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Fiscal Calculus in a New Keynesian Model with Labor Market Frictions

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Fiscal Calculus in a New Keynesian Model with Labor Market Frictions*

(Költségvetési számítások egy új-keynesianus, munkapiaci súrlódásokat tartalmazó modellben)

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Abstract

During the Great Recession following the recent financial crisis large fiscal stimuli were implemented to counteract labor market sclerosis. We explore the effectiveness of various fiscal packages in a matching model featuring inefficient unemployment and a rich fiscal sector employing distortionary taxation and government debt. Results show that only stimuli directed toward the labor market, such as hiring subsidies, deliver large multipliers. Those policies can, indeed, abate the congestion externality, pervasive in the labor market. Various robustness checks confirm the results. The results obtained in the calibrated model are also confirmed through Bayesian estimation.

Keywords: fiscal calculus, taxation, matching frictions, bayesian estimation.

Összefoglalás

A közelmúlt pénzügyi válságát követő „Nagy Recesszióban” a munkapiacokat erőteljes fiskális expanzióval próbálták kizökésteneni a pangás állapotából. Ebben a tanulmányban különböző fiskális intézkedési csomagok hatékonyságát vizsgáljuk egy olyan modellben, amelyet torzító hatású adók és kormányzati adósság, illetve nem hatékony munkanélküliség jellemez. Az eredményeink azt mutatják, hogy csak a kifejezetten a munkapiacot célzó intézkedések (pl. foglalkoztatási támogatások) járnak nagy multiplikátor-hatással. Az ilyen típusú intézkedések valóban alkalmassak arra, hogy enyhítsék a munkapiacokra általában jellemző telítettségi externálitát. A különböző érzékenységi vizsgálatok is alátámasztják az eredményeinket. A kalibrálás mellett a modell bayes-i becslése is a fenti következtetésekre vezet.
The endorsement of expansionary fiscal packages has often been based on the idea that large multipliers can contrast rising unemployment. Following the 2007-2008 crisis, various national governments around the globe have passed expansionary fiscal packages arguing that, with nominal interest rates at the zero lower bound, only a strong fiscal stimuli could help in fighting the consequences of a recession associated with rising unemployment. We examine how effective are those fiscal packages using a New Keynesian model in which inefficient unemployment arises because of matching frictions and departure from Hosios’ efficiency. Additionally, we elaborate our results thoroughly by including endogenous participation decisions, something which brings along a labor supply friction. In sum, our final model economy will include both a labor demand and a labor supply friction. In this environment we compare alternative fiscal packages, both in terms of target for the fiscal stimulus and in terms of source of financing. We consider two forms of government spending: a traditional increase in aggregate demand and an increase in firms’ hiring subsidy. Furthermore, various forms of government financing are considered, namely lump sum taxation versus distortionary taxation on labor. The analysis of the fiscal multiplier is initially conducted by using the calibrated model: this will help in guiding through the intuition on how the model functions. At last, we perform a Bayesian estimation of our DSGE model and comment on the observed size of the fiscal multipliers and on the statistical performance of the structural model.

The results from our calibrated model are as follows. Government expenditure in the form of aggregate demand stimuli produces low to nearly zero multipliers. Thus, in comparison to the standard New Keynesian model the expansionary effects of aggregate demand stimuli are much lower in a model with matching frictions. When distortionary taxation is used, multipliers become even negative. To understand the reason for this results we compare our model with an RBC model with non-walrasian labor markets. In such a model an increase in aggregate demand can be accommodated if firms post more vacancies. For this to become an equilibrium outcome, the continuation value of a filled vacancy needs to be higher than its steady state value, which in turn requires an increase in the stochastic discount factor and in current consumption. However, due to the crowding-out effect, current consumption falls, therefore implying a fall in current vacancy posting. The fall in vacancy posting brings about a fall in employment and output. Our results show, on the other side, that when the fiscal stimulus takes the form of subsidy to cost of posting vacancies, fiscal multipliers turn positive and become significantly large. A reduction in the cost of posting vacancies boosts job creation, which in turn induces an increase in employment and output. This effect is particularly powerful when the model features inefficient unemployment fluctuations, which occur to the extent that the Hosios condition does not hold. In this case indeed the fall in the cost of posting vacancies also reduces the distortions present in the economy, therefore moving the long run level of output toward the potential one. We re-examine our results by adding to the model an endogenous participation decision, which induces frictions on the labor supply and involuntary unemployment on top and above inefficient fluctuations in unemployment. Even in this case multipliers are smaller than one and turn negative with distortionary taxation, showing that the fiscal stimulus is ineffective in boosting workers’ participation decisions.

To further check robustness of our results we consider fiscal stimuli in combination with an interest rate peg. This case is of interest since over the last year countries like the U.S. were close to the zero lower bound on interest rate and have thus adopted a policy mix in which the monetary authority kept the interest rate fixed at low levels, while the fiscal authority had passed large fiscal packages. In this case a temporary interest rate peg (the monetary authority keeps the nominal interest rate constant for one year), coupled with fiscal stimulus, increases the multiplier both, in the standard New Keynesian model and in the model with search and matching frictions, although less in the model with matching frictions. This result echoes the ones reported in Christiano, Eichenbaum and Rebelo (2009). Finally, the main results go through even when the economy starts from a recession scenario, under different degrees of price stickiness and under the assumption of real wage rigidity (that we considered in order to obtain larger fluctuations in unemployment consistently with the implications of the Shimer puzzle).
In order to measure the significance of our results in the data, we perform Bayesian estimation of the full DSGE model by including all fiscal shocks considered in the calibrated model, alongside with shocks to productivity, monetary policy, marginal cost and the matching function. The estimation is run by using U.S. data for GDP, inflation, the Federal funds rate, employment, wages and vacancies. The results are encouraging since i) the estimated parameters are significant and the posterior likelihood distribution shows that the data are informative and ii) using the estimated parameters we confirm results from our calibrated model.

Our paper is close to a number of papers which consider fiscal stimuli with a particular focus on policies directed toward the welfare state: those contributions include Davig and Leeper (2011), Davig, Eric and Walker (2010) and Leeper, Walker and Yang (2010). Our paper is generally related to the recent literature on fiscal multipliers which includes, among others, Romer and Bernstein (2009), Cogan, Cwik, Taylor and Wieland (2010), Uhlig (2009) and Christiano et al. (2009).

Recently other authors have analyzed fiscal multipliers in models with labor market frictions. Faia, Lechthaler and Merkl (2010) use a model with workers' heterogeneity and labor turnover costs, but in absence of search and matching frictions. They analyze fiscal multipliers and show that they are small, unless the fiscal expansion involves a subsidy to turnover costs or short-time work policies. Monacelli, Perotti and Trigari (2011) also computed independently fiscal multipliers in a model with search and matching frictions and find that they are small. The results in our paper are complementary to their for three reasons. First, while they consider model modifications, such as endogenous job destruction and capital, we focus more on the participation margin and introduce labor supply frictions. Second, while they focus on traditional fiscal policies, our analysis is more tailored to labor market oriented fiscal policies. Third, they compare results from the calibrated model with estimates from VAR, while we bring the model to the data by direct Bayesian estimation of the DSGE model. Finally, Pappa and Brückner (2011), use an insider-outsider model with matching frictions and analyze the effects of government expenditure shocks with a special focus on unemployment.

