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OPTIMAL INDEBTEDNESS OF A SMALL OPEN ECONOMY WITH PRECAUTIONARY BEHAVIOR[1]

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Abstract

The model is an application of the precautionary consumer saving model to the external debt policy of a small open economy.

Let us assume that the welfare criterion of macroeconomic policy is the utility function of a representative infinitely living dynasty. This approach is in line with the intertemporal optimization model of the current account of *Obstfeld-Rogoff* [1995]. It is known that assuming differences in tastes or growth rates across countries imply unacceptably extreme long-run predictions in this model. We show that a model with uninsurable wage risks and precautionary behaviour leads to stable stationary indebtedness levels within the range of magnitudes observed in reality.

Let us assume that the consumption function of the representative dinasty has the form of a CES function. The positive third derivative of this function and uncertainty together give rise to a precautionary behavior. As a result, countries who grow fast relative to their own time preference, will borrow, but their debt will be constrained by the risk that indebtedness implies. By similar reasoning, a patient or slow-growing country will lend but its lending will converge to an amount where the gained security that its reserves offer is equal to its opportunity cost.

We parameterized the model assuming a drifting random-walk aggregate income with a standard error of 2%, a habit factor of 80% of the previous years income incorporated into the CES funtion, risk-aversion and time preference parameters taken from the literature, and found that typical indebtedness ratios observed in the world can be replicated as a policy outcome of our model, in contrast to deterministic models where the rate of the optimal indebtedness was in the range of 20-30 times GDP.

The calculations are based on adaptation of the Taylor-series approximation of *Skinner* [1989]. A sensitivity analysis of the stationary solution to various parameters and various scenarios for effects of assumed shocks and consequences of catch-up growth paths for a converging country are calculated.

In another section the problem is discussed whether the level of indebtedness can be considered a goal of macroeconomic policy and if it was a goal how could it be achieved.

Let us assume agents with idiosynchrasic uninsured wage risks. These risks are correlated which generates an aggregate risk for the country as a whole. Aggregate income risks affect individuals differently (more deeply) than individual risks, however, individuals cannot separately hedge aggregate and individual risk. Aggregate risk has to be handled by macroeconomic policy. In this sense the task and the optimum intertemporal consumption choice of the social planner (macro-policy maker) does not necessarily coincide with the sum of the optimal decisions of individual agents. Fiscal policy is the tool that creates consistency between the two.

It is shown that the model with the parametrizations given above and a proportional income tax system implies that government debt nearly fully appears in external debt, i.e. Ricardian compensation is very close to 0. This means, that the fiscal tool is effective in enforcing the social optimum of indebtedness.

Finally, the risk consequences of equity financing rather than bond financing of current account deficits are discussed. In this analysis equity capital is considered to be a component of a portfolio consisting of human capital, bonds and equities.

Introduction

Precautionary behavior is a well-known phenomenon with a long history in the literature that goes back to *Leland* [1968], but it has gained interest in the last 15 years by the analyses of *Skinner* [1988], *Zeldes* [1989a] *Kimball* [1990], *Carroll* [1992], *Ayiagari* [1994] and many others. In this paper we use their results for the purpose of explaining the rate of indebtedness of an open economy.

The starting point of our paper is the result of *Skinner* [1988] that is stated in the first section. In Section 2

– we reformulate the problem assuming a social planner who maximizes a social utility function and calculate stationary debt levels for calibrated parameters,

– show that in the case of incomplete labor markets and idiosyncratic risks the social optimum cannot be derived from adding up individual decisions,

– we prove that the social optimum can be enforced by fiscal policy because Ricardian tax imputation is close to 0.

– we show that assuming equity as a second asset does not change the main conclusions of the model.

In Section 3 we give results of various simulations

In Appendix A we review the model of precautionary saving, in Appendix B we give details of mathematical derivations of the main paper.

1 The problem of the precautious agent

Let assume that an infinitely living dynasty maximizes the following CRRA utility function:

$$\sum_{s=0}^{\infty} \beta^s \frac{c_s^{1-1/\gamma}}{1-1/\gamma}, \tag{1}$$

where c_s is consumption in period s, β time preference factor, and γ parameter of risk aversion.

Let assume that the endowment process of the agent is the following stochastic process:

$$y_t = (1+g)y_{t-1}\varepsilon_t \qquad \ln(\varepsilon_t) \sim N(0, \sigma_\varepsilon^2 I), \tag{2}$$

where σ_ε^2 is known and g is the drift of the random walk process.

Let assume that the income (endowment) shocks cannot be diversified.

Let assume that the agent keeps only one asset, a riskless financial asset with a constant r interest rate. The agent solves the following problem:

$$\max_{\{c_s\}} E_0 \left[\sum_{s=0}^{\infty} \beta^s \frac{c_s^{1-\gamma}}{1-\gamma} \right] \tag{3}$$

$$W_s = (W_{s-1} - c_{s-1})(1+r) + y_s \qquad s = 1, 2, 3, \dots \quad , \tag{4}$$

where W_s is the financial asset of the agent after the inflow of income but before consumption expenditure.

Lifetime wealth L is the following:

$$L_t = W_t + H_t, \tag{5}$$

where $H_t = E_t \left[\sum_{s=t+1}^{\infty} y_s / (1+r)^{s-t} \right]$ is human wealth.

Skinner [1988] derived by Taylor-approximation the optimal path of consumption:

$$c_t = \left[(1+r) \beta (1+v_t) \right]^{1/\gamma} \frac{L_t}{E_{t-1}[L_t]} c_{t-1}, \tag{6}$$

$$v_t = \frac{\gamma(1+\gamma)}{2} \sigma_{L_t}^2, \tag{7}$$

where σ_L^2 is the relative variance of lifetime wealth, which in case of a random walk with drift is the following:

$$\sigma_{L_t}^2 = \frac{\operatorname{var}(L_t)}{L_t^2} = \left(\frac{\sigma_\varepsilon \frac{1+r}{1+g} H_t}{W_t + H_t} \right)^2 . \tag{8}$$

The steady state value of W/y and c/y can be determined in the following way.

In steady state consumption grows along income:

$$1 + g = (1+r)^{1/\gamma} \beta^{1/\gamma} (1+v)^{1/\gamma}. \tag{9}$$

Expressing v and by substitution equations (7)-(8) lead to the following:

$$\begin{aligned} \overline{W/y} &= \left(\sigma_\varepsilon \frac{1+r}{1+g} \sqrt{\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta} - 1\right)}} - 1 \right) H/y = \\ &= \left(\frac{\operatorname{var}(L)}{y^2} \frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta} - 1\right)} \right)^{0.5} - H/y. \end{aligned} \tag{10}$$

and from (5):

$$\overline{L/y} = \left(\frac{\operatorname{var}(L)}{y^2} \frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta} - 1\right)} \right)^{0.5} , \tag{11}$$

$$\overline{c/y} = \frac{1+g}{1+r}\overline{L/H}, \tag{12}$$

where $\overline{W/y}$, $\overline{L/y}$, and $\overline{c/y}$ are steady-state values of W/y, L/y and c/y.

To approach applicability of the model, we introduce habit behavior. Let assume that agents cannot tolerate a consumption that is less than ρc_{t-1}. As derived in Appendix A.3, this means that in equation (10) σ_ε has to be replaced by $\sigma_\varepsilon/\left(1-\rho\right)$.

2 The social optimum problem

The agent's problem can be interpreted as a problem of a social planner who maximizes a social welfare function over an infinite horizon. Formally there is no difference from the general formulation but to distinguish the results let us denote the aggregate fiscal position by F or net foreign assets in contrast to W of the agent's problem.

Table 1: **Steady state financial wealth positions**

Parameters			
		Baseline	
Standard deviation of income	0.018	*0.020*	0.022
β	0.94	*0.95*	0.96
γ	2	*3*	4
g	0.015	*0.020*	0.025
r	0.04	*0.05*	0.06
Habit	0.79	*0.80*	0.81
The values of $(F-y)/y$ are calculated with the parameter of their own row. The rest of the parameteres are from baseline:			
Standard deviation of income	−3.950	*−0.500*	2.950
β	−3.203	*−0.500*	2.949
γ	−5.272	*−0.500*	3.575
g	4.005	*−0.500*	−4.914
r	−4.412	*−0.500*	1.953
Habit	−2.692	*−0.500*	0.709

$(F-y)/y$ is the steady state value of indebtedness (if it is negative) or foreign lending (if it is positive) in proprotion to GNP.

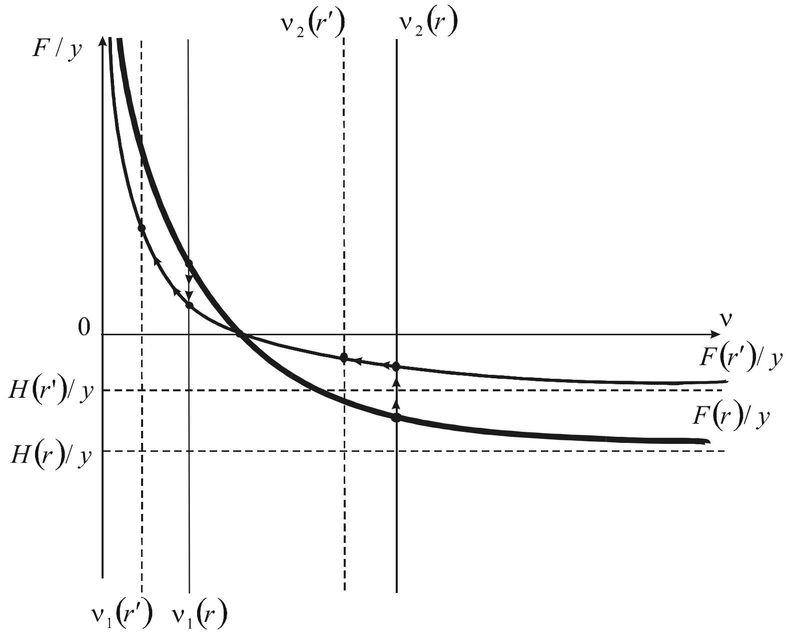
The Table 1 gives the steady state values calculated on the basis of equation (10) depending on various values of the parameters. The values are related to GNP.

The effects of changes in the parameters are easy to interpret. Faster growth results in larger indebtedness, similar effect has larger impatience. Risk aversion (γ and ρ) decreases the steady state debt level.

The sign of the effect of interest rate change is ambiguous because of its wealth effect. It is interesting, that in contrast to the point expectation model the wealth effect here works through the larger absolute risk that is acceptable by higher wealth.

In Figure 1 higher wealth means an upward shift in H/y and a shrinkage of F/y in absolute value. Intuitively it means that in case of a positive financial position the increase in the interest rate has a positive effect on the ratio of her buffer stock to her human capital which cuts risk and allows her to cut reserves. In case of a negative financial position her debt becomes too high compared to human capital and forces her to cut the level of indebtedness.

An increase in r makes future consumption cheaper and shifts v to the left. This works against the wealth effect in the case of a creditor and adds to the wealth effect in case of a debtor.



The figure shows two cases, a debtor and a creditor case. In the first case the risk premium is denoted as v_2 in the second case as v_1.

Figure 1: **Effect of a change in the interest rate on savings**

In Figure 1 the substitution effect dominates as it is implied by the results given in the table.

The steady state financial asset rate is very sensitive to the parameters. For β and γ we took values assumed in most cases in the literature[1]. For the habit parameter we had no empirical or theoretical basis for our assumption. We used this parameter to calibrate the desired output, a reasonable rate of indebtedness. This procedure was similar to that of *Ayiagari-McGratten* [1997], who calibrated β and arrived at a 3-digit figure to ensure consistence with an assumed interest rate.

As we see, the model is not suitable for working out advices for economic policy about the optimal level of indebtedness. It only shows that the model might explain observed magnitudes with reasonable parameters[2].

This cannot be said for alternative, point estimation models. In this class of models the uniform agent infinite horizon model leads to a level of optimal indebtedness 20-30 times GDP in the stable case, while overlapping generation models have difficulties in bringing observed asset ratios in line with reasonable parameters.

2.1 Individual decisions do not aggregate to a social optimum

We believe that it is reasonable to assume that the labor market is not complete. One can contrive schemes of insurance arrangements for labor income and use these as a basis of models but it seems better to comply with the fact that such insurance schemes do not exist or they have marginal importance, whatever its reason. In this case even if we assume that agents have identical and homothetic utility functions, the aggregate maximization problem is not the sum of the individual problems. Because of the covariances of individual shocks the aggregate income variance will differ from the sum of the individual variances.

Another difference between the aggregate problem of the social planner and the individual problems come from the difference in the planning horizon and in accounting for new generations. The consequences of these differences have been exhaustingly discussed in the literature for the point-expectation model[3]. To exclude this problem from our discussion and concentrate on the consequences of idiosyncratic shocks, we assume the economy to consist of a constant number of infinitely lived dynasties.

Let assume that the dynasties are hit by synchronized permanent shocks and idiosyncratic transitory shocks[4]. This means, that the (log of the) income process of the "representative" dynasty is described by the sum of a random walk with aggregate

[1] Some authors think that much higher values for v and accordingly much lower values for β are possible. *Friedman* [1957; 1963] assumed 0.8 for β and *Hayashi* [1982], *Weale* [1990], *Carrol* [1990] considered similar magnitudes.

[2] We know that the way we modelled habit creates too many puzzles to be satisfactory. However, we preferred here a simple approach and refer to *Campbell – Cochrane* [1995] for a discussion of the problem.

