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2008/1

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January 2008

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MNB Working Papers 2008/1

Are the exchange rates of EMU candidate countries anchored by their expected euro locking rates?

(Lehorgonyozza-e az EMU-ba belépő országok deviza árfolyamát a várt konverziós ráta?)

Written by: Anna Naszódi*

Published by the Magyar Nemzeti Bank

Szabadság tér 8–9, H–1850 Budapest

http://www.mnb.hu

ISSN 1585 5600 (online)

* Magyar Nemzeti Bank, Budapest, Hungary, Central European University, Budapest, Hungary, email: naszodia@mnb.hu, cphnaa01@phd.ceu.hu.
The views expressed are those of the authors and do not necessarily reflect the official view of the Magyar Nemzeti Bank. This disclaimer is particularly important in the case of the future euro locking rate of the Hungarian forint. The filtered market expectations concerning the euro locking rate do not necessarily coincide with the preferred euro locking rate of the Magyar Nemzeti Bank.
A previous version of this paper received the Olga Radzyner Award from the National Bank of Austria in year 2006.
The author gratefully acknowledges comments and suggestions from Péter Benczúr, Giuseppe Bertola, Attila Csajbók, Balázs Égert, András Fülöp, Christian Gourieroux, Csilla Horváth, Tamás Kollányi, Péter Kondor, István Kónya, László Mátyás, Ádám Reiff, Jean-Marc Robin and Ákos Valentinyi.

Contents

Abstract

This paper tests whether the exchange rates of the Czech koruna, the Hungarian forint, and the Polish złoty were anchored by market expectations concerning their euro locking rates. First, the process of the exchange rate is derived as a function of the following factors: (i) latent exchange rate, (ii) market expectations concerning locking rate, (iii) market expectations concerning locking date. Then, the locking dates and rates are filtered from historical exchange rates, currency option prices and yield curves. The main finding of the paper is that the relatively stable market expectations concerning the locking rates have substantially stabilized the three analyzed exchange rates.

JEL: F31, F36, G13.

Keywords: Monetary union, eurozone entry, factor model, Kalman filter, exchange rate stabilization, asset-pricing exchange rate model.

Összefoglaló

A tanulmány azt teszteli, hogy vajon lehorgonyozza-e a cseh korona, a magyar forint és a lengyel zlotyi árfolyamát az euro konverziós rátára vonatkozó piaci várakozás. Először levezetem az árfolyam folyamatát a következő három faktor függvényeként: (i) látens árfolyam, (ii) a konverziós rátára vonatkozó piaci várakozás, (iii) a végső konverzió idejére vonatkozó piaci várakozás. Majd historikus árfolyam, deviza opció és hozam görbe adatokból filterezem a konverziós rátára és a konverzió idejére vonatkozó várakozásokat. A tanulmány legfontosabb eredménye, hogy a konverziós rátára vonatkozó várakozások stabilitása jelentősen csökkentette a három vizsgált árfolyam volatilitását.

JEL-kód: F31, F36, G13.

Kulcsszavak: Monetáris unió, eurozóna-csatlakozás, faktor modell, Kálmán filter, árfolyam stabilizáció, eszköz-árazási árfolyam modell.

Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?

Anna Naszodi[1]

1 Introduction

This paper investigates the stabilizing feature of the market expectations concerning the euro locking rate. I apply the analysis to three countries, Czech Republic, Hungary and Poland, which are expected to join the Economic and Monetary Union (EMU) in a couple of years.

First, I extend the conventional asset-pricing exchange rate model with the final locking assumption. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks. Similarly, in the model with final locking the exchange rate is a linear combination of the fundamental and the market expectations concerning the euro locking rate. The relative weights of the linear combination depend on the market expectations concerning the locking date. The asset-pricing model with final locking is a three-factor [2] model, where the factors are the market expectations concerning the locking rate and date. The third factor is the fundamental, which drives the exchange rates even if no locking is expected in the future. Therefore, I will refer to the fundamental as the latent exchange rate, i.e. the exchange rate that would prevail if the currency was never going to be locked against the euro.

Then, in the empirical part of the paper, I filter out the three factors from the historical exchange rate, interest rate and currency option data. I estimate the expected date of locking from the euro and domestic yield curves by following the method suggested by Bates (1999). [3]

The other two factors are filtered out from the historical exchange rate by the Kalman filter technique. The Kalman filter decomposes the exchange rate changes into changes in the remaining two factors by utilizing the identification through the variances. To estimate the time-varying variances of the innovations of the factors, I use a theoretical option pricing model derived in the paper and cross-sectional data on option prices with different maturities. The identification of the time-varying variances is based on the followings. Option prices both with long and short maturities are functions of the variances of innovations of the latent exchange

[1] Magyar Nemzeti Bank, Budapest, Hungary Central European University, Budapest, Hungary
email: naszodia@mnb.hu, cphnaa01@phd.ceu.hu

[2] The stabilizing effect of locking is analyzed in a similar, but simpler model by De Grauwe et al. (1999/a,b) and by Wilfling and Maennig (2001). Both papers demonstrate the impact of locking with two-factor models. The factors in De Grauwe et al. (1999/a,b) are the fundamental and locking rate and the announced locking date is assumed to be credible. Whereas the factors of Wilfling and Maennig (2001) are the fundamental and locking date and no uncertainty is assumed concerning the locking rate. Therefore, the theoretical contribution of this paper to the literature is that all three sources of uncertainties are taken into account, moreover, the comovements between the factors are modeled as well.

[3] The yield curve based EMU probability calculator approach has been implemented inter alia by Lund (1999) and Favero et al. (2000). An alternative of the yield curve based EMU probability calculator is the currency options based calculator suggested by Driessen and Perotti (2004).

rate and the market expectation concerning the locking rate. However, the long end of the option term structure is influenced more by the variance of the market expectation concerning the locking rate, than the short end.

By investigating the filtered market expectation concerning the euro locking rate I make inferences on the exchange rate stabilizing effect of the locking. I find that the market expectation concerning the euro locking rate is less volatile than either the historical exchange rate or the filtered latent exchange rate. This result has the important implication that the prospect of locking has a stabilizing effect on the exchange rate in all three analyzed countries. Moreover, this stabilizing effect is substantial despite that locking in these countries will take place in the relatively far future and the locking date is highly uncertain yet.

Rather than filtering market expectations, I could, theoretically, have obtained the market expectations from an alternative source (Reuters polls) or with an alternative method (estimating equilibrium exchange rates). However, filtering has some advantages over its alternatives, for the reasons outlined below.

Reuters regularly[4] surveys the expectations of analysts concerning the dates of EMU and ERM II entries and concerning the central parities in ERM II of all three analyzed countries. The central parity expected by respondents may be considered as the market expectations concerning the final conversion rate. And the reported expectations concerning either the date of EMU or ERM II entries may be considered as the market expectations concerning the locking date. Yet extracting market expectations from daily historical exchange rate and interest rate data may yield more accurate and more up-to-date information than the monthly or quarterly Reuters polls. Moreover, the higher frequency of the filtered expectations enables us to investigate the stabilizing effect of the prospect of locking has on the exchange rate.

As the EMU candidate countries aim at having their irrevocable conversion rates set equal to their equilibrium exchange rates, reliable estimates on the latter also reflect market expectations concerning the final conversion rate. In the context of our research this concept poses, however, at least three kinds of problems.[5] First, economists use a number of different concepts to define and a number of different methods[6] to estimate the equilibrium real exchange rate. Second, these estimates refer to the real rather than the nominal exchange rate. Third, market expectations might deviate from the estimated nominal equilibrium exchange rate, especially if the choice of the final conversion rate is based not only on observable fundamentals, but also on unobservables. For instance, if not only economic, but also political considerations do play a role in the negotiations over the locking rates. Another example is, when the locking rate is determined by a backward looking manner, i.e. by a kind of average of past exchange rates. [7] In that case, speculative price pressure[8] can push away the exchange rate from its equilibrium level and the bubble will never burst due the locking.

The novelty of this paper is that it filters the subjective market expectation concerning the final conversion rate from exchange rate, interest rate and currency option data, which should

[4]The frequency of these surveys are monthly for Hungary and quarterly for Czech Republic and Poland.

[5]Egert, Halpern and MacDonald (2006) survey a number of issues related to the equilibrium exchange rates of transition economies. They conclude that "deriving a precise figure for the equilibrium real exchange rates in general and also for the transition economies is close to mission impossible as there is a great deal of model uncertainty related to the theoretical background and to the set of fundamentals chosen."

[6]Williamson (1994) gives an overview of the widely used FEER, BEER, NATREX methods.

[7]This rule for setting the conversion rate is better known as the Lamfalussy rule, named after the former president of the European Monetary Institute.

[8]The personal view of the author is that the chances for the analyzed currencies to be subject to speculative exchange rate pressure before they enter the ERM II or even in the ERM II system have increased by experiencing the Slovakian ERM II band shift on 16th of March 2007.

mirror all the information that the market use at forming their expectations and at pricing bonds, options and currencies. This set of information is likely to be wider than the one that is used in the literature of estimating equilibrium exchange rate.

One of the important contributions of the paper to the theoretical exchange rate literature is that it finds empirical support for the conventional asset-pricing exchange rate model. By comparing the filtered market expectations concerning the locking rate with the survey-based expectations I find that the difference between the two is small for all three countries. Further, if we consider the survey-based expectations to be unbiased estimates[9] on the general view of the markets, then the model used for filtering should be taken to be successful provided the filtered expectations are close to the reported expectations of the surveys. Moreover, the filtered locking rate for Hungary correlates somewhat more with the survey-based average expectations than the historical exchange rate. This finding can be interpreted, as the model helped to denoise the exchange rate, which is just a rough proxy of the expected future locking rate. This test of the model is similar in spirit to the one applied by Engel and West (2005). They use the functional relationship of the model between the current fundamental and the discounted future fundamental on one hand and the exchange rate on the other hand to forecast future fundamental. They find evidence for forecasting ability, what can be interpreted as a general support for the conventional asset-pricing exchange rate models and also for this specific model.

The paper is structured as follows. Section 2 presents the exchange rate model. Section 3 derives an option pricing formula, which is utilized for parameter estimation in the empirical part of the paper. Section 4 shows some stylized facts supporting the stabilizing effect. Section 5 presents the filtering methods and their applications. Finally, Section 6 concludes.

2 Exchange Rate Model

The exchange rate model is the conventional asset-pricing exchange rate model extended with the assumption of final locking. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks. Another extended version of the asset-pricing model is the target-zone model of Krugman (1991), which shows some similarities to this model.

First, in Krugman's model, the exchange rate would be equal to the fundamental if there would be no target-zone. Similarly, in our model, the log exchange rate would be equal to the fundamental in the absence of future locking. Given this relationship, I refer to the fundamental as the latent exchange rate, i.e. the exchange rate that would prevail if the currency was never going to be locked against the euro. While Krugman investigates the stabilizing feature of the target-zone with a floating regime as a bench-mark, I explore the stabilizing effect of future locking with the "no locking" as a bench-mark.

Second, the implicit relationship between the exchange rate subject to future locking and the latent exchange rate in this model is the same as the relationship between the target-zone exchange rate and the fundamental in Krugman's model. In general, this relationship between the exchange rate and the fundamental is common across all asset-pricing approach based representative agent models. I can formulate this relationship as follows in a reduced form:

$$s_t = v_t + c\frac{E_t(ds_t)}{dt} \quad . \tag{1}$$

[9] Our confidence on this result should depend, however, on how reliable are the survey data. The survey data usually tend to show systematic bias in the reported expectations as it has been documented by the early papers of Frankel and Froot (1987) and Froot and Frankel (1989).

Here, s_t is the log exchange rate, and v_t is the log latent exchange rate. The constant c is the time scale. Engel and West (2005) and Svensson (1991) presents the Money Income Model [10] as one possible structural model that rationalizes the reduced form (1), where c is the interest rate semi-elasticity of the money demand. The term $\frac{E_t(ds_t)}{dt}$ is the expected[11] instantaneous change of the exchange rate. As I will derive it, the expected instantaneous change of the exchange rate depends on the log latent exchange rate v_t, the market expectation concerning the log final conversion rate x_t and concerning the time of locking T_t.

In the following, I define the latent exchange rate as a function of some macro variables based on the Money Income Model:

$$v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^* \quad . \tag{2}$$

In this respect, y denotes the domestic real output, q is the real log exchange rate, ψ is the risk premium, p^* is the eurozone log price, m denotes the domestic nominal money supply and i^* is the euro interest rate.

As mentioned above, it is aimed to lock the exchange rates at their equilibrium levels. I use a concept of equilibrium exchange rate under which the strong law of purchasing power parity (PPP) holds for the locking rate, i.e. the log nominal exchange rate at the time of locking is equal to the difference between the domestic and eurozone log prices: $s_T = p_T - p_T^*$. Under rational expectation the market expects the log final conversion rate at time t to be $x_t = E_t(s_T)$, which gives

$$x_t = p_t - p_t^* + \int_t^{T_t} E_t(\pi_\tau - \pi_\tau^*)d\tau \quad . \tag{3}$$

Where π and π^* denote domestic and eurozone inflation rates respectively.

