the economy that typically exert influence on the way monetary policy should optimally be conducted. As in the closed economy both countries are characterized by two internal distortions: because of monopolistic competition output is inefficiently low and firms face nominal rigidities. In addition, there is an external distortion which stems from a country's monopoly power on the relative price of their exports to imports, that is, on its own terms of trade (TOT). In open economies monetary authorities may have an incentive to manipulate this price in their advantage if acting uncoordinated. The size and direction of the terms of trade externality crucially depends on the degree of international risk sharing and the degree of substitutability between domestic and foreign goods, and as a result these policy spillovers may influence optimal monetary policy very differently.

In such a framework I analyze Ramsey optimal monetary policy under policy coordination and under Nash competition. I do so under three stylized assumptions on international financial markets, - namely, complete markets financial autarky and an incomplete marketsbond economy- and for a wide range of the intratemporal elasticity of substitution where goods are allowed to be substitutes or complements. I assume that policymakers can commit and choose the welfare maximizing policy in response to equilibrium fluctuations as induced by country-specific shocks. For all cases but the one of perfect risk sharing and coordination, the implications for monetary policy are that deviations from full (producer) price stability are optimal. These are brought about by considerations about the optimal variability of international prices, such as the terms of trade or the real exchange rate.¹ The characterization of the optimal policy problems under coordination versus under Nash competition also allows us to look at the welfare implications of policy cooperation under the different financial market assumptions. The results can be summarized as follows:

In response to a productivity increase in, say, the domestic economy, I find that under *complete markets* (producer) price stability is always optimal when policymakers coordinate. However, deviations from price stability are found to be optimal in response to the productivity increase if policymakers act uncoordinated: when goods are substitutes producer price inflation is negative and the *TOT* is more appreciated *relative to a flexible price outcome*.

¹Throughout the paper, I refer the terms of trade when talking about the influence of international prices on policy decisions. However, it should be noted that, equivalently, I could have referred to the real exchange rate (which in this model moves always proportionally to the terms of trade).

Policymakers, when acting independently have an incentive to appreciate the terms of trade a bit, increasing the price of their own goods by reducing output and employment. As consumption risk is shared and domestic goods can easily be substituted by foreign goods, the reduction in employment increases their welfare. When goods are complements, producer price inflation is positive and the TOT is more depreciated relative to its flexible price response. Only in the case of a unit elasticity of intratemporal substitution policymakers the Nash outcome and coordination deliver the same result (of price stability as the prescription of optimal policy).

When financial markets are incomplete (financial autarky or incomplete markets-bond case) the TOT is found to be more depreciated (compared to a flexible price scenario) when goods are substitutes, and the inflation response is positive. If policymakers now were to reduce employment, this would still benefit agents by increasing the utility of leisure; however, unlike under complete markets, consumption risk is not shared and consumption is much more closely tied to current output. As such, policymakers find it optimal to expand output so much when productivity is temporarily higher that the terms of trade depreciate even more that when compared to a flexible price world. The prescription of optimal policy flips again when moving to the region of complementary goods: in that case the TOT is found to be more appreciated relative to the flexible price response and producer price inflation decreases in response to a productivity shock. It is interesting to note, that even if policymakers act coordinately price stability is, in general, not found to be the optimal outcome. The reason for this finding is that the flexible price financial autarky economy is not efficient as countries do not involve in any risk sharing. A policymaker that can, because of sticky prices, influence the terms of trade/ the real exchange rate finds it optimal to let it respond more closely to the way it would under complete financial markets, such that the equilibrium responses of the real exchange rate under the optimal policy is also doing some risk sharing.

The fact that the policy prescription under a Nash generally differs from the policy prescription under coordination implies that there are welfare gains from coordination. These are found increasing for elasticities of substitution away from unity and typically much larger in the case of complementarity between domestic and foreign goods. In addition, I find that welfare gains from coordination are bigger under complete markets when goods are substitutes, but turn out to be bigger under financial autarky when goods are complements. This is likely due to the fact that the lack of risk sharing becomes even more important when, because of a low elastiticity, international prices move strongly and wealth effects are large.

This paper is not the first to point out the important role of the trade elasticity for optimal monetary policy. In a setup of complete markets Benigno and Benigno (2002) find that "special conditions on levels of country specific intratemporal and intertemporal elasticities of substitution need to be satisfied" in order to conclude that price stability is the optimal policy with independent policymakers. Yet they do not characterize the specific patterns in which producer prices deviate from stability, and they do not consider the role of financial market structure. De Paoli (2009) discusses the optimal monetary policy of a small open economy in the form of comparing different targeting rules, considering different financial market assumptions. In particular, she computes on welfare based ranking of producer price inflation targeting, consumer price inflation targeting, and a fixed exchange rate regime. In contrast, the current paper focuses on describing the optimal amount of deviation from price stability as the outcome of a Ramsey optimal policy problem. In addition, the two-country setup allows for an explicit consideration of Nash versus coordinated optimal policies, which allows to also draw conclusions on the gains from policy cooperation. Faia and Monacelli (2004) study the role of the terms of trade externality in a two-country world under complete markets, but do not focus on alternative financial market structures (nor on the possibility of complementarity of domestic and foreign goods). Sutherland (2004) looks at welfare gains from coordination contrasting complete markets and financial autarky in a simple static model, but does not draw conclusions about the implications for (deviations from) price stability. In addition he only considers the case when goods are substitutes. As I show, for the case of complementary goods welfare gains from coordination are found to be an order of magnitude larger. A number of recent contributions have also emphasized the role of a low elasticity of intratemporal substitution (or trade elasticity) together with an incomplete financial markets structure in the transmission of productivity shocks across countries, in particular in addressing stylized facts on international relative prices such as exchange rate volatility, terms of trade volatility or the sign of transmission (see, among others e.g., Corsetti, Dedola and Leduc (2008), Thoenissen (2008), Enders and Mueller (2006)).

The rest of the paper is organized as follows. Section 2 describes the model and sets up the Ramsey problems for policymakers that act either under coordination or independently. Section 3 discusses the results for optimal monetary policy depending on the degree of intratemporal elasticity and depending on the financial market structure. It then continues to look at the implications for the optimality of price stability and the gains from policy coordination. Section 4 concludes.

2 The Model

The world economy consists of a Home country (H) and a Foreign country (F), each of which is specialized in one type of tradable good. Households and firms are defined over a continuum of unit mass. Home and Foreign households are indexed by $j \in [0, 1]$ and $j^* \in [0, 1]$ respectively. Each good is produced by firms in a number of varieties, indexed by h in the Home country and by f in the Foreign country. Each variety is an imperfect substitute to all other varieties and is produced under conditions of monopolistic competition. Firms face quadratic adjustment costs in their price setting decision and are assumed to set the price in the foreign market in their own currency (producer currency pricing). I abstract from modeling monetary frictions by considering a cashless economy. Unless necessary otherwise, in the following I only discuss the problem of Home agents, with an understanding that the problem for Foreign agents is symmetric - variables of Foreign agents are marked with an asterisk.

2.1 Model Setup

2.1.1 Preferences and Budget Constraint

Household j maximizes her lifetime expected utility:

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}(j)}{1-\sigma} - \frac{L_t^{1+\kappa}(j)}{1+\kappa} \right\}$$
(1)

where β is the discount factor, C(j) is consumption and L(j) is labor effort. Consumption C(j) is a constant-elasticity-of-substitution (CES) basket over domestic and foreign goods:

$$C_t(j) = \left[\gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}}(j) + (1-\gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}}(j)\right]^{\frac{\omega}{\omega-1}},\tag{2}$$

where ω denotes the trade elasticity, that is, the intratemporal elasticity of substitution between domestic and foreign goods, and where parameter $\gamma \geq \frac{1}{2}$ is the degree of home bias in consumption. For each household *j* the consumption indices of Home varieties and Foreign varieties are defined as:

$$C_{H,t} = \left[\int_{0}^{1} C_t \left(h,j\right)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \left[\int_{0}^{1} C_t \left(f,j\right)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}$$
(3)

where $C_t(h, j)$ and $C_t(f, j)$ are respectively consumption of Home variety h and Foreign variety f by agent j at time t.

Household j maximizes equation (1) subject to the budget constraint. Each period household j receives wage income, $W_t L_t(j)$, and dividends from the monopolistic firms they own, $\Pi_t(j)$, pays non-distortionary (lump-sum) taxes, $T_t(j)$, to the government, purchases consumption, $P_t C_t(j)$. The availability of any assets of domestic household j depends on the assumptions of the structure of international financial markets. Throughout the paper, I consider three possible scenarios: complete markets, financial autarky and an incomplete markets bond economy.