[3] See the development of this line of thinking in *Blanchard* [1985], *Buiter* [1988] and *Weil* [1989].

[4] In this section we only make a shortcut amendment to the problem of the social planner, stressing the differences in the stochastic properties of the individual and the aggregate income. Research efforts aimed specifically at explaining lifetime income and consumption patterns use much richer specifications of the income processes and arrive at much more sophisticated conclusions. See Hubbard–Skinner–Zeldes [1993], Storesletten–Telmer–Yaron [2000], Gourinchas–Parker[1999] as examples from the recent huge literature on the topic.

shocks and an autocorrelated process with idiosyncratic shocks. This way we approximate the observation confirmed by *Jenks* [1972] that intergenerational correlation of incomes within dynasties is rather low (0.12-0.15 between parents and children), but on the other hand we allow for persistent differences in individual life cycles. According to this assumption the income process of the representative dynasty can be described the following way:

$$\begin{aligned}
\breve{y}_s &= (1+g)\breve{y}_{s-1}\varepsilon_s & & \ln(\varepsilon_s) \sim N(0,\sigma_\varepsilon^2 I), \quad (13)\\
q_s &= (q_{s-1})^\alpha \xi_s & 0<\alpha<1 \quad & \ln(\xi_s) \sim N(0,\sigma_\xi^2 I), \quad (14)\\
y_s &= q_s \breve{y}_s & & \quad (15)
\end{aligned}$$

Empirical estimates on the variance of American incomes are in the range of 30 percent with an autocorrelation of 0.4 [5] For the aggregate shocks we assumed a 2.5 percent relative variance with an autocorrelation of 1. [6]

Let assume that ε and ξ are independent: $\sigma_{\varepsilon\xi} = 0$. Logarithmic variance of life cycle wealth is then the following (See Appendix B for details):

$$\sigma_L^2 = \frac{\left(\frac{1+r}{1+g}H\right)^2 \sigma_\varepsilon^2 + \left(\frac{1+r}{1-\alpha+(r-\alpha g)}\right)^2 \sigma_\xi^2}{L^2}. \quad (16)$$

If $r = 0.05$, $\alpha = 0.4$, $g = 0.03$, $\sigma_\xi = 0.3$, $\sigma_\varepsilon = 0.02$, then we can see by substitution that total variance of individual wealth consists of the aggregate variance term and a variance of idiosyncratic shocks that is about half as much as the aggregate variance. If the parameters of the individual utility functions would be the same as those of the social planner, this would mean that individual precaution would lead to higher saving than it is justifiable from the point of view of a social optimum. However, there is no reason to assume that the parameters of aggregate and the individual utility function are the same. Individuals might be willing to bear higher risks of income loss than society as a whole. A family may endure a 20-30 percent loss in income within a year, while the same loss for a whole country would probably be considered disastrous.

Whatever differences exist in the parameters of the two problems, it is clear that aggregating individual decisions does not lead to a meaningful social optimum. If we accept the idea or the existence of a social optimum it is interesting to tell whether it is attainable by fiscal policy. In the next section we will see, that within the framework of the model fiscal policy can efficiently determine rate of steady state net foreign assets because domestic bonds issues appear in the net foreign asset position of the country nearly 100 percently.

[5] See for example *Lillard–Willis* [1978] and *Abowd–Card* [1989].

[6] We know that a calculation aimed at a best fit to actual data would have to take into account that aggregate income has a transitory component as well.

2.2 The irrelevance of the Ricardian equivalence principle

We keep the assumption that the number of infinitely living dynasties is constant. This assumption allows us to separate the effect of precautionary behavior from the effect of the existence of disconnected generations on Ricardian non-equivalence that was analyzed by *Blanchard* [1985], *Buiter* [1989], *Weil* [1989].

Let assume that the government issues D risk-free bonds at an international $r = r^*$ interest rate. Because of the assumption on intertemporally linked dynasties Ricardian neutrality regarding life-cycle wealth prevails. However, the question is open how much the outstanding stock of bonds effects the variance of life-cycle wealth. If it changes this variance it will not be neutral for the desired total holding of assets by individuals.

Let assume that our country is small and open. Total financial assets of domestic individual residents are the following:

$$W = F + D, \tag{17}$$

where W is total financial wealth,

D net domestic bonds held by domestic residents,

F net foreign bonds held by domestic residents..

Let assume for sake of simplicity that only labor income is taxed.

After-tax human wealth:

$$\widetilde{H} = H - D. \tag{18}$$

Let consider two cases, lump-sum and proportional taxation.

1. Lump-sum taxes. The interest burden of the debt is distributed among individuals according to their expected income in the period of the issuance of the bond. This does not effect the absolute variance of lifetime wealth and because of the neutrality in terms of wealth the effect on relative variance is unchanged as well.

Specifically, in equation (10) substituting W from (17) and replacing H with $\widetilde{H}$ from (18):

$$\overline{F/y} + D/y = \frac{1}{y}\left(\operatorname{var}(L)\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}\right)^{0.5} - (H/y - D/y)\,. \tag{19}$$

From this (Ricardian) total debt neutrality is a straightforward consequence:

$$\frac{\Delta\overline{F/y}}{\Delta D/y} = 0,$$

2. Proportional taxes. Let assume that the interest burden of debt is distributed proportionally to actual rather than expected income. In this case the tax decreases the absolute variance of income by the rate $(H-D)^2/H^2$ while wealth-neutrality is maintained as before. Equation (10) modifies the following way:

$$\overline{F/y} + D/y = \frac{1}{y}\left(\frac{(H-D)^2\operatorname{var}(L)}{H^2}\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}\right)^{0.5} - (H/y - D/y)\,. \tag{20}$$

Differentiating by D/y the effect on total external position:

$$\frac{\Delta \overline{F/y}}{\Delta D/y} = -\frac{1}{H}\left(\mathrm{var}L\frac{\gamma\left(1+\gamma\right)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}\right)^{0.5} + 1 - 1 = -\overline{L/H}. \tag{21}$$

This means that the effect depends on the ratio of (steady state) total wealth to human wealth. Without having to discuss whether we should include equity into non-human wealth in a more realistic extended model or not, we can safely say that in practice human wealth far exceeds non-human wealth that makes the $\overline{L/H}$ ratio close to 1. This means that domestic debt displaces foreign assets in agents' wealth at a rate of around one for one. If agents are net creditors, the replacement rate is even larger than 1. The reason is, that net creditors have a relatively smaller human capital, therefore the same amount of "insurance" of labor income that the tax system provides means more for them in relative variance terms.

The fact that the tax system has an insurance effect against income risks has been long known. Analyses started with *Chan* [1983] and *Barsky–Mankiw–Zeldes* [1986] *Kimball-Mankiw* [1989] calculated coefficients that can be compared with our results. Barsky et al. showed in a simple model that the insurance effect may be important and their results are much conform with our "close to 1" coefficient. On the other hand, Kimball-Mankiw when assuming infinitely living agents and random walk income arrived at a coefficient significantly smaller than 1. Part of the difference might come from the CARA utility function that they assume. However we believe that the main reason of the difference comes from the calibration criteria of the two calculations. Our calibration criterion for the parameters was that in addition to being in line with former research results or reasoning the financial asset outcome that they imply should be comparable to actual observed levels. Because of the high sensitivity of results, we could have chosen other parameters that do not contradict our a priori knowledge regarding *parameter values* but would lead to unreasonably extreme debt levels and consequently to a displacement ratio that is far from 1. Kimball-Mankiw started with a set of reasonable assumptions on the parameters themselves but did not calibrate those parameters to produce reasonable asset levels at the same time. This has led to a displacement ratio in the magnitude of 0.5, convincing enough to show that insurance is important but not as much to show that precautionary behavior and the tax system together may bring full displacement.

It is probable that similarly to precaution liquidity constraints are important in determining savings. Unfortunately our analytical method based on the Taylor-approximation is not suitable to handle liquidity constraints. *Ayiagari-McGratten* [1998] took both effects into account by explicitly solving a stochastic dynamic programming problem. In choosing the parameters they used a calibration principle similar to that of ours. They allowed parameters that reproduces the observed 0.66 level for D/y and the 4.5% level of r in the US. Unfortunately we could not infer on a displacement coefficient similar to ours, because their exercise assumes a closed economy and looks for the relation of the interest rate and public debt.

We know without calculations [7] that government debt has an effect of loosening liquidity constraints of agents by broadening their collateral basis. The effect of government debt through this channel has the same sign as the effect of income insurance of taxes incurred by the debt have. It would need further research to determine the relative weights of these two channels in the decisions of agents. To an empirically relevant answer we probably have to drop the assumption of uniform agents with homothetic utility function.[8]

2.3 Equity as an alternative asset

We do not undertake solving the maximization problem of an agent by two assets, bond and equity. However, we are able to make a simple extension.

Let us assume that there exists a choice of the agent about his equity stock and calculate the optimum financial asset with this constraint. If the level of equity holdings was part of the unconstrained optimum, the constrained solution to the financial asset is part of the unconstrained optimum as well. With this additional assumption on the level of equity holdings we may recalculate the model in order to demonstrate our conclusion in the first section, that the model is able to reproduce a reasonable level of financial asset holding with acceptable parameters.

The model is a simple extension of the Skinner-model. The detailed mathematical derivation is given in Appendix „C". The idea behind the model is that in equation (10) income is augmented by profits and the variance of lifetime wealth is a linear function of the variances of labor and equity income and their covariance.

For statistical properties of labor income we kept the description of equation (2) while the stock of equities increases proportionately with labor income, yielding a rate of profit π where $\pi = \overline{\pi} + \eta$, $\eta \sim N(0, \sigma_\eta^2 I)$ and $Corr(\eta, \varepsilon) = \sigma_{\varepsilon\eta}$

According to *Mehra-Prescott* [1985] and *Mankiw-Zeldes* [1991] the rate of equity income exceeds the riskfree interest rate by about 6 percentage points and its variance is about 16.7% in the US between 1890 and 1979 and 14.0 percentage point between 1948-1988. Correlation with consumption is 0.4.

The variance of aggregate income is 1.5-3.6 percent. If we accept the random walk assumption this means a relative variance for human capital of $0.025/(r-g) = 1$ which is much higher than the variance of equity income.

In our calculations we assumed $\sigma_\eta = 0.15$, $\sigma_\varepsilon = 0.025$, $\sigma_{\varepsilon\xi} = 0.4\,(0.15)\,(0.025) = 0.0015$ and an equity/income ratio of 2. Using these parameters together with β, g, r, γ assumed before and a habit parameter of 0.81 in equation (151) produces the same financial asset rate that we arrived at in the simpler model.

The conclusion is analogous to the previous one. There exists a plausible set of parameters that reproduces reasonable financial asset rates. Calculations again are sensitive to the choice of parameter values. If we assume habit to be less by a percentage point, we arrive at rates of indebtedness in the range of multiples of income. [9]

[7] See *Woodford* [1990] and *Ayiagari-McGratten* [1990].

[8] Ongoing research seems to follow this line. See for example *Carroll* [2000] and *Mankiw* [2000].

[9] This sensitivity is the small open economy form of the sensitivity of the interest rate that the applied habit model implies. See *Campbell-Cochrane* [1995] and *Campbell* [1996] for remedies on the

It would be an attractive idea to use the model for making inferences on the optimal share of equity financing of a current account deficit. Unfortunately our model is not suitable for such practical application because of several reasons:

(1) Large sensitivity on parameters that can be estimated by a wide margin of error anyway.

(2) The model explains only the position in riskless assets and considers equity holdings as exogenous.

(3) Even if we would model the decision of equity holdings, it would be an uninteresting exercise when profits and labor income are exogenous. In most cases foreign equity financing goes along with direct investments which bring know-how. The effect of this know-how import on income would be the relevant variable to explain when assessing the level of foreign equity financing. This would need endogenizing of income.

3 Simulation exercises and conclusions

3.1 Comparative illustrations of model behavior

We made calculations to determine the speed of convergence to the steady state. Calculations were run by using the *turnpike*-principle on a finite (1000 period) sample which is however long enough for numerical convergence. .[10] We defined a continuous function $f : \mathbb{R}^{1000} \to \mathbb{R}^{1000}$ which maps a convex and compact subset of $\mathbb{R}^{1000}$ into itself. We defined the f function to have the optimal solution of the Skinner-model as its fix point.

According to our experience starting from a suitable starting value for $\mathbf{k}_0$ the iteration $\mathbf{k}_{i+1} = f(\mathbf{k}_i)$ converged always to the fixed-point, i.e. to the optimal solution.

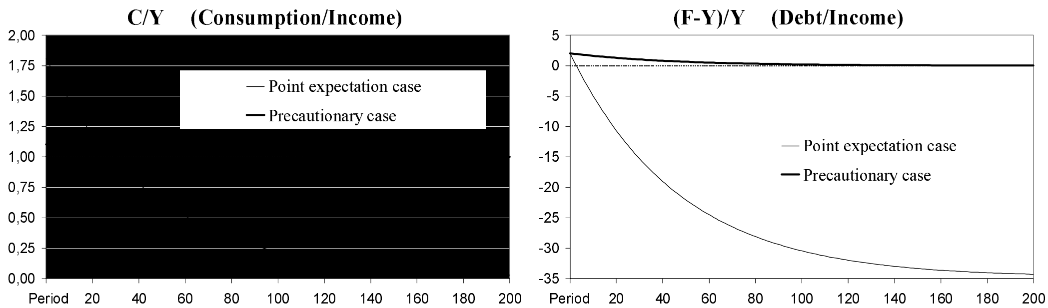


Figure 2: **Convergence towards steady state**

In Figure 2 we compared the convergence of the precautionary model with that of the point expectation model. Parameter values are taken from the baseline case of the

problem.