Neither the definitions (3) and (2) nor the corresponding macro data are used directly in the empirical part of the paper - mainly because of the low frequency of these data, but also because of a possible misspecification of the underlying macro models. For instance, the examples of the current EMU countries show that the locking rate can deviate from its PPP value. Still, it is the equilibrium real exchange rate that should be the key determinant of the locking rate chosen on the basis of economic considerations. Therefore, while not used directly in the rest of the paper, these definitions motivate the calibration of the model, the choice of the processes of the underlying factors and the interpretation of the results.

2.1 Dynamics

In this subsection I specify the processes of the factors. These processes will be used to derive the process of the exchange rate.

I start by assuming that the factors follow Brownian motions. This assumption can be decomposed into an assumption on the martingale property of the processes and into the Gaussian

[10] The Money Income Model is the following.

(1f) $m_t - p_t = \alpha y_t - ci_t \quad \alpha > 0 \quad c > 0$ money market equilibrium

(2f) $q_t = s_t + p_t^* - p_t$ real exchange rate

(3f) $\psi_t = i_t - i_t^* - \frac{E(ds_t)}{dt}$ risk premium

(4f) $v_t = -\alpha y_t + q_t + c\psi_t - p_t^* + m_t + ci_t^*$ fundamental/latent exchange rate.

[11] I consider two different types of expectations in the paper. One is the subjective market expectation, and the other is the mathematical expected value of a random variable. Here, I refer to the latter one. In order to distinguish between the two, I refer to the first type of expectation as the market expectation. However, under rational expectation the two are the same.

distribution of the innovations. The Gaussian distribution of the innovations is assumed for technical reason. The martingale property of these processes, however, can be easily explained:

- Under rational expectation, the *expectation of the market participants concerning the log final conversion rate* is the expected value of the log final conversion rate given all the information available at the time the expectation is formed by the market ($x_t = E_t(s_T)$). And also the *market expectation concerning the time of locking* is the expected value of the true time of locking T^* given all the information available at the time the expectation is formed ($T_t = E_t(T^*)$). The *law of iterated expectations* implies that the process of both T_t and x_t are martingales, since $E_t(E_{t+1}(s_T)) = E_t(s_T)$ and $E_t(E_{t+1}(T^*)) = E_t(T^*)$.

- The martingale property of the process of the *log latent exchange rate* can be derived from the Money Income Model under the assumption that the right hand side variables of equation (2) have martingale processes. The martingale property of v_t allows us to focus entirely on the dynamics caused by the future final locking, as opposed to the effects of predictable future changes in the latent exchange rate.

The process [12] of the market expectations concerning the log euro locking rate x_t

$$dx_t = \begin{cases} \sigma_{x,t} dz_{x,t} \ , & \text{if } t < T_t \\ 0 \ , & \text{otherwise} \end{cases} \tag{4}$$

where $dz_{x,t}$ is a Wiener process.

Second, if the right hand side variables of equation (2) follow Brownian motion,[13] then so does v_t. Hence, its process can be written as

$$dv_t = \sigma_{v,t} dz_{v,t} \quad . \tag{5}$$

Where $dz_{v,t}$ is a Wiener process, which is not necessarily independent from $dz_{x,t}$. I assume linear, contemporaneous relationship between the two shocks, what can be captured by their time-varying correlation denoted by $\rho\left(dz_{v,t}, dz_{x,t}\right)$.

The third factor, the market expectation concerning locking time T_t is modeled as a stochastic variable. In the empirical part of the paper I filter the factors not only with stochastic T_t, but also with constant T_t in order to see to what extent does the dynamics of T_t influence the dynamics of the other factors. One can think of the constant locking time as having stochastic locking time with volatility $\sigma_{T,t}$ restricted to zero.

The assumed process of the market expectations concerning locking time is the following martingale,

$$dT_t = \begin{cases} (T_t - t)\sigma_{T,t} dz_{T,t} \ , & \text{if } t < T_t \\ 0 \ , & \text{otherwise.} \end{cases} \tag{6}$$

[12] In a discrete time framework the process can be derived from equation (3).

$$\Delta x_t = [\pi_{t+1} - E_t(\pi_{t+1})] + \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)] - [\pi^*_{t+1} - E_t(\pi^*_{t+1})] - \sum_{i=t+2}^{T_t} [E_{t+1}(\pi^*_i) - E_t(\pi^*_i)] \quad .$$

If both the expectation errors and the change of expectations are independent and normally distributed with zero mean, then the process of x_t is a Brownian motion in a continuous time framework.

[13] The assumption that the growth rate of GDP and the real appreciation rate are normally distributed comes from two additional equations of the underlying macro model. One is the supply curve equation and the other captures the Balassa-Samuelson effect:

(5f) $y_t - y_{t-1} = \beta(\pi_t - E_{t-1}(\pi_t)) \quad \beta > 0$ supply curve

(6f) $dq_t = -\gamma dy_t \quad \gamma > 0$ Balassa-Samuelson effect (real appreciation).

The growth rate and the real appreciation rate should have normal distribution, because both are linear functions of the normally distributed expectation error $\pi_t - E_{t-1}(\pi_t)$.

Where $dz_{T,t}$ is a Wiener process which may correlate with the other two shocks $dz_{v,t}$ and $dz_{x,t}$.

Given that the EMU candidate countries are not eligible to join the eurozone unless the Maastricht criteria are fulfilled, the market expectations concerning the time of locking should depend on the expected future inflation, debt-to-GDP and deficit-to-GDP ratios. For the sake of simplicity I assume that the Maastricht criteria can be simplified to one nominal and one real criterion. Moreover, the nominal criterion is assumed to be captured by the expected locking rate which is being driven by the future expected inflation rate according to equation (3). The higher is the cumulated expected excess inflation rate, the later will the locking take place. The real criteria are captured by the fundamental, which is a decreasing function of the log output according to equation (2). The lower is the log output and therefore the higher is the fundamental, the later will the country be eligible to join the euro area. This simplified view on the Maastricht criteria can be described by the following Taylor-rule-type functional relationship between the expected time until locking on one hand and the expected locking rate and fundamental on the other hand.

$$T_t - t = \left(\frac{1}{2\lambda_t} \frac{\sigma_t^2}{c\sigma_{x,t}^2} x_t^2 + \frac{1}{2\lambda_t} \frac{\sigma_t^2}{c\sigma_{v,t}^2} v_t^2 \right)^{\lambda_t} \qquad \lambda_t > 0 \quad . \tag{7}$$

The instantaneous volatility of the expected locking date is the following function of the expected time until locking:

$$\sigma_{T,t}^2 = \sigma_t^2 (T_t - t)^{-\frac{1}{\lambda_t}} \quad . \tag{8}$$

The parameters σ_t^2 and λ_t are time-varying, therefore the number of factors is not reduced by the restrictions of (7) and (8). [14]

Besides equations (7) and (8), I pose another restriction on the processes. I assume that there is no such shock that affects both v_t and x_t but not T_t, because v_t and x_t are interrelated through T_t as the correlation between nominal and real shocks helps to make the more binding Maastricht criteria to earlier meet. Or in other words, shocks to x_t and v_t that are orthogonal to shocks to T_t are assumed to be independent from each other as well. This assumption can be formalized as

$$\rho(dz_{x,t}, dz_{v,t}) = \rho(dz_{T,t}, dz_{v,t}) \rho(dz_{T,t}, dz_{x,t}) \quad . \tag{9}$$

The exact functional form of (7) representing the convergence criteria and the restrictions (8) and (9) can be motivated by the followings. As it is shown by the next Subsection 2.2, the log exchange rate can be derived as a closed-form function $s_t = f(t, v_t, x_t, T_t)$ of the factors under these restrictions. Moreover, this functional relationship is also easily interpretable.

2.2 Functional Relationship Between The Exchange Rate And Underlying Factors

Here, I derive the functional relationship between the exchange rate on the one hand and the latent exchange rate, the market expectations concerning the locking time and locking rate on the other hand. Subsection 2.2.1 presents the derivation under the assumption of having constant locking time. Then I relax this assumption in Subsection 2.2.2 to derive the function in the general case with stochastic locking time.

The derivation has the following two steps in both cases with constant and stochastic locking time. First, I derive the process of the log exchange rate s_t from the processes of the factors by

[14] The three-eqution restriction of (7), (8) and (9) with λ_t and σ_t^2 is used in Appendix A to derive the following single-equation restriction without λ_t and σ_t^2: $\rho\left(dz_{T,t}, dz_{v,t}\right) \sigma_{v,t} - \rho\left(dz_{T,t}, dz_{x,t}\right) \sigma_{x,t} = (v_t - x_t) \frac{\sigma_{T,t}(T_t - t)}{c}$.

using Ito's stochastic change-of-variable formula. Second, I obtain that the function satisfying the derived process and the terminal condition $s_{T^*} = x_{T^*}$ and equation (1) is given by

$$s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{T_t - t}{c}}\right) v_t + e^{-\frac{T_t - t}{c}} x_t \quad . \tag{10}$$

2.2.1 Constant Locking Time

Here I assume that the locking time T is constant. According to Ito's formula, the function $f(t, v_t, x_t, T)$ should satisfy (11).

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t \partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\right]dt + \\ + \frac{\partial f}{\partial v_t}\sigma_{v,t} dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t} dz_{x,t} \quad . \tag{11}$$

The different μ's denote the drift terms, whose values are zero in equations (4) and (5).

At time T^* the exchange rate s_{T^*} is equal to the market expectation concerning the final conversion rate x_{T^*}, because at the time of locking the market already knows the conversion rate. Consequently, the function $f(t, v_t, x_t, T)$ should satisfy the terminal condition

$$f(T^*, v_{T^*}, x_{T^*}, T^*) = x_{T^*} \quad . \tag{12}$$

The solution is given by (10). The proof is provided in the general case with stochastic locking time in Appendix A.

2.2.2 Stochastic Locking Time

Here I assume that the locking time T_t is stochastic and its process is given by (6). The function $f(t, v_t, x_t, T_t)$ is derived under the assumption of stochastic locking time similarly to the deterministic case. The solution is again given by (10), however, this finding depends on how the convergence criteria are modeled by (7) and also on restrictions (8) and (9). The convergence criteria are modeled so to have exactly the same solution with stochastic locking time as with constant one.

The Ito's formula can be used again to find the function $f(t, v_t, x_t, T_t)$. By using Ito's stochastic change-of-variable formula, we will get a similar expression for *df* as previously with constant locking time, however some new terms appear in the formula.

$$df = \left[\frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t}\mu_{v,t} + \frac{\partial f}{\partial x_t}\mu_{x,t} + \frac{\partial f}{\partial T_t}\mu_{T,t} + \frac{1}{2}\frac{\partial^2 f}{\partial v_t^2}\sigma_{v,t}^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_t^2}\sigma_{x,t}^2 + \right. \\ + \frac{1}{2}\frac{\partial^2 f}{\partial T_t^2}\sigma_{T,t}^2(T_t - t)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial T_t \partial x_t}\rho\left(dz_{T,t}, dz_{x,t}\right)(T_t - t)\sigma_{T,t}\sigma_{x,t} + \\ \left. + \frac{1}{2}\frac{\partial^2 f}{\partial T_t \partial v_t}\rho\left(dz_{T,t}, dz_{v,t}\right)(T_t - t)\sigma_{T,t}\sigma_{v,t} + \frac{1}{2}\frac{\partial^2 f}{\partial x_t \partial v_t}\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t}\right]dt + \\ + \frac{\partial f}{\partial v_t}\sigma_{v,t} dz_{v,t} + \frac{\partial f}{\partial x_t}\sigma_{x,t} dz_{x,t} + \frac{\partial f}{\partial T_t}(T_t - t)\sigma_{T,t} dz_{T,t}. \tag{13}$$

The different μ's denote the drift terms, whose values are zero in equations (4), (5) and (6).

I obtain again that the function satisfying the derived process (13), the terminal condition (12) and the equation (1) borrowed from Krugman (1991) [15] is given by (10). The proof can be found in Appendix A.

Equation (10) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights are changing over time; if the time until locking is infinite, or in other words, if the currency will never be locked, then the weight of the latent exchange rate is one, and the weight of the expected conversion rate is zero. As the time until locking decreases, the weight of the expected conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected conversion rate approaches one.

In order to examine the exchange rate dynamics of the model, I substitute (10),(4), (5) and (6) into equation (13).

$$ds_t = \frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}\left(x_t-s_t\right)dt + \left(1-e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}dz_{v,t} + \tag{14}$$

$$+e^{-\frac{T_t-t}{c}}\sigma_{x,t}dz_{x,t} - \frac{1}{c}\frac{e^{-\frac{T_t-t}{c}}}{1-e^{-\frac{T_t-t}{c}}}\left(x_t-s_t\right)(T_t-t)\sigma_{T,t}dz_{T,t} \quad .$$

Equation (14) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation concerning the final conversion rate. Moreover, the closer the time of locking, the faster the convergence is.

Equations (4), (5), (6) and (10) define a three-factor model. One factor is the market expectation concerning the final conversion rate; another factor is the market expectation concerning the time of locking; the third factor is the latent exchange rate.

3 Option Pricing

In this section I show a pricing formula for European-type options. The pricing formula is consistent with the exchange rate model with constant locking time and factor volatilities. Although not being fully consistent with the stochastic locking time, the pricing formula is used also in the general framework with stochastic locking time to estimate parameters. The parameters to be estimated are the variances of the innovations of the factors v_t and x_t. The historical option prices are given in terms of implied volatility; consequently, I derive the pricing formula in terms of volatility as well.

