

Should Currency Be Taxed?

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August 2002

Preliminary Version: please do not quote without consultation

Paper presented at the Macro Money Finance Conference, University of Warwick,

September 2002

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Introduction

Abilities to earn differ. Lump-sum taxes based on individuals' own reports of their own earning ability fall foul of the incentive for the able to lie. So marginal taxes on income become unavoidable, despite their adverse effects on work incentives. These marginal taxes are needed to defray public spending and redistributive transfer costs.

Taxing money involves a positive nominal interest rate. This, too, is distortionary, and would of course be avoided under ideal conditions. But in the second best world where marginal income taxes are inescapable, should money stay untaxed? This is the question this paper attempts to throw some new light upon².

This paper adopts the view that people hold real money because it is convenient. It saves them time, time that could be devoted to (enjoyable) leisure and/or remunerated work that helps pay for (enjoyable) consumption. The paper goes beyond the standard model of optimum inflation in a representative agent setting where government meets its outlays through a *constant* marginal income tax rate and possibly seignorage. It does this

² This is an old and very lively debate, which has stimulated much valuable work – explored most recently by Lucas (2000). Earlier contributions of note include Woodford's survey (1990). Other contributions include, among others, Feldstein (1996), and, from scholars in the Bank of England, Briault (1995), Bakhshi et al (1998) and Haldane (1998).

in two ways. First agents differ in earning power, and their utilities will differ too. Second, marginal income tax rates *decline* with income (as do transfers). Agents pick their personal tax-transfer package to suit themselves, as in Mirrlees (1971).

One particular issue is what if anything inflation might do to help the poor – by transferring seignorage revenue from the better off to benefit payments received in larger amounts by the less well off. Easterley and Fischer (2001) find that inflation in practice tends to hurt the poor more than others. But need this be true if optimal taxes on incomes and money are set under a social welfare function that gives added weight to the less well off?

Assumptions

The key ideas underlying the paper are as follows.

- i. earning abilities, reflected accurately by market wage rates, display a given distribution
- ii. agents enjoy consumption and leisure, with similar preferences
- iii. (real) money holdings reduce the time devoted to “shopping” activities
- iv. labour supplies and money demands are governed by individual optimization

- v. the marginal income tax rate, t , contains two components, one (ζ) exogenous and common, and a second which is positive and proportional³ to the individual's transfer requested, b: $t = \zeta + \theta b$
- vi. the government's public goods expenditure is exogenous, and financed exactly by the sum of seignorage, and by marginal income taxes levied on income plus transfer minus seignorage.
- vii. the key policy parameters θ , u (the unemployment rate) and N (the nominal interest rate) are set by the authorities, subject to (iv), (v) and (vi), to maximise some weighted average of mean and minimum utilities.

Beginning with Phelps (1973), there have been numerous previous attempts⁴ to probe the public finance argument for taxing money. Nearly all of them rely, however, on a representative agent setting, which is unhelpful as it ignores the main respectable justification for distortionary taxes (problems of redistribution); and then only in a setting with a single income tax rate⁵. The main novelty in this paper, then, consists in applying

³ This makes the individual's consumption a quadratic function of the benefit, and the marginal income tax rate declines linearly with the sum of earnings and benefit net of seignorage. More general functions are of course possible; but this parabolic form has the great merit of tractability and closed form solutions for the variables of interest.

⁴ Later contributions of note include Kimbrough (1986), Guidotti and Vegh (1993) and Mulligan and Sala-I-Martin (1997).

⁵ The standard model of money as a time-saver in a representative agent framework with government revenue from a constant marginal income tax rate (t) and (possibly) seignorage emerges with the following key imperative. *Never* tax money if: money demand is sufficiently interest-elastic, *or* if government spending is low enough, *or* if utility is sufficiently concave in consumption and leisure, *or* if money demand is sufficiently consumption-elastic (certainly if this elasticity is unitary or higher). The analysis runs as follows. With shopping time $s(m)$ dependent on real money alone, decreasing and convex, and consumption taken to equal the net of tax sum of wage income and seignorage, the agent optimizes with respect to work-time and money, yielding indirect utility as a function of t and the nominal interest rate n . The government's revenue $R(t,n)$ must balance government spending, g/w (w is the given real wage rate). As far as the Friedman Optimum Quantity of Money Rule is concerned the key question is whether n should be zero as in Friedman or positive. The answer is that everything depends on whether the agent's marginal rate of substitution between the two bads, t and n , is equal to the authorities' marginal rate of substitution between those two instruments. This resolves into a comparison of g/w and qE , where E (positive) is the nominal interest elasticity of money demand, and q is the sum of the elasticities of the marginal utility of consumption and the marginal disutility of work (or some generalizing incorporating cross effects if utility is not additively separable). If g/w is less than qE , money should *not* be taxed. If qE is less than g/w , money should be taxed, and an interior optimum exists where both the nominal rate of interest and the marginal rate of income tax are positive.

