The behavior of the nominal exchange rate at the beginning of disinflations *

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Abstract

A standard rational expectations model would give very strong predictions about the behavior of the nominal exchange rate at the beginning of a disinflation (monetary restriction): a substantial initial appreciation, followed by a steady depreciation. It largely conflicts actual observations, like the current experience of Poland and Hungary, where an initial appreciation was not followed by any systematic depreciation. The paper tries to explore whether rational expectations can be rescued by introducing noise and parameter learning into such a model. An optimistic learning case (worse than expected inflation data every period), or the combination of a pessimistic learning case (better than expected data every period) and a declining proportional risk content of the interest rate offers a potential explanation.

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1 Introduction

This paper tries to resolve the inability of a frictionless rational expectations model (one that builds on interest parity) to match the observed behavior of the nominal exchange rate at the beginning of disinflations. Interest parity would imply a large initial appreciation (following a surprise interest rate hike) and then a gradual depreciation (reflecting the equalization of

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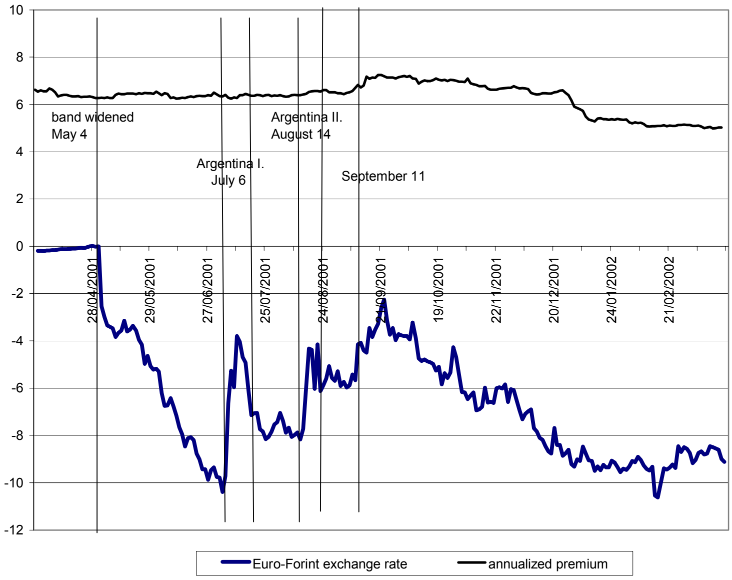

Figure 1: The Forint-Euro exchange rate and the interest differential

expected returns on domestic and foreign bonds, including the capital loss of holding domestic currency). Actual observations also show the initial strengthening, but then the exchange rate shows no clear sign of a reversal.

Figure 1 depicts the evolution of the Forint-Euro exchange rate and the difference of 3-month Forint and Euro benchmark yields, in 2001-2002. As shown on the picture, the current phase of disinflation started on May 4, 2001: the Forint, which used to have a $\pm 2.25\%$ band, was allowed to move freely within a $\pm 15\%$ band. The immediate response was a heavy appreciation (though part of it might have reflected an initial undervaluation), in line with interest parity (reflecting the attractive bond yield, which could not have appreciated the Forint further in the previous narrow band). Later on, however, actual and predicted behavior diverged: apart from three large depreciation episodes (two turmoils related to Argentina, and the consequences of September 11), the exchange rate showed a general appreciating tendency (though after the first two episodes, there seems to have been a correction, but it clearly vanished after the third episode). On the other hand, the excess bond yield was stable and substantial.

Figure 2 conveys the similar experience of Poland. The beginning of the sample is the first interest rate hike within the inflation targeting regime – the first indication of a true shift in monetary policy –, which was followed by three further contractionary steps. This period observed a gradual strengthening of the Zloty, up to June 2000 (a quarter before the last rate increase), which was followed by large swings, but without any tendency of reversal. Interest

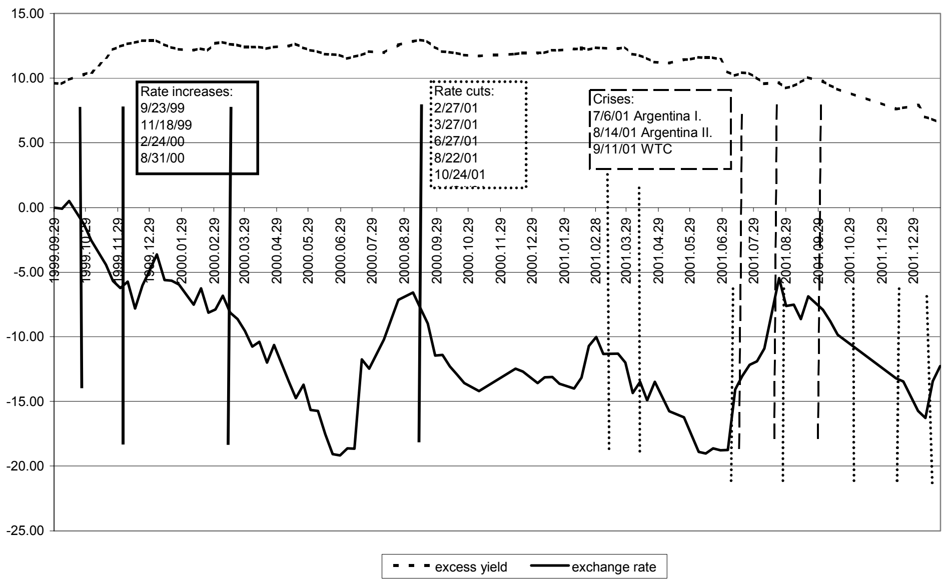

Figure 2: The Zloty-Euro exchange rate and the interest differential

rate differentials, however, remained large, regardless of Zloty interest rate changes.

A similar puzzle is the delayed overshooting finding of Eichenbaum and Evans (1995). They document that after a monetary contraction, nominal exchange rates show a gradual appreciation, followed by a gradual reversal. The first finding is in line with the Hungarian and Polish experience as well, but the similarity of the gradual reversal is not.

As put by Obstfeld and Rogoff (1996), page 622: "While conventional wisdom holds the Mundell-Fleming-Dornbusch model to be useful in predicting the effects of major shifts in policy, its ability to predict systematically interest-rate and exchange rate movements is more debatable." The underlying force in that model is again the uncovered interest parity condition, which predicts the same strong but empirically questionable behavior of the exchange rate.

The puzzlingly poor track record of uncovered interest parity is well known: a classical documentation and interpretation is offered in Fama (1984), and surveyed in Froot and Thaler (1990) and Isard (1995), among others. This paper does not aim at any general evaluation or rescue of the UIP hypothesis: the narrow objective is to focus on marked disinflation episodes.

To frame the discussion and main points, I will adopt a forward-looking "small macromodel" of an open economy (along the lines of Svensson (2000)). The motivation for this choice is at least twofold: it allows for an explicit treatment of the disinflation process, highlighting the relation between inflation, interest rates and exchange rate behavior; besides, this is also in line with the current major monetary framework for modeling inflation. Benczúr, Simon and Várpalotai

(2002b) offers a general but simple description of inflationary dynamics in small macromodels. In this paper, I will just summarize the necessary results, and using a substantially simplified version, I address the behavior of the nominal exchange rate, under Bayesian learning.

In particular, I want to investigate whether the following consideration can be a qualitatively and quantitatively important factor in determining the nominal exchange rate. At the beginning of a disinflation, it should be quite clear for investors that the currency will offer a medium-term excess yield, thus leading to massive capital inflows, and a large initial appreciation (reflecting not just the immediate excess yield, but also its "persistence"). Apart from this "obvious" step, a disinflation then continues with many uncertainties: about the determination of the central bank, the effectiveness of monetary policy tools (like the exchange rate pass-through, the effect of real interest rates on the output gap, and the disinflationary effect of the output gap, etc.), the persistence of inflationary expectations, just to name a few. This means that every major data announcement represents an additional surprise, which can counteract the trend depreciation of the currency. In a modeling language, this means that I maintain rational expectations, but introduce noisy signals: I relax the assumption of perfect foresight, model-consistent expectations.

A traditional channel for such uncertainties is the behavior of the central bank itself: how much costs it is ready to tolerate, consequently, how aggressively it would react to changes in inflation. There is a potential signal extraction problem here as well: markets do observe inflation and interest rate data, but the central bank may (and often should) react to changes only in core inflation, and neglect temporary inflationary disturbances. It means that markets have to "reverse" the decomposition done by the central bank to infer its true behavior.

In an inflation targeting framework, this uncertainty is likely to be less serious, since the regime operates under a high degree of transparency. The central bank clearly communicates its motivations, the decomposition of inflation into permanent and temporary components is made public knowledge. There is still room for uncertainties, which are in fact shared by the central bank and market participants: after a regime shift (moving into a disinflation, or changing the monetary policy framework), many of the monetary mechanisms might have changed, or forces that used to be non-operational might have become active, so their size or strength is not known precisely. However, as shown in Benczúr, Simon and Várpalotai (2002b), the dynamics of inflation, hence the behavior of interest rates and exchange rates can be quite sensitive to such parameters. Among many others, the effect of the output gap on inflation, or the strength of the exchange rate pass-through can be the source of such aggregate uncertainty.

An akin idea is followed by Lewis (1989): there is uncertainty about the change in the money demand process, and markets learn the new situation only gradually. That paper succeeds in quantifying the bias this learning causes (ex post), using actual data. Though my approach is similar, the focus is shifted in many ways: first, I want to concentrate on the specifics of disinflations, which is a clear restrictive shock, and it is only its effect but not the change itself that is uncertain. Second, by considering the links between inflation, interest rates and exchange rates, the source of uncertainty will be more structural. For this reason, I need to model their determinants and interdependence, in particular, to use an interest rate rule for the central bank.This also enables me to track the performance and the components of the uncovered interest parity condition itself (changes in the long-run nominal exchange rate, the cumulative excess yield, and its risk content). Third, in my model, there is a feedback from imperfect expectations to the inflation process, thus back to the signal extraction problem as well. Finally, current monetary regimes use the nominal interest rate as their policy tool, so the endogeneity of interest rates needs to be incorporated into the analysis.[1] Unfortunately, the short time period of potential observations, and the complications implied by my version of parameter learning made it impossible to quantify this argument in any econometric sense.[2]

Since my objective is to explain an approximately year-long episode, the speed of learning is a key concern. One may accept that it took financial markets some weeks to digest changes, but is it reasonable to have learning even after a year? In my view, I am on safe ground here: to learn about structural parameters of previously inactive (or different) forces, effects, one essentially needs new observations. For aggregate links like between inflation and output, or real exchange rates, the relevant frequency is monthly, or rather, quarterly. Moreover, the exact nature of such relationships is often unclear, and there is enormous noise in the observations. So even after an entire year, one still has only twelve (or four) noisy observations, which will not yield precise estimates. In Lewis (1989), where learning was about the money demand process, the data suggested a 1-3 years span of learning. For aggregate inflation, I would expect the speed of learning to be even slower.

Can the analysis of such a particular episode add anything to the general uncovered interest parity debate? My story tells us that in these episodes, where a parameter changes (due to some

[1] It is not straightforward to determine which interest rate should be used in the interest parity condition, or how such an interest rate is influenced by the rate decisions of the central bank. To avoid these complications, I assume that the central bank sets the relevant interest rate directly, and any additional interest rate can be obtained by using the expected future rates.

[2] The main complication is caused by the nonlinearity of the structural model: all variables are linear in inflation, but highly nonlinear in the underlying structural parameters. This makes the explicit calculation of conditional expectations practically impossible.

regime switch), a previously inactive channel becomes operational, or market participants need to tell persistent supply and short-lived demand fluctuations apart, there is a substantial ex post bias in interest parity. If such episodes are relatively long – measured in years –, then we may run into sample size problems even with ten years of monthly or weekly data: the long-run average of the ex post bias is zero, but we will have only 5-6 such episodes in our data, insufficient for cancelling the bias. A further indication is the finding of Lewis (1989), that learning can explain half of the observed bias of interest parity in a nearly 3-year long period.

The uncertainties associated with early stages of a disinflation (on top of "regular" shocks to output etc.) may also lead to a gradual entry of foreign bond investors. Though this can be compatible with rationality,[3] relaxing the perfect market assumption may end up being necessary: Figures 1 and 2 may also suggest that investors were responding only gradually to high interest rates. That could have decreased the initial appreciation, and allowed the strengthening later, or at least no deprecation. Still, it my view it is important to relax the perfect foresight assumption first, instead of relaxing the rationality of expectations.

The paper is organized as follows. The next section examines the perfect foresight behavior of the nominal exchange rate. Section 3 describes the small macromodel framework for endogenizing inflation. In Section 4, parameter learning is introduced into this framework. The behavior of the realized exchange rate is analyzed in Sections 5 and 6, first under the assumption of no risk premium (a fixed proportion), and then allowing for a systematically changing risk premium content. Finally, Section 7 offers some concluding remarks, and the Appendix contains a formal but not fully rigorous treatment of the explicit learning process.

