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A joint macroeconomic-yield curve model for Hungary

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A joint macroeconomic-yield curve model for Hungary

(Együttes makro-hozamgörbe modell becslése magyar adatokon)

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Contents

Abstract

The main goal of this paper is to examine the relationship between macroeconomic shocks and yield curve movements in Hungary. To this end, we apply a Nelson-Siegel type dynamic yield curve model, where changes of the yield curve are driven by two latent factors and some key macro variables that follow a VAR(1) process. The structural macroeconomic shocks are identified by sign restrictions.

According to the model, more than sixty percent of the variation of the yield curve factors can be explained by macro shocks. In particular, the monetary policy shock is the most important determinant of the level factor, while the slope factor is mainly driven by risk premium and demand shocks.

As for the direction of the responses, monetary policy and supply shocks decrease long forward rates, while premium and demand shocks increase short forward rates. The effect of the premium and monetary policy shocks is strongest in the period when the shock occurs, while for the demand and supply shocks the responses reach their peak only after some delay.

JEL Classification: C32, E43, E44, G12.

Keywords: yield curve, Nelson-Siegel, factor models, state space models, structural identification.

Összefoglalás

A tanulmányban azt vizsgáljuk, hogy milyen kapcsolat áll fenn a magyarországi makrogazdasági környezet és a hozamgörbe változásai között. Ehhez egy dinamikus Nelson-Siegel típusú hozamgörbe modellt alkalmazunk, ahol a hozamgörbe időbeni alakulását két nem megfigyelt faktor és néhány fontos makrogazdasági változó határozza meg, a faktorok és a megfigyelt változók együttes dinamikája pedig egy VAR(1) folyamatot követ. A strukturális sokkokat előjelmegkötések segítségével identifikáljuk.

A becslési eredmények szerint a látens faktorok varianciájának közel hatvan százaléka megmagyarázható a makrosokkok segítségével. Ezen belül a hozamok szintjét leginkább a monetáris politikai sokkok mozgatják, míg a hozamgörbe meredekségére főleg a kockázati prémium és a keresleti sokkok vannak hatással.

Ami a hatások irányát illeti, a monetáris politikai és a kínálati sokkok a hosszú forward hozamokat csökkentik, míg a prémium és keresleti sokkok a rövid forward hozamokat növelik. A prémium és monetáris politikai sokkok hatása a sokk bekövetkeztének időpontjában a legerősebb, a keresleti és kínálati sokkok viszont csak késleltetve fejtik ki teljes hatásukat.

1 Introduction

In inflation targeting monetary policy regimes, central banks achieve their inflation goals by setting short interest rates. Interest rates in turn affect consumption, investment and pricing decisions, which then drive inflation back to target.

However, it is obvious that these decisions are not determined directly by the short rate only, but also by longer rates of various maturities. The efficiency of monetary policy therefore depends on the response of longer rates to changes in the short rate, and understanding this relationship is important from a central banking perspective.

To this end, we first have to realize that the effects of monetary policy shocks can only be correctly assessed if they can be separated from other major economic determinants of the yield curve. This means that we need to build a dynamic model that is able to explain the joint evolution of the yield curve and some key macroeconomic variables.

This paper is an effort to set up such a model for the Hungarian economy. The model is a Nelson-Siegel type factor model, where the joint dynamics of the yield curve factors and the macro variables is described by a VAR process. As the yield curve factors are not observed, the model is estimated by state-space methods, using monthly data on the MNB base rate,[1] the forint/euro exchange rate, inflation and output. Four structural shocks are identified, which are monetary policy, risk premium, demand and supply shocks. The identification method is a combination of short term exclusions and sign restrictions.

There have been several attempts at the MNB to assess the effect of monetary policy shocks on interest rates. Vonnák (2005) applies an SVAR identification based on sign restriction of responses, but examines only short (three-month) interest rates, while Rezessy (2005) identifies the shocks by exploiting heteroscedasticity using daily data, and considers responses of one, five and ten year yields. Kiss (2004) goes even further and uses intraday data to estimate the effects of MNB decisions and various other macroeconomic news announcements. The present paper can be regarded as a combination and extension of these earlier efforts: the identification strategy is similar to that of Vonnák (2005), and responses of yields with maturities up to ten years are examined. The novelty is that the reaction of the whole yield curve can be analyzed, and responses to shocks other than monetary policy are also discussed.

In the next section we give a short overview of the various approaches extant in the literature on macro - yield curve modeling, and give a more detailed description of the model used in this paper. Section 3 provides some basic properties of our data set. Section 4 contains the main results and several robustness checks, and Section 5 concludes. Some graphs and technical details are deferred to the Appendix.

[1] MNB is the abbreviation of Magyar Nemzeti Bank, the central bank of Hungary.

2 Dynamic yield curve models

In this section we give a brief overview of affine arbitrage free and Nelson-Siegel type models. Arbitrage free models are based on firm theoretical grounds set by such classics as Black & Scholes (1973), Harrison & Kreps (1979) and Harrison & Pliska (1981), and the majority of yield curve models used in academics and the financial industry fall in this class. Nelson-Siegel models were introduced recently in Diebold & Li (2006) and are therefore much less prevalent. On the other hand, they are a direct dynamic extension of established yield curve fitting methods, are relatively easy to estimate, and the results are easy to interpret. The main point of what follows in this section is to compare the two approaches, and motivate our choice for the latter. The discussion here closely follows Björk (1998), Duffie & Kan (1996) and Diebold *et al.* (2006); a comparison of the two frameworks can also be found in Diebold *et al.* (2005).

AFFINE ARBITRAGE FREE MODELS

Affine arbitrage free models assume frictionless markets and absence of arbitrage. Uncertainty in the market is captured by an n dimensional factor process:

$$dX(t) = \mu(t, X(t))dt + \sigma(t, X(t))dW(t), \tag{1}$$

where $W(t)$ is a standard n-dimensional Wiener process, μ and σ are functions of appropriate dimensions. One of the components of X is usually interpreted as the short rate, denoted by $r(t)$. On this market, zero coupon bonds are interpreted as contingent claims, i.e. we assume that for each $T \geq t$ there is a frictionless market for zero coupon bonds maturing at T, and the price of such a bond, $p(t, T)$ (with principal equal to one) is a function of the risk factors and time to maturity:

$$p(t, T) = F^T(t, X(t)), \quad F^T(T, X(T)) = 1. \tag{2}$$

Since there are $n \geq 1$ sources of uncertainty and no risky underlying assets, this market is not complete. Hence, a risk premium process λ also needs to be defined, which is an implicit assumption about the preferences of market participants.

For analytical convenience, it is usually assumed that F^T has the exponential affine form:

$$F^T(t, x) = \exp\left\{A^T(t) + B^T(t)x\right\}, \tag{3}$$

which also means that yields, defined as $y(\tau) = -\log p(t, t+\tau)/\tau$, are linear functions of the factors. Duffie & Kan (1996) and Dai & Singleton (2000) provide conditions on μ, σ and λ which ensure that the pricing function have the form in (3). These conditions imply that the factor dynamics in (1) is nonlinear.

Using standard techniques and absence of arbitrage, the functions $A^T(t)$ and $B^T(t)$ can be analytically calculated from the parameters μ, σ and λ. This means that if, for some reason, we prefer to have a specific functional form for the yield curve, this can only be achieved (if at all) by restricting the dynamics of the factors and the risk premium.

Equations (1) and (3) form a state space system, with a transition equation that is nonlinear in the factors.[2] Since this forbids the use of the Kalman-filter for calculating the likelihood function, one usually assumes that some (usually n) yields are observed without error, which means that, modulo a regularity condition, the unobserved factors can be calculated as linear combination of these yields. However, the choice of the observed yields is ad hoc.

[2] Unless we assume that σ is constant.

NELSON-SIEGEL MODELS

We have just seen that in arbitrage free models the form of the yield curve is determined by the parameters describing the dynamics of the underlying factors and the risk premium. The model introduced in Diebold & Li (2006) takes a different approach, and defines the form of the yield curve independently of the factor dynamics as

$$y(t,h) = \frac{-\log(p(t,t+h))}{h} = L_t + S_t\,\frac{1-e^{-\tau h}}{\tau h} + C_t\left(\frac{1-e^{-\tau h}}{\tau h} - e^{-\tau h}\right). \tag{4}$$

Here L_t, S_t and C_t are part of the factors, h is time to maturity, and τ is a parameter that governs the rate at the which the loadings of the yields on S_t and C_t are increasing/decreasing. The coefficient of L_t is independent of τ, and we also have $S_t = y(t,0) - y(t,\infty)$; these properties imply the interpretation of L_t as level and of S_t as slope, defined as short minus long rates. Also, the coefficient of C_t is zero at $\tau = 0,\infty$, hence the interpretation of C_t as curvature. This form of the yield curve is motivated by the Nelson-Siegel functional form introduced in Nelson & Siegel (1987) and widely used in day-to-day yield curve fitting exercises both by central banks and in the financial industry.

It is possible to fit the yield curve in (4) to the observed yields in every period, and then use the resulting time series of the coefficients in a VAR with macro variables to derive the dynamic properties of the system. This way, however, the sampling uncertainty of the factors would not be accounted for in the VAR estimation, and for this reason Diebold *et al.* (2006) treats the factors as unobserved variables, and estimates the model in a single step using the Kalman-filter.

Compared to arbitrage free models, the Nelson-Siegel model is more statistical in nature, and therefore requires less stringent assumptions about market frictions and investor preferences. Furthermore, there is no need to arbitrarily pick out yields that are treated as factors, and are assumed to be observed without error. Also, the dynamics of the factors can be defined arbitrarily, yet preserving the simple form of the yield curve.

In practice, estimation of the Nelson-Siegel model is straightforward, while Ang & Piazzesi (2003) and Dai & Singleton (2000), who use arbitrage free models, have to rely on complicated methods or use unrealistic restrictions, and still they report serious difficulties with the estimation. Bolder (2006) and Bolder & Liu (2007) carries out a systematic comparison of several dynamic yield curve models, which include the Ang & Piazzesi (2003) and the Nelson-Siegel model. According to their practice oriented criteria, the Nelson-Siegel model is found to be superior to the others.

We finally note that the Nelson-Siegel yield curve is in fact an affine function of the factors: if we introduce the notation

$$\tilde{A}^T(t) = 0, \quad \tilde{B}^T(t) = -\left(T-t,\ \frac{1-e^{-\tau(T-t)}}{\tau},\ \frac{1-e^{-\tau(T-t)}}{\tau} - (T-t)e^{-\tau(T-t)}\right), \quad X_t = (L_t, S_t, C_t)',$$

then the Nelson-Siegel bond prices will be given by

$$p(t,T) = \exp\left\{\tilde{A}^T(t) + \tilde{B}^T(t)X_t\right\}.$$

Christensen *et al.* (2007) cite Filipović (1999) to show that there is no factor dynamics that would imply this particular affine term structure, but they also show that with small modifications (in which $A^T(t)$ is modified to have nonzero values), the resulting yield curve can be supported by a three factor affine arbitrage free model (with restricted dynamics).

THE ESTIMATED MODEL

Both Diebold & Li (2006) and Diebold *et al.* (2006) find that the level and slope factors have strong correlation with macro variables, but the interpretation of the curvature factor is much less straightforward. Also, statistical tests suggest that the changes of the Hungarian yield curve can almost totally be explained by only two factors (see later). We therefore decided to follow Diebold *et al.* (2007), and use only the level and slope factors in the model.

In a more operational fashion, the structure of the model estimated in this paper can be summarized as follows:

- there are four observed macroeconomic variables, denoted by the vector $m_t \in \mathbb{R}^4$: inflation, output, MNB base rate and the exchange rate, described in detail in the next section,
- there are two unobserved yield curve factors, denoted by $f_t \in \mathbb{R}^2$,
- there are observed yields for 19 horizons, denoted by $y_t \in \mathbb{R}^{19}$,
- the joint evolution of the macro variables and the yield curve factors is described by an unrestricted VAR(1) process,
- the observed yields are linear combinations of the unobserved yield curve factors plus an error.