In the theoretical model the uncertainty is present due to the stochastic innovations ($dz_{v,t}$, $dz_{x,t}$, $dz_{T,t}$) of the factors; consequently, the price of an option is a function of the variances and covariances of these normally distributed innovations. From equation (14) and (10), we can derive, that the instantaneous variance of returns at time t is

$$\begin{aligned}\sigma_{s,t}^2 = {}& \left(1-e^{-\frac{T_t-t}{c}}\right)^2\sigma_{v,t}^2 + \left(e^{-\frac{T_t-t}{c}}\right)^2\sigma_{x,t}^2 + \left(\frac{1}{c}e^{-\frac{T_t-t}{c}}\right)^2\left(x_t-v_t\right)^2(T_t-t)^2\sigma_{T,t}^2 + \\ & + 2\left(1-e^{-\frac{T_t-t}{c}}\right)\left(e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}\sigma_{x,t}\rho\left(dz_{v,t},dz_{x,t}\right) + \\ & - 2\frac{1}{c}e^{-\frac{T_t-t}{c}}\left(x_t-v_t\right)(T_t-t)\sigma_{T,t}\left(1-e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}\rho\left(dz_{T,t},dz_{v,t}\right) + \\ & - 2\frac{1}{c}e^{-\frac{T_t-t}{c}}\left(x_t-v_t\right)(T_t-t)\sigma_{T,t}e^{-\frac{T_t-t}{c}}\sigma_{x,t}\rho\left(dz_{T,t},dz_{x,t}\right) \quad .\end{aligned} \tag{15}$$

[15] Similarly to the solution in the Krugman (1991) paper, equation (10) also satisfies a smooth pasting condition $\frac{ds_{T^*}}{dv_{T^*}} = 0$.

In order to derive a closed form option pricing formula, I work with the simple constant volatility model. I assume that the covariance matrix of the three shocks is constant over the life of the option. Until now, I allowed $\sigma_{v,t}$, $\sigma_{x,t}$ and $\sigma_{T,t}$ and the correlations to change over time. While I do not rule out the possibility, that the covariance matrix used to price option can vary over time, I do not model the dynamics of the covariance matrix. Obviously, the price of options in the stochastic volatility framework, is different from the one of the constant volatility framework, however the latter is a good approximation for the theoretical value of at-the-money options with a maximum of one-year maturity.[16]

In order to have a closed form option pricing formula, I fix not only the covariance matrix over the life of the option, but also the market expectation concerning the locking date T_t.

By applying this final simplification and by calculating the integral we obtain the option pricing formula

$$g(t, m, \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho\left(dz_{v,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{v,t}\right)) = \left[\int_t^{t+m} \sigma_{s,\tau}^2 d\tau\right]^{\frac{1}{2}} =$$

$$\Bigg(\sigma_{v,t}^2 m + \Gamma_1 \Big[\sigma_{v,t}^2 + \sigma_{x,t}^2 - 2\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t} +$$

$$- \rho^2\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t}^2 - \rho^2\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t}^2 + 2\rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t}\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t}\Big] +$$

$$+ \Gamma_2 \Big[-2\sigma_{v,t}^2 + 2\rho\left(dz_{v,t}, dz_{x,t}\right)\sigma_{v,t}\sigma_{x,t} + 2\rho^2\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t}^2 +$$

$$- 2\rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t}\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t}\Big]\Bigg)^{\frac{1}{2}} \quad . \tag{16}$$

The option is sold at time t and the maturity of the option is denoted by m. The $\Gamma_1 = \frac{c}{2}e^{-\frac{2}{c}(T_t - t - m)} - \frac{c}{2}e^{-\frac{2}{c}(T_t - t)}$ and the $\Gamma_2 = ce^{-\frac{1}{c}(T_t - t - m)} - ce^{-\frac{1}{c}(T_t - t)}$.

4 Stylized Facts On The Stabilizing Effect

This section investigates the stabilizing effect of locking by looking at some stylized facts on the term structure of options and by comparing estimated constant factor volatilities.

The stabilizing effect of locking can be detected by comparing the volatility of the exchange rate with future locking σ_s on the one hand and the volatility of exchange rate without locking σ_v on the other hand. Obviously, if the former is smaller than the latter, then we can infer that the prospect of locking stabilizes the exchange rate.

Under the assumption of having constant locking time and independent factors with constant volatilities, the instantaneous variance of the exchange rate $\sigma_{s,t}^2$ is the weighted average of the variances of the factors x and v (see equation (15)). Consequently, it is sufficient to show $\sigma_v > \sigma_x$ in order to prove the existence of stabilizing effect. It is important to notice, that the condition $\sigma_v > \sigma_x$ is sufficient only under the highly restrictive assumptions of having constant locking time and independent factors with constant volatilities. If these restrictive assumptions are not

[16]As it is pointed out by Hull (1997, p. 620): "For options that last less then a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases."

fulfilled, then it is possible to have no exchange rate stabilization $\sigma_s > \sigma_v$ despite of having more volatile latent exchange rate than locking rate $\sigma_v > \sigma_x$. For instance, high uncertainty related to the locking time can increase the volatility of the exchange rate with future locking even to be more than the volatility of the latent exchange rate.

In the next sections, these restrictive assumptions will be relaxed and the magnitude of the stabilizing effect will be estimated by controlling for the uncertainty of the locking date, the dynamics of the market expectation concerning the locking date, correlations between the factors, and time-varying volatilities of the factors.

Unlike next sections, here it is assumed to have constant locking time and independent factors with constant volatilities. The advantage of this framework, is that it is consistent with the option pricing formula (17), therefore estimates on the factor volatilities are not subject to bias due to model misspecification. Moreover, even the term structure of implied volatilities can provide us insight into the stabilizing feature of locking. In this framework the stabilizing effect is analyzed by applying two simple approaches. First, we look at the term structure of options. Second, we compare estimated volatilities of the factors.

In this framework, the option pricing formula (16) reduces to

$$g(t, m, \sigma_x, \sigma_v) = \left(\sigma_v^2 (m + \Gamma_1 - 2\Gamma_2) + \sigma_x^2 \Gamma_1 \right)^{\frac{1}{2}} \quad . \tag{17}$$

The $\Gamma_1 = \frac{c}{2} e^{-\frac{2}{c}(T-t-m)} - \frac{c}{2} e^{-\frac{2}{c}(T-t)}$ and the $\Gamma_2 = ce^{-\frac{1}{c}(T-t-m)} - ce^{-\frac{1}{c}(T-t)}$.

The option pricing formula (17), but also our intuition suggest that longer options are more exposed to shocks occurring in the far future than options with shorter maturities. Or in other words, σ_x has higher relative weight in a longer option, then in a shorter one. And the opposite holds for σ_v. Consequently, if the term structure of options is downward sloping, then $\sigma_v > \sigma_x$. Therefore, a decreasing term structure can be interpreted as evidence for the stabilizing feature of locking. This relationship between the term structure and the stabilizing effect is not an if and only if relationship, because even if the term structure is upward sloping, it is possible to have $\sigma_v > \sigma_x$.

Table 4 shows the average implied volatilities for each of the six maturities for the three countries. The six options are at-the-money (ATM) options and have different maturities. In case of Czech koruna and Polish złoty the maturities are one-month, two-months, three-months, six-months, nine-months and one-year. Whereas in case of the Hungarian forint the currency options have one-week, one-month, two-months, three-months, six-months and one-year $m(6)$ maturities. For more details on the data see Csávás and Gereben (2005).

The average term structure of options is clearly downward sloping for Czech koruna and Polish złoty, but not for Hungarian forint. In order to have a more detailed picture on the Hungarian term structure, I divide the sample into two equal sized subsamples and calculate the average implied volatilities for the two subsamples separately. In the second half of the sample, the Hungarian term structure is downward sloping as well, but not in the first one.

All in all, the average term structure of these countries suggest, that the prospect of locking has stabilized the Czech koruna and Polish złoty, and also the Hungarian forint in the second half of the sample. Purely based on the investigation of the term structure, we can not rule out, that locking have stabilized the forint even in the first half on the sample. In order to judge the stabilization in that period and to make inferences on the magnitude of the stabilizing effects, I present estimates on the factor volatilities.

The option pricing formula (17) and historical option prices can be used to estimate the constant volatilities σ_v, σ_x of the factors. I assume that the pricing errors, i.e. the difference between the historical prices and the theoretical prices, are independent identically distributed (IID) with Gaussian distribution. Under this assumption, the volatilities σ_v, σ_x can be estimated

Table 1: The term structure of implied volatilities (Source: Reuters)

	CZ	HU			PL
sample	full	full	1st half	2nd half	full
σ_{1W}^{imp}	-	7.71%	6.37%	9.06%	-
σ_{1M}^{imp}	4.8%	7.69%	6.48%	8.9%	8.64%
σ_{2M}^{imp}	4.76%	7.71%	6.72%	8.7%	8.5%
σ_{3M}^{imp}	4.72%	7.78%	6.99%	8.57%	8.41%
σ_{6M}^{imp}	4.71%	7.9%	7.35%	8.46%	8.3%
σ_{9M}^{imp}	4.75%	-	-	-	8.29%
σ_{1Y}^{imp}	4.71%	8.04%	7.65%	8.43%	8.25%
Num. obs.	3168	3012	1506	1506	3162

by maximum likelihood method. The constant market expectation concerning the locking date T is calibrated to 1.Feb.2008, 13.Feb.2010 and 8.Jun.2010. for Czech Republic, Hungary and Poland respectively. The calibration is based on the filtered $\hat{T}_{t_0}$ where $t_0 = 5.Jan.2005.$ is the beginning of the sample (see Subsection 5.1 on the filtering of T_t).

Table 4 presents the estimated factor volatilities. These point estimates support the existence of the stabilizing effect in all three countries as having $\hat{\sigma}_v > \hat{\sigma}_x$. Moreover, the differences between the volatilities are significant. Based on the likelihood ratio test, we can reject the null that the volatilities are equal at 10% significance level for each country and for both periods for Hungary. And we can also reject the null even at 1% significance level in three cases out of four.

This section have shown some empirical evidences on the stabilizing effect under the highly restrictive assumptions of having constant locking time and independent factors with constant volatilities. In the followings, I will relax these assumptions and investigate whether our findings on the stabilizing effect remains still valid even under more general conditions.

5 Filtering Factors

I apply the Kalman filter technique to extract the time series of the factors from the time series of the observable exchange rate. However, filtering all three factors from only one series would be overambitious and unlikely to provide robust results. Moreover, the Kalman filter technique can be applied only if the model is linear[17] in all the factors to be filtered. In fact, the log exchange rate s_t is linear in two of the factors, namely the latent exchange rate v_t and the market expectation concerning the final conversion rate x_t, but not in the expected date of locking T_t. However, the expected date of locking T_t can be filtered independently from the other two factors.

Once T_t is given, the Kalman filter can be interpreted as a method for decomposing the exchange rate changes ds into changes in the state variables v and x. Under the assumption of having constant locking date T, the decomposition is such that the higher is the relative weight

[17] To filter all three factors from the time series of the exchange rate, one should apply a different technique then the Kalman filter, because the model is not linear in T_t. The Extended Kalman filter and the particle filter are possible candidates.

Table 2: The estimated constant volatilities of the latent exchange rate v and the market expectation concerning the locking rate x – with independent factors and constant locking time

	CZ	HU		PL
sample	full	1st half	2nd half	full
$\hat{\sigma}_v$	19.87%	12.55%	13.97%	20.06%
(t-stat)	(201.16)	(11.79)	(9.06)	(145.71)
$\hat{\sigma}_x$	4.14%	8.26%	10.69%	7.87%
(t-stat)	(183.16)	(18.24)	(33.54)	(94.74)
LR-test $\sigma_x = \sigma_v$	4315.64	15.2	2.77	1365.36
critical value - 1% signif level	6.63	6.63	6.63	6.63
critical value - 10% signif level	2.71	2.71	2.71	2.71
Num. obs.	3168	1506	1506	3162

of v in s, and the higher is its volatility relative to that of the other factor $\frac{\sigma_v}{\sigma_x}$, the higher change will be attributed to v by the Kalman filter. (See Appendix D on the Kalman filter in general and also on this special feature of it.)

The next Subsection 5.1 introduces the applied method and the results of filtering T_t. Then, I set up the filtering problem for the remaining two factors in Subsection 5.2. Subsection 5.3 describes how the parameters of the filtering problem are set. Finally, Subsection 5.4 presents the filtered market expectation concerning the locking rate x_t and its estimated stabilizing effect on the exchange rate.

5.1 Filtering The Expected Locking Date

The expected date of locking can be estimated independently from the other two factors from the euro and domestic yield curves. The yield curve method has been popularized by Bates (1999) and it has been applied for Hungary by Csajbók and Rezessy (2006). The yield curve method is based on the fact that after adopting the euro, the interest rate differential will become marginal, but not before the adoption. The higher probability is attached by the market participant to the scenario that a country is already a full-fledged member of the eurozone by a given year, the lower is the forward differential for that particular year. Along these lines the market expectation on the date of locking can be estimated from the forward differentials. Appendix B describes the method in details.

Figure 5.1 shows the smoothed and the original filtered market expectations concerning the locking dates. The smoothed time series is the moving average of the original time series where the window size is 21. In the followings, I consider the smoothed time series to be the filtered $\hat{T}_t$, because it is cleaned from daily noises.