[10]The 1000 period was long enough to reach steady state and stay there before breaking out to reach the 0-asset end-condition.

Table, except that for the point expectation model we set $\beta = 0.97$. The change is purely cosmetic to avoid a steady decline of consumption. Otherwise the two models differ only in the variance of income, which is 0 in the point expectation model. In both cases the starting value of the financial asset (defined without current income): $(F_0 - y_0)/y_0 = 2$. For the calculation of the effect of habit we needed a starting value of consumption that we assumed to be: $c_0/y_0 = 1$.

The horizontal axis of the figure comprises 200 years. As we see the convergence is slow in both cases although in the precautionary case it is faster. The difference can be measured by the halving time, which is 34 years for the point expectation model and 31 years for the precautionary model.

The figure illustrates well the basic difference between the predictions of the two models that was stressed and demonstrated by *Carroll* [1992] and *Deaton* [1992] who solved the problem by dynamic optimization. While in the point expectation model the rate of increase in consumption is independent form the rate of increase in income, in the precautionary model people try to keep a constant c/y ratio in the long run. Smoothing out exists in the precautionary model as well, but smoothness means not a constant rate of increase but a gradual adjustment to income.

The same difference in behavior can be seen in the next figure where responses to shocks are demonstrated. The graph shows 150 years but simulations were run for 1000 years.

Let us assume that the growth rate of income is 2% .

Version A: Unexpected shift in the income path The starting position is steady state. In the 20th year the growth path is shifted unexpectedly up, but the previous growth rate remains.

In the *point expectation case* the consumption path shifts upwards. The shock improves the net wealth temporarily.

In the *precautionary case* the adjustment of consumption is gradual. The initial improvement of net wealth is followed by a transitory further improvement, as consumption shift lags behind the shift in income.

Version B: Expected shift in the path of income *Assumption.* The starting position is steady state. The upward shift of the income path in the 20th year is known from the 2nd year.

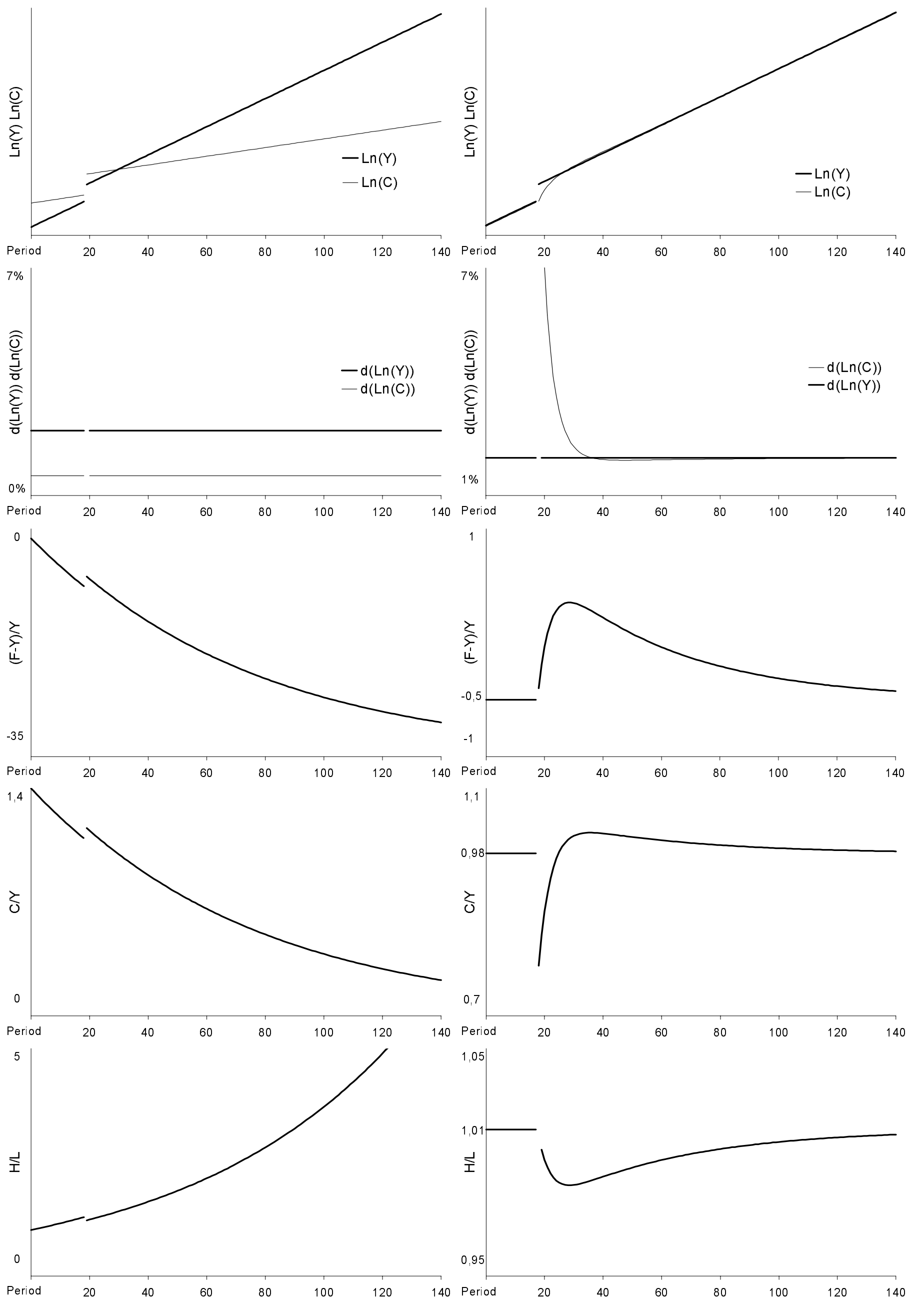


Figure 3: **Simulation results of Version A**

The point expectation case is on the left side, the precautionary case is on the right side.

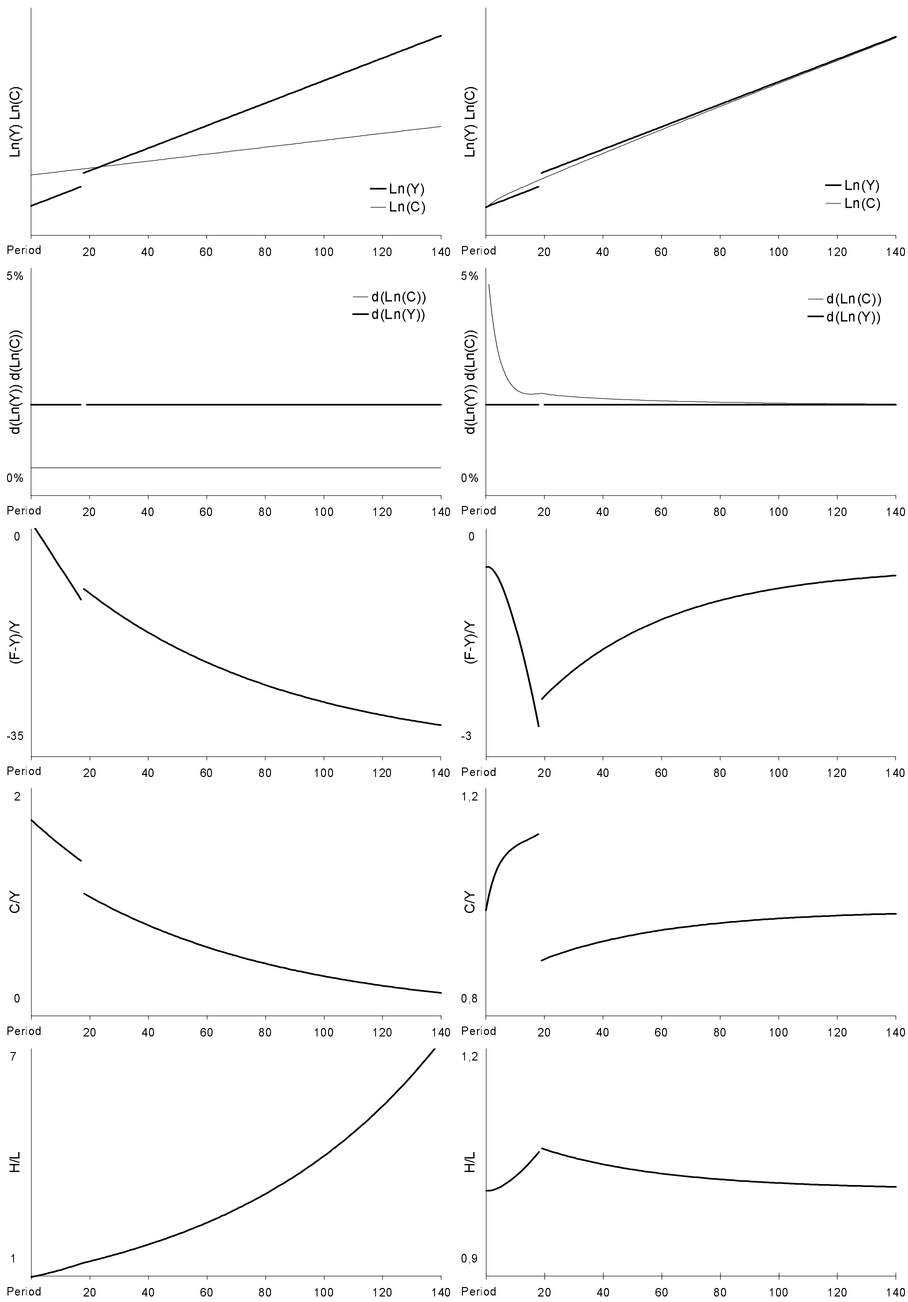


Figure 4: **Simulation results of Version B**

The point expectation case is on the left side, the precautionary case is on the right side.

In the point expectation case consumption shifts at the moment when information is available and stays on a constant growth path.

In the precautionary model the adjustment process is best seen in the second row where the rate of change is given. In the second year the rate of growth in consumption shoots up to adjust to the higher human capital that became known. The rate is approaching the steady state rate for a while but turns around when getting closer to the date when the actual shift in income will take place. The revival of the growth rate does not come from new information but from the already known fact that the higher income level is close and it will allow a higher level of consumption at an unchanged level of risk. The consumer is preparing for this new situation in advance in order to smooth out consumption, even though her risk, H/L is increasing (F/y goes down). When the income shift is over consumption approaches the path of income again and F/y and H/L gradually take up their steady state value.

3.1.1 Hypothetical scenarios

Even though we know that the sensitivity of the optimal asset ratio on the parameters of the model is too large to make any normative assessment for policy makers, it is tempting to compute a scenario for a country in a similar situation as Hungary was in the last 10 years and is expected to be in the next 20 years. The stylized facts and hypothetical prospects for this scenario are, that it starts with a recession (transition period) and follows with a fast growth period (catching-up) and growth gradually adjusts to the world average rate (mature country period).

During the recession and catching up intertemporal utility maximization suggests an accumulation of liabilities. The real picture is complicated by the fact that several types of liabilities exist but as we told earlier we cannot follow this richness of reality and have to resign from modelling optimal equity holdings. Considering this loss we can comfort us with the argument that because of the sensitivity problems of the model the calculated levels of indebtedness have no normative relevance anyway and if there are lessons to draw from the exercise they regard only to the dynamics of calculated trade deficits and not their levels.

Before transition Hungary grew slower than the world and it was still in debt, which means that the starting position before transition was not a steady state. To follow stylized Hungarian facts let us assume a country in a position where its steady state asset ratio would have been $F/y = 0.5$, but its actual was $F/y = -0.5$. We calibrated the parameters of the model to reproduce this situation.

In the second year "transition" starts. We assume – overstraining reality a bit – that the country was fully informed about the expected value of the future income path. For 5 years output decreased (we take actual Hungarian figures here). Then the period of catching-up started. We assume that catching-up takes place according to a logistic curve calculated in *Darvas–Simon* [2000] with peak growth rates of 5 percent and a smoothing into the world average growth rate of 2.3 percent after 25 years. [11]. We calculate the optimal consumption path according to these assumptions.

Note that income growth is exogenous. To prevent investments from complicating the model we can interpret income as the rest of output after subtracting necessary investments for the given growth path.

We calculate catching-up in three variants. In Version 1 we have all information about the expected path at the starting year in 1989 and the catching-up to the 70

[11] This would happen at the 70 percent level of Austrian GDP. See *Darvas–Simon* [1999].

percent level of Austria takes place by 2030. In Version 2 we assume that in 1998 we gain new information that the catching-up by 2030 will be 100 percent. In Version 3 catching-up will be 70 percent, but faster, than in Version 1.

In the calculations we made a refinement to the model assuming that the variance of income depends on the rate of growth. We assume that every percentage point of difference in the growth rate from the world average of 2.3% creates an additional variance of $(0.3)^2$.

In Figure 5. the columns show the scenarios:

– first column: catching-up by 2030 to 70 % of Austria (Baseline),

– second column: catching-up by 2030 to 100 % of Austria

– third column: catching-up faster to 70 % of Austria.

Actual data for output are shaded, but domestic expenditure data are calculated throughout the time period.

As it comes straightforwardly from theory, consumption follows income, but it does some smoothing out. Therefore, during the recession indebtedness increases but later on repayment of the debt follows.

