Precaution, Optimal External Debt and Fully non-Ricardian Behavior*

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Abstract

The model is based on the Taylor-series approximation of Skinner (1989). It is shown that in a model of infinite horizon, uninsurable labor income risk, and precautionary bahavior stable steady state solutions of plausible magnitudes for foreign financial asset ratios exist for both creditor and debtor countries with idiosyncratic growth rates. In this case individual decisions do not necessarily add up to social optimum. However, social optimum can be enforced efficiently through fiscal policy, because with an income-tax system behavior is fully non-Ricardian: fiscal saving appears approximately one-for-one in aggregate saving.

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1 Introduction

There are two topics in the literature that our paper is contributing to. One of them is the optimal indebtedness of a country, an issue of open economy macroeconomics, the other is the Ricardian equivalence principle. However different issues they might seem, they get much related in this paper.

The open economy macroeconomics problem is the following. Let assume that a social planner of an open economy maximizes an expected utility function on an infinite horizon given an infinitely large world economy with an exogenous world interest rate. What is the steady state net asset position of the country? The use of certainty equivalent or point expectations with infinite planning horizon leads to very unattractive predictions (Obstfeld-Rogoff (1996)). Those countries, where growth is slow will infinitely accumulate financial assets while fast growing countries will take debts to the level of their human capital which might be 20-40 times their GDP. Obstfeld-Rogoff (1996) suggests a model that constrains indebtedness by the limited scope of enforcing international financial contracts. However, this approach implies a certain asymmetry in prudence which is difficult to maintain. In this world the fast growing borrower is always interested in high indebtedness and only the borrowing constraint imposed by the lender prevents her from choosing a debt level equal to her human wealth.

Another way out of the puzzle might be the assumption that the decision-makers about debt levels have finite lives and therefore finite horizons which constrains the level of human wealth that they are willing to burden with the debt. However, this does not alter the infinite horizon of the assumed sovereign policy maker. We might refer to the Ricardian equivalence principle and say that such sovereign policy maker does not exist, but this principle would imply the assumption of infinitely living agents (among others assumptions) and we would get back to the original puzzle.
This is the point where the Ricardian equivalence problem and the optimal indebtedness problem get related. A model built around the indebtedness problem is incomplete unless it encompasses the issue whether the policy choice can be put in effect through fiscal policy. This is the first reason why we are addressing both issues in this paper. On the other hand it will fortunately turn out that the framework suitable to explain the plausible debt ratios for a country is at the same time suitable to explain an extreme case of Ricardian non-neutrality, (nearly) full debt-independence.

In the optimal indebtedness problem we can rely on well-established basis, the precautionary saving model framework.\(^1\) There are two approaches to model precautionary behavior in a world of stochastic endowments. Some models (Carrol (1994), Ayiagari (1994)) solve the underlying stochastic dynamic optimization problem directly. This approach requires some assumption on a minimum possible wealth level (liquidity constraint) and/or a minimum possible income level in order to avoid the occurrence of outcomes that imply utility losses approaching infinity or negative consumption levels. The Taylor approximation approach of Skinner (1988) uses only the first two moments of the utility function which has the advantage that it does not need an explicit borrowing constraint to be built into the model. This allows the model builder to avoid the task of giving a value to a parameter that he has no information on. In an analysis of household behavior the assumption of Carroll (1994) that the consumer will consider a 0 minimum level of financial wealth to avoid the occurrence of 0 consumption might seem reasonable and in line with the observation that net household assets are positive. However, using the same assumption in an open economy intertemporal optimization framework would lead to a conclusion that borrower countries do not exist. Therefore it is probably better to distance ourselves from the fineries of the CRRA utility function and use only

its two moments when making inferences. This is done in the Skinner-approach that we follow in this paper. An appropriate parameterization and extension to an infinite planning horizon of the model of Skinner (1988) gives steady state net external financial asset results that are within the magnitude of observed data.