The state-space representation of this system is the following:

$$\begin{pmatrix} \hat{f}_t \\ \hat{m}_t \end{pmatrix} = F \begin{pmatrix} \hat{f}_{t-1} \\ \hat{m}_{t-1} \end{pmatrix} + w_t, \tag{5}$$

$$\begin{pmatrix} y_t \\ m_t \end{pmatrix} = \begin{pmatrix} H(\tau) & 0_{19\times 4} \\ 0_{4\times 2} & I_4 \end{pmatrix} \begin{pmatrix} \hat{f}_t \\ \hat{m}_t \end{pmatrix} + \begin{pmatrix} H(\tau) & 0_{19\times 4} \\ 0_{4\times 2} & I_4 \end{pmatrix} \begin{pmatrix} \mu_f \\ \mu_m \end{pmatrix} + v_t, \tag{6}$$

where $\hat{f}_t$ and $\hat{m}_t$ are the demeaned factors and macro variables, and μ_f and μ_m are the means. We assume that the error terms have the usual properties:

$$E\left(w_t' w_t\right) = \Omega, \qquad E\left(w_s' w_t\right) = 0 \quad \text{for } t \neq s, \tag{7}$$

$$E\left(v_t' v_t\right) = \begin{pmatrix} \Gamma & 0_{19\times 4} \\ 0_{4\times 19} & 0_{4\times 4} \end{pmatrix}, \qquad E\left(v_s' v_t\right) = 0 \quad \text{for } t \neq s. \tag{8}$$

The last four rows of the signal equation (6) and the signal variance equation (8) reflect the fact that the macro variables are observed without error. Also, the second term in (6) is included since the state equation describes the dynamics of the demeaned state variables. The readout matrix $H(\tau)$ relates the unobserved yield curve factors to observed yields, and has the following structure:

$$H(\tau) = \begin{pmatrix} 1 & \frac{1-e^{-h_1/\tau}}{h_1/\tau} \\ \vdots & \vdots \\ 1 & \frac{1-e^{-h_n/\tau}}{h_n/\tau} \end{pmatrix} \in \mathbb{R}^{19\times 2}. \tag{9}$$

That is, for a given horizon h_i, the observed h_i-horizon yield in period t is given by

$$y_t(h_i) = f_{1t} + f_{2t} \frac{1 - e^{-h_i/\tau}}{h_i/\tau},$$

which of course is just (4) without the curvature term, with the identification $L_t = f_{1t}$ and $S_t = f_{2t}$, and τ redefined as $1/\tau$.

3 The data

We used monthly (end-of-month) spot yields for horizons 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 42, 48, 54, 60, 72, 84, 96, 108 and 120 months. The yields were calculated from Hungarian government bond prices using the Fama-Bliss method described in Fama & Bliss (1987) and Diebold & Li (2006).[3]

As the discount factor based on Fama-Bliss rates perfectly prices the bonds, these rates are very sensitive to pricing errors, therefore some filtering is required to obtain reasonable values. We decided to drop all bonds whose prices implied a forward rate that was more than one hundred basis point higher or lower than the mean of the forward rates for the neighbouring maturities. On average, this resulted in dropping 5.7 bonds per observation period, and the average number of bonds remaining in the sample was 20.9. Since the Fama-Bliss method only provides yields for the maturities of the bonds used in the calculations, yields for the above listed horizons were calculated by linear interpolation. The yields are plotted in Figure 1.

The sample period covered the interval between December 2001 and June 2008 consisting of 79 months. The start of the sample period was chosen such that the maturity range covered by the bonds be longer than ten years. Prices of bonds with maturity longer than eight years are available from January 1999, so if we were willing to accept eight years as the end of the maturity spectrum, then the sample could be extended to contain 114 months. However, the introduction of inflation targeting and the widening of floating range of the forint/euro exchange rate in early 2001 was definitely a significant structural break, making the use of a reduced form VAR model with constant coefficients hard to justify.

Figure 1
Fama-Bliss spot yields

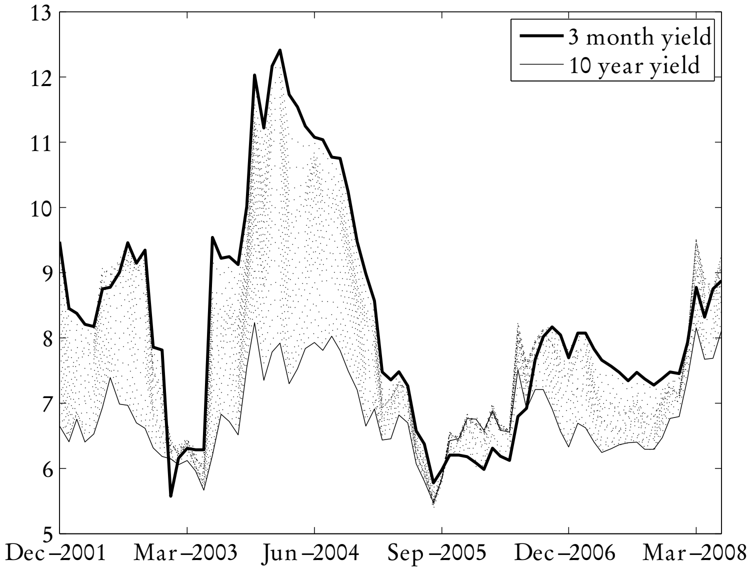


Figure 1 clearly indicates that most of the variation in the yields comes from parallel shifts, i.e. from changes of the level factor. This is confirmed by the results of a principal component analysis: indeed, as Table 1 shows, two factors are enough to explain almost all of the yield curve variability. The loadings of the first two principal components plotted in Figure 2 clearly justify the interpretation of these factors as level and slope. We also see that in this period the average yield curve was downward sloping.

The macro variables are the MNB base rate, the monthly percentage change of the HUF/EUR exchange rate, the annualized monthly percentage changes of CPI core inflation and the cyclical component of industrial

[3] The earlier studies of Csávás *et al.* (2007) and Reppa (2008) showed that, due to their being less noisy, interest rate swap yields might be a better choice, but swap yields are available only after the middle of 2006.

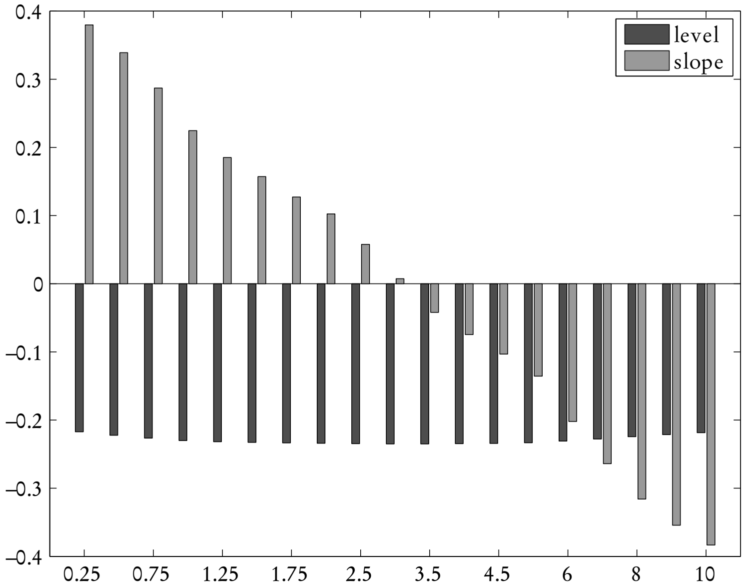
Table 1

Principal component latent variances in percentage points

Factor	variance explained	cumulative
1	94.74	95.74
2	4.71	99.45
3	0.32	99.77
4	0.10	99.87

Figure 2

Principal component loadings

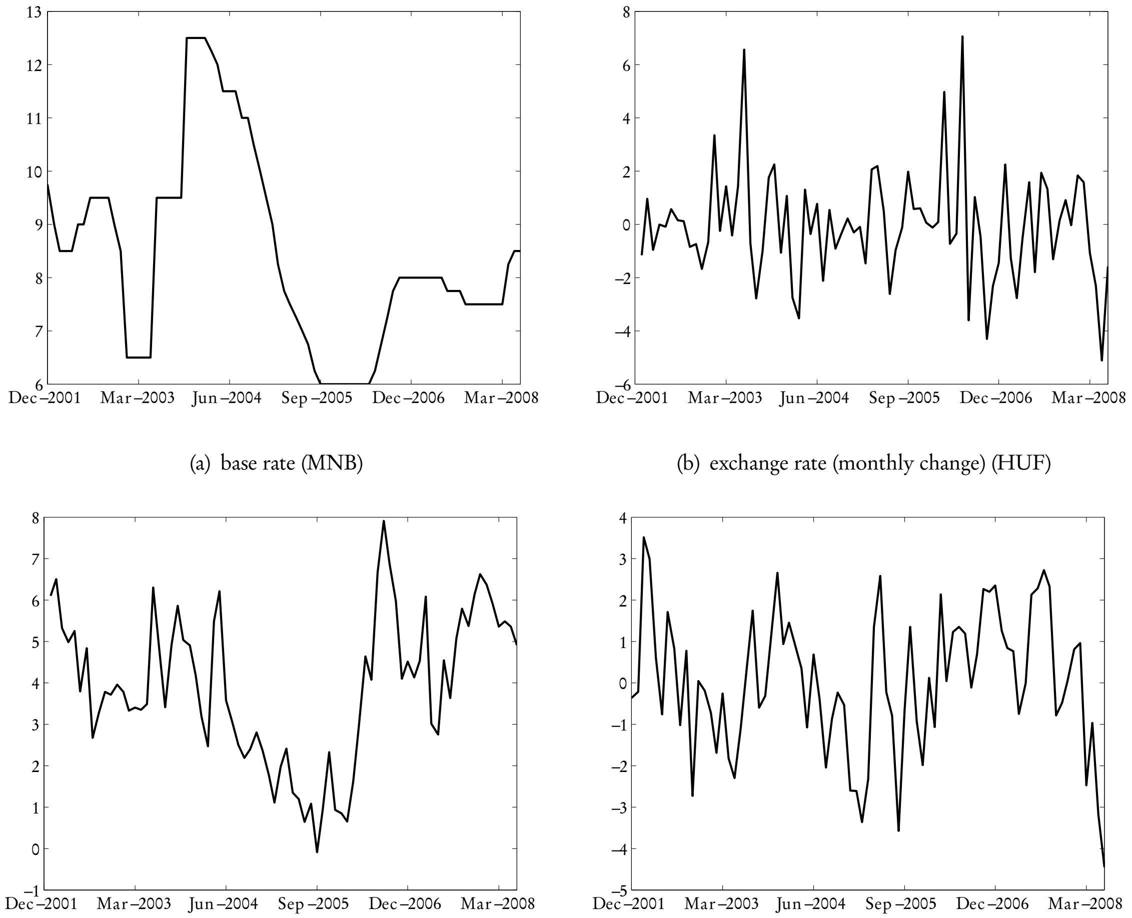


production (HP filtered), as shown by Figure 3.[4] Inflation and industrial production are supposed to capture supply and demand shocks, while the exchange rate is included to account for the fact that Hungary is a small open economy, and—together with the base rate—is supposed to capture monetary policy and risk premium shocks.

As Figure 3(a) shows, in 2003 there were three extraordinarily large jumps in the base rate, which might cause problems both in terms of estimation and identification. We therefore decided to preform a robustness check by estimating the model for the shorter sample starting in January 2004. The results of this and several other robustness checks are given at the end of the next Section.

[4] We used end of month data for the base rate and the exchange rate.

Figure 3
Macro variables



(a) base rate (MNB)

(b) exchange rate (monthly change) (HUF)

(c) inflation (annualized monthly change) (CORE)

(d) industrial production (HP filtered cyclical component) (IP)

4 Estimation results

The model was estimated by maximum likelihood, and the likelihood function was evaluated by the Kalman filter. The signal error covariance matrix Γ was restricted to be diagonal, and after that the number of parameters to be estimated was 83: 36 parameters in the state transition matrix F, 6 parameters in the state mean vector μ, the τ parameter in the readout matrix H, 21 parameters describing the state error covariance matrix Ω, and 19 signal error variances in Γ.[5] To make sure that Ω and Γ are positive definite, we reparameterized Ω in terms of its Cholesky factor, and used log-variances in Γ.

For inference and simulation we used the asymptotic normal distribution of the estimated parameters. The covariance matrix of this distribution was calculated by inverting the negative of the Hessian evaluated at the optimum, where the Hessian itself was approximated by finite differences after reverting back to the original parameterization, as suggested by Hamilton (1994).