Under complete markets the household has access to a full set of state-contingent (Arrow-Debreu) securities. Let $Q(s_{t+1}|s_t)$ denote the price of one unit of Home currency delivered in period t + 1 contingent on the state of nature at t + 1 being s_{t+1} . With complete markets, $Q(s_{t+1}|s_t)$ is the same for all individuals. Let $B_{H,t}(j, s_{t+1})$ denote the claim to $B_{H,t}(j, s_{t+1})$ units of Home currency at time t + 1 in the state of nature s_{t+1} , that household j buys at time t and brings into time t + 1. $Q^*(s_{t+1}|s_t)$ and $B_{F,t}(j, s_{t+1})$ are defined similarly in terms of units of Foreign currency. ε_t denotes the nominal exchange rate (units of Home currency per unit of Foreign currency). The budget constraint under complete markets is then given by:

$$\sum_{s_{t+1}} Q(s_{t+1}|s_t) B_{H,t}(j, s_{t+1}) + \sum_{s^{t+1}} Q^*(s_{t+1}|s_t) \varepsilon_t B_{F,t}(j, s_{t+1})$$

$$\leq B_{H,t-1}(j, s_t) + \varepsilon_t B_{F,t-1}(j, s_t) + W_t L_t(j) + \Pi_t(j) + T_t(j) - P_t C_t(j)$$
(4)

If the two economies are in *financial autarky* no assets can be traded internationally. Let $B_{H,t}(j)$ and $B_{F,t}(j)$ denote bonds denominated in either domestic and foreign currency. Under international financial autarky, the domestic currency bond, $B_{H,t}$, that can be traded only domestically. Equivalently, foreign agents can trade a foreign currency bond, $B_{F,t}^*$, but also only within their country.² The budget constraint of domestic household j under financial autarky then becomes:

²That is,
$$\int_{0}^{1} B_{H,t}(j) dj = 0$$
 and $\int_{0}^{1} B_{F,t}^{*}(j^{*}) dj^{*} = 0$.

$$B_{H,t}(j) = B_{H,t-1}(j) R_{t-1} + W_t L_t(j) + \Pi_t(j) + T_t(j) - P_t C_t(j)$$
(5)

Finally, I consider the case of the *incomplete markets-bond economy*. We now assume that both countries can now engage in financial trade through one of the one-period nominal bonds. In particular, I assume that the foreign currency denominated bond, $B_{F,t}$, can be traded internationally (and net foreign wealth is initially zero).³ Following Schmitt-Grohe and Uribe (2003) and Benigno (2001), to render the incomplete markets economy stationary, I assume that domestic agents face a quadratic adjustment cost when taking on an international asset position different from their long-run (zero) position.⁴ The budget constraint under the assumption of the incomplete markets-bond economy is:

$$B_{H,t}(j) + \varepsilon_t B_{F,t-1}(j) + \frac{\phi}{2} \left(\frac{\varepsilon_t B_{F,t-1}(j)}{P_t}\right)^2 P_t$$

$$\leq B_{H,t-1}(j) R_{t-1} + \varepsilon_t B_{F,t-1}(j) R_{t-1}^* + W_t L_t(j) + \Pi_t(j) + T_t(j) - P_t C_t(j)$$
(6)

2.1.2 Households' Intratemporal Consumption Allocation

Household j minimizes, each period, its consumption expenditure subject to obtaining a unit of the final consumption good. Denoting with P_t the Lagrange multiplier to that problem⁵ gives the following optimal demand functions:

$$c_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}(j) = \gamma \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t(j),$$
(7)

$$c_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}(j) = (1-\gamma) \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t(j), \qquad (8)$$

For given Home-currency prices of varieties, $p_t(h)$ and $p_t(f)$ the utility-based CPI, P_t , is given by:

$$P_{t} = \left[\gamma P_{H,t}^{1-\omega} + (1-\gamma) P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$
(9)

$$\int_{0}^{1} B_{H,t}(j) \, dj = 0, \int_{0}^{1} B_{F,t}(j) \, dj + \int_{0}^{1} B_{F,t}^{*}(j^{*}) \, dj^{*} = 0$$

⁴It is important to note that the internationally traded asset is exogeneously restricted to be the foreign currency bond only, for which a long-run zero position is simply assumed. In particular, this setup does not enter the recent literature of portfolio choice issues and endogenous non-zero positions (see, e.g. Devereux and Sutherland (2008) and Tille and van Wincoop (2007)).

⁵Formally,

$$\min \int_{0}^{1} p_{t}(h) C_{t}(h, j) dh + \int_{0}^{1} p_{t}(f) C_{t}(f, j) df - P_{t}C_{t}(j).$$

³The nominal bonds are in zero net-supply worldwide, so that:

where

$$P_{H,t} = \left[\int_{0}^{1} p_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, P_{F,t} = \left[\int_{0}^{1} p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}.$$
 (10)

2.1.3 Households' Labor Supply and Intertemporal Allocation

Denote with $\lambda_t(j)$ the Lagrange multiplier of the household's budget constraint. Household j's first order conditions with respect to $C_t(j)$ and $L_t(j)$ are identical for all possible financial market assumptions and are given by:

$$\lambda_t \left(j \right) = \frac{1}{P_t C_t^\sigma \left(j \right)} \tag{11}$$

$$L_t^{\kappa}(j) = \lambda_t(j) W_t \tag{12}$$

Under *complete financial market*, the first order condition w.r.t. home and foreign Arrow-Debreu securities are given by:

$$Q(s_{t+1}|s_t) = \beta E_t \left\{ \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, Q^*(s_{t+1}|s_t) = \beta E_t \left\{ \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right\}$$
(13)

which can be combined to obtain the risk sharing equation:

$$\frac{\varepsilon_t P_t^*}{P_t} = \left(\frac{C_t^*(j)}{C_t(j)}\right)^{-\sigma} \tag{14}$$

under *financial autarky* the domestic currency bond can only be held domestically such that

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$
(15)

under the *incomplete markets bond economy*, the first order condition w.r.t. home and foreign bond are similarly given by:

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, 1 = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} \right\}$$
(16)

The nominal interest rate R_t and R_t^* can be thought of as the underlying instruments of monetary policy in the two economies.

2.1.4 Production and Price Setting

The production function is assumed to be linear in labor:

$$Y_t(h) = Z_t L_t(h) \tag{17}$$

where Z_t is a country-specific productivity process. Firms operate under conditions of monopolistic competition taking into account the downward-sloping demand for their product and set prices to maximize their profit. They are assumed to set the prices in the foreign market in their own currency, that is, I consider the scenario of producer currency pricing (PCP). Firms are small, in the sense that they ignore the impact of their pricing and production decisions on aggregate variables and price indices. When firms set their prices they have to consider a quadratic adjustment cost, with parameter α measuring the degree of price stickiness:

$$\phi_t(h) = \frac{\alpha}{2} \left(\frac{p_t(h)}{p_{t-1}(h)} - 1 \right)^2$$
(18)

The presence of Rotemberg adjustment costs makes the firms' price setting dynamic, which introduces richer and arguably more realistic equilibrium dynamic effects of monetary policy than in a setup where prices are set one period in advance. The richer description of price stickiness is also likely to be more appropriate for quantitive welfare analysis. I assume throughout that the law of one price holds, such that for each variety h we have $\varepsilon_t p_t^*(h) = p_t(h)$. Each producer chooses its price $p_t(h)$ such as to maximize its total market value:

$$E_{t}\left\{\sum_{t=0}^{\infty}\Lambda_{0,t}\left[p_{t}\left(h\right)\left(1+\tau\right)-MC_{t}\left(h\right)\right]\left[\left(\frac{p_{t}\left(h\right)}{P_{H,t}}\right)^{-\theta}\left(C_{H,t}+C_{H,t}^{*}\right)\right]-\frac{\alpha}{2}\left(\frac{p_{t}\left(h\right)}{p_{t-1}\left(h\right)}-1\right)^{2}P_{H,t}\right\}$$
(19)

where MC_t is the marginal cost that minimizes labor input, which is equal to all firms, $MC_t(h) = MC_t = W_t/Z_t$, and where τ stand for a production subsidy that offsets the distortion from monopolistic competition.

2.1.5 Firms' Optimality Conditions

The firm's optimal price setting condition is derived as:

$$0 = \left[\left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} \left(C_{H,t} + C_{H,t}^* \right) \right] \left(\frac{p_t(h)}{P_{H,t}} \right)^{-1} \left[\theta \frac{MC_t(h)}{P_{H,t}} - (\theta - 1)(1 + \tau) \right] - (20)$$

$$\alpha \left(\frac{p_t(h)}{p_{t-1}(h)} - 1 \right) \frac{P_{H,t}}{p_{t-1}(h)} + E_t \Lambda_{t,t+1} \alpha \left(\frac{p_{t+1}(h)}{p_t(h)} - 1 \right) \frac{p_{t+1}(h)P_{H,t+1}}{p_t^2(h)}$$

Parameter $\alpha = 0$ corresponds to the case of flexible prices, in which case the price is set as the a simple markup over current marginal costs.⁶

⁶That is, the resulting first order conditions under flexible prices are:

We focus our attention on a symmetric equilibrium where all domestic producers charge the same price, adopt the same technology and therefore choose the same demand for labor. This implies $p_t(h) = P_{H,t}$, $p_t^*(h) = P_{H,t}^*$, $L_t(h) = L_t$, $\Pi_t(j) = \Pi_t$.

2.1.6 Resource Constraints and Aggregate Budget Constraints

The resource constraint for each variety h and each variety f are given by:

$$Y_t(h) = \int_0^1 c_t(h, j) dj + \int_0^1 c_t(h, j^*) dj^* + \int_0^1 \phi_t(j) dj = C_{H,t} + C_{H,t}^* + \phi_t$$
(21)

$$Y_t^*(f) = \int_0^1 c_t(f,j)dj + \int_0^1 c_t(f,j^*)dj^* + \int_0^1 \phi_t^*(j^*)dj^* = C_{F,t} + C_{F,t}^* + \phi_t^*$$
(22)

Symmetry across all households j gives $C_t(j) = C_t$, $L_t(j) = L_t$, $\lambda_t(j) = \lambda_t$, and implies that conditions (7)-(8), (11)-(14), (15) and (16) must also hold for aggregate variables and indices j can be dropped.