The structure of the paper is as follows. Section 2 presents the model economy and the fiscal policy packages considered. Section 3 provides the intuition for the main mechanism at work for the real business cycle version of the model. Section 4 shows simulation results on the fiscal multipliers for the general version of the model, providing also robustness checks. In Section 5 we re-examine the fiscal multipliers by including endogenous participation in the model. Section 6 shows the results of the Bayesian estimation. Finally, Section 7 concludes. Tables and Figures follow.
2 A New Keynesian Model with Matching Frictions

There is a continuum of agents whose total measure is normalized to one. The economy is populated by households who consume different varieties of goods, save and work. Households save in both non-state contingent securities and in an insurance fund that allows them to smooth income fluctuations associated with periods of unemployment. Each agent can indeed be either employed or unemployed. In the first case he receives a wage that is determined according to a Nash bargaining, in the second case he receives an unemployment benefit. The labor market is characterized by matching frictions and exogenous job separation. The production sector acts as a monopolistic competitive sector which produces a differentiated good using labor as input and faces adjustment costs a la Rotemberg (1982).

2.1 HOUSEHOLDS

Let \( c_t \equiv \left[ \int_{c_{t-1}}^{c_{t+1}} c_t(l) \frac{c_t}{\tau} dl \right]^{\frac{1}{\gamma}} \) be a Dixit-Stiglitz aggregator of different varieties of goods. The optimal allocation of expenditure on each variety is given by \( c_t(l) = \left( \frac{p_t(l)}{p_t} \right)^{-1+\epsilon} c_t \), where \( p_t \equiv \left[ \int_{c_{t-1}}^{c_{t+1}} p_t(l)^{1+\epsilon} dl \right]^{\frac{1}{1+\epsilon}} \) is the price index. There is continuum of agents who maximize the expected lifetime utility\(^1\).

Households supply labor hours, \( N \), inelastically (normalized to 1). Unemployed households members receive an unemployment benefit \( b \) that can be interpreted as including also the output of a home production activity. Total real labor income is given by \( \omega c_t \). The contract signed between the worker and the firm specifies the wage and is obtained through a Nash bargaining process. In order to finance consumption at time \( t \) each agent also invests in non-state contingent nominal government bonds \( B_t \), which pay a gross nominal interest rate \( (1 + r_t^n) \) one period later. As in Andolfatto (1996) and Merz (1995) it is assumed that workers can insure themselves against earning uncertainty and unemployment. For this reason the wage earnings have to be interpreted as net of insurance costs. Finally agents receive profits from the monopolistic sector which they own, \( \Theta_t \). Agents are subject to the following tax system: \( \tau_t^c \) represents tax on wages, \( \tau_t^n \) represents tax on consumption and \( \tau_t \) represents lump sum taxation. The sequence of real budget constraints reads as follows:

\[
(1 + \tau_t^c) c_t - \frac{B_t}{p_t} \leq (1 - \tau_t^r) \omega c_t + b(1 - e_t) + \frac{\Theta_t}{p_t} - \tau_t + (1 + r_t^n) \frac{B_{t-1}}{p_{t-1}}
\]

Households choose the set of processes \( \{c_t, B_t\}_{t=0}^{\infty} \), taking as given the set of processes \( \{p_t, \omega, r_t^n, \tau_t^c, \tau_t^n, \tau_t, \Theta_t\}_{t=0}^{\infty} \) and the initial wealth \( B_0 \), so as to maximize (1) subject to (2). Let’s define \( \lambda_t \) as the Lagrange multiplier on constraint (2). The following optimality conditions must hold:

\[
\lambda_t = \frac{c_t^{1-\sigma}}{1 + \tau_t^c}
\]

\[
\frac{c_t^{1-\sigma}}{1 + \tau_t^c} = \beta(1 + r_t^n) E_t \left[ \frac{c_{t+1}^{1-\sigma} p_t}{1 + \tau_{t+1}^c \rho_{t+1}} \right]
\]

Equation (3) is the marginal utility of consumption and equation (4) is the Euler condition with respect to bonds. Optimality requires that No-Ponzi condition on wealth is also satisfied.

\(^1\) Let \( s^t = \{s_0, \ldots, s_t\} \) denote the history of events up to date \( t \), where \( s_t \) denotes the event realization at date \( t \). The date 0 probability of observing history \( s^t \) is given by \( \rho_t \). The initial state \( \phi \) is given so that \( \rho_0 = 1 \). Henceforth, and for the sake of simplifying the notation, let’s define the operator \( E_t \{ \cdot \} \equiv \sum_{s_{t+1}} \rho(s_{t+1} | s^t) \) as the mathematical expectations over all possible states of nature conditional on history \( s^t \).
2.2 THE PRODUCTION SECTOR

Firms in the production sector sell their output in a monopolistic competitive market and meet workers on a matching market. The labor relations are determined according to a standard Mortensen and Pissarides (1999) framework. Workers must be hired from the unemployment pool and searching for a worker involves a fixed cost. Workers wages are determined through a Nash decentralized bargaining process which takes place on an individual basis.

2.2.1 Search and Matching in the Labor Market

The search for a worker involves a fixed cost $\kappa$ and the probability of finding a worker depends on a constant return to scale matching technology which converts unemployed workers $u$ and vacancies $v$ into matches, $m(u_t, v_t)$:

$$m(u_t, v_t) = m_0 u_t^\xi v_t^{1-\xi}$$

where $v_t = \int_0^t v_i(i)\,di$ and $m_t$ is a shock to the matching efficiency which follows a first order autoregressive process $\log(m_0/m_t) = \rho_\mu \log(m_0/m_{t-1}) + \xi_t$. Defining labor market tightness as $\theta_t \equiv \frac{\nu_t}{m_t}$, the firm meets unemployed workers at rate $q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = m_0 \theta_t^{\xi_t}$, while the unemployed workers meet vacancies at rate $f_t \equiv \theta_t q(\theta_t) = m_0 \theta_t^{\xi_t-\xi}$, which we refer to as the job finding rate. If the search process is successful, the firm in the monopolistic good sector operates the technology $z_t e_i(l)$, where $z_t$ is the aggregate productivity shock which follows a first order autoregressive process, $\log(z_t) = \rho_z \log(z_{t-1} + \varepsilon_{z,t})$, and $e_i(l)$ is the number of workers hired by firm $i$. Matches are destroyed at an exogenous rate $2\rho$. Labor force is normalized to unity. The number of employed people at time $t$ in each firm $i$ is given by the number of employed people at time $t - 1$ plus the flow of new matches concluded in period $t$ who did not discontinue the match:

$$e_i(t) = (1 - \rho)e_{i,t-1}(l) + v_i(t)q(\theta_t)$$

Hiring in this model is instantaneous. At the beginning of period $t$ firms observe the realization of the stochastic variables and post vacancies accordingly. Those vacancies are matched with the pool of searching workers which is given by the workers not employed at the end of period $t - 1$, $u_t = 1 - (1 - \rho)e_{t-1}$.