the theory of self-selection constraints for income taxes and transfers to the central question of long-term monetary policy – should currency be taxed?

Analysis

Assumption (v) above allows us to write the individual's consumption as:

$$c(w) = [1 - \zeta - \theta b(w)][wh + b(w) - S(w)] \quad (1)$$

Where c denotes consumption, $S(w) = Nm(w)$ represents seignorage, and h work-time.

Utility, U , is increasing in c and L , leisure, and concave in the first argument. Leisure competes with work time, h , and shopping time, $s(m)$, subject to a time constraint $T - L - h - s(m) = 0$. Utility functions are common across agents; $s'(m) < 0 < s''(m)$. The individual chooses $b(w)$ to maximise utility, so that

$$b = \frac{1}{2} \left[\frac{1-\zeta}{\theta} wh + S \right] \quad (2)$$

$$wh + b(w) - S = \frac{1-\zeta}{\theta} wh - b(w) = \frac{1}{2} \left[\frac{1-\zeta}{\theta} + wh - S \right]$$

Substitution of (2) into (1) implies

$$c(w) = \frac{\theta}{4} \left[\frac{1-\zeta}{\theta} + wh - S \right]^2 \quad (3)$$

Given (3), maximisation of $U(c, L)$ with respect to h and m subject to the time constraint

$$T - L - h - s(m) = 0 \text{ implies}$$

$$-s'(m) = N/w \text{ for labour force participants} \quad (4)$$

$$-s'(\tilde{m}) = N / \tilde{w} \text{ for non-participants} \quad (5)$$

$$\text{and } h^* = \text{Max}\{0, \text{argmax}(h) U(c, T - h - s(m))\} \quad (6)$$

Non-participants supply no labour, and the ablest of these has a wage rate \tilde{w} . The tilde refers to all variables associated with this group, which will typically be non-empty if the minimum ability is low enough.

Define $v^* = wh^* + 1 - \zeta / \theta - S^*$ for those for whom $h^* > 0$ and where S^* is the optimised level of seignorage ($S^* = Nm^*$, where m^* is given implicitly by (4)). Similarly, let $\tilde{v}^* = 1 - \zeta / \theta - \tilde{S}^*$ for non-participants. Hence

$$h^* = \frac{1}{w} [v^* - \tilde{v}^* + S^* - \tilde{S}^*] \quad (7)$$

For those in work, maximised utility will be

$$U^*(\theta v^{*2} / 4, T - s^* - [v^* - \tilde{v}^* + S^* - \tilde{S}^*] / w) \quad (8)$$

and, for those who do not work

$$\tilde{U}^*(\theta \tilde{v}^{*2} / 4, T - \tilde{s}^*) \quad (9)$$

The government budget constraint – assumed balanced – is formally equivalent to an aggregate resource constraint

$$g - \int_w f(w) [wh^*(w) - \theta^{*2} / 4] dw + \theta \tilde{v}^{*2} u / 4 = 0 \quad (10)$$

where $u(\tilde{w})$ is given by $\int^{\tilde{w}} f(w) dw = u(\tilde{w})$. The variable u can be interpreted as an unemployment rate. (10) reflects the assumption that, as in Mirrlees (1971), labour is the

sole factor of production, and aggregate output is simply the sum of all labour inputs multiplied by their abilities. Innocuously, and for convenience, we shall assume that $f(w)$ has unit mass.