In the intertemporal optimization problem we assume that a social planner of a country maximizes a CRRA utility function of consumption over an infinite planning horizon constrained by a budget where labor income follows a random walk with drift. We will see that using reasonable parameters with a simple habit assumption leads to net financial asset ratios that are in the range of observed values.

In a world of uninsurable idiosyncratic personal risks the optimization problem of the social planner cannot be decentralized. The aggregate of household decisions will not necessarily coincide with the social optimum. However, if Ricardian debt neutrality does not hold, the fiscal authority is able to enforce any aggregate financial asset ratio it wants. The arguments against Ricardian debt-neutrality are well known. However these arguments all allow a partial imputation of a future tax burden. The rate of imputation depends on the difference between internal and market discount rates (Blanchard (1985)—Buiter(1985)—Weil(1989)). In our model this imputation is (approximately) 0. Our approach relies on the idea of Barsky et al. (1986) and Kimball–Mankiw (1989), who showed that the assumption of income taxes and uninsured idiosyncratic risks lead to Ricardian non-neutrality even if agents live infinitely long. Assuming a priori reasonable parameters for the utility functions they arrive at the partially non-Ricardian result of a coefficient of 0.5, which means that a 1 dollar permanent increase in government debt decreases household savings by 0.5 dollar. In our model this coefficient is approximately 0.

The clue to the difference in the results is in the model parameters. We will see that calibrating the infinite-horizon, random-walk income, uninsured idiosyncratic risk framework to explain financial asset levels observed in the real world necessarily
leads to a fully non-Ricardian result.

In the first section of the paper we calibrate the Skinner-model to reproduce reasonable indebtedness rates. In the second section we shortly recall the differences between the household’s problem and the problem of the social planner. The discussion of these differences only motivate our interest to look for the potential of the fiscal authority to enforce its preference. The outcome on debt independence relies on the assumption that the model alone explains observed facts.

2 The Model

Let assume that a social planner of a small open economy maximizes the following CRRA utility function of a country:

$$\sum_{s=0}^{\infty} \beta^s \frac{c_s^{1-1/\gamma}}{1-1/\gamma},$$

where $c_s$ is consumption in period $s$, $\beta$ time preference factor, and $\gamma$ parameter of risk aversion.

Let assume that the endowment process of the economy is the following stochastic process:

$$y_t = (1 + g) y_{t-1} \varepsilon_t \quad \ln(\varepsilon_t) \sim N(0, \sigma^2 I),$$

where $\sigma^2$ is known and $g$ is the drift of the random walk process.

Let assume that the income (endowment) shocks cannot be diversified.

Let assume that there is only one asset, a riskless financial asset with a constant
The social planner solves the following problem:

$$\max_{\{c_s\}} E_0 \left[ \sum_{s=0}^{\infty} \beta^s \frac{c_s^{1-\gamma}}{1-\gamma} \right]$$  \hspace{1cm} (3)$$

$$F_s = (F_{s-1} - c_{s-1}) (1 + r) + y_s \quad s = 1, 2, 3, \ldots ,$$  \hspace{1cm} (4)$$

where $F_s$ is the (net) financial asset of the economy after the inflow of income but before consumption expenditure.

Lifetime wealth $L$ is the following:

$$L_t = F_t + H_t,$$  \hspace{1cm} (5)$$

where $H_t = E_t \left[ \sum_{s=t+1}^{\infty} y_s / (1 + r)^{s-t} \right]$ is human wealth.

Skinner (1988) derived by Taylor-approximation the optimal path of consumption:

$$c_t = \left[ (1 + r) \beta (1 + v_t) \right]^{1/\gamma} \frac{L_t}{E_{t-1} [L_t]} c_{t-1},$$  \hspace{1cm} (6)$$

$$v_t = \frac{\gamma (1 + \gamma)}{2} \frac{\sigma_L^2}{L_t},$$  \hspace{1cm} (7)$$

where $\sigma_L^2$ is the relative variance of lifetime wealth, which in case of a random walk with drift is the following:
\[ \sigma_{L_t}^2 = \frac{\text{var}(L_t)}{L_t^2} = \left( \frac{\sigma \frac{1+r}{1+g} H_t}{F_t + H_t} \right)^2. \] 

(8)

The steady state value of \( F/y \) and \( c/y \) can be determined in the following way.