2.2.2 Monopolistic Firms

There is a continuum of firms which hire a continuum of workers. Firms in the monopolistic sector use labor to produce different varieties of consumption good and face a quadratic cost of adjusting prices and costs of posting vacancies which are linear in the number of vacancies. Due to the constant return to scale of vacancy posting technology, firms can take wages as given when choosing prices and employment. Wages are determined through the bargaining problem analyzed in the next section. Here we develop the dynamic optimization decision of firms choosing prices, $p_i(t)$, number of employees, $e_i(t)$, and number of vacancies, $v_i(t)$, to maximize the discounted value of future profits and taking as given the wage schedule. The representative firm maximization problem (in real terms):

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \lambda_0 \left\{ \frac{p_i(t)}{p_t} z_t e_i(l) - w_i(t) e_i(t) - (1 - \tau^*_t) v_i(t) - \frac{\psi}{2} \left( \frac{p_i(t)}{p_{t-1}} - 1 \right)^2 d_t \right\}$$

subject to

$$z_t e_i(l) = \left( \frac{p_i(t)}{p_t} \right)^{-\xi} d_t$$

$$e_i(t) = (1 - \rho)e_{i,t-1}(l) + v_i(t)q(\theta_t)$$

where $d_t$ is the aggregate demand inclusive of resources wasted for search activity and price adjustment, $\frac{\psi}{2} \left( \frac{p_i(t)}{p_{t-1}} - 1 \right)^2 d_t$ represents the cost of adjusting prices, $\psi$ can be thought as the sluggishness in the price adjustment process, $\kappa$ as the cost of posting vacancies, $\tau^*_t$ is a subsidy to the cost of posting vacancies, and $w_i(t)$ denotes the fact that the bargained wage

---

2 The alternative assumption of endogenous job destruction would induce, consistently with empirical observations, additional persistence as shown in den Haan, Ramey and Watson (2000). However due to the normative focus of this paper we choose the more simple assumption of exogenous job separation.
might depend on time varying factors. Let’s define $mc_t$, the Lagrange multiplier on constraint (8), as the marginal cost of firms and $\mu_t$, the Lagrange multiplier on constraint (9), as the marginal value of one worker. Since all firms will choose in equilibrium the same price and allocation we can now assume symmetry and drop the index $i$. First order conditions for the above problem read as follows:

- $e_t$:
  \[ \mu_t = mc_t z_t - w_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \mu_{t+1} \right) \]  
  \[ (10) \]

- $v_t$:
  \[ \frac{\kappa(1 - \tau^k_t)}{q(\theta_t)} = \mu_t \]  
  \[ (11) \]

- $p_t$:
  \[ \psi(\pi_t - 1) \pi_t - \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \psi(\pi_{t+1} - 1) \pi_{t+1} \frac{z_{t+1} e_{t+1}}{z_t e_t} \right) = 1 - \varepsilon(1 - mc_t) \]  
  \[ (12) \]

By merging (10) and (11) we obtain the job creation condition:

\[ \frac{\kappa(1 - \tau^k_t)}{q(\theta_t)} = mc_t z_t - w_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \frac{\kappa(1 - \tau^k_{t+1})}{q(\theta_{t+1})} \right) \]  
\[ (13) \]

Rearranging equation (13) we obtain the following expression for the marginal cost:

\[ mc_t = \frac{w_t}{z_t} + \frac{\kappa(1 - \tau^k_t)}{q(\theta_t)} - \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \frac{\kappa(1 - \tau^k_{t+1})}{q(\theta_{t+1})} \right) \]  
\[ (14) \]

As already noticed in Krause and Lubik (2007) in a matching model the marginal cost of firms is given by the marginal productivity of each single employee, $\frac{w_t}{z_t}$, as it is in a standard Walrasian model but it also contains an extra component, which depends on the future value of a match. Posting vacancy is costly hence a successful match today is valuable also since it reduces future search costs. In this context wages maintain their allocative role only for future matches.3

### 2.2.3 Bellman Equations, Wage Setting and Nash Bargaining

The wage schedule is obtained through the solution to an individual Nash bargaining process. To solve for it we need first to derive the marginal values of a match for both, firms and workers. Those values will indeed enter the sharing rule of the bargaining process. Let’s denote by $V^f_t$ the marginal discounted value of a vacancy for a firm. From equation (10) and noticing that $V^f_t = \mu_t$ we obtain:

\[ V^f_t = mc_t z_t - w_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) V^f_{t+1} \right) \]  
\[ (15) \]

The marginal value of a vacancy depends on real revenues minus the real wage plus the discounted continuation value. With probability $(1 - \rho)$ the job remains filled and earns the expected value and with probability, $\rho$, the job is destroyed and has zero value. In equilibrium the marginal discounted value of a vacancy should equalize the expected cost of posting vacancies. For each worker, the values of being employed and unemployed are given by $V^e_t$ and $V^u_t$:

\[ V^e_t = (1 - \tau^e_t) w_t + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( (1 - \rho) V^e_{t+1} + \rho V^u_{t+1} \right) \right] \]  
\[ (16) \]

\[ V^u_t = b + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( f_{t+1}(1 - \rho) V^e_{t+1} + (1 - f_{t+1}(1 - \rho)) V^u_{t+1} \right) \right] \]  
\[ (17) \]

---

3 See Goodfriend and King (2001).
where $b$ denotes real unemployment benefits. Workers and firms are engaged in a Nash bargaining process to determine wages. Let $\zeta$ being the workers’ bargaining power. The optimal sharing rule of the standard Nash bargaining is given by:

$$(V^i_t - V^j_t) = \frac{\zeta}{1 - \zeta} (1 - \tau^i_t) V^j_t$$ (18)

After substituting the previously defined value functions it is possible to derive the following wage schedule:

$$w_t = \zeta mc_t z_t + \left(1 - \zeta \right) b + \left(1 - \tau^i_t \right) \beta E_t \left(1 - f_{t+1} \right) \left(1 - \tau^m_t \right) \kappa \left(1 - \tau^p_t \right) \left(1 - \tau^q_t \right) \left(1 - \tau^r_t \right)$$ (19)

2.3 EQUILIBRIUM CONDITIONS

Good market clearing condition requires aggregate production to be equal to aggregate demand, $y_t = c_t + g_t$, plus the resources wasted into the search activity and the cost of adjusting prices:

$$z_{te} = y_t + \kappa v_{te} + \psi_{te}$$ (20)

2.4 MONETARY POLICY

Monetary policy is conducted by means of an interest rate reaction function of this form:

$$\ln \left(1 + r_t^m \right) = (1 - \phi_r) \left( \phi_y \ln \left( \frac{y_t}{\pi} \right) + \phi_c \ln \left( \frac{c_t}{\pi} \right) + \phi_{te} \ln \left(1 + r_{te}^m \right) \right)$$ (21)

