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MAGYAR NEMZETI BANK

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Testing the asset pricing model of exchange rates with survey data

(Az árfolyam eszközárazási modelljének tesztelése szakértői előrejelzéseken)

Written by: Anna Naszodi*

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Abstract

This paper proposes a new test for the asset pricing model of the exchange rate. It examines whether the way market analysts generate their forecasts is closer to the one implied by the asset pricing model, or to any of those implied by some alternative models. The asset pricing model is supported by the test since it has significantly better out-of-sample fit on survey data than simpler models including the random walk. The traditional test based on forecasting ability is applied as well. The asset pricing model proves to have better forecast accuracy in case of some exchange rates and forecast horizons than the random walk.

JEL: F31, F36, G13.
Keywords: asset pricing exchange rate model, present value model of exchange rate, survey data.

Összefoglalás

A devizaárfolyam eszközárazási modelljének tesztelésére egy új megközelítést javasolok a tanulmányban. Ennek keretében azt vizsgálom, hogy vajon a piaci elemzők előrejelzése az eszközárazási modell logikájához áll-e közelebb, avagy annak egyik, vagy másik alternatívájához. A teszt empirikusan igazolja az elméleti eszközárazási modellt: ennek a modellnek szignifikánsan jobb a szakértői előrejelzési adatokon való mintán kívüli illeszkedése, mint a vizsgált alternatív modelleknek, így például a véletlen bolyongás modelljének. Az új teszt mellett a hagyományos, előrejelzési képességen alapuló tesztet is alkalmazom. Eszerint az eszközárazási modell csupán csak néhány devizaárfolyam és előrejelzési horizont esetében képes pontosabb előrejelzést adni, mint a véletlen bolyongás modellje.

Non-Technical Summary

This paper tests the standard asset pricing model by looking at an empirical implication of the model that has not been investigated before in this context.

As to previous studies in the exchange rate literature, papers test the asset pricing model in different ways. The *traditional test* is based on the forecasting performance of the model. By applying this test, the seminal paper by Meese and Rogoff (1983) came up with the disappointing result that the naive forecast predicting no change in the exchange rate is not worse than the model-based forecasts in the short and medium horizons.

Engel and West (2005) claim that the traditional test has only limited applicability. They argue that the failure of a model at beating the random walk can not be taken as evidence against the model, because under some general conditions the process of the exchange rate is near random walk. They propose an *alternative test* that relies on the following. If the spot exchange rate is determined in a forward-looking way, as it is suggested by the asset pricing view, then the spot exchange rate contains valuable information about the expected future fundamentals. Engel and West show that the exchange rate is useful in forecasting some of the observed fundamentals. This empirical finding supports the asset pricing model as being consistent with it.

Rogoff (2007) challenges the previous findings of Engel and West (2005) by the following claim. If the relationship between the observable fundamentals and the exchange rate is strong, then it is not clear why it does not show up more strongly in the traditional test. Moreover, Engel and West (2005) note that the forecasting ability can be the result of some alternative mechanisms. For instance, if the monetary authority reacts to changes in the exchange rate and the changing policy rate influences the fundamentals, then the active monetary policy creates a link between the unforeseeable changes in the exchange rate and the fundamentals.

For reasons outlined above, neither of the previous two tests is fully satisfactory. This paper contributes to the literature by developing a *third test* that uses survey data on exchange rate forecasts. The logic of the test can be illustrated by a magician's trick. The magician asks someone from the audience to tell her 1-year and 2-year forecast. The person is asked to make a 3-month forecast as well, but instead of telling it to the magician, she should write it down on a piece of paper and hide it in an envelope. If the magician can find out the secret 3-month forecast, then he is likely to know how the forecasts were generated. Moreover, if the forecaster is rational, the model she has in mind is identical to the data generating process of the exchange rate. Therefore, a successful magician knows not only what the forecaster thinks, but also how the exchange rate is determined.

In the paper, the survey forecasts are assumed to be generated by one of the following models. The first one is the *asset pricing model*, where the exchange rate is determined by the weighted average of the fundamentals and the expected exchange rate at some future point in time. The relative weights depend on the forecast horizon so that the expected future exchange rate is an exponential function of the forecast horizon. The second model is a *linear model*, where the forecast is a linear function of the forecast horizon. The third model is the *random walk model*. Here, the exchange rate is equal to the expected future exchange rate that follows a random walk process due to the law of iterated expectations.

The proposed test suggests that the asset pricing model can be used by our hypothetical magician in his show with greater success than any of its alternatives. Or, in other words, this model is the closest to the one that the representative forecaster of the surveys has in mind.

This empirical finding has some further implications. First, the representative forecaster does not think that the process of the exchange rate is random walk; in other words, we can reject the static expectation hypothesis. Second, according to the forecaster's thinking, it is not only the expected future exchange rate that determines the spot exchange rate, but also the factor of fundamentals. Third, the representative forecaster thinks that there is a non-linear relationship between the forecast horizon and the forecast.

These findings are robust in the sense that the asset pricing model has better out-of-sample fit on *survey expectations* than the other models for all the investigated exchange rates on a long sample spanned by January 1999 and April 2009. The investigated rates are the Canadian Dollar, the Egyptian Pound, the Euro, the Israeli Shekel, the Japanese Yen, the Nigerian Naira, the South African Rand, the United Kingdom Pound versus the US Dollar; and the Norwegian Krone, the Swedish Krona, and the Swiss Franc versus the Euro.

The paper applies the traditional test as well. We obtain the usual result: the asset pricing model can not systematically out-perform the random walk at forecasting the exchange rate. However, for some of the investigated exchange rates, and forecast horizons, the asset pricing model proved to be better.

1 Introduction

Although the asset pricing view has become a widely used building block in the exchange rate literature, it has been rejected by the empirical studies of the disconnect literature.[1]

This paper tests the asset pricing model[2] by using survey data on exchange rate forecasts of multiple horizons. The test is based on an empirical implication of the model that has not been investigated before in this context. The implication is that the log exchange rate forecast is an exponential function of the forecast horizon. Whereas the term-structure of forecast is either constant, or linear in the two alternative models, which are the random walk model and the linear model. The three models are fit on three points of the term-structure, and it is checked whether a fourth point is closest to the predicted forecast of the exponential curve, or to that of any of the alternatives. The forth point of the term-structure is the survey forecast with the shortest forecast horizon. This out-of-sample test clearly favors the asset pricing model against its two alternatives. This finding is remarkable, because usually the traditional time series tests can not reject that the exchange rate follows a random walk on the short-run.

As a second test of the asset pricing model, the commonly used test is applied as well that is based on the forecasting ability of the models. First, the competing models are fit on the survey data and then the forecast accuracies of the fitted forecasts are compared. We obtain the usual result. Unfortunately, the asset pricing model can not systematically out-perform the random walk model, however, it has turned out to be better for some of the analyzed exchange rates and forecast horizons. Therefore, the asset pricing model can represent not only the model used by the surveyed respondents, but it also captures some important properties of the data generating process of some exchange rates.

The rejection of the asset pricing model by the previous empirical literature is likely to be attributable to the misspecification of the structural macro models that define the fundamentals. In view of this problem leading to the false rejection of the model, this paper treats the fundamental term with special care. Although it is a common practice in the exchange rate literature to start with a structural macro model and define the fundamentals accordingly, this paper estimates the fundamentals by using a statistical model. Similar empirical approaches are applied also by some previous papers in the literature. See Burda and Gerlach (1993), De Grauwe et al. (1999), Gardeazabal et al. (1997), Naszodi (2011). The advantage of this approach is that it allows us to bypass the problem of choosing among the plenty of competing structural macro models. There is no reason to believe that it is not the macro variables and the expectations on their future evolutions that are the most important determinants of the exchange rate. However, it is unlikely that the commonly used structural models can sufficiently capture the rich dynamics of the fundamentals. These ideas have gained empirical support by Sarno and Valente (2009). They claim that the exchange rate disconnect puzzle is unlikely to be caused by the lack of information in the macro fundamentals, and it is more likely due to frequent shifts in the set of fundamentals driving the exchange rates. Moreover, the commonly used fundamental term constructed from macro data do not only fail to keep track on these frequent shifts, but also fail to contain information on some factors directly unobservable to the econometrician. The typical example for the unobservable variable is the risk premium. Professional forecasters, however, do possess information on both the risk premium and also on all the relevant factors, and they incorporate them into their forecasts. All these motivate me to treat the fundamental as being unobservable and estimate it from survey data on exchange rate forecasts.

[1] The papers by Engel and West (2005) and Engel et al. (2007) find weak empirical evidences that are consistent with the asset pricing view, but their findings can result also from some alternative mechanisms.

[2] The asset pricing model of the exchange rate has different names in the literature. It is called the “asset market view model”by Frenkel and Mussa (1980), the “canonical model”by Krugman (1992) and by Gardeazabal et al. (1997) and the “rational expectations present-value model”by Engel and West (2005).

The paper is structured as follows. Section 2 introduces the competing exchange rate models. Section 3 tests the asset pricing model based on the out-of-sample fit on survey data and the out-of-sample forecasting ability. Finally, Section 4 concludes.

2 Exchange Rate Models

This Section introduces the three exchange rate models: the asset pricing model, the random walk model, and the linear model. In the *asset pricing model*, the exchange rate is the linear combination of the fundamentals and the expected discounted value of future shocks.

$$s_t = v_t + c\frac{E_t(ds_t)}{dt} . \tag{1}$$

Here, s_t is the log exchange rate at time t, v_t is the term of fundamentals at time t, and $\frac{E_t(ds_t)}{dt}$ is the expected instantaneous change of the log exchange rate at time t.[3] The only parameter of this model is c that determines the relative importance of the forward-looking term $\frac{E_t(ds_t)}{dt}$ in the exchange rate.

Macro models that rationalize the asset pricing exchange rate model offer different interpretations of parameter c. For instance, Engel and West (2005) review some standard models, where parameter c is either the semi-elasticity of money demand, or the transformed discount rate, or the inverse of the relative weight of the exchange rate in the Taylor rule. Without committing myself to any of these definitions, I will refer to it neutrally as the *scaling parameter*. Still, the results will be interpreted by using all three definitions.