ESTIMATED PARAMETERS

Tables 2 and 3 show the estimated state transition and error covariance matrices, with standard deviations in parentheses. We see that, with the exception of the exchange rate, all variables depend strongly on their lagged values, but the estimated autoregressive coefficients are far from one (in absolute value, the largest eigenvalue of F is 0.86). Also, many of the off-diagonal elements are significant according to a simple two-sigma rule; in particular, the exchange rate reacts negatively (appreciates) to both yield curve factors, and the base rate also shows positive reaction to these factors, and also reacts positively to lagged inflation.

Table 2

Estimated state mean vector and transition matrix μ and F

	level	slope	MNB	HUF	CORE	IP	μ
level	0.66	−0.04	−0.02	0.00	0.05	−0.11	5.93
	(0.11)	(0.10)	(0.10)	(0.03)	(0.03)	(0.04)	(0.29)
slope	0.23	0.64	0.34	0.03	0.09	0.09	2.52
	(0.20)	(0.19)	(0.20)	(0.05)	(0.06)	(0.07)	(0.80)
MNB	0.23	0.21	0.76	0.04	0.08	−0.02	8.54
	(0.12)	(0.11)	(0.11)	(0.03)	(0.04)	(0.04)	(0.72)
HUF	−1.42	−1.47	1.02	0.03	0.05	−0.20	−0.11
	(0.38)	(0.35)	(0.36)	(0.10)	(0.12)	(0.13)	(0.22)
CORE	−0.05	0.05	−0.13	0.05	0.79	0.09	3.82
	(0.22)	(0.21)	(0.22)	(0.06)	(0.07)	(0.08)	(0.44)
IP	−0.30	−0.08	0.03	0.17	0.13	0.47	−0.05
	(0.31)	(0.29)	(0.30)	(0.08)	(0.10)	(0.10)	(0.42)

As Table 3 shows, many of the off diagonal elements of the estimated residual covariance matrix are significantly different from zero, indicating a strong contemporaneous interaction between the VAR residuals, and clearly pointing to the necessity of structural identification in order to calculate impulse responses, variance decompositions and structural shocks.

The estimated signal error standard deviations are shown in Figure 4, and their proper interpretation will be discussed in the next section. We note here only that the obvious dependence of the variances on the maturity of the yields seems to imply that more than two factors are necessary to fit the yield curve data. However, this does not contradict our earlier finding that two factors are enough to explain nearly all of the yield curve variance. This is because the former statement is about the cross-section fit, while the latter is

[5] The large number of parameters is another motivation to carefully check the robustness of the results.

Table 3

Estimated state error covariance matrix Ω

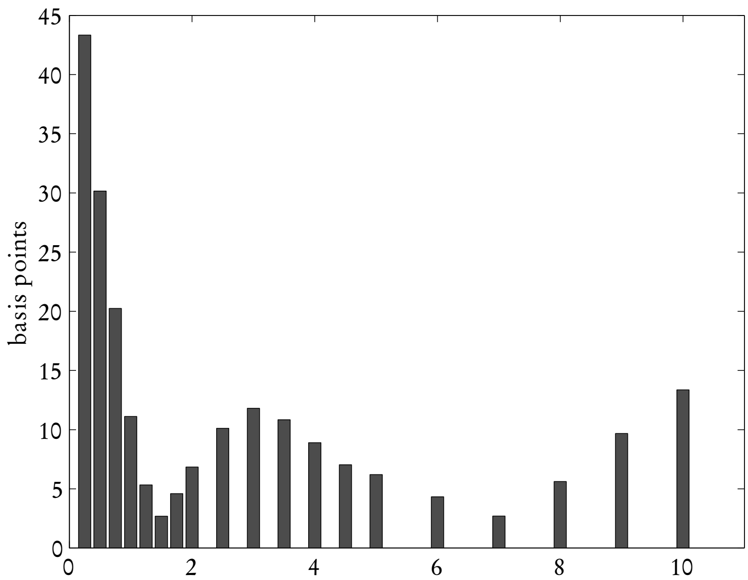
	level	slope	MNB	HUF	CORE	IP
level	0.23	−0.30	−0.12	0.06	−0.03	0.07
	(0.07)	(0.03)	(0.10)	(0.06)	(0.08)	(0.14)
slope	−0.30	0.87	0.39	0.63	0.12	−0.08
	(0.03)	(0.07)	(0.20)	(0.11)	(0.15)	(0.05)
MNB	−0.12	0.39	0.29	0.32	0.12	0.06
	(0.10)	(0.20)	(0.11)	(0.06)	(0.09)	(0.48)
HUF	0.06	0.63	0.32	3.01	0.18	0.44
	(0.06)	(0.11)	(0.06)	(0.20)	(0.28)	(0.17)
CORE	−0.03	0.12	0.12	0.18	1.07	0.06
	(0.08)	(0.15)	(0.09)	(0.28)	(0.17)	(0.32)
IP	0.07	−0.08	0.06	0.44	0.06	2.00
	(0.14)	(0.05)	(0.48)	(0.17)	(0.32)	(0.03)

about the dynamics. In other words, although a third factor might be useful to describe yields on a given day, its change from one day to the other would add very little to the explanation of the dynamics of the yield curve, because its variability is dwarfed by the variability of the level and slope factors.

Nevertheless, the relatively poor fit at short maturities is worrisome, therefore we include a comparison of the results with those of the three factor model in our robustness checks.

Figure 4

Signal error standard deviations



Finally, we note that estimated value of τ is 4.30, with a standard deviation of 0.14.

DIAGNOSTICS

Table 4 gives the results of several Wald tests. The first four rows indicate that the restriction that certain blocks of the estimated state transition matrix are zero are rejected at every reasonably significance level. In particular, the third and four rows confirm that there is indeed a dynamic interaction between the yield curve factors and the macro variables, in both directions.

The last row shows that the diagonality of Ω is also rejected, which is not surprising given that many off-diagonal elements are individually significant by Table 3.

Table 4

Wald test statistics

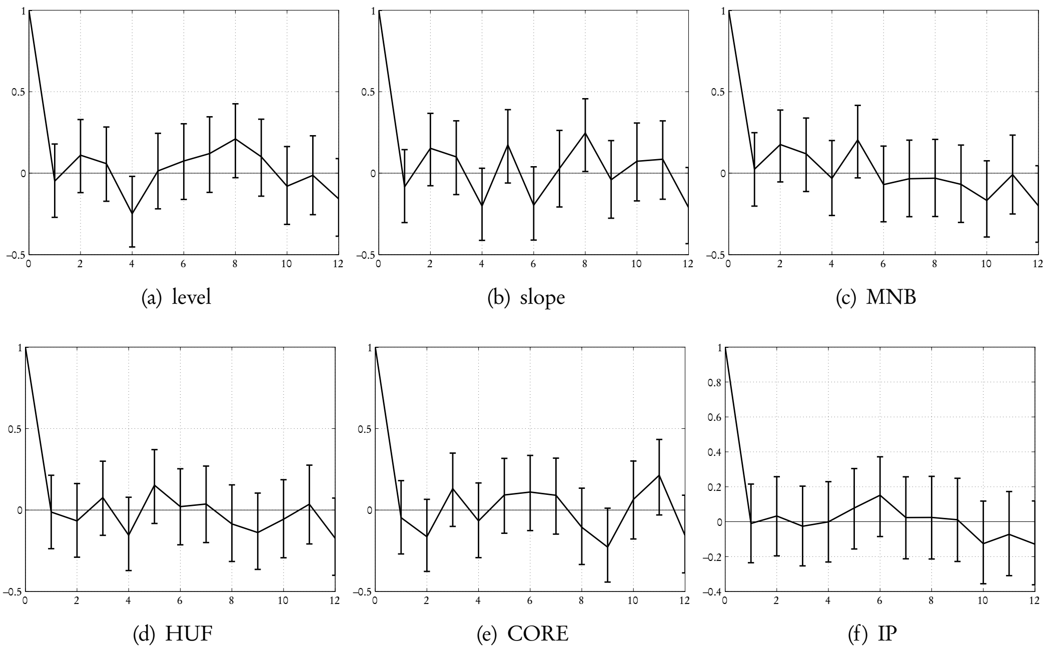
	Wald statistic	p-value
YC to YC	116.28	0.00
MAC to MAC	249.36	0.00
YC to MAC	36.09	0.00
MAC to YC	23.75	0.00
Ω diag	61.61	0.00

It is also important to make sure that there is no autocorrelation left in the state residuals. A proper way of testing this should account for the filtering uncertainty in the state estimates from which the residuals are calculated. However, this would probably have little effect on the point estimates of the autocorrelations, and would probably widen the confidence bands. As Figure 5 shows, the autocorrelations of the residuals calculated from the smoothed state estimates are already insignificant, so it seems safe to conclude that the model with one lag is correctly specified.

Nevertheless, we estimated the model with two and three lags as well, and found that the impulse responses and variance decompositions were very similar to those with one lag, and the Bayesian information criteria also selected the one lag model.

Figure 5

Autocorrelation of smoothed state residuals and 95% confidence bands



(a) level (b) slope (c) MNB

(d) HUF (e) CORE (f) IP

IN-SAMPLE FIT

At first glance, Figure 4 and Table 5 seem to imply that model has a very good fit, at least at longer maturities: the signal error standard deviation is in many cases only one tenth of the signal variation. However, one should be careful with this interpretation: the signal error variances should be thought of as measurement error variances, and as such they show the size of the error when the states are know with certainty. This is of course not the case, and the prediction error standard deviations, given in the last column of Table 5, account for this uncertainty. Comparing them with the second column we see that the model explains roughly half of the variation of the yield curve. Comparing the last two columns imply that, especially for longer horizons, most of the prediction error results from the error in predicting the yield curve factors.

Table 5

Model fit

Horizon	Mean yields (in pp)	Std of yields (in bp)	Sig err std (in bp)	Pred err std (in bp)
0.25	8.25	170.64	43.33	69.15
0.50	8.19	161.68	30.16	63.26
0.75	8.09	153.13	20.24	58.89
1.00	8.03	147.59	11.11	58.98
1.25	7.98	143.46	5.34	58.92
1.50	7.94	139.86	2.69	58.50
1.75	7.90	136.64	4.59	58.48
2.00	7.86	133.17	6.84	57.88
2.50	7.78	126.44	10.11	56.26
3.00	7.70	119.65	11.81	54.98
3.50	7.59	112.66	10.83	52.52
4.00	7.49	105.67	8.90	49.77
4.50	7.40	99.34	7.03	47.11
5.00	7.33	94.17	6.21	45.00
6.00	7.20	85.18	4.33	40.82
7.00	7.09	77.75	2.70	37.90
8.00	6.99	72.35	5.62	36.56
9.00	6.90	67.14	9.68	35.46
10.00	6.82	63.24	13.37	34.97

An interesting feature of the yields is that the standard deviation declines rapidly as the maturity gets longer; this is not the case for US data, at least not to this extent, see Diebold & Li (2006). The large jumps in the base rate seen in Figure 3(a) could explain the large volatility of the short yields, while the reason behind the (relatively) stable long yields might be the anchoring effect of the introduction of the euro, which also explains the negative slope of the average yield curve.

IDENTIFICATION STRATEGY

As we have seen before, the diagonality of the estimated Ω matrix is strongly rejected, which means that the estimated VAR residuals are correlated, and this correlation must be taken into account when calculating impulse responses and variance decompositions. Also, economically meaningful structural shocks are usually assumed to be uncorrelated, so this again points to the necessity of structural identification.

As is well known,[6] structural identification is the same as finding a matrix S such that $\Omega = SS'$. The problem is that many such decomposition exist, and therefore restrictions must be applied. We follow Uhlig (2005) and identify structural shocks based on sign restrictions with a Bayesian re-estimation algorithm.

This algorithm works as follows. First, it is easy to see that if S_1 and S_2 are two decomposition of Ω, then there is an orthogonal matrix O such that $S_2 = S_1 O$. We then make random draws from F, Ω and O, calculate the corresponding responses, and keep the draw if all the sign restrictions are satisfied. This will give us a random sample of responses, from which statistics, such as means and percentiles can be calculated. Notice that this procedure does not give us identified estimates of F, Ω and S, but it does provide identified values of the impulse responses and variance decompositions, which is after all our final goal (see Fry & Pagan (2007) for more on this point).