2.5 FISCAL POLICY REGIMES

The government faces the following budget constraint:

$$g_t + b(1 - e_t) + (1 + r_t^m) \frac{B_{t-1}}{p_t} + \tau^k_t v_t = \tau^c_t c_t + \tau^p_t w_t e_t + \frac{\beta_r t}{p_t} + \tau_t$$ (22)

Since government expenditure is financed partly with taxes and partly with government bonds we need some rule to pin down the evolution of government bonds. First we assume that $\tau_t = \tau^c_t$, and that government expenditure follows an exogenous process. Furthermore, following Uhlig (2009) we assume that a fraction $\phi$ of expenditure is financed through tax and the remaining part through government bonds:

$$\tau^p_t w_t e_t = \phi \left( g_t + b(1 - e_t) + \tau^k_t v_t - \tau_t + (1 + r_t^m) \frac{B_{t-1}}{p_t} \right)$$ (23)

We consider two possible targets for government expenditure: aggregate demand and subsidy to cost of posting vacancies. They are both modeled through temporary shocks. Government expenditure in the form of demand stimuli takes the following form:

$$\frac{g_t}{g} = \left( \frac{g_{t-1}}{g} \right)^{\rho_g} + \epsilon^g_t$$ (24)

where $\epsilon^g_t$ is a surprise increase. A hiring subsidy takes instead the following form:

$$\frac{\tau^k_t}{\tau^k} = \left( \frac{\tau_{t-1}^k}{\tau^k} \right)^{\rho_{\tau^k}} + \epsilon^\tau^k_t$$ (25)

where $\epsilon^\tau^k_t$ is a surprise increase.
2.6 CALIBRATION

Preferences. Time is measured in quarters. The discount factor $\beta$ is set equal to 0.99, so that the annual real interest rate is equal to 4 percent. The parameter on consumption in the utility function, $\sigma$, is set equal to 2.

Production. Following Basu and Fernald (1997), the value added mark-up of prices over marginal cost is set to 0.2. This generates a value for the price elasticity of demand $\varepsilon$ of 6. In order to parameterize the cost of adjusting prices, $\psi$, we observe that by log-linearizing equation (12) we can obtain an elasticity of inflation to real marginal cost that takes the form $\frac{\varepsilon-1}{\psi}$. This allows a direct comparison with empirical studies on the New-Keynesian Phillips curve such as Gali and Gertler (1999) and Sbordone (2002) using the Calvo-Yun approach. In those studies, the slope coefficient of the log-linear Phillips curve can be expressed as $(1-\bar{\omega}(1-\beta))$, where $\bar{\omega}$ is the probability of not resetting the price in any given period in the Calvo-Yun model. For any given values of $\varepsilon$, one can build a mapping between the frequency of price adjustment in the Calvo-Yun model, $\frac{1}{1-\beta}$, and the cost of adjusting prices, $\psi$, in the Rotemberg setup. The recent New Keynesian literature has usually considered a frequency of price adjustment between two and four quarters as realistic. As a benchmark, we parameterize $\frac{1}{1-\beta} = 3$, which implies $\bar{\omega} = 2/3$. Given $\varepsilon = 6$, the resulting stickiness parameter satisfies $\psi = \frac{\bar{\omega}(\varepsilon-1)}{(1-\omega)(1-\beta)} = 30$.

Labor market frictions parameters. The matching technology is a homogenous of degree one function and is characterized by the parameter $\xi$. Consistently with estimates by Blanchard and Diamond (1991) this parameter is set to 0.4. The exogenous separation probability, $\rho$, is set to 0.07 consistently with estimates from Hall (1995); this value is also compatible with those used in the literature which range from 0.07 (Merz (1995)) to 0.15 (Andolfatto (1996)). The steady state unemployment rate is set equal to 12% which is the value used by Krause and Lubik (2007). With those two values we can compute the steady state for employment. We set the steady state firm matching rate, $q(\theta)$, to 0.7 which is the value used by den Haan et al. (2000). With those values and using the fact that steady state number of matches is given by $\frac{\rho}{1-\theta}(1-u)$, it is possible to determine the number of vacancies, the vacancy/unemployment ratio as well as the scale parameter, $m$. The value for $b$ is set so as to generate a steady state ratio $\frac{\bar{\omega}}{m}$ of 0.5 which corresponds to the average value observed for industrialized countries (see Nickell and Nunziata (2001)). The bargaining power of workers, $\varsigma$, is set to 0.5 as in most papers in the literature, while the value for the cost of posting vacancies is obtained from the steady state version of labor market tightness evolution. Note that we set $\varsigma > \xi$ implying that the Hosios (1990) condition for efficiency is not satisfied. The rationale for this choice is that we want to analyze a fiscal package in form of a hiring subsidy which is only a reasonable instrument when not imposing an efficient unemployment rate.

Monetary policy parameters. The coefficient on inflation, $\phi_p$, is set to 1.5, the coefficient on output, $\phi_y$, is set 0.5/4. Finally the parameter $\phi_r$ is set equal to zero in the baseline calibration.

Fiscal policy parameters. The constant fraction of public spending, $g$, financed by current taxes is calibrated so as to match $g/y = 0.15$. Steady state taxes are set to $\tau^c = 0.05$ and $\tau^o = 0.28$ which are values calculated for the U.S. by Trabandt and Uhlig (2009). The steady state level of the hiring subsidy is set to $\tau^h = 0.01$. Given those values, the steady state value of the government debt will depend on $\phi$. As a baseline we set $\phi = 0.275$ implying $\frac{\tau^h}{y} = 0.6$, but we also consider the following limiting cases: 0 and 1.

Shocks. The autocorrelation of government spending, $\rho_g$, and the hiring subsidy, $\rho_{\tau^h}$, are calibrated to 0.9.
3 Special Case: Dynamics under Flexible Prices

Before analyzing the fiscal multipliers in the general New Keynesian model, it is useful to understand how the presence of matching frictions alters the transmission of a government spending shock in a real business cycle framework. When studying the model with Walrasian labor market, we consider the following utility function:

\[ U_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\sigma}}{1+\sigma} \]

i.e. we substitute employment \( e_t \) with hours \( N_t \). In what follows we assume a constant technology \( z \) and a constant matching efficiency \( m \), so that movements in the variables are driven only by changes in fiscal policy.

3.1 RBC Model with Walrasian Labor Market

The model is characterized by the following labor supply equation:

\[ \frac{\varepsilon - 1}{\varepsilon} \frac{1}{z} (1 - \tau_t^o) = N_t^\sigma c_t^\sigma \]

Suppose there is an increase in government spending financed by lump-sum taxes, so that \( \tau_t^o \) is constant. Then the left hand side of this equation remains constant. This implies that in equilibrium the increase in \( N \), as required by an increase in production, is accompanied by a decrease in consumption. Hence an increase in government spending tends to crowd out private consumption. The more so the higher the parameter \( \phi \). Thus, in this model the fiscal multiplier is always below one and decreases when the elasticity of labor supply, \( \phi \), increases. If the spending is financed, fully or in part, with distortionary labor taxes, the left hand side decreases when \( \tau_t^o \) increases. Thus, for a given level of government spending and for given elasticity of labor supply, \( \phi \), the drop in consumption is larger the higher the increase in taxes, \( \tau_t^o \). Hence, financing government spending with distortionary taxation reduces the fiscal multiplier even more than the use of lump sum taxation. The next step we undertake is to analyze the impact of matching frictions on the fiscal multiplier.