2 The rational expectations behavior of the nominal exchange rate

In this section I explore the behavior of the nominal exchange rate at the beginning of a disinflation, under the assumption of model-consistent expectations (perfect foresight if there is no uncertainty, rational expectations if there is any noise). These results are completely independent from whether we believe that the economy is described accurately by a small macromodel or not: I will use only rational interest rate and inflation forecasts, and the uncovered inter-

[3] With risk-aversion, investors would have a finite (less than perfectly elastic) demand for the currency. If they receive a positive flow of fresh funds to invest, that would imply a slow but steady capital inflow, but this effect is likely to be weak. Positive surprises (decreasing the riskiness of the currency) can lead to a continuous and sizable inflow, thus a maintained appreciation period. This, however, is already within the area of our analysis: one of our scenarios will have the same feature, with a changing risk content of domestic interest rates.

est parity condition (thus assuming risk neutrality and no market frictions), and some form of purchasing power parity.[4]

2.1 Perfect foresight (no uncertainty)

Start from the interest parity condition for the nominal exchange rate (in logarithmic form):

$$s_t = s_{t+1|t} - i_t + \phi_t,$$

where s_t denotes the current value of the nominal exchange rate, i_t is the current nominal interest rate (in excess of world interest rates) ϕ_t is a risk premium term,[5] and $s_{t+1|t}$ is the expected exchange rate one period ahead. All time t expectations are taken at the *beginning* of period t.

At the beginning of disinflation, there is a surprise regime change: in the case of Hungary, it was to let the currency out of a narrow band.[6] Using rational expectations again, market participants now have the ability to predict the central bank's behavior under the new regime, and all of its consequences (assuming that the wide band does not become binding). This means that they form an infinite sequence of expected interest rates, inflation, output gap etc. Iterate interest parity along this expected path:

$$s_t = s_{t+2|t} - \left(i_{t+1|t} - \phi_{t+1|t}\right) - (i_t - \phi_t) = ... = s_{\infty|t} - \left((i_t - \phi_t) + \left(i_{t+1|t} - \phi_{t+1|t}\right) + ...\right). \tag{1}$$

The current exchange rate is the difference of the long-run expected exchange rate (we will see its existence later on) and the cumulative (risk-free) excess interest rate. Interest rates need to be adjusted for the *current ("expected") levels* of current and future risk premia. Since it is possible that new information emerges during the process of disinflation, for example there

[4] My assumption about the rationality of inflation expectations also involves a long-term stability assumption: inflation must converge to its equilibrium value sufficiently fast, thus the sum $\sum \pi_t$ – which defines the long-run price level – exists. With noise in the model, it applies to expected values. For a linear model, it is a relatively weak requirement: for the solvability and stability of such a model in general, inflation must disappear asymptotically, which happens with an exponential speed. The role of this assumption is to ensure that the nominal exchange rate has a long run value: with the real exchange rate reaching an equilibrium value, and the price level converging to some constant, the nominal exchange rate is also constant.

[5] One can include a nonzero risk premium even under "perfect foresight": markets may price in the probability of an event that changes the model itself. This event never happens along the equilibrium path, but its risk is incorporated into the exchange rate every period. Section 6 offers a more detailed interpretation and discussion of the risk premium term.

[6] Under the narrow currency band, the currency had no room for further appreciation, or a substantial reversal later on. This situation was altered by the increased bandwidth. Besides, there were signs of an undervaluation: see, for example, Halpern and Wyplosz (1997) on the real undervaluation in transition economies in general. Kovács (2001) estimates the initial undervaluation around 5%, which is smaller than the initial appreciation of 10%.

is learning about the strength of certain monetary effects, or how much costs the central bank is ready to accept, $\phi_{t+1|t}$ and ϕ_{t+1} may be in general different from each other (and the same applies to $\phi_{t+2|t}$ and $\phi_{t+2|t+1}$ etc.).

The long-run value of the nominal exchange rate is determined by the equilibrium level of the real exchange rate, implied by some form of purchasing power parity. Normalize this level to zero. Since the real exchange rate satisfies

$$q_t = s_t + p_t^* - p_t, \tag{2}$$

we must have

$$0 = q_{\infty|t} = s_{\infty|t} + p_{\infty|t}^* - p_{\infty|t} = s_{\infty|t} + p_{t-1}^* - p_{t-1} - \left(\pi_t + \pi_{t+1|t} + ...\right).$$

This implies

$$s_t = p_{t-1} - p_{t-1}^* + \left(\pi_t + \pi_{t+1|t} + ...\right) - \left((i_t - \phi_t) + \left(i_{t+1|t} - \phi_{t+1|t}\right) + ...\right). \tag{3}$$

The current level of the nominal exchange rate is thus determined by the current price level differential (it is only a matter of normalization), the cumulative excess expected inflation and expected interest rates. Therefore, the long run value of the nominal exchange is well-defined if there is a limit of the price level at infinity, i.e., the series of excess inflation is summable (converges to zero fast enough). In contrast to the real exchange rate and inflation, the long run value of the nominal exchange rate cannot be considered as an equilibrium variable: for example, it depends on initial conditions.

How does the exchange rate evolve through time? Assume that initially $p_{t-1} = p_{t-1}^*$, $q_{t-1} = 0$ (the process starts from the current equilibrium value of the real exchange rate[7]), then $s_{t-1} = 0$. Markets learn the regime change at the "middle" of period t, and then s_t is set by (3). It seems safe to assume that interest rates are overall restrictive, meaning that the riskless real interest rate is positive, then (3) implies an initial appreciation (nominal and real as well): not just based on the current high level of the nominal interest rate, but the entire cumulative excess

[7] In case of an equilibrium real appreciation (or depreciation), let q_t denote the deviation from the equilibrium level. This appreciation then must be matched in the equilibrium path of inflation, thus π_t is also the deviation from the sum of foreign inflation and the structural excess inflation, caused by the real appreciation; and p_t^* is the hypothetical equilibrium price path, starting from the initial price level. Then (2) remains valid, and the long run behavior of inflation and the real exchange rate is still consistent with a fixed nominal exchange rate. This procedure essentially means that we think of inflation and the real exchange rate only of domestically produced and consumed goods.

yield should have its full effect immediately.

After this first surprise, without further news or noise – thus all expectations being equal to the model-consistent realizations –, high interest rates imply a steady depreciation:

$$s_{t+1} = s_{t+1|t} = s_t + i_t,$$

so if i_t is positive, there is a depreciation. This follows from interest parity: if high interest rates are foreseen, then indifference between domestic and foreign bonds requires a capital loss on domestic bonds, i.e., a depreciation. In the long run, the interest differential converges to zero, and the exchange rate becomes constant.

This constant level (s_∞) is necessarily weaker than the initial value $s_{t-1} = 0$: for the real exchange rate to return to its original level, any cumulative excess inflation must be exactly offset by the long-run nominal depreciation. We have positive excess inflation at the beginning, so unless it becomes heavily negative for quite some time, the increase in the domestic price level will be larger than that of foreign, so we must have a long-run nominal depreciation. If the real exchange rate was undervalued by, say, 5% initially, then the same argument applies relative to a long-run appreciation of 5% (and not to 0%).[8] Having a negative cumulative inflation is not necessarily unreasonable, since this would refer to a negative inflation on top of "structural" inflation, as implied by foreign inflation and the Balassa-Samuelson effect (or any other factor that causes the equilibrium real exchange rate to appreciate). Therefore, it need not mean a true deflation, but only a smaller than "equilibrium" level of inflation.[9]

2.2 Introducing noise

These considerations remain mostly unaltered if there is noise in the economy, but without any informational content. If both market participants and the central bank know the true parameters of the system from the very beginning, then we can write identical equations for the expected values of the same variables, and we get only mean zero deviations, with some potential persistence though. The nominal exchange rate might show fluctuations around the depreciating trend, but its trend should be a gradual weakening.

To get a rational deviation from this strong prediction, one needs to introduce informative

[8] We still have that $s_\infty < 0$, but $s_{t-1} = q_{t-1}$ implies that $s_\infty < s_{t-1} - q_{t-1}$, which means an at most q_{t-1} appreciation.

[9] The equilibrium level of inflation is in general the foreign inflation level. Under an equilibrium real appreciation (due to excess productivity growth, for example), the equilibrium level of inflation becomes higher as well. See also footnote 7 on page 8.

surprises into the economy (rational learning). A straightforward first interpretation is that there are many parameters uncertain in the economy, and each new observation leads to new estimates. If there is really new information in new data points, then this is a better estimate, but if the new information comes in the form of a noisy signal, then the new estimate is still not perfect.

Given the new estimate, market participants would rationally update their interest rate and inflation forecasts, and the new level of the exchange rate would reflect this information, thus being different from the level of the exchange rate predicted in the previous period.

The interest parity condition would still hold between the current and the expected next period exchange rate, but not between realized exchange rates.[10] Moreover, the forecast based on the old estimates will look biased *ex post*: knowing the direction of the update, one would have predicted the prevailing appreciation or depreciation correctly. Now suppose that information is always about a faster than expected disinflation, matched by a more than proportional reduction in anticipated future interest rates (assuming a higher than one inflation coefficient in the reaction function of the central bank). Then every period comes with a decrease in cumulative real interest rates, thus an overall monetary easing, leading to a depreciation bias of the actual exchange rate.

Looking ex post at many periods of the exchange rate (current and predicted) , one would find a significant bias in the predictions, but that statement involves an ex post conditioning on the fact that new information was always about an even faster disinflation. At any moment, the market formed its best forecast based on current observations, and this forecast was updated systematically in one direction.[11] Ex post we do know that some uncertain parameter was higher than initially expected, but due to noisy signals, the best feasible estimate was only converging to this true value.

In ex post terms, Gourinchas and Tornell (2001) explores a very similar, though more general idea behind the forward premium puzzle: in their scenario, market participants underestimate *ex ante* the persistence of interest rate changes, which leads to the failure of interest parity. In my model, people can be subject to a similar but *ex post* underestimation: based on their too optimistic expected disinflation path, they also expected interest rates to return to normal faster

[10] Looking at Reuters polls about market expectations of the HUF/EUR exchange rate, it shows no reversal either, thus casting doubts even on this form of interest parity.

[11] This can be also interpreted as a peso-problem: we would need a sufficiently large number of observations corresponding to both directions of updates, yielding a zero (unconditional) expectation of the exchange rate prediction error. From a sample selection approach, when we observe mostly one direction of updating, we face a nonzero conditional expectation of this prediction error – conditional on some parameter being higher than originally expected.

than the realization. The first key difference is that they can also overestimate interest rates ex post, if they were too pessimistic in their inflation forecast, and the second is that there are no ex ante misperceptions.

To give this rough idea a formal framework, I will have to be explicit about the determinants of inflation and interest rates. For this reason, I adopt a small macromodel description of the economy, and introduce a potential for gradual learning.

3 The small macromodel framework

Here I briefly describe the dynamic system defining the economy, and summarize the behavior of such a model based on Benczúr, Simon and Várpalotai (2002b). Then parameter uncertainty and learning can be introduced explicitly.

3.1 The single equation reduced form

As explained in many places (like Svensson (2000), or Gali and Monacelli (2002)), the Calvo sticky price model can be reduced to a convenient dynamic system, consisting of an aggregate supply equation (Phillips curve), and aggregate demand relation, and a reaction function. The key ingredient of these models is the "new keynesian Phillips curve"

$$\pi_t = \beta y_t + \lambda E\pi_{t+1|t}, \tag{4}$$

where y_t is the output gap, and λ is approximately one. This equation, however, means the persistence of the price level, but not inflation itself. For this reason, practitioners usually replace λ by one and $E\pi_{t+1|t}$ by $\alpha\pi_{t-1} + (1-\alpha) E\pi_{t+1|t}$. Though this form can no longer be given such solid microfoundations like the original expression (4), I would give two motivations. One is followed by Svensson (2000): the original Phillips curve defines only a fraction $1-\alpha$ of inflation, and the rest is set by inertia (π_{t-1}). This also transforms β into $(1-\alpha)\beta$. The other motivation is the sticky information framework of Mankiw and Reis (2001), (2002). It replaces the fully rational expectation term $E\pi_{t+1|t}$ the following way. A fraction $1-\alpha$ of market participants uses the correct expectation, and the rest adopts an obsolete forecast. Instead of iterating the $1-\alpha$ fractions back to infinity, one can simply replace the imperfect expectation part by π_{t-1}.

After introducing some convenient but irrelevant changes in the timing of variables relative to Svensson (2000), our "core" system can be written as follows:

$$
\begin{aligned}
\pi_t &= \alpha\pi_{t-1} + (1-\alpha)\pi_{t+1|t} + \beta y_t \\
y_t &= \gamma y_{t-1} - \eta\left(i_t - \pi_{t+1|t}\right) \\
i_t &= \tau\pi_{t+1|t} + \psi y_t.
\end{aligned}
$$

Inflation (π) is given by a Phillips curve relation: there is a pure persistence term $\alpha\pi_{t-1}$, and the remainder is determined by expected inflation. Price rigidities, however, lead to a positive effect of output gap (y) on inflation. Such Phillips curves can be derived from microfoundations (with the exception of the inertia term). For my purposes, its simplified semi-structural (or semi-reduced) form is all I need.

Output gap is set by aggregate demand: there is potentially an autoregressive term y_{t-1}, and a positive real interest rate (nominal minus expected inflation: $i_t - \pi_{t+1|t}$) has a dampening effect (through investment or consumption). Later on, when considering an open economy, the real exchange rate will also have an effect on the output gap (and also on inflation directly). Assuming that it is not just current interest rates but the infinite sum of future interest rates that matters, would give exactly the same modification as the real exchange rate.

The last equation is the reaction function of the central bank. In its present form, it is a Taylor rule: interest rates respond to inflation (in particular: to expected future inflation) and the output gap. Though this might look restrictive, but it is sufficient for my exploratory purposes. Moreover, as explicitly argued in Benczúr, Simon and Várpalotai (2002b), any quadratic objective function of the central bank would lead to a linear reaction function, with as many arguments as state variables (including persistent shocks).