[6] See Hamilton (1994) for the concept and details of SVAR identification.

The distribution that we draw from is the estimated asymptotic normal distribution for F and Ω, and the uniform distribution for O. It is not at all evident how to define a uniform distribution on the set of orthogonal matrices, and we spend some time describing and motivating our calculations in the Appendix.

Table 6

Restrictions on the structural decomposition of Ω

	L	S	PREM	MP	DEM	SUPP
level	+	*	*	*	*	*
slope	*	+	*	*	*	*
MNB	0	0	+	+	*	*
HUF	*	*	+	−	*	*
CORE	0	0	0	0	+	−
IP	0	0	0	0	+	+

We identify four shocks, labelled as, *monetary policy* (MP), *risk premium* (PREM), *demand* (DEM) and *supply* (SUPP), and the restrictions are listed in Table 6. The responses were required to satisfy the restrictions only for $t = 0$, i.e. in the period when the shock occurred. In fact, we use both sign and zero (exclusion) restrictions, and all these are standard in the literature: for the demand and supply shocks see Fry & Pagan (2007), for the monetary policy and premium shocks see Vonnák (2005). Since there are six variables in the VAR there are two more shocks that could be identified, namely a *level* shock (L) and a *slope* shock (S). However, we could not think of well-accepted interpretations of these shocks, so we impose only a minimum set of restriction in this case: as these shocks are financial in nature, they are likely to have no contemporaneous effect on inflation and output, and we assumed that there is no immediate reaction of the base rate to these shocks, which ensures that they are properly distinguished from the other four shocks. As a robustness check, we also carried out the calculations with the additions restrictions that the level and slope shocks have no contemporaneous effect on the exchange rate.

IMPULSE RESPONSES

The mean and 16-84 percent percentiles of the responses, based on 5000 successful draws, are shown in Figure 6, with the horizontal axis showing, in months, the number of periods after the shock.

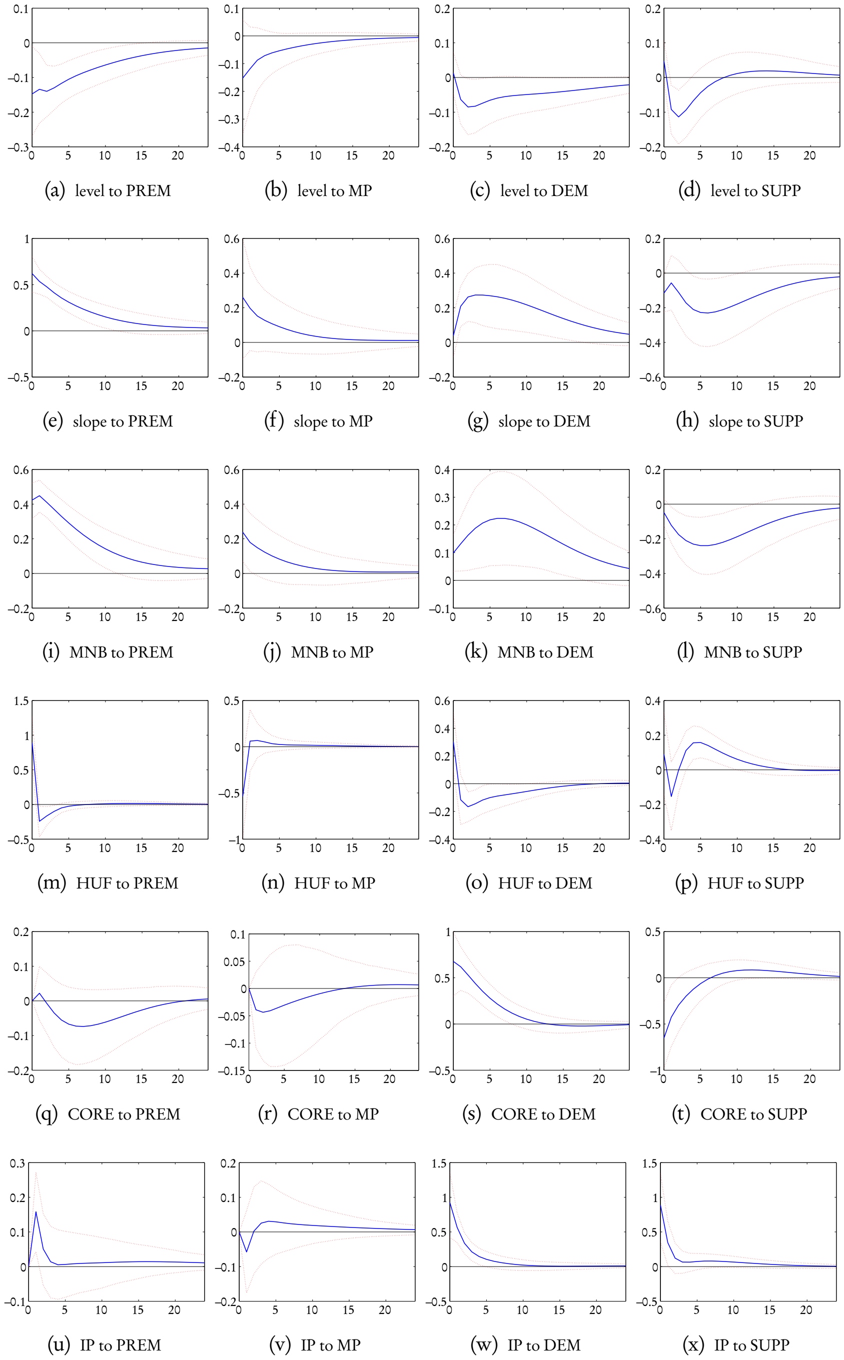
We see that all four shocks decrease the level factor. This response is somewhat puzzling in case of the premium and demand shocks, but we should not forget that the responses of yields are combinations of the responses of the level and the slope factor, and the latter responds positively to all shocks except for the supply shock.

The responses of the base rate and the exchange rate are very different in dynamics. As for the base rate, the responses, not surprisingly, are very similar to the responses of the slope factor, and are among the responses that are the most persistent. On the other hand, the exchange rate shows strong immediate reactions to all shocks, but the response totally disappears after a couple of months. Note that the exchange rate enters the model in log-differences, therefore the reaction of the level is calculated by cumulating the results in Figures 6(m)-(p).

Finally, inflation and output seems to go on their own way, showing negligible responses the all shocks other than demand and supply. This however does not mean that these variables are not influenced by monetary policy, since in this model monetary policy is endogenous, and it might very well be the case that demand and supply shocks exert some of their effect through the reaction of monetary policy.

Figure 6

Impulse responses, mean and 16-84% confidence bands



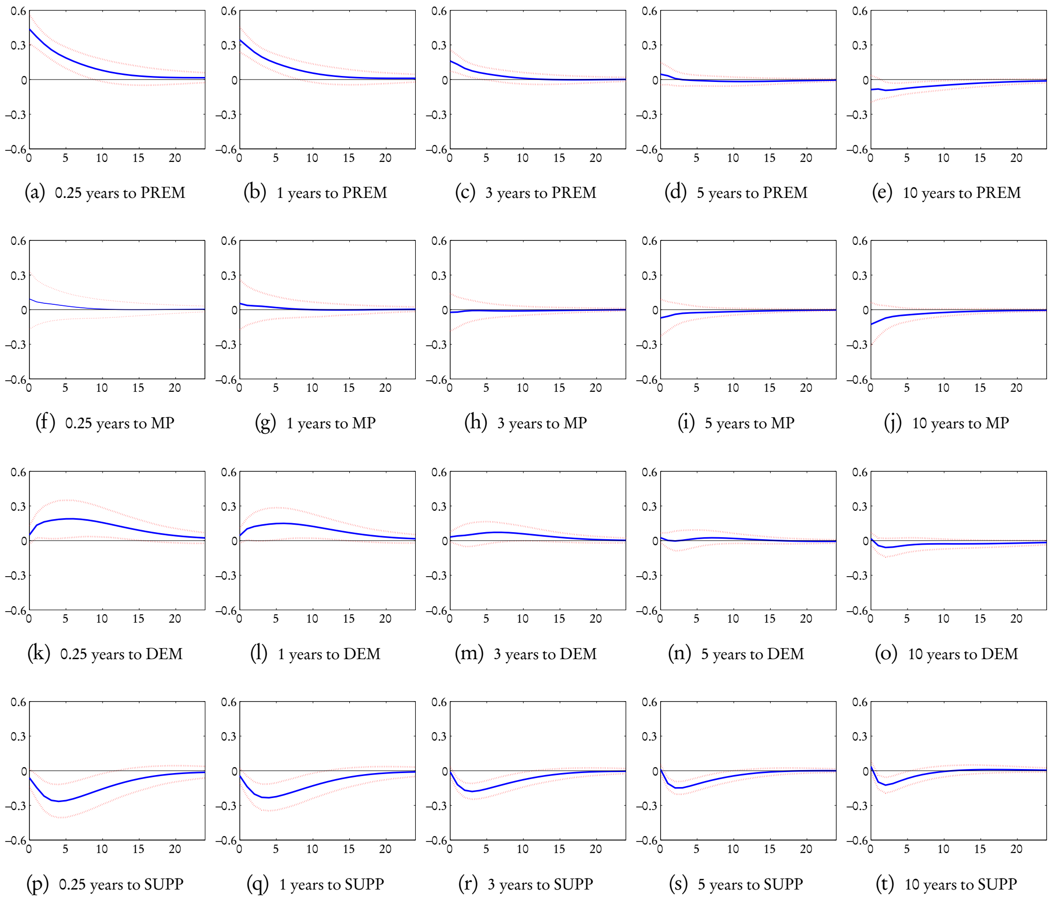
It is also possible to calculate responses of the whole yield curve to the identified shocks: we only have to multiply the responses of the two yield curve factors by the readout matrix H, taking also into account the uncertainty of the parameter τ. Figure 7 shows the responses of instantaneous forward yields for selected maturities, and Figure 8 gives the responses of the corresponding spot yields.

We see that short horizon forward rates react very little to a MP shock, but they decline after the three year horizon. On the other hand, the reaction of forward rates to premium shocks is strongly positive at short horizons, while it is similar to the monetary policy shock at long horizons; this also means that the puzzle of the long forward rates declining in response to an increase in the risk premium is not solved. It is also true for both shocks that the reaction is strongest in the month when the shock occurs. This is not the case for the demand and supply shocks: the strongest reaction is about four-five months after the shock.

As spot rates are time averages of instantaneous forward rates, their reaction can be predicted from those of the forward rate: when the shock mainly affects the long forward yields (MP shock), the reaction of the corresponding long spot yields is dampened relative to the reaction of the forward yields, and when the shock affect mainly the short forward yields (premium and demand shocks), the reaction of the long spot yields is amplified relative to the reaction of the long forward yields.