It is remarkable that the repayment starts already during the catching-up period (3. row). This is intuitively understandable, because at the end of the process when catching-up is accomplished, the debt rate has to be 0. Indebtedness is justified only by high income in the *future*. Fast growth itself does not warrant high debt. On the contrary, the country has to borrow when present income is low but the future is bright, and repay the loan *during* the rise in the level of relative income, i.e. within the catching-up period.

The faster the growth rate the higher the necessary saving rate. This does not mean that consumption would not grow faster in this case (2. row), but income growth has to succeed consumption growth.

In the second and third scenario F/y swings up into a positive range before converging to 0. This is a consequence of our specific assumption about the dependence of risk on the rate of growth: fast growth requires a higher buffer stock.

The bottom two rows reflect developments in income and consumption.

The favorable news about growth (2nd and 3rd scenario) increase human capital. This allows

– higher consumption compared to the baseline (row 2),

– decreases risk through a larger buffer of human capital (row 5),

– increases assumed risk through fast growth.

The net effect of the last two factors lead to a higher accumulation of assets. This result is depends strongly on the assumed parameters.

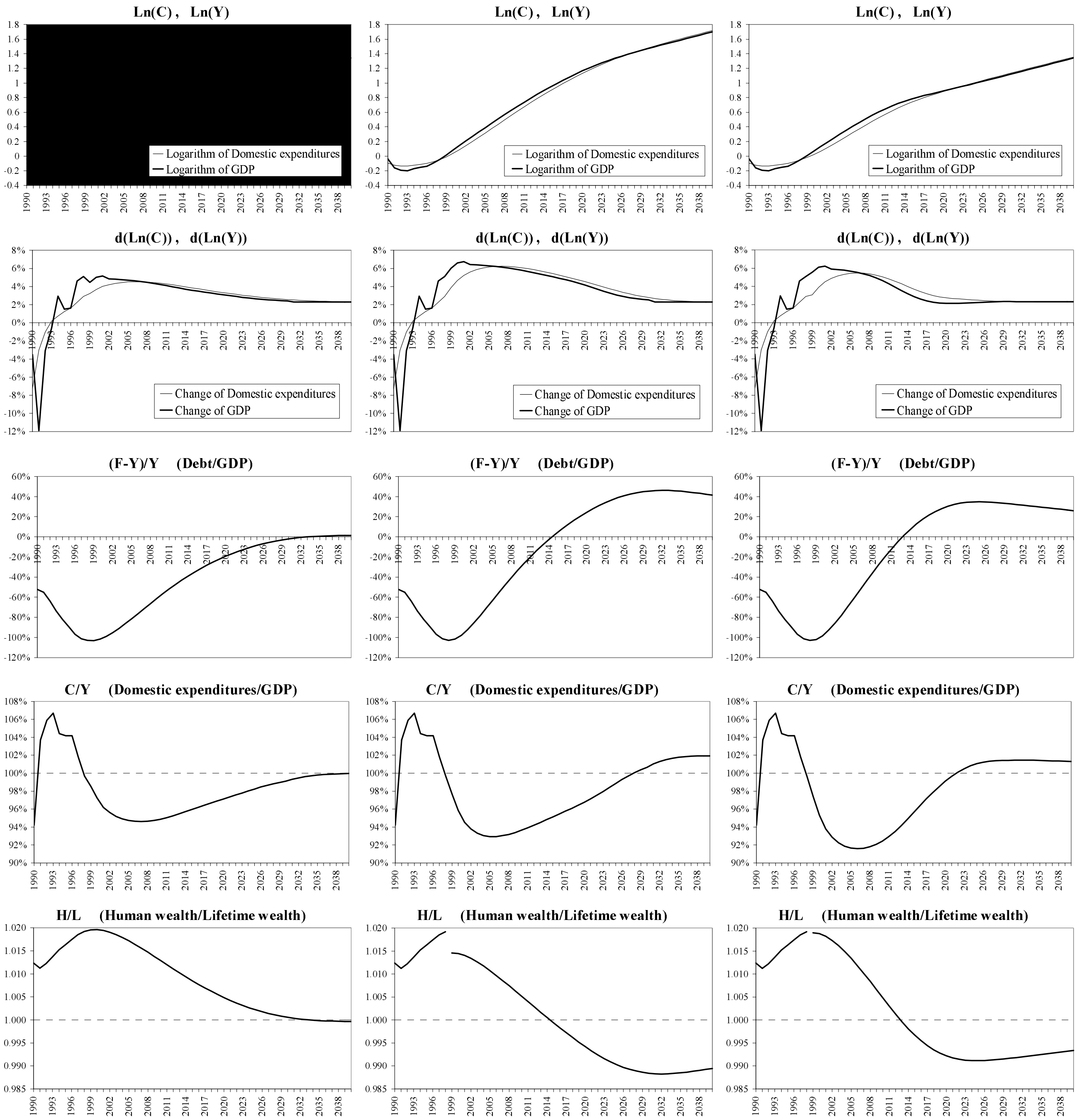


Figure 5: **Simulation of catch-up scenarios**

4 Conclusions

We have analyzed the saving behavior of a small open economy where infinitely-living agents have CRRA utility functions, idiosyncrasic labor income risks together with aggregate income shocks.

In this model the sum of the agents' decisions cannot be formulated as the solution to an aggregate optimization problem. However, an optimization problem of the social planner can be formulated and its solution can be enforced by an associated suitable fiscal policy. With an income-tax system this fiscal policy is very efficient in the sense that fiscal saving appears approximately one-for-one in aggregate saving.

The model can be calibrated with plausible parameters to reproduce aggregate debt ratios for countries that are in the range of actual ratios. This demonstrates that precautionary behavior alone can give an explanation to the relatively low levels of net foreign financial assets that are observed in contrast to the predictions of the point expectation model with infinite horizon. This does not mean that other determinants may not play important roles. Liquidity constraints both in the agents' problem and in the country-problem (arising from international law-enforcement difficulties) are presumably important. The answer how a full explanation based on precautionary behavior and a multiple explanation can be reconciled lies in the fact that the simulation results on the steady-state asset ratios are highly sensitive on the parameters. We could easily change parameters remaining within the range of reasonable values that produce results that give room for additional explanatory factors.

The model leaves open several related questions and puzzles. The high sensitivity of the results on the parameters might lead to highly volatile predictions. We did not undertake to investigate or refine if necessary the model considering its behavior from this point of view.

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Appendix

A Precautionary saving behavior. Review

It is generally accepted approach among economists to assume that consumers take their whole lifetime income into account when they decide on consumption. However, in most of the literature this behavior is analyzed in a framework where it is forgotten that consumers act as a risk-averse agent in a stochastic world. Neglecting this feature leads to predictions that are often out of line with facts. In this review we describe the how the model of the agent's consumption-saving behavior is enriched by incorporating risk and risk-aversion and show how this features lead to radically different predictions.

A.1 The point-expectation model

The intertemporal optimization model of the agent (household) in a deterministic world is based on the following assumptions:[12]

The household consumes one good. Every price is expressed in this good. There is only one financial asset which yields an interest rate. The value of this asset is called financial wealth, in contrast to human wealth.

The household maximizes an intertemporally additive separable utility function in a world of transaction-cost free unconstrained credit market:

$$U(c_0, c_1, c_2, ...) = \sum_{s=0}^{\infty} u_s(c_s), \tag{22}$$

where we assume that

$$u_s(c_s) = \beta^s u(c_s) \qquad u'(.) > 0 \text{ and } u'' < 0. \tag{23}$$

For future income and interest rates the agent expects a value that has no variance. Therefore it is convenient to call the model "point expectation model".

The maximization problem of the agent is the following:

$$\max_{\{c_s\}} \sum_{s=0}^{\infty} \beta^s u(c_s) \tag{24}$$

$$W_s = (W_{s-1} - c_{s-1})\,(1 + r_s^e) + y_s^e \qquad s = 1, 2, 3, \dots \quad , \tag{25}$$

where

c_s consumption in period s,

[12] Our description is a brief summary. For those who wish to see a more didactic and comprehensive survey we suggest *Muellbauer–Lattimore* [1995] and *Deaton* [1992].

$u(c_s)$ utility of c_s,

β time discount factor (if $\beta < 1$ the agent prefers today's consumption to next years' consumption),

r_s^e expected interest rate in period s,

y_s^e expected income in period s,

W_s financial wealth in period s after income has been realized but before consumption expenditure.

Because of the point expectation assumption we may simplify notation by deleting superscript ^{e} denoting expectations: $r_s^e = r_s$ és $y_s^e = y_s$. Let us assume the interest rate to be constant ($r_s = r$). We will keep this assumption when switching to the stochastic model. Its inclusion would not lead to important new insights.

Notice that in the maximization problem the intertemporal constraints (25) are formulated in a way financial wealth is defined for the point in time when time point when income has been realized but before consumption expenditure[13].

If we assume that debt does not grow faster than income (excluding Ponzi-games):

$$\lim_{s\to\infty} \frac{W_s}{(1+r_s)^s} = 0, \tag{26}$$

then (25) can be added up through continuous substitution:

$$\sum_{s=0}^{\infty} \frac{c_s}{(1+r_s)^s} = W_0 + \sum_{s=1}^{\infty} \frac{y_s}{(1+r_s)^s} \equiv W_0 + H_0 \equiv L_0, \tag{27}$$

where H is total *future* income accumulated in lifetime discounted by the interest rate and called human capital or human wealth. The sum of financial and human wealth is denoted by L. Thus (27) condition means that the present value of consumption is equal to total lifetime wealth.

With the inclusion of condition (27) the Lagrange-function of the maximization problem id the following:

$$\Lambda = \sum_{s=0}^{\infty} \beta^s u(c_s) - \lambda \left(\sum_{s=0}^{\infty} \frac{c_s}{(1+r)^s} - W_0 - \sum_{s=1}^{\infty} \frac{y_s}{(1+r)^s} \right), \tag{28}$$

and derivatives in respect to c_s give the first order conditions:

$$\beta^s u'(c_s) = \lambda \left(\frac{1}{1+r} \right)^s. \tag{29}$$

[13] There are two alternative interpretations of financial wealth in dinstinct time period models. One of them considers weath at the beginning of the period, the other after income has been realized but no interest is earned yet. In the dynamic programming formulation of the problem – like in *Skinner* [1989] the basis of our model – the latter results in more convenient expressions. Usually W and B are used to denote the latter and former interpretation respectively.

Similarly:

$$\beta^{s+1} u'(c_{s+1}) = \lambda \left(\frac{1}{1+r} \right)^{s+1}, \tag{30}$$

and dividing (30) by (29) we arrive at the ratio of the marginal utilities of neighboring periods (Euler-equation):

$$u'(c_s) = (1+r)\,\beta u'(c_{s+1}). \tag{31}$$

The result depends on the additivity of the intertemporal utility function. In this case the marginal rate of substitution depends only on the time preference rate. In the following we use a specific assumption for the utility function, the CRRA-function,[14]:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \qquad \text{if } \gamma > 0,\ \gamma \neq 1, \text{ and} \tag{32}$$

$$u(c) = \ln c, \qquad \text{if } \gamma = 1. \tag{33}$$

The function implies that the marginal utility of consumption is $c^{-\gamma}$, the intertemporal elasticity of substitution is constant and $1/\gamma$, and risk aversion γ.

Substituting $u'(c) = c^{-\gamma}$ for the Euler-equation (31) we get:

$$c_{s+1} = (1+r)^{1/\gamma} \beta^{1/\gamma} c_s. \tag{34}$$

Using the Euler-equation and the budget equation the optimal consumption c_0 can be expressed as a proportion of lifetime wealth.[15].

By substituting (34) for (27):

$$c_0 \sum_{s=0}^{\infty} \left(\frac{(1+r)^{1/\gamma} \beta^{1/\gamma}}{1+r} \right)^s = L_0. \tag{35}$$

The left hand side is a geometric series which is finite if $(1+r)^{1/\gamma} \beta^{1/\gamma} < 1+r$. Let us assume that this is fulfilled. If consumption grows according to equation (34) this means that the rate of consumption growth has to be below the rate of interest. Then:

$$c_0 = \frac{1}{\varphi} L_0 \equiv \frac{r+\vartheta}{1+r} L_0, \tag{36}$$

[14] CRRA: Constant Relative Risk Aversion

[15] From c_0 we can calculate consumption for each period by using equation (34).

where:

$$\varphi = \sum_{s=0}^{\infty} \left(\frac{(1+r)^{1/\gamma} \beta^{1/\gamma}}{1+r} \right)^{s} = \frac{1}{1 - \frac{(1+r)^{1/\gamma} \beta^{1/\gamma}}{1+r}} = \frac{1+r}{r+\vartheta}, \tag{37}$$

$$\vartheta = 1 - (1+r)^{1/\gamma} \beta^{1/\gamma}. \tag{38}$$

The slope of the consumption path depends on the rate of time preference (β), on the interest rate (r), and risk aversion (γ) according to the formula $(1+r)^{1/\gamma}\beta$. If the subjective discount rate equals to the interest rate, $(1+r)^{1/\gamma}\beta^{1/\gamma} = 1$, then $\vartheta = 0$ and consumption equals the revenue from total wealth, $rL/(1+r)$.

To analyze the implied steady state stock of saving we assume that income growth rate is expected to be constant and equal g. We retain the point expectation assumption.

Then human capital is the following:

$$H_t = y_t \sum_{s=1}^{\infty} \left(\frac{1+g}{1+r} \right)^{s} = y_t \frac{1+g}{r-g}, \tag{39}$$

assuming that $g < r$.