These structural models provide different definitions for the *fundamental* v_t as well. However, I opt for using neither the definition of c, nor that of v_t in any of these models, nor the corresponding macro data, mainly because of the possibility of misspecification of the underlying macro model, but also because of the substantial measurement errors and the low frequency of these data. However, these definitions of v_t motivate the choice of its processes. Unit-root tests of the previous empirical studies are hardly able to reject the hypothesis that the fundamentals are integrated of order one, no matter how exactly they are defined. Therefore, the assumed process of v is

$$dv_t = \sigma_{v,t} dw_{v,t} . \tag{2}$$

Where $\sigma_{v,t}$ is the volatility of the fundamental, and $w_{v,t}$ is a Wiener process.

In this model, the *expected instantaneous change of the exchange rate* $\frac{E_t(ds_t)}{dt}$ depends on the fundamental v_t, the scaling parameter c, and another factor not mentioned yet. This factor is the market expectation for the log exchange rate of a given future point of time. I denote this future time by T^* and the expected log exchange rate by x_{t,T^*}. I assume that expectations are formed rationally in the sense that the *subjective expectation of the market for the* $T^* - t$ *ahead log exchange rate* is the mathematical expected value conditional on all the information available at the time the expectation is formed

$$x_{t,T^*} = E_t(s_{T^*}) . \tag{3}$$

If the *law of iterated expectations* holds, then the process of x_{t,T^*} is martingale. In order to show this, I formalize the law of iterated expectations as follows

$$E_t(E_{t+1}(s_{T^*})) = E_t(s_{T^*}) . \tag{4}$$

[3] The reasons for writing the model in continuous time instead of discrete time are the following. First, holding foreign currency generates continuous flow of returns. Second, our data is sampled irregularly that can be handled easier in the continuous time model. For those readers, who are more familiar with the discrete time model, Appendix A derives the link between the two.

By substituting Equation (3), the definition of x_{t,T^*}, into the previous formula, we get that $E_t(x_{t+1,T^*}) = x_{t,T^*}$. That is, the process of x_{t,T^*} is martingale. Motivated by that, the assumed process of the factor x_{t,T^*} is

$$dx_{t,T^*} = \begin{cases} \sigma_{x,t,T^*} dw_{x,t,T^*}, & \text{if } t < T^* \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Where w_{x,t,T^*} is a Wiener process. The parameter σ_{x,t,T^*} is the time-varying volatility.

The Wiener process w_{x,t,T^*} is not necessarily independent of $w_{v,t}$. Similarly, x_{t,T^*} is not necessarily independent of v_t. For instance, the latter two are identical in a special case, under the no-bubble condition. If the no-bubble condition holds, then neither the spot exchange rate deviates from the fundamental, nor the expected future exchange rate deviates from the expected future fundamental. The expected future fundamental is equal to its current value, because the fundamental is assumed to follow a martingale process, therefore, the expected future exchange rate should not deviate from the current fundamental either. In that case the changes in x_{t,T^*} and v_t are equal making the Wiener processes w_{x,t,T^*} and $w_{v,t}$ positively correlated. While whenever a gap appears between x_{t,T^*} and v_t, the exchange rate starts to deviate from the fundamental, and it can be interpreted as evidence for the presence of bubbles. Along these lines, an alternative but equivalent approach would be to have a time-varying bubble term in the model instead of the expected exchange rate of a given future point of time. However, it is more convenient to work with the latter, because our survey data are on exchange rate expectations.

One can show, that the process of the exchange rate s_t of (6) is a solution of the model given by Equation (1), and the assumed processes of (2), and (5).

$$ds_t = \frac{1}{c}\frac{e^{-\frac{T^*-t}{c}}}{1-e^{-\frac{T^*-t}{c}}}\left(x_{t,T^*} - s_t\right)dt + \left(1 - e^{-\frac{T^*-t}{c}}\right)\sigma_{v,t}dw_{v,t} + e^{-\frac{T^*-t}{c}}\sigma_{x,t,T^*}dw_{x,t,T^*}\,. \tag{6}$$

Moreover, the exchange rate s_t is the weighted average of the fundamental v_t, and the expectation x_{t,T^*}:[4]

$$s_t = f\left(t, v_t, x_{t,T^*}, \frac{1}{c}\right) = \left(1 - e^{-\frac{T^*-t}{c}}\right)v_t + e^{-\frac{T^*-t}{c}}x_{t,T^*}\,. \tag{7}$$

Equation (6) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation for the future exchange rate. Moreover, the shorter the expectation horizon, the faster the convergence is. The deviation from this trend is due to the stochastic innovations, $dw_{v,t}$, dw_{x,t,T^*}. It is important to notice that the expectation is self-fulfilling in this model, because no matter what the expected exchange rate is, the exchange rate converges to it.

The asset pricing model can be thought of as a two-factor model, where the fundamental v_t and the expected exchange rate for horizon $T^* - t$ are the two factors. These two factors determine not only the exchange rate, but also the expected exchange rate for horizons shorter than $T^* - t$. As it is proved in Appendix C, the expected log exchange rate of any time T, $T^* > T > t$, is

$$E_t(s_T) = e^{\frac{T-t}{c}}(s_t - v_t) + v_t \qquad \forall T, \quad T^* > T > t\,. \tag{8}$$

Besides the asset pricing model, I consider two alternative models, the random walk model, and the linear model.

The *random walk model* is nested by the asset pricing model. Under the parameter restriction $e^{-\frac{T^*-t}{c}} = 1$, the exchange rate s_t is driven exclusively by the expected future exchange rate x_{t,T^*}. (See Equation (7)). And Equation (8) reduces to

$$E_t(s_T) = s_t \qquad \forall T, \quad T^* > T > t\,. \tag{9}$$

Or, in other words, the expected future exchange rates for all horizons until T^* are equal to the spot exchange rate. This feature of the model motivates me to call this model the random walk model.

[4] Appendix B derives Equations (6) and (7).

The third model is the *linear model*, where the exchange rate expectation is the following linear function of the forecast horizon:

$$E_t(s_T) = s_t + (T - t)\mu \qquad \forall T > t\,. \tag{10}$$

We obtain this model under the following assumptions. First, the fundamental follows a Brownian-motion with drift

$$dv_t = \mu dt + \sigma_{v,t} dw_{v,t}\,. \tag{11}$$

In general, the trend parameter μ is different from zero. Therefore, the linear model is not nested by the asset pricing model.

The second assumption is that there is no bubble in the exchange rate, which could make the exchange rate s_t to deviate from its fundamental value v_t. In contrast to the linear model, bubbles are not ruled out in the asset pricing model. Therefore, by comparing the goodness of fit of the linear model with that of the asset pricing model, one can infer whether *bubbles* are thought to drive the exchange rates.

3 Survey-Based Tests of the Exchange Rate Models

This Section tests the three competing exchange rate models against each other in two different ways. First, it is tested whether the way market analysts generate their exchange rate expectations is closer to the one implied by the asset pricing model or to any of those implied by the simpler models. Second, the asset pricing model is tested against the random walk model by the criterion of their forecasting ability.

The survey data that are used for these tests are from the monthly surveys of the Consensus Economics on the expected 3 months, 1 year, and 2 years ahead exchange rates.[5] [6] The survey data is the mean of the forecasts of the survey participants, therefore it mirrors the consensus view of the professional forecasters. The forecasted exchange rates are the Canadian Dollar, the Egyptian Pound, the Euro, the Israeli Shekel, the Japanese Yen, the Nigerian Naira, the South African Rand, the United Kingdom Pound versus the US Dollar; and the Norwegian Krone, the Swedish Krona, and the Swiss Franc versus the Euro. The sample spans from January 1999 to April 2009. The size of the time series dimension of the sample is 123, because the data for March 1999 are missing. The size of the cross-section is 3 as having 3 different forecast horizons for each of the 11 exchange rates. And estimations are carried out separately for each exchange rate.

3.1 TESTING THE EXCHANGE RATE MODELS BASED ON THEIR OUT-OF-SAMPLE FIT ON SURVEY FORECASTS

One way of testing the competing models is to compare their out-of-sample fit. This Section explains how the competing models are estimated for this test, what the test statistics are, and interprets the result.

The estimation method is based on minimizing the sum of squares of differences between the model implied forecasts and the survey forecasts. One reason why the survey forecasts can deviate from the forecasts consistent with the model is that the individual forecasters may have noisy information, like in the model of Bacchetta and Wincoop (2004). The average of the noises in the private information is not necessarily zero. Therefore, even the consensus forecasts, the average of the individual forecasts, can contain errors.

The parameters of each of the models are estimated by using only the 1-year (1Y) and 2-year (2Y) forecasts in the survey, but not the 3-month (0.25Y) forecast. The data on the 3-month forecast is saved to measure the *out-of-sample fit*. After estimating the models, I investigate how close the fitted 3-month forecasts to their survey counterparts are.

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[5] The reported forecasts are not the expected log exchange rates, but the expected exchange rates. I approximate the expected log exchange rates by the log of the reported expected exchange rates in all calculations and estimations. An even more precise approximation would be based on adjusting by half of the variance. This approach is often applied under the assumption that the percentage change of the exchange rate has Gaussian distribution. Both approximations work well according to a simulation-based test. The difference between the approximations are negligible, therefore, I apply the simple one. All results obtained with the other approximation are available from the author upon request.

[6] The forecast horizons usually differ from 3 months, 1 year, and 2 years by a few days, because the surveys do not take place exactly at the end of each month, while the forecasts refer to the end-of-month exchange rates. For instance, the survey can be on the 15th of December of a given year and the participants of that survey should forecast the end-of-March, end-of-December exchange rates of the coming year and the end-of-December exchange rate of the year after. When estimating the model, I treat the forecast horizons rigorously by using the exact number of days in the calculations. While in the theoretical part of the paper and even at deriving the estimators, the above mentioned differences are disregarded.