In addition, using equilibrium in the asset markets we can write the aggregate budget constraint under the case of financial autarky, having imposed clearing conditions, $\int_{0}^{1} B_{H,t}(j) dj =$

0 and
$$\int_{0}^{1} B_{F,t}^{*}(j^{*}) dj^{*} = 0$$
, as:

$$0 = W_{t}L_{t} + \Pi_{t} - T_{t} - P_{t}C_{t}$$
(23)

In the incomplete markets bond economy, with asset market clearing conditions, $\int_{0}^{1} B_{F,t}(j)dj = -\int_{0}^{1} B_{F,t}^{*}(j^{*})dj^{*}$ and $\int_{0}^{1} B_{H,t}^{*}(j^{*})dj^{*} = 0$, the budget constraint becomes:

$$\varepsilon_t B_{F,t} + \frac{\phi}{2} \left(\frac{\varepsilon_t B_{F,t-1}}{P_t} \right)^2 P_t = B_{F,t-1} R_{t-1}^* + W_t L_t + \Pi_t - T_t - P_t C_t$$
(24)

2.1.7 Relative Prices and The Terms of Trade

The terms of trade is defined as the price of imports to exports, $\frac{P_{F,t}}{\varepsilon_t P_{H,t}^*}$, which given the law of one price can be written as:

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} \tag{25}$$

Using the optimal consumer price level resulting from the intratemporal allocation problem, it is possible to express all relative prices as a function of the terms of trade only. In

$$p_t(h) = \varepsilon_t p_t^*(h) = \frac{\theta}{(\theta - 1)(1 + \tau)} MC_t(h)$$

particular, the real exchange rate, which is the price of a foreign consumption bundle relative to domestic consumption bundle, that is, $RER_t = (\varepsilon_t P_t^*)/P_t$, is related to the terms of trade by:

$$RER_{t} = f^{RER} \left(TOT_{t} \right) = \frac{\left[\gamma^{*} + (1 - \gamma^{*}) TOT_{t}^{1-\omega} \right]^{\frac{1}{1-\omega}}}{\left[\gamma + (1 - \gamma) TOT_{t}^{1-\omega} \right]^{\frac{1}{1-\omega}}}$$
(26)

The PPI-to-CPI ratios are defined as $p_{H,t} \equiv P_{H,t}/P_t$ and $p_{F,t}^* \equiv P_{F,t}^*/P_t^*$ and can also be written as functions of the terms of trade only:

$$p_{H,t} = f^{p_H} \left(TOT_t \right) = \left[\gamma + (1 - \gamma) TOT_t^{1-\omega} \right]^{-\frac{1}{1-\omega}}, p_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t \right) = \left[\gamma^* TOT_t^{\omega-1} + (1 - \gamma^*) \right]^{-\frac{1}{1-\omega}}, q_{F,t}^* = f^{p_F^*} \left(TOT_t^{\omega-1}$$

2.2 Optimal Monetary Policy

Having completed the description of the model economy, we now turn to studying the optimal monetary policy in this two-country imperfectly competitive sticky price economy. For this reason, it is useful to reflect on the distortions that characterize the economy. As in the closed economy both countries are characterized by two internal distortions: price stickiness and monopolistic competition. The latter produces an inefficient level of output. The other internal distortion is price stickingess which prevents efficient adjustment to the disturbances that affect the economy. A procyclical policy can remove the sticky-price distortion by making production supply-determined and can replicate the flex-price equilibrium. In addition, there is an external distortion which stems from a country's monopoly power on the relative price of their exports to imports, that is, on its own terms of trade (TOT). In open economies monetary authorities may have an incentive to manipulate it in their advantage if acting uncoordinated. The size and direction of this terms of trade externality crucially depends on a) the degree of international risk sharing and b) the degree of substitutability between domestic and foreign goods. This is because the effects of monetary policy in an open economy depend to a great extent on the influence it has on the nominal exchange rate, which in turn depends very much on the assumptions on the structure of international financial markets. On the other hand the degree of substitutability between goods produced in different countries determines the strength of the expenditure switching effect of exchange rate changes and therefore determines the impact of monetary policy on goods demand in different countries. Also, the elasticity influences the degree to which countries are subject to asymmetric income shocks. If the elasticity is close to unity then relative price changes are largely offset by changes in output volumes and the terms of trade provide strong automatic risk sharing (see Cole and Obstfeld (1991)).

To study the policy spillovers of the international price externality and its influences on optimal monetary policy I consider a production subsidy that offsets the distortion from monopolistic competition which therefore isolates the effects of the terms of trade externality on optimal policy. I assume throughout that policymakers can credibly commit to a chosen rule.⁷ I compare optimal commitment policy under Nash competition and under cooperation. By deriving second order accurate solutions to the policy functions, it is also possible to characterize the welfare gains from international policy cooperation.

To study these issues I follow a Ramsey type approach, which is the classical approach to the study of optimal policy in dynamic economies (see e.g., (Ramsey (1927), Atkinson and Stiglitz (1976), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991)). In this setup the optimal monetary policy entails a Ramsey planner which maximizes a social objective function subject to the private sector's constraints. While most studies of optimal monetary policy in the recent literature build on a linear-quadratic approximation approach in the spirit of Rotemberg and Woodford (1997), Woodford (2003), and Benigno and Woodford (2004), recently, the Ramsey type approach has been employed in an increasing number of dynamic equilibium models with monopolistic competition and nominal rigidities. Examples include, in the context of closed economy models, Adao et. al (2003), Khan, King and Wolman (2003), Schmitt-Grohe and Uribe (2003, 2004), and Siu (2004). In the open economy a Ramseytype approach has been employed by Faia and Monacelli (2003) and Arsenau (2004), open economy applications employing the linear-quadratic approach are, among others, Benigno and Benigno (2004) and De Paoli (2004).

2.3 Definition of Equilibrium and Description of Constraints for Ramsey Problem

An equilibrium requires that households and firms behave optimally, as described by the above optimality conditions. Specifically, given exogenous processes for Z_t and Z_t^* , a policy for R_t and R_t^* and given initial conditions, a symmetric world competitive equilibrium is a set of prices and quantities that

- satisfy the Home and Foreign consumers' optimality conditions, equations (7)-(10), (11)-(12), (15) and their foreign counterparts, together with:
 - the risk sharing equation (14) under *complete financial markets*
 - equation (15) and the budget constraint, equation (24), under the *incomplete* markets-bond economy
 - the budget constraint, equation (23), under the *financial autarky*
- maximize firms profits, meaning that prices are set according to (20) and similarly in the foreign economy,
- satisfy the market clearing conditions for each asset and each good, in all the markets where it is traded, and
- satisfy the resource constraints.

⁷As stressed by Corsetti and Pesenti (2001) and Benigno and Benigno (2003) policymakers face an ex post temptation to deviate from any pre-announced policy rule. This can generate either an inflationary or a deflationary bias depending on the balance between the monopoly distortion (if not offset by a production subsidy) and the terms of trade. here the complications arising from these issues are avoided by assuming that policymakers can commit to the ex ante choice of policy rules.

It is possible to reduce the system of equilibrium conditions to a system of equations in $C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*$, and TOT_t only (given exogenous processes for Z_t and Z_t^* , and for a policy for R_t and R_t^*). In particular, plugging in for the demand functions (7) and (8) together with their foreign counterparts, making use of the fact that $\pi_t = \frac{p_{H,t-1}}{p_{H,t}}\pi_{H,t}, \pi_t^* = \frac{p_{F,t-1}^*}{p_{F,t}^*}\pi_{F,t}^*$, and by using the functional relationships between the real exchange rate and the terms of trade (equation (26)) and the PPI-to-CPI ratio and the terms of trade (equation (27)), we can write the equilibrium as being described by equations (28)-(34) below. Equations (28)-(29) are the two Euler equations, equations (30)-(31) the two price setting equations, equations (32)-(33) the two resource constraints, and optimality conditions (34a), (34b) or (34c) that hold under complete markets, financial autarky or the bond economy respectively.

$$1 = \beta E_t \left\{ R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right\}$$
(28)

$$1 = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \right\}$$
(29)

$$\alpha (\pi_{H,t} - 1) \pi_{H,t} = (p_{H,t})^{-\omega} [C_t + RER_t^{\omega} C_t^*] \left[\theta \left(\frac{L_t^{\kappa} C_t^{\sigma}}{Z_t p_{H,t}} \right) - (\theta - 1) (1 + \tau) \right]$$
(30)
+ $E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{p_{H,t+1}}{p_{H,t}} \alpha (\pi_{H,t+1} - 1) \pi_{H,t+1}$

$$\alpha \left(\pi_{F,t}^{*}-1\right) \pi_{F,t}^{*} = \left(p_{F,t}^{*}\right)^{-\omega} \left[RER_{t}^{-\omega}C_{t}+C_{t}^{*}\right] \left[\theta \left(\frac{L_{t}^{*\kappa}C_{t}^{*\sigma}}{Z_{t}^{*}p_{F,t}^{*}}\right) - (\theta-1)\left(1+\tau\right)\right]$$
(31)
+ $E_{t} \left(\frac{C_{t+1}}{C_{t}}\right)^{-\sigma} \frac{p_{F,t+1}^{*}}{p_{F,t}^{*}} \alpha \left(\pi_{F,t+1}^{*}-1\right) \pi_{F,t+1}^{*}$
 $Z_{t}L_{t} = (p_{H,t})^{-\omega} \left[C_{t}+RER_{t}^{\omega}C_{t}^{*}\right]$ (32)

$$Z_t^* L_t^* = \left(p_{F,t}^*\right)^{-\omega} \left[RER_t^{-\omega}C_t + C_t^*\right]$$
(33)

under complete markets:

$$RER_t = \left(\frac{C_t^*}{C_t}\right)^{-\sigma} \tag{34a}$$

under financial autarky:

$$p_{H,t}\left(Z_t L_t\right) - \phi_t = C_t \tag{34b}$$

under incomplete markets, bond economy⁸:

⁸The budget constraint is also expressed in real terms, where $b_{F,t} = B_{F,t}/P_t^*$.

$$(1 + \psi RER_t b_{F,t}) = \beta E_t \left\{ R_t^* \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}^*} \frac{RER_{t+1}}{RER_t} \right\}$$
(34c)
$$RER_t b_{F,t} + \frac{\phi}{2} \left(RER_t b_{F,t} \right)^2 = RER_t b_{Ft-1} \frac{R_{t-1}^*}{\pi_t^*} + p_{H,t} \left(Z_t L_t \right) - C_t - \phi_t$$

2.3.1 Definition of Ramsey problem under cooperation

To derive the Ramsey optimal monetary policy under cooperation, I set up the problem of a world social planner that aims to maximize the country-sized weighted average measure of welfare, which are given by the lifetime expected utilities:

$$W_t^{average} = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right\}$$
(35)

Under complete markets, define the vector of constraints as $CONSTR_t^{CMC}$, by vertically stacking equations (28)-(34a). Superscript CMC refers to the case of complete markets and policymakers acting under coordination. Also, define the vector of Lagrange multipliers at time t attached to constraints (28)-(34a) by Ξ_t^{CMC} , where $\Xi_t^{CMC} = [\xi_{H1,t}^{CMC}, \xi_{F1,t}^{CMC}, \xi_{H2,t}^{CMC}]$. The optimal policy can then be described by taking derivatives of the Lagrangian:

$$L^{CMC} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right] + \Xi_t^{CMC} CONST R_t^{CMC} \right\}$$
(36)
with respect to $\left\{ \Xi_t^{CM, coord} \right\}_{t=0}^{\infty}$, and $\left\{ C_t, C_t^*, L_t, L_t^*, \pi_{H,t}, \pi_{F,t}^*, TOT_t, R_t, R_t^* \right\}_{t=0}^{\infty}$.