For simplicity, I neglect the autoregressive term ($\gamma = 0$). As explained in Benczúr, Simon and Várpalotai (2002b), the dynamic properties of the autoregressive system would remain identical. The full system reduces to

$$
\pi_t = \alpha\pi_{t-1} + \left(1 - \alpha - \beta\eta\left(\tau - 1\right)\right)\pi_{t+1|t}.
$$

Now extend the basic model to an open economy: this includes an uncovered interest parity equation, and a role for the real exchange rate, either through the output gap, or directly on inflation. We shall soon see that this modification also covers the case when the output gap is influenced not only by the current level of the real interest rate, but also its cumulative future values.

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha)\pi_{t+1|t} + \beta y_t + \kappa q_t + \delta\left(q_{t+1} - q_t\right)$$
$$y_t = -\eta\left(i_t - \pi_{t+1|t}\right) + \phi q_t$$
$$i_t = \tau\pi_{t+1|t}$$
$$q_t = q_{t+1} - \left(i_t - \pi_{t+1|t}\right).$$

Two main modifications were introduced here: the real exchange rate has an effect on certain variables, and we need to determine the value of the real exchange rate. Inflation is potentially reduced by a strong real exchange rate (like in Leitemo (2000), Svensson (2000)), a real appreciation (like in Buiter–Clemens (2001)), and a strong real exchange rate also depresses output (through the worsening of the trade balance, for example). The real exchange rate is then set by a real interest parity condition (which is a simple rearrangement of a nominal interest parity here).

The term Δq_t in the Phillips curve is only an addition to an already existing channel: from interest parity, it is equal to the real interest rate, so the total reduced form effect of the real interest rate on inflation becomes $-\beta(\eta - \delta)$, instead of $-\beta\eta$.

From the viewpoint of inflation and exchange rate dynamics, it does not matter whether q_t affects the output gap or inflation directly: its total effect on inflation is $\kappa + \beta\phi > 0$, and the relative size of these two terms does not change the speed of disinflation (it is of course influential for the output gap cost of disinflation). For inflationary dynamics, one can thus assume that $\phi = \delta = 0$, and q_t enters only through the aggregate supply equation.

Substituting the reaction function into interest parity and iterating yields

$$q_t = q_\infty - (\tau - 1)\sum_{s=t+1}^{\infty} \pi_{s|t}.$$

Assume that the long run value of the real exchange rate is determined by purchasing power parity (an exogenous assumption), thus $q_\infty = 0$. Then the final form of the inflation equation is:

$$\pi_t = \alpha\pi_{t-1} + \left(1 - \alpha - \beta\eta\left(\tau - 1\right) - \kappa\left(\tau - 1\right)\right)\pi_{t+1|t} - \kappa\left(\tau - 1\right)\sum_{s=t+2}^{\infty}\pi_{s|t} \tag{5}$$

3.2 Transforming Svensson[2000] into this framework

In the previous part, I have deliberately stripped down the shock part of the model. Mean zero shocks would lead only to mean zero deviations, but if there are some predetermined variables (like prices), then a shock would have a more complicated effect during those periods when some variables are preset. Therefore, the trend behavior of a disinflation can be described by such a deterministic system, once the initial preset variables are allowed to incorporate shocks (which happens after a fixed number of periods). As an example, let us take a look at the deterministic but potentially preset part of Svensson (2000).

Phillips curve:

$$\pi_{t+2} = \alpha_\pi \pi_{t+1} + (1-\alpha_\pi)\,\pi_{t+3|t} + \alpha_y \left(y_{t+2|t} + \beta_y \left(y_{t+1} - y_{t+1|t}\right)\right) + \alpha_q q_{t+2|t}.$$

In an impulse response, there is only one initial shock, which fully enters all expectations after 3 periods. Thus any expectation is the same as the realization, which leads to

$$\pi_{t+2} = \alpha_\pi \pi_{t+1} + (1-\alpha_\pi)\,\pi_{t+3} + \alpha_y y_{t+2} + \alpha_q q_{t+2}.$$

The real exchange rate is determined by real interest parity:

$$q_{t+1|t} = q_t + i_t - \pi_{t+1|t}.$$

Assuming that any foreign variable is in equilibrium, aggregate demand is given by

$$y_{t+1} = \beta_y y_t - \beta_\rho \left(i_{t+1|t} - \pi_{t+2|t} + i_{t+2|t} - \pi_{t+3|t} + \dots\right) + \beta_q q_{t+1|t} + \left(\gamma_y^n - \beta_y\right) y_t^n$$
$$y_{t+1} = \beta_y y_t - \beta_\rho \left(i_{t+1} - \pi_{t+2} + i_{t+2} - \pi_{t+3} + \dots\right) + \beta_q q_{t+1} + \left(\gamma_y^n - \beta_y\right) y_t^n.$$

Assume first that potential output follows its trend, so y_t^n is identically zero. If there was an initial shock in potential output, dying out with some exponential speed, it would add an extra equation to the system, but we shall see that it would introduce only an extra, exogenous term in inflation.

Writing all equations as of time t:

$$\pi_t = \alpha_\pi \pi_{t-1} + (1-\alpha_\pi)\,\pi_{t+1} + \alpha_y y_t + \alpha_q q_t$$

$$y_t = \beta_y y_{t-1} - \beta_\rho \underbrace{(i_t - \pi_{t+1} + i_{t+1} - \pi_{t+2} + \dots)}_{-q_t} + \beta_q q_t$$

$$q_t = q_{t+1} - (i_t - \pi_{t+1})\,.$$

It is clear that we have a two dimensional system here (assuming that the constraint on long-run behavior pins down the future), so any previous shocks are summarized by π_0 and y_0. Then any linear reaction function can be written as

$$i_t = \tau \pi_{t+1} + \psi y_t.$$

This is exactly our specification with nonzero autoregression ($\gamma \neq 0$) and openness ($\beta\phi + \delta > 0$).

One needs to be more careful when collapsing all previous shocks into the two state variables, π_t and y_t. If a shock has a nonzero persistence, then its value constitutes an extra state variable. This is exactly the same issue as a shock to y_t^n, the natural rate. Such an extra state variable, say, z_t, has its own exponential dynamics, leading to the closed form of $z_t = (\gamma_z)^t z_0$. Introducing a modified inflation variable $\tilde{\pi}_t = \pi_t - x \cdot z_t$, an appropriate choice of x implies that $\tilde{\pi}_t$ follows the dynamics with $z_t = 0$, i.e., $\tilde{\pi}_t = A_1\lambda_1^t + A_2\lambda_2^t$. Changing back to the original inflation variable, its time path becomes $\pi_t = A_1\lambda_1^t + A_2\lambda_2^t + x(\gamma_z)^t z_0$. The only effect of the term z_t is an extra component in inflation, but its dynamics is completely exogenous: the parameters of the reduced form Phillips curve will influence x, but not γ_z; and the dynamics of $\tilde{\pi}_t$ is unaffected by the presence of z_t. Over the long run, it also means a mean zero disturbance term, though it disappears only gradually, and not in a finite number of periods.

3.3 The general solution of such a model

Benczúr, Simon and Várpalotai (2002b) offer a detailed analysis of the convergence and stability properties of such models. Here I only restate the broad results. Decompose first the model's behavior into an intrinsic component (essentially: setting $\tau = 1$), and the modifications coming from monetary mechanisms (when $\tau \neq 1$, there are many "product terms" in the reduced form inflation equation, like $\beta\eta$ – reflecting the effect of interest rates on inflation, via the output gap variable; etc.). The key observation is that these modifications are relatively small: each monetary effect is small, and the combined (product) effect is even smaller. This means that one should understand the intrinsic part first, and then look at the effects of moderate perturbations around it.

Intrinsic dynamics are heavily influenced by the relative weight of backward and forward looking inflation terms in the Phillips curve. Though this relative weight can be viewed as reflecting price and wage adjustments, formal microfoundations usually give such an interpretation to the output gap parameter, leaving the inflation persistence reflecting an empirical regularity.

If the forward looking part dominates the backward looking term (its weight, $1-\alpha$, is larger than half), then inflation is programmed to disappear: from any starting point (initial inflation, output gap etc.), it converges to zero with an exponential rate of $\frac{\alpha}{1-\alpha}$ per period. If the backward looking part dominates, then the intrinsic dynamics does not take the economy to zero inflation: inflation is either constant or explosive.

This phenomena is in fact not due to the success or failure of a credibility-based disinflation. For given relative weights, market participants must have model-consistent expectations (full model-credibility), and they must believe that the economy will not start along an explosive path. This belief is already sufficient for disinflation in the $\alpha < 0.5$ case, there is no need for any active interest rate policy. If $\alpha \geq 0.5$, then a neutral interest rate policy is no longer sufficient for eliminating inflation. Credibility might be influencing α, the weight of the backward looking term, but since we do not have any good theory for the presence of the persistence term, it is even more difficult to argue for the determinants of its importance.

Adding the extra effects of monetary mechanisms can only slightly modify the dynamic behavior (formally: the eigenvalues of the dynamic system are continuous, thus they cannot change much if we slightly perturb the system). If $\alpha < 0.5$, then the speed of inflation remains around $\frac{\alpha}{1-\alpha}$ (one can write this as $\lambda = \frac{\alpha}{1-\alpha} \pm o(\tau - 1)$) This speed is not "too sensitive" to the parameters of monetary mechanisms, or the activism of the central bank's reaction function: once the speed is below 1, a further acceleration (decrease) will have relatively little effect on the halving time of inflation.

A completely different picture emerges if the backward looking term dominates: intrinsic dynamics gives an eigenvalue of one, so we need strong monetary mechanisms, active interest rate policies to decrease this eigenvalue (it can be written as $\lambda = 1 - o(\tau - 1)$). Any small cut below one will have dramatic effects on the halving time of inflation and on cumulative output gaps: the dynamic behavior of the system is very sensitive to precise estimates of monetary effects, which offers an important role for learning and surprises.

Benczúr, Simon and Várpalotai (2002b) also give a characterization of stable reaction functions. If the system is at most two-dimensional, then any linear reaction function (in particular: any optimal reaction function which comes from a constant coefficient quadratic objective function) can be viewed as a general Taylor rule: $i_t = \tau\pi_{t+1} + \psi y_t$. Partly in contrast with the standard view, the condition of $\tau > 1$ is required for the saddle path stability of the solutions, but not necessarily for the asymptotic boundedness of inflation ($\pi_\infty = 0$): the saddle path stability here means that π_0, y_0 (if there is autoregression in the aggregate demand equation) and the well-behaved asymptotics (π_∞ being zero, or at least bounded) of the system uniquely determine the models's behavior. If there is an autoregressive component in aggregate demand, then the exact condition on the reaction function becomes $\tau > \tau^{crit} = 1 + \frac{2\psi\gamma}{\beta(1+\eta\psi)}$.

Should this condition fail, the system then becomes either unstable (from general initial conditions, it explodes) or globally stable: the two initial conditions and the asymptotic (terminal) condition is not sufficient to pin down the system. From any set of initial conditions, there is a continuum of inflationary paths, all converging to zero. Fixing one further condition (e.g., π_1) resolves the indeterminacy, but market participants need to coordinate on such a particular solution. In the case of an open economy model, we usually get global stability, while the closed economy case gives instability. The difference between these two lies in the long-run assumption of (relative) purchasing power parity.

For my purposes, the most canonical parameter choice is the most suitable: $\tau > 1$ (or $\tau > \tau^{crit}$) and $\alpha > 0.5$. Then we have a well-behaved dynamic system, and do not have to worry about whether some of the results are related to the indeterminacy property. Moreover, the high sensitivity of inflation dynamics to τ-τ^{crit}, and more importantly, to further monetary parameters (η, κ etc.), offers an excellent room for a coexistence of large inflation surprises and relatively slow learning.

4 Parameter learning in the Phillips curve

4.1 The learning process

For convenience, assume that the only influence of the central bank over inflation is through the direct real exchange rate channel (the coefficient κ from the reduced form equation). Now suppose that there is a "true" parameter κ in the Phillips curve $\pi_t = \alpha\pi_{t-1}(1+\varepsilon_t) + (1-\alpha)\pi_{t+1|t} + \kappa q_t$, but its precise value is not known to the market or the central bank. For simplicity, assume that the prior distribution is also common for market participants and the central bank. Then every period constitutes a new observation for estimating (learning) κ: but due to some additional pure noise, there is a signal extraction problem, leading to a common Bayesian update of the distribution of κ. As time goes on, this distribution should converge to the truth.

One can explicitly model this learning process: start from some prior distribution about κ, and a true value. We also need a source of noise, with its distribution. Then it is possible to derive the updating rules. This unfortunately gives us only a random variable, a function of the per period realization of the noise. It should in general move towards the true value of κ, unless we have some extreme realizations of the noise, when agents rationally attribute such an observation more to κ than to noise. Simulating many potential time profiles, the sample average then describes the average evolution of inflation and the exchange rate (alternatively, one might be able to calculate this expected value explicitly). Note that this average is an expectation conditional on the true κ, so it is not equal to the inflation path expected by agents at any point in time.

A convenient and tractable shortcut is the following: assume that for any time t, there is a value κ_t which describes the current knowledge of the market. This means that the market forms a point estimate of κ, and uses that parameter to obtain its forecast. That might not lead to fully correct expected value calculations: in period $t + 2$, the expected value of inflation already depends on higher moments of the κ distribution. Using the point estimate implicitly assumes that these higher order terms are relatively small. In the Appendix, I will sketch a formal but incomplete argument that κ_t can be constructed as a certainty equivalent of the current distribution of κ, at least for the current average value of realized inflation, real and nominal exchange rates, and the nominal interest rate: calculating the true expected future real interest rate path and inserting it into real interest parity, the implied q_t and π_t is equal to the values calculated using κ_t. For any other variables in the more distant future (q_{t+1}, for example), one should use a different point estimate κ'_t. For such variables, using κ_t gives us a biased, but still reasonable forecast. Forecasted real exchange rates and real interest rates still satisfy the real uncovered interest parity condition.