In steady state consumption grows along income:

\[ 1 + g = (1 + r)^{1/\gamma} \beta^{1/\gamma} (1 + v)^{1/\gamma}. \] 

(9)

Expressing \( v \) and by substitution equations (7)-(8) lead to the following:

\[ F/y = \left( \frac{1 + r}{1 + g} \right)^{\frac{\gamma}{2}} \left( \frac{\gamma (1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\gamma} - 1 \right)} - 1 \right) H/y = \] 

\[ = \left( \frac{\text{var}(L)}{y^2} \frac{\gamma (1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\gamma} - 1 \right)} \right)^{0.5} - H/y. \] 

and from (5):

\[ L/y = \left( \frac{\text{var}(L)}{y^2} \frac{\gamma (1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\gamma} - 1 \right)} \right)^{0.5}, \] 

(11)

\[ c/y = \frac{1 + g}{1 + r} L/H, \] 

(12)

where \( F/y, L/y, \) and \( c/y \) are steady-state values of \( F/y, L/y \) and \( c/y. \)

To improve the applicability of the model, in our calibration exercise we introduced habit behavior\(^2\) on the assumption that the social planner – the fiscal

\(^2\)We know that the way we modelled habit (along the lines of Constantinides (1990) ) creates
authority — cannot tolerate a consumption that is less than $\rho c_{t-1}$. This means that in equation (10) $\sigma_z$ were replaced by $\sigma_z/(1 - \rho)$.

3 The social optimum and individual optima

Table 1 gives the steady state values calculated on the basis of equation (10) depending on various values of the parameters. The values are related to GNP.

<table>
<thead>
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<th>Table 1: Steady state financial wealth positions</th>
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<td>Parameters</td>
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<td>Standard deviation of income</td>
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<td>$\beta$</td>
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The values of $(F - y)/y$ are calculated with the parameter of their own row. The rest of the parameters are from baseline:

| Parameters                  | Baseline |           |
| Standard deviation of income | -3.950   | -0.500    | 2.950     |
| $\beta$                     | -3.203   | -0.500    | 2.949     |
| $\gamma$                    | -5.272   | -0.500    | 3.575     |
| $g$                         | 4.005    | -0.500    | -4.914    |
| $r$                         | -4.412   | -0.500    | 1.953     |
| Habit                       | -2.692   | -0.500    | 0.709     |

$(F - y)/y$ is the steady state value of indebtedness (if it is negative) or foreign lending (if it is positive) in proportion to GNP.

The effects of changes in the parameters are easy to interpret. Faster growth results in larger indebtedness, similar effect has larger impatience. Risk aversion ($\gamma$ and $\rho$) decreases the steady state debt level.

The sign of the effect of interest rate change is ambiguous because of its wealth effect. It is interesting, that in contrast to the point expectation model the wealth effect here works through the larger absolute risk that is acceptable by higher wealth.

too many puzzles to be satisfactory. However, we preferred here a simple approach and refer to Campbell—Cochrane (1995) for a discussion of the problem.

3 For a derivation of this amendment see Simon—Várpalotaí (2001).
The steady state financial asset rate is very sensitive to the parameters. For $\beta$ and $\gamma$ we took values assumed in most cases in the literature$^4$. For the habit parameter we had no empirical or theoretical basis for our assumption. We used this parameter to calibrate the desired output, a reasonable rate of indebtedness. This calibration strategy was similar to that of Ayiagari-McGrattan (1998), who aimed consistence with an assumed (observed) interest rate and had to use 3-digit detail to determine the proper $\beta$.