Figure 7

Impulse responses of forward yields, mean and 16-84% confidence bands



(a) 0.25 years to PREM (b) 1 years to PREM (c) 3 years to PREM (d) 5 years to PREM (e) 10 years to PREM

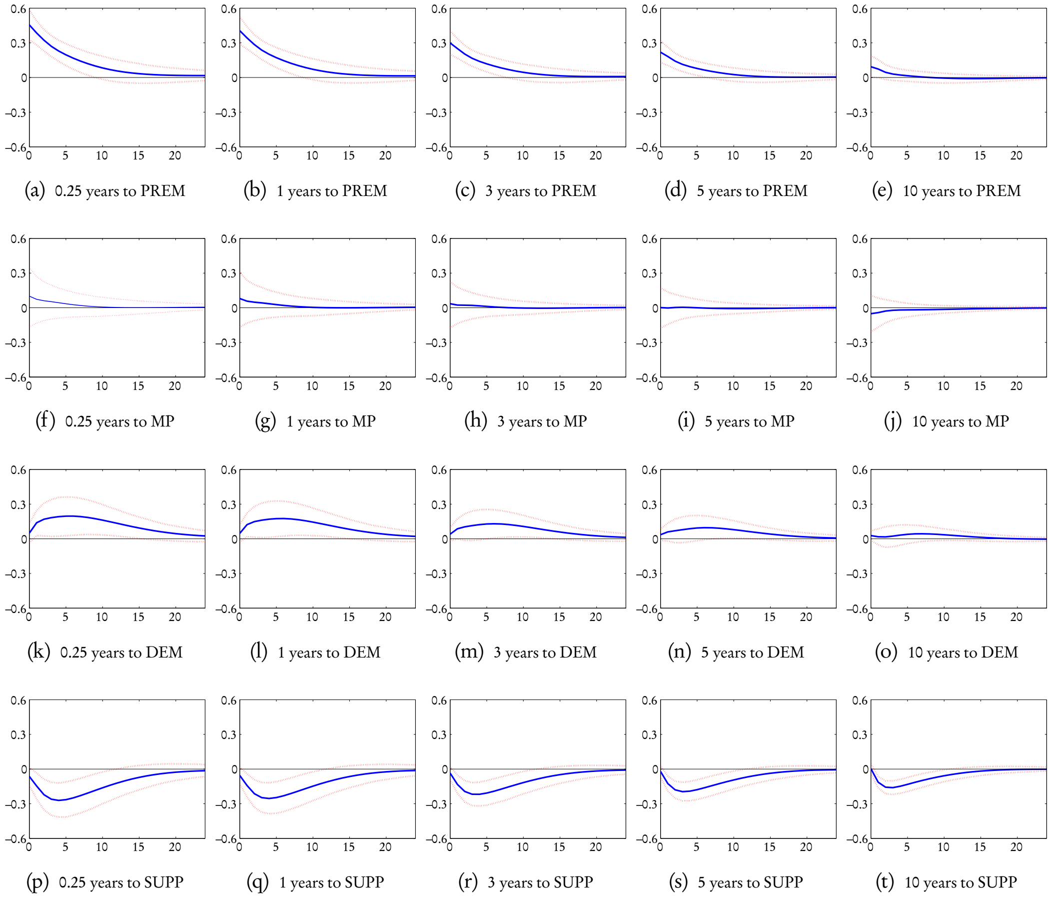
(f) 0.25 years to MP (g) 1 years to MP (h) 3 years to MP (i) 5 years to MP (j) 10 years to MP

(k) 0.25 years to DEM (l) 1 years to DEM (m) 3 years to DEM (n) 5 years to DEM (o) 10 years to DEM

(p) 0.25 years to SUPP (q) 1 years to SUPP (r) 3 years to SUPP (s) 5 years to SUPP (t) 10 years to SUPP

Figure 8

Impulse responses of spot yields, mean and 16-84% confidence bands



(a) 0.25 years to PREM (b) 1 years to PREM (c) 3 years to PREM (d) 5 years to PREM (e) 10 years to PREM

(f) 0.25 years to MP (g) 1 years to MP (h) 3 years to MP (i) 5 years to MP (j) 10 years to MP

(k) 0.25 years to DEM (l) 1 years to DEM (m) 3 years to DEM (n) 5 years to DEM (o) 10 years to DEM

(p) 0.25 years to SUPP (q) 1 years to SUPP (r) 3 years to SUPP (s) 5 years to SUPP (t) 10 years to SUPP

The same phenomenon is shown in Figures 9 and 10 where mean changes in the respective type of yield curves are shown, the darkest (red) line corresponding the the immediate reaction, and the lighter lines to reactions in three month steps ahead, up until twelve months.

Figure 9

Mean impulse responses of forward yields (after 0, 3, 6, 9 and 12 months)

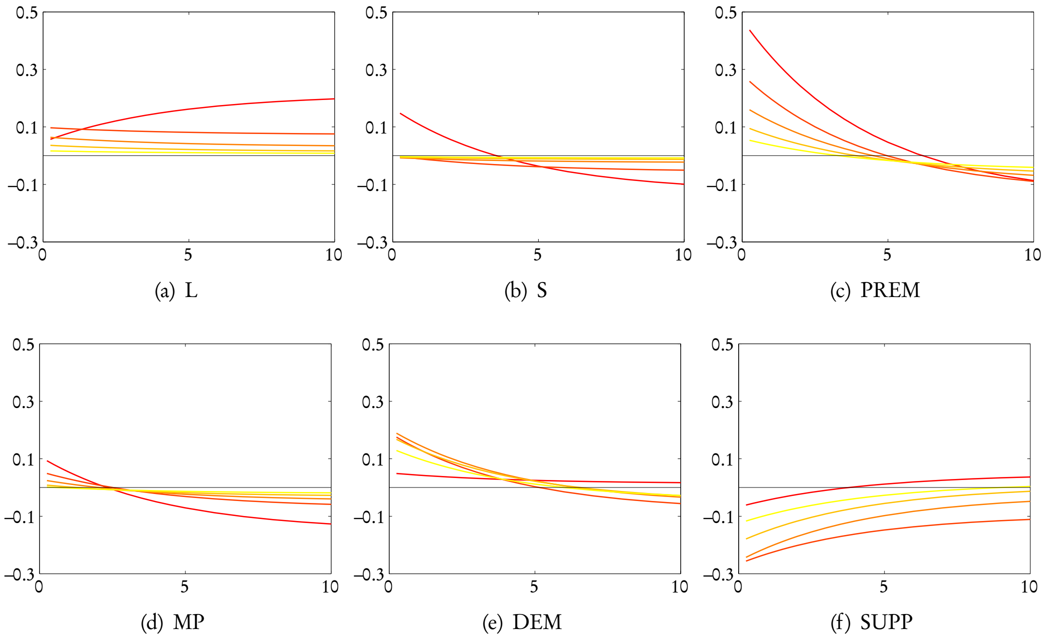
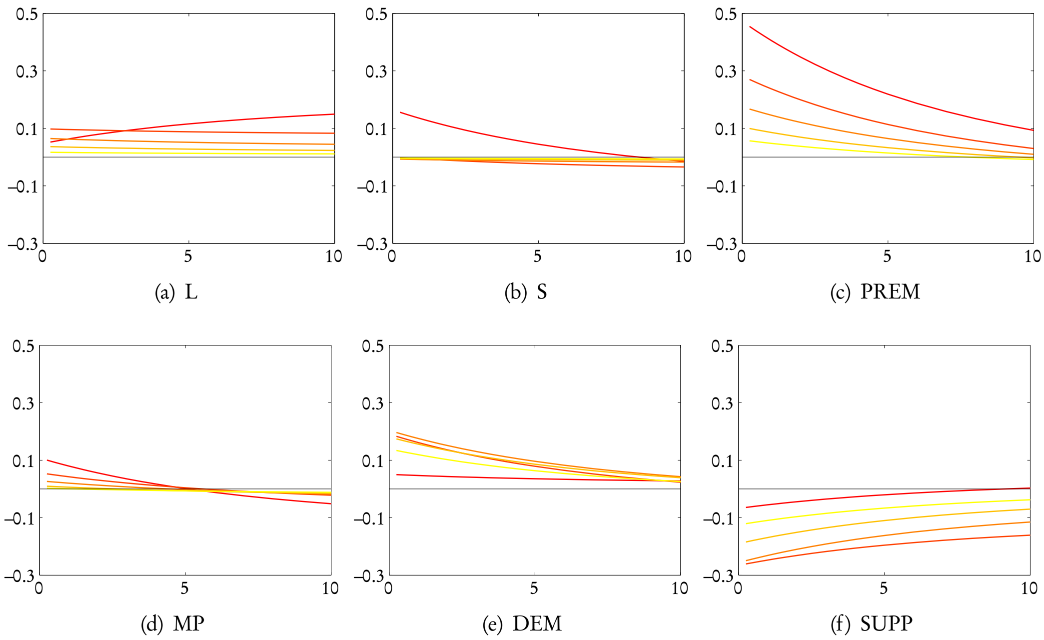


Figure 10

Mean impulse responses of spot yields (after 0, 3, 6, 9 and 12 months)



VARIANCE DECOMPOSITIONS AND STRUCTURAL SHOCK SERIES

Another way of presenting information from a VAR estimation is to calculate variance decompositions, which indicate the portion of the forecast error variance of the VAR variables that can be attributed to the shocks. Table 7 shows the results for the one month and two years ahead forecast variances.

Table 7

Variance decomposition, mean

(a) One month ahead

	$PREM$	MP	DEM	$SUPP$	sum
level	17.67	25.08	3.49	5.08	51.32
slope	50.56	18.43	5.12	3.68	77.78
MNB	64.05	21.62	6.42	5.30	97.39
HUF	27.92	15.08	4.99	3.47	51.46
CORE	0.40	0.40	54.30	43.81	98.91
IP	1.45	0.70	51.60	43.95	97.70

(b) Two years ahead

	$PREM$	MP	DEM	$SUPP$	sum
level	22.67	19.19	12.62	12.80	67.28
slope	34.92	11.63	21.06	17.34	84.95
MNB	35.05	11.15	21.80	21.93	89.92
HUF	24.87	14.17	8.41	7.59	55.04
CORE	4.88	3.56	48.80	34.38	91.63
IP	4.18	3.34	45.74	38.29	91.55

A comparison of the two tables clearly shows that the main driving forces behind the financial variables in the short run are the risk premium and monetary policy shocks; this, however, changes in the long run, when demand and supply shocks outweigh the monetary policy shock. On the other hand, the long term impact of policy and premium shocks on output and inflation, although still quite small, is much larger than their short run effects.

The contributions of the four shocks to the long term variance of the level factor are broadly similar, with the demand and supply shocks having a somewhat lesser role. The four shocks explain only about two thirds of the total variation of the level factor, which indicates that some important sources of uncertainty might not have been accounted for in the model, an obvious candidate being the date of introducing the euro in Hungary. This might also explain that for the base rate, output and inflation, the explained proportion of variance decreases with time.

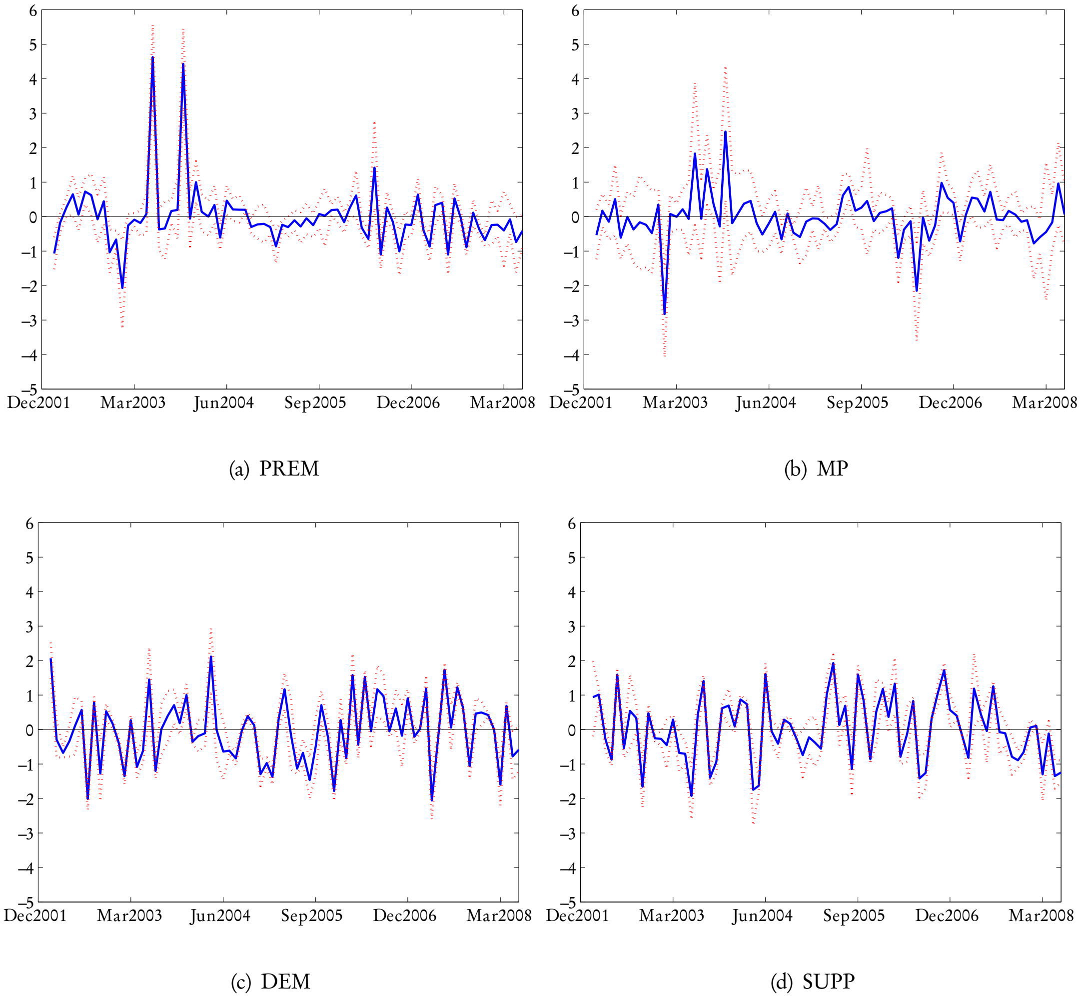
As we have noted earlier, the responses of the slope factor and the base rate are very similar, and this is again seen in the variance decompositions. In the long run, more than eighty percent of the variation of these two variables can be explained by the identified shocks, with the monetary policy shock being the least and premium shocks being the most important driving forces.