Asymptotic learning then requires that $\kappa_t \to \kappa$.[12] We can insert this parameter into the deterministic model: whenever there is a time t expectation term, expectations are obtained in a near-rational but not model consistent way, by replacing κ with κ_t.[13] There can be two

[12] One can easily argue that true Bayesian learning would lead to complete asymptotic learning: rewrite the Phillips curve as $\pi_t/\pi_{t-1} - \alpha - (1-\alpha)\,\pi_{t+1|t}/\pi_{t-1} = \kappa q_t/\pi_{t-1} + \varepsilon'_t$. All variables are observed every period, the parameter α is known, and ε'_t is an orthogonal mean zero error term, with a known distribution. As $t \to \infty$, even the OLS estimate of κ from this equation is consistent – and that estimator even neglects the specific distribution of ε' and the prior of κ.

[13] One might want to call them predicted values, instead of expected values. Since κ influences most variables in a nonlinear way, no single, common point estimate can yield unbiased predictions for all variables. I will define

main cases for learning: pessimistic – when the starting value of κ_t is smaller than the truth, and learning means a gradual revision upwards ($\kappa_t \nearrow \kappa$), and optimistic – $\kappa_t \searrow \kappa$. Again, the Appendix contains a formal but incomplete argument for the certainty equivalent κ_t converging monotonically to the true κ value.

As explained in Section 3, if $\alpha > 0.5$ (the backward looking term dominates in the Phillips curve), then the speed of disinflation is increasing in κ. So one would expect that the optimistic case would imply too low expected inflation (too fast expected disinflation), and each observation would push κ down, decreasing the expected speed of further disinflation. Optimistic agents are continuously subject to negative surprises, they keep updating their inflationary and interest rate expectations upwards; and exactly the opposite for the pessimistic case.

4.2 Optimism, pessimism and the speed of inflation

What is the effect of parameter uncertainty and learning on the speed of disinflation? One can interpret the question at least two ways. The first concerns the effect of parameter uncertainty relative to full information. The answer to this interpretation is ambiguous: in the pessimistic case, expected inflation is large, but that implies a large real interest rate and also a stronger real exchange rate. The first effect indeed increases inflation (relative to the perfect foresight $\kappa = \kappa_t$ case), but the latter works against it. Everything is flipped in the optimistic case, but we get the same ambiguity at the end.

The following example shows that the real exchange rate effect might dominate in the pessimistic case, implying that too much pessimism initially achieves faster disinflation than perfect foresight. If $\kappa_t \approx 0$, then $\pi_{t+1|t} \approx \pi_{t-1}$, so $\pi_t \approx \pi_{t-1} + \kappa q_t$. On the other hand, $i_{t+j|t} - \phi_{t+j|t} - \pi_{t+j|t} \approx (\tau - 1 - \phi)\, \pi_{t+j|t}$, and inflation converges to zero at a speed of $(1 - o(\kappa_t))^j$, so the sum of the geometric inflation series can be arbitrarily large as $\kappa_t \to 0$. This means that as $\kappa_t \to 0$, $q_t \to \infty$. The inflationary effect of pessimism relates to the coefficient $\lambda = 1$ of π_{t-1} (instead of $\lambda < 1$), and it is finite, while the anti-inflationary effect (the nominal and also the real exchange rate) can be arbitrarily large.

From our purposes, however, the relevant question is whether a smaller than true point estimate implies a lower than expected inflation. For a given real exchange rate, the pessimistic case indeed means that inflation is smaller than expected by market participants or the central bank (due to $\kappa > \kappa_t$). There is, however, a link between inflation and the real exchange rate:

κ_t in a way that the prediction of the infinite sum of real interest rates is unbiased, thus the resulting value of q_t (and hence π_t) is consistent with rational expectations.

using (3), we get

$$
\begin{aligned}
q_t &= s_t + p^*_{t-1} - p_{t-1} \\
&= p_{t-1} - p^*_{t-1} + \pi_t + \underbrace{(1+\phi-\tau)\left(\pi_{t+1|t} + \pi_{t+2|t} + \dots\right)}_{I_{t|t}} + p^*_{t-1} - p_{t-1} =: \pi_t + I_{t|t}.
\end{aligned}
$$

Notice that under reasonable assumptions (cumulative inflation is positive, and $\tau > 1+\phi$), $I_{t|t} < 0$. Writing it back to the inflation equation, we get

$$
\begin{aligned}
\pi_t &= \underbrace{\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t}}_{J_{t|t}} + \kappa\pi_t + \kappa I_{t|t} \\
\pi_t &= \frac{J_{t|t} + \kappa I_{t|t}}{1-\kappa}.
\end{aligned}
$$

Similarly

$$
\begin{aligned}
q_{t|t} &= \pi_{t|t} + I_{t|t} \\
\pi_{t|t} &= \alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + \kappa_t\pi_{t|t} + \kappa_t I_{t|t} \\
\pi_{t|t} &= \frac{J_{t|t} + \kappa_t I_{t|t}}{1-\kappa_t}.
\end{aligned}
$$

Comparing realized and expected inflation, their relation is still not clear: if $\kappa > \kappa_t$ (pessimism), then π_t has both a smaller numerator and a smaller denominator (and the opposite for $\kappa < \kappa_t$). The relation $\pi_t > \pi_{t|t}$ boils down to

$$
\kappa_t\left(\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + I_{t|t}\right) < \kappa\left(\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + I_{t|t}\right).
$$

The bracket term equals $\alpha\pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + q_t - \pi_t = q_t\,(1-\kappa)$, which is negative if $\kappa < 1$ and $q_t < 0$. The realistic order of magnitude for κ is evidently $\kappa < 1$ (one cannot expect a one percentage point immediate decrease in inflation from a real exchange rate being one percentage point stronger than its equilibrium level). If we further assume that the central bank wants to maintain a positive real interest rate all the time (which looks plausible during a disinflation), then the real exchange rate shows an initial appreciation, then a gradual return to zero – thus it is always negative.

This implies that $\kappa_t > \kappa$ is equivalent to $\pi_t > \pi_{t|t}$, and $\kappa_t < \kappa$ to $\pi_t < \pi_{t|t}$. One can easily show that a similar result applies to the $i_t = \tau\pi_t$ choice of the reaction function: the only difference is that $\kappa_t > \kappa$ is equivalent to $|\pi_t| > \left|\pi_{t|t}\right|$, and vice versa. Using the explicit solutions of Section 5.2, one can give also give a formal proof for the $i_t = \tau\pi_t$ case. In summary, it is true that parameter-pessimism is also inflation-pessimism (a too low expected κ_t implies a too high expected inflation), therefore a lower than expected inflation number should rationally lead to an update of κ_t upwards.

5 The behavior of the realized exchange rate

5.1 Realized exchange rate movements

Let us turn now to realized exchange rate movements:

$$s_t = p_{t-1} + \left(\pi_t + \pi_{t+1|t} + ...\right) - \left((i_t - \phi_t) + \left(i_{t+1|t} - \phi_{t+1|t}\right) + ...\right)$$

$$s_{t+1} = p_t + \left(\pi_{t+1} + \pi_{t+2|t+1} + ...\right) - \left(\left(i_{t+1} - \phi_{t+1}\right) + \left(i_{t+2|t+1} - \phi_{t+2|t+1}\right) + ...\right)$$

$$\begin{aligned} s_{t+1} - s_t = & \underbrace{i_t - \phi_t}_{s_{t+1|t}-s_t} + \underbrace{\pi_{t+1} - \pi_{t+1|t}}_{\substack{\text{inflation surprise having} \\ \text{no effect on interest rates}}} + \underbrace{\sum_{i=0}^{\infty} \left(\phi_{t+1+i|t+1} - \phi_{t+1+i|t}\right)}_{\text{change in risk premia}} \\ & - \underbrace{\sum_{i=0}^{\infty} \left(i_{t+1+i|t+1} - i_{t+1+i|t}\right)}_{\substack{\text{change in the entire} \\ \text{time profile of interest rates}}} + \underbrace{\sum_{i=1}^{\infty} \left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right)}_{\substack{\text{change in the long-run} \\ \text{nominal exchange rate}}}. \end{aligned}$$

Suppose first that there is no risk premia, or at least, it is constant.. Section 6 offers a detailed interpretation and investigation of the risk premium term. Then the nominal exchange rate changes by the (riskless) interest rate plus the cumulative effect of inflation and interest rate surprises. In the pessimistic case, the sum of inflation differentials is negative. If we assume that the central bank follows some sort of a Taylor rule, in the sense of reacting more than one in one to inflation, then the interest rate sum is also negative, and its absolute value is larger than that of the inflation sum (note that the interest rate sum goes from $t+1$, while the inflation sum starts at $t+2$, since the interest rate is assumed to depend on expected inflation next period). Altogether, these two terms act as a surprise monetary easing, leading to an even larger depreciation of the currency than implied by i_t.

There is a "free" inflation surprise term as well, corresponding to period $t+1$. This surprise, due to the assumption on the reaction function, does not imply any interest rate change. Should this term dominate the total effect of the two infinite sums, then the nominal depreciation will be smaller than i_t. If, however, we further assume that the time t real interest rate is at least as large as the risk premium, then $i_t - \phi_t - \pi_{t+1|t} > 0$. This implies that there is still a nominal depreciation, not less than realized inflation and the total (positive) effect of the two infinite sums.

If we assume that i_t depends on π_t (and not on $\pi_{t+1|t}$), then we do not have this extra "free" term, because $i_{t+1} - i_{t+1|t}$ is exactly (more than) proportional to this surprise. Therefore, in the pessimistic case, we always have a steady weakening of the currency after the initial appreciation, usually even on top of i_t.

A similar argument shows that everything is reversed in the optimistic case: inflation is higher than expected (predicted), so the two infinite sums are positive, and their joint effect is negative. Due to negative inflation surprises, there is an overall restrictive monetary surprise, so the nominal depreciation is smaller than implied by the nominal interest rate (assuming that the "free" term does not dominate the infinite sums).

At a first glance, one might say that the first year of the current Hungarian disinflation story corresponds to the pessimistic case: inflation was decreasing faster than expected by market participants (based on Reuters poll observations), maybe even faster than predicted by the central bank. This would offer no remedy for the permanently strong nominal exchange rate.

There are, however, signs of the optimistic case as well: initial bank forecasts were using a high parameter of exchange rate pass-through, which, according to the forecasting framework, implied a fast disinflation. By the end of 2001, these estimates were under downward revision, in spite of better than expected inflation data for 2001, thus attributing a large part of the current success to favorable exogenous shocks. The "deterministic" part of inflation – the one affected by monetary policy – may actually have shown the symptoms of the optimistic case.

5.2 Operationalizing the small macromodel framework

Given any initial condition π_{t-1}, inflation is determined by the conditional-expectations Phillips curve:

$$\pi_t = \alpha \pi_{t-1} + (1-\alpha)\,\pi_{t+1|t} + \kappa q_t.$$

Working with the $i_t = \tau \pi_t$ reaction function, the real interest parity condition becomes

$$q_t = q_{t+1|t} - \left(i_t - \pi_{t+1|t}\right) = q_\infty - \tau \pi_t - (\tau - 1) \sum_{s=t+1}^{\infty} \pi_{s|t} = -\tau \pi_t + (1-\tau) \sum_{s=t+1}^{\infty} \pi_{s|t},$$

so

$$\pi_t = \alpha\pi_{t-1} + (1-\alpha-\kappa(\tau-1))\,\pi_{t+1|t} - \kappa\tau\pi_t - \kappa(\tau-1)\sum_{s=t+2}^{\infty}\pi_{s|t}.$$

Market expectations (predictions) are formed through a similar equation, but with κ replaced by κ_t:

$$\pi_{t|t} = \alpha\pi_{t-1} + (1-\alpha-\kappa_t(\tau-1))\,\pi_{t+1|t} - \kappa_t\tau\pi_{t+1|t} - \kappa_t(\tau-1)\sum_{s=t+2}^{\infty}\pi_{s|t}.$$

The process $\pi_{s|t}$ is thus follows a perfect foresight small macromodel with parameter κ_t. The characteristic equation of this system is

$$\begin{aligned}\lambda(1+\kappa_t\tau) &= \alpha + (1-\alpha-\kappa_t(\tau-1))\,\lambda^2 - \kappa_t(\tau-1)\frac{\lambda^3}{1-\lambda}\\ 0 &= \alpha - (1+\alpha+\kappa_t\tau)\,\lambda + (2-\alpha+\kappa_t)\,\lambda^2 - (1-\alpha)\,\lambda^3.\end{aligned}$$

In the same fashion as explained earlier for the general case, if $\alpha > 0.5$ (the backward-looking term dominates), $\tau > 1$ (active monetary policy), and $\kappa_t \approx 0$ (small magnitude of exchange rate effect), then this equation has two divergent and one convergent roots.[14] Denote the convergent root by $1-\lambda(\kappa_t)$, which can be computed easily for any given value of α and τ. Therefore, $\pi_{t-1+j|t} = \pi_{t-1}(1-\lambda(\kappa_t))^j$. We also have

$$q_{t+j|t} = -\tau\pi_{t+j|t} + (1-\tau)\sum_{s=t+1+j}^{\infty}\pi_{s|t}.$$

Plug this back to the original Phillips curve:

$$\begin{aligned}\pi_t &= \alpha\pi_{t-1} + (1-\alpha-\kappa(\tau-1))\,\pi_{t+1|t} - \kappa\tau\pi_t - \kappa(\tau-1)\sum_{s=t+2}^{\infty}\pi_{s|t}\\ &= \alpha\pi_{t-1} + (1-\alpha-\kappa(\tau-1))\,\pi_{t-1}(1-\lambda(\kappa_t))^2 - \kappa\tau\pi_t - \kappa(\tau-1)\sum_{s=t+2}^{\infty}\pi_{t-1}(1-\lambda(\kappa_t))^{s-t+1}\\ (1+\kappa\tau)\,\pi_t &= \pi_{t-1}\left(\alpha + (1-\alpha-\kappa(\tau-1))(1-\lambda(\kappa_t))^2 + \kappa(\tau-1)\frac{(1-\lambda(\kappa_t))^3}{\lambda(\kappa_t)}\right)\\ \pi_t &= \pi_{t-1}\left(1-\mu_{t-1}\right).\end{aligned}$$

Again, this can be computed easily, thus we can get π_t, q_t given π_{t-1}. Based on the previous

[14] Due to the extra term $-\tau\pi_t$ in q_t, this remains true even for values of τ slightly below one.

discussion, and explicitly shown in the Appendix, the values of π_t, i_t are indeed equal to the conditional expectation of π_t and i_t, where expectation is taken with respect to period t noise (ε_t), but conditional on the true value of κ. Also, q_t and s_t are equal to the real (and nominal) exchange rate coming from interest parity using the correct expectations – but all the other time t and future (expected) variables are only biased predictions.