As we see, the model is not suitable for working out advices for economic policy about the optimal level of indebtedness. It only shows that the model might explain observed magnitudes with reasonable parameters.

The sensitivity of parameters is probably still a puzzle. However, the progress in contrast to earlier models is marked. The point estimation model leads to a level of optimal indebtedness 20-30 times GDP in the stable case (See Obstfeld–Rogoff (1995)). Overlapping generations models have difficulties in bringing observed asset ratios in line with reasonable parameters.

Let assume that households bear idiosyncratic labor income risks. We believe that it is reasonable to assume that the labor market is not complete. One can contrive schemes of insurance arrangements for labor income and use these as a basis of models but it seems better to comply with the fact that such insurance schemes do not exist or they have marginal importance, whatever its reason. In this case even if we assume that agents have identical and homothetic utility functions, the aggregate maximization problem is not the sum of the individual problems. Aggregate variance depends on the covariances of individual risks as well.

Another difference between the problem of the social planner and the individual problems come from the difference in the planning horizon and in accounting for new

$^4$Some authors think that much higher values for $v$ and accordingly much lower values for $\beta$ are possible. Friedman (1957; 1963) assumed 0.8 for $\beta$ and Hayashi (1982), Weale (1990), Carrol (1990) considered similar magnitudes.
generations. The consequences of these differences have been intensively discussed in the literature for the point-expectation model\(^5\). To exclude this problem from our discussion and concentrate on the consequences of idiosyncratic shocks, we assume the economy to consist of a constant number of infinitely lived dynasties. There is a large literature on lifetime income and consumption patterns. (See for example Hubbard–Skinner–Zeldes (1993), Storesletten–Telmer–Yaron (2000), Gourinchas–Parker (1999)) We do not contribute to this literature only point out the main parameters that differ in the problem of households and the social planner.

Let assume that the dynasties are hit by synchronized permanent shocks and idiosyncratic transitory shocks. This means, that the (log of the) income process of the "representative" dynasty is described by the sum of a random walk with aggregate shocks and an autocorrelated process with idiosyncratic shocks. This way we can approximate the observation confirmed by Jenks (1972) that intergenerational correlation of incomes within dynasties is rather low (0.12-0.15 between parents and children), but on the other hand we allow for persistent differences in individual life cycles. According to this assumption the income process of the representative dynasty can be described the following way:

\[
\dot{y}_s = (1 + g)\dot{y}_{s-1} + \xi_s \quad \ln(\xi_s) \sim N(0, \sigma^2_\xi I), \quad (13)
\]

\[
g_s = (q_{s-1})^{10} \xi_s \quad 0 < \alpha < 1 \quad \ln (\xi_s) \sim N(0, \sigma^2_\xi I), \quad (14)
\]

\[
y_s = g_s \dot{y}_s \quad (15)
\]

Empirical estimates on the variance of American incomes are in the range of 30 percent with an autocorrelation of 0.4 \(^6\). For the aggregate shocks we assumed a 2.5

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\(^5\)See the development of this line of thinking in Blanchard (1985), Buiter (1988) and Weil (1989).

percent relative variance with an autocorrelation of 1. 7

Let assume that $\varepsilon$ and $\xi$ are independent: $\sigma_{\varepsilon \xi} = 0$. Logarithmic variance of life cycle wealth is then the following: 8

$$\sigma^2_L = \left( \frac{1+r}{1+r^2} H \right)^2 \sigma^2_\varepsilon + \left( \frac{1+r}{1+\alpha+(r\alpha)} \right)^2 \sigma^2_\xi.$$

(16)

If $r = 0.05$, $\alpha = 0.4$, $g = 0.03$, $\sigma_\xi = 0.3$, $\sigma_\varepsilon = 0.02$, then we can see by substitution that total variance of individual wealth consists of the aggregate variance term and a variance of idiosyncratic shocks that is about half as much as the aggregate variance. If the parameters of the individual utility functions would be the same as those of the social planner, this would mean that individual precaution would lead to higher saving than it is justifiable from the point of view of a social optimum. However, there is no reason to assume that the parameters of the social and the individual utility functions are the same. Individuals might be willing to bear higher risks of income loss than society as a whole. A family may endure a 20-30 percent loss in income within a year, while the same loss for a whole country would probably be considered disastrous.