Accordingly, the estimates for the fundamental v_t and the scaling parameter c in the *asset pricing model* can be obtained by solving the following minimization problem.

$$\min_{c,v_{\underline{\tau}},\ldots,v_{\overline{\tau}}} \sum_{\Theta=\underline{\tau}}^{\overline{\tau}} [\tilde{x}_{\Theta,\Theta+1Y} - E_\Theta(s_{\Theta+1Y})]^2 + [\tilde{x}_{\Theta,\Theta+2Y} - E_\Theta(s_{\Theta+2Y})]^2 \ , \tag{12}$$

where the sample period is between date $\underline{\tau}$ and $\overline{\tau}$. And $E_\Theta(s_{\Theta+1Y})$, $E_\Theta(s_{\Theta+2Y})$ are the model consistent forecasts for 1 year and 2 years given by Equation (8). The survey data on the forecasted 1 year, and 2 years ahead exchange rates at time Θ are denoted by $\tilde{x}_{\Theta,\Theta+1Y}$, and $\tilde{x}_{\Theta,\Theta+2Y}$ respectively.

The above minimization problem of (12) aims at matching the moment conditions given only by the measurement Equation (8), while some other moment conditions determined by the transition Equations (2), and (5) are disregarded. Given that the data contain only very limited information on the variances and covariances of the disturbances in the transition equations, we would not gain much by taking into account even these moments.[7]

The minimization problem of (12) is multi-dimensional. In general, multi-dimensional optimizations raise numerical problems. Luckily, this problem can be reduced to single dimension by utilizing the following analytical solution. Equation (13) solves the minimization problem of (12) for any given constant c, $e^{-\frac{1Y}{c}} \notin \{0,1\}$, and time $t \in [\underline{\tau}, \overline{\tau}]$.[8]

$$\widehat{v}_t^{AP} = \frac{(e^{-\frac{2Y}{c}} + e^{-\frac{1Y}{c}} + 1)s_t - e^{-\frac{3Y}{c}}\tilde{x}_{t,t+1Y} - (e^{-\frac{3Y}{c}} + e^{-\frac{2Y}{c}})\tilde{x}_{t,t+2Y}}{-2e^{-\frac{3Y}{c}} + e^{-\frac{1Y}{c}} + 1} \ . \tag{13}$$

With the analytical solution of Equation (13) in hand, what remains to be done numerically, is only the optimization with respect to parameter c. Once we have the estimates $\widehat{c}^{AP}$ and $\widehat{v}_t^{AP}$, we can calculate the fitted value of the 3-month forecast by using Equation (8).

$$\widehat{x}_{t,t+0.25Y}^{AP} = e^{\frac{T-t}{\widehat{c}^{AP}}}(s_t - \widehat{v}_t^{AP}) + \widehat{v}_t^{AP} \ , \tag{14}$$

where $\widehat{x}_{t,t+0.25Y}^{AP}$ is the estimated 3-month forecast consistent with the asset pricing model.

In case of the *linear model*, parameter μ is estimated by the least squares:

$$\min_\mu \sum_{\Theta=\underline{\tau}}^{\overline{\tau}} [\tilde{x}_{\Theta,\Theta+1Y} - E_\Theta(s_{\Theta+1Y})]^2 + [\tilde{x}_{\Theta,\Theta+2Y} - E_\Theta(s_{\Theta+2Y})]^2 \ , \tag{15}$$

where $E_\Theta(s_{\Theta+1Y})$, $E_\Theta(s_{\Theta+2Y})$ are the model consistent forecasts given by Equation (10). We can calculate the time series of the fitted 3-month forecast consistent with the linear model, $\widehat{x}_{t,t+0.25Y}^{linear}$, by substituting the estimated trend parameter $\widehat{\mu}$ into Equation (10):

$$\widehat{x}_{t,t+0.25Y}^{linear} = s_t + 0.25Y\widehat{\mu} \ . \tag{16}$$

Finally, the 3-month forecast consistent with the *random walk model*, $\widehat{x}_{t,t+0.25Y}^{RW}$, is equal to the spot exchange rate no matter what the survey data are.

$$\widehat{x}_{t,t+0.25Y}^{RW} = s_t \ . \tag{17}$$

After estimating the forecasts consistent with the competing models, I compare the models by using some standard measures on how well they fit the survey data on the expected 3-month ahead exchange rate $\tilde{x}_{t,t+0.25Y}$. These measures inform us about the out-of-sample performance of the competing models, because the survey data $\tilde{x}_{t,t+0.25Y}$ has not been used at estimating $x_{t,t+0.25Y}$. The goodness of fit is measured by the mean absolute error (MAE) and the root mean square error (RMSE).

$$\text{MAE} = P^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}} |\tilde{x}_{t,t+0.25Y} - \widehat{x}_{t,t+0.25Y}| \ , \tag{18}$$

[7] Future research however will aim at filtering the factors by using both the measurement equations and the transition equations and a richer dataset including currency option prices. As it is shown by Naszodi (2008), option prices with different maturities can be used to estimate the volatilities of the short term determinant, v_t, and the long term determinant, x_{t,T^*}, of the exchange rate.

[8] See Appendix D.

$$\text{RMSE} = \left[P^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}} (\tilde{x}_{t,t+0.25Y} - \widehat{x}_{t,t+0.25Y})^2 \right]^{\frac{1}{2}} , \tag{19}$$

where P is the size of the time series dimension of the sample, *i.e.*, the number of 3-month forecasts between time $\underline{\tau}$ and $\overline{\tau}$.

Whether the out-of-sample prediction performance of any of the competing models is significantly different from that of the asset pricing model is tested. In case of the RMSE, the hypothesis to be tested is that the squared errors of the asset pricing model are equal to that of the alternative model. If the alternative model is the random walk for instance, then the hypothesis can be formalized as

$$H_0 : E\left[(\tilde{x}_{t,t+0.25Y} - \widehat{x}^{RW}_{t,t+0.25Y})^2 - (\tilde{x}_{t,t+0.25Y} - \widehat{x}^{AP}_{t,t+0.25Y})^2 \right] = 0 \quad \forall t \in [\underline{\tau}, \overline{\tau}] \, . \tag{20}$$

Under the null

$$\frac{\sqrt{P}\bar{g}}{\sqrt{\widehat{V}}} \sim_A N(0,1) \, , \tag{21}$$

where $\bar{g} = P^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}} g_t$ is the average of the differences between the squared errors. While $g_t = (\tilde{x}_{t,t+0.25Y} - \widehat{x}^{RW}_{t,t+0.25Y})^2 - (\tilde{x}_{t,t+0.25Y} - \widehat{x}^{AP}_{t,t+0.25Y})^2$ is the difference between the squared errors at time t. Finally, $\widehat{V}$ is the estimated variance of g_t. It is calculated as $\widehat{V} = P^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}} (g_t - \bar{g})^2$. The hypotheses and test statistics can be obtained analogously for the MAE and for the other alternative model, the linear model.

Table 1 shows that the asset pricing model has the best out-of-sample fit for all currency pairs according to both measures (MAE, RMSE). The difference between the performances of the competing models is always significant, pointing towards the rejection of nulls of equal predictability.

It is worth to remark that the asset pricing model dominates the random walk model not simply because of being broader. A model that is complex enough can fit the data in-sample even perfectly as an extreme example of overfitting. However, the same model usually performs poorly out-of-sample. The intuitive explanation for this finding is that a sufficiently broad model is flexible enough to learn sample specific regularities and consider them falsely as part of the underlying relationship. Since the goodness of fit is measured out-of-sample in our test, the good performance of the asset pricing model can not be attributed to the model complexity and to the potential problem of overfitting.

The test suggests that the data generating process of the surveys is closer to the asset pricing model than to any of the alternative models. This result can be interpreted as follows. First, the representative forecaster thinks that there is a non-linear relationship between the forecast horizon and the exchange rate forecast, because the asset pricing model has better out-of-sample fit than the linear model. Moreover, we can reject the *static expectation hypothesis*, *i.e.*, the representative forecaster does not think that the process of the exchange rate is random walk.[9] Finally, we can make inferences about the presence of bubbles and the process of the fundamentals given that the linear model and the asset pricing model differ in the assumptions on the existence of bubbles and the trend in the fundamentals. Provided that the linear model has been found to have poorer out-of-sample fit than the asset pricing model, the representative forecaster either thinks that bubbles drive the exchange rate, or she thinks that the trend of the fundamentals is zero. As a third alternative, the assumption that one of the three models is thought be the right one, is violated. For instance, she may believe that there are no bubbles and the trend of the fundamental is stochastic. It is beyond the scope of this paper to consider such models.

We can also learn the magnitude of parameter c from the test. Theoretically, the parameter restriction of the random walk model is fulfilled, if $e^{-\frac{T^*-t}{c}} = 1$. By rejecting the random walk model, we can also reject that parameter c is infinitely large. By following Engel and West (2005), we can think of the scaling parameter c of being either the *inverse of the relative*

[9] Frankel and Froot (1987) also reject the static expectation hypothesis.

weight of the exchange rate in the Taylor rule, or the *semi-elasticity of money demand*.[10] Depending on which of these interpretations is preferred, one of the following conclusions can be drawn from rejecting $c = \infty$. First, the representative forecaster thinks that the monetary policy reacts to the deviation of the exchange rate from the targeted level. Second, she thinks that the country is not in liquidity trap.

3.2 TESTING THE EXCHANGE RATE MODELS BASED ON THEIR OUT-OF-SAMPLE FORECASTING PERFORMANCES

This Section tests the models based on their forecasting abilities. First, the models are fitted on the survey data by applying a similar method to the one described in Section 3.1. Then, the forecasting performances of the fitted values are compared.

In contrast to Section 3.1, here I use not only the survey data on the 1-year and 2-year forecasts, but also the 3-month forecast for estimation. Another difference is that I apply the *recursive estimation method*, where only the data available until time t are used to estimate the model-consistent forecast of time t. As t increases, the parameters are re-estimated on a larger sample.