Under *financial autarky* the Ramsey problem under cooperation can similarly be defined by:

$$L^{FAC} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right] + \Xi_t^{FAC} CONST R_t^{FAC} \right\}$$
(37)

where superscript FAC refers to the financial autarky scenario and coordinated policymakers. The vector of constraints $CONSTR_t^{FAC}$ is given by vertically stacking equations (28)-(33) and (34b), and Ξ_t^{FAC} is the vector of Lagrange multipliers to the constraints under FA.

Finally, under *incomplete markets bond economy* the Ramsey problem of coordinated policymakers can be summarized in a similar fashion as:

$$L^{IMC} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right] + \Xi_t^{IMC} \cdot CONSTR_t^{IMC} \right\}$$
(38)

where the incomplete markets-bond economy financial market structure now implies that the vector of constraints includes two financial market specific equations (given by (34c)) and where the size of Lagrange multipliers is accordingly enlarged.

2.3.2 Definition of Ramsey problem under independently acting monetary authorities

If monetary authorities act uncoordinated, the home policymaker does not internalize that the relative price, TOT_t , does also depend on the level of consumption in the Foreign country, and that, symmetrically, the relative price is affected by its own consumption choice. The Ramsey problem for policymakers that act uncoordinated implies that in each country the policymaker takes as given the other country's variables and optimizes own welfare, taking into consideration only the country's own optimality conditions as constraints. The Nash equilibrium can then be found by combining the Ramsey optimality conditions of both country's uncoordinated policymakers. In particular, the domestic and the foreign policymaker maximize, respectively:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} \right\}$$
(39)

$$W_t^* = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right\}$$
(40)

Under complete markets, construct the vector of constraints for the domestic agent, $CONSTR_t^{CMN}$, by vertically stacking equations (28), (30), (32) and (34a), the vector of constraints for the foreign agent, $CONSTR_t^{CMN*}$, by stacking (29), (31), (33) and (34a). Also, define the sequence of the vector of Lagrange multipliers attached to constraints in $CONSTR_t^{CMN}$ and $CONSTR_t^{CMN*}$ by $\Xi_t^{CMN} = [\xi_{H1,t}^{CMN}, \xi_{H2,t}^{CMN}, \xi_{H3,t}^{CMN}]$ and $\Xi_t^{CMN*} = [\xi_{F1,t}^{CMN}, \xi_{F2,t}^{CMN}, \xi_{F3,t}^{CMN}, \xi_{F4,t}^{CMN}]$. The Lagrangians of the optimal policy problem for the domestic and foreign policymaker can be set up as:

$$L^{CMN,H} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \Xi_t^{CMN} CONST R_t^{CMN} \right\}$$
(41)

$$L^{CMN,F} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} + \Xi_t^{CMN*} CONSTR_t^{CMN*} \right\}$$
(42)

The domestic monetary authority maximizes equation (41) w.r.t. Ξ_t^{CMN} , and $C_t, L_t, \pi_{H,t}, TOT_t$ and R_t , the foreign authority maximizes equation (43) w.r.t. $\Xi_t^{CMN*}, C_t^*, L_t^*, \pi_{F,t}^*, TOT_t$ and R_t^* . It should be noted that the first order condition of the domestic policymaker w.r.t. $\xi_{4H,t}^{CMN}$ and the first order condition of the foreign policymaker w.r.t. $\xi_{4F,t}^{CMN}$ both give back the same optimality condition defining the equilibrium terms of trade (over the risk sharing equation), one of which therefore can be safely dropped.

Under *financial autarky* the uncoordinated Ramsey problem can similarly be defined by:

$$L^{FAN,H} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \Xi_t^{FAN} CONST R_t^{FAN} \right\}$$
(43)

$$L^{FAN,F} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} + \Xi_t^{FAN*} CONSTR_t^{FAN*} \right\}$$
(44)

where the vectors of constraints $CONSTR_t^{FAN}$ and $CONSTR_t^{FAN*}$ are defined as under complete markets, replacing the last equation with the corresponding equilibrium condition under financial autarky, equation (34b) and its foreign counterpart respectively. Ξ_t^{FAN} and Ξ_t^{FAN*} again refer to the vectors of Lagrange multiplier on the constraints under FA. The first order condition w.r.t. $\xi_{4H,t}^{FAN}$ and w.r.t. $\xi_{4F,t}^{FAN}$ return the budget constraints of the Home and Foreign country, one of which is redundant (by Walras' law).

Under the *incomplete markets bond economy* the Lagrangians of the uncoordinated policymakers are given by, respectively:

$$L^{IMN,H} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} + \Xi_t^{IMN} CONST R_t^{IMN} \right\}$$
(45)

$$L^{IMN,F} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} + \Xi_t^{IMN*} CONST R_t^{IMN*} \right\}$$
(46)

where the vectors of constraints $CONSTR_t^{IMN}$ now contains equations (28), (30), (32) and (34c). $CONSTR_t^{IMN*}$ contains equations (29), (31), (33) and the relevant financial market related constraints for the foreign country. In particular, since the domestic currency bond is not traded internationally, there is no foreign country equivalent to equation (34c). The right constraint to consider for the uncoordinated Ramsey problem of the Foreign country is instead given by the equation relating the expected changes in marginal utilities across countries to the expected exchange rate changes⁹, together with the foreign country's budget constraint. Ξ_t^{IMN} is now a 1x5 vector, with Lagrange multipliers $\xi_{1H,t}^{IMN}$ to $\xi_{5H,t}^{IMN}$ as elements, and similarly for Ξ_t^{IMN*} . The first order conditions w.r.t. $\xi_{4H,t}^{IMN}$ and $\xi_{4F,t}^{IMN}$ return 2 equations for the internationally traded asset, the first order conditions w.r.t. $\xi_{5H,t}^{IMN}$ and $\xi_{5F,t}^{IMN}$ return the Home and Foreign budget contraint, one of each of which is redundant.

Further details about the exact setup of the Ramsey optimal policy problems can be found in the appendix.

$$\left(1 + \psi RER_t b_{F,t}\right) \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} = \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{RER_{t+1}}{RER_t}$$

 $^{^{9}}$ This is obtained from combining the Foreign and Home agent's first order condition w.r.t. the foreign currency bond, that is:

2.4 Parameterization

The parameterization of the model is summarized in Table 1. The discount factor β is taken to be 0.99, implying an annual interest rate of about 4 percent. Parameter θ is taken to be 6, which implies a markup over marginal cost of about 20 percent. Parameter γ (γ^*), which is the weight on domestic good in the domestic (foreign) consumption basket, is set to 0.75 (0.25) in the baseline case, implying that there is positive home bias. Following Bergin et al. (2007) and Faia and Monacelli (2004) the parameter of the quadratic adjustment cost in price setting is taken to be 50. The degree of risk aversion, σ , is considered to be 1 in the baseline parameterization (which implies log utility in consumption), the inverse Frisch elasticity of labor supply, κ , is equal to 3, a value commonly used in the real business cycle literature. The production subsidy parameter is set such that it offsets the monopolistic competition distortion, that is, τ , is set equal to $1/(\theta - 1)$. As for the exogenous processes, I consider a technology shock persistence of ρ_Z , $\rho_Z^* = 0.95$ and standard deviation of the shock of σ_Z , $\sigma_Z^* = 0.01$.

Finally, I consider a wide range for the value of the trade elasticity, ranging from goods being very complementary in consumption to goods being very substituable. There is no consensus on the choice of the value of this elasticity in the literature. In the trade literature, Trefler and Lai (1999) estimate, for individual goods, very high trade elasticities ranging between 1.2 and 21.4. In the (real) business cycle literature, the trade elasticity is typically taken to be lower. Backus, Kehoe and Kydland (1995) use elasticities between 0 and 5, Chari et al. (2002) assume a value of 1.5, Anderson and van Wincoop choose values between 5 and 10. A number of recent contributions have also emphasized the role of a low elasticity of intratemporal substitution (well into the complementarity region) together with an incomplete financial markets structure in the transmission of productivity shocks across countries, in particular in addressing stylized facts on international relative prices such as exchange rate volatility, terms of trade volatility or the sign of transmission (see e.g., Heathcote and Perri (2002), Corsetti, Dedola and Leduc (2008), Thoenissen (2008), Enders and Mueller (2006)).

As I show, the value if the trade elasticity is a most crucial parameter in determining the influence of the terms of trade volatility on optimal monetary policy in an open economy, and is generally responsable for deviating from prescriptions of price stability as an optimal policy. Also, I will show that for the case of complementary goods welfare gains from coordination are found to be an order of magnitude larger.