Whatever differences exist in the parameters of the two problems, it is clear that aggregating individual decisions does not lead to a meaningful social optimum. If we accept the idea or the existence of a social optimum it is interesting to tell whether it is attainable by fiscal policy. In the next section we will see, that within the framework of the model fiscal policy can efficiently determine the rate of steady state net foreign assets because there is an approximately one-to-one correspondence between domestic bond issues and net foreign asset changes.

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7 We know that a calculation aimed at a best fit to actual data would have to take into account that aggregate income has a transitory component as well.

8 For a derivation of this amendment see Simon–Várpalotai (2001).
4 The irrelevance of the Ricardian equivalence principle

We keep the assumption of a constant number of infinitely living dynasties. This allows us to separate the effect of precautionary behavior from the effect of the existence of disconnected generations on Ricardian non-equivalence that was analyzed by Blanchard (1985), Buiter (1989), Weil (1989).

Let assume that the government issues $D$ risk-free bonds at an international $r = r^*$ interest rate. Because of the assumption on intertemporally linked dynasties Ricardian neutrality regarding life-cycle wealth prevails. However, the question is open how much the outstanding stock of bonds effects the variance of life-cycle wealth. If it changes this variance it will not be neutral for the desired total holding of assets by individuals.

Let assume that our country is small and open. Total financial assets of domestic residents are the following:

$$ W = F + D, \quad (17) $$

where $W$ is total financial wealth,

$D$ net domestic bonds held by domestic residents,

$F$ net foreign bonds held by domestic residents.

Let assume for sake of simplicity that only labor income is taxed.

After-tax human wealth:

$$ \tilde{H} = H - D. \quad (18) $$

Let consider two cases, lump-sum and proportional taxation.
1. Lump-sum taxes. The interest burden of the debt is distributed among individuals according to their expected income in the period of the issuance of the bond. This does not affect the absolute variance of lifetime wealth and because of the neutrality in terms of wealth the effect on relative variance is unchanged as well.

Specifically, in equation (10) substituting \( W \) from (17) and replacing \( H \) with \( \bar{H} \) from (18):

\[
\frac{F/y + D/y}{y} = \frac{1}{y} \left( \text{var}(L) \frac{\gamma(1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\beta} - 1 \right)} \right)^{0.5} - (H/y - D/y). \tag{19}
\]

From this (Ricardian) total debt neutrality is a straightforward consequence:

\[
\frac{\Delta F/y}{\Delta D/y} = 0.
\]

2. Proportional taxes. Let assume that the interest burden of debt is distributed proportionally to actual rather than expected income. In this case the tax decreases the absolute variance of income by the rate \( (H - D)^2 / H^2 \) while wealth-neutrality is maintained as before. Equation (10) modifies the following way:

\[
\frac{F/y + D/y}{y} = \frac{1}{y} \left( \frac{(H - D)^2}{H^2} \text{var}(L) \frac{\gamma(1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\beta} - 1 \right)} \right)^{0.5} - (H/y - D/y). \tag{20}
\]

Differentiating by \( D/y \) the effect on total external position:

\[
\frac{\Delta F/y}{\Delta D/y} = \frac{1}{H} \left( \text{var}L \frac{\gamma(1 + \gamma)}{2 \left( \frac{(1+g)^\gamma}{(1+r)^\beta} - 1 \right)} \right)^{0.5} + 1 - 1 = -L/H. \tag{21}
\]

This means that the effect depends on the ratio of (steady state) total wealth to human wealth. Without having to discuss whether we should include equity into non-human wealth in a more realistic extended model or not, we can safely say that
in practice human wealth far exceeds non-human wealth that makes the $L/H$ ratio close to 1. This means that domestic debt displaces foreign assets in agents' wealth at a rate of around one for one. If agents are net creditors, the replacement rate is even larger than 1. The reason is, that net creditors have a relatively smaller human capital, therefore the same amount of "insurance" of labor income that the tax system provides means more for them in relative variance terms.