3.2 RBC Model with Matching Frictions

As we will prove in this section, in presence of matching frictions the fiscal multipliers turn even negative, the reason being that an increase in government spending induces not only a crowding out of private consumption but also a fall in employment\(^3\).

To simplify the analysis, we assume that workers have zero bargaining power \( \varsigma = 0 \). Under flexible prices the marginal cost stays constant, \( m c_t = \frac{\varepsilon - 1}{\varepsilon} \). Using the latter relation and combining the wage equation (19) with the job creation condition (13), we obtain:

\[ \frac{\varepsilon - 1}{\varepsilon} \frac{1}{z} \frac{b}{1 - \tau_t^o} = \frac{k(1 - \tau_k)}{m} \beta_t^E - \frac{(1 - \rho)\beta}{m} E_t \left( \left( \frac{c_{t+1} - 1}{c_t} \right)^{1-\sigma} \right) \]

Under lump-sum taxation, and maintaining \( \tau_t^o \) constant, the left hand side of (27) stays constant. In order to accommodate the increase in \( g \), firms will have to post more vacancies at time \( t \). Therefore, given that \( u_t \) adjusts one period later,

\[^4\text{Recall that under flexible prices the optimal pricing rule implies } m c_t = \frac{\varepsilon - 1}{\varepsilon}. \text{ With linear production technology we have } m c_t = \frac{z}{m} \text{ and from the household problem the (net) real wage has to equate the marginal rate of substitution between consumption and leisure, } (1 - \tau_t^o)w_t = N_t^\sigma c_t^\sigma.\]

\[^5\text{A similar point has also been noted in Yuan and Li (2000).}\]
labor market tightness $\theta_t$ will increase. As we consider a temporary government spending shock, we expect firms to post less vacancies in future periods, hence we expect $\theta_{t+1} < \theta_t$. Thus, for firms to post more vacancies today, we need the continuation value of a filled vacancy, $E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \theta_{t+1}^2 \right\}$, to be higher than its steady state value. This would require an increase in the stochastic discount factor, $\left( \frac{c_{t+1}}{c_t} \right)^{-\sigma}$, which in turn requires an increase in current consumption. However, due to the crowding out effect of government spending, current consumption will actually decrease, thus inducing a fall in vacancy posting below steady state value. Overall, the fall in vacancies induces a fall in employment which brings about negative fiscal multipliers.

If labor taxes are used to finance the increase in government spending, the left hand side decreases, thus requiring an even bigger reduction in vacancy posting. Therefore, in such a framework an increase in government spending will actually induce a negative multiplier since firms will reduce vacancy posting below the steady state value.

The presence of sticky prices will induce higher multipliers in both models. However, we can already expect the multipliers of a government spending shock to be lower in a model with matching frictions.
4 Fiscal multipliers

For each fiscal intervention considered we compute short run and long run multipliers both in terms of GDP, \(z_t e_t\), and in terms of aggregate demand, \(y_t\). Short-run multipliers are calculated as output effects during the impact period divided by costs during the impact period. Long-run multipliers are the discounted output effects divided by the discounted costs. Results are summarized in Tables 1 and 2. Furthermore, we follow Uhlig (2009) and compute dynamic multipliers whose value at time \(t\) is equal to the sum of discounted GDP (or aggregate demand) changes until time \(t\) divided by the sum of discounted cost changes until time \(t\).

4.1 DEMAND STIMULUS

We start by considering a temporary one percent increase in government spending which is either financed by lump sum taxes or by distortionary labor taxes. As baseline value we set \(\phi = 0.275\). We also consider two alternative values for this parameter: \(\phi = 0\) (equivalent to lump sum taxation) and \(\phi = 1\). To highlight the role of search and matching frictions we compare the fiscal multipliers in the New Keynesian model with search and matching with the ones in the standard New Keynesian model. Recall that in the standard New Keynesian model, equilibrium unemployment is absent, hence fiscal stimuli occur solely through increases in aggregate demand.

Figure 1 shows the fiscal multipliers in the New Keynesian model with search and matching frictions. The exact numbers are also summarized in Tables 1 and 2 for both, short run and long run multipliers. The multipliers are very small (nearly zero) when the expansion in government expenditure is financed with lump sum taxation; they even turn negative when a certain fraction is financed with distortionary taxation. To understand the reason for such bad performance of an expansionary fiscal policy, we examine the impulse responses of a set of selected variables to the same temporary government expenditure shock. Figure 2 shows that, in response to a temporary increase in government expenditure, employment and output drop, the more so the higher the \(\phi\). As explained for the RBC model with search and matching frictions, an increase in current vacancy posting requires an increase in the continuation value of a filled vacancy. Such an increase however cannot take place as it would require an increase in current consumption, which, on the contrary falls due to the crowding out effect. The fall in current vacancies brings about a fall in job creation and employment. The fall in employment triggers a fall in output, which more than compensates the increase in aggregate demand. Differently from the RBC version, since prices will not adjust completely, there will be a temporary reduction in price markups that will alleviate the crowding out effect on consumption. This is why, compared to the RBC case, multipliers are in general bigger. Our model highlights a novel dimension in the analysis of fiscal multipliers, as in a model with a non-Walrasian labor market changes in output are determined also by changes in labor demand, alongside with changes in aggregate demand. The positive effect on aggregate demand due to price rigidity is barely enough to compensate for the labor demand effect under lump sum taxation (multipliers are nearly zero). As soon as distortionary taxation is introduced, the multipliers turn negative.

Figure 3 shows the fiscal multipliers in the standard New Keynesian model with Walrasian labor market. Once again the exact numbers are also summarized in Tables 1 and 2 for both, short run and long run multipliers. Figure 4 shows impulse responses of selected variables to the usual temporary shock to government spending while Figure 5 plots a set of selected variables for both models in the same panels to simplify the comparison. Two considerations emerge. First, multipliers in this case are all positive and larger than in the model with a non-Walrasian labor market. Second, with Walrasian labor markets, and contrary to the previous case, we observe an increase in employment and in output. Interestingly, multipliers

---

6 When considering a Walrasian labor market \(N\) denotes hours worked and not employed workers (intensive instead of extensive margin).
7 In this picture for simplicity we consider only lump sum taxation.
are positive even when financing occurs with distortionary taxation. This is in contrast with the results that Uhlig (2009) finds for the RBC model with capital, in which an increase in government spending financed through distortionary taxes leads to negative long-run multipliers.

One clear conclusion that emerges from those first experiments is that the evaluation of fiscal stimuli might be too optimistic when the model employed neglects frictions in the labor market. This is in sharp contrast to the common wisdom which fosters the idea that large fiscal packages are needed exactly when unemployment is high.

4.2 ALTERNATIVE FORMS OF FISCAL STIMULI

In practice fiscal stimuli have taken various forms, which go beyond the mere increase of aggregate demand. Our model allows in particular to highlight the role of subsidies to the labor market, which is one very popular form of fiscal stimuli.