The estimated parameters of the *asset pricing model* are obtained by solving the following minimization problem of a given t:

$$\min_{c,v_{\underline{\tau}},\ldots,v_t} \sum_{\Theta=\underline{\tau}}^{t} [\tilde{x}_{\Theta,\Theta+0.25Y} - E_\Theta(s_{\Theta+0.25Y})]^2 + [\tilde{x}_{\Theta,\Theta+1Y} - E_\Theta(s_{\Theta+1Y})]^2 + [\tilde{x}_{\Theta,\Theta+2Y} - E_\Theta(s_{\Theta+2Y})]^2 \ , \tag{22}$$

where $E_\Theta(s_{\Theta+0.25Y})$, $E_\Theta(s_{\Theta+1Y})$, and $E_\Theta(s_{\Theta+2Y})$ are the model consistent forecasts given by Equation (8). By substituting the estimates $\widehat{c}^{AP}$, $\widehat{v}_t^{AP}$, the spot exchange rate s_t, and the forecast horizons of 3 months, 1 year, and 2 years into Equation (8), we obtain the fitted forecasts consistent with the asset pricing model, $\widehat{x}^{AP}_{t,t+0.25Y}$, $\widehat{x}^{AP}_{t,t+1Y}$, $\widehat{x}^{AP}_{t,t+2Y}$.

Similarly, the estimated forecasts at time t consistent with the *linear model*, $\widehat{x}^{linear}_{t,t+0.25Y}$, $\widehat{x}^{linear}_{t,t+1Y}$, $\widehat{x}^{linear}_{t,t+2Y}$, are calculated by solving the minimization problem of (23) for the same t and substituting the estimates of μ into (10).

$$\min_{\mu} \sum_{\Theta=\underline{\tau}}^{t} [\tilde{x}_{\Theta,\Theta+0.25Y} - E_\Theta(s_{\Theta+0.25Y})]^2 + [\tilde{x}_{\Theta,\Theta+1Y} - E_\Theta(s_{\Theta+1Y})]^2 + [\tilde{x}_{\Theta,\Theta+2Y} - E_\Theta(s_{\Theta+2Y})]^2 \ , \tag{23}$$

where $E_\Theta(s_{\Theta+0.25Y})$, $E_\Theta(s_{\Theta+1Y})$, and $E_\Theta(s_{\Theta+2Y})$ are given by Equation (10).

While, the fitted forecasts consistent with the *random walk model*, $\widehat{x}^{RW}_{t,t+0.25Y}$, $\widehat{x}^{RW}_{t,t+1Y}$, $\widehat{x}^{RW}_{t,t+2Y}$, are simply equal to the spot exchange rate s_t.

The fitted forecasts $\widehat{x}^{AP}_{t,t+0.25Y}$, $\widehat{x}^{AP}_{t,t+1Y}$, $\widehat{x}^{AP}_{t,t+2Y}$, $\widehat{x}^{linear}_{t,t+0.25Y}$, $\widehat{x}^{linear}_{t,t+1Y}$, $\widehat{x}^{linear}_{t,t+2Y}$, $\widehat{x}^{RW}_{t,t+0.25Y}$, $\widehat{x}^{RW}_{t,t+1Y}$, $\widehat{x}^{RW}_{t,t+2Y}$, and the realized log exchange rates $s_{t+0.25Y}$, s_{t+1Y}, s_{t+2Y} are used to calculate some measures of the forecast accuracy. The traditional measures are the mean absolute forecast error (MAE), and the root mean square forecast error (RMSE).

$$\text{MAE} = R^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}-\theta} |s_{t+\theta} - \widehat{x}_{t,t+\theta}| \ , \tag{24}$$

$$\text{RMSE} = \left[R^{-1} \sum_{t=\underline{\tau}}^{\overline{\tau}-\theta} (s_{t+\theta} - \widehat{x}_{t,t+\theta})^2 \right]^{\frac{1}{2}} \ , \tag{25}$$

where R denotes the size of the subsample between time $\underline{\tau}$ and $\overline{\tau} - \theta$.

Besides the MAE and the RMSE, a third measure, the weighted measure of sign prediction WSP, is calculated as well. The WSP depends on whether the direction-of-changes are forecasted correctly, and also on the weight assign to each forecast.

[10] See the Taylor rule model and the money income model presented by Engel and West (2005) page 492–496, where parameter α is the semi-elasticity of money demand, and β_0 denotes the relative weight of the exchange rate in the Taylor rule.

Since we care more about forecasting the sign of the changes when the changes are large, the weight depends on the magnitude of changes.

$$\begin{aligned} \text{WSP} &= R^{-1}\sum_{t=\underline{\tau}}^{\overline{\tau}-\theta}(-I_{t,\theta}W_{t,\theta}) \\ I_{t,\theta} &= \text{sign}(\widehat{x}_{t,t+\theta}-s_t)\text{sign}(s_{t+\theta}-s_t) \\ W_{t,\theta} &= |s_{t+\theta}-s_t|\frac{1Y}{\theta}, \end{aligned} \tag{26}$$

where $I_{t,\theta}$ is zero either if no change has been forecasted, or if the exchange rate has not changed. While it is $+1$ if the forecasted direction-of-change is right; -1 if the forecasted direction-of-change is wrong. The minus sign in front of $I_{t,\theta}$ in Equation (26) makes the WSP to be similar to the MAE and the RMSE in the sense that the smaller its value, the better the forecast is. Finally, the weight $W_{t,\theta}$ is equal to the annualized absolute value of the ex post realized percentage change of the exchange rate.

One can think of the WSP not only as a weighted measure of the performance of the direction-of-change forecasts, but also as a proxy for the negative profit generated by a simple trading strategy. The trading strategy suggests to buy the currency that is forecasted to appreciate, and sell the one that is forecasted to depreciate no matter what the magnitude of the forecasted change is. A similar profitability measure is used by Macdonald and Marsh (1996), Boothe (1983), Boothe and Glassman (1987). In contrast to the WSP, their measure takes into account the interest rate differential as well.

It is tested whether the forecasting performances of the models are the same as that of the random walk on the horizon θ. If the forecasting performance is measured by the RMSE, and the alternative model of the random walk is the asset pricing model, then the hypothesis to be tested is that the square realized forecast errors are the same. The H_0 can be formalized as:[11]

$$H_0 : E\left[(s_{t+\theta}-\widehat{x}^{RW}_{t,t+\theta})^2-(s_{t+\theta}-\widehat{x}^{AP}_{t,t+\theta})^2\right]=0 \qquad \forall t\in[\underline{\tau},\overline{\tau}]\,. \tag{27}$$

The adequate test for this hypothesis is the Diebold-Mariano test. (See Diebold and Mariano (1995).) Under the null

$$\frac{\sqrt{R}\,\bar{g}}{\sqrt{\widehat{V}}}\sim_A N(0,1)\,, \tag{28}$$

where $g_t=(s_{t+\theta}-\widehat{x}^{RW}_{t,t+\theta})^2-(s_{t+\theta}-\widehat{x}^{AP}_{t,t+\theta})^2$ is the difference between the square forecast errors at time t, and $\bar{g}$ is the time average of these differences. Finally, $\widehat{V}$ is the estimated long-run variance of g_t. If the forecast horizon θ is γ number of months, then the number of overlapping months for two consecutive monthly forecasts is $\gamma-1$. The forecast errors follow moving average processes of order $\gamma-1$ making the forecast errors autocorrelated. The autocorrelation consistent variance can be estimated by $\widehat{V}=\sum_{j=-\gamma+1}^{\gamma-1}\widehat{\Gamma}_j$, where $\widehat{\Gamma}_j=R^{-1}\sum_{t>|j|}(g_t-\bar{g})(g_{t-|j|}-\bar{g})$.

Tables 2 and 3 summarize some of the empirical results.[12] Our first finding is that the relative rankings of the models vary across the three measures. The asset pricing model is surprisingly successful relative to the random walk model, when the measure takes into account only whether the direction-of-change in the exchange rate is forecasted correctly. While the asset pricing model does not perform so well at forecasting the magnitude of changes. Based on the WSP, the asset pricing model is better than the random walk model for almost all pairs of exchange rates and forecast horizons. There are only 6 exceptions out of the 33 pairs of exchange rates and horizons. For all the remaining 27 cases the fitted forecasts of the asset pricing model can be used successfully at forecasting the direction-of-changes in the exchange rate. The asset pricing model is better than the random walk model only for 14 cases out of the 33 if the forecast accuracy is measured by the RMSE. While the same number is 12 for the MAE.

The asset pricing model is significantly better than the random walk model at 10% for at least one of the measures of the MAE, the RMSE, and the WSP for the following cases: Egyptian Pound 3-month, Nigerian Naira 3-month, Egyptian Pound 1-year, Nigerian Naira 1-year, Egyptian Pound 2-year, Euro 2-year, and UK Pound 2-year. Still, we can not say that the asset pricing model is clearly superior to the random walk at forecasting, because the latter is significantly better at the same

[11] For the MAE, the WSP, and for the other alternative models the hypotheses and test statistics can be obtained analogously.

[12] The detailed results are reported by Tables 4, 5, 6, 7, 8, 9, 10, 11, and 12.

10% level for at least one of the measures for the following pairs of exchange rates and horizons: Japan Yen 3-month, South African Rand 3-month, Swedish Krona 3-month, Swiss Franc 3-month, Japan Yen 1-year, Swedish Krona 1-year, Nigerian Naira 2-year, Swedish Krona 2-year. For the majority of the pairs of exchange rates and forecast horizons, neither the asset pricing model nor the random walk model out-performs the other significantly. The lack of significance can be, however, due to having too short samples.

In order to compare the models on an even larger sample, I calculate also the aggregated MAE, RMSE, and WSP by pooling the forecast errors for all the eleven exchange rates. (See either Tables 2 and 3, or the last rows in Tables 4, 5, 6, 7, 8, 9, 10, 11, and 12.) According to the aggregated measures, the only case when the random walk model is significantly better than the asset pricing model is when the forecast horizon is 3 months and the distance is measured by the MAE. While the differences of the squared forecast errors on the same horizon are not significant. As the horizon gets longer, the differences of both the absolute and the squared forecast errors are negative but insignificant. One reason for this improvement of the forecasting performance of the asset pricing model is that the relative importance of the expectation factor x_{t,T^*} at determining the exchange rate is decreasing in the forecast horizon. Since the expectation factor x_{t,T^*} follows a random walk, the higher its weight in the exchange rate, the closer the process of the exchange rate to the random walk is. Therefore, the random walk model is less likely to be beaten on the shorter horizons.