The $g < r$ assumption is needed for human wealth to be finite. Without this assumption consumption as a linear function os human wealth would became meaningless.

The steady state value of financial assets is determined from (25) by dividing both sides by y_{s-1}:

$$\frac{W_s}{y_{s-1}} = \left(\frac{W_{s-1}}{y_{s-1}} - \frac{c_{s-1}}{y_{s-1}} \right) (1+r) + \frac{y_s}{y_{s-1}} \tag{40}$$

$$\frac{W_s}{y_s} \frac{y_s}{y_{s-1}} = \left(\frac{W_{s-1}}{y_{s-1}} - \frac{c_{s-1}}{y_{s-1}} \right) (1+r) + \frac{y_s}{y_{s-1}} \tag{41}$$

$$(1+g) \frac{W_s}{y_s} = (1+r) \frac{W_{s-1}}{y_{s-1}} - (1+r) \frac{c_{s-1}}{y_{s-1}} + (1+g). \tag{42}$$

Substituting (36) in (42) for c_{s-1} and using (39):

$$(1+g) \frac{W_s}{y_s} = (1+r) \frac{W_{s-1}}{y_{s-1}} - (1+r) \frac{\frac{r+\vartheta}{1+r} (W_{s-1} + H_{s-1})}{y_{s-1}} + (1+g)$$

$$\frac{W_s}{y_s} = \frac{1+r}{1+g}\frac{W_{s-1}}{y_{s-1}} - \frac{1+r}{1+g}\frac{\frac{r+\vartheta}{1+r}\left(W_{s-1}+H_{s-1}\right)}{y_{s-1}} + 1$$

$$\frac{W_s}{y_s} = \frac{1+r}{1+g}\frac{W_{s-1}}{y_{s-1}} - \frac{1+r}{1+g}\frac{r+\vartheta}{1+r}\frac{W_{s-1}}{y_{s-1}} - \frac{r+\vartheta}{1+g}\frac{H_{s-1}}{y_{s-1}} + 1$$

$$\frac{W_s}{y_s} = \frac{1-\vartheta}{1+g}\frac{W_{s-1}}{y_{s-1}} - \frac{r+\vartheta}{r-g} + 1$$

$$\frac{W_s}{y_s} = \frac{1-\vartheta}{1+g}\frac{W_{s-1}}{y_{s-1}} - \frac{\vartheta+g}{r-g}. \tag{43}$$

The steady-state financial asset ratio:

$$\overline{W/y} = -\frac{1+g}{r-g} = -\frac{H}{y}. \tag{44}$$

From (44) we can express the steady state consumption ratio:

$$(1+g)\overline{W/y} = \left(\overline{W/y} - \overline{c/y}\right)(1+r) + (1+g) \tag{45}$$

$$\overline{c/y} = \frac{\overline{W/y}(r-g)+1+g}{1+r} = \frac{-\frac{1+g}{r-g}(r-g)+1+g}{1+r} = 0. \tag{46}$$

A.1.1 Some unfavorable features

In this model the path of consumption is independent from the path of income. The level of consumption depends on total wealth and current income determines consumption only as part of total wealth and a possible indicator of future income.

This feature contrasts empirical findings. In fact agents increase consumption not only as a response to new information on their future income but as a response to a previously expected increase in income as well. In fast-growing countries consumption grows together with income even though fast growth is not a surprise from year to year. Similarly, consumers in fast-growing Japan adjusted their intertemporal consumption patterns within a lifetime followed actual income.

The infinite horizon version of the model has features that need heroic efforts to interpret meaningfully:

1. If the constellation of parameters produces incidentally that $(1+r)^{1/\gamma}\beta^{1/\gamma} = 1+g$, then consumption growth rate equals income growth rate and from equation (43) W/y is indeterminate.

2. If $(1+r)^{1/\gamma}\beta^{1/\gamma} > 1+g$, then W/y becomes unstable, i.e. the accumulation of financial wealth goes to infinity: $\lim_{t\to\infty} W/y = \infty$.

3. If $(1+r)^{1/\gamma}\beta^{1/\gamma} < 1+g$, financial wealth converges, $\lim_{t\to\infty} W/y = \overline{W/y}$, but the rate of indebtedness becomes implausibly high (20-30 times income).

A.1.2 Overlapping generations

The unfavorable features of 1-3. disappear if we assume intertemporally disconnected generations. Most of the macroeconomic models consider the model developed by *Blanchard* [1985], *Buiter* [1989], *Weil* [1989] as a basis of their simulation exercises[16] In these models the stability of the steady state financial asset ratio is assured by the new generations starting with 0 assets. In this case by assuming appropriate birth and death rates both infinite accumulation of assets and excessive accumulation of debt can be prevented by the phase out of the old and the entering of new generations. This way the model is able to imitate observed aggregate behavior. The effectiveness of fiscal policy is ensured too as policy is able to reallocate incomes between disconnected generations.

This way, from a pragmatic point of view the overlapping generations model seems to be a useful tool in simulating aggregate behavior. However, we have to see that behind the good simulation the microeconomic foundations are wrong. Firstly, to create meaningful simulations in the aggregate in practical applications the Blanchard–Buiter–Weil model is difficult to parameterize [17] meaningfully. Secondly, even if we construct a life-cycle income path that is both reasonable and leads to desirable macroeconomic features[18], the example of the Japanese consumer who followed income in his lifetime allocation of consumption remains a strong counterexample and the doubt that we calibrate a wrong model to facts remains.

A.2 Introducing precaution

Let us resolve the assumption on point expectations.

In the modified model we still consider the interest rate as constant but income has a variance.

Maximum utility is understood as an expected value. Therefore, problem (24)-(25) modifies to the following:

$$\max_{\{c_s\}} E_0 \left[\sum_{s=0}^{\infty} \beta^s u\left(c_s\right) \right] \tag{47}$$

$$W_s = \left(W_{s-1} - c_{s-1}\right)\left(1+r\right) + y_s \qquad s = 1, 2, 3, \ldots \quad , \tag{48}$$

where E_0 means expectations according to informations of the time in period 0 when income is already known. In the following we discard the suffix $_0$ from the notation

[16] Multimod (*Laxton* et al. [1998]), the Bank of Canada (*Black et al.*[1994]) and New-Zealand model (*Black* et al. [1997]), the Bank of Finland model (*Willman* et al. [1998]) are models of this sort.

[17] In the Bank of Canada model (*Black et al.*[1994]) for example a death rate of 4 percent together with a birth rate of 4 percent has to be assumed. Of course the model builders realize the problem and interpret it as a convenient way of describing the "myopic" behavior of consumers. In our opinion the most convenient way of simulating aggregate behavior is to find a model that explains this myopic nature. This is exactly what the precautionary saving model is for.

[18] As it is claimed in *Laxton* et al. [1998].

and we put the suffix indication time of information only if we want to stress it or if it is different from 0.

In this model y_s *is a random variable.* This means that in the budget equation the income constraint of the consumer is not one value but a distribution function. Therefore $u(c_s)$ is a random variable too.

Analogously to equation (34) the first order condition(Euler-equation):

$$u'(c_0) = (1+r)\,\beta E\left[u'(c_1)\right]. \tag{49}$$

The problem can be traced back to the point expectation model if $E[u'(c)] = u'(E[c])$. This is true only if $u'(c)$ is linear. Such a $u(.)$ function is easy to construct, for example the quadratic function $u(c) = bc - c^2/2$. In this case the expected value gives the *certainty equivalence.* Introducing uncertainty in this form may be convenient in discussing some problems but it escapes the question what happens when the optimum point of expected utility differs from the point chosen in a riskless world. In addition because of its implausible features the quadratic utility function inappropriately describes consumer behavior[19].

In the following we apply the CRRA-function that was used earlier. Here $E[c^{-\gamma}] > [E(c)]^{-\gamma}$, which means that the marginal utility of the expected value of consumption is smaller than the expected value of its marginal utility. This means that in equation (49) we cannot replace $E[u'(c_1)]$ with $u'(E[c_1])$ but rather with $(1+v_1)u'(E[c_1])$ where $1+v_1$ $\{v_1 > 0\}$ depends on the risk and risk aversion.

The Euler-equation becomes:

$$c_0^{-\gamma} = (1+r)\,\beta(1+v_1)\left(E[c_1]\right)^{-\gamma}, \tag{50}$$

or

$$E[c_1] = (1+r)^{1/\gamma}\beta^{1/\gamma}(1+v_1)^{1/\gamma}c_0. \tag{51}$$

We can see that with the same r and β parameters the rate of increase in consumption becomes higher. The intuitive meaning of this is that the consumer is "precautious" and saves more. We can see from equation (51) too, that the certainty equivalent consumption – future certain consumption that renders the same marginal utility as expected marginal utility – is $Ec_{t+1}/(1+v_{t+1})^{1/\gamma}$.

In the following we determine the value of v. The approach of *Skinner* [1988], *Kimball* [1990] that is surveyed in *Muellbauer–Lattimore* [1995], uses a second-order Taylor expansion approximation[20]. We follow their line. For two periods the derivation is simple and we reproduce it. For the general case we refer to *Skinner* [1988].

[19] For example it attributes positive marginal utility to negative consumption.

[20] *Lattimore* [1993] considers higher-order expansions as well, calculating consequences of higher-order momentums of the distribution function as well.

A.2.1 Certainty equivalent consumption for two periods

Let us assume that the world exists for two periods, 0 and 1. The budget equation of the consumer:

$$W_1 = (W_0 - c_0)(1+r) + y_1. \tag{52}$$

In equation (52) y_1 is a random variable with known expected value and variance. The Euler-equation for the case of CRRA utility:

$$c_0^{-\gamma} = (1+r)\beta E\left[W_1^{-\gamma}\right], \tag{53}$$

where we used that in period 1 – the last period – all wealth is consumed ($c_1 = W_1$).

For determining the certainty equivalence of future wealth W_1^* let expand $W_1^{-\gamma}$ around $E[W_1]$, using second-order Taylor approximation[21], which means that we use only the variance and expected value of W_1[22]:

$$\begin{aligned} W_1^{-\gamma} \;\approx\; & (E[W_1])^{-\gamma} - \gamma (E[W_1])^{-(1+\gamma)} (W_1 - E[W_1]) + \\ & + \frac{\gamma(1+\gamma)}{2} (E[W_1])^{-(2+\gamma)} (W_1 - E[W_1])^2 . \end{aligned} \tag{54}$$

Taking expected values the second term becomes 0:

$$E\left[W_1^{-\gamma}\right] \approx (E[W_1])^{-\gamma} + \frac{\gamma(1+\gamma)}{2} (E[W_1])^{-(2+\gamma)} E\left[(W_1 - E[W_1])^2\right]. \tag{55}$$

We used that the expected value of the second term in brackets is 0. On the right side the first term of the Taylor series shows the marginal utility of the expected value of consumption that is smaller than the expected value of marginal utility, the whole right side of the equation. The higher order terms of the polinom give the difference[23]. Reformulating it as a proportional relation:

$$E\left[W_1^{-\gamma}\right] \approx (E[W_1])^{-\gamma} \left(1 + \frac{\gamma(1+\gamma)}{2} \sigma_{W_1}^2\right) \tag{56}$$

[21] For memo: the Taylor expansion around $E[y]$:

$$f(y) = f(E[y]) + f'(E[y])(y - E[y]) + f''(E[y])\frac{(y - E[y])^2}{2} + o^2(y),$$

where $o^2(y)$ second order function.

[22] We assume of course that income has a finite expectation and variance.

[23] We can check that the rest of the terms of the quadratic utility function are 0, that is why the expected value of the marginal utility is equal to the marginal utility of the expected value in this case.

$$E\left[W_1^{-\gamma}\right] \approx \left(E\left[W_1\right]\right)^{-\gamma}\left(1+v_1\right). \tag{57}$$

Thus the certainty equivalence to wealth in period 1:

$$W_1^* = \frac{E\left[W_1\right]}{\left(1+v_1\right)^{1/\gamma}}, \tag{58}$$

where $v_1 = \frac{\gamma(1+\gamma)}{2}\sigma_W^2$ and σ_W^2 is relative variance of wealth.

In the numerator of σ_W^2 it is the variance of income while in the denominator it is wealth. The intuition behind is clear: the higher is income variance the more cautious the consumer has to be while its wealth hedges him against the risk that this variance means.

A.2.2 Certainty equivalent: general case

Skinner [1988] derived the value of v for the case of n periods and arrived to results analogous to the two-period case. The formulation of his results for an infinite horizon is straightforward.

Let L be lifetime wealth:

$$L_t = W_t + E_t\left[\sum_{s=t+1}^{\infty}\frac{y_s}{\left(1+r\right)^{s-t}}\right]. \tag{59}$$

If autocorrelation of income is 0, for the optimal path of consumption *Skinner* [1988] arrived at the following differential equation[24]:

$$c_t = \left[\left(1+r\right)\beta\left(1+v_t\right)\right]^{1/\gamma}\frac{L_t}{E_{t-1}\left[L_t\right]}c_{t-1}, \tag{60}$$

where:

$$v_t = \frac{\gamma\left(1+\gamma\right)}{2}\mu_t^2\sigma_{\varepsilon t}^2, \tag{61}$$

$$\mu_t = \frac{E_{t-1}[y_t]}{E_{t-1}\left[L_t\right]}, \tag{62}$$

and $\sigma_{\varepsilon t}^2$ is the relative variance of y_t income. If annual income flows are independent, then μ_t may be interpreted as the relative effect of a unit income shock on total wealth. This effect in general would depend on the ratio of y_t to wealth and the persistence of

[24]The 0 autocorrelation assumption makes mathematics simpler but the results can be generalized as we will see later.

the shock. This interpretation generalizes the relation of v and risk for the case when the income flows are not independent.