One alternative form of fiscal stimuli which can be usefully analyzed in our model consists in an increase in the subsidy to the cost of posting vacancies. Figure 6 shows fiscal multipliers for a temporary increase in the subsidy to the cost of posting vacancies for different values of $\phi$. Again Tables 1 and 2 report exact numbers for both, short run and long run multipliers. A temporary one percentage point increase in the subsidy to the cost of posting a vacancy generates a strong positive fiscal multiplier, both, in the short and in the long run. This is so independently of the financing strategy adopted, lump sum versus distortionary labor tax. The multipliers are actually larger under the benchmark scenario of partial financing with distortionary taxation ($\phi = 0.275$) and deficit. Reducing the cost of posting a vacancy increases the number of vacancies, which in turn raises job creation and reduces unemployment. This has a direct positive impact on employment and consumption, as shown by the impulse response functions shown in Figure 7. The cost of posting vacancies introduces a gap between the model with Walrasian and non-Walrasian labor markets, hence reducing this gap brings the model closer to the standard New Keynesian model which features larger multipliers. In addition our model features inefficient unemployment fluctuations as our parametrization brings the model away from the Hosios (1990) condition. In these circumstances a fall in the cost of posting vacancies can reduce the wedge in the labor demand schedule, therefore bringing employment and output closer to the pareto efficient level.

We conclude that when frictions operate mainly along the labor demand side and when inefficient fluctuations in unemployment occur, policy makers should direct their fiscal stimuli toward measures that directly increase the number of available jobs.

4.3 ROBUSTNESS CHECKS

Recession scenario. Governments have recently passed large fiscal packages to fight the recessionary impact of the current financial crisis. The question arises whether the effectiveness of fiscal stimuli changes when the economy starts from a recession scenario. Figure 8 and Figure 9 show the responses of GDP and employment in the model with search and matching frictions for the case in which the recession is followed by a policy intervention (solid lines) and for the case in which no policy intervention takes place (dashed lines). We consider policy interventions in terms of a temporary one percent increase in government spending and in terms of an increase in the subsidy to the cost of posting vacancies of 2 percentage points. We restrict the analysis by considering two cases: lump sum taxation, which is depicted in the first two panels, and distortionary taxation with $\phi = 0.275$, which is depicted in the bottom panels.

The analysis confirms our previous findings. A pure demand stimulus has negligible effects when financed by raising lump sum taxes. Moreover, when financed partly with distortionary taxes, an expansion in government spending even deepens the economic downturn. A hiring subsidy, on the other hand, dampens the recession triggered by the drop in productivity. This policy intervention is particularly powerful in fighting the drop in employment. Remarkably, the costs incurred by a 2 percentage point increase in the hiring subsidy accounts for only 0.04 percent of GDP, whereas a one percent increase in government spending costs approximately 0.15 percent of GDP.

\[8\] Contrary to other authors, our recession scenario does not bring the economy toward the zero lower bound. While this might sound like a realistic outcome at least from the point of view of the Federal Reserve Board policy, it might be more easily obtained in models which also include financial frictions.
Interest rate peg. Over the last year, countries like the U.S. have adopted a policy mix in which the monetary authority has kept the interest rate fixed at low levels, while the fiscal authority had passed large fiscal packages. There has been a widespread consensus that, when the interest rate is close to the zero lower bound, fiscal policy might have a stronger leverage than monetary policy. To verify whether those statements are correct we repeat our experiments in combination with a temporary interest rate peg (the monetary authority keeps the nominal interest rate constant for one year).

As shown in Table 1 and 2, in this case a demand stimulus induces large positive multipliers both, in the standard New Keynesian model and in the model with search and matching frictions. Additionally, in the model with search and matching frictions, an interest rate peg tends to dampen the crowding out effects generated in the long run by an increase in aggregate demand financed through distortionary taxation. These results echo the ones reported in Christiano et al. (2009).

It is worth noticing, however, that, even when an interest rate peg is considered, it remains true that search and matching frictions dampen the positive effects of expansionary fiscal policies.

Next, we analyze the effects of subsidizing the cost of posting vacancies in presence of an interest rate peg. Again, results are reported in Tables 1 and 2. Surprisingly, this type of policy mix is very unsuccessful as multipliers are nearly zero and they even turn negative. A reduction in the cost of posting vacancies allows firms to increase the number of vacancies only when accompanied by a fall in the nominal interest rate. When the interest rate stays constant, the discounted future values of new vacancies as well as firms’ incentives to post new vacancies tend to decrease.

Price and Wage Rigidity. Furthermore, we re-examine fiscal multipliers for different degrees of price rigidity. We set the cost of adjusting prices, \( \psi \), to 9.9 and 58 to generate a slope of the log-linear Phillips curve consistent with an average price duration of two and four quarters, respectively. When considering the demand stimulus, higher price rigidity increases the short and long run fiscal multipliers (both for the aggregate demand and for the GDP). This is so both, for the case of lump sum taxation and for distortionary taxation. This is in line with the results found by the previous literature examining New Keynesian models with Walrasian labor markets. When considering an increase in the hiring subsidy, some degree of price rigidity seems to make the fiscal stimulus more powerful, although the relation does not seem to be always monotonic. Importantly, results show that, independently of the degree of price rigidity, matching frictions dampen fiscal multipliers when demand stimuli are considered.

This finding is also true when considering real wage rigidity or when varying the worker’s bargaining power. We consider those two additional cases as they are the hypothesis considered in the literature to solve the so-called Shimer puzzle. First, we assume wages to be set according to the partial adjustment equation \( w_t = \gamma w_{t-1} + (1 - \gamma)w^*_t \), where \( w^*_t \) is described by the baseline wage equation (19) and the degree of real wage rigidity, \( \gamma \), is calibrated to 0.9. An increase in aggregate demand produces an increase in wages which induces a fall in vacancy posting and thus employment. Under real wage rigidity, the upward pressure on wages is dampened which gives rise to larger multipliers. However, our results show that the effects of a demand stimulus are still considerably smaller when the model with search frictions is compared to a model with a Walrasian labor market.

Finally, we vary the workers’ bargaining power, \( \varsigma \). Even in this case previous results are largely confirmed.
5 Endogenous Labor Market Participation

As standard practice in the New Keynesian models with search and matching frictions, we assumed inelastic labor supply in the model. One however could ask whether our results are robust to the introduction of an intensive margin. Fiscal policy could deliver higher multipliers by increasing labor market participation, particularly in models featuring involuntary unemployment. To test whether this is indeed the case, we introduce endogenous labor market participation as in Campolmi and Gnocchi (2011). While we refer to that paper for a thorough explanation of the model with endogenous participation, here we underline the main differences with respect to the baseline model used so far.

Household's size is still normalized to 1 but now only a fraction $n_t$ of family members actively participate into the labor market while $1 - n_t$ voluntarily choose non-employment. The non-participant family members allocate their time to home production activity\(^9\). Expected life time utility of the household can be written as:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} + \Phi \frac{[1-n_t]^{1+\nu}}{1+\nu} \right] \right\}$$

where $\Phi$ is a scaling parameter used to pin down the steady state participation rate and $-\nu$ is the inverse of the Frisch elasticity of labor supply.