The Reuters polls queries the analysts about their opinion on the most likely year of EMU and ERM II entry of the three analyzed countries. These are the only references for the filtered T_t to be compared with. Consequently, I put also these survey data on Figure 5.1. The filtered time series and the survey-based one show similar trends. One can see that analysts expectations were relatively stable until the autumn of 2005 and have changed subsequently between the quarterly polls of August and November for the Czech Republic and Poland, and between September and October 2005 for Hungary. Until the autumn of 2005 the three countries had been expected to enter ERM II in the year 2007. Thereafter, the expectations changed dramatically, as reflected by the monthly and quarterly Reuters polls, pointing to a postponement of ERM II entry to 2008 for the Czech Republic, to 2009 for Poland, and to 2010 for Hungary. Similar trend can be detected in the expected EMU entry date. The filtered locking dates suggest that expectations were subject to both positive and negative shocks during 2006 and the first quarter of 2007 in all three countries. The positive shocks shifting the expected locking date of Hungary and Poland substantially upwards have started in July 2005 according to the filtered locking date, whereas the survey-based expectations have changed only a few months later. This finding suggests that the estimates on the expected locking date provide us earlier warnings on the changes in the market sentiment than the surveys. In the first half of 2006 the filtered expected locking dates have downward trends in Hungary and Poland, which is either due to some favorable news on the EMU outlook, or due to a correction of the previous overshooting of the expectations.

The locking of the exchange rate can precede the EMU entry, but not vice versa. For instance, the exchange rates of almost all the countries that entered the ERM II system in recent years[18] are almost fixed: the volatility of the Estonian kroon, the Lithuanian lita, the Slovenian tolar, the Cyprus pound and the Maltese lira dropped below 1% after entering the ERM II regime. These facts imply that practically locking does not necessarily take place when a country enters the monetary union, but rather when it enters the ERM II regime. The filtered locking date of the Czech Republic has fluctuated between the narrow band given by the survey-based expected EMU and ERM II entry dates, which is in accordance with the sequence of locking and EMU entry. Whereas in case of Hungary and Poland the survey-based expected EMU entry dates precede sometimes the yield curve based locking dates. Or in other words, the survey-based expectations are more optimistic relative to the yield curve based estimates. Since I have more confidence in the filtered expectations then in the survey-based expectations, I attribute the difference to systematic bias in the survey data. This systematic bias can be explained, for instance, with the skewed distribution of the entry date. If market analysts report the most likely entry date of a country, i.e. the mode of the distribution, instead of the expected value and if the subjective distribution of the entry date is skewed towards a later entry date, then the survey-based expectations are downward biased estimates on the expected locking dates.

Once we have estimates on T_t, we can filter the other two factors from the historical exchange rate by Kalman filter. In the Kalman filter I treat T_t as being exogenously given.

5.2 Filtering Problem

Here, I set up the filtering problem for x_t and v_t under the general case of modeling T_t as a stochastic variable. I do not derive the filtering problem for the case with constant locking time separately, because it is just a special case of the general one with the zero restriction for the correlation parameters and $\sigma_{T,t}$.

In contrast to the case with constant locking date, the shocks to the factors do correlate under the assumption of having stochastic locking date. Since the third factor T_t is exogenous and it is not independent from the other two factors, I have to use the conditional distributions of x_t and v_t, where I condition on the realization of T_t. The conditional expected innovations of x_t and v_t are $\sigma_{x,t}\rho(dz_{T,t}, dz_{x,t})dz_{T,t}$ and $\sigma_{v,t}\rho(dz_{T,t}, dz_{v,t})dz_{T,t}$ respectively. The expected values of the innovations are taken into account in the model by having a constant as a third state

[18] The Estonian kroon, the Lithuanian lita and the Slovenian tolar joined ERM II on 27 June 2004. On 2 May 2005 three other Member States joined ERM II: Cyprus, Latvia and Malta.

variable. Moreover, the system covariance matrix $Q(t)$ is also conditional on T_t.

The filtering problem can be written in the usual form:

$$\Lambda(t+1) = A(t)\Lambda(t) + w_1(t+1) \tag{18}$$

$$\Omega(t) = C(t)\Lambda(t) + w_2(t) \tag{19}$$

$$E\left[\begin{pmatrix} w_1(t+1) \\ w_2(t) \end{pmatrix} \begin{pmatrix} w_1(t+1) & w_2(t) \end{pmatrix}\right] = \begin{pmatrix} Q(t) & 0 \\ 0 & R \end{pmatrix} \tag{20}$$

In our problem, the vector of states is $\Lambda^{'}(t) = \begin{pmatrix} v_t & x_t & 1 \end{pmatrix}$. The system matrix is

$$A(t) = \begin{pmatrix} 1 & 0 & \sigma_{v,t}\rho(dz_{T,t}, dz_{v,t})\frac{dT_t}{\sigma_{T,t}(T_t - t)} \\ 0 & 1 & \sigma_{x,t}\rho(dz_{T,t}, dz_{x,t})\frac{dT_t}{\sigma_{T,t}(T_t - t)} \\ 0 & 0 & 1 \end{pmatrix}.$$

The vector $w_1(t)$ is assumed to be a Gaussian vector white noise. The observable variable is the log exchange rate $\Omega(t) = s_t$. Equation (10) implies that the observation matrix is $C(t) = \begin{pmatrix} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{T_t - t}{c}} & 0 \end{pmatrix}$.

The system covariance matrix can be written as

$$Q(t) = \begin{pmatrix} Q_{1,1}(t) & Q_{1,2}(t) & 0 \\ Q_{1,2}(t) & Q_{2,2}(t) & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

where the covariance is conditional on the observed T_t, therefore

$$Q_{1,1}(t) = \sigma^2_{v,t}\left[1 - \rho^2(dz_{T,t}, dz_{v,t})\right],$$

$$Q_{1,2}(t) = \sigma_{v,t}\sigma_{x,t}\left[\rho(dz_{x,t}, dz_{v,t}) - \rho(dz_{T,t}, dz_{v,t})\rho(dz_{T,t}, dz_{x,t})\right],$$

$$Q_{2,2}(t) = \sigma^2_{x,t}\left[1 - \rho^2(dz_{T,t}, dz_{x,t})\right].$$

The conditional covariance $Q_{1,2}$ between $dz_{x,t}$ and $dz_{v,t}$ is zero due to the restriction (9). In contrast to the conditional covariance, the unconditional covariance between $dz_{x,t}$ and $dz_{v,t}$ is not zero, which is captured by the system matrix $A(t)$.

I assume that the error term $w_2(t)$ is zero. In other words, the exchange rate is assumed to be observed without error and the model (10) perfectly describes the relationship between the factors and the exchange rate. Hence, the variance of the observation error term R is set to zero. The Kalman filter remains valid even in this case.[19]

In our problem, the observation matrix $C(t)$, the system matrix $A(t)$ and the system covariance $Q(t)$ are changing over time.[20]

The parameters c, $\sigma_{v,t}$, $\sigma_{x,t}$, $\sigma_{T,t}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$ of the system covariance $Q(t)$ and of the system matrix $A(t)$ need to be either calibrated or estimated. Moreover, the initial values x_{t_0} and v_{t_0} of the factors belonging to the beginning of the sample period, $t_0 = 5.Jan.2005.$, need to be set as well. The next Subsection describes how these parameters are estimated or calibrated.

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[19] See Harvey (1990) page 108 for a detailed discussion.

[20] I used the Kalman filter toolbox for Matlab written by Kevin Murphy. Its advantage is that it allows for time-varying parameters. (See http://www.ai.mit.edu/ murphyk/Software/kalman.html for details.)

5.3 Parameters

This Subsection shows how the parameters c, $\sigma_{T,t}$, x_{t_0}, v_{t_0}, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{v,t})$, $\sigma_{v,t}$ and $\sigma_{x,t}$ are calibrated or estimated in the general case with stochastic locking time. The calibration of parameters is almost the same with the constant locking time as with the stochastic locking time. The calibration in the constant case deviates from that with stochastic locking time only by having the constant market expectation concerning the locking date T set equal to the initial value of the filtered time series T_{t_0} and the parameters $\sigma_{T,t}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$ are zero.

By following Engel and West (2005), I *calibrate the parameter c based on estimates on the interest rate semi-elasticity of money demand*. Frankel (1979), Stock and Watson (1993, 802, table 2, panel 1) and Bilson (1978) estimate the semi-elasticity approximately 29, 40, 60 respectively. These estimates are either on quarterly data or rescaled to quarterly basis. By dividing the quarterly semi-elasticity parameter by 4 we get the annualized c parameter. It should be in the range of 7.25–15. I calibrate c to the average ($\hat{c} = 10.75$) of the estimates of the three cited studies and I use the two extreme values of the range for sensitivity analyses.

One can interpret the parameter value of c around 10.75 as follows. If a country will lock its exchange rate to the euro in four years, then the elasticity of the exchange rate with respect to market expectations concerning the final conversion rate ($e^{-\frac{T-t}{c}} = e^{-\frac{4}{10.75}}$) is almost 70%. If locking is expected to occur in two years, then this elasticity or the relative weight of the log final conversion rate in the log exchange rate is more than 80%.

Figure 5.3 shows the relative weight of the locking rate component of the log exchange rate in the investigated period. Under the assumption of constant locking date, it is a monotonous exponential increasing function of time. More interesting is the case with stochastic locking date, where positive shocks to T_t can even decrease the relative weight of x. In that case, the relative weights are stable for the Czech Republic, just like the expected time until locking. For Hungary and Poland, the largest change in the relative weights took place after July 2005, when the market sentiment on the EMU entry chances has started to become more and more pessimistic. Thereafter, the upward shift in the relative weight of x in the first half of 2006 is due to the decreasing trend of the market expectation concerning the time until locking.

The *market expectation concerning the locking date* T_t is estimated from the yield curves as it is discussed in Subsection 5.1. Under the assumption of stochastic locking date, its *volatility* $\sigma_{T,t}$ is estimated from the filtered $\hat{T}_t$. Since estimates on $\sigma_{T,t}$ may be sensitive to the window size of the moving average used to smooth the original filtered locking time, I calculated the stabilizing effect of future locking with different window sizes as part of the sensitivity analysis. In case of over-smoothing the original time series, the estimated volatility becomes downward biased, whereas failure to average out some errors leads to upward bias. Each value of the time-varying volatility is estimated from 11 data points

$$\hat{\sigma}_{T,t} = \sqrt{\sum_{\tau=t-5days}^{t+5days} \left(\frac{d\hat{T}_\tau}{\hat{T}_\tau - \tau} \right)^2} \quad . \tag{21}$$

For a given value of c, one can calibrate the *initial values of the factors* x_{t_0} and v_{t_0}. What makes these calibrations somewhat difficult is that I have no direct information on the latent exchange rate. For the calibration of the *initial states*, x_{t_0} and v_{t_0} I use the Reuters polls. I assume that x_{t_0} is equal to the log of averaged expectations on the central parity reported by the last Reuters polls of 2004. The initial value of v_{t_0} is calculated by plugging s_{t_0}, $\hat{c}$ and $\hat{T}_{t_0}$ into equations (10).

The calibrated initial values are utilized in the Kalman filter not only as the initial state variables, but also to estimate the *correlations*. These estimated correlations are plugged into the system matrix and the system covariance matrix used to filter the next period state variables. And these filtered state variables are used again for estimating the correlations between the next period's shocks. These steps of estimation and filtering follow each other until the end of the Universe or until the end of the sample.

By rearranging (4) and (6) we obtain that the correlation between the shocks to market expectation concerning the locking rate and locking date is equal to $\rho(dz_{T,t}, dz_{x,t}) = \frac{dx_t}{dT_t}\frac{\sigma_{T,t}(T_t-t)}{\sigma_{x,t}}$. This formula suggests that it is sufficient to calibrate or estimate T_t, $\sigma_{T,t}$, $\frac{dx_t}{dT_t}$ and $\sigma_{x,t}$ in order to obtain the correlation.

Among these terms $\frac{dx_t}{dT_t}$ is calibrated based on the inflation dynamics. Equation (3) of the exchange rate model assumes that the market expectation concerning the log locking rate x_t equals to the difference between the current domestic and eurozone log prices plus the cumulated expected excess domestic inflation rate over the eurozone inflation rate. Based on a plausible view on the inflation dynamics until locking, we can calibrate $\frac{dx_t}{dT_t}$. This term is likely to depend more on the reaction of the domestic inflation path to a change in the expected locking date due to real shocks, rather then on the reaction of the expected locking date to inflationary shocks. The calibration of $\frac{dx_t}{dT_t}$ depends on which of these effects is considered to be more relevant. Both effects and the corresponding calibrations are discussed below.

My view on the first effect is that the targeted path of the inflation of the domestic central bank depends on the outlook for the locking date. Following a shift in the expected locking date, the targeted path of inflation is assumed to be adjusted so that the inflationary Maastricht criterion will be met only at the new expected locking date. Further, I assume, that the targeted inflation at the locking date is 2 % no matter when the locking will be realized and whether the country had higher or even lower inflation rate before. Moreover, the eurozone inflation rate is expected to be constant 2 % as well until the locking date.

My assumption on the second effect is that if the locking is postponed due to an inflationary shock, then the rate of disinflation will remain unchanged.