One would then specify various choices of α, τ, and "learning scenarios" $\kappa_t \to \kappa$ (either from above, or from below). That leads to a numerical path of q_t, π_t over time, from which we can infer $p_t = p_{t-1} + \pi_t$ ($p_0 = 0$ from normalization), and finally, the evolution of the nominal exchange rate s_t. The object of interest is its behavior: the size of the initial appreciation, and how much reversal follows.

Similar though somewhat more complicated general results apply to the case where there is an extra real interest rate channel (through the output gap), and there is autoregression in the output gap. From a numerical solution point of view, the modifications are straightforward: using the modified characteristic equation, we get a different function $\lambda(\kappa_t)$, but we can use that expression for $\pi_{t+j|t}$, plug those values into the Phillips curve, and get the numerical path of π_t, q_t and s_t.

5.3 Numerical results for the optimistic case

Accepting first the optimistic case, the next question is whether one can find reasonable parameter values, or at least orders of magnitude, at which the nominal exchange rate can stay nearly constant. For simplicity, assume that $i_t = \tau\pi_t$, i.e., neglect the "free" term. Then, using again the general results summarized in Section 5.2, we have $\pi_{t+1+i|t} = \pi_t (1 - \lambda_t)^{i+1}$, $\pi_{t+1+i|t+1} = \pi_{t+1} (1 - \lambda_{t+1})^i = \pi_t (1 - \mu_t) (1 - \lambda_{t+1})^i$. Here $1 - \lambda_t$ is the eigenvalue of the time-t expected inflation (with the exchange rate coefficient being κ_t), $1 - \lambda_{t+1}$ is the same but as of time $t+1$, while $1 - \mu_t$ describes the change from π_t to π_{t+1}(with the inflation expectation term coming from a κ_{t+1}-equation, but the real exchange rate is multiplied by the true coefficient κ).

Then we must have $\lambda_t > \lambda_{t+1} > \bar{\lambda}$, since market predictions about the speed of disinflation are continuously downgraded, but the realized speed still remains above the perfect foresight speed, $\bar{\lambda}$. It is also expected that $\mu_t > \bar{\lambda}$, since optimism should "usually" mean that inflation disappears faster than under perfect foresight (if the real exchange rate channel is not too strong, the expected inflation term should dominate). Finally, we have $\mu_t < \lambda_t$, since realized inflation is higher than expected (predicted) inflation.

Plugging everything into the equation for nominal exchange rate movements:

$$s_{t+1} - s_t = \pi_t \left(\tau \left(1 + \frac{1-\lambda_t}{\lambda_t} - \frac{1-\mu_t}{\lambda_{t+1}} \right) + \frac{1-\mu_t}{\lambda_{t+1}} - \frac{1-\lambda_t}{\lambda_t} \right).$$

It immediately shows that as inflation becomes smaller and smaller, the nominal exchange rate path becomes flatter and flatter. We would like to ensure that the coefficient of inflation is negative, or at least near zero, since that would imply an appreciating, or nearly stable currency. This is equivalent to

$$\tau \left(\frac{1}{\lambda_t} - \frac{1}{\lambda_{t+1}} + \frac{\mu_t}{\lambda_{t+1}} \right) \lessapprox \frac{1}{\lambda_t} - \frac{1}{\lambda_{t+1}} + \frac{\mu_t}{\lambda_{t+1}} - 1.$$

It is easy to see that the right hand side is negative:

$$\frac{1}{\lambda_t} + \frac{\mu_t}{\lambda_{t+1}} - 1 - \frac{1}{\lambda_{t+1}} = \frac{(\lambda_t - 1)(\lambda_t - \lambda_{t+1})}{\lambda_t \lambda_{t+1}} + \frac{\mu_t - \lambda_t}{\lambda_{t+1}} < 0,$$

since $\lambda_t < 1$, $\lambda_t > \lambda_{t+1}$ and $\mu_t < \lambda_t$. If the left hand side is positive, then it is necessarily larger than the right hand side, so there will be a nominal depreciation, at least as large as realized inflation.

Is it possible for the left hand side to be negative? It would imply $\frac{\lambda_{t+1}}{\lambda_t} + \mu_t < 1$. In other words, the updated prediction about the speed of disinflation must be much smaller than the previous prediction, and realized disinflation must also be relatively slow. So we need a large inflation surprise and a slow disinflation.[15]

On the long run, it cannot be maintained: $\lim_{t\to\infty} \lambda_t > 0$, so $\frac{\lambda_{t+1}}{\lambda_t} \to 1$ and $\mu_t > 0$, but the condition may hold at early stages of the disinflation. Even then, we need the additional constraint that τ times the left hand side ($\tau \approx 1.5$) is greater than the right hand side. Repeating the approximate calculations from footnote 15, we would get a positive left hand side (λ_t and λ_{t+1} are nearly the same); while if the difference of the λs is also around 0.1 (which makes the right hand side even more negative!), the left hand side is approximately -0.4, so even $\tau = 2$ is not enough for keeping the exchange rate constant.

The real issue is whether we can find a suitable set of model parameters (α, τ κ^{true}) and a learning path ($\kappa_t \searrow \kappa^{true}$) which would give the desired nominal (and real) exchange rate profile. Matching the previous discussion, it turned out that such a scenario should exhibit a relatively

[15] Making the left hand side negative may not be enough, because it might also involve further decreasing the right hand side. For example, if λ_t, λ_{t+1} and μ_t are "small", the right hand side is not "too small": if all terms are approximately 0.1, their differences are around 0.01, then its value is around -1.

slow disinflation (high persistence – α, and a weak exchange rate channel – κ), large inflation surprises (substantial drops in κ_t each period), and an aggressive reaction function (high τ).

The following choice leads to a particularly good-looking exchange rate behavior: $\alpha = 0.8$, $\tau = 2$, $\kappa_1 = 0.019$, $\kappa_2 = 0.011$, $\kappa_3 = 0.007$, $\kappa_4 = 0.005$, $\kappa_5 = 0.004$, $\kappa_6 = \kappa^{true} = 0.003$. The choice of $\alpha = 0.8$ is in line with the calibration of Svensson (2000), and it implies a relatively large persistence of the inflation process. The Taylor parameter τ is somewhat larger than the "standard" choice of 1.5, but it is not very far. It is hard to access the values of κ directly, since it is an extremely reduced form parameter. Its quantitative meaning is that a 5% real overvaluation leads to a quarterly disinflation of $10 - 1.5$ basis points. The best guide is to look at the implied speed of disinflation: the halving time of inflation is around three years, which may be slightly slow, but not unreasonable.[16]

The initial condition of quarterly (excess) inflation is 1.5%, roughly matching the corresponding Hungarian number of early 2001. The time unit is a quarter of a year. All data is in percentage points, at a quarterly level. The results are depicted on Figure 3.

Before discussing the results, let me emphasize once more that all the future (expected) variables are in fact forecasts, and it is only the one-period ahead forecast of the nominal exchange rate which is constructed to be unbiased. The true expected values would show a similar time profile (decreasing inflation, a depreciating nominal and real exchange rate), but the actual numbers would slightly differ. Those expectations would satisfy the "expected value" version of the interest parity conditions (linking expected exchange rate movements and expected interest rates). Nevertheless, the forecasted variables also satisfy interest parity, but in a "prediction" version, linking predicted exchange rate movements and predicted interest rates.

Panels A and B show the behavior of the real and nominal exchange rate: after a large initial appreciation, though there is an expected nominal and real depreciation every period, realizations show a further real appreciation, and a near steady nominal exchange rate. By construction, both exchange rates switch to the perfect foresight depreciation path after period six, when parameter learning ceases (or at least, becomes negligible).

Panel C illustrates the deviation from interest parity: from period 2, both exchange rates are expected to depreciate: the nominal rate should change according to the excess domestic interest rate, while the true real interest rate is different from the predicted one, but its sign and order of magnitude is similar. In periods 2-6, however, there is an inflation surprise every period, thus a shock to the expected (predicted) inflation and interest rate path, and the long run expected

[16] Benczúr, Simon and Várpalotai (2002a) discusses the adaptation of a similar macromodel to Hungary.

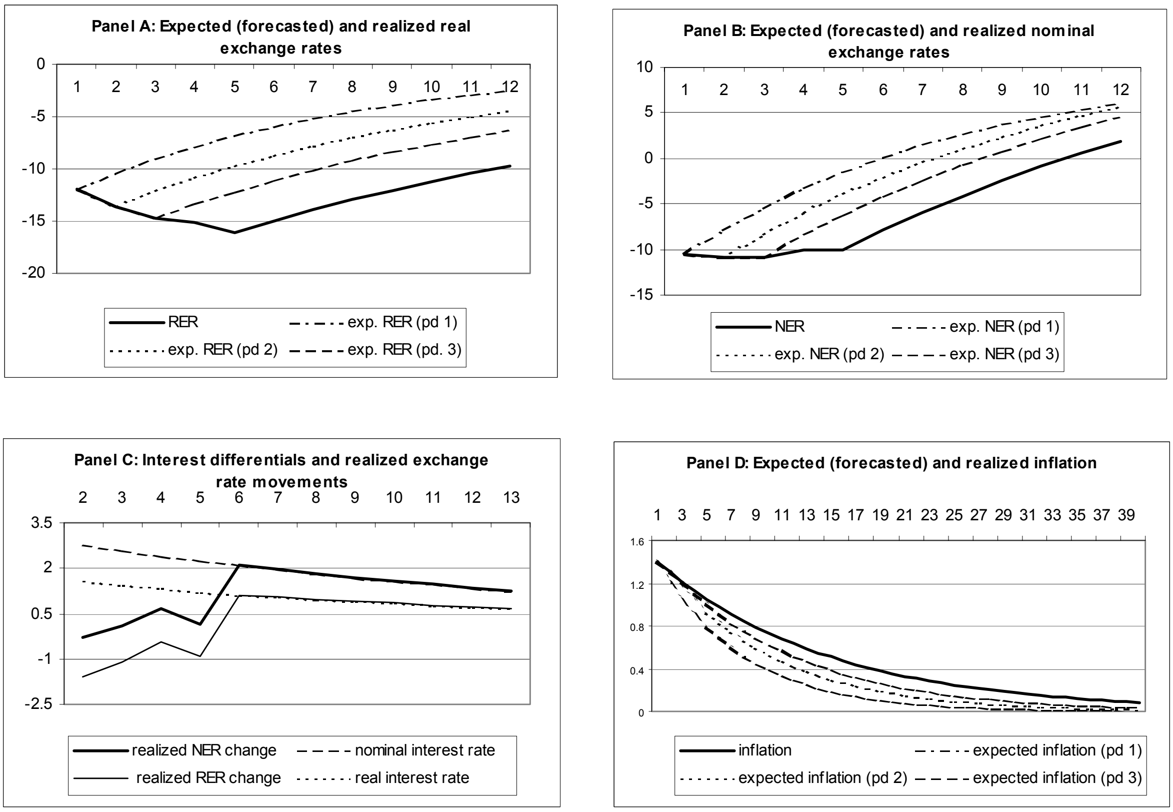

Figure 3: Numerical results for an optimistic learning scenario

nominal exchange rate. Consequently, realized exchange rate movements are systematically lower than interest differentials.

Panel D depicts the speed of disinflation and size of the inflation surprises. The halving time of inflation is approximately 12 periods (3 years), which is not extremely fast, but not implausible. Every period, there is an expected (predicted) inflation path starting from the current realization. One period later, the realization is higher than originally expected. This leads to an upward revision of the inflation forecast (based on a lower point estimate of κ). Changes in inflation forecasts lead to changes in interest rate forecasts (by the mechanical link of $i_{s|t} = \tau \pi_{s|t}$) and $s_{\infty|t}$. This process continues until the true value of κ is learned. In theory, this would require infinite periods, but practically, we can assume that all agents learn κ accurately in a couple of periods, and further updates are negligible.

6 Changing the risk premium

6.1 "Endogenizing" the risk premium

The results so far suggest that inflation surprises alone can account for the behavior of the exchange rate, keeping the risk premium fixed. Still, for many reasons, I also want to explore the implications of a changing (partly endogenous) risk premium term. The starting point is to clarify the meaning and interpretation of risk premium in the model.