3 Results

3.1 Ramsey Steady State

Section 2 has shown that the Ramsey equilibrium under the various financial market assumption and under coordination or Nash is obtained as the system of equations of first order conditions derived from the appropriate Ramsey problem. To determine the long-run inflation rate associated to the optimal policy problem above, one needs to solve the steady-state version of the set of efficiency conditions. In that steady-state, from the first order condition with respect to $\pi_{H,t}$ and $\pi_{F,t}^*$ it can be seen that the steady state (gross) inflation rate as-

sociated the to optimal policy problem equals 1.¹⁰ Hence the Ramsey planner would like to generate an average (net) inflation rate of zero. The intuition for why the long-run optimal inflation rate is zero is simple. Under commitment, the planner cannot resort to ex-post inflation as a device for eliminating the inefficiency related to market power in the goods market. Hence the planner aims at choosing that rate of inflation that minimizes the cost of adjusting prices and is summarized by the quadratic term.

One may wonder why the openness dimension does not apparently exert any influence on the desired optimal long-run inflation rate. The desire of adjusting the terms of trade can drive the planner's behavior only in the presence of equilibrium fluctuations (as induced by country-specific shocks) around the same long-run steady state. In other words, under commitment, the planner cannot on average resort to movements in inflation to alter the relative purchasing power of domestic residents. Thus, under commitment, the desire to influence the terms of trade and/or the real exchange rate shapes the optimal policy behavior only outside the long-run steady state.

3.2 Transmission under flexible versus sticky prices

To facilitate the analysis of optimal monetary policy, I first examine a useful benchmark in which price adjustment is flexible, and then describe the dynamics under sticky prices. Under flexible prices a productivity increase in the domestic economy lead to a higher abundance of domestic goods. This translates into a decrease in the price of domestic goods resulting in a depreciation of the domestic terms of trade, making domestic goods relatively cheaper and channeling world demand to the demand for domestic goods. Figure (1) shows the responses to the domestic productivity shock of major variables for the three financial market structures (CM, FA, and IM-Bond) and for the case where goods are either substitutes ($\omega = 3$), complements ($\omega = 0.7$) or are unit-elastic ($\omega = 1$).

Let's focus first on the case of goods being *substitutes* and consider the scenario of complete financial markets. The increase in domestic productivity leads to a domestic consumption increase, labor effort rise as the home economy gets more productive and the terms of trade deteriorate. Enjoying a more favorable price and because it is easy to sustitute to the now more abundant domestic good the foreign country also benefits from the domestic productivity shock. In particular, under complete markets, the terms of trade depreciate just enough to equalize the marginal utility benefit from the productivity shock in both countries, as dictated by the risk sharing equation. In the other extreme case of financial autarky, the response of the terms of trade is somewhat less pronounced. While the terms of trade still depreciates as an equilibrium response to the now more abundant domestic goods, it does so to a much

$$0 = \xi_{2H,t}^{i} \alpha \left(\pi_{H,t} - 1 \right) + \left(\xi_{3H,t}^{i} - \xi_{3H,t-1}^{i} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}} \left(TOT_{t} \right)}{f^{p_{H}} \left(TOT_{t-1} \right)} \right) \left[\alpha \left(2\pi_{H,t} - 1 \right) \right] - \xi_{1H,t-1}^{i} R_{t-1} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}} \left(TOT_{t} \right)}{f^{p_{H}} \left(TOT_{t-1} \right)} \right) \left[\alpha \left(2\pi_{H,t} - 1 \right) \right] - \xi_{1H,t-1}^{i} R_{t-1} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}} \left(TOT_{t} \right)}{f^{p_{H}} \left(TOT_{t-1} \right)} \right) \left[\alpha \left(2\pi_{H,t} - 1 \right) \right] - \xi_{1H,t-1}^{i} R_{t-1} \left(\frac{C_{t}}{C_{t-1}} \right)^{-\sigma} \frac{f^{p_{H}} \left(TOT_{t-1} \right)}{f^{p_{H}} \left(TOT_{t-1} \right)} \right]$$

which, at steady state (as $\xi_{3H,t}^i = \xi_{3H,t-1}^i$ and $\xi_{1H,t}^i = 0$) implies

$$0 = \xi_{2H,t}^i \alpha \left(\pi_{H,t} - 1 \right).$$

¹⁰This is the case even if the monopolistic distortion were not offset. In particular, the Ramsey first order condition w.r.t. $\pi_{H,t}$ is given by:

lesser extent than where the marginal utility gain in the foreign economy is as high as in the home country. As no state-contingent assets have been traded promising Foreign part of the benefits, Home labor effort does not increase, the expansion in domestic output is therefore lower than in the complete markets case, and the fall in the price of domestic goods relative to foreign goods (that is, the terms of trade deterioration) in turn less pronounced.

The transmission of the productivity shock is somewhat different when goods are *complements*. Generally speaking, a lower elasticity of substitution implies that for any given change in quantities, higher movements in the price are necessary to bring about these movements in quantities. That is, under all financial market structures, the terms of trade responses are now much stronger than in the case where goods are substitutes. In addition, the TOT now depreciates more in the case of incomplete financial markets than under complete markets. Because home and foreign goods are complementary in utility from consumption, the (productivity-induced) higher abundance of domestic goods also leads to a higher demand for foreign goods. If markets are complete the foreign country is therefore bound to expand its output by increasing its labor effort which tends to take some of the pressure of the terms of trade increasing. Under financial autarky such an increase in foreign output is absent, as a result the increased demand for the foreign goods without a counterbalancing increase in supply for it leads to a deterioration of the terms of trade that is even stronger. The lower the trade elasticity, the stronger is the terms of trade depreciation, and the foreign country increasingly benefits from the domestic productivity increase.

Finally, we turn to the case in which goods are *unit-elastic*. If the elasticity is unity then relative price changes are completely offset by changes in output volumes. In this knifeedge case, the income effect of the required terms of trade depreciation (given the relatively higher productivity in Home) balances the incentive to switch expenditure towards Home goods. they imply that relative wealth is always unaffected in response to country specific shocks and that complete risk sharing is always obtained independent of the financial market structure assumed.¹¹

Under sticky prices, it is costly for firms to change their prices which cannot adjust instantaneously. As is well known in the literature, a policy of producer price targeting would, however, lead to an exact replication of the flexible price allocation. In such case, firms choose prices so as to insure that, on average, they will operate on their flex-price supply curve, stabilizing their marginal cost. If prices were set at a level below that, market demand for the firms' goods turns out to be excessively high, and firms would need to hire excessive labor to meet demand at unchanged prices, sacrificing their profits. If prices were set at an excessively high level, firms' sales revenue turns out to be too low and labor (and therfore output) would be too low. In particular, under sticky prices the monetary policymaker has control over the nominal exchange rate. If P_H and P_F^* are rigid, the policymaker can initiate a nominal depreciation of the home currency (a higher ε) such that the home terms of trade worsens, such as the its response matches the one under flexible prices. When the home currency weakens, Home goods are cheaper relative to Foreign goods in both Home and the Foreign country. As demand shifts in favor of the goods with the lowest relative price, world consumption of Home goods increases relative to consumption of Foreign goods, which is known as "expenditure switching" effect of the exchange rate. While the replication if the

¹¹Strictly speaking, the threshold where relative price changes are completely offset by changes in output volumes lies only at unity because of my assumption of log-utility (that is a coefficient of relative risk aversion, $\sigma = 1$). More generally, as shown by Benigno and Benigno (2003) this threshold depends on both the intraand the intertemporal elasticity of substitution, and lies at $\omega = \frac{1}{\sigma}$.

flex-price allocation is possible, the adjustment under sticky prices requires action on the part of the monetary policymaker.

But in the open economy, firms ignore the impact of their pricing and production decisions on the country's overall terms of trade. A decentralized equilibrium reflects this inefficiency, adding a further dimension to the policy problem. Furthermore, having inspected the striking differences in the behavior of the terms of trade across different financial market stuctures, it should be expected that the policy incentives from the terms of trade inefficiency have also strikingly different implications on the optimal policy prescription.

Also, the fact that the TOT under incomplete markets depreciate too little (relative to the efficient economy) when goods are substitutes, but depreciate too much when goods are complements will help us understand why under incomplete markets even a planner acting under coordination will find it optimal to deviate from price stability.

3.3 The Role of Financial Market Structure and the Trade Elasticity for Stabilization

Allowing for a non-unitary elasticity implies that terms-of-trade volatility becomes important in the consideration for optimal policy, which many previous contributions to the literature have not addressed. This section presents results on how the structure of international asset markets can change the way monetary policy should be conducted and analyzes the implications of the terms of trade externality. In particular, such externality generally affects the optimality of inward-looking policies, unless the economies are insular to terms of trade movements. The desire of adjusting the terms of trade (or the real exchange rate) is generally sufficient to induce the planner to deviate from choosing a constant markup allocation. However, as outlined in section 3.1 these considerations can drive the planner's behavior only in the presence of equilibrium fluctuations (coming from country-specific shocks) around the long-run steady state. Therefore, the "optimal policy" is studied here in the sense of optimal stabilization in response to shocks.

Figure (2) studies the optimal producer price inflation responses on impact of a 1% domestic productivity shock, that is, unlike the regular impulse responses it ignores the time dimension of the shock. With the setup of our model, the responses decline out relatively smoothly such that it suffices to study the responses on impact of the shock to gain an understanding of the workings of the model. The optimal impact responses of domestic and foreign producer price inflation are depicted over a large range of the elasticity of substitution between domestic and foreign goods (ranging from very complementary goods to very substitutable goods), and for the various scenarios of financial markets.

As a general result, it can be observed that, for all cases but the one of perfect risk sharing and coordination, the implications are that deviations from full (producer) price stability are optimal. While, independent of the financial market assumption, a policy of keeping producer price inflation at zero would replicate the flexible price outcome, this is found to be the optimal policy only in the case of complete markets and coordination, or in the special case of a unit elasticity and therefore automatic full risk sharing.