The intuition is the following: for any ARMA y_t variable with known σ_ε^2 variance, there exists a corresponding white noise process which generates the same risk for wealth. Let assume for example that income is a random walk with drift[25]:

$$y_t = (1+g)y_{t-1}\varepsilon_t \qquad \ln(\varepsilon_t) \sim N(0, \sigma_\varepsilon^2 I), \tag{63}$$

where σ_ε^2 is known. Then the total effect of ε_t on wealth:

$$(1+g)y_{t-1}\varepsilon_t + \frac{(1+g)^2 y_{t-1}\varepsilon_t}{(1+r)} + \frac{(1+g)^3 y_{t-1}\varepsilon_t}{(1+r)^2} + \dots \quad , \tag{64}$$

which is a finite value if ha $g < r$:

$$(1+g)\, y_{t-1}\frac{1+r}{r-g}\varepsilon_t. \tag{65}$$

This means, that the white noise process that creates the same risk for wealth is the following:

$$y_t = \overline{y}_t + (1+g)y_{t-1}\frac{1+r}{r-g}\varepsilon_t, \tag{66}$$

where $\overline{y}_t = (1+g)^t y_0$. However, the variance of this white noise process is not σ_ε^2 any more, but $(1+g)^2 y_{t-1}^2 \left[(1+r)/(r-g)\right]^2 \sigma_\varepsilon^2$. The relative variance of this process will be constant, $\left[(1+r)/(r-g)\right]^2 \sigma_\varepsilon^2$. Thus if income is generated by an AR(1)process, then in the (61) formula of v_t the variance term has to be replaced by the variance of the white noise process that is equivalent from the aspect of generating the same risk on wealth.

This justifies the interpretation of μ; by augmenting the formula of μ with the term that provides the transition from the general process to the white noise process – in the case of AR(1) this was $\left[(1+r)/(r-g)\right]^2$ – we arrive in stead of (62) to the following formula[26]:

$$\mu_t = \frac{E_{t-1}[y_t \frac{1+r}{r-g}]}{E_{t-1}\left[L_t\right]}. \tag{67}$$

The $\mu^2\sigma_\varepsilon^2$ term means the relative variance of lifetime wealth.

[25] The idea is applicable to any other ARMA process, the random walk assumption makes only the formulation somewhat simpler.

[26] Naturally, in (61) $\sigma_{\varepsilon_t}^2$ has to be replaced by the variance of the original process σ_ε^2. See equation (63).

From equation (39) μ can be defined in terms of human wealth:

$$\mu_t = \frac{E_{t-1}[H_t \frac{1+r}{1+g}]}{E_{t-1}\left[L_t\right]}. \tag{68}$$

As we see, the path of consumption is described well by equation (51) if expectations on future income would not change from period to period. In fact the consumer has to adjust to new and new surprises. .

Similarly to the point expectation model, the optimal consumption level can be determined as a fraction of lifetime wealth.

$$c_t = \left[\sum_{j=t}^{\infty}(1+r)^{t-j}\prod_{s=t+1}^{j}\left(\frac{(1+r)(1+v_s)}{(1+\beta)}\right)^{1/\gamma}\right]^{-1} L_t \tag{69}$$

For first glance it might seem that not much has changed. Consumption is proportional to wealth, the only difference is that because of the term $(1+v_t)$ this proportion is lower. In other words the difference is the same as if β would be higher. In fact there is another important difference: v_t is not a constant but it depends on lifetime wealth. Let us see the consequences.

In the point expectation model the rate of growth of income and consumption may differ in the long run. As we have seen when discussing features 1.–3. (page26), the impatient (relatively to income growth) consumer has a lower consumption growth rate than income growth rate (because it starts from a high level of consumption from loans) while the relatively patient consumer accumulates wealth which allows a higher growth rate of consumption. Those who have $(1+r)^{1/\gamma}\beta^{1/\gamma} < 1+g$ will be the debtors and those who have $(1+r)^{1/\gamma}\beta^{1/\gamma} > 1+g$ will be the creditors. This model leads to extreme solutions in the infinite horizon case. Debtors get indebted to the level of their human wealth and creditors will increase their wealth until they crowd out debtors on the market and drive down interest rates to create the homogenous world where $(1+r)^{1/\gamma}\beta^{1/\gamma} = 1+g$ and the level of financial assets is indeterminate.

Introducing risk and precautionary behavior creates a new world where a stable steady state of the financial asset/income ratio is achieved by every agent already before the world would become homogeneous. For this world to come about we have to discard the possibility that $(1+r)^{1/\gamma}\beta^{1/\gamma} > 1+g$ might be true and look for lenders and borrowers within those agents who all have $(1+r)^{1/\gamma}\beta^{1/\gamma} < 1+g$. This is always possible by assuming β to be lower than previously assumed. The value of β is anyway a parameter that has not much empirical foundation and its value is usually derived from the model used. When *Carroll* [1992] proposed the higher time discount rate for parameterizing the precautionary saving model he could already refer to *Friedman*[1957] who was not constrained by the infinite horizon point expectation model and was thinking in terms of 30 percent.

If we accept this assumed new constraint, it is easy to see that risk and risk aversion lead to a steady state level of W/y that might be both positive or negative depending on the parameters of the idiosyncratic income stream or utility function of the agent.

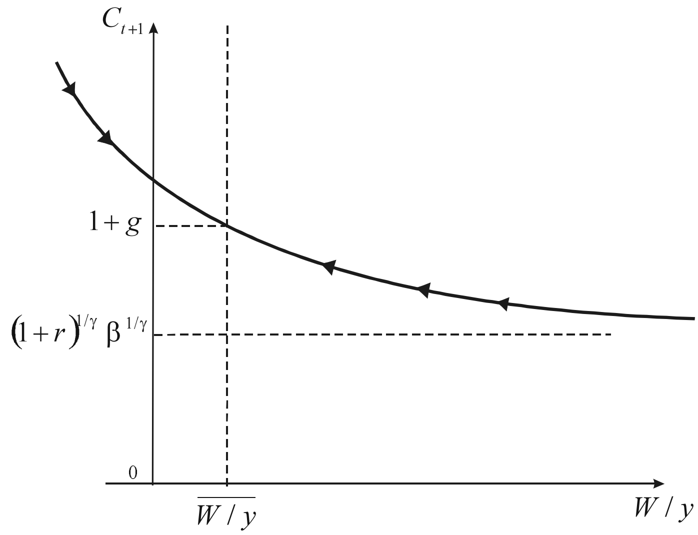
Specifically, the assumption in the stochastic case is that the agent is always impatient enough that the growth rate of consumption be less than that of income in the long run:

$$(1+r)^{1/\gamma}\beta^{1/\gamma}(1+v)^{1/\gamma} \leq 1+g. \tag{70}$$

The strong inequality cannot hold because it would mean that c/y approaches 0. This happens in the point expectation model, but the precautious consumer would not allow it because it would mean an infinite risk (relative variance of income available for consumption). The precautious consumer determines a W/y ratio that creates an equilibrium between expected value of consumption and risk. If a surprise brings W/y lower than targeted, then risk and v becomes larger and according to equation (51) c_t decreases. However, a drop in c_t brings about saving and the equilibrium of W/y is restored. In the long run both W and c grow proportionally to y at a rate of g:

$$(1+r)^{1/\gamma}\beta^{1/\gamma}(1+v)^{1/\gamma} = 1+g. \tag{71}$$

On Figure 6. we can see $E(c_{t+1})/c_t$ as a function of W/y. If W/y is smaller than the equilibrium value, than risk is too large, present (t) consumption is less to accumulate W . Accumulated W cuts risk and consumption can grow decreasing $E(c_{t+1})/c_t$.



If W/y goes to infinity than $\nu \to 0$ and the growth rate of consuption is $(1+r)^{1/\gamma}\beta^{1/\gamma}$.

Figure 6: **The adjustment of consumption to income**

Let us give more rigor to the intuitive notion of equilibrium mentioned above.

Definition 1 *The (60)-(62) stochastic path of optimal consumption is in equilibrium if* $W_{t-1}/y_{t-1} = E_{t-1}[W_t]/E_{t-1}[y_t]$ *és* $c_{t-1}/y_{t-1} = E_{t-1}[c_t]/E_{t-1}[y_t]$.[27]

Let us assume that income is a random walk with drift:

$$y_s = (1+g)y_{s-1}\varepsilon_s \qquad \ln(\varepsilon_s) \sim N\left(0, \sigma_\varepsilon^2 I\right), \tag{72}$$

where I is the unit matrix. Then instead of (62) we may use equation (67). As $E_{t-1}[y_t] = (1+g)y_{t-1}$, because of the definition of equilibrium $E_{t-1}[W_t] = (1+g)W_{t-1}$ and $E_{t-1}[c_t] = (1+g)c_{t-1}$ has to be true as well, and it is easy to see that $E_{t-1}[H_t] = (1+g)H_{t-1}$.

Let us take the expected value of c_t based on information in $t-1$ according to equation (60) where on the right hand side L_t is the only random variable.

$$E_{t-1}[c_t] = E_{t-1}\left[\left[(1+r)\beta(1+v_t)\right]^{1/\gamma}\frac{L_t}{E_{t-1}[L_t]}c_{t-1}\right] \tag{73}$$

$$(1+g)c_{t-1} = \left[(1+r)\beta(1+v_t)\right]^{1/\gamma}\frac{E_{t-1}[L_t]}{E_{t-1}[L_t]}c_{t-1} \tag{74}$$

$$(1+g)c_{t-1} = \left[(1+r)\beta(1+v_t)\right]^{1/\gamma}c_{t-1} \tag{75}$$

$$1+g = \left[(1+r)\beta(1+v_t)\right]^{1/\gamma}. \tag{76}$$

Write (67) in detail, using the definition on equilibrium:

$$\begin{aligned}
\mu_t &= \frac{E_{t-1}\left[y_t\frac{1+r}{r-g}\right]}{E_{t-1}[L_t]} = \frac{E_{t-1}\left[y_t\frac{1+r}{r-g}\right]}{E_{t-1}[W_t+H_t]} = \\
&= \frac{E_{t-1}\left[y_t\frac{1+r}{r-g}\right]}{E_{t-1}[W_t]+E_{t-1}[H_t]} = \frac{(1+g)y_{t-1}(1+r)/(r-g)}{(1+g)W_{t-1}+(1+g)H_{t-1}} = \\
&= \frac{y_{t-1}(1+r)/(r-g)}{W_{t-1}+H_{t-1}} = \frac{y_{t-1}(1+r)/(r-g)}{W_{t-1}+y_{t-1}(1+g)/(r-g)}.
\end{aligned} \tag{77}$$

The equation system that gives the equilibrium does not contain random variable:

[27]Notice that we define the ratio of expected values and not the expected value of ratios.

$$1+g=\left[(1+r)\,\beta\,(1+v_t)\right]^{1/\gamma} \tag{78}$$

$$v_t=\frac{\gamma\,(1+\gamma)}{2}\mu_t^2\sigma_{\varepsilon_t}^2 \tag{79}$$

$$\mu_t=\frac{y_{t-1}(1+r)/(r-g)}{W_{t-1}+y_{t-1}(1+g)/(r-g)}. \tag{80}$$

Express (78) to v_t, equation (79) to μ_t, and equation (80) to W_{t-1}/y_{t-1}:

$$v_t=\frac{(1+g)^{\gamma}}{(1+r)\,\beta}-1 \tag{81}$$

$$\mu_t=\frac{1}{\sigma_{\varepsilon}}\sqrt{\frac{2v_t}{\gamma\,(1+\gamma)}} \tag{82}$$

$$\frac{W_{t-1}}{y_{t-1}}=\frac{\mu(1+g)-(1+r)}{\mu\,(g-r)}. \tag{83}$$

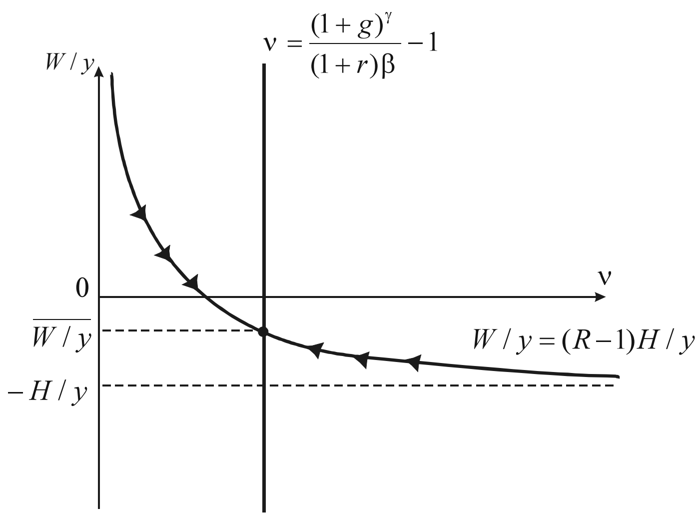
As in equilibrium W_t/y_t is constant for every t therefore v_t and μ_t are constant for every period and we can discard the t subscript. Substituting v from (81) into (82) and the generated μ into (83) we get the steady state value of W_{t-1}/y_{t-1}, $\overline{W/y}$ is the following:

$$\overline{W/y}=\frac{W_{t-1}}{y_{t-1}}=\sigma_{\varepsilon}\frac{1+r}{r-g}\sqrt{\frac{\gamma\,(1+\gamma)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}}-\frac{1+g}{r-g} \tag{84}$$

$$\overline{W/y}=\sigma_{\varepsilon}\frac{1+r}{1+g}H/y\sqrt{\frac{\gamma\,(1+\gamma)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}}-H/y. \tag{85}$$

Because $r>g$ the upper limit on indebtedness is human wealth but it is reached only if $\sigma_{\varepsilon}=0$.