The presence of endogenous participation does not alter the optimal choice of $\{c_t, B_t\}_{t=0}^{\infty}$, thus conditions (3) and (4) are unchanged.\(^10\) Maximizing (28) with respect to $\{n_t\}_{t=0}^{\infty}$ subject to the budget constraint and the low of motion for employment \(^11\) $e_t = (1-\rho)(1-f_t)e_{t-1} + f_t n_t$, gives the following new first order condition\(^12\):

\[
\begin{align*}
\left[ \frac{1-f_t}{f_t} \right] (1 + \tau^c) \Phi h_t^\nu c_t^\sigma - b = w_t (1 - \tau^c) - (1 + \tau^c) \Phi h_t^\nu c_t^\sigma \\
+ \beta (1-\rho) E_t \left\{ \left( \frac{c_t}{c_{t+1}} \right)^\sigma \frac{(1-f_{t+1})}{f_{t+1}} ((1 + \tau^c) \Phi h_{t+1}^\nu c_{t+1}^\sigma - b) \right\}
\end{align*}
\]

where $h_t \equiv 1 - n_t$ is the home production. This is the first difference with respect to the basic model. The second difference is the low of motion for searching workers at the beginning of period $t$, that now includes also new participants, $u_t = n_t - (1-\rho)e_{t-1}$. The introduction of endogenous participation has no direct consequences on the production sector and on the wage formation.

Compared to the baseline model, we now have one more endogenous variable, $n_t$, and two parameters, $\Phi$ and $\nu$. $\Phi$ is chosen using (29) to target a steady state participation rate $n = 0.66$, consistent with the evidence for the U.S. We set the inverse of the Frisch labor supply elasticity to $\nu = -5$, a standard value in the literature.

\(^9\) Here we assume that only non-participants can allocate their time to home production. A more general specification where also unemployed workers contribute to home production is presented in the appendix.

\(^10\) When solving their consumption/saving problem, households are subject to the same budget constraint as in the previous model, the only difference being that the unemployment benefit is perceived only by family members actively searching for a job, i.e. the total unemployment benefits payed to the household is now given by $b(n_t - e_t)$. This however has no consequences for the optimal consumption/saving decision.

\(^11\) Note that this is just an alternative way of writing equation (6).

\(^12\) We are already imposing a constant consumption tax, consistently with the analysis conducted with the benchmark model.
With this richer framework we reassess the fiscal multipliers associated to both a government spending shock and a temporary increase in the subsidy to the cost of posting a vacancy, again considering different financing schemes. As Figure 10 and Figure 11 show, indeed both shocks temporarily increase labor market participation. This increases the fiscal multiplier of a government expenditure shock though they still remain considerably below 1 under lump sum taxation and negative when financing is done using distortionary taxation (Figure 12). Multipliers associated to a temporary increase in the subsidy are practically unchanged with respect to the previous case (Figure 13). The main reason is that labor market participation increases but only marginally and thus cannot push up multipliers in a quantitatively significant way.
6 Bayesian Estimation of the Fiscal Multipliers

To verify whether the results of the baseline model are consistent with empirical evidence we estimate the structural model using Bayesian estimation methods. The estimated model will indeed allow us to measure the actual size of the fiscal multipliers and to compare them with the ones obtained out of the calibration exercise. The choice of the estimation method is motivated by the belief that measures of the fiscal multipliers, obtained through the Bayesian estimation of the DSGE model, are more reliable than estimates based, for instance, on VAR evidence. In the latter indeed the assumptions imposed to identify the shocks are not enough to allow direct comparison with the structural model.

The estimation of the model is based on U.S. data for real GDP, civilian employment, GDP deflator, the Federal fund rate, the real wage and vacancies. The sample period goes from 1965Q1 to 2009Q4. All series are logged and de-trended with Hodrick-Prescott filter except the inflation and the interest rate for which we compute first differences. We estimate the baseline model by including all fiscal shocks considered in the calibrated model, alongside with shocks to productivity, monetary policy (e.g. to the nominal interest rate), marginal cost and matching efficiency. Fiscal shocks, shocks to productivity and to matching efficiency are modeled as AR(1) processes. The cost-push shock, $\varepsilon_{\text{inf}}^t$, and the shock to monetary policy, $\varepsilon_r^t$, follow a white noise process.

Table 3 shows the estimated parameters, as means of the posteriori distribution, alongside with the prior distribution and the confidence interval. The prior distributions have been chosen as follows. All parameters with values lower than one have been assigned a Beta distribution. For the Taylor coefficient on output we have chosen the Gamma distribution to impose a positivity constraint. Finally, priors for the shocks have been chosen following previous literature, more specifically Smets and Wouters (2007) and Lubik (2009). The values for the estimated parameters are in line with the ones assigned in our calibrated model and they are significant considering a 90% error band. Particularly the shock to government spending and to the subsidy to the cost of posting vacancies are large and persistent. The parameter characterizing price rigidity, $\hat{\vartheta}$, and the bargaining power are quite high perhaps capturing other forms of rigidity of persistence characterizing respectively nominal variables and labor market variables. Figure 14 shows the posterior density distributions (black lines) together with the prior distributions (grey lines) and the posterior modes (green dashed lines). Two things are worth noticing. First, the posterior distribution is shifted relatively to the prior showing that the data are informative. Second, all posterior distributions are smooth and unimodal.

We then feed the estimated parameters into the non-linear model and compute the output multipliers for the government expenditure shock and for the hiring subsidy. The first is equal to 0.1186 in the short run and to 0.0438 in the long run. Those values therefore confirm the previous result concerning the fact that multipliers associated with pure demand stimuli are very small. Repeating the exercise for the hiring subsidy we obtain multipliers even larger than the one delivered by the baseline calibration, thus confirming that a subsidy to the cost of posting vacancies may deliver quite big effects on output. Figures 15 and 16 show the impulse responses to a government expenditure shock and to a subsidy shock, respectively.

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13 Data for real GDP are taken from OECD statistics.
14 Data for civilian employment are taken from Bureau of Labor Statistics.
15 Data are taken from Bureau of Economic Analysis.
16 Data are taken from FRED database of the Federal Reserve Board.
17 Real wages are computed by dividing the hourly compensation by the GDP deflator. The hourly compensation is obtained from the Bureau of Labor Statistics.
18 Vacancies are measured from the Help Wanted Index taken from the Bureau of Labor Statistics.
Both have been computed using the estimated parameters. It is encouraging to notice that the qualitative and quantitative behavior of all variables is quite sensible and in line with the one of the calibrated model.\textsuperscript{19}

\textsuperscript{19} This also holds when estimating the model with endogenous labor market participation. Results are available upon request.
7 Conclusions