Strictly speaking, the only model-consistent source of uncertainty of domestic bond returns comes from the parameter uncertainty and the noise ε, but the implied exchange rate volatility should not matter for risk neutral investors. One could still think about the premium as a correction term, reflecting some risk aversion. There can be many further, not modeled sources of risk: liquidity, or default risk, for example. These factors can be taken as exogenous from the viewpoint of the model, so they can be considered as fixed.

A major part of the risk premium, however, is likely to come directly from the disinflation process itself: if its evolution (in terms of speed, costs etc.) substantially differs from predictions, then the central bank may decide to implement another regime change (or, within the model, it may change the Taylor coefficients). This means some form of less than perfect credibility, but not necessarily a distrust in the central bank itself: based on new information, an update on model parameters should imply an update of the optimal reaction function, even for the same central bank objective function.

This premium,therefore, represents expected losses from a hypothetical monetary realignment, which takes the economy "out of the model". This event never occurs along the equilibrium path, but its risk is incorporated into the exchange rate every period. It does not necessarily implies a loss for all investors, but it looks more plausible that the possibility of such a realignment means an expected loss for bond investors.

Since the probability of such a realignment decreases as the disinflation process matures, it is reasonable to set the risk premium proportional to inflation. As a starting point, assume that this ratio is a constant: $\phi_{t+i|t} = \phi\pi_{t+i|t}$. Keeping the same reaction function $i_t = \tau\pi_t$ as before, the total effect of inflation, interest rate and risk premium surprises becomes

$$-\left(\tau - 1 - \phi\right)\sum\left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right).$$

For a fixed ϕ, it is identical to a less aggressive reaction function, therefore, all previous considerations equally apply here. We saw that, in the optimistic case, a larger τ is more likely to give constant exchange rates, so we do not get any help from the risk premium term.

In the pessimistic case, it may look possible that $\tau - 1 - \phi$ and $\tau - 1$ have opposite signs, meaning that the no-premium behavior shows a continuous depreciation, while the risk premium behavior implies further appreciation. The problem with this argument is that $\tau - 1 - \phi < 0$ would mean that the riskless real interest rate is *negative* for positive inflation levels, but then the central bank is in fact not following a restrictive policy. This is not very sensible, moreover, if the backward looking term dominates in the Phillips curve, then such a reaction function does not lead to asymptotic disinflation.

Another problem here is that the assumption of $\tau - 1 - \phi \approx 0$ indeed implies near constant exchange rates (for a *fixed* inflation path), but it also eliminates the initial appreciation (unless it is entirely due to the correction of real undervaluation). Besides, a too mild reaction function leads to slow disinflation, increasing the expected inflation path $\pi_{t+j|t}$, and the effect on the real interest rate $(\tau - 1 - \phi)\,\pi$ is ambiguous. A less aggressive reaction function might even imply a bigger initial appreciation, and then a severe reversal, if the increase in π dwarfs the decrease in $\tau - 1 - \phi$.

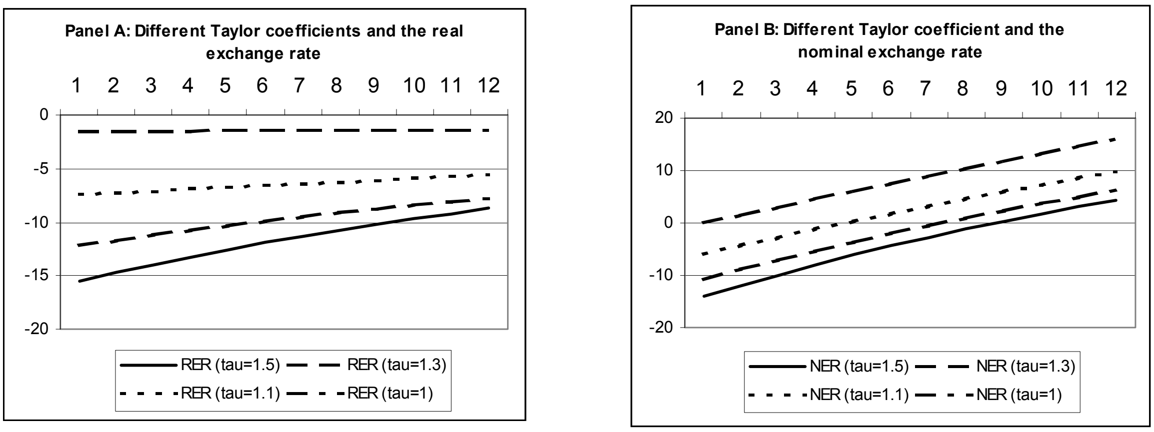

Figure 4: Varying the risk proportion

Figure 4 illustrates the implications of a nonzero but fixed risk proportion ϕ, without any parameter learning. Formally, it is equivalent to working with different τ Taylor coefficients, since it is only $\tau - \phi - 1$ that matters. For the parameter choice of $\alpha = 0.8$, $\kappa = 0.003$, the figure contains the results of four different $\tau - \phi$ values: 1.5, 1.3, 1.1 and 1.[17] As transparent from Panel A, there is a reasonable tradeoff between the initial appreciation and the speed of the

[17] Because of the $i_t = \tau\pi_t$ assumption, $q_t = -\tau\pi_t + (1-\tau)\sum \pi_{s|t}$, which is nonzero even for $\tau = 1$. With this reaction function, one can get a disinflation even for $\tau = 1$, and the critical level is approximately $\tau = 1 - \kappa$.

following depreciation for the *real* exchange rate: decreasing τ leads to a flatter real exchange rate path, but still allows for some initial appreciation.

Panel B, however, shows that this is not the case for the nominal exchange rate: a decrease in τ leaves the slope nearly unchanged, while it decreases the initial appreciation. Since I am particularly interested in the behavior of the nominal exchange rate, the assumption of nonzero but fixed ϕ seems to be insufficient for my objectives.

One can relax the constant proportion assumption in at least two ways. One is that even for a given set of current information, this proportion changes through time: $\frac{\phi_{t+i|t}}{\pi_{t+i|t}} \neq \frac{\phi_{t+j|t}}{\pi_{t+j|t}}$. Put differently, the risk premium content (proportion) of the current interest rate is not the same as that of tomorrow's. This ratio may be declining (as disinflation continues, inflation gradually declines, decreasing the probability of a monetary correction), increasing (due to parameter uncertainty, there is a "probationary" period at the beginning, when the central bank may tolerate relatively large initial costs, and engage in a correction only later, once it is clear that the large costs are not due to bad luck); so a combination of these effects may imply any arbitrary pattern (for example, an initial increase, reflecting the probationary period, and then a gradual decline).

Another approach could be to postulate $\frac{\phi_{t+i|t}}{\pi_{t+i|t}} = \phi_t$: for given information, the ratio is fixed, but new information brings about a change in the risk proportion. In the optimistic case, it seems plausible that the arrival of negative surprises increases ϕ, and the opposite for the pessimistic case. The next section elaborates this framework.

6.2 The proportional risk premium case

The optimistic scenario comes with a per period increase in the risk content of interest rates, which points to a further weakening of the exchange rate: the effect of the interest rate surprise (increase) is partly offset by a higher risk content. Since I did manage to explain the exchange rate behavior without this increase of the risk premium, those results would stay similar though weaker with a changing premium. Overall, this would not provide us with a new explanation for the observed exchange rate behavior: negative inflation surprises alone would remain the driving force, and changes in the risk premium would only work against this finding.

In the pessimistic case, however, the effect of the risk premium is in the desired direction: as a result of better than expected data, the predicted interest rate path is revised downwards, but its risk content also decreases. So it is possible that the total change in the "riskless" interest rate is equal to the total surprise in inflation, which would lead to an unchanged exchange rate (or if it dominates the inflation surprise, it leads to an appreciation). This would constitute an alternative explanation: the inflation surprise itself would point to further depreciation, but the drop in the risk content can potentially counter this effect, and keep the nominal exchange rate constant.

Plugging $\phi_{t+i|t} = \phi_t \pi_{t+i|t}$ into the expression for the change in the exchange rate:

$$\begin{aligned} s_{t+1} - s_t &= i_t + \phi_{t+1} \sum \pi_{t+1+i|t+1} - \phi_t \sum \pi_{t+1+i|t} - (\tau - 1) \sum \left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right) \\ &= i_t + (1 - \tau + \phi_t) \sum \left(\pi_{t+1+i|t+1} - \pi_{t+1+i|t}\right) + \left(\phi_{t+1} - \phi_t\right) \sum \pi_{t+1+i|t+1}. \end{aligned}$$

Note that the time t "effective" Taylor coefficient, $1 - \tau + \phi_t$, multiplies the difference of two inflation sums, while the change in the risk content multiplies such an inflation sum alone. If the surprise in inflation is small compared to total inflation, and if both the level and the change of the risk proportion is relatively large, then we may get $s_{t+1} \lessapprox s_t$.

Use the expressions for $\pi_{t+1+i|t}$ and $\pi_{t+1+i|t+1}$ we obtained earlier:

$$\begin{aligned} s_{t+1} - s_t &= \underbrace{\tau \pi_{t-1} \left(1 - \mu_{t-1}\right)}_{i_t} + \underbrace{\pi_{t-1} \left(1 - \mu_{t-1}\right)}_{\pi_t} \cdot \\ &\quad \cdot \left((\tau - 1 - \phi_t) \frac{(1 - \lambda_t)(\lambda_{t+1} - \lambda_t) + \lambda_t (\mu_t - \lambda_t)}{\lambda_t \lambda_{t+1}} + \left(\phi_{t+1} - \phi_t\right) \frac{1 - \mu_t}{\lambda_{t+1}} \right) \\ s_t &= p_t - p_t^* + \pi_{t-1} (1 - \tau + \phi_t) \frac{1 - \lambda_t}{\lambda_t}. \end{aligned}$$

As a promising first calibration exercise, suppose that $\lambda_t, \lambda_{t+1}, \mu_t, \mu_{t-1} \approx 0.05$, any difference is approximately 0.01, $\tau = 1.5$, $\phi_t \approx 0.4$, $\phi_{t+1} \approx 0.3$, $p_t - p_t^* = 0$ (normalization). Then $s_t \approx -1.9\pi_{t-1}$ (a large initial appreciation) and $s_{t+1} - s_t \approx 0$ (no depreciation next period).

Finding realistic structural parameters, learning and risk premium scenarios proved to be a much less fruitful experiment than in the optimistic case with fixed risk content. The major reason is that one needs a relatively large drop in the risk content (ϕ_t), which in turn requires a high Taylor coefficient (τ), thus high initial interest rates. This either gives too large inflation surprises (which work against the desired appreciation), or too low inflation (which weakens the effect of the decreasing risk premium). One may produce higher inflation by weakening the real exchange rate channel, but that also leads to a higher sensitivity of inflation to estimates of κ, hence, larger inflation surprises again. It is still possible to balance these two effects, and achieve an initial appreciation, followed by 2-3 periods of nearly flat nominal exchange rate,[18] but then

[18] Having substantial drops in ϕ_t for many periods requires a very high τ, thus unrealistically high risky interest rates.

it quickly reverts to depreciation.

Figure 5 reports numerical results for the following set of parameters: $\alpha = 0.7$, $\tau = 1.5$, $\phi_1 = 0.4$, $\phi_2 = 0.3$, $\phi_3 = 0.2$, $\phi_4 = 0.1$, $\phi_5 = 0.1$, $\phi_6 = 0.05$, $\phi_7 = \ldots = 0$, $\kappa_1 = 0.003$, $\kappa_2 = 0.0032$, $\kappa_3 = 0.0035$, $\kappa_4 = 0.004$, $\kappa_5 = 0.0045$, $\kappa_6 = \kappa^{true} = 0.005$.