This strategy of the central bank is illustrated by Figure 5.3 under the scenario of having higher initial inflation rate then the targeted inflation rate. The initial inflation rate π of Figure 5.3 is expected to decrease linearly to the targeted inflation rate π^* until time T. Hence, the initial cumulated excess inflation rate is the checkboard area. The economy can be hit either by inflationary or by real shocks.

If the expected locking date shifts to T^s due to a *real shock*, then the cumulated inflation rate will increase by the dotted area. Under this scenario the term $\frac{dx_t}{dT_t}$ is half of the excess inflation $\frac{\pi-\pi^*}{2}$.

If the expected locking date shifts to T^s due to an *inflationary shock* increasing the inflation rate to π^s, then the cumulated inflation rate will increase not only by the dotted area, but also by the striped one. Under this scenario, the term $\frac{dx_t}{dT_t}$ is the product of the time until locking and the constant rate of disinflation ν.

I consider the real shocks to be more relevant, because EMU entry delays were mainly due to fiscal slippages in Hungary. Moreover, in Czech Republic and Poland the inflation rates were already moderate in the analyzed period. Table 5.3 presents the inflation rates by country and by year and also the average inflation rate used for calibration. I calibrate $\frac{dx_t}{dT_t}$ equal to half of the average excess inflation rate over 2%: -0.085 %, 1.15% and -0.125 % for Czech Republic, Hungary and Poland respectively.

As part of the sensitivity analysis I use the alternative approach of calibration, where $\frac{dx_t}{dT_t}$ is the function of the disinflation rate ν and the time until locking $T-t$. Table 5.3 reports the disinflation rate calculated as the ratio of the average excess inflation rate over 2% and the average filtered time until locking. With this approach, the average calibrated values of $\frac{dx_t}{dT_t}$ are twice as big as with the other calibration approach.

Table 3: Harmonized consumer price index HCPI (Source: IFS)

	2005 YoY	**2006 YoY**	**2007 Q1 QoQ**	**Average[21] (2005-2007Q1)**
CZ	1.60%	2.09%	1.71%	**1.83**%
HU	3.48%	4.04%	8.81%	**4.32**%
PL	2.18%	1.27%	1.96%	**1.75**%

Table 4: The average time until locking and the disinflation rate

	Average time until locking	**Disinflation rate[22]ν**
CZ	3.21 years	-0.05%
HU	5.57 years	0.41%
PL	6.06 years	-0.04%

Once I have calibrated or estimated c, T_t, $\sigma_{T,t}$, $\frac{dx_t}{dT_t}$ and the factors x and v have already been filtered until time t, the only terms that need to be estimated are the volatilities $\sigma_{x,t}$ and $\sigma_{v,t}$. To see this, I show that the correlations can be expressed as functions of some already calibrated or estimated parameters and the volatilities $\sigma_{x,t}$ and $\sigma_{v,t}$. The calibrated or estimated variables are denoted by hats.

$$\rho(dz_{T,t}, dz_{x,t}) = \frac{d\hat{x}_t}{dT_t}\frac{\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{x,t}} \quad . \tag{22}$$

By using (7), (8) and (9) and (22), we obtain

$$\rho(dz_{T,t}, dz_{v,t}) = \frac{(\hat{v}_t - \hat{x}_t)\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t}\hat{c}} + \frac{d\hat{x}_t}{dT_t}\frac{\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t}} \quad . \tag{23}$$

Finally, by substituting (22) and (23) into equation (9), we obtain

$$\rho(dz_{v,t}, dz_{x,t}) = \frac{d\hat{x}_t}{dT_t}\frac{\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{x,t}}\left[\frac{(\hat{v}_t - \hat{x}_t)\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t}\hat{c}} + \frac{d\hat{x}_t}{dT_t}\frac{\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t}}\right] \quad . \tag{24}$$

The time-varying *volatilities* $\sigma_{x,t}$ and $\sigma_{v,t}$ are estimated from six implied volatilities $\sigma_{t,i}^{imp}$ for each time t by ordinary least squares (OLS). The six options have different maturities $m(i)$. The basic idea of the estimation is to minimize the distance between the theoretical and the historical option prices.

[21] The average inflation rate is calculated as the weighted average of the yearly and quarterly inflation rates, where the weight of the quarterly inflation rate is one-forth of that of the yearly rates.

[22] The disinflation rate is calculated as the ratio of the excess average inflation rate over 2% and the average expected time until locking. For instance, for Czech Republic $\nu = \frac{1.83\% - 2\%}{3.21} = -0.05\%$. The average time until locking is the mean of the filtered time until locking.

The OLS estimates of $\sigma_{v,t}$ and $\sigma_{x,t}$ satisfy the minimization problem

$$\min_{\sigma_{v,t},\sigma_{x,t}} \sum_{i=1}^{6} \left[g(t, m(i), \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho\left(dz_{v,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{x,t}\right), \rho\left(dz_{T,t}, dz_{v,t}\right)) - \sigma_{t,i}^{imp} \right]^2 . \tag{25}$$

The option pricing formula for $g()$ is given by (16). All the input parameters of the pricing formula should be substituted by their estimated or calibrated values. The only exceptions are the correlations, which are substituted by the expressions of (22),(23) and (24), where the correlations are functions of the volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$.

Figure 5.3 shows the historical option prices with the shortest and longest maturities and also the fitted values. One can see from Figure 5.3 that the implied volatilities of the options with the shortest maturity fluctuate around the implied volatilities of the options with the longest maturity. In those periods, when the short implied volatility exceeds the long one, the fitted short volatility exceeds the fitted long one as well. This is reflected by the increased estimated volatility of v_t and the decreased estimated volatility of x_t in these periods (see Figure 5.3). The higher than average $\sigma_{v,t}$ and the substantially lower than average $\sigma_{x,t}$ in these periods can be explained by the followings: first, the relative option prices suggests that uncertainty concerning shocks in the short horizon increases relatively to that in the long horizon; second, $\sigma_{v,t}$ can be thought of as a proxy for the short-run uncertainty, whereas $\sigma_{x,t}$ reflects more the expected magnitude of shocks occurring in the far future.

Figure 5.3 shows that on average $\bar{\sigma}_{v,t} > \bar{\sigma}_{x,t}$ for all three countries, providing further support for the stabilizing effect of locking. In the followings, I will calculate not only the factor volatilities estimated from option prices, but also the volatilities of the filtered factors. Moreover, I will measure the stabilizing effect as the relative difference between the realized volatility and the volatility of the filtered latent exchange rate. The advantage of this measure of the stabilizing effect is that it better reflects the magnitude of the ex post stabilization as being calculated from the exchange rate.

Figure 5.3 shows not only the estimated volatilities of v_t and x_t, but also that of the third factors T_t. The estimated volatility of T_t is higher than average in those periods when there are big changes in the filtered locking date. Among the three countries, Czech Republic has the highest average volatility of T_t, despite of the fact that the filtered locking date $\hat{T}_t$ is the most stable for Czech Republic (see Figure 5.1). The magnitude of the volatility can be explained by the followings. First, Czech Republic is likely to be the first among these countries that will be eligible to join the EMU. Second, the volatility measures the variation of the relative change of the expectations and not the absolute. The relative change of the expectation is a decreasing function of the market expectation concerning the time until locking.

5.4 The Filtered Expected Locking Rate And The Stabilizing Effect Of Locking

Figure 5.4 shows the historical exchange rates of the koruna, the forint and the złoty against the euro, the filtered states and analysts' average expectations concerning the central parity as polled by Reuters. Market expectations about the final conversion rate may be thought to be close to the expected central parity of the ERM II regime. In that case, the expected central parity is a good reference for the filtered expected final conversion rate to be compared with. Here, we compare the filtered market expectations with the average expectations reported by the Reuters polls.

Respondents' views on central ERM II parities vary a lot in each poll (see: Figure 5.4). There is at least 6% difference between the two extreme views of the analysts, and even differences of more than 20% being nothing rare. These differences indicate that uncertainty around the reported expectations is likely to be big, which one needs to bear in mind when taking the average reported expectations as the general view of the market on central parity.

As is evident from Figure 5.4, the patterns of the filtered expected final conversion rate and of the reported expected central parity are similar for each of the three countries. The difference between the filtered and the survey-based expectation is always less than 5% for Czech Republic. For Hungary and Poland this difference is more than 5% in a few times, but never exceeds 10%. Although being higher than for Czech Republic, still the difference can be considered to be moderate for Hungary and Poland as well.

According to a naive approach, the spot exchange rate can be considered as noisy estimates on the locking rate. By taking this approach as a bench-mark, the criterion for filtering the expected locking rate successfully, is that the filtered locking rate moves closer together with some alternative measures of the locking rate than the historical exchange rate. Table 5.4 shows that the innovations in the filtered log locking rate for Hungary correlate somewhat more with the changes in the survey-based log locking rate than the exchange rate returns. This finding can be interpreted, as the model helped to de-noise the naive estimates on the locking rate. For the other two countries, the filtering can not be taken to be successful on the same basis, however, there the substantially fewer number of observations disables us to make any firm conclusion.

It is worth to compare the filtered locking rate $\hat{x}_t$ not only with the survey-based expectation, but also with the filtered latent exchange rate $\hat{v}_t$. In this model higher $\hat{v}_t$ than $\hat{x}_t$ suggest that inflation is less of a problem for the economy. Whereas in the opposite case, a later EMU entry is partly due to the high current and future expected inflation.

The filtered locking rate is weaker than the filtered latent exchange rate in Hungary, supporting the view that not only the criteria on the real variables, but also the inflationary Maastricht criterion is binding. In contrast to Hungary, the filtered locking rate is stronger than the filtered latent exchange rate in Poland, what is due to the low inflation rate in the analyzed period. For Czech Republic, there is no significant difference between the filtered locking rate and the latent exchange rate.

We have some important findings on the volatility of the expected final conversion rate as well. The volatility of market expectations concerning the final conversion rate is important, because relatively stable market expectations can stabilize the exchange rate. The locking rate is often referred to as the nominal anchor of the exchange rate that can decrease the volatility of the exchange rate through the expectation channel.

Table 5: The volatility of filtered and survey-based locking rate (X) and the correlations of their percentage changes with the exchange rate returns (ds) – All data are on the frequency of the survey data

	CZ	**HU**	**PL**
$\sigma(X_{survey})$	2.4%	3.93%	4.31%
$\sigma(X_{filtered})$	4%	5.7%	7.48%
$\rho(ds, dx_{survey})$	72.43%	23.18%	42.35%
$\rho(dx_{filtered}, dx_{survey})$	33.33%	24.22%	11.96%
Frequency	Quarterly	Montly	Quarterly
Num. obs. of X_{survey}, $X_{filtered}$, S	7	19	7

Regarding the volatilities, one can see from Table 5.4 that the filtered x is more volatile than the survey-based expectation both calculated from data on the same frequencies. This finding might adversely modify our previous view based purely on the Reuters polls on the stabilizing feature of locking. Still, if the volatility of x is lower than that of s, then market expectations concerning the final conversion rate might have some stabilizing effect on the exchange rate. The advantage of filtering the market expectations over using the survey-based expectations is that no reliable estimate can be given on the volatility of x, and on its relative weight in the exchange rate from the low frequency Reuters polls data, therefore no estimate can be given for the stabilizing effect of the locking either purely from survey data.

The big picture on the stabilizing feature of locking in the entire sample period is provided by Table 5.4. The stabilizing effect of locking is calculated as the relative difference between the volatilities of the historical exchange rate and the filtered latent exchange rate. Across the sample period, the stabilizing effect of locking is found to be the highest in Czech Republic. And somewhat lower, but still high in Poland and Hungary. Even in these two countries, the stabilizing effect is substantial, the volatility without future locking would be approximately twice as large.

Table 6: The volatility of the exchange rate (S) and the latent exchange rate (V) – with stochastic T_t and time-varying volatilities

	CZ	**HU**	**PL**
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	12.88%	17.71%	21.26%
$\frac{\sigma_S-\sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-57.52%	-46.41%	-51.37%

Window size for smoothing the original filtered $\hat{T_t}$ is 21, $c = 10.75$,

$\frac{dx_t}{dT_t}$ equals to half of the average excess inflation rate over 2%: -0.085 %,

1.15%, and -0.125 % for Czech Republic, Hungary and Poland respectively

The estimated stabilizing effect is robust to the calibration of the parameter c. The expectations concerning the locking decrease the volatility at least by 30% even under a smaller parameter of 7.25 for each of the countries (see Table 6 in Appendix C). The stabilizing effect is increasing in c, for $c = 15$ it is close 60% (see Table 6 in Appendix C).

Appendix C presents the results of some other sensitivity analysis as well. The estimates are robust not only to the calibration of c, but also to the calibration of $\frac{dx_t}{dT_t}$ (see Table 6) . Moreover, it is robust to the window size used to smooth the original filtered T_t (see Tables 6 and 6).

The sensitivity of the stabilizing effect to the locking date is also investigated. Instead of assuming stochastic locking time, one may assume to have either constant or deterministic locking time. In the constant case the market expectation concerning the locking date is set equal to the initial value of the filtered locking time ($T_t = T_{t_0}$ where $t_0 = 5.Jan.2005.$). Whereas in the deterministic case, we have time-varying locking time with zero volatility, i.e. the market expectation concerning the locking date is set equal to the filtered one and it is assumed to be independent from the other factors. In order to see, to what extent does the stabilizing effect depend on the dynamics of the volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$, I calculate the stabilizing effect also with constant volatilities. These constant volatilities are set equal to the averages of their time-varying estimates. Tables 6, 6 and 6 show that the magnitude of the stabilizing effect is robust even to the locking date and the dynamics of the volatilities.