Not surprisingly, the identified shocks explain only about fifty percent of exchange rate forecast variation, and the relative shares are similar to what we have seen in case of the level factor. More than ninety percent in the forecast error of inflation and output can be attributed to the four shocks, and the majority here comes from demand and supply.

The historical values of the structural shocks with 16–84 percent confidence bands are plotted in Figure 11. We see that the really large premium and monetary policy shocks occurred in 2003, with a somewhat smaller monetary policy shock in the summer of 2006.

Figure 11

Structural shocks, mean and 16-84% confidence bands



ROBUSTNESS CHECKS

Sample period

As we have noted earlier, the large jumps in the MNB base rate in 2003 might distort the estimation and identification results. In this section we compare the earlier results with those that we get when we use the shorter sample starting in January 2004.

The basic impulse responses are shown in Figure 12, with the full sample responses plotted in green. We see that the reaction of the exchange rate, inflation and output are broadly the same. On the other hand, the reaction of the yield curve factors and the base rate to the demand and supply shocks are more persistent than in the case of the full sample estimation. Nevertheless, the full sample results are within the confidence band of the short sample estimates.

This is not the case for the responses of these variables to the MP and PREM shocks. In the first few periods after the shock, the magnitude of these reactions is much smaller for the short sample, and this difference, with the exception of the level to MP response, is significant.

These findings are corroborated by the comparison of the variance decompositions. As Tables 7(a) and 8(a) show, the weight of the premium and policy shocks in the near forecast error variances of the yield variables

Table 8

Variance decomposition, mean, short sample

(a) One month ahead

	PREM	*MP*	*DEM*	*SUPP*	sum
level	8.72	18.40	5.99	14.57	47.68
slope	26.67	15.11	9.74	6.43	57.94
MNB	44.60	17.26	18.52	5.84	86.22
HUF	23.52	13.69	7.30	7.45	51.96
CORE	1.31	0.79	54.25	40.61	96.97
IP	1.63	1.23	48.08	44.76	95.70

(b) Two years ahead

	PREM	*MP*	*DEM*	*SUPP*	sum
level	7.94	10.89	21.13	27.91	67.87
slope	9.55	6.69	29.61	31.20	77.04
MNB	9.22	6.27	30.82	32.19	78.51
HUF	19.06	12.09	12.46	14.44	58.05
CORE	3.22	3.20	48.54	34.53	89.50
IP	3.18	3.43	42.21	40.00	88.81

is lower in the short sample, corresponding to the lower magnitude of the near term responses. The extra weight is mostly allocated to the demand and supply shocks, the only exception being the slope factor, where the weight of the unidentified shocks is increased.

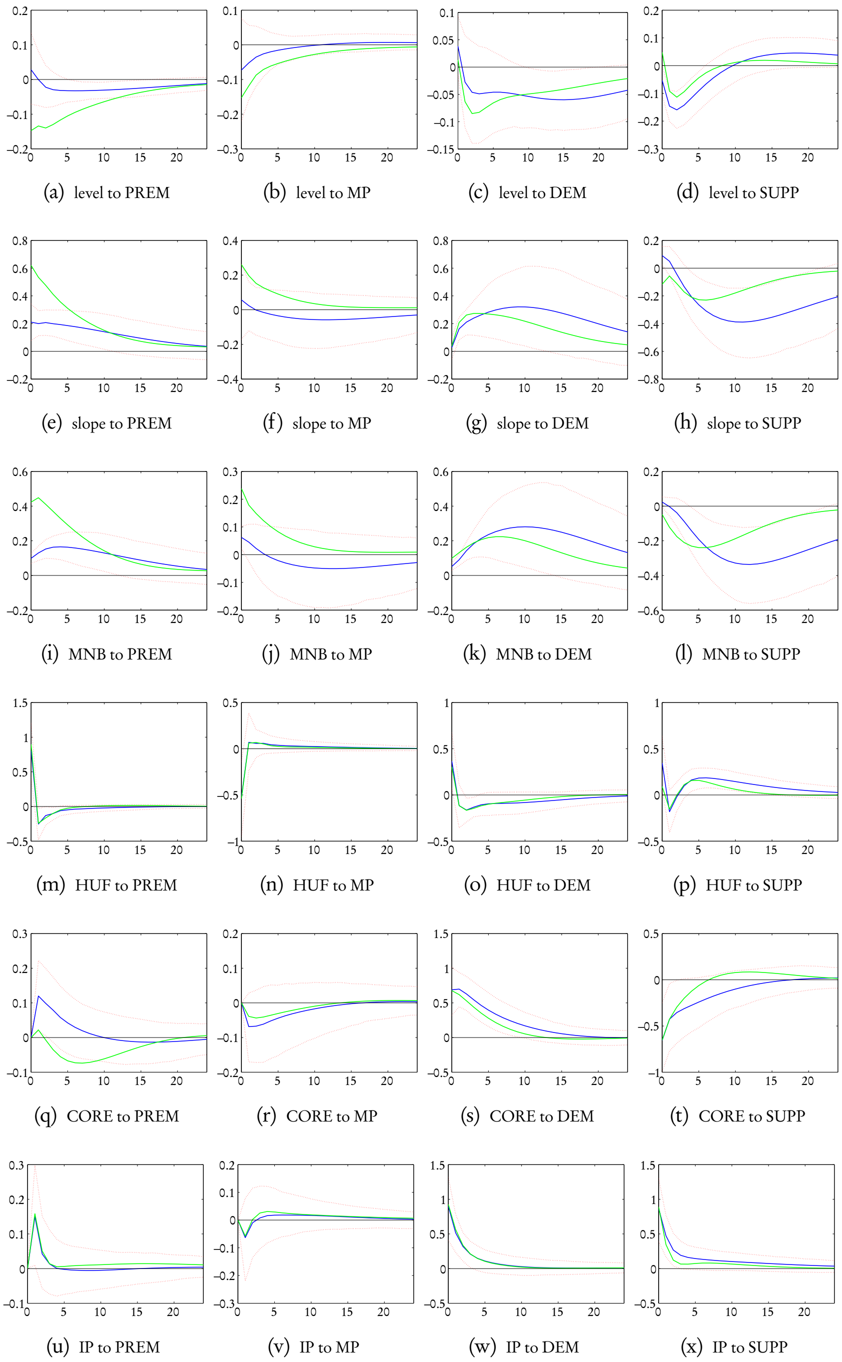
The big difference, however, is in the long term decomposition. Now both the MP and the PREM shocks are outweighed by the demand and supply shocks in the decomposition of all variables but the exchange rate. Also, a sizable portion of the forecast error variance of the slope factor and the base rate (eight and eleven percentage points, respectively) is reattributed to the unidentified shocks.

The reaction of the forward rates shown in Figure 13 reveals that the puzzling increase of the forward rates after an increase in the risk premium has disappeared. However, the price is the emergence of a new puzzle, namely the negative reaction of all forward rates to a positive monetary policy shock. Nevertheless, while the "old puzzle" was significant both statistically and economically, the "new puzzle" is highly insignificant in either sense of the word.[7]

[7] The short sample responses of the spot rates and the forward and spot yield curves are shown in the Appendix.

Figure 12

Impulse responses, mean and 16-84% confidence bands, short sample



(a) level to PREM (b) level to MP (c) level to DEM (d) level to SUPP

(e) slope to PREM (f) slope to MP (g) slope to DEM (h) slope to SUPP

(i) MNB to PREM (j) MNB to MP (k) MNB to DEM (l) MNB to SUPP

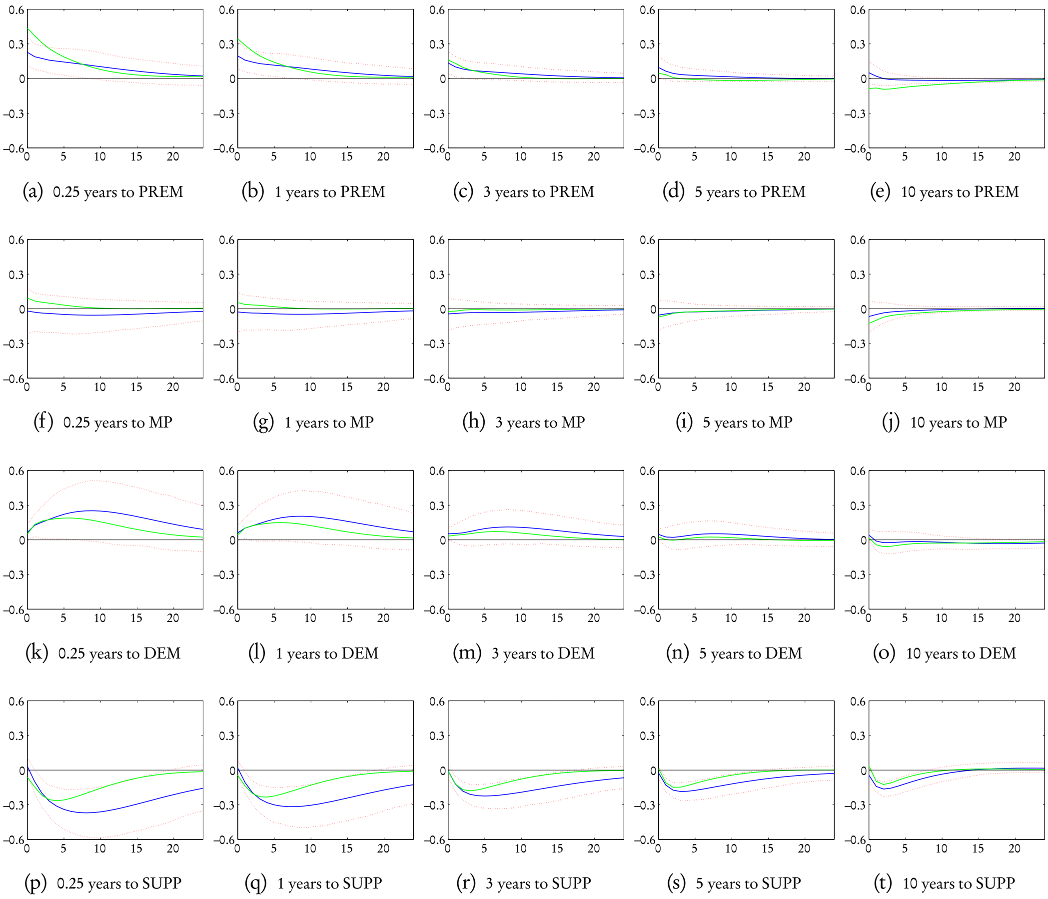
(m) HUF to PREM (n) HUF to MP (o) HUF to DEM (p) HUF to SUPP

(q) CORE to PREM (r) CORE to MP (s) CORE to DEM (t) CORE to SUPP

(u) IP to PREM (v) IP to MP (w) IP to DEM (x) IP to SUPP

Figure 13

Impulse responses of forward yields, mean and 16-84% confidence bands, short sample



(a) 0.25 years to PREM (b) 1 years to PREM (c) 3 years to PREM (d) 5 years to PREM (e) 10 years to PREM

(f) 0.25 years to MP (g) 1 years to MP (h) 3 years to MP (i) 5 years to MP (j) 10 years to MP