In contrast to the MAE and the RMSE, the WSP of the pooled forecasts is significantly smaller at 5% for the asset pricing model, than for the random walk model for all the three horizons. It is important to notice that the success at forecasting the direction-of-change on the aggregated level is not only due to some exotic currencies. The WSP is significantly smaller than zero also for the Euro and the UK Pound versus the US Dollar on the 2 years horizon.

In order to see where the forecasting ability of the asset pricing model comes from, I calculate the MAE, the RMSE, and the WSP also for the raw survey data. Tables 4, 5, 6, 7, 8, 9, 10, 11, and 12 show that when the asset pricing model beats the random walk, so does almost always the raw survey data. And similarly, whenever the asset pricing model is dominated by the random walk, so is the raw survey data with two exceptions. These findings suggest that the forecasting ability or inability of the fitted forecasts consistent with the asset pricing model is mainly attributable to the raw survey data. However, it is also evident that the asset pricing model improves substantially the forecast accuracy of the survey for most of the exchange rates and horizons by cleaning the data from errors. For instance, the aggregated WSP is -0.86%, -1.49%, -1.76% for the raw survey data for the horizons 3 months, 1 year, and 2 years respectively. While it is -2.2%, -1.62%, and -1.81% for the asset pricing model for the same horizons. That is, one can make more accurate forecast with the asset pricing model fitted on the survey data than with the raw survey data. If the asset pricing model were the only model capable of mitigating the error and enhancing the forecasting power, then it could be taken as a clear evidence favoring this model. Unfortunately, an alternative model, the linear model can deliver the same improvement. The aggregated WSP is -3.11%, -2.47%, -2.25% for the linear model for the horizons 3 months, 1 year, and 2 years respectively.

Based on the forecast accuracy of the survey data measured on the aggregate level by the WSP, we can say that the raw survey data out-performs the random walk model on all the three horizons, and its forecasting ability is significantly better on the 1 year, and 2 years horizons. While, the survey data are not found to be better than the random walk based on the other measures, the MAE, and the RMSE.

All in all, our results on the forecasting performances are not conclusive about the ranking of the models. The lack of a definite answer to which of the models represents the best the dynamics of the exchange rate is not surprising for the following reasons.

First, as it is argued by Engel and West (2005), it is almost impossible to come up with a better forecast than the random walk. They point out that if one of the factors driving the exchange rate follows a random walk process and the relative weight of this factor is high enough, then the exchange rate follows a process that is indistinguishable from the random walk on the usual sample sizes. I add the following to their theoretical consideration. Once we assume rational expectations, it is redundant to assume also that at least one of the factors follows a random walk process. Under rational expectations it is automatically fulfilled for the expectation factor, x_{t,T^*} with a fixed T^*, because of the law of iterated expectations. While the random walk behavior of the fundamental, although assumed in this paper, is less obvious. The unit root tests are usually of low power. Therefore, the empirical evidences supporting the random walk of the fundamental are weak. Regarding the second condition of Engel and West (2005), the estimates in this paper suggest that the exchange rate is thought to be driven mainly by the expectation x_{t,T^*} as the estimated relative weight of x_{t,T^*} in s_t is high.[13] The estimates of the relative weights, defined as $\frac{\left|e^{-\frac{T^*-t}{c}}\right|}{\left|e^{-\frac{T^*-t}{c}}\right|+\left|1-e^{-\frac{T^*-t}{c}}\right|}$ in the asset pricing model, are reported by Table 13.

Another reason why the asset pricing model has not proved to be superior to the random walk is the limitation of the test. As it is pointed out by Clark and West (2006), the Diebold-Mariano-test is undersized. Or, in other words, this test rejects the nested model rarer than it should for a given significance level.[14] For this reason, even if the test formally rejects the asset pricing model against the random walk model in some of the cases examined here, the former can still be the right one.

[13] This finding is supported also by Sarno and Sojli (2009), who estimate the relative weight of the expectations from survey data.

[14] For linear models, the problem can be fixed by adjusting the test statistics by an easily computable term. Whereas for non-linear models, like the asset pricing model, the simple adjustment is not applicable.

4 Conclusion

This paper has proposed a new test for the asset pricing model. This test uses survey data on exchange rate expectations. It examines whether the way market analysts generate their forecasts is closer to the one implied by the asset pricing model, or to any of those implied by simpler models. The simpler models are the linear model, and the random walk model. The three models differ in their predictions on the term-structure of forecasts. The forecast is an exponential function of the forecast horizon in the asset pricing model, while it is a linear function in the linear model. Since the random walk model predicts no change in the exchange rate in any horizon, this model is consistent with a flat term-structure.

The remarkable result of the test is that the asset pricing model with the exponential term-structure has been found to have significantly better fit on the survey data, than the simpler models. The goodness of fit is measured out-of-sample and not in-sample, therefore the dominance of the most complex model, the asset pricing model, can not be attributed to overfitting. We can interpret the result of the test as follows. What the representative professional exchange rate forecaster has in mind about the exchange rate can be represented by the asset pricing model far the best. If we believe that the model used by the forecasters is identical to the data generating process of the exchange rate, then the asset pricing model can capture the best way not only how the expectations on the exchange rate are formed, but also how the exchange rate is actually determined.

Whether the asset pricing model provides the most realistic description on the dynamics of the exchange rate itself has been tested directly as well. For this purpose, the conventional test has been applied that investigates the relative forecasting abilities of the models. First, the models have been fitted on the survey forecasts. Then, the forecasting performances of the fitted forecasts have been compared. We have found that the forecasting ability of the raw survey data can be enhanced by fitting either the asset pricing model, or the linear model on the data. However, even the fitted forecasts are rarely significantly better than the random walk forecasts. Still, for some pairs of exchange rates and forecast horizons, the asset pricing model performs better than its alternative. But for some others, the random walk provides the most accurate forecast. As it has been argued by Engel and West (2005), the failure of a model at systematically out-performing the random walk can not be taken as evidence against the model, because under some general conditions the process of the exchange rate is near random walk. As it has been shown by this paper, these conditions are fulfilled. First, the process of the expected exchange rate is random walk because of the law of iterated expectations. Second, the estimated relative importance of the expected exchange rate at determining the exchange rate is high. Therefore, when judging the asset pricing model one should not rely too much on the mixed results of the second test, and should use alternative tests as well. An example of these alternative tests is the one introduced and implemented in this paper that strongly favors the asset pricing model against the random walk.

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Appendix A

This Appendix derives the link between two commonly used asset pricing equations. One is in *continuous time* and it is used by Froot and Obstfeld (1991) among others. The other is a *discrete time* model that is equally popular in the exchange rate literature. The latter has been used by Engel and West (2005) for instance. Here, it is demonstrated that the discrete time model is compatible with the continuous time model.

The first model is given by Equation (1) that I repeat here for convenience.

$$s_t = v_t + c\frac{E_t(ds_t)}{dt} \, .$$

The second model is the following discrete time model (see Equation (7) in Engel and West (2005), where I have translated their notation to be consistent with the one in this paper):

$$s_t = (1-b)v_t + bE_t(s_{t+\Delta t}) \, . \tag{29}$$

Parameter b is the discount factor. Although the discount factor b has no index, it corresponds to the Δt period. In order to make it explicit, I substitute $b = e^{-\rho\Delta t}$ into Equation (29), where ρ is the discount rate.

$$s_t = (1-e^{-\rho\Delta t})v_t + e^{-\rho\Delta t}E_t(s_{t+\Delta t}) \, . \tag{30}$$

By subtracting $e^{-\rho\Delta t}s_t$ from both sides of Equation (30), we obtain

$$(1-e^{-\rho\Delta t})s_t = (1-e^{-\rho\Delta t})v_t + e^{-\rho\Delta t}E_t(s_{t+\Delta t} - s_t) \, . \tag{31}$$

After dividing by $1-e^{-\rho\Delta t}$, we get

$$s_t = v_t + \frac{e^{-\rho\Delta t}}{1-e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t) \, . \tag{32}$$

In order to make the second model in discrete time comparable to the first model in continuous time, we take the limit.

$$s_t = v_t + \lim_{\Delta t\to 0}\left(\frac{e^{-\rho\Delta t}}{1-e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t)\right) \, . \tag{33}$$

The second term on the RHS of Equation (33) can be rearranged along the following lines

$$\lim_{\Delta t\to 0}\left(\frac{e^{-\rho\Delta t}}{1-e^{-\rho\Delta t}}E_t(s_{t+\Delta t} - s_t)\right) = \lim_{\Delta t\to 0}\left(\frac{e^{-\rho\Delta t}\Delta t}{1-e^{-\rho\Delta t}}\frac{E_t(s_{t+\Delta t} - s_t)}{\Delta t}\right) =$$

$$= \lim_{\Delta t\to 0}\left(\frac{\Delta t}{e^{\rho\Delta t}-1}\frac{E_t(s_{t+\Delta t} - s_t)}{\Delta t}\right) = \frac{1}{\rho}\frac{E_t(ds_t)}{dt} \, . \tag{34}$$

By substituting $\frac{1}{\rho}\frac{E_t(ds_t)}{dt}$ into Equation (33), we obtain the continuous version of the second model that can be directly compared to the first model of Equation (1).

$$s_t = v_t + \frac{1}{\rho}\frac{E_t(ds_t)}{dt} \, . \tag{35}$$

It is straightforward from the comparison of (35) and (1) that the two are identical under the condition $c = \frac{1}{\rho}$. By substituting the definition $b = e^{-\rho\Delta t}$ of parameter ρ into this condition, we obtain an equivalent condition for the original parameters c and b:

$$b = e^{-\frac{1}{c}\Delta t} \, . \tag{36}$$

We get another form of the condition, if we express the relationship between the discount rate ρ and the discount factor b in discrete time, *i.e.*, $b = \left(\frac{1}{1+\rho}\right)^{\Delta t}$. I repeat the derivation from Equation (30) by using this latter definition of the discount rate ρ, and by applying the following approximation $\frac{1}{\log(1+\rho)} \approx \frac{1}{\rho}$. Then, the model compatibility condition is of the form of

$$b = \left(\frac{c}{1+c}\right)^{\Delta t}. \tag{37}$$

Appendix B

This Appendix proves that the function $s_t = f\left(t, v_t, x_{t,T^*}, \frac{1}{c}\right)$ of (7) satisfies the implicit relationship (1) between the exchange rate and the fundamentals, and the process of the log exchange rate is given by (6).