This model is rich in dynamics: we have time-varying volatilities $\sigma_{v,t}$ $\sigma_{x,t}$ $\sigma_{T,t}$, and stochastic T_t. Each of these dynamics may have important effect on the filtered time series of the factors. In order to see what drives the dynamics of the filtered states I shut down each of these effects one by one. First, I filter the states with deterministic T_t , i.e. with zero $\sigma_{T,t}$, but with time-varying T_t. Second, I filter the states with constant T_t, by shutting down not only the dynamics due to $\sigma_{T,t}$, but also the one due to time-varying T_t. Finally, I look at the effect of time-varying volatilities on the dynamics of the filtered x_t by filtering with zero $\sigma_{T,t}$ and with constant volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$. These constant volatilities are set equal to the averages of their time-varying estimates. Figure 5.4 clearly shows that most of the dynamics of the filtered x is due to the dynamics of the exchange rate as other sources of dynamics do not seem to change the patterns of the filtered factors substantially.

6 Conclusion

This paper investigated market expectations concerning the locking rate of EMU candidate countries based on a theoretical model for exchange rate subject to future locking. The model is the conventional asset-pricing exchange rate model extended with the future locking assumption. In this model, the exchange rate converges to the actual market expectation concerning the locking rate in expected term. The asset-pricing model with final locking is a three-factor model, where the factors are the market expectations concerning the locking rate, the locking date and the latent exchange rate. In the empirical part of the paper, I used the model to filter out the subjective expectation of market participants concerning the locking rate for Czech Republic, Hungary and Poland.

One of the important contributions of the paper to the theoretical exchange rate literature is the following: it finds empirical support for the conventional asset-pricing exchange rate model. If we consider the survey-based expectation to be unbiased estimates on the expectation of the markets, then the exchange rate model used to filter the market expectation should be taken to be successful provided the filtered expectations are close to the reported expectations of the surveys. This test of the model is akin to the one applied by Engel and West (2005). Engel and West provide also some empirical support for this type of models based on the forecasting ability of the future fundamentals.[23]

Another important finding of the paper is that the stabilizing effect of the future locking is substantial. This finding is robust to the calibration of the parameters. Moreover, this qualitative result is in line with our view based on the term structure of option prices and also with our view purely based on Reuters polls data. In contrast to the low frequency Reuters polls data, our high frequency data on the filtered state variables enables us to measure the magnitude of the stabilizing effect. The estimated effect is that the volatility without future locking would be approximately twice as large as its current level in all three countries. The magnitude of the stabilizing effect depends on two determinants - the stability of market expectations concerning the locking rate, and the importance of expectations in determining the exchange rate. In case of an earlier euro area entry, the stabilizing effect is likely to be more substantial, because market expectations concerning the locking rate are themselves more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this intuitive argument, the prospect of locking should contribute the most to the stabilization of the koruna, compared to the other two currencies. Actually, the estimated stabilizing effect of locking is found to be the highest in Czech Republic.

As a next step, my future research will aim at examining the exchange rates of the main currency pairs by using the same idea presented in this paper. The exchange rate of the main currency pairs can also be expressed as a weighted average of a short and a long term component in accordance with the asset-pricing view. The long term component of the exchange rate can be justified not only by the possibility of final locking, but also by the long run reversion of the exchange rate to its PPP level. To complete the analogy between the locking and the no locking case, the uncertainty related to the half-lives of exchange rate deviations from PPP plays

[23] For many years, the standard criterion for judging the exchange rate models has been based on the forecasting power relative to the random-walk model. This criterion was first proposed by the seminal paper of Meese and Rogoff (1983). Recently, Engel and West (2005) has questioned the standard criterion. They have demonstrated on a similar model to the one in this paper that if fundamentals are integrated of order one and the discount factor is close to unity then the exchange rate will approximately follow random-walk. The important implication of this result is that the standard criterion is not useful at judging the performance of exchange rate models. Engel and West (2005) have proposed an alternative criterion, which is based on the forecasting ability of the future fundamental.

a similar role in case of the main exchange rates as the uncertainty related to the locking time of the analyzed currencies.

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Appendix A

Here, I prove, that the derived function $s_t = f(t, v_t, x_t, T_t)$ of (10) satisfies the implicit relationship between the exchange rate and fundamental (1).

By calculating the partial derivatives of (10) and by substituting these derivatives and $\mu_{v,t} = \mu_{x,t} = \mu_{T,t} = 0$ into (13), we obtain

$$ds_t = \left[\frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t) + \frac{1}{2}\frac{1}{c^2}e^{-\frac{T_t-t}{c}}(x_t - v_t)\sigma_{T,t}^2(T_t - t)^2 + \right.$$
$$\left. - \frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{x,t}\right)(T_t - t)\sigma_{T,t}\sigma_{x,t} + \frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{v,t}\right)(T_t - t)\sigma_{T,t}\sigma_{v,t}\right]dt +$$
$$+ \left(1 - e^{-\frac{T_t-t}{c}}\right)\sigma_{v,t}dz_{v,t} + e^{-\frac{T_t-t}{c}}\sigma_{x,t}dz_{x,t} - \frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t)(T_t - t)\sigma_{T,t}dz_{T,t}. \quad (26)$$

By using (26), the expected instantaneous change of the exchange rate can be expressed as

$$\frac{E_t(ds_t)}{dt} = \left[\frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t) \quad + \quad \frac{1}{2}\frac{1}{c^2}e^{-\frac{T_t-t}{c}}(x_t - v_t)\sigma_{T,t}^2(T_t - t)^2 \quad + \right.$$
$$\left. - \quad \frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{x,t}\right)(T_t - t)\sigma_{T,t}\sigma_{x,t} \quad + \quad \frac{1}{2}\frac{1}{c}e^{-\frac{T_t-t}{c}}\rho\left(dz_{T,t}, dz_{v,t}\right)(T_t - t)\sigma_{T,t}\sigma_{v,t}\right] . \quad (27)$$

The implicit function (1) can be rewritten as

$$\frac{E_t(ds_t)}{dt} = \frac{1}{c}(s_t - v_t) \quad . \quad (28)$$

Consequently, if the RHS of equation (27) is equal to the RHS of equation (28), then the implicit function (1) is satisfied by (10). In order to prove the equality, it is sufficient to show that the first term of the RHS of (27) is equal to the RHS of (28), whereas the other three terms of (27) sum up to zero.

By rearranging (10), we obtain that the first term of the RHS of (27) is equal to the RHS of (28).

$$\frac{1}{c}(s_t - v_t) = \frac{1}{c}e^{-\frac{T_t-t}{c}}(x_t - v_t) \quad . \quad (29)$$

What remains to prove is that the other three terms of (27) sum up to zero. After some simplification, we get that this zero condition is equivalent with

$$(x_t \quad - \quad v_t)\frac{\sigma_{T,t}(T_t - t)}{c} \quad - \quad \rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} \quad + \quad \rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} \quad = \quad 0 \quad . \quad (30)$$

In order to prove (30), I derive $\rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t}$ and $\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t}$ from equations (7), (8) and (9). Equation (9) restricts the partial effect of x_t on v_t to be zero ($\frac{\partial v_t}{\partial x_t} = 0$).

$$\rho\left(dz_{T,t}, dz_{x,t}\right)\sigma_{x,t} = \frac{dT_t}{dx}\frac{\sigma_{x,t}^2}{\sigma_{T,t}} = \left(\frac{\partial T_t}{\partial x_t} + \frac{\partial T_t}{\partial v_t}\frac{\partial v_t}{\partial x_t}\right)\frac{\sigma_{x,t}^2}{\sigma_{T,t}} = \left(\frac{\partial T_t}{\partial x_t}\right)\frac{\sigma_{x,t}^2}{\sigma_{T,t}} =$$
$$= \lambda_t(T_t - t)^{1-\frac{1}{\lambda_t}}\frac{1}{2\lambda_t}\frac{\sigma_t^2}{c\sigma_{x,t}^2}2x_t\frac{\sigma_{x,t}^2}{\sigma_{T,t}} = \frac{\sigma_{T,t}(T_t - t)}{c}x_t \quad . \quad (31)$$

$$\rho\left(dz_{T,t}, dz_{v,t}\right)\sigma_{v,t} = \frac{dT_t}{dv}\frac{\sigma_{v,t}^2}{\sigma_{T,t}} = \left(\frac{\partial T_t}{\partial v_t} + \frac{\partial T_t}{\partial x_t}\frac{\partial x_t}{\partial v_t}\right)\frac{\sigma_{v,t}^2}{\sigma_{T,t}} = \left(\frac{\partial T_t}{\partial v_t}\right)\frac{\sigma_{v,t}^2}{\sigma_{T,t}} =$$

$$= \lambda_t(T_t - t)^{1-\frac{1}{\lambda_t}}\frac{1}{2\lambda_t}\frac{\sigma_t^2}{c\sigma_{v,t}^2}2v_t\frac{\sigma_{v,t}^2}{\sigma_{T,t}} = \frac{\sigma_{T,t}(T_t - t)}{c}v_t \quad . \tag{32}$$

The derived formulas of (32) and (31) prove (30). Consequently, the three terms of (27) sum up to zero. Therefore, the RHS of equation (27) is equal to the RHS of equation (28). This equality proves that the implicit function (1) is satisfied by (10).

Appendix B

This appendix shortly introduces the yield curve method applied to estimate the market expectation concerning the locking date. The paper by Bates (1999) and Csajbók and Rezessy (2006) provide a more detailed description on this method.

The market expectation, formed at time t, on the date of locking T_t can be estimated from the forward differentials as follows. The expected value is calculated from the subjective probability distribution of the year the country enters the EMU.

$$T_t = \sum_{i=t}^{\bar{T}} p_t(EMU_i)\ \ i \tag{33}$$

where $p_t(EMU_i)$ is the probability that the market attaches at time t to the scenario in which the country becomes a full member of eurozone in the ith year. The distribution is assumed to have finite support, the country will join the eurozone in the year $\bar{T}$ at the latest. The marginal probability $p_t(EMU_i)$ can be calculated from the cumulative probability distribution $P_t(EMU_i)$. The interpretation of $P_t(EMU_i)$ is straightforward, this is the probability that the market attaches at time t to the scenario in which the country is in the EMU by the ith year. The cumulative probability can be derived from the pricing equation of the one-year forward interest differential.

$$FS_{t,i} = (1 - P(EMU_i))FS_{t,i}^{non-EMU_i} + P(EMU_i)FS_{t,i}^{EMU_i} \tag{34}$$

where $FS_{t,i}$ is the one-year forward interest rate differential for year i observed at time t. The $FS_{t,i}^{non-EMU_i}$ and $FS_{t,i}^{EMU_i}$ are the expected interest rate differentials under the two alternative scenarios, i.e. the accession country is either in or out the eurozone by year i. By rearranging (34) we obtain

$$P(EMU_i) = \frac{FS_{t,i}^{non-EMU_i} - FS_{t,i}}{FS_{t,i}^{non-EMU_i} - FS_{t,i}^{EMU_i}} \tag{35}$$

Among the right-hand-side variables of (35) only $FS_{t,i}$ is observable. However, by assuming that the analyzed countries will surly enter the EMU in nine years, and have almost zero chance to become an EMU member in one year, we can set $FS_{t,i}^{non-EMU_i} = FS_{t,t+1}$ and $FS_{t,i}^{EMU_i} = FS_{t,t+9}$.

Appendix C

This appendix presents the results of some sensitivity analysis. The parameters which are not subject of the analysis are calibrated exactly the same as in the baseline case presented in Table 5.4.

Table 7: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter c is calibrated to 7.25

	CZ	HU	PL
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	10.09%	13.72%	16.88%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-45.78%	-30.83%	-38.78%

Table 8: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter c is calibrated to 15

	CZ	HU	PL
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	16.31%	22.61%	26.79%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-66.45%	-58.03%	-61.41%

Table 9: The volatility of the exchange rate (S) and the latent exchange rate (V) – Parameter $\frac{dx_t}{dT_t}$ is calibrated under the assumption that changes in the expected locking date are due to inflationary shocks

	CZ	HU	PL
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	12.81%	17.65%	21.28%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-57.29%	-46.24%	-51.43%

Table 10: The volatility of the exchange rate (S) and the latent exchange rate (V) The window size used to smooth the original filtered T_t is 41

	CZ	**HU**	**PL**
σ_S	5.47%	9.47%	10.41%
$\sigma_{\hat{V}}$	13.09%	17.65%	20.5%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-58.18%	-46.31%	-49.24%

Table 11: The volatility of the exchange rate (S) and the latent exchange rate (V) The window size used to smooth the original filtered T_t is 11

	CZ	**HU**	**PL**
σ_S	5.45%	9.5%	10.31%
$\sigma_{\hat{V}}$	13.19%	18.13%	21.5%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-58.66%	-47.61%	-52.06%

Table 12: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with deterministic locking time

	CZ	**HU**	**PL**
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	13.03%	17.7%	21.04%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-58.01%	-46.4%	-50.87%

Table 13: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with constant locking time ($T_t = T_{t_0}$ where $t_0 = 5.Jan.2005.$)

	CZ	**HU**	**PL**
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	12.46%	23.72%	21.87%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-56.09%	-59.99%	-52.73%

Table 14: The volatility of the exchange rate (S) and of the latent exchange rate (V) – with deterministic locking time and constant volatilities $\sigma_{x,t} = \bar{\hat{\sigma}}_{x,t}$, $\sigma_{v,t} = \bar{\hat{\sigma}}_{v,t}$

	CZ	**HU**	**PL**
σ_S	5.47%	9.49%	10.34%
$\sigma_{\hat{V}}$	11.68%	13.34%	18.6%
$\frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}}$	-53.14%	-28.84%	-44.42%

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Appendix D

This appendix shortly introduces the Kalman filter. It derives the estimator of the state vector $\Lambda(t)$ for the general model and also for the restricted model with constant T_t. The estimator of the restricted model clearly shows that Kalman filter can be interpreted as follows. It decomposes the exchange rate changes ds into changes in the state variables v and x. The higher is the relative weight of v in s, and the higher is its volatility relative to that of the other factor $\frac{\sigma_v}{\sigma_x}$, the higher change is attributed to v by the Kalman filter.