(k) 0.25 years to DEM (l) 1 years to DEM (m) 3 years to DEM (n) 5 years to DEM (o) 10 years to DEM

(p) 0.25 years to SUPP (q) 1 years to SUPP (r) 3 years to SUPP (s) 5 years to SUPP (t) 10 years to SUPP

Table 9

Restrictions on the structural decomposition of Ω, alternative version

	L	S	PREM	MP	DEM	SUPP
level	$+$	$*$	$*$	$*$	$*$	$*$
slope	$*$	$+$	$*$	$*$	$*$	$*$
MNB	0	0	$+$	$+$	$*$	$*$
HUF	0	0	$+$	$-$	$*$	$*$
CORE	0	0	0	0	$+$	$-$
IP	0	0	0	0	$+$	$+$

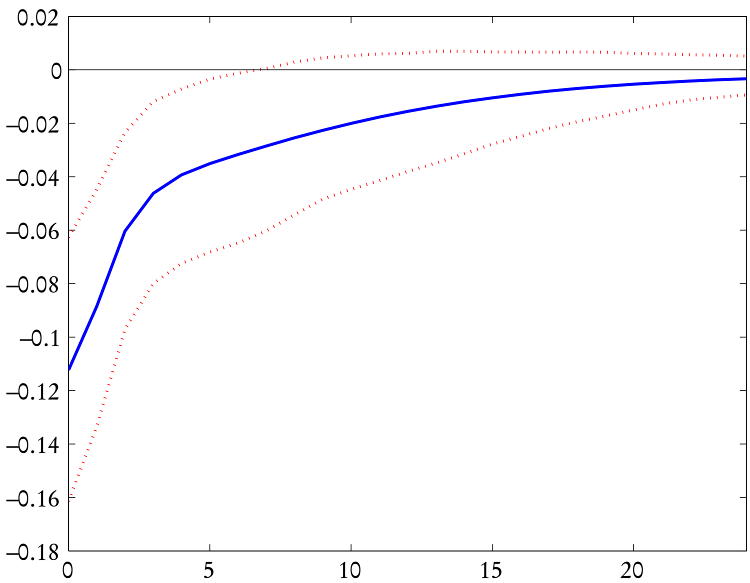
Identifying restrictions

As we said earlier, the economic interpretation of the level and slope factors is not straightforward, and the identifying restrictions reflect this fact: we used only a minimum set of restriction to separate these shocks from the four identified shocks. On the other hand, one might argue that these factors represent shocks that are unrelated to the current macroeconomic situation, and this interpretation would justify the restriction that the exchange rate has no contemporaneous reaction to the level and slope shocks. We therefore calculated the impulse responses and variance decompositions with the alternative set of restriction given in Table 9.

The mean responses from this alternative identification scheme are very similar to the original ones[8], but there are differences in the significance of the responses. This can most clearly be seen in the responses of the long forward yields to monetary policy shocks; as Figure 14 shows, these reactions now become significant, although the magnitude of the response still remains small.

Figure 14

Impulse responses of 5 year forward yields to a monetary policy shock, mean and 16-84% confidence bands, alternative version

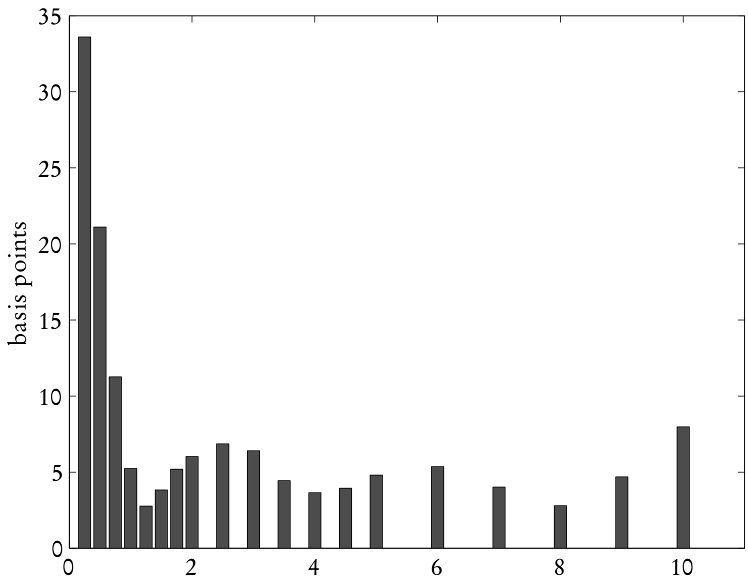


[8] This is not surprising, since the mean responses of the exchange rate to the yield curve shocks were already close to zero.

Number of factors

The poor fit of the model to the short end of the yield curve motivated our next robustness check, which involved estimating the model with three factors. Figure 15 shows that the fit is indeed improved at the short end, but the pricing errors remain large. Moreover, the dynamic properties of the model do not change significantly, reaffirming our earlier observation that a third factor is not necessary to describe the dynamic interaction among the variables of the model.

Figure 15
Signal error standard deviations, three factor model



Yields

Finally, we examined whether the results are sensitive to the maturities used in the estimation. We followed the suggestion in Anderson & Sleath (2001), and estimated the model using alternate maturities, i.e. $0.25, 0.75, \ldots, 10$ year yields in one estimation and $0.5, 1, \ldots, 9$ year yields in the other. The parameters from the the two estimations were very similar to the base values, and the impulse responses and the variance decompositions were virtually indistinguishable. This robustness can most likely be attributed to the use of only two factors: any two short and long yield would determine essentially the same slope and level.

5 Conclusion

We estimated a Nelson-Siegel type dynamic yield curve model for monthly Hungarian data. Such a model does not impose the restriction that are necessary to exclude arbitrage opportunities, but does not require the ad hoc specification of a risk premium process and that some yields are observed without error. It is also statistically very simple and is a direct generalization of popular static yield curve fitting models, therefore seems to be good first choice in an explanatory exercise.

The model was estimated by state-space methods using the Kalman filter. Besides the latent factors, we incorporated the MNB base rate, the forint euro exchange rate, inflation and output as observable factors. We found that a two latent factor version of the model can explain about half of the variation of the yields.

We calculated impulse responses, variance decompositions and historical shock series by an SVAR identification procedure where we have used one period sign restrictions as well as exclusion restrictions. We identified monetary policy, risk premium, demand and supply shocks. The generation of random orthogonal matrices that satisfy the zero restrictions was examined in detail.

The sign of the (unrestricted) impulse responses were consistent with theory, and the macro factors explained a sizable portion of the yield curve movements. The variance decomposition showed that the the risk premium is the most important driving force behind interest rates and the exchange rate. Monetary policy is second in case of the level factor and the exchange rate, while for the slope factor and the base rate demand and supply shocks outweigh monetary policy shocks. Risk premium and monetary policy shocks only have a minor role in explaining the forecast error variance of output and inflation.

Appendices

A Generating random orthogonal matrices

Given a random draw from the distribution of the covariance matrix Ω and a random orthogonal matrix O, the identification matrix S can be calculated as $S = PO$, where P is the Cholesky factor of Ω.

The key technical ingredient in the sign restriction method is the generation of random orthogonal matrices. To do this, we first have to decide about the distribution that these random matrices should come from. There are two natural requirements: first, since the data does not provide any information about the structural decomposition of the estimated error covariance matrix Ω, the distribution should be uniform over the set of orthogonal matrices; second, the resulting distribution of the unrestricted impulses should not depend on the initial decomposition of Ω.

It is easy to see that these requirements are in fact closely related. Since the set $\mathcal{O}_n$ of n-dimensional orthogonal matrices is a compact topological group, the definition of uniformity of a measure μ is that for every measurable (Borel) subset $A \subseteq \mathcal{O}_n$ and every element $O \in \mathcal{O}_n$ we have $\mu(OA) = \mu(A)$. It can be shown (see Halmos (1950)), that there is a unique probability measure, called the *Haar measure*, on the Borel sets of $\mathcal{O}_n$ that satisfies this property.

As for the second requirement, if P and $\widetilde{P}$ are two decompositions of Ω, then there is an orthogonal matrix $\widetilde{O}$ such that $\widetilde{P} = P\widetilde{O}$, see Faust (1998). Now, if O comes from the Haar measure, then $\widetilde{P}O = P\widetilde{O}O$ will have the same distribution as PO, so the second requirement will also be satisfied.

The next question is how to sample from the Haar distribution. There are two kinds of algorithms in the literature: Heiberger (1978) and Stewart (1980) generate a random orthogonal matrix from the QR-decomposition of a matrix X whose elements are independent standard normal variables, while Anderson *et al.* (1987) construct it as the product of Givens rotations, where the angles come from beta distributions. We opted for the first method since the QR-decomposition is efficiently implemented in MATLAB, and, as we show next, it can easily be extended to the case of zero restrictions.

ZERO RESTRICTIONS

Imposing zero restriction on some responses effectively decreases the dimensionality of the possible decompositions of Ω, and this should be taken into account when generating the random matrices O.

To solve this problem let us first look at more closely how the QR-decomposition of a matrix X can be interpreted. First of all, the QR-decomposition factorizes X as $X = QR$, where Q is an orthogonal and R is an upper triangular matrix. This factorization is unique if the diagonal elements of R are positive. But in this case, the column vectors are the results of a Gram-Schmidt orthogonalization procedure conducted on the column vectors of X, i.e., the j^{th} column of Q is the normalized residual of the linear projection of the j^{th} column of X onto the previous columns of Q. On the other hand, the restriction that the contemporaneous response of variable i to shock j is zero is equivalent to requiring that the j^{th} column of O is orthogonal to the i^{th} row of P.

A straightforward algorithm to produce an orthogonal matrix O that satisfies the zero restrictions is therefore the following: calculate the j^{th} column of O as the normalized residual of the linear projection of the j^{th} column of X onto the previous columns of O *and* the corresponding rows of P.

In order to formulate this algorithm recursively, let us denote by P_j the rows of P which are orthogonal to shock j, i.e., if the j^{th} shock is to have no contemporaneous effect on k_j variables, then $P_j \in \mathbb{R}^{k_j \times n}$. Next, define

$$A_j = \begin{bmatrix} P'_j & O_{j-1} \end{bmatrix} \in \mathbb{R}^{n\times(k_j+j-1)}, \tag{10}$$

where O_{j-1} is the (already calculated) first $j-1$ columns of O. Then the residual from projecting $X_{\cdot j}$, the j^{th} column of X on the columns of A_j is given by

$$r_j = X_{\cdot j} - A_j \left(A'_j A_j\right)^{-1} A'_j X_{\cdot j}, \tag{11}$$

and the j^{th} column of O will be $r_j/||r_j||$.

We have to check that the resulting distribution of the unrestricted elements of PO do not depend on the choice of P. Let $\widetilde{P} = P\widetilde{O}$ be another decomposition of Ω; we will show that if the algorithm is carried out with $\widetilde{P}$ and $\widetilde{O}'X$, then the result will be $\widetilde{O}'O$, and since $\widetilde{P}\widetilde{O}'O = P\widetilde{O}\widetilde{O}'O = PO$, this—together with the fact that $\widetilde{O}'X$ and X have the same distribution—will deliver the result.