The Roberts Social Welfare Function (Roberts, 1980), ψ , will be a weighted mean of minimum utility (the Rawlsian maximand) and mean utilities (the Benthamite maximand):

$$\begin{aligned} \psi = & [\alpha + u(1 - \alpha)]\tilde{U}^*(\theta\tilde{v}^{x^2}/4, T - \tilde{s}^*) \\ & + (1 - \alpha) \int_{\bar{w}} f(w)U^*(\theta v^{*2}/4, T - s^* - [v^* - \tilde{v}^* + S^* - \tilde{S}^*]/w)dw \end{aligned} \quad (11)$$

This is to be maximised subject to (10) with respect to the parameters⁶ u , θ and N . Our central question is, of course, what role if any taxes on money have in the resulting optimum.

To make further progress, it is helpful to specify the shape of three key functions – those for utility, shopping time, and the ability distribution.

The simplest utility function is logarithmic in c and quasi-linear in L :

$$U(c, L) = \ln c + L \quad (12)$$

The simplest form for $f(w)$ is uniform on $[1,0]$. As for shopping time, we shall work with three forms:

⁶ The parameter ζ is not involved, since $(1 - \zeta)/\theta = \tilde{v}^* + \tilde{S}^*$, and (11), like (7) and (8), makes use of this substitution.

$$(a) \ s(m) = \gamma - \delta m + \frac{\varepsilon}{2} m^2 \quad (\text{quadratic}) \quad (13)$$

$$(b) \ s(m) = \frac{\gamma m^{1-\frac{1}{\varepsilon}}}{\frac{1}{\varepsilon} - 1} \quad (\text{isoelastic}) \quad (13')$$

$$(c) \ s(m) = \gamma e^{-\delta m} \quad (\text{exponential}) \quad (13'')$$

Parameter restrictions⁷ in (a) are $\gamma, \delta, \varepsilon > 0$ and $\gamma = \delta^2 / 2\varepsilon$ (so that $s(m) \rightarrow 0$ if money is untaxed). Those in (b) are $1 > \varepsilon > 0$ and $\gamma > 0$, needed to satisfy the principle that $s(m)$ is decreasing and convex; similarly, $\gamma, \delta > 0$ in (c).

With utility given by (12), $v^* = 2w$ and $\tilde{v}^* = 2\tilde{w}$. The assumption on $f(w)$ implies $\tilde{w} = u$.

Shopping time is $N^2 / 2\varepsilon w^2$ for those in work, and $N^2 / 2\varepsilon u^2$ for non-participants, under (a).

With (b), $s(m) = \frac{\gamma}{\frac{1}{\varepsilon} - 1} \left[\frac{N}{\gamma w} \right]^{1-\varepsilon}$, and with (c), $N / w\delta$. Seignorage in these three cases equals:

$$\frac{N}{\varepsilon} \left(\delta - \frac{N}{w} \right), N^{1-\varepsilon} (\gamma w)^\varepsilon, \frac{N}{\delta} \ln \frac{\gamma \delta w}{N}$$

Our specific assumptions about utility and $f(w)$ imply that utilities for those in and out of work are, respectively:

$$U^*(w) = 2 \ln w + \ln \theta + T - s^* - 2 \left(1 - \frac{u}{w} \right) - (S^* - \tilde{S}^*) / w \quad (14)$$

$$\tilde{U}^*(u) = 2 \ln u + \ln \theta + T - \tilde{s}^* \quad (15)$$

⁷ It follows from (4) that money demands for those in works in the three cases are $m = (\delta - N/w) / \varepsilon$, $m = [\gamma w / N]^\varepsilon$ and $m = \frac{1}{\delta} \ln[\gamma \delta w / N]$, respectively. For the unemployed, \tilde{w} replaces w .