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector $\Lambda(t)$, based on information available at time t. The state vector of time t is estimated from the previous periods' estimated state $\hat{\Lambda}(t-1)$ and the estimated covariance matrix of the estimation error $\hat{Cov}(v(t-1))$.

The predicted value of state $\Lambda(t)$ is $A(t-1)\hat{\Lambda}(t-1)$ based on equation (18) and the prediction error is $v(t|t-1) = \Lambda(t) - A(t-1)\hat{\Lambda}(t-1) = A(t-1)(\Lambda(t-1) - \hat{\Lambda}(t-1)) + w_1(t-1)$. The predicted state is different from the estimated state, because the former is calculated purely from the estimated state of the previous periods and does not utilize the time t realization of the observable variable $\Omega(t)$. Therefore, the prediction error is different from the estimation error, that is $v(t-1) = \Lambda(t-1) - \hat{\Lambda}(t-1)$ for the previous period. The prediction error can be expressed as a function of the previous periods' estimation error $v(t|t-1) = A(t-1)v(t-1) + w_1(t-1)$ and its covariance matrix can be written as

$$Cov(v(t|t-1)) = A(t-1)Cov(v(t-1))A^{'}(t-1) + Q(t-1) \quad . \tag{36}$$

Now, the estimator of the state vector of time t is conditional not only on the estimates of the previous state vector, but also on the contemporaneous observable variable $\Omega(t)$. In our system $\Lambda(t)$ and $\Omega(t)$ are jointly normally distributed, consequently, the expected value of the vector of states $\Lambda(t)$ conditional on the observation $\Omega(t)$ is

$$E(\Lambda(t)|\Omega(t)) = E(\Lambda(t)) + Cov(\Lambda(t), \Omega(t))Var^{-1}(\Omega(t))(\Omega(t) - E(\Omega(t))) \quad . \tag{37}$$

Equation (37) is the key for the estimator of the state vector. The estimator of the state vector of time t can be obtained by substituting the estimators of the terms on the right-hand-side into equation (37). First, let us see what are the estimators for these terms, than we can put together the estimator for $\Lambda(t)$. Equation (18) shows that the unconditional expected value

$$E(\Lambda(t)) = A(t-1)\Lambda(t-1) \quad . \tag{38}$$

The covariance is $Cov(\Lambda(t), \Omega(t)) = E[(\Lambda(t) - E(\Lambda(t)))(\Omega(t) - E(\Omega(t)))^{'}]$, therefore, one can estimate it by

$$\begin{aligned} &\hat{Cov}(\Lambda(t), \Omega(t)) = E[(\Lambda(t) - A(t-1)\hat{\Lambda}(t-1))(C(t)\Lambda(t) + w_2(t) - C(t)\hat{\Lambda}(t))^{'}] = \\ &= E[(\Lambda(t) - A(t-1)\hat{\Lambda}(t-1))(\Lambda(t) - A(t-1)\hat{\Lambda}(t-1))^{'}C^{'}(t)] = \hat{Cov}(v(t|t-1))C^{'}(t) \quad . \end{aligned} \tag{39}$$

The observable $\Omega(t) = C(t)\Lambda(t) + w_2(t)$, therefore its variance is

$$Var(\Omega(t)) = C(t)Cov(v(t|t-1))C^{'}(t) + R(t) \quad . \tag{40}$$

By substituting equations (38), (39), and (40) into equation (37) we get that the estimator for the state vector at time t is

$$\begin{aligned} &\hat{\Lambda}(t) = A(t-1)\hat{\Lambda}(t-1) + \\ &+ \hat{Cov}(v(t|t-1))C^{'}(t)(C(t)\hat{Cov}(v(t|t-1))C^{'}(t) + R(t))^{-1}(\Omega(t) - C(t)A(t-1)\hat{\Lambda}(t-1)) \quad . \end{aligned} \tag{41}$$

Where $\hat{Cov}(v(t|t-1)) = A(t-1)\hat{Cov}(v(t-1))A^{'}(t-1) + Q(t-1)$.

What we are left with is to provide estimates for $Cov(v(t))$ that will be used in the next step of the recursive process of the Kalman filter. The estimator is given by

$Cov(v(t)) = Cov(v(t|t-1)) - Cov(v(t|t-1))C^{'}(t)(C(t)Cov(v(t|t-1))C^{'}(t) +$
$+ R(t))^{-1}C(t)Cov(v(t|t-1))$.

After deriving the Kalman estimator for the state vector in the general case, I will show that the estimator reduces to a simple, easily interpretable formula in a special case. This special case is the one, where we have constant or deterministic T_t, the noise w_2 is restricted to zero. For the sake of simplicity, I assume also that $Cov(v(t-1)) = 0$, or in other words, the state of the previous period is known. Under these assumptions the variance of the observation error R is zero, the system matrix A is the identity matrix; and $Cov(v(t|t-1)) = Q(t-1)$ is diagonal. In this special case the general formula (41) simplifies to

$$\hat{\Lambda}(t) = \hat{\Lambda}(t-1) + Q(t-1)C^{'}(t)(C(t)Q(t-1)C^{'}(t))^{-1}(\Omega(t) - \Omega(t-1)) \quad . \tag{42}$$

By substituting $\Lambda^{'}(t) = \begin{pmatrix} v_t & x_t \end{pmatrix}$, $\Omega(t) = s_t$, $C(t) = \begin{pmatrix} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{T_t - t}{c}} \end{pmatrix}$, and $Q(t-1) = \begin{pmatrix} \sigma^2_{v,t-1} & 0 \\ 0 & \sigma^2_{x,t-1} \end{pmatrix}$ into (42) we get the following estimator for the two state variables of time t:

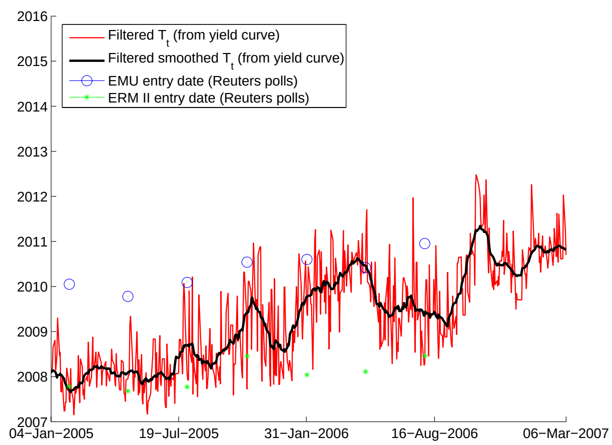
$$\hat{v}_t = \hat{v}_{t-1} + \frac{1 - e^{-\frac{T_t - t}{c}}}{(1 - e^{-\frac{T_t - t}{c}})^2\sigma^2_{v,t-1} + (e^{-\frac{T_t - t}{c}})^2\sigma^2_{x,t-1}}\sigma^2_{v,t-1}ds_t \tag{43}$$

$$\hat{x}_t = \hat{x}_{t-1} + \frac{e^{-\frac{T_t - t}{c}}}{(1 - e^{-\frac{T_t - t}{c}})^2\sigma^2_{v,t-1} + (e^{-\frac{T_t - t}{c}})^2\sigma^2_{x,t-1}}\sigma^2_{x,t-1}ds_t \tag{44}$$

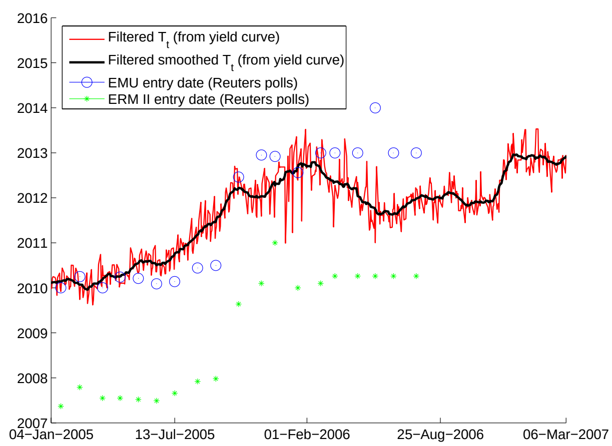
These estimators provide an intuitive interpretation of the Kalman filter in this exchange rate model. It decomposes the exchange rate changes ds into changes in v and x. The higher is the relative weight of v in s, i.e the higher is $1 - e^{-\frac{T_t - t}{c}}$, and the higher is its volatility relative to that of the other factor $\frac{\sigma_v}{\sigma_x}$, the higher is the estimated innovation in v. Similarly, the higher is the relative weight of x in s, and the higher is its volatility relative to that of the other factor $\frac{\sigma_x}{\sigma_v}$, the higher is the estimated innovation in x.

Figure 1: The average expectation of the analysts concerning the EMU and the ERM II entry date and the filtered locking date T_t

Czech Republic



Hungary



Poland

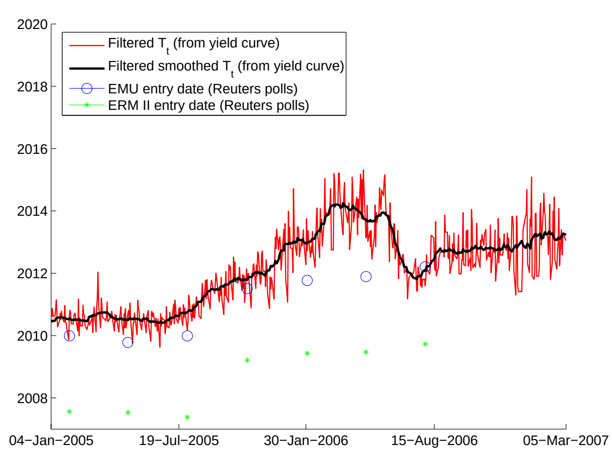
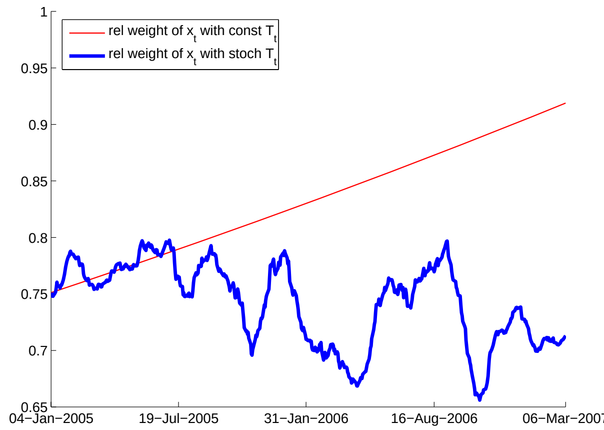


Figure 2: The relative weight of the expected log locking rate x_t in the log exchange rate s_t – with constant and stochastic locking date

Czech Republic



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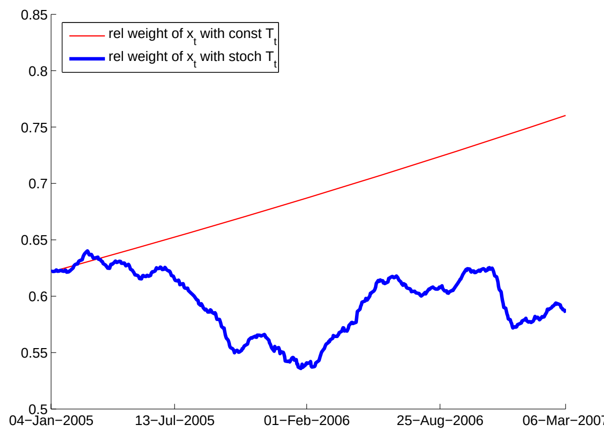