We will use induction. To start off, we have $\widetilde{P}_1 = P_1\widetilde{O}$, $\widetilde{A}_1 = \widetilde{O}'P'_1$, and therefore

$$\begin{aligned} \widetilde{r}_1 &= \widetilde{O}'X_{\cdot 1} - \widetilde{A}_1 \left(\widetilde{A}'_1\widetilde{A}_1\right)^{-1} \widetilde{A}'_1 \widetilde{O}'X_{\cdot 1} = \widetilde{O}'X_{\cdot 1} - \widetilde{O}'P'_1 \left(P_1\widetilde{O}\widetilde{O}'P'_1\right)^{-1} P_1\widetilde{O}\widetilde{O}'X_{\cdot 1} = \\ &= \widetilde{O}'X_{\cdot 1} - \widetilde{O}'P'_1\left(P_1P'_1\right)^{-1} P_1X_{\cdot 1} = \widetilde{O}'r_1, \end{aligned} \tag{12}$$

and therefore the first column will be $\widetilde{O}'$ times the first column of O. Now, if we suppose that the first $j-1$ columns are given by $\widetilde{O}O_{j-1}$, then we will have

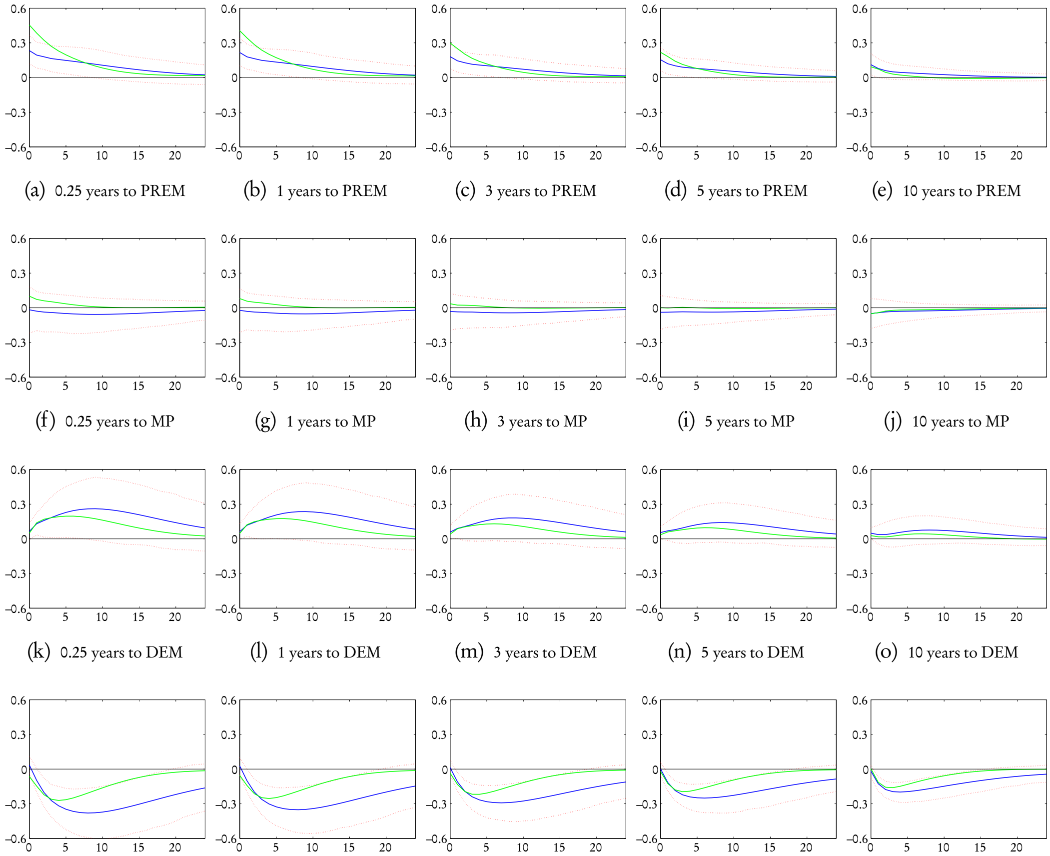
$$\widetilde{A}_j = \begin{bmatrix} \widetilde{P}'_j & \widetilde{O}'O_{j-1} \end{bmatrix} = \begin{bmatrix} \widetilde{O}'P'_j & \widetilde{O}'O_{j-1} \end{bmatrix} = \widetilde{O}' \begin{bmatrix} P'_j & O_{j-1} \end{bmatrix} = \widetilde{O}'A_j, \tag{13}$$

and the same calculation as in equation (12) gives that $\widetilde{r}_j = \widetilde{O}'r_j$, completing the proof.

B Additional graphs

Figure 16

Impulse responses of spot yields, mean and 16-84% confidence bands, short sample



(a) 0.25 years to PREM (b) 1 years to PREM (c) 3 years to PREM (d) 5 years to PREM (e) 10 years to PREM

(f) 0.25 years to MP (g) 1 years to MP (h) 3 years to MP (i) 5 years to MP (j) 10 years to MP

(k) 0.25 years to DEM (l) 1 years to DEM (m) 3 years to DEM (n) 5 years to DEM (o) 10 years to DEM

(p) 0.25 years to SUPP (q) 1 years to SUPP (r) 3 years to SUPP (s) 5 years to SUPP (t) 10 years to SUPP

Figure 17

Mean impulse responses of forward yields (after 0, 3, 6, 9 and 12 months), short sample

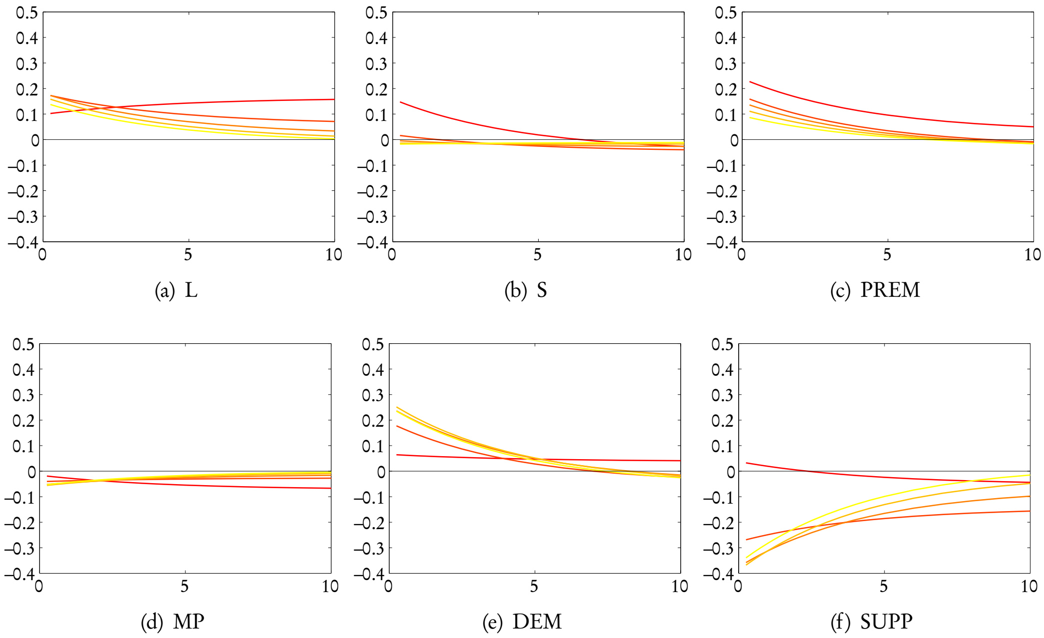


Figure 18

Mean impulse responses of spot yields (after 0, 3, 6, 9 and 12 months), short sample

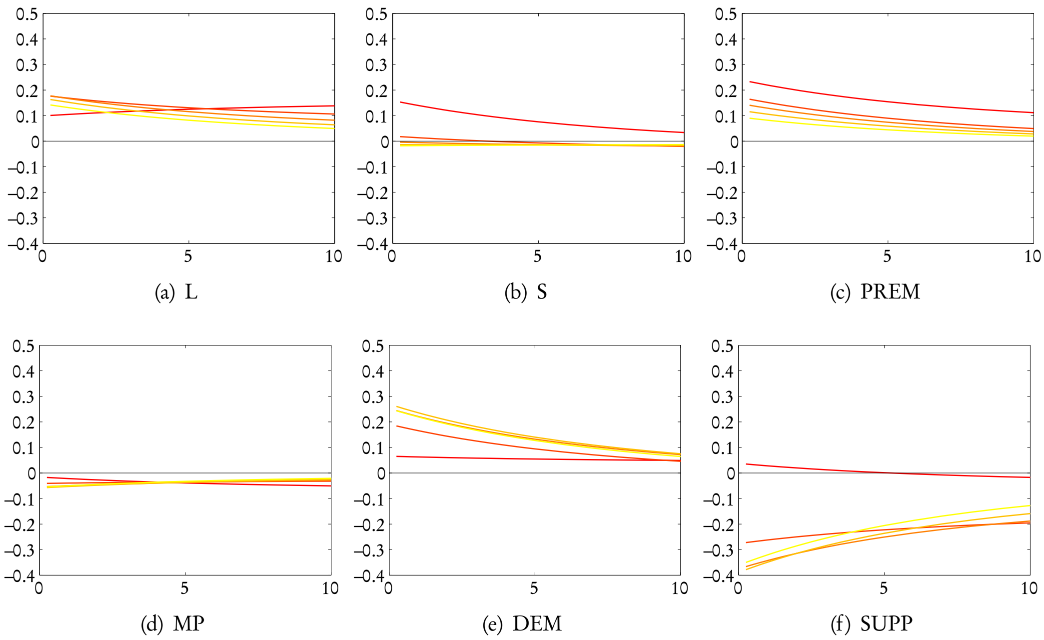
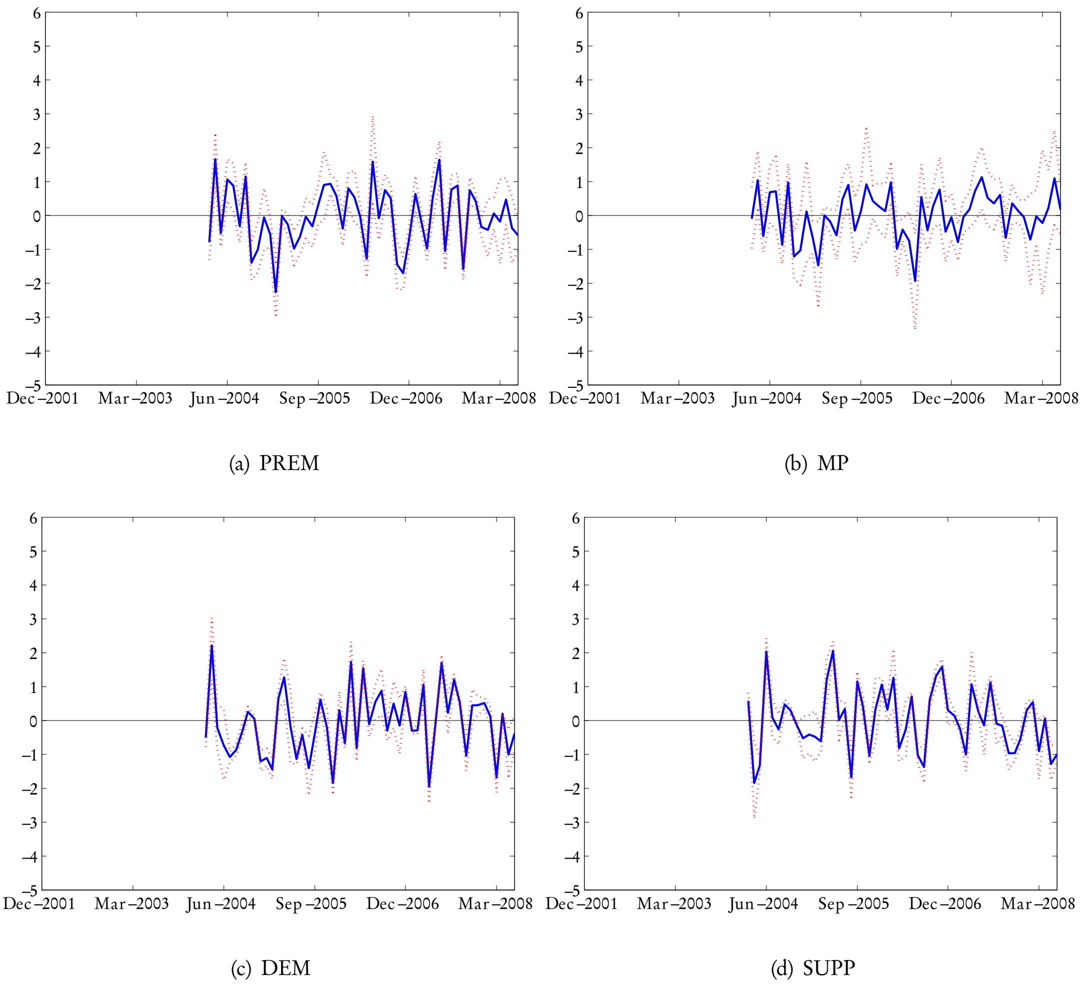


Figure 19

Structural shocks, mean and 16-84% confidence bands, short sample



(a) PREM

(b) MP

(c) DEM

(d) SUPP

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