Knowing the steady state value of W_{t-1}/y_{t-1} we may determine the steady state of c_{t-1}/y_{t-1}. Recall equation (52):



If indebtedness approaches human wealth then the risk becomes infinite. The risk disappears if the reserve goes to infinity. In the figure $R = \sigma_{\varepsilon}\frac{1+r}{1+g}\sqrt{\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^{\gamma}}{(1+r)\beta}-1\right)}}$.

Figure 7: **Steady state indebtedness ratio as a function of risk.**

$$W_t = (W_{t-1} - c_{t-1})(1+r) + y_t. \tag{86}$$

Take $E_{t-1}[.]$ expectations when W_{t-1} and c_{t-1} are already known, non-random:

$$E_{t-1}[W_t] = (W_{t-1} - c_{t-1})(1+r) + E_{t-1}[y_t]. \tag{87}$$

Divide (87) by y_{t-1} and restructure the left hand side:

$$\frac{E_{t-1}[W_t]}{E_{t-1}[y_t]}\frac{E_{t-1}[y_t]}{y_{t-1}} = \left(\frac{W_{t-1}}{y_{t-1}} - \frac{c_{t-1}}{y_{t-1}}\right)(1+r) + \frac{E_{t-1}[y_t]}{y_{t-1}}. \tag{88}$$

Considering that in our case $E_{t-1}[y_t]/y_{t-1} = 1+g$ and in equilibrium $W_{t-1}/y_{t-1} = E_{t-1}[W_t]/E_{t-1}[y_t] = \overline{W/y}$, we get:

$$(1+g)\overline{W/y} = \left(\overline{W/y} - \frac{c_{t-1}}{y_{t-1}}\right)(1+r) + (1+g), \tag{89}$$

and $\overline{c/y}$ the equilibrium value of c_{t-1}/y_{t-1} id the following:

$$\overline{c/y} = \frac{c_{t-1}}{y_{t-1}} = \overline{W/y}\frac{r-g}{1+r} + \frac{1+g}{1+r}. \tag{90}$$

The interpretation of this formula becomes easier if we introduce human wealth into the formula:

$$\overline{c/y} = \frac{W_{t-1}}{H_{t-1}\frac{r-g}{1+g}}\frac{r-g}{1+r} + \frac{1+g}{1+r}. \tag{91}$$

Simplifying and using that $W/H + 1 = L/H$ we receive:

$$\overline{c/y} = \frac{1+g}{1+r}\overline{L/H}. \tag{92}$$

A.3 Refinement: habit persistence

Let us introduce habit persistence into the model in a most simple way. Let us assume that the period utility function is the following:

$$u_s(c_s) = \beta^s \frac{(c_s - h_s)^{1-\gamma}}{1-\gamma}, \qquad \text{if } \gamma > 0 \text{ and } \gamma \neq 1, \tag{93}$$

$$u_s(c_s) = \beta^s \ln(c_s - h_s), \qquad \text{if } \gamma = 1. \tag{94}$$

where h is habit.

For the definition of habit we assume that consumption cannot drop below a portion of previous years' consumption[28]: $h_s = \rho c_{s-1}$. Marginal utility of consumption in t :

$$\frac{\partial u(c_t, c_{t-1})}{\partial c_t} = (c_t - \rho c_{t-1})^{-\gamma}, \tag{95}$$

where ρ is the rate of persistence in consumer behavior. For a household this parameter may be interpreted as the level of consumption that the household would never allow to happen whatever high the expected value of a risky saving strategy. For a country it may mean the minimum level that society would tolerate without revolts or the level where society would be inflicted with unacceptable damages. In the parameterization we assumed this level to be 80 percent. This means that a more than 20 percent drop of aggregate income within a year is assumed to be unacceptable (in the long run, any rate of decrease is possible of course)

The introduction of habit persistence modifies the model somewhat.

It is useful to define income, consumption, and wealth "above habit".

[28]This approach of introducing habit persistence has unfavorable side effects that might be important but probably do not hurt the message of our analysis. (See *Campbell-Cochrane* [1995]) for a proposed modification)

$$\begin{aligned}
\widetilde{c}_s &= c_s - \rho c_{s-1}, && (96)\\
\widetilde{y}_s &= y_s - \rho y_{s-1}, && (97)\\
\widetilde{W}_s &= W_s - \rho W_{s-1}. && (98)
\end{aligned}$$

Then analogously to (47)-(48) the maximization problem of the agent becomes:

$$\max_{\{\widetilde{c}_s\}} E_0 \left[\sum_{s=0}^{\infty} \beta^s u\left(\widetilde{c}_s\right) \right] \tag{99}$$

$$W_s = \left(W_{s-1} - c_{s-1}\right)(1+r) + y_s \qquad s = 1, 2, 3, \ldots \ . \tag{100}$$

Notice that in the objective function we have $\widetilde{c}_s$ while in the intertemporal budget constraint c_s. Let us restructure (100) by replacing c_s with terms that contain only $\widetilde{c}_s$. By subsequent substitution we have

$$c_s = \rho^{s+1} c_{-1} + \sum_{i=0}^{s} \rho^{s-i} \widetilde{c}_i. \tag{101}$$

Using (101) we can write the budget constraint analogously to (27) in the following way:

$$\sum_{s=0}^{\infty} \frac{\rho^{s+1} c_{-1} + \sum_{i=0}^{s} \rho^{s-i} \widetilde{c}_i}{(1+r)^s} = W_0 + \sum_{s=1}^{\infty} \frac{y_s}{(1+r)^s}. \tag{102}$$

The Lagrange function based on (99), (102):

$$\Lambda = E_0 \left[\sum_{s=0}^{\infty} \beta^s u(\widetilde{c}_s) \right] - \lambda \left(\sum_{s=0}^{\infty} \frac{\rho^{s+1} c_{-1} + \sum_{i=0}^{s} \rho^{s-i} \widetilde{c}_i}{(1+r)^s} - W_0 - \sum_{s=1}^{\infty} \frac{y_s}{(1+r)^s} \right), \tag{103}$$

differentiating(103) with respect to $\widetilde{c}_s$:

$$\beta^s E_0 \left[u'(\widetilde{c}_s) \right] = -\lambda \sum_{i=s}^{\infty} \frac{\rho^{i-s}}{(1+r)^i}, \tag{104}$$

and similarly (103) with respect to $\widetilde{c}_{s+1}$:

$$\beta^{s+1} E_0 \left[u'(\widetilde{c}_{s+1}) \right] = -\lambda \sum_{i=s+1}^{\infty} \frac{\rho^{i-(s+1)}}{(1+r)^i}. \tag{105}$$

Considering that

$$\sum_{i=s}^{\infty} \frac{\rho^{i-s}}{(1+r)^{i}} = \frac{1}{(1+r)^{s-1}(1+r-\rho)} \tag{106}$$

$$\sum_{i=s+1}^{\infty} \frac{\rho^{i-(s+1)}}{(1+r)^{i}} = \frac{1}{(1+r)^{s}(1+r-\rho)}, \tag{107}$$

the Euler equation from (104)-(105):

$$u'(\widetilde{c}_s) = (1+r)\beta E\left[u'(\widetilde{c}_{s+1})\right], \tag{108}$$

which is analogous to the former Euler equations except that it is written for the above habit consumption. We get further analogies if in equation (96) we substitute the expressions

$$c_{s-1} = \frac{y_s - W_s}{1+r} + W_{s-1} \tag{109}$$

$$c_s = \frac{y_{s+1} - W_{s+1}}{1+r} + W_s \tag{110}$$

derived from (100):

$$\widetilde{c}_s = \left(\frac{y_{s+1} - W_{s+1}}{1+r} + W_s\right) - \rho\left(\frac{y_s - W_s}{1+r} + W_{s-1}\right). \tag{111}$$

Restructuring:

$$\widetilde{c}_s = \frac{(y_{s+1} - \rho y_s) - (W_{s+1} - \rho W_s)}{1+r} + (W_s - \rho W_{s-1}). \tag{112}$$

Using notations of (96)-(98) we see that the same equations hold for the above habit values as for the original values:

$$\widetilde{c}_s = \frac{\widetilde{y}_{s+1} - \widetilde{W}_{s+1}}{1+r} - \widetilde{W}_s, \tag{113}$$

rewriting into the form analogous to (100):

$$\widetilde{W}_s = \left(\widetilde{W}_{s-1} - \widetilde{c}_{s-1}\right)(1+r) + \widetilde{y}_s. \tag{114}$$

Because of the formulae in (108) and (114) the solution is analogous too[29]:

$$\widetilde{c}_t = \left[(1+r)\,\beta\,(1+\widetilde{v}_t)\right]^{1/\gamma} \frac{\widetilde{L}_t}{E_{t-1}\left[\widetilde{L}_t\right]}\widetilde{c}_{t-1}, \tag{115}$$

where

$$\widetilde{v}_t = \frac{\gamma\,(1+\gamma)}{2}\widetilde{\mu}_t^2\sigma_{\widetilde{y}_t}^2, \tag{116}$$

$$\widetilde{\mu}_t = \frac{E_{t-1}[\widetilde{y}_t]}{E_{t-1}\left[\widetilde{L}_t\right]}. \tag{117}$$

Because of the analogy to the habit free model it is informative to stress the differences:

(1) In solution (115)-(117) there are everywhere above habit values[30]

(2) In equation (116) instead of the relative variance of income we get the relative variance of above habit variance. If the relative variance of total income is σ_ε^2, then the relative variance of above habit income is $\sigma_{\widetilde{y}}^2 = \sigma_\varepsilon^2/(1-\rho)^2$.

We can formulate the steady state conditions in the same way as in the habit-free model. We have to recall that in steady state growth rates of above habit variables are equal to the growth rates of the gross values of variables. As the derivation is exactly the same as in (73)-(85) we give only the steady state value of $\overline{W/y}$:

$$\overline{W/y} = \frac{\sigma_\varepsilon}{1-\rho}\frac{1+r}{1+g}\frac{H}{y}\sqrt{\frac{\gamma\,(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta}-1\right)}} - H/y. \tag{118}$$

The formula for the steady state consumption does not change compared with (90):

$$\overline{c/y} = \frac{1+g}{1+r}\overline{L/H}. \tag{119}$$

A.4 Some features

A.4.1 The Taylor-approximation

The Taylor-approximation has the advantage that the calculations are simpler than an explicit solution of the dynamic programming problem. It might be considered

[29] Formulae (115)-(117) change analogously as well is we assume that log of income is a unit root process with drift. Then in (116) $\sigma_{\widetilde{y}}^2$ is replaced by $\left(\frac{\sigma_\varepsilon}{1-\rho}\right)^2$ where σ_ε is relative variance of income and the numerator of (117) changes to $E_{t-1}\left[\widetilde{y}_t\frac{1+r}{r-g}\right]$.

[30] The above habit lifetime wealth is defined as $\widetilde{L}_t = L_t - \rho L_{t-1}$.

an advantage too that in the Taylor-approximation only the expected value and the variance of the utility function matter. This allows to get around the problem arising from the infiniteness of utility at 0 consumption. However this advantage may turn into a disadvantage if we want to extend the model to take liquidity constraints into account.

In the explicit approach we have to assume an income band where the probability that income is outside the band is 0. Credit constraints have to be defined correspondingly. Otherwise consumption could take 0 with non-zero probability. These assumptions however allow to run exercises with changing liquidity constraints that the Taylor approximation is not suitable for. *Zeldes* [1989b], *Deaton* [1992], *Carroll [1992], Ayiagari* [1994], *Ayiagari-McGratten* [1998], and others made calculations of this sort.

A.4.2 Comparison with the model of Obstfeld–Rogoff

Obstfeld-Rogoff[31] starts from an optimum-horizon point expectation model of a social planner and arrives to indebtedness levels in the range 15-20 times GDP. To arrive at ratios closer to observed facts they endogenize credit constrains referring to the difficulties in international law enforcement. In this model debtors have an interest of raising debts to the above cosmic levels but they are constrained by the willingness of creditors who are afraid of default. In this model the transaction bears risks but creditors are the only risk takers.

The model is probably a good approximation of the way of thinking of debtors before the debt crises in the 1980-ies. Since then it has turned out that debtors take risks as well. This is reflected in the precautionary model. In our model it has been shown that this risk might be large enough to explain in itself the relative moderate levels of indebtedness. This lesson is probably useful nowadays as well when the transition and catching-up of the Central and East European countries raises again the question of optimal indebtedness.

B Individual and aggregate risks

Let us assume that labor income of a country is a random walk with drift:

$$\breve{y}_s = (1+g)\breve{y}_{s-1}\varepsilon_s \qquad\qquad \ln(\varepsilon_s) \sim N(0, \sigma_\varepsilon^2 I), \tag{120}$$

where $\breve{y}_s$ is permanent per capita (dynasty) income .