To better understand why this is the case we would like to also study the responses of other variables of interest. Figures (4) and (5) study the behavior of the terms of trade, consumption in Home and Foreign and labor effort in Home and Foreign, by looking at differences of the responses of these variables to the responses that would occur in a flexible price version.

As discussed previously, in response to a 1% productivity increase in the domestic economy, the terms of trade depreciates channeling demand to the now more abundant domestic good, both under flexible prices or the sticky price optimal monetary policy economy. Figure 2 shows that in the case of complete financial markets under policy coordination the *TOT* responds *exactly* as in the flexible price world, the differences between the responses under the two scenarios (sticky-price optimal and the flexible-price) being zero. For all other cases we observe that the terms of trade either appreciate (CM, Nash) or depreciate (FA, Nash and Cooperation) relative to the flexible price responses, in line with the observation that pure producer price inflation targeting is not found to be optimal.

In particular, under *complete markets* I find that (producer) price stability is always optimal when policymakers coordinate, as seen by the firm black line from Figure (2). However, deviations from price stability are found to be optimal in response to a productivity shock if policymakers act uncoordinately: producer price inflation is negative and the TOT is more appreciated relative to a flexible price outcome when goods are substitutes. Policymakers have an incentive to let the terms of trade (or the real exchange rate) fluctuate less that what would be dictated by perfect risk sharing, thereby aiming at generating a lower volatility of labor effort. When acting independently, each policymaker has an incentive to appreciate the price of the terms of trade a bit, increasing the price of their own goods by reducing output and employment. As consumption risk is shared and domestic goods can easily be substituted by foreign goods, the reduction in employment would be increasing their welfare, with the prospect of keeping the same utility from consumption. In a Nash equilibrium, however, this attempt is unsuccessful. As the policymakers of both countries follow this incentive, in a Nash equilibrium employment, and therefore output, will be too low on average, leading to a decrease also in consumption (relative to the efficient policy of producer price stability), which can be seen in Figure 3. When goods are complements, the incentive for the home policymaker to contract employment and push some of the output to the foreign economy is absent, as foreign goods consumption cannot substitute consumption of domestic goods. On the contrary, the incentive is to render foreign goods even cheaper. As a result, when goods are complements, producer price inflation is positive following the domestic productivity increase, and the TOT is more depreciated relative to its flexible price response. Only in the case of a unit elasticity of intratemporal substitution the economies are insular with respect to TOT movements and the Nash outcome and coordination deliver the same result (of prescription of price stability as optimal policy).

When we consider the scenario of *financial autarky* Figure (2) shows that the TOT is found to be more depreciated (compared to a flexible price scenario) when goods are substitutes, and the inflation response is positive. If a non-coordinated policymakers now were to reduce employment, this would still benefit agents by increasing the utility of leisure; unlike under complete markets, consumption risk is not shared and consumption is much more closely tied to current output. As productivity is currently high it pays off to increase output so much that the terms of trade depreciate even more than in the flexible price scenario. Only in the case where goods are complements domestic agents have an incentive to appreciate their terms of trade again and to contract output relative to the flexible price outcome. As a result, the prescription of an optimal policy flips again when crossing the area over from goods being substitutes into the complementarity region: in the latter case the TOT is found to be more appreciated relative to the flexible price inflation decreases in response to a productivity shock. It is interesting to note, that even if we consider the optimal policy of a policymakers that acts coordinately, we find the same qualitative implications of

deviating from price stability. In particular, a coordinated policymaker will find it optimal to depreciate the terms of trade even more when goods are complements, taking account of the fact that production should take place in the more productive economy and that over a lower price of domestic goods both economies benefit (similarly, the coordinated planner will find it optimal to appreciate the terms of trade even more relative to the flex price case when goods are substitutes). The finding that not even a coordinated policymaker will find it optimal to replicate the (non-distorted) flexible price equilibrium, may appear puzzling at first. It can be understood however, by realizing that the flex price world under financial autarky is not a first best world, as the two countries do not involve in any risk sharing. From studying flexible price impulse responses in section 3.2 under the various financial market scenarios, we have seen that the TOT under incomplete markets (financial autarky or bond economy) depreciate too little (relative to the efficient economy with risk sharing) when goods are substitutes, but depreciate too much when goods are complements. A planner that, because of the presence of price rigidities, has some control over the terms of trade (or the real exchange rate), will therefore find it optimal to push it closer to the responses that would prevail in the complete markets case, thereby obtaining some risk sharing through the relative price. Policymakers under a sticky-price incomplete financial markets can therefore improve over the flex-price (but incomplete markets) outcome.

Finally, in the *incomplete markets-bond economy* case, the optimal responses to a domestic productivity shock lie somewhere in between the cases of complete markets and financial autarky. This finding is not surprising, considering that the availability of the international bond allows for some consumption smoothing. In turn, how easily the bond can be used in consumption smoothing depend crucially on the parameter of the portfolio adjustment cost, ψ . As ψ becomes very large, the policy prescriptions will closely follow the ones under financial autarky, if ψ is very small the optimal policy in the bond economy will be closer to the complete markets case. With the chosen value, it turns out that a policymaker under Nash competition follows a policy that is closer to the full risk sharing case, while a coordinated policymaker's policy matches closer that under financial autarky.

3.4 The Role of Financial Market Structure and the Trade Elasticity for Gains from Coordination

The fact that the policy prescription under Nash competition generally differs from the policy prescription under coordination implies that there are welfare gains from coordination. Figure (6) shows that these are found increasing for elasticities of substitution away from unity and typically by an order of magnitude larger in the case of complementarity between domestic and foreign goods. The plots depict both conditional and unconditional welfare measures, expressed as a percent over the world average steady state consumption. In addition, while in the case where domestic and foreign goods can easily be substituted welfare gains from coordination are typically larger under complete markets than under financial autarky, the case is different when domestic and foreign goods are complementary in consumption. When goods are complements and the elasticity of substitution is very low, wealth effects from the large movements in the relative price become very large under financial autarky (while they are absent under complete markets). As a result, a coordinated planner, taking into account the relative price distortion, can acchieve much larger welfare gains.

Finally, as section 3.3 outlined, it is generally the case that the optimal policy under

incomplete markets (financial autarky or the bond economy) can improve upon the flexible price allocation by pushing the real exchange towards the case of perfect risk sharing. It is therefore of interest to consider the welfare gains of the various financial market regimes over the flexible price allocation which are depicted in Figure (7). Clearly, under complete markets Nash competition leads to welfare losses over a flexible price (and therefore efficient) allocation. Under financial autarky, however, Nash policymakers, even though they choose an inefficient level of the terms of trade volatility are able to achieve welfare gains over a flexible price allocation.

4 Conclusion

The analysis of this paper has shown that the elasticity of intratemporal substitution and assumptions on the international financial market structure are important determinants of optimal monetary policy in the open economy. In particular, a purely inward-looking policy of producer price stability is found to be optimal only in the very special case in which financial markets are complete and policymakers act coordinately, or in the case of a unit trade elasticity which provides automatic perfect risk sharing. In all other cases it is optimal for monetary policymaker to not only consider stabilizing internal prices but also the variability of international prices as the terms of trade (or the real exchange rate) in shaping their policy. In all but the special case where an inward-looking policy is optimal, there are gains from coordination to be achieved.

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6 Appendix

In the following the setup of the various Ramsey optimal policy problems are more explicitly discussed. The appendix discusses the different financial market assumptions (CM, FA, IM-Bond), and the case of coordinated vs. independent policymakers. In all cases, relative prices (the PPI-to-CPI ratio in the Home and Foreign economy and the real exchange rate) are expressed as functions of the terms of trade only, as described in section 2.1.7.

6.1 Complete Markets

6.1.1 Coordination

The coordinated planner maximizes:

$$\begin{split} \max_{\{C,t,C_{*}^{*},Lt,L_{t}^{*},\pi_{t}^{*},\pi_{t}^{*},TOT_{t},R_{t},R_{t}^{*},\Xi_{t}^{CMC}\}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{L_{t}^{1+\kappa}}{1+\kappa} + \frac{C_{t}^{*1-\sigma}}{1-\sigma} - \frac{L_{t}^{*1+\kappa}}{1+\kappa} \right\} \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H1,t}^{CMC} \left[\beta E_{t} \left\{ R_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{pH}(TOT_{t+1})}{f^{p^{*}}(TOT_{t})\pi_{H,t+1}} \right\} - 1 \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H2,t}^{CMC} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{p^{*}}f(TOT_{t+1})}{f^{p^{*}}(TOT_{t})\pi_{H,t+1}} \right\} - 1 \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H2,t}^{CMC} \left[\frac{\alpha(\pi_{H,t} - 1)\pi_{H,t} + [f^{pH}(TOT_{t})]^{-\omega} \left[C_{t} + [f^{RER}(TOT_{t})]^{\omega} C_{t}^{*} \right] \\ \left[\frac{\theta[L_{t}^{\kappa}C_{t}^{\sigma}]}{2t_{t}[f^{PH}(TOT_{t})]} - (\theta - 1)(1+\tau) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{p^{*}}f(TOT_{t+1})}{f^{p^{*}}(TOT_{t})} \alpha(\pi_{H,t+1} - 1)\pi_{H,t+1} \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{CMC} \left[\frac{\theta[L_{t}^{\kappa}C_{t}^{\sigma}]}{\left[\frac{\theta[L_{t}^{\kappa}C_{t}^{\sigma}]}{2t_{t}f^{p^{*}}(TOT_{t})} - (\theta - 1)(1+\tau) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{p^{*}}f(TOT_{t+1})}{f^{p^{*}}(TOT_{t})} \alpha\left(\pi^{*}_{F,t+1} - 1 \right) \pi^{*}_{F,t+1} \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{CMC} \left[[f^{PH}(TOT_{t})]^{-\omega} \left[C_{t} + [f^{RER}(TOT_{t})]^{\omega} C_{t}^{*} \right] - Z_{t}L_{t} \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H3,t}^{CMC} \left[[f^{p^{*}}(TOT_{t})]^{-\omega} \left[(f^{RER}(TOT_{t})]^{-\omega} C_{t} + C_{t}^{*} \right] - Z_{t}L_{t} \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H3,t}^{CMC} \left[[f^{p^{*}}(TOT_{t})]^{-\omega} \left[(f^{RER}(TOT_{t})]^{-\omega} C_{t} + C_{t}^{*} \right] - Z_{t}L_{t}^{*} \right] \\ + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H3,t}^{CMC} \left[f^{RER}(TOT_{t}) - \left(\frac{C_{t}^{*}}{C_{t}} \right)^{-\sigma} \right] \right]$$