The fact that the tax system has an insurance effect against income risks has been long known. Analyses started with Chan (1983) and Barsky–Mankiw–Zeldes (1986) Kimball–Mankiw (1989) calculated coefficients that can be compared with our results. Barsky et al. showed in a simple model that the insurance effect may be important and their results are much conform with our "close to 1" coefficient. On the other hand, Kimball–Mankiw when assuming infinitely living agents and random walk income arrived at a coefficient significantly smaller than 1. Part of the difference might come from the CARA utility function that they assume. However we believe that the main reason of the difference comes from the calibration criteria of the two calculations. Our calibration criterion for the parameters was that in addition to being in line with former research results or reasoning the financial asset outcome that they imply should be comparable to actual observed levels. Because of the high sensitivity of results, we could have chosen other parameters that do not contradict our a priori knowledge regarding parameter values but would lead to unreasonably extreme debt levels and consequently to a displacement ratio that is far from 1. Kimball–Mankiw started with a set of reasonable assumptions on the parameters themselves but did not calibrate those parameters to produce reasonable asset levels at the same time. This has led to a displacement ratio in the magnitude of 0.5, convincing enough to show that insurance is important but not as much to show that precautionary behavior and the tax system together may bring full displacement.
It is probable that similarly to precaution liquidity constraints are important in determining savings. Unfortunately our analytical method based on the Taylor-approximation is not suitable to handle liquidity constraints. Ayiagari-McGrattan (1998) took both effects into account by explicitly solving a stochastic dynamic programming problem. In choosing the parameters they used a calibration principle similar to that of ours. They allowed parameters that reproduces the observed 0.66 level for $D/y$ and the 4.5% level of $r$ in the US. Unfortunately we could not infer on a displacement coefficient similar to ours, because their exercise assumes a closed economy and looks for the relation of the interest rate and public debt.

We know without calculations that government debt has an effect of loosening liquidity constraints of agents by broadening their collateral basis. The effect of government debt through this channel has the same sign as the effect of income insurance of taxes incurred by the debt has. It would need further research to determine the relative weights of these two channels in the decisions of agents. To an empirically relevant answer we probably have to drop the assumption of uniform agents with homothetic utility function.\footnote{9See Woodford (1990) and Ayiagari-McGrattan (1998).}

5 Conclusions

We have analyzed the saving behavior of a small open economy where infinitely-living agents have CRRA utility functions, idiosyncratic labor income risks together with aggregate income shocks.

In this model the sum of the agents’ decisions cannot be formulated as the solution to an aggregate optimization problem. However, an optimization problem of the social planner can be formulated and its solution can be enforced by an associated suitable fiscal policy. With an income-tax system this fiscal policy is

\footnote{10Ongoing research seems to follow this line. See for example Carroll (2000) and Mankiw (2000).}
very efficient in the sense that fiscal saving appears approximately one-for-one in aggregate saving.

The model can be calibrated with plausible parameters to reproduce aggregate debt ratios that are in the range of actual ratios. This demonstrates that precautionary behavior alone can give an explanation to the relatively low levels of net foreign financial assets that are observed in contrast to the predictions of the point expectation model with infinite horizon. This does not mean that other determinants may not play important roles. Liquidity constraints both in the agents’ problem and in the country-problem (arising from international law-enforcement difficulties) are presumably important. As simulation results on the steady-state asset ratios are highly sensitive on the parameters we could easily change them while remaining within the range of reasonable values to give room for additional explanatory factors.

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