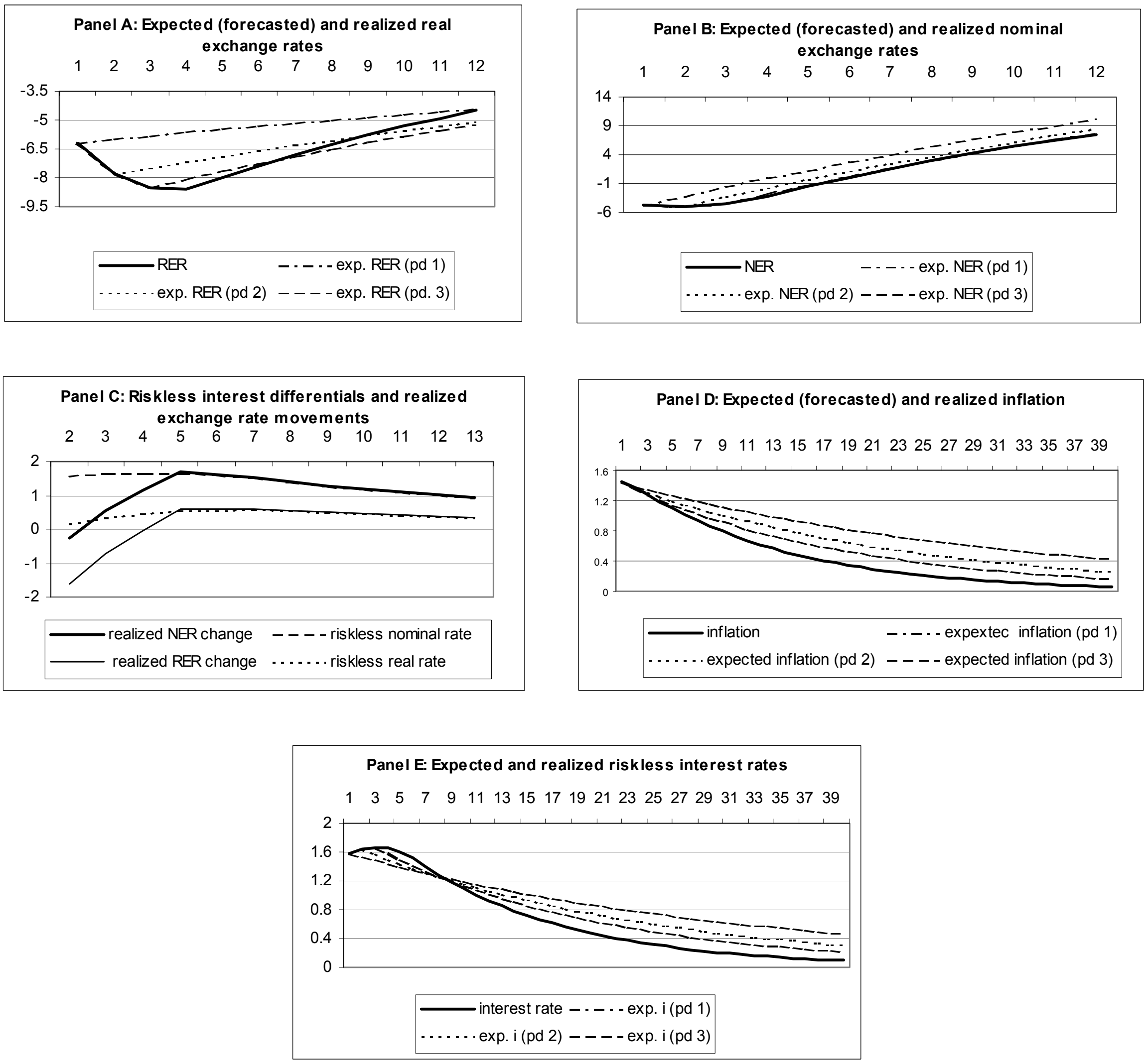

Figure 5: Numerical results for a pessimistic learning scenario with declining risk content

Panels A and B show the evolution of realized and expected (forecasted) exchange rates. The real exchange rate initially appreciates, and then is expected to depreciate slowly (since riskless real interest rates are very low: $\tau - \phi_1 - 1 = 0.1$). In period 2, because of the inflation surprise, realized risky rates are lower than expected, but realized riskless rates are higher, due to our assumption of the surprise drop in the risk premium. This leads to an even stronger real exchange rate, and a slightly appreciating nominal exchange rate. The same applies to period

3, but after that, the change in ϕ_t is too small to reverse the depreciation.

Panel C depicts the violation of uncovered interest parity ex post: instead of the heavy depreciation implied by excess yields, realized exchange rates exhibit some further appreciation, and then return to the perfect foresight path.

Panel D shows the path of realized inflation, and inflation surprises. Risky rates are proportional to inflation (both expected and realized), but due to changes in ϕ_t, the evolution of riskless rates is more complicated. One can see in Panel E that the overall change in riskless rates is initially restrictive (higher than expected), and then it becomes expansionary (lower than expected).

7 Concluding remarks

There are many potential qualifications of the analysis. From the viewpoint of theoretical attractiveness, one could treat the learning effect precisely (either by Monte Carlo simulations for the average realized variables, or proving the missing pieces in the construction of the "certainty equivalent"), or model the source of the risk premium. The first is likely to leave most of the results unaltered; while the second may involve nontrivial interactions between the risk premium and the rest of the model. Unfortunately, there is no obvious choice for an endogenous risk premium term.

An unpleasant feature of my model is the long-term behavior of the nominal exchange rate: plugging back all the "structural" terms, the long-term nominal exchange rate is determined by the long-term equilibrium real exchange rate (equilibrium real appreciation, plus a potential initial undervaluation), and the cumulative total inflation. If we are above structural inflation, then the long run nominal exchange rate can at most reflect the initial undervaluation of the real exchange rate – and markets seem to expect a larger long-run appreciation.

One can of course drive inflation below the structural level (thus get a long-run appreciation), in fact, the EMU criteria is likely to involve such a decision. This requires some modification of the reaction function: $i = \tau\pi$ would make little sense, because a negative π would imply a negative (below equilibrium) level of the nominal exchange rate. Instead, one can use $i = \tau\left(\pi - \pi^{tar}\right)$, with $\pi^{tar} < 0$.

Besides this point about parameter uncertainty and learning, the actual behavior of nominal exchange rates around disinflations are likely to be influenced by a large number of additional factors. Some of them might not be restricted to disinflations: for example, Benczúr (2002) estimates a bond pricing version of interest parity in a developing country sample, of the form

$$i_t = \alpha + \beta i_t^* + \lambda \triangle e_{t+1}^e,$$

and finds a significantly less than one coefficient of the benchmark interest rate ($\beta \approx 0$), but a near one coefficient of the exchange rate risk ($\lambda \approx 1$). Since here we keep foreign rates roughly fixed, such a modification can give at most a "level shift" of the exchange rate profile, but not a "slope shift".

Can we get to market expectations directly – from Reuters polls, for example? Then one could check whether UIP holds with measured expectations. If yes, then one should try to explain why it makes sense to have biased forecasts and trade based on them, or whether the bias is only an "ex post bias".[19] If it fails, then we need even further explanations, like slow capital inflows, or EMU speculation (see below).

If capital inflows respond slowly to extra gains (which is in fact related to a changing risk premium explanation) – with some adjustment cost-type story, one might get exactly the desired outcome. This is already more specific to a disinflation: there is a foreseen medium-term excess yield on the currency, and still, it is arbitraged away only slowly. Can we make such a story consistent and rational (e.g., with cost of convincing German small investors to let their money go into accession countries)?

Finally, in the case of Hungary (and the other former or current EU accession countries), the level at which the country might join the monetary union offers an extra scope for exchange rate speculation: at entry, there will be a "forever fixed" exchange rate, the level of which is uncertain, moreover, the evolution of the "free market" exchange rate might influence this entry parity.

8 Appendix

8.1 Defining the "certainty equivalent" of the realized exchange rate and inflation

I will illustrate the procedure for a simplified Phillips curve specification:

$$\pi_t = \pi_{t-1}(1 + \varepsilon_t) + \kappa q_{t-1}.$$

[19] Gourinchas and Tornell (2001) address the issue of ex ante misperceptions about future interest rates, which can be translated into misperceptions about future exchange rates.

Inflation equals last period's inflation, a random shock proportional to past inflation, and the disinflating effect of an overvalued real exchange rate (of the previous period). The assumptions of full inflation inertia and a lag in the effect of the real exchange rate simplify the derivations and the signal extraction problem, while the proportionality of the noise is necessary to have a tractable solution (it essentially makes the evolution of the entire system proportional to the initial level of inflation).

The real exchange rate is still determined by real interest parity:

$$q_{t-1} = E_{t-1}[q_t] - (i_{t-1} - E_{t-1}\pi_t) = E_{t-1}[q_t] - \tau\pi_{t-1} + E_{t-1}[\pi_t]. \tag{6}$$

Assume that κ can take two values: κ_H or κ_L.[20] As of time $t-1$, the current (posterior) distribution of κ is binary, with $P_{t-1}(\kappa = \kappa_L) = \lambda_{t-1}$, $P_{t-1}(\kappa = \kappa_H) = 1-\lambda_{t-1}$. After observing π_t, all agents (market participants and the central bank as well) update their distribution for κ, which can be summarized in the probability $\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t, q_{t-1})$.

Let us look for a solution of the form $q_{t-1} = f(\lambda_{t-1})\pi_{t-1}$. Then $\pi_t = \pi_{t-1}(1+\varepsilon_t) + \kappa f(\lambda_{t-1})\pi_{t-1}$, so $E_{t-1}[\pi_t] = \pi_{t-1} + E_{t-1}[\kappa]f(\lambda_{t-1})\pi_{t-1}$. Plugging everything into (6):

$$\begin{aligned} f(\lambda_{t-1})\pi_{t-1} &= E_{t-1}[f(\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t))(\pi_{t-1} + \pi_{t-1}\varepsilon_t + \kappa f(\lambda_{t-1})\pi_{t-1})] \\ &\quad - \tau\pi_{t-1} + \pi_{t-1} + E_{t-1}[\kappa]f(\lambda_{t-1})\pi_{t-1} \\ f(\lambda_{t-1}) &= E_{t-1}[f(\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t))(1 + \varepsilon_t + \kappa f(\lambda_{t-1}))] - \tau + 1 + E_{t-1}[\kappa]f(\lambda_{t-1}). \end{aligned} \tag{7}$$

I will show in a minute that $\lambda_t(\lambda_{t-1}, \pi_{t-1}, \pi_t) = \lambda_t(\lambda_{t-1}, \varepsilon_t, \kappa)$ – that is, the level of inflation cancels from the inference problem. This is the consequence of the proportional noise assumption. Then (7) gives an equation for the function f :

$$\begin{aligned} f(\lambda_{t-1}) = {} & \lambda_{t-1}E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_L))(1 + \varepsilon + \kappa_L f(\lambda_{t-1}))] \\ & + (1-\lambda_{t-1})E_\varepsilon[f(\lambda_t(\lambda_{t-1}, \varepsilon, \kappa_H))(1 + \varepsilon + \kappa_H f(\lambda_{t-1}))] \\ & - \tau + 1 + (\lambda_{t-1}\kappa_L + (1-\lambda_{t-1})\kappa_H) f(\lambda_{t-1}). \end{aligned} \tag{8}$$

Specifying the distribution of ε, one can calculate (or at least set up) the two expected values, which completes the equation defining f.

[20] The general case could be treated in a similar though more complicated way. One should then derive the posterior density of κ, and all functions of the probability λ_{t-1} would become functionals of the pdf $\lambda_{t-1}(x)$.

Before proceeding, I need to derive the learning rules. Using Bayes' rule:

$$
\begin{aligned}
&\lambda_t\left(\lambda_{t-1}, \pi_t, \pi_{t-1}\right) \\
&= P\left(\kappa = \kappa_L | \lambda_{t-1}, \pi_t, \pi_{t-1}\right) = \frac{P_{t-1}\left(\kappa = \kappa_L, \pi_t | \pi_{t-1}\right)}{P_{t-1}\left(\pi_t | \pi_{t-1}\right)} \\
&= \frac{P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_L\right) P_{t-1}\left(\kappa = \kappa_L | \pi_{t-1}\right)}{P_{t-1}\left(\kappa = \kappa_L | \pi_{t-1}\right) P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_L\right) + P_{t-1}\left(\kappa = \kappa_H | \pi_{t-1}\right) P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_H\right)}.
\end{aligned}
$$

By definition, $P_{t-1}\left(\kappa = \kappa_L | \pi_{t-1}\right) = \lambda_{t-1}$ and $P_{t-1}\left(\kappa = \kappa_H | \pi_{t-1}\right) = 1 - \lambda_{t-1}$. Plugging this back:

$$
\begin{aligned}
\lambda_t &= \frac{\lambda_{t-1} P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_L\right)}{\lambda_{t-1} P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_L\right) + \left(1 - \lambda_{t-1}\right) P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_H\right)} \\
&= \frac{\lambda_{t-1}}{\lambda_{t-1} + \left(1 - \lambda_{t-1}\right) \frac{P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_H\right)}{P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_L\right)}}.
\end{aligned}
\tag{9}
$$

The two probabilities can be calculated by recalling $\pi_t = \pi_{t-1} + \pi_{t-1}\varepsilon_t + \kappa f\left(\lambda_{t-1}\right)\pi_{t-1}$: then $P_{t-1}\left(\pi_t | \pi_{t-1}, \kappa = \kappa_x\right) = P\left(\varepsilon_t = \frac{\pi_t - \pi_{t-1} - \kappa_x f\left(\lambda_{t-1}\right)\pi_{t-1}}{\pi_{t-1}}\right) = P\left(\varepsilon_t = \frac{\pi_t}{\pi_{t-1}} - 1 - \kappa_x f\left(\lambda_{t-1}\right)\right)$. If the density function of ε is f_ε, then this probability becomes $f_\varepsilon\left(\frac{\pi_t}{\pi_{t-1}} - 1 - \kappa_x f\left(\lambda_{t-1}\right)\right)$.

As of time $t-1$, $\pi_t = \pi_{t-1}\left(1 + \varepsilon_t + \kappa f\left(\lambda_{t-1}\right)\right)$. For a given κ, λ_t is the following function of ε_t (thus a random variable):

$$
\lambda_t\left(\lambda_{t-1}, \pi_{t-1}, \varepsilon_t, \kappa\right) = \frac{\lambda_{t-1}}{\lambda_{t-1} + \left(1 - \lambda_{t-1}\right) \frac{f_\varepsilon\left(\varepsilon_t + \left(\kappa - \kappa_H\right) f\left(\lambda_{t-1}\right)\right)}{f_\varepsilon\left(\varepsilon_t + \left(\kappa - \kappa_L\right) f\left(\lambda_{t-1}\right)\right)}} = \lambda_t\left(\lambda_{t-1}, \varepsilon_t, \kappa\right). \tag{10}
$$

Then the unconditional distribution of λ_t (without the assumption of a fixed κ) is the following: it is equal to $\lambda_t\left(\lambda_{t-1}, \varepsilon, \kappa_L\right)$ with probability $f_\varepsilon\left(\varepsilon\right)\lambda_{t-1}$, and $\lambda_t\left(\lambda_{t-1}, \varepsilon, \kappa_H\right)$ with $\left(1 - \lambda_{t-1}\right) f_\varepsilon\left(\varepsilon\right)$. This establishes the properties of the learning rule which were necessary for deriving (8).