6.1.2 Nash

The home policymaker maximizes:

$$\max_{\{C,t,L_t,\pi_{H,t},TOT_t,R_t,\Xi_t^{CMN}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} \right\}$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H1,t}^{CMN} \left[\beta E_t \left\{ R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)\pi_{H,t+1}} \right\} - 1 \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H2,t}^{CMN} \left[\frac{-\alpha \left(\pi_{H,t} - 1 \right) \pi_{H,t} + \left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] \right] }{\left[\frac{\theta[L_t^{\kappa} C_t^{\sigma}]}{Z_t [f^{p_H}(TOT_t)]} - (\theta - 1) \left(1 + \tau \right) \right] + E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)} \alpha \left(\pi_{H,t+1} - 1 \right) \pi_{H,t+1} \right] }$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H3,t}^{CMN} \left[\left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] - Z_t L_t \right]$$

$$+E_0\sum_{t=0}^{\infty}\beta^t\xi_{H4,t}^{CMN}\left[f^{RER}\left(TOT_t\right)-\left(\frac{C_t^*}{C_t}\right)^{-\sigma}\right]$$

The foreign policymaker maximizes:

$$\max_{\{C_{t}^{*}, L_{t}^{*}, \pi_{F,t}^{*}, TOT_{t}, R_{t}^{*}, \Xi_{t}^{CMN*}\}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{C_{t}^{*1-\sigma}}{1-\sigma} - \frac{L_{t}^{*1+\kappa}}{1+\kappa} \right\}$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F1,t}^{CMN} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{p_{F}^{*}}(TOT_{t+1})}{f^{p_{F}^{*}}(TOT_{t})\pi_{F,t+1}^{*}} \right\} - 1 \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{CMN} \left[\left[-\alpha \left(\pi_{F,t}^{*} - 1 \right) \pi_{F,t}^{*} + \left[f^{p_{F}^{*}} \left(TOT_{t} \right) \right]^{-\omega} \left[\left[f^{RER} \left(TOT_{t} \right) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{CMN} \left[\left[\frac{\theta [L_{t}^{*\kappa} C_{t}^{*\sigma}]}{Z_{t}^{*} f^{p_{F}^{*}}(TOT_{t})} - \left(\theta - 1 \right) \left(1 + \tau \right) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{p_{F}^{*}}(TOT_{t+1})}{f^{p_{F}^{*}}(TOT_{t})} \alpha \left(\pi_{F,t+1}^{*} - 1 \right) \pi_{F,t+1}^{*} \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{CMN} \left[\left[f^{p_{F}^{*}} \left(TOT_{t} \right) \right]^{-\omega} \left[\left[f^{RER} \left(TOT_{t} \right) \right]^{-\omega} C_{t} + C_{t}^{*} \right] - Z_{t}^{*} L_{t}^{*} \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F4,t}^{CMN} \left[f^{RER} \left(TOT_{t} \right) - \left(\frac{C_{t}^{*}}{C_{t}} \right)^{-\sigma} \right]$$

6.2 Financial Autarky

6.2.1 Coordination

The coordinated planner maximizes:

$$\begin{aligned} & \text{for continue cut plainies interactions} \\ & \text{max} \\ & \{C_{t}, C_{t}^{*}, L_{t}, L_{t}^{*}, \pi_{H,t}, \pi_{F,t}^{*}, TOT_{t}, R_{t}, R_{t}^{*}, \Xi_{t}^{FAC}\} \\ & E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \sum_{t=0}^{1-\sigma} - \frac{L_{t}^{1+\kappa}}{1+\kappa} + \frac{C_{t}^{*1-\sigma}}{1-\sigma} - \frac{L_{t}^{*1+\kappa}}{1+\kappa} \right\} \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H1,t}^{FAC} \left[\beta E_{t} \left\{ R_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{pH}(TOT_{t+1})}{f^{p_{F}^{*}}(TOT_{t})\pi_{H,t+1}} \right\} - 1 \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F1,t}^{FAC} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}^{*}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{p_{F}^{*}}(TOT_{t})\pi_{H,t+1}}{f^{p_{F}^{*}}(TOT_{t})\pi_{H,t+1}^{*}} \right\} - 1 \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H2,t}^{FAC} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}^{*}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{p_{F}^{*}}(TOT_{t})\pi_{H,t+1}^{*}}{f^{p_{F}^{*}}(TOT_{t})} \right] - \left(\theta - 1 \right) (1+\tau) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{pH}(TOT_{t+1})}{f^{pH}(TOT_{t})} \alpha \left(\pi_{H,t+1} - 1 \right) \pi_{H,t+1} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{FAC} \left[\left[\frac{\theta[L_{t}^{*}C_{t}^{*\sigma}]}{(2t|f^{pH}(TOT_{t})} - \left(\theta - 1 \right) (1+\tau) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{p_{F}^{*}}(TOT_{t+1})}{f^{p_{F}^{*}}(TOT_{t})} \alpha \left(\pi_{F,t+1}^{*} - 1 \right) \pi_{F,t+1}^{*} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAC} \left[\left[f^{pH}(TOT_{t}) \right]^{-\omega} \left[C_{t} + \left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAC} \left[\left[f^{p_{F}}(TOT_{t}) \right]^{-\omega} \left[C_{t} + \left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAC} \left[\left[f^{p_{F}}(TOT_{t}) \right]^{-\omega} \left[\left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \\ & - Z_{t}^{*} L_{t}^{*} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAC} \left[f^{p_{F}}(TOT_{t}) \right]^{-\omega} \left[\left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \\ & - Z_{t}^{*} L_{t}^{*} \right] \\ & + E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAC} \left[f^{p_{F}}(TOT_{t}) \right]^{-\omega} \left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \\ & - Z_{t}^{*} L_{t}^{*} \right] \\ & - Z_{t}^{*} L_{t}^{*} \right] \\ & - Z_{t}^{*} L_{t}^{*} \left[f^{p_{F}}(TOT_{t}) \right] \\ & - Z_{t}^{*} L_{t}^{*} L_{t}^{*} L_{t}^{$$

6.2.2 Nash

The home policymaker maximizes:

$$\max_{\{C,t,L_t,\pi_{H,t},TOT_t,R_t,\Xi_t^{FAN}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} \right\}$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H1,t}^{FAN} \left[\beta E_t \left\{ R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)\pi_{H,t+1}} \right\} - 1 \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H2,t}^{FAN} \left[\frac{-\alpha \left(\pi_{H,t} - 1 \right) \pi_{H,t} + \left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] \right] }{\left[\frac{\theta [L_t^{\kappa} C_t^{\sigma}]}{Z_t [f^{p_H}(TOT_t)]} - \left(\theta - 1 \right) \left(1 + \tau \right) \right] + E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)} \alpha \left(\pi_{H,t+1} - 1 \right) \pi_{H,t+1} \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H3,t}^{FAN} \left[\left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] - Z_t L_t \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H4,t}^{FAN} \left[f^{p_H} \left(TOT_t \right) Z_t L_t - C_t - \phi_t \right]$$

The foreign policymaker maximizes:

$$\max_{\{C_{t}^{*}, L_{t}^{*}, \pi_{F,t}^{*}, TOT_{t}, R_{t}^{*}, \Xi_{t}^{FAN*}\}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{C_{t}^{*1-\sigma}}{1-\sigma} - \frac{L_{t}^{*1+\kappa}}{1+\kappa} \right\}$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F1,t}^{FAN} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}^{*}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{p}_{F}(TOT_{t+1})}{f^{p}_{F}(TOT_{t})\pi_{F,t+1}} \right\} - 1 \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{FAN} \left[-\alpha \left(\pi_{F,t}^{*} - 1 \right) \pi_{F,t}^{*} + \left[f^{p}_{F}(TOT_{t}) \right]^{-\omega} \left[\left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] \right] \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F2,t}^{FAN} \left[\frac{\theta[L_{t}^{*\kappa}C_{t}^{*\sigma}]}{\left[\frac{\theta[L_{t}^{*\kappa}C_{t}^{*\sigma}]}{Z_{t}^{*}f^{p}_{F}(TOT_{t})} - (\theta - 1)(1 + \tau) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{p}_{F}(TOT_{t+1})}{f^{p}_{F}(TOT_{t})} \alpha \left(\pi_{F,t+1}^{*} - 1 \right) \pi_{F,t+1}^{*}$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F3,t}^{FAN} \left[\left[f^{p}_{F}(TOT_{t}) \right]^{-\omega} \left[\left[f^{RER}(TOT_{t}) \right]^{-\omega} C_{t} + C_{t}^{*} \right] - Z_{t}^{*} L_{t}^{*} \right]$$