According to Ito's stochastic change-of-variable formula, the function $f\left(t, v_t, x_{t,T^*}, \frac{1}{c}\right)$ should satisfy (38).

$$df = \left[\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_{t,T^*}^2}\sigma_{x,t,T^*}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_{t,T^*}\partial v_t}\rho\left(dw_{v,t}, dw_{x,t,T^*}\right)\sigma_{v,t}\sigma_{x,t,T^*}\right]dt + \\ + \frac{\partial f}{\partial v_t}\sigma_{v,t}dw_{v,t} + \frac{\partial f}{\partial x_{t,T^*}}\sigma_{x,t,T^*}dw_{x,t,T^*}\ , \quad (38)$$

where $\rho\left(dw_{v,t}, dw_{x,t,T^*}\right)$ denotes the correlation between $dw_{v,t}$ and dw_{x,t,T^*}.

By calculating the partial derivatives of (7) and by substituting these derivatives into (38), we obtain

$$ds_t = \frac{1}{c}e^{-\frac{T^*-t}{c}}\left(x_{t,T^*} - v_t\right)dt + \left(1 - e^{-\frac{T^*-t}{c}}\right)\sigma_{v,t}dw_{v,t} + e^{-\frac{T^*-t}{c}}\sigma_{x,t,T^*}dw_{x,t,T^*}. \quad (39)$$

Equation (39) gives that the expected instantaneous change of the exchange rate is

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c}e^{-\frac{T^*-t}{c}}\left(x_{t,T^*} - v_t\right)\ . \quad (40)$$

The fundamental v_t can be expressed from (7) as:

$$v_t = \frac{s_t - e^{-\frac{T^*-t}{c}}x_{t,T^*}}{1 - e^{-\frac{T^*-t}{c}}}\ . \quad (41)$$

Then, by plugging Equation (41) into (40) , we get (1). Thereby, Equation (7) proved to satisfy the implicit relationship (1).

What remains to be proved is that the dynamics of the exchange rate is given by (6). However, by plugging Equation (41) into (39), we obtain (6).

Appendix C

This Appendix proves that Equation (8) holds in our two-factor model. For convenience, Equation (8) is repeated here:

$$E_t(s_T) = e^{\frac{T-t}{c}}(s_t - v_t) + v_t \qquad \forall T, \quad T^* > T > t\,.$$

Let us choose a T between t and T^*, and replace t by T in Equation (7). We get

$$s_T = \left(1 - e^{-\frac{T^*-T}{c}}\right) v_T + e^{-\frac{T^*-T}{c}} x_{T,T^*}\,. \tag{42}$$

By taking the expected value of both sides of Equation (42), we obtain

$$E_t(s_T) = \left(1 - e^{-\frac{T^*-T}{c}}\right) E_t(v_T) + e^{-\frac{T^*-T}{c}} E_t(x_{T,T^*})\,. \tag{43}$$

The processes of the fundamental and the expectation factor are martingales. Therefore, Equation (43) can be simplified to

$$E_t(s_T) = \left(1 - e^{-\frac{T^*-T}{c}}\right) v_t + e^{-\frac{T^*-T}{c}} x_{t,T^*}\,. \tag{44}$$

We can express x_{t,T^*} from Equation (44) as

$$x_{t,T^*} = e^{\frac{T^*-T}{c}} E_t(s_T) + \left(1 - e^{\frac{T^*-T}{c}}\right) v_t\,. \tag{45}$$

By substituting Equation (45) into Equation (7), we obtain (8).

Appendix D

This Appendix proves that Equation (13) solves the optimization problem given by the objective function of Equation (12), and the constraint of (8), if parameter c is a given constant such that $e^{-\frac{1Y}{c}} \neq 0$, $e^{-\frac{1Y}{c}} \neq 1$.

By substituting the expressions for $E_\Theta(s_{\Theta+1Y})$ and $E_\Theta(s_{\Theta+2Y})$ given by (8) into the objective function of (12), we get

$$\left[\tilde{x}_{\Theta,\Theta+1Y} - e^{\frac{1Y}{c}}(s_\Theta - v_\Theta) - v_\Theta\right]^2 + \left[\tilde{x}_{\Theta,\Theta+2Y} - e^{\frac{2Y}{c}}(s_\Theta - v_\Theta) - v_\Theta\right]^2 . \tag{46}$$

This objective function is quadratic in v_Θ, and the coefficient of v_Θ^2 is positive for any given constant c, $e^{-\frac{1Y}{c}} \neq 0$, $e^{-\frac{1Y}{c}} \neq 1$.

$$\begin{aligned} & v_\Theta^2(e^{-\frac{4Y}{c}} - e^{-\frac{2Y}{c}} - 2e^{-\frac{1Y}{c}} + 2) + \\ & \quad + v_\Theta\left[(-2e^{-\frac{4Y}{c}} + 2e^{-\frac{1Y}{c}})s_\Theta + (2e^{-\frac{1Y}{c}} - 2)\tilde{x}_{\Theta,\Theta+1Y} + (2e^{-\frac{2Y}{c}} - 2)\tilde{x}_{\Theta,\Theta+2Y}\right] + \text{const} . \end{aligned} \tag{47}$$

Therefore, it has a unique minimum at

$$v_\Theta = \frac{(e^{-\frac{2Y}{c}} + e^{-\frac{1Y}{c}} + 1)s_\Theta - e^{-\frac{3Y}{c}}\tilde{x}_{\Theta,\Theta+1Y} - (e^{-\frac{3Y}{c}} + e^{-\frac{2Y}{c}})\tilde{x}_{\Theta,\Theta+2Y}}{-2e^{-\frac{3Y}{c}} + e^{-\frac{1Y}{c}} + 1} . \tag{48}$$

Appendix E Tables

Table 1
Out-of-sample fit of the asset pricing model, the linear model, and the random walk model

		Mean absolute error			Root mean square error		
Exch.	Num.	Model:			Model:		
rate	obs.	AP	linear	RW	AP	linear	RW
$\frac{\text{CAD}}{\text{USD}}$	123	**0.0085**	0.0132	0.0136	**0.0118**	0.0166	0.0169
(stat)			(5.6668)***	(5.9499)***		(5.1875)***	(5.3398)***
$\frac{\text{EGP}}{\text{USD}}$	123	**0.0086**	0.0104	0.0124	**0.0125**	0.0155	0.0184
(stat)			(3.7205)***	(4.3898)***		(3.5752)***	(3.9799)***
$\frac{\text{USD}}{\text{EUR}}$	123	**0.0111**	0.0179	0.0195	**0.0136**	0.0228	0.025
(stat)			(4.9784)***	(5.719)***		(4.5729)***	(5.0809)***
$\frac{\text{ILS}}{\text{USD}}$	123	**0.0099**	0.0136	0.0155	**0.0131**	0.0175	0.02
(stat)			(7.1695)***	(7.1373)***		(5.3498)***	(6.2883)***
$\frac{\text{JPY}}{\text{USD}}$	123	**0.0147**	0.0219	0.022	**0.0193**	0.0271	0.0276
(stat)			(5.4804)***	(4.984)***		(4.4788)***	(4.3588)***
$\frac{\text{NGN}}{\text{USD}}$	123	**0.0164**	0.023	0.0338	**0.027**	0.0332	0.042
(stat)			(5.0791)***	(9.1989)***		(3.4786)***	(6.8504)***
$\frac{\text{NOK}}{\text{EUR}}$	123	**0.0075**	0.012	0.012	**0.0109**	0.0157	0.0158
(stat)			(5.651)***	(5.5846)***		(3.8247)***	(3.803)***
$\frac{\text{ZAR}}{\text{USD}}$	123	**0.0205**	0.0307	0.0313	**0.0266**	0.0387	0.04
(stat)			(6.8361)***	(5.6988)***		(5.8029)***	(5.1466)***
$\frac{\text{SEK}}{\text{EUR}}$	123	**0.0062**	0.0107	0.0149	**0.0082**	0.0133	0.0174
(stat)			(6.375)***	(10.0264)***		(3.9929)***	(6.3914)***
$\frac{\text{CHF}}{\text{EUR}}$	123	**0.0048**	0.009	0.009	**0.0065**	0.011	0.011
(stat)			(9.0968)***	(8.9247)***		(7.346)***	(7.0873)***
$\frac{\text{USD}}{\text{GBP}}$	123	**0.0087**	0.0114	0.0113	**0.0121**	0.0149	0.0149
(stat)			(4.0216)***	(4.0059)***		(3.0339)***	(3.0322)***

The 3-month fitted forecast of the asset pricing model, the linear model, and the random walk model are given by Equations (14), (16), and (17). The reported MAE and RMSE measure the distance of these fitted forecasts from the 3-month survey forecast. The test statistics in parentheses compares the performance of the asset pricing model with those of the alternative models.
$***$: *significant at* 1%.

Table 2
The exchange rates for which the asset pricing model gives better forecast than the random walk for various horizons

	3 months horizon			1 year horizon			2 years horizon		
	MAE	RMSE	WSP	MAE	RMSE	WSP	MAE	RMSE	WSP
1	ILS/USD	EGP/USD	CAD/USD	EGP/USD	CAD/USD	CAD/USD	EGP/USD	CAD/USD	CAD/USD
2	NGN/USD	ILS/USD	EGP/USD**	ILS/USD	EGP/USD	EGP/USD*	USD/EUR	EGP/USD*	EGP/USD
3	NOK/EUR	NGN/USD*	USD/EUR	NOK/EUR	NOK/EUR	USD/EUR	NOK/EUR	USD/EUR	USD/EUR**
4	USD/GBP	NOK/EUR	ILS/USD	USD/GBP	USD/GBP	ILS/USD	USD/GBP	NOK/EUR	ILS/USD
5		USD/GBP	JPY/USD			NGN/USD**		USD/GBP	NGN/USD
6			NGN/USD***			NOK/EUR			NOK/EUR
7			NOK/EUR			ZAR/USD			ZAR/USD
8			ZAR/USD			CHF/EUR			USD/GBP***
9			CHF/EUR			USD/GBP			Aggr**
10			USD/GBP			Aggr**			
11			Aggr**						

*: *significant at* 10%, **: *significant at* 5%, ***: *significant at* 1%.