Instead of trying to solve for f for a given distribution of ε (say, a mean zero normal), let us proceed in a qualitative, though not fully rigorous manner. The number $f\left(\lambda_{t-1}\right)$ is equal to $\frac{q_{t-1}}{\pi_{t-1}}$. Iterating interest parity yields

$$
q_{t-1} = q_{\infty|t} - \tau\pi_{t-1} + (1 - \tau)\sum_{s=t}^{\infty} \pi_{s|t-1} = -\tau\pi_{t-1} + (1 - \tau)\sum_{s=t}^{\infty} \pi_{s|t-1}.
$$

It looks plausible that a higher λ_{t-1}, which means a weaker real exchange rate effect, leads to a slower disinflation, hence, a stronger (more negative, so smaller) initial q_{t-1}. It implies that the function f is decreasing in λ. One would expect that this feature could be proven in general, but I have not succeeded. The intuitive logic, however, I find clear and convincing.

From this monotonicity, we get that $f(\lambda)$ is between $f(1)$ and $f(0)$. Let us look at these two extreme values: both correspond to a degenerate distribution of κ, being equal to κ_H or κ_L, and there is no further learning. It is then easy to derive the corresponding values of f: if $\kappa = \kappa^*$ for sure, than we are back to a perfect foresight equilibrium with mean zero noise, which cancels from any expected value calculation. For a given level of κ^*, let us denote the corresponding value of f by $g(\kappa^*) = g$. Then

$$\begin{aligned} q_{t-1} = g\pi_{t-1} &= gE_{t-1}[\pi_t] - \tau\pi_{t-1} + E_{t-1}[\pi_t] = g(\pi_{t-1} + \kappa^* g\pi_{t-1}) - \tau\pi_{t-1} + \pi_{t-1} + \kappa^* g\pi_{t-1} \\ 0 &= k^* g^2 + 1 - \tau + \kappa^* g \\ g_{1,2} &= -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{\tau - 1}{\kappa^*}}. \end{aligned}$$

From the two roots, we must pick the one which is consistent with $E_{t-1}\pi_\infty = 0$. Substituting $q_{t-1} = g\pi_{t-1}$ back into the Phillips curve gives us

$$E_{t-1}\pi_t = \pi_{t-1}(1 + \kappa^* g).$$

So we must pick a root between $\frac{-2}{\kappa^*}$ and 0. It is clear that the bigger root is positive (using $\tau > 1$), while the smaller root is less than -1. It is straightforward to check that it stays above the lower bound as long as $\kappa^* < \frac{4}{1+\tau}$, and this holds for any reasonable choice of τ and κ^*.

All this implies that $g(\kappa^*) = -\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{\tau-1}{\kappa^*}}$, and $f(0) = g(\kappa_H)$, $f(1) = g(\kappa_L)$. Since $f(\lambda)$ is decreasing, and g is increasing, there is a unique value of κ_{t-1} for every λ_{t-1} such that $f(\lambda_{t-1}) = g(\kappa_{t-1})$. This means that one can calculate the realized real exchange rate based on κ_{t-1}, instead of calculating all the complicated expected values for λ_{t-1}. In this sense, κ_{t-1} is the certainty equivalent of λ_{t-1}: picking κ_{t-1} as a point estimate of κ, and forecasting all future variables with $\kappa = \kappa_{t-1}$ and no uncertainty, leads to $q_{t-1} = g(\kappa_{t-1})\pi_{t-1} = f(\lambda_{t-1})\pi_{t-1}$. Though all the predicted future real exchange rates, interest rates and inflation figures are biased predictors, κ_{t-1} is picked in such a way that the resulting q_{t-1} equals the correct value.

To define our final version of the certainty equivalent, we need to do one extra step. Our model starts at time $t-1$, with π_{t-1} and a prior distribution for κ, summarized by λ_{t-1}, and an assumption about the true value of $\kappa = \kappa^*$ (this is what the market will learn gradually, but the inflation process is governed by this true value). The time t values of π_t and q_t are random variables: they depend on ε. What we want to calculate is the average value of π_t and q_t for different realizations of ε. Formally, these averages are equal to

$$\widetilde{\pi}_t = E_\varepsilon[\pi_{t-1}\left(1+\varepsilon_t+\kappa^* q_{t-1}\right)] = \pi_{t-1}+\kappa^* q_{t-1}$$

and

$$\begin{aligned}\widetilde{q}_t &= E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa^*\right)\right)\pi_{t-1}\left(1+\varepsilon_t+\kappa^* q_{t-1}\right)] \\ &= E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa^*\right)\right)]\left(\pi_{t-1}+\kappa^* q_{t-1}\right)+E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa^*\right)\right)\varepsilon_t]\pi_{t-1}.\end{aligned}$$

Now define κ_t such that $\widetilde{q}_t = g\left(\kappa_t\right)\widetilde{\pi}_t$. It means that if we have the average realization of π_t – which is easy to get, given π_{t-1}, κ^* and λ_{t-1}–, then we can calculate the average realization of q_t by using κ_t. Having calculated the average realization as of time t, we restart the system from this average point.

Finally, we want to use that if $\kappa^* = \kappa_H$ (which corresponds to our pessimistic learning case: the true effect is larger, so true disinflation is faster than originally thought), then $\kappa_{t-1} < \kappa_t$ holds. This looks plausible: when the true real exchange rate effect is stronger than currently expected, then new data dominantly move the distribution of κ upwards. This higher distribution leads to a faster expected disinflation path, therefore a weaker real exchange rate (relative to inflation). This argument suggests that the average of $\frac{q_t}{\pi_t}$ should be smaller than $f\left(\lambda_{t-1}\right)$.

What we need, however, is not the average of $\frac{q_t}{\pi_t}$, but the ratio of the two averages. The difference between the two can be seen by noting that $\kappa_{t-1} < \kappa_t$ is equivalent to $g\left(\kappa_{t-1}\right) = f\left(\lambda_{t-1}\right) < g\left(\kappa_t\right) = \frac{\widetilde{q}_t}{\widetilde{\pi}_t}$. Rearranging:

$$\begin{aligned}f\left(\lambda_{t-1}\right)\left(\pi_{t-1}+\kappa_H q_{t-1}\right) &< E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa_H\right)\right)\pi_{t-1}\left(1+\varepsilon_t+\kappa_H f\left(\lambda_{t-1}\right)\right)] \\ f\left(\lambda_{t-1}\right)\left(1+\kappa_H f\left(\lambda_{t-1}\right)\right) &< E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa_H\right)\right)]\left(1+\kappa_H f\left(\lambda_{t-1}\right)\right) \\ &\quad + E_\varepsilon[f\left(\lambda_t\left(\lambda_{t-1},\varepsilon_t,\kappa_H\right)\right)\varepsilon_t].\end{aligned}$$

The number $1+\kappa_H f\left(\lambda_{t-1}\right)$ is the average realized speed of disinflation – which is less than one, but since it is a quarterly speed, by not too much. All the $f\left(\lambda\right)$ terms are in absolute values around or greater than one, while the expected value of the product term $\left(\varepsilon f\left(\lambda_t\right)\right)$ is in general much smaller (though can be shown to be negative). Note that $E_\varepsilon[f\left(\lambda_t\right)|\kappa_H] = E_\varepsilon[\frac{q_t}{\pi_t}|\kappa_H]$ – the average of the ratio of q_t and π_t. The certainty equivalent κ_t was defined to match the ratio of the averages. We can see that the two are quite close to each other, and the comparison $\kappa_{t-1} < \kappa_t$ is closely related to $f\left(\lambda_{t-1}\right) < E_\varepsilon[f\left(\lambda_t\right)|\kappa_H]$. Again, though I could not prove formally that $\kappa^* = \kappa_H$ implies that κ_t is increasing, it looks plausible that it holds, at least if the starting distribution is not too far from the truth.

This property would imply that the average evolution of our variables in the pessimistic learning case is equivalent to some increasing time path of $\kappa_t \to \kappa_H$. In principle, the distribution of ε and the starting value κ_0 fully fixes the time path of κ_t – but for any specified values of $\kappa_1 < \kappa_2 < \ldots < \kappa_T$, there is in general a distribution of ε that leads to this evolution of κ_t (until time T): select a family of distributions with at least T free parameters (say, moments), then we get T equations with T unknowns. Unless the distribution family we choose is degenerate, these T equations should have full rank, so there should be a set of parameters yielding exactly the prescribed evolution of κ_t.

Let us now check the implications of this certainty equivalent approach for the validity of interest parity. We do know that the average observed real exchange rate follows the path that we determine with the certainty equivalent. Average realized real interest rates, on the other hand, are not precisely equal to the values obtained with κ_t. Denote the average realization of any variable x by $\widetilde{x}$, and the certainty equivalent (predicted) value, which is calculated by using κ_t, of x by x^f. Then

$$\begin{aligned}
\widetilde{r_t} &= \widetilde{i_t} - \widetilde{\pi_{t+1|t}} = \tau\widetilde{\pi_t} - \pi_t\left(\widetilde{1 + E_t[\kappa]q_t}\right) = \\
&\tau\pi_t^f - E_\varepsilon[\pi_t\left(1 + \lambda_t\left(\varepsilon\right)\kappa_L q_t + \left(1 - \lambda_t\left(\varepsilon\right)\right)\kappa_H q_t\right)] \\
&= i_t^f - \pi_t\left(1 + \kappa_H q_t + \left(\kappa_L - \kappa_H\right) q_t E_\varepsilon[\lambda_t\left(\varepsilon\right)]\right) \\
&\neq i_t^f - \pi_t\left(1 + \kappa_t q_t\right) = i_t^f - \pi_{t+1|t}^f = r_t^f.
\end{aligned}$$

However, if we get a flat initial real exchange rate, or a further real appreciation following the initial strengthening, we can be sure that interest parity is violated ex post, since $\widetilde{r_t} > 0$ remains true, so the real exchange rate was expected to depreciate. The numerical difference between r_t^f and the realized real exchange rate movement, however, is not equal to the average ex post deviation from real interest parity.

For the nominal interest rate, we are on safe ground: $\widetilde{i_t} = \tau\widetilde{\pi_t} = \tau\pi_t^f = i_t^f$. One also needs to check the certainty equivalent values of the nominal exchange rate, s :

$$\widetilde{s_t} = \widetilde{q_t + p_t} = \widetilde{q_t} + p_0 + \sum_{s=1}^{t}\widetilde{\pi_s} = q_t^f + p_0 + \sum_{s=1}^{t}\pi_s^f = s_t^f,$$

so the nominal exchange rate is fine. This means that the difference between realized nominal exchange rate movements and i_t^f is equal to the average deviation from nominal interest parity.

Finally, let me illustrate the extra difficulties this approach would be facing if there was any simultaneity in the Phillips curve: suppose that the system is given by

$$\pi_t = \pi_{t-1}(1+\varepsilon_t) + \kappa q_t$$
$$q_t = q_{t+1|t} - \tau\pi_t + \pi_{t+1|t}.$$

Again, we want to look for a solution of the form $q_t = f(\lambda_t)\pi_t$: since q_t depends directly on π_t, it will be determined using the updated distribution for κ. Then

$$\pi_t = \pi_{t-1}(1+\varepsilon_t + \kappa f(\lambda_t)\pi_t)$$
$$\pi_t = \frac{\pi_{t-1}(1+\varepsilon_t)}{1-\kappa f(\lambda_t(\lambda_{t-1},\varepsilon_t))}.$$

The signal extraction problem leads to the same expression as in (9). The conditional probability $P = P(\pi_t|\pi_{t-1},\lambda_{t-1},\kappa^*)$, however, will turn out to be problematic:

$$P = P\left(\frac{\pi_{t-1}(1+\varepsilon_t)}{1-\kappa f(\lambda_t)} = \pi_t\right).$$

The issue is that λ_t also contains ε_t, so the realization π_t is a complicated function of ε_t. For a given π_t, there should be a unique ε_t being compatible with a fixed value of κ^*, but this one to one mapping depends on the updating rule $\lambda_t(\lambda_{t-1},\varepsilon_t,\kappa^*)$. So in the expression for the learning rule, $\lambda_t(\lambda_{t-1},\varepsilon_t,\kappa^*)$, both sides will contain this function.

In fact, we were having a similar issue for the $\pi_t = \pi_{t-1}(1+\varepsilon_t)+\kappa q_{t-1}$ Phillips curve as well: $\lambda_t(\lambda_{t-1},\varepsilon_t,\kappa^*)$ contained the real exchange rate function $f(\lambda_{t-1})$, and the equation for $f(\lambda_{t-1})$ contained $\lambda_t(\lambda_{t-1},\varepsilon_t,\kappa^*)$. There, however, the system was recursive: we could substitute the learning rule into the interest parity equation and get a single equation defining $f(\lambda_{t-1})$, and then plug the result back into the learning rule. Here, the two equations are not recursive.

All this means that it would be quite hopeless to prove all the necessary properties of the certainty equivalent approach for a forward looking and highly simultaneous Phillips curve, like $\pi_t = (\alpha+\varepsilon_t)\pi_{t-1} + (1-\alpha)\pi_{t+1|t} + \kappa q_t$. Still, it looks plausible that the average behavior of this system under the true learning process is closely resembled by the certainty equivalent results. Since this more complex Phillips curve enables a bigger role for inflation surprises, I will use this specification for my numerical simulations, keeping in mind that those results might be quantitatively different from the outcome of the true learning process. I believe, however, that the qualitative behavior of the two systems is quite similar.

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