We provide some fiscal calculus for a New Keynesian model with search and matching frictions. We do so by considering fiscal packages in the form of demand stimuli and in the form of subsidies to the cost of posting vacancies. We also analyze the effects of alternative forms of financing in terms of lump sum versus distortionary taxation. By comparing the results obtained in our model with the ones obtained in a standard New Keynesian model, we find that frictions in the labor market tend to dampen fiscal multipliers when demand stimuli are considered. This seems to contrast the common wisdom which sees fiscal stimuli particularly helpful exactly in situations of high unemployment. On the other side, policies targeted more specifically toward the labor market, particularly those aimed at reducing labor wedges, such as cost of hiring, are particularly effective in boosting employment and output. Results are confirmed by estimating the structural model with Bayesian methods.
References


Appendix A  Endogenous Participation

Following Campolmi and Gnocchi (2011), a more general specification of the model with endogenous participation allows for the presence of a search cost smaller than one, i.e. unemployed workers allocate only a fraction of their time to the search activity while the rest is used for home production. The expected life time utility reads as follows:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{e_1 - \sigma}{1 - \sigma} \Phi \left( \frac{1 - e_1 - \Gamma(n_t - e_t)}{1 + \nu} \right) \right] \right\}$$

(30)

where $\Gamma$ is the fraction of time that unemployed workers devote to the search activity. When $\Gamma = 1$ searching is very costly because unemployed people spend all their time searching for a job and thus can not contribute to home production, same as employed people. This is the assumption used in section 5. When $\Gamma = 0$ searching for a job requires no time and thus unemployed members not matched with a job at time $t$ contribute to home production like a non-participant. Under this more general specification the first order condition for the participation decision is given by:

$$\left[ \frac{1 - f_t}{f_t} \right] ((1 + \tau_t) \Phi \Gamma h_t^\nu c_t^0 - b) = w_t (1 - \tau_t) (1 + \tau_t) \Phi h_t^\nu c_t^0$$

+ $\beta (1 - \rho) E_t \left\{ \left( \frac{c_t}{c_{t+1}} \right) \beta \left[ \frac{(1 - f_{t+1})}{f_{t+1}} \right] \left( (1 + \tau_t) \Phi \Gamma h_t^\nu c_t^0 c_{t+1} - b \right) \right\}$

(31)

where $h_t \equiv 1 - e_t - \Gamma(n_t - e_t)$ is the home production. The introduction of the search cost smaller than one affects the value function of being unemployed given that now we decomposed the income of unemployed members into unemployment benefit $b$ and home production:

$$V_t^w = (1 - \tau_t^v) w_t - b - \Phi(1 - \Gamma) h_t^\nu c_t^0 (1 + \tau_t^v) + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho)(1 - f_{t+1}) V_{t+1}^w \right\}$$

(32)

Solving the Nash bargaining problem between firms and workers gives the following wage schedule:

$$w_t = \zeta mc_t z_t + \frac{1 - \zeta}{1 - \tau_t^v} [b + \Phi(1 - \Gamma) h_t^\nu c_t^0 (1 + \tau_t^v)]$$

+ $\zeta (1 - \rho) \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ 1 - (1 - f_{t+1}) \frac{1 - \tau_t^v}{1 - \tau_t} \right] \kappa (1 - \tau_t^v) \right\}$

(33)

We are now left with one parameter to be calibrated, the cost of search $\Gamma$. In the baseline model we set the replacement ratio to $b/w = 0.5$, a standard value in the literature when the home production is not considered separately. However, since here unemployed family members receive the unemployment benefit plus contribute to home production, in order not to increase their outside option with respect to the baseline specification we set $b/w = 0.2$ being $b$ only the social security benefit, and choose $\Gamma$ together with the cost of posting a vacancy $\kappa$ so as to get a cost of posting a vacancy per filled job equal to 5% of the real wage.\(^{20}\) If we follow this calibration strategy we obtain $\Gamma = 0.32$ meaning that searching workers allocate 1/3 of their time to the search activity. Using this more general specification we re-compute the multipliers and find them to be quantitatively the same as the one obtained by setting $\Gamma = 1$, like in the main text. Results are available upon request.

\(^{20}\) This calibration strategy is the same as in Campolmi and Gnocchi (2011) and Galí (2010).
### Table 1
GDP multiplier

<table>
<thead>
<tr>
<th></th>
<th>$\phi = 0$</th>
<th></th>
<th>$\phi = 0.275$</th>
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<td>LR Multiplier</td>
<td>SR Multiplier</td>
<td>LR Multiplier</td>
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<tr>
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<td>-0.1070</td>
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<tr>
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### Table 2
#### Aggregate demand multiplier

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<td>Std. dev. cost shock $\varepsilon_{\text{inf}}$</td>
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Appendix C Figures

Figure 1
Government spending multipliers in a model with search frictions

![Figure 1](image1)

Figure 2
Impulse responses to a one percent temporary increase in government spending in a model with search frictions

![Figure 2](image2)
Figure 3
Government spending multipliers in a model with a Walrasian labor market

- **AGGREGATE DEMAND MULTIPLIER**
  - $q=0$
  - $q=0.275$
  - $q=1$

- **GDP MULTIPLIER**

Figure 4
Impulse responses to a one percent temporary increase in government spending in a model with a Walrasian labor market

- **AGGREGATE DEMAND**
  - % deviation from s.s

- **CONSUMPTION**
  - % deviation from s.s

- **HOURS WORKED**
  - % deviation from s.s

- **NOMINAL INTEREST RATE**
  - % deviation from s.s

- **REAL INTEREST RATE**
  - % deviation from s.s

- **INFLATION**
  - % deviation from s.s
**Figure 5**
Impulse responses to a one percent temporary increase in government spending with lump sum taxation

![Graph showing impulse responses to government spending](image1)

**Figure 6**
Fiscal multipliers for a 1 percentage point temporary increase in the subsidy to the cost of posting vacancies

![Graph showing fiscal multipliers](image2)
Figure 7
Impulse responses to a one percentage point temporary increase in the subsidy to the cost of posting vacancies

Figure 8
Response of GDP under a temporary increase in government spending (left column) and under a temporary subsidy to the cost of posting vacancies (right column) starting from a recession scenario for $\phi = 0$ (first row) and $\phi = 0.275$ (second row)
Figure 9
Response of employment under a temporary increase in government spending (left column) and under a temporary subsidy to the cost of posting vacancies (right column) starting from a recession scenario for $\phi = 0$ (first row) and $\phi = 0.275$ (second row)

Figure 10
Impulse responses to a one percent temporary increase in government spending in a model with search frictions and endogenous participation
Figure 11
Impulse responses to a one percentage point temporary increase in the subsidy to the cost of posting vacancies in a model with search frictions and endogenous participation

Figure 12
Government spending multipliers in a model with search frictions and Endogenous Participation
Figure 13
Fiscal multipliers for a temporary increase in the subsidy to the cost of posting vacancies in a model with search frictions and Endogenous Participation.

AGGREGATE DEMAND MULTIPLIER

GDP MULTIPLIER

Figure 14
Estimated Parameter Distribution
Figure 15
Impulse responses to a one percent temporary increase in government spending using estimated parameters

Figure 16
Impulse responses to a one percentage point temporary increase in the subsidy to the cost of posting vacancies using estimated parameters
MNB Working Papers 2011/5
Fiscal Calculus in a New Keynesian Model with Labor Market Frictions