Poland

Figure 3: Stylized inflation paths until locking

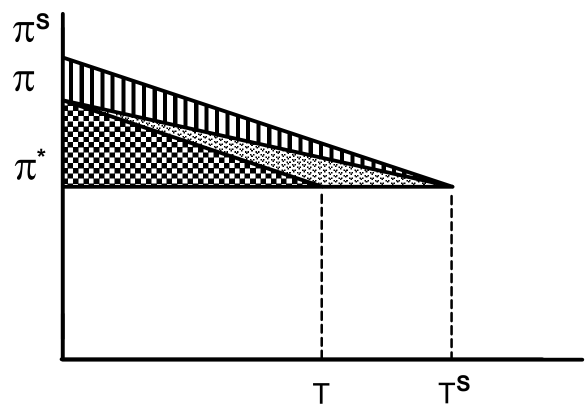
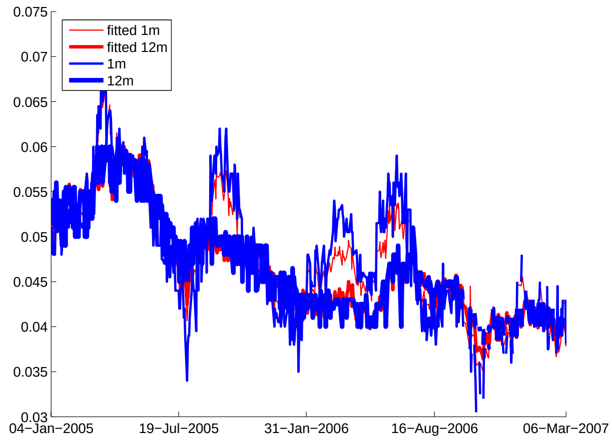


Figure 4: The option prices with the shortest and longest maturities and the fitted values in terms of volatility – with stochastic locking date

Czech Republic



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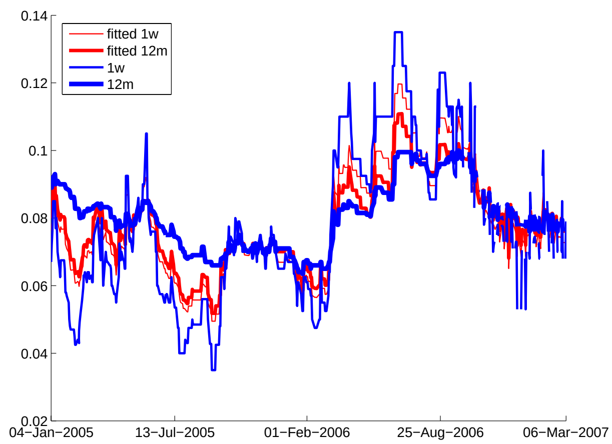

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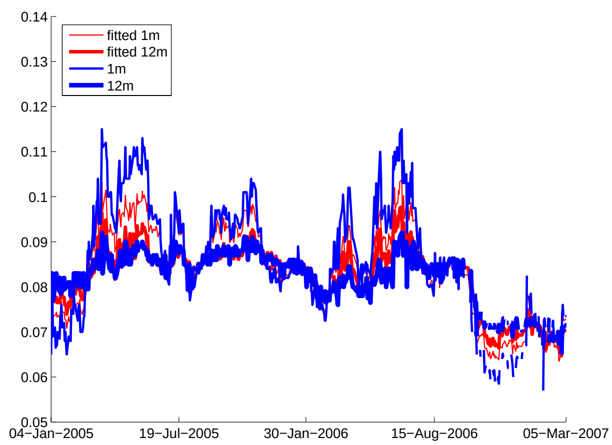
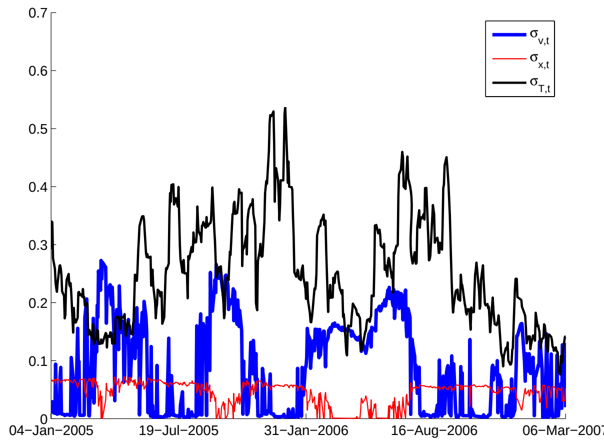


Figure 5: The estimated volatilities of x_t, v_t and T_t − with stochastic locking date

Czech Republic



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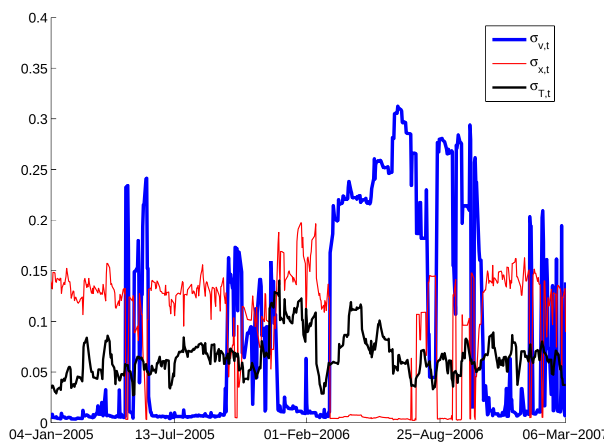

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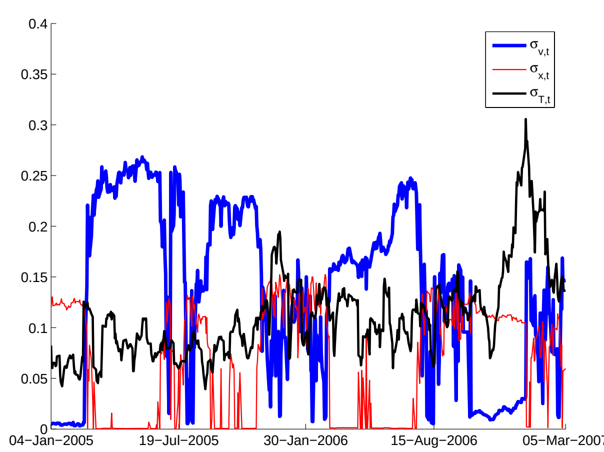
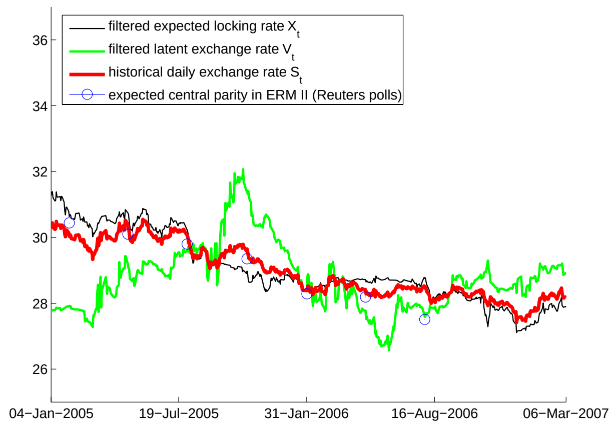
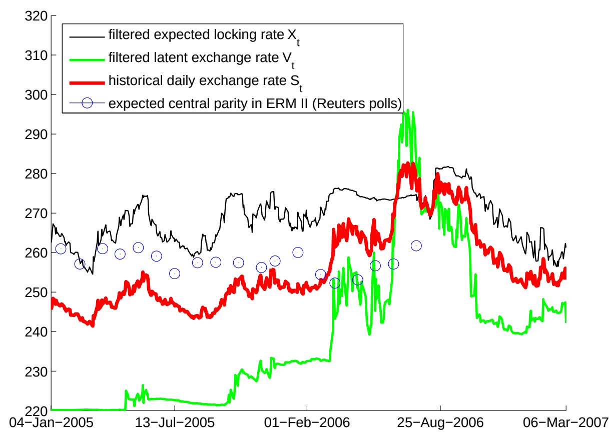


Figure 6: The filtered expected locking rate, the latent exchange rate, the historical exchange rate and the survey-based expected locking rate – with stochastic locking date

Czech Republic



Hungary



Poland

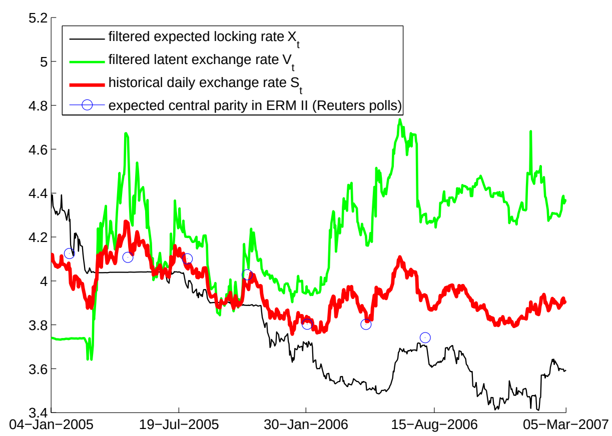
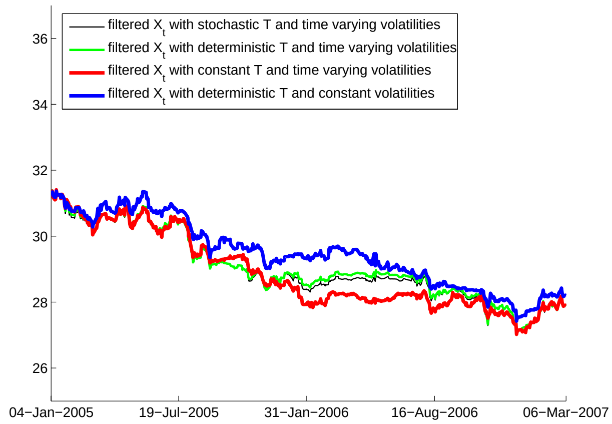


Figure 7: The filtered expected locking rate with stochastic, deterministic and constant T_t and with time-varying and constant $\sigma_{x,t}$ and $\sigma_{v,t}$

Czech Republic



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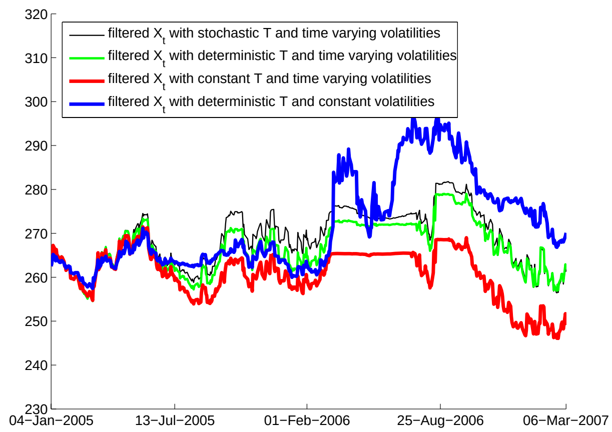


Poland

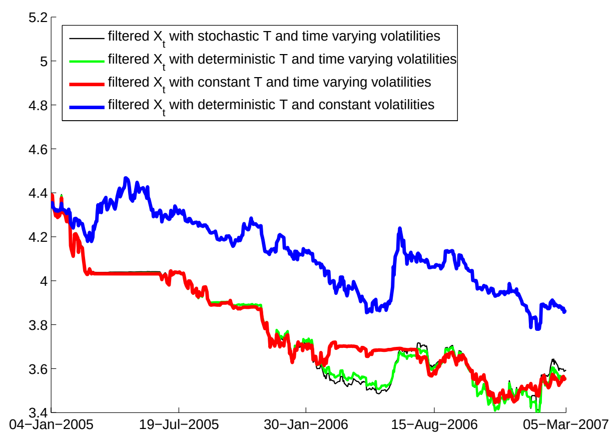
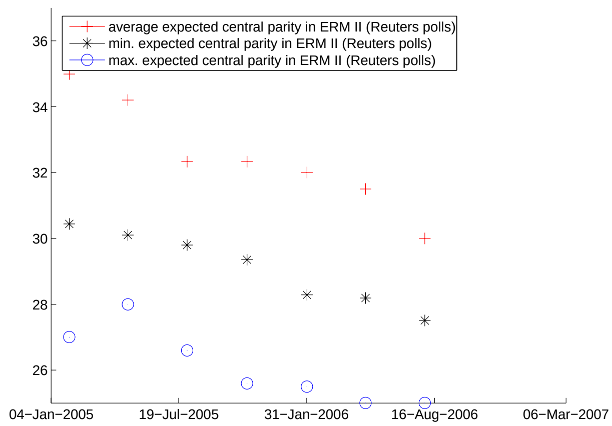
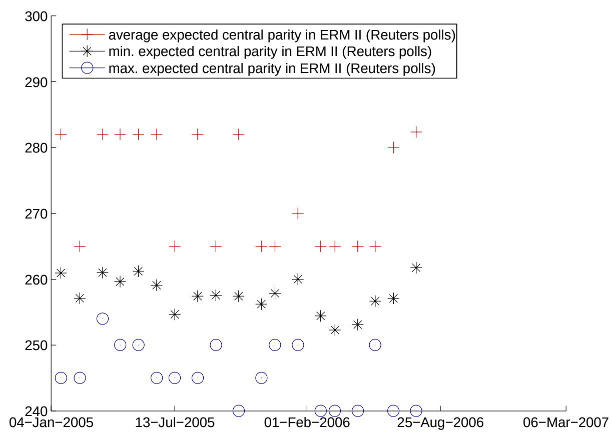


Figure 8: Minimum, maximum and average expectation of the market analysts concerning the central parity in the ERM II

Czech Republic



Hungary



Poland