Table 3
The exchange rates for which the random walk model gives better forecast than the asset pricing model for various horizons

	3 months horizon			1 year horizon			2 years horizon		
	MAE	RMSE	WSP	MAE	RMSE	WSP	MAE	RMSE	WSP
1	CAD/USD	CAD/USD	SEK/EUR	CAD/USD	USD/EUR	JPY/USD	CAD/USD	ILS/USD	JPY/USD
2	EGP/USD	USD/EUR		USD/EUR	ILS/USD	SEK/EUR	ILS/USD	JPY/USD	SEK/EUR
3	USD/EUR	JPY/USD*		JPY/USD*	JPY/USD		JPY/USD	NGN/USD**	CHF/EUR
4	JPY/USD*	ZAR/USD		NGN/USD	NGN/USD		NGN/USD**	ZAR/USD	
5	ZAR/USD*	SEK/EUR**		ZAR/USD	ZAR/USD		ZAR/USD	SEK/EUR***	
6	SEK/EUR***	CHF/EUR		SEK/EUR***	SEK/EUR**		SEK/EUR***	CHF/EUR	
7	CHF/EUR**	Aggr		CHF/EUR	CHF/EUR		CHF/EUR	Aggr	
8	Aggr**			Aggr	Aggr		Aggr		

*: *significant at* 10%, **: *significant at* 5%, ***: *significant at* 1%.

Table 4
Forecasting performance on the 3 months horizon measured by the MAE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Mean absolute error			
CAD/USD	119	0.0353	0.0362	**0.0352**	0.0384
(stat)			(-0.6793)	(0.1125)	$(-2.0404)^{**}$
EGP/USD	119	**0.0256**	0.0279	0.0277	0.0298
(stat)			(-1.1323)	(-1.151)	$(-1.6521)^{**}$
USD/EUR	119	0.0497	0.0531	**0.0494**	0.0554
(stat)			(-0.9778)	(0.154)	$(-1.7513)^{**}$
ILS/USD	119	0.035	**0.0343**	0.0352	0.0363
(stat)			(0.4476)	(-0.1401)	(-0.6032)
JPY/USD	119	0.0458	0.0508	**0.0456**	0.0527
(stat)			$(-1.4579)^{*}$	(0.384)	$(-2.1947)^{**}$
NGN/USD	119	0.0417	**0.0369**	0.0453	0.0397
(stat)			(1.1905)	(-0.5678)	(0.4121)
NOK/EUR	119	0.0268	**0.0261**	0.0266	0.0281
(stat)			(0.4984)	(0.6384)	(-0.7934)
ZAR/USD	119	**0.0758**	0.0826	0.0758	0.0881
(stat)			$(-1.637)^{*}$	(-0.0115)	$(-2.5591)^{***}$
SEK/EUR	119	**0.0214**	0.025	0.0224	0.0274
(stat)			$(-2.3789)^{***}$	$(-1.5945)^{*}$	$(-3.6258)^{***}$
CHF/EUR	119	0.0149	0.0168	**0.0149**	0.0177
(stat)			$(-1.8682)^{**}$	(0.1131)	$(-2.4105)^{***}$
USD/GBP	119	0.0394	0.0392	**0.0389**	0.039
(stat)			(0.132)	$(1.411)^{*}$	(0.2405)
Aggregated	1309	**0.0374**	0.039	0.0379	0.0412
(stat)			$(-2.0162)^{**}$	(-0.7537)	$(-4.2589)^{***}$

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 5
Forecasting performance on the 3 months horizon measured by the RMSE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Root mean square error			
CAD/USD	119	0.0484	0.0488	**0.0477**	0.0517
(stat)			(-0.2141)	(0.9756)	(−1.7635)**
EGP/USD	119	0.0492	**0.0459**	0.0465	0.0467
(stat)			(1.0519)	(1.2215)	(0.5219)
USD/EUR	119	**0.0617**	0.0655	0.0619	0.0679
(stat)			(-0.9075)	(-0.1139)	(−1.6391)*
ILS/USD	119	0.0464	**0.0462**	0.0471	0.0489
(stat)			(0.144)	(-0.446)	(-1.0111)
JPY/USD	119	0.0589	0.0631	**0.0584**	0.0656
(stat)			(−1.3731)*	(0.6492)	(−2.0683)**
NGN/USD	119	0.0605	**0.0551**	0.0588	0.058
(stat)			(1.2969)*	(0.2533)	(0.577)
NOK/EUR	119	0.0363	0.0361	**0.0361**	0.039
(stat)			(0.1026)	(0.7318)	(-1.1041)
ZAR/USD	119	0.1002	0.1054	**0.0989**	0.1123
(stat)			(−1.2499)	(0.8039)	(−2.402)***
SEK/EUR	119	**0.0318**	0.0361	0.0332	0.0376
(stat)			(−1.8859)**	(−1.7182)**	(−2.6393)***
CHF/EUR	119	0.0215	0.0218	**0.0213**	0.0225
(stat)			(-0.2374)	(0.5448)	(-0.7812)
USD/GBP	119	0.0557	**0.0548**	0.0552	0.0555
(stat)			(0.6453)	(1.8397)**	(0.1551)
Aggregated	1309	0.0554	0.0565	**0.0548**	0.0593
(stat)			(-1.0123)	(0.7795)	(−3.1089)***

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 6
Forecasting performance on the 3 months horizon measured by the WSP

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Weighted measure of sign prediction			
CAD/USD	119	0	-0.0125	**-0.027**	0.0046
(stat)			(0.5975)	(0.8103)	(-0.2352)
EGP/USD	119	0	-0.0587	**-0.0589**	-0.0397
(stat)			(1.7315)**	(1.7392)**	(1.3612)*
USD/EUR	119	0	-0.011	**-0.018**	0.0003
(stat)			(0.2813)	(0.4429)	(-0.0113)
ILS/USD	119	0	**-0.0089**	0.0048	-0.0048
(stat)			(0.2922)	(-0.1456)	(0.2232)
JPY/USD	119	0	**-0.0187**	-0.0102	0.0262
(stat)			(0.6249)	(0.2728)	(-0.9397)
NGN/USD	119	0	-0.078	**-0.1092**	-0.0729
(stat)			(2.7955)***	(3.045)***	(2.6288)***
NOK/EUR	119	0	-0.0164	**-0.0186**	-0.0007
(stat)			(0.7327)	(0.795)	(0.0322)
ZAR/USD	119	0	-0.0195	**-0.0485**	-0.0011
(stat)			(0.3339)	(0.7049)	(0.0188)
SEK/EUR	119	**0**	0.0201	0.0315	0.0194
(stat)			(-0.9614)	(-1.4922)	(-0.9539)
CHF/EUR	119	0	-0.0039	**-0.0128**	-0.0008
(stat)			(0.2914)	(0.9234)	(0.0816)
USD/GBP	119	0	-0.0342	**-0.075**	-0.0247
(stat)			(1.0313)	(2.0607)**	(1.0862)
Aggregated	1309	0	-0.022	**-0.0311**	-0.0086
(stat)			(2.2412)**	(2.7431)***	(0.9737)

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: significant at 10%, $**$: significant at 5%, $***$: significant at 1%.

Table 7
Forecasting performance on the 1 year horizon measured by the MAE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Mean absolute error			
CAD/USD	110	0.0734	0.0773	**0.0687**	0.076
(stat)			(-0.5111)	(0.6697)	(-0.3116)
EGP/USD	110	0.0847	0.0765	0.0843	**0.0738**
(stat)			(0.5338)	(0.0205)	(0.723)
USD/EUR	110	0.1021	0.1072	**0.093**	0.1066
(stat)			(-0.2698)	(0.568)	(-0.2278)
ILS/USD	110	0.0675	0.0672	0.0758	**0.0654**
(stat)			(0.0329)	(-1.0126)	(0.2416)
JPY/USD	110	0.0774	0.0929	**0.0761**	0.0954
(stat)			$(-1.2918)^{*}$	(0.3804)	$(-1.6464)^{**}$
NGN/USD	110	**0.0703**	0.0838	0.1356	0.0839
(stat)			(-0.5355)	$(-1.8843)^{**}$	(-0.5894)
NOK/EUR	110	0.0471	**0.0418**	0.047	0.0424
(stat)			(1.0799)	(0.0842)	(0.8577)
ZAR/USD	110	0.1731	0.1825	**0.1576**	0.1857
(stat)			(-0.3993)	$(1.3462)^{*}$	(-0.5731)
SEK/EUR	110	**0.0346**	0.0489	0.0425	0.0498
(stat)			$(-3.0911)^{***}$	$(-2.3369)^{***}$	$(-3.0734)^{***}$
CHF/EUR	110	0.0326	0.0361	**0.0325**	0.0367
(stat)			(-0.7084)	(0.0565)	(-0.842)
USD/GBP	110	0.0836	0.0827	**0.0799**	0.0818
(stat)			(0.203)	$(1.5978)^{*}$	(0.3388)
Aggregated	1210	**0.077**	0.0815	0.0812	0.0816
(stat)			(-1.0496)	(-0.8439)	(-1.0878)

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 8
Forecasting performance on the 1 year horizon measured by the RMSE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Root mean square error			
CAD/USD	110	0.0941	0.0894	**0.0883**	0.0883
(stat)			(0.6033)	(1.0433)	(0.6964)
EGP/USD	110	0.118	0.094	0.1036	**0.0917**
(stat)			(1.2479)	(0.9315)	(1.3014)*
USD/EUR	110	0.1177	0.1277	**0.1123**	0.1278
(stat)			(-0.4611)	(0.3003)	(-0.4542)
ILS/USD	110	0.0861	0.0871	0.0942	**0.0853**
(stat)			(-0.0977)	(-1.0022)	(0.0957)
JPY/USD	110	0.0921	0.1055	**0.0897**	0.108
(stat)			(−1.2075)	(0.572)	(−1.6082)*
NGN/USD	110	**0.0912**	0.0978	0.1477	0.0982
(stat)			(-0.3141)	(−1.8964)**	(-0.3673)
NOK/EUR	110	0.0621	**0.0572**	0.0611	0.0584
(stat)			(0.7477)	(0.4439)	(0.5446)
ZAR/USD	110	0.209	0.2225	**0.1996**	0.2256
(stat)			(-0.6123)	(0.9058)	(-0.8156)
SEK/EUR	110	**0.0527**	0.0629	0.0591	0.0636
(stat)			(−1.6806)**	(−1.7864)**	(−1.7614)**
CHF/EUR	110	0.0396	0.0419	**0.039**	0.0425
(stat)			(-0.367)	(0.4135)	(-0.5093)
USD/GBP	110	0.1075	0.1006	0.1039	**0.0998**
(stat)			(0.9192)	(1.8222)**	(0.98)
Aggregated	1210	**0.1063**	0.1087	0.1084	0.1092
(stat)			(-0.4214)	(-0.4058)	(-0.5223)

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
∗: significant at 10%, ∗∗: significant at 5%, ∗∗∗: significant at 1%.