In addition to the aggregate shock each dynasty is hit by transitory shocks. Let us denote the ratio of income/permanent income as q and assume that it is generated by the following process:

$$q_s = (q_{s-1})^{\alpha}\,\xi_s \qquad 0 < \alpha < 1 \qquad \ln(\xi_s) \sim N(0, \sigma_\xi^2 I). \tag{121}$$

[31] *Obstfeld-Rogoff* [1995a] Appendix 2A and section 6.2. .

Assume that ε_s and ξ_s are independent: $\mathrm{Cov}[\varepsilon_s, \xi_s] = 0$. Individual income is determined the following way:

$$y_s = q_s \breve{y}_s = (q_{s-1})^{\alpha} \xi_s (1+g) \breve{y}_{s-1} \varepsilon_s. \tag{122}$$

The individual maximizes utility constrained by the budget:

$$\max_{\{c_s\}} E_0 \left[\sum_{s=0}^{\infty} \beta^s u(c_s) \right] \tag{123}$$

$$W_s = (W_{s-1} - c_{s-1})(1+r) + y_s \qquad s = 1, 2, 3, \ldots \ . \tag{124}$$

The solution procedure is similar to the baseline case only the risk effects on human wealth have to be determined. Let us formulate human wealth the following way:

$$H_{s-1} = \sum_{i=0}^{\infty} \frac{y_{s+i}(\varepsilon_s, \eta_s)}{(1+r)^i}. \tag{125}$$

As

$$y_{s+i}(\varepsilon_s, \eta_s) = q_{s+i}(\xi_s) \breve{y}_{s+i}(\varepsilon_s) = (q_{s-1})^{\alpha^{i+1}} \xi_s^{\alpha^i} (1+g)^{i+1} \breve{y}_{s-1} \varepsilon_s, \tag{126}$$

we get by substitution:

$$\begin{aligned} H_{s-1} &= \sum_{i=0}^{\infty} \frac{(q_{s-1})^{\alpha^{i+1}} \xi_s^{\alpha^i} (1+g)^{i+1} y_{s-1} \varepsilon_s}{(1+r)^i} \\ &= \sum_{i=0}^{\infty} \frac{\left[(q_{s-1})^{\alpha} (1+g)\right]^{i+1} y_{s-1} \varepsilon_s \xi_s^{\alpha^i}}{(1+r)^i}. \end{aligned} \tag{127}$$

Expanding $\varepsilon \xi^b$ into first order Taylor series around $\varepsilon = \xi = 1$:

$$\begin{aligned} \varepsilon\xi^b &\approx \left. \varepsilon\xi^b \right|_{\varepsilon=\xi=1} + \left. \frac{\partial \varepsilon\xi^b}{\partial \varepsilon} \right|_{\varepsilon=\xi=1} (\varepsilon - 1) + \left. \frac{\partial \varepsilon\xi^b}{\partial \xi} \right|_{\varepsilon=\xi=1} (\xi - 1) \\ \varepsilon\xi^b &\approx 1 + (\varepsilon - 1) + b(\xi - 1). \end{aligned} \tag{128}$$

As we are interested in the effect of shocks on wealth only we may drop constant terms in (128):

$$\mathrm{Var}\left[\varepsilon\xi^b\right] \approx \mathrm{Var}\left[1 + (\varepsilon - 1) + b(\xi - 1)\right] = \mathrm{Var}\left[\varepsilon + b\xi\right]. \tag{129}$$

Using (129) we reformulate (127):

$$
\begin{aligned}
H_{s-1} &= \sum_{i=0}^{\infty} \frac{\left[(q_{s-1})^{\alpha}(1+g)\right]^{i+1} y_{s-1}\left[\varepsilon_s+\alpha^i \xi_s\right]}{(1+r)^i} \qquad (130)\\
&= (q_{s-1})^{\alpha}(1+g) y_{s-1}\left(\frac{1+r}{1+r-(q_{s-1})^{\alpha}(1+g)} \varepsilon_s+\frac{1+r}{1+r-\alpha(q_{s-1})^{\alpha}(1+g)} \xi_s\right).
\end{aligned}
$$

The first term is expected income $E_{s-1}[y_s] = (q_{s-1})^{\alpha}(1+g)y_{s-1}$. Introducing notation $\Gamma = \frac{1+r}{1+r-(q_{s-1})^{\alpha}(1+g)}$ and $\Delta = \frac{1+r}{1+r-\alpha(q_{s-1})^{\alpha}(1+g)}$ the risk generated by ε_s and η_s can be written shortly:

$$
\operatorname{Var}(H_{s-1}) = E_{s-1}[y_s]^2\left(\Gamma^2 \operatorname{Var}[\varepsilon_s]+\Delta^2 \operatorname{Var}[\xi_s]+2\Gamma\Delta \operatorname{Cov}[\varepsilon_s, \xi_s]\right). \qquad (131)
$$

As we assumed $\operatorname{Cov}[\varepsilon_s, \xi_s] = 0$,

$$
\operatorname{Var}(H_{s-1}) = E_{s-1}[y_s]^2\left(\Gamma^2 \operatorname{Var}[\varepsilon_s]+\Delta^2 \operatorname{Var}[\xi_s]\right). \qquad (132)
$$

As we see relative variance is constant $\breve{\sigma} = \Gamma^2\operatorname{Var}[\varepsilon_s] + \Delta^2\operatorname{Var}[\xi_s]$[32].

C Introducing equities

There are two sorts of financial assets, bonds and equities. Bonds pay a fixed interest, equities pay a stochastic profit. We assume that residents continuously invest into domestic equities to keep the ratio of equities to labor income k constant. Foreign capital markets are closed to residents. Foreigners may have a share in domestic equities but this does not change the model analytically. If they do, k has to be interpreted as a (total capital-foreigner's capital)/labor ratio.

We assume that profits are always at a rate that make residents willing to keep ky stock of equities and then we calculate that how much bond holding is consistent with this choice.

A maximization problem is formally the same independently whether the agent is a social planner or a household. [33]:

$$
\max_{\{c_s\}} E_0\left[\sum_{s=0}^{\infty} \beta^s u(c_s)\right] \qquad (133)
$$

[32] It is easy to see that if $h_{s-1} = 1$, $\operatorname{Var}[\xi_s] = 0$, $\operatorname{Cov}[\varepsilon_s, \xi_s] = 0$, then we receive the variance in the baseline model:

$$
(1+g)^2 y_{s-1}^2\left(\frac{1+r}{r-g}\right)^2 \operatorname{Var}[\varepsilon_s].
$$

[33] The difference lies in the parameters.

$$F_s = (F_{s-1} - c_{s-1})(1 + r_s) + y_s - i_s + s_s \qquad s = 1, 2, 3, \dots \quad , \tag{134}$$

where y_s is labor income and $y_s = (1+g)y_{s-1}\varepsilon_s,\ \ln(\varepsilon_s) \sim N(0, \sigma_\varepsilon^2 I)$
c_s consumption
s_s profit income
i_s net investment into equities.

Let capital stock be K_s that pays a rate of profit $\pi_s = \overline{\pi} + \eta_s$, where $\eta_{s,} \sim N(0, \sigma_\pi^2 I)$ and $Corr(\pi_s, \varepsilon_s) = \sigma_{\varepsilon\pi}$. Thus profit income $s_s = \pi_s K_s$. Assume that the production function is constant and capital/labor income ratio $k = K_s/y_s$ is constant. As labor income is a unit root process with g rate of growth, K_s has to be a similar process. Assume that investments in period s are chosen to fulfil: $K_{s+1}/E_s[y_{s+1}] = k$. As $E_s[y_{s+1}] = (1+g)y_s$ and $K_{s+1} = K_s + i_s$, therefore [34]

$$i_s = k(1+g)y_s - K_s. \tag{135}$$

From (134) using the assumptions:

$$F_s = (F_{s-1} - c_{s-1})(1 + r_s) + y_s - [k(1+g)y_s - K_s] + \pi_s K_s \qquad s = 1, 2, 3, \dots \quad . \tag{136}$$

Let us define:

$$\begin{aligned}
H_s &= y_s \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i = y_s \frac{1+g}{r-g}, \\
I_s &= i_s \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i = i_s \frac{1+g}{r-g}, \\
\Pi_s &= \pi_s K_s \sum_{i=1}^{\infty} \left(\frac{1+g}{1+r}\right)^i = \pi_s K_s \frac{1+g}{r-g},
\end{aligned}$$

where the sums are human wealth, investment liabilities and capital (equity) wealth. Total wealth L_s is the following:

$$L_s = F_s + H_s - I_s + \Pi_s. \tag{137}$$

Let us see the effect of ε_t and η_s on total wealth by components.

On labor income:

[34] We assume that investments take place after y_s has been realized.

$$
\begin{aligned}
& (1+g)y_{s-1}\varepsilon_s + \frac{(1+g)^2 y_{s-1}\varepsilon_s}{(1+r)} + \frac{(1+g)^3 y_{s-1}\varepsilon_s}{(1+r)^2} + \ldots && (138)\\
= \ & \frac{1+r}{r-g}(1+g)\, y_{s-1}\varepsilon_s = H_{s-1}\varepsilon_s && (139)
\end{aligned}
$$

On investments:
Using (135):

$$
i_s = k(1+g)^2 y_{s-1}\varepsilon_s - K_s, \tag{140}
$$

which means that:

$$
K_{s+1} = K_s + i_s = K_s + k(1+g)^2 y_{s-1}\varepsilon_s - K_s = k(1+g)^2 y_{s-1}\varepsilon_s, \tag{141}
$$

$$
i_{s+1} = k(1+g)^3 y_{s-1}\varepsilon_s - k(1+g)^2 y_{s-1}\varepsilon_s, \tag{142}
$$

$$
K_{s+2} = K_{s+1} + i_{s+1} = k(1+g)^3 y_{s-1}\varepsilon_s, \tag{143}
$$

$$
i_{s+2} = k(1+g)^4 y_{s-1}\varepsilon_s - k(1+g)^3 y_{s-1}\varepsilon_s. \tag{144}
$$

Following up it means that the effect of ε_s on the present value of all future investments:

$$
\begin{aligned}
& k(1+g)^2 y_{s-1}\varepsilon_s - K_s + \frac{k(1+g)^3 y_{s-1}\varepsilon_s - k(1+g)^2 y_{s-1}\varepsilon_s}{1+r} + \\
& + \frac{k(1+g)^4 y_{s-1}\varepsilon_s - k(1+g)^3 y_{s-1}\varepsilon_s}{(1+r)^2} + \ldots \\
= \ & rk\,(1+g)^2 y_{s-1}\varepsilon_s \frac{1}{r-g} - K_s = rk(1+g)\varepsilon_s H_{s-1} - K_s. && (145)
\end{aligned}
$$

Capital income:
Let substitute (140) into the expression of the present value of capital income $\pi_s K_s + \frac{\overline{\pi} K_{s+1}}{1+r} + \frac{\overline{\pi} K_{s+2}}{(1+r)^2} + \ldots$ and use equation $K_{s+1} = K_s + i_s$:

$$
\begin{aligned}
& \pi_s K_s + \frac{\overline{\pi} k(1+g)^2 y_{s-1}\varepsilon_s}{1+r} + \frac{\overline{\pi} k(1+g)^3 y_{s-1}\varepsilon_s}{(1+r)^2} + \ldots \\
= \ & \pi_s K_s + \overline{\pi} k\,(1+g)^2 y_{s-1}\varepsilon_s \frac{1}{r-g} = \pi_s K_s + \overline{\pi} k\,(1+g)\, H_{s-1}\varepsilon_s && (146)
\end{aligned}
$$

Summing up (138)-(145)-(146) the effect of the two random terms ε_s and η_s (using $\pi = \overline{\pi} + \eta$):

$$(1+r) H_{s-1}\varepsilon_s - [rk(1+g) H_{s-1}\varepsilon_s - K_{s-1}] + (\overline{\pi}+\eta) K_s + \overline{\pi}k(1+g) H_{s-1}\varepsilon_s. \quad (147)$$

Simplifying:

$$[(1+r) + (\overline{\pi} - r) k (1+g)] H_{s-1}\varepsilon_s + K_{s-1} + \pi_s K_s. \quad (148)$$

Let[35] $\Psi_s = [(1+r) + (\pi - r) k (1+g)] H_{s-1}$, then the variance of lifetime wealth:

$$\mathrm{Var}[L_s] = \Psi_s^2 \mathrm{Var}[\varepsilon_s] + K_s^2 \mathrm{Var}[\pi_s] + 2\Psi_s K_s \mathrm{Cov}[\varepsilon_s, \pi_s]. \quad (149)$$

The (10) formula for the steady state financial asset ration modifies to the following:

$$F/y = \left(\mathrm{Var}(L) \frac{1+r}{1+g} \sqrt{\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta} - 1\right)}} - 1 \right) (H/y + \Pi/y - I/y), \quad (150)$$

where $\mathrm{Var}(L) = [\Psi^2 \mathrm{Var}(\varepsilon) + K_s^2 \mathrm{Var}(\pi) + 2\Psi K \mathrm{Cov}(\varepsilon, \pi)]$. By substitution:

$$F/y = \left(\mathrm{Var}(L) \frac{1+r}{1+g} \sqrt{\frac{\gamma(1+\gamma)}{2\left(\frac{(1+g)^\gamma}{(1+r)\beta} - 1\right)}} - 1 \right) (1 + k(\overline{\pi} - g)) H/y. \quad (151)$$

Risk parameters of equities do not depend on whether we consider a social planner or a household because it is reasonable to assume that equity income risks are diversificable. In our model this diversification is only within the country, which means that risks are country-specific.

[35] As we see if $k = 0$ we get the baseline model.

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