6.3 Incomplete Markets Bond Economy

6.3.1 Coordination

The coordinated planner maximizes:

$$\max_{\{C,t,C_{t}^{*},L_{t},L_{t}^{*},\pi_{H,t},\pi_{F,t}^{*},TOT_{t},R_{t},R_{t}^{*},\Xi_{t}^{IMC}\}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{C_{t}^{1-\sigma}}{1-\sigma} - \frac{L_{t}^{1+\kappa}}{1+\kappa} + \frac{C_{t}^{*1-\sigma}}{1-\sigma} - \frac{L_{t}^{*1+\kappa}}{1+\kappa} \right\}$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H1,t}^{IMC} \left[\beta E_{t} \left\{ R_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{pH}(TOT_{t+1})}{f^{pH}(TOT_{t})\pi_{H,t+1}} \right\} - 1 \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{F1,t}^{IMC} \left[\beta E_{t} \left\{ R_{t}^{*} \left(\frac{C_{t+1}^{*}}{C_{t}^{*}} \right)^{-\sigma} \frac{f^{pF}(TOT_{t+1})}{f^{pF}(TOT_{t})\pi_{F,t+1}^{*}} \right\} - 1 \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H2,t}^{IMC} \left[\left[-\alpha \left(\pi_{H,t} - 1 \right) \pi_{H,t} + \left[f^{pH} \left(TOT_{t} \right) \right]^{-\omega} \left[C_{t} + \left[f^{RER} \left(TOT_{t} \right) \right]^{\omega} C_{t}^{*} \right] \right] \right]$$

$$+ E_{0} \sum_{t=0}^{\infty} \beta^{t} \xi_{H2,t}^{IMC} \left[\left[\frac{\theta[L_{t}^{\kappa}C_{t}^{\sigma}]}{\left[\frac{\ell[L_{t}^{\kappa}C_{t}^{\sigma}]}{Z_{t}[f^{pH}(TOT_{t})]} - \left(\theta - 1 \right) \left(1 + \tau \right) \right] + E_{t} \left(\frac{C_{t+1}}{C_{t}} \right)^{-\sigma} \frac{f^{pH}(TOT_{t+1})}{f^{pH}(TOT_{t})} \alpha \left(\pi_{H,t+1} - 1 \right) \pi_{H,t+1} \right]$$

$$+E_{0}\sum_{t=0}^{\infty}\beta^{t}\xi_{F2,t}^{IMC}\left[\begin{array}{c}-\alpha\left(\pi_{F,t}^{*}-1\right)\pi_{F,t}^{*}+\left[f^{p_{F}^{*}}\left(TOT_{t}\right)\right]^{-\omega}\left[\left[f^{RER}\left(TOT_{t}\right)\right]^{-\omega}C_{t}+C_{t}^{*}\right]\right]\right]\\\left[\frac{\theta\left[L_{t}^{*\kappa}C_{t}^{*\sigma}\right]}{\left[Z_{t}^{*}f^{p_{F}^{*}}\left(TOT_{t}\right)}-\left(\theta-1\right)\left(1+\tau\right)\right]+E_{t}\left(\frac{C_{t+1}}{C_{t}}\right)^{-\sigma}\frac{f^{p_{F}^{*}}\left(TOT_{t+1}\right)}{f^{p_{F}^{*}}\left(TOT_{t}\right)}\alpha\left(\pi_{F,t+1}^{*}-1\right)\pi_{F,t+1}^{*}\right]\right]\\+E_{0}\sum_{t=0}^{\infty}\beta^{t}\xi_{H3,t}^{IMC}\left[\left[f^{p_{H}}\left(TOT_{t}\right)\right]^{-\omega}\left[C_{t}+\left[f^{RER}\left(TOT_{t}\right)\right]^{-\omega}C_{t}+C_{t}^{*}\right]-Z_{t}L_{t}\right]\right]\\+E_{0}\sum_{t=0}^{\infty}\beta^{t}\xi_{F3,t}^{IMC}\left[\left[f^{p_{F}^{*}}\left(TOT_{t}\right)\right]^{-\omega}\left[\left[f^{RER}\left(TOT_{t}\right)\right]^{-\omega}C_{t}+C_{t}^{*}\right]-\left(1+\psi RER_{t}b_{F,t}\right)\right]\right]\\+E_{0}\sum_{t=0}^{\infty}\beta^{t}\xi_{4,t}^{IMC}\left[\beta E_{t}\left\{R_{t}^{*}\left(\frac{C_{t+1}}{C_{t}}\right)^{-\sigma}\frac{f^{p_{F}^{*}}\left(TOT_{t+1}\right)}{f^{p_{F}^{*}}\left(TOT_{t}\right)\pi_{F,t+1}^{*}}\frac{f^{RER}\left(TOT_{t+1}\right)}{f^{RER}\left(TOT_{t}\right)}\right\}-\left(1+\psi RER_{t}b_{F,t}\right)\right]\\+E_{0}\sum_{t=0}^{\infty}\beta^{t}\xi_{5,t}^{IMC}\left[RER_{t}b_{Ft-1}\frac{R_{t-1}^{*}}{\pi_{t}^{*}}+f^{p_{H}}\left(TOT_{t}\right)Z_{t}L_{t}-C_{t}-\phi_{t}-RER_{t}b_{F,t}-\frac{\phi}{2}\left(RER_{t}b_{F,t}\right)^{2}\right]$$

6.3.2 Nash

The home policymaker maximizes:

$$\max_{\{C,t,L_t,\pi_{H,t},TOT_t,R_t,\Xi_t^{IMN}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\kappa}}{1+\kappa} \right\}$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H1,t}^{IMN} \left[\beta E_t \left\{ R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)} \frac{1}{\pi_{H,t+1}} \right\} - 1 \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H2,t}^{IMN} \left[\frac{-\alpha \left(\pi_{H,t} - 1 \right) \pi_{H,t} + \left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H3,t}^{IMC} \left[\frac{\theta[L_t^{\kappa} C_t^{\sigma}]}{\left[\frac{\mathcal{U}_t^{\Gamma} TOT_{t-1}}{2t[f^{p_H}(TOT_t)]} - \left(\theta - 1 \right) \left(1 + \tau \right) \right] + E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_H}(TOT_{t+1})}{f^{p_H}(TOT_t)} \alpha \left(\pi_{H,t+1} - 1 \right) \pi_{H,t+1} \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H3,t}^{IMC} \left[\left[f^{p_H} \left(TOT_t \right) \right]^{-\omega} \left[C_t + \left[f^{RER} \left(TOT_t \right) \right]^{\omega} C_t^* \right] - Z_t L_t \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H4,t}^{IMN} \left[\beta E_t \left\{ R_t^* \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_F}(TOT_{t+1})}{f^{p_F}(TOT_t) \pi_{F,t+1}^*} \frac{f^{RER}(TOT_{t+1})}{f^{RER}(TOT_t)} \right\} - \left(1 + \psi RER_t b_{F,t} \right) \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{H5,t}^{IMN} \left[RER_t b_{Ft-1} \frac{R_{t-1}^*}{\pi_t^*} + f^{p_H} \left(TOT_t \right) Z_t L_t - C_t - \phi_t - RER_t b_{F,t} - \frac{\phi}{2} \left(RER_t b_{F,t} \right)^2 \right]$$

The foreign policymaker maximizes:

$$\max_{\{C_t^*, L_t^*, \pi_{F,t}^*, TOT_t, R_t^*, \Xi_t^{IMN*}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{*1-\sigma}}{1-\sigma} - \frac{L_t^{*1+\kappa}}{1+\kappa} \right\}$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{F1,t}^{IMN} \left[\beta E_t \left\{ R_t^* \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{f^{p_F^*}(TOT_{t+1})}{f^{p_F^*}(TOT_t)} \frac{1}{\pi_{F,t+1}^*} \right\} - 1 \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{F2,t}^{IMN} \left[-\alpha \left(\pi_{F,t}^* - 1 \right) \pi_{F,t}^* + \left[f^{p_F^*} \left(TOT_t \right) \right]^{-\omega} \left[\left[f^{RER} \left(TOT_t \right) \right]^{-\omega} C_t + C_t^* \right] \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{F2,t}^{IMN} \left[\frac{\theta[L_t^{*\kappa}C_t^{*\sigma}]}{Z_t^* f^{p_F^*}(TOT_t)} - (\theta - 1) \left(1 + \tau \right) \right] + E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{p_F^*}(TOT_{t+1})}{f^{p_F^*}(TOT_t)} \alpha \left(\pi_{F,t+1}^* - 1 \right) \pi_{F,t+1}^* \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{F3,t}^{IMN} \left[\left[f^{p_F^*} \left(TOT_t \right) \right]^{-\omega} \left[\left[f^{RER} \left(TOT_t \right) \right]^{-\omega} - \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{RER}(TOT_{t+1})}{f^{RER}(TOT_t)} \right] \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \xi_{F4,t}^{IMN} \left[\left(1 + \psi RER_t b_{F,t} \right) \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} - \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{f^{RER}(TOT_{t+1})}{f^{RER}(TOT_t)} \right] \right]$$

7 Tables

Table 1: Model Parameters

discount factor	β	0.99
elasticity between varieties	θ	6
home bias	$\gamma, 1 - \gamma^*$	0.75
persistence of productivity shock	ρ_Z, ρ_Z^*	0.95
standard deviation of productivity shock	σ_Z, σ_Z^*	0.01
coefficient of relative risk aversion	σ	1
Rotemberg price adjustment cost parameter	α	50
Production subsidy offsetting monopolistic competition distortion	au	$1/(\theta - 1)$
Portfolio adjustment cost parameter	ψ	0.0007
trade elasticity between H and F consumption goods	ω	$\epsilon~[0.7,3]$

8 Figures



Figure 1: Impulse responses to a domestic productivity shock under flexible prices



Figure 2: Impact responses of optimal domestic and foreign producer price inflation to a domestic 1 % productivity shock



Figure 3: Impact responses of optimal domestic and foreign nominal interest rates to a domestic 1 % productivity shock



Figure 4: Differences of optimal TOT impact responses over flexible price TOT impact responses (to a domestic 1 % productivity shock), depending on the trade elasticity



Figure 5: Differences of optimal consumption and labor impact responses over flexible price impact responses (to a domestic 1 % productivity shock), depending on the trade elasticity



Figure 6: Welfare gains from coordination



Figure 7: Welfare gains over policy of price stability