Table 9
Forecasting performance on the 1 year horizon measured by the WSP

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Weighted measure of sign prediction			
CAD/USD	110	0	-0.0152	**-0.0323**	-0.0218
(stat)			(0.7493)	(1.3207)*	(1.0104)
EGP/USD	110	0	-0.0536	-0.0536	**-0.0555**
(stat)			(1.2885)*	(1.2885)*	(1.3626)*
USD/EUR	110	0	-0.0242	**-0.0329**	-0.0243
(stat)			(0.7335)	(0.86)	(0.7721)
ILS/USD	110	0	-0.0001	0.0138	**-0.0048**
(stat)			(0.0048)	(-0.522)	(0.1864)
JPY/USD	110	0	0.004	**-0.0253**	0.0139
(stat)			(-0.1863)	(1.0794)	(-0.802)
NGN/USD	110	0	**-0.0453**	**-0.0453**	**-0.0453**
(stat)			(1.8098)**	(1.8098)**	(1.8098)**
NOK/EUR	110	0	**-0.0114**	-0.007	-0.009
(stat)			(0.7104)	(0.4139)	(0.5616)
ZAR/USD	110	0	-0.0074	**-0.0382**	0.0185
(stat)			(0.1548)	(0.5131)	(-0.3699)
SEK/EUR	110	**0**	0.0033	0.0168	0.0045
(stat)			(-0.2965)	(−1.3628)*	(-0.4309)
CHF/EUR	110	0	-0.0024	**-0.0094**	-0.0002
(stat)			(0.2002)	(0.7005)	(0.0173)
USD/GBP	110	0	-0.026	**-0.0584**	-0.0401
(stat)			(1.2328)	(2.811)***	(2.1777)**
Aggregated	1210	0	-0.0162	**-0.0247**	-0.0149
(stat)			(1.8773)**	(2.3334)***	(1.6979)**

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 10
Forecasting performance on the 2 year horizon measured by the MAE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Mean absolute error			
CAD/USD	98	0.1151	0.116	**0.0991**	0.117
(stat)			(-0.0766)	(0.9158)	(-0.1695)
EGP/USD	98	0.1646	**0.1371**	0.163	0.139
(stat)			(0.7647)	(0.0353)	(0.7076)
USD/EUR	98	0.1453	0.1339	**0.1155**	0.1347
(stat)			(0.7802)	(1.2051)	(0.7304)
ILS/USD	98	**0.0857**	0.092	0.1074	0.0932
(stat)			(-0.8589)	$(-1.9016)^{**}$	(-0.9804)
JPY/USD	98	0.1034	0.1071	**0.0962**	0.1057
(stat)			(-0.4362)	$(1.9772)^{**}$	(-0.3072)
NGN/USD	98	**0.1003**	0.2185	0.2809	0.2182
(stat)			$(-1.898)^{**}$	$(-2.1959)^{**}$	$(-1.846)^{**}$
NOK/EUR	98	0.0516	0.0468	0.0506	**0.046**
(stat)			(0.6706)	(0.4258)	(0.758)
ZAR/USD	98	0.2532	0.2567	**0.2246**	0.256
(stat)			(-0.1086)	(1.0091)	(-0.0791)
SEK/EUR	98	**0.0389**	0.0648	0.064	0.0633
(stat)			$(-5.9409)^{***}$	$(-4.2844)^{***}$	$(-6.08)^{***}$
CHF/EUR	98	0.0435	0.0459	**0.0407**	0.0462
(stat)			(-0.3463)	(1.2747)	(-0.4196)
USD/GBP	98	0.1158	0.1103	**0.104**	0.1116
(stat)			(1.225)	$(2.2438)^{**}$	(0.6408)
Aggregated	1078	**0.1107**	0.1208	0.1224	0.121
(stat)			(-0.9029)	(-0.7577)	(-0.9134)

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 11
Forecasting performance on the 2 year horizon measured by the RMSE

Exchange rate	Num. obs.	Model: RW	AP	linear	survey data
		Root mean square error			
CAD/USD	98	0.1314	0.1251	**0.1093**	0.1264
(stat)			(0.552)	(1.3737)*	(0.4626)
EGP/USD	98	0.2103	**0.1607**	0.178	0.1622
(stat)			(1.2973)*	(0.7988)	(1.2704)
USD/EUR	98	0.1745	0.1643	**0.1407**	0.1639
(stat)			(0.471)	(1.0361)	(0.4897)
ILS/USD	98	**0.1138**	0.1184	0.1336	0.1197
(stat)			(-0.3849)	(-1.1126)	(-0.4694)
JPY/USD	98	0.1207	0.1238	**0.1122**	0.1209
(stat)			(-0.4238)	(2.0494)**	(-0.0249)
NGN/USD	98	**0.1279**	0.2374	0.2949	0.2377
(stat)			(−2.0964)**	(−2.223)**	(−2.0429)**
NOK/EUR	98	0.0649	0.0618	0.0631	**0.061**
(stat)			(0.3328)	(1.3252)*	(0.4049)
ZAR/USD	98	0.3054	0.3145	**0.2821**	0.3135
(stat)			(-0.2828)	(0.8193)	(-0.2339)
SEK/EUR	98	**0.0586**	0.0759	0.0793	0.075
(stat)			(−4.5497)***	(−3.5419)***	(−3.8412)***
CHF/EUR	98	0.0498	0.0541	**0.0477**	0.0539
(stat)			(-0.8077)	(0.706)	(-0.78)
USD/GBP	98	0.1364	0.1289	**0.125**	0.1296
(stat)			(2.4358)***	(1.8424)**	(1.5499)*
Aggregated	1078	**0.1531**	0.1602	0.162	0.1601
(stat)			(-0.5244)	(-0.4614)	(-0.5102)

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: *significant at* 10%, $**$: *significant at* 5%, $***$: *significant at* 1%.

Table 12
Forecasting performance on the 2 year horizon measured by the WSP

Exchange rate	Num. obs.	RW	AP	linear	survey data
		Model:			
		Weighted measure of sign prediction			
CAD/USD	98	0	-0.0112	**-0.0407**	-0.0078
(stat)			(0.6701)	(1.8691)**	(0.4381)
EGP/USD	98	0	**-0.0572**	**-0.0572**	**-0.0572**
(stat)			(1.1346)	(1.1346)	(1.1346)
USD/EUR	98	0	-0.0387	**-0.0451**	-0.0377
(stat)			(1.7622)**	(1.8964)**	(1.7168)**
ILS/USD	98	0	**-0.0036**	0.0093	0.0021
(stat)			(0.2038)	(-0.39)	(-0.0974)
JPY/USD	98	0	0.0013	**-0.0281**	-0.0055
(stat)			(-0.1293)	(1.9175)**	(0.6308)
NGN/USD	98	0	**-0.0338**	**-0.0338**	**-0.0338**
(stat)			(1.1652)	(1.1652)	(1.1652)
NOK/EUR	98	0	-0.0048	**-0.0061**	-0.0059
(stat)			(0.5925)	(1.8881)**	(0.6658)
ZAR/USD	98	0	-0.0347	-0.0217	**-0.0371**
(stat)			(0.9881)	(0.3331)	(0.9076)
SEK/EUR	98	**0**	0.0069	0.0147	0.0082
(stat)			(-1.2778)	(−2.6315)***	(−1.5518)*
CHF/EUR	98	0	0.0004	**-0.0033**	0.0029
(stat)			(-0.0791)	(0.3346)	(-0.4648)
USD/GBP	98	0	-0.024	**-0.0358**	-0.0213
(stat)			(2.9212)***	(2.7098)***	(2.8093)***
Aggregated	1078	0	-0.0181	**-0.0225**	-0.0176
(stat)			(2.2284)**	(2.2111)**	(2.084)**

The test statistics in parentheses compares the forecasting ability of the random walk model with those of the alternative models.
$*$: significant at 10%, $**$: significant at 5%, $***$: significant at 1%.

Table 13
The relative importance of the expected exchange rate at determining the exchange rate calculated as $\frac{\left|e^{-\frac{T-t}{c}}\right|}{\left|e^{-\frac{T-t}{c}}\right|+\left|1-e^{-\frac{T-t}{c}}\right|}$, where $T-t$ is the forecast horizon

Exchange rate	Forecast horizon: 3-month	1-year	2-year
CAD/USD	0.8013	0.5952	0.527
EGP/USD	0.9648	0.8788	0.7956
USD/EUR	0.8239	0.6182	0.5394
ILS/USD	0.9143	0.7546	0.6473
CAD/USD	0.7999	0.5939	0.5263
NGN/USD	0.9334	0.759	0.5761
NOK/EUR	0.7591	0.5609	0.5121
ZAR/USD	0.8865	0.7032	0.6003
SEK/EUR	0.826	0.6204	0.5407
CHF/EUR	0.8484	0.6473	0.5578
USD/GBP	0.8823	0.6964	0.5946

Parameter c is estimated from the full sample of survey data consisting of the 3-month, 1-year and 2-year forecasts.

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