Consequently, integrating (14) with respect to w between l and u , with a weight of $l - \alpha$, and adding (15) with a weight of $\alpha + u(1 - \alpha)$, gives social welfare:

$$\begin{aligned} \psi = & \ln \theta + T - \tilde{s}^* + 2\alpha \ln u \\ & - (1 - \alpha)[2 \ln u + 4(1 - u) + \int_u^1 [s^*(w) - \tilde{s}^* + (S^* - \tilde{S})/w] dw] \end{aligned} \quad (16)$$

and the government budget or resource constraint (10) gives

$$\theta = 3[(1 - u)^2 - g + \int_u^1 (S^* - \tilde{S}^*) dw](1 + 2u^3)^{-1} \quad (17)$$

The Implications for Monetary Policy

What can we discern here about monetary policy? Consider a small increase in the nominal interest rate (which must betoken a rise in the rate in the rate of inflation, or lower deflation, in the long run). The harm this does is direct, and easily apparent in (16). First, it increases the shopping time of those out of work, \tilde{S}^* , and this is a negative effect – the only negative effect, in fact, when $\alpha = 1$, so that the social welfare function is Rawlsian. When $l > \alpha$, we must add the hurt wrought to the utilities of those in work, which are reflected in two terms: the differential rise in workers', as against non-workers', shopping time, $s^* - \tilde{s}^*$; and the differential increase in wage-deflated seignorage, $(S^* - \tilde{S}^*)/w$. The last effect damages utility by tending to increase workers' hours of work.

But our monetary policy change is not all bad. Because the authorities gain some extra revenue, in the form of higher seignorage, there is a rise in the proportion of this income

that individuals can retain after tax. Private consumption is proportional to θ , for both workers and non-workers alike. So optimum long-run monetary policy could involve some departure from Friedman's Optimum Quantity of Money Rule ($N=0$). N should be positive if, and only if, the beneficial effect of N on θ outweighs the harm done, in the neighbourhood of $N=0$.

Consider, first, the quadratic shopping-time function (13). Writing $\kappa = N^2/2\varepsilon$, we have

$$\int_u^1 [S^*(w) - \tilde{S}^*]dw = \frac{2\kappa}{u}(1-u + u \ln u), \tilde{s}^* = \kappa u^{-2}, \text{ and}$$

$$\int_u^1 [s^*(w) - \tilde{s}^* + (S^*(w) - \tilde{S}^*/w)]dw = \frac{k}{u^2}[1-u^2 + 2u \ln u]$$

Suppose N , hence κ , to be low enough for

$$\ln[(1-u)^2 - g + \frac{2\kappa}{u}(1-u + u \ln u)] \sim \ln[(1-u)^2 - g] + \frac{2\kappa}{u} \frac{1-u + u \ln u}{(1-u)^2 - g}$$

The partial derivatives $\partial\psi/\partial\kappa$ in the neighbourhood of $N=0$ will be

$$[-1 + 2 \frac{u(1-u + u \ln u)}{(1-u)^2 - g} + (1-a)[1-u^2 + 2u \ln u]]$$

From $\partial\psi/\partial u = 0$ when $N \sim 0$ we have

$$\frac{1-u}{(1-u)^2 - g} = \alpha/u - \frac{3u^2}{1+2u^3} + (1-\alpha)[2 - \ln u]$$

Substituting, it is apparent that

$$\frac{\partial\psi}{\partial\kappa} \Big|_{N \sim 0} \rightarrow 2A \left[B + \frac{1+u+u^2}{1+2u^3} \right] - [1 + B((1-u)^3 + 2A^2)] \quad (18)$$

Where $A = 1 - u + u \ln u$ and $B = \frac{1 - a}{1 - u}$.

Results

A number of results can be obtained here. One is that if optimal unemployment falls to zero (a consequence of very high levels of exogenous government spending), $\alpha > 0$ is both sufficient and necessary for departure from Friedman's rule. A strictly Benthamite social welfare function will prescribe no taxation on money even in this extreme case. But any positive weight on minimum utility will suffice for taxing money.

A second result is that if α goes to its other extreme value of unity, so that social welfare is now strictly Rawlsian, money should be untaxed if, and only if, government spending is sufficiently low. The threshold value of u below which money *should* be taxed is some 23%, just over half the value it would display if government spending were zero. So high enough government spending triggers a more inflationary, looser, monetary policy, once this threshold is crossed.

A third conclusion is that optimal policy is likelier to involve taxing money, the greater the social welfare weight on minimum as opposed to mean utility. If left-wing administrations operate with a higher value of α than right-wing ones, we should expect a greater probability of inflation with the former. This inference is reinforced if left wing administrations can be expected to spend more on public goods.

Let us now turn to the negative exponential shopping time function. Here, shopping time reduces to $\frac{N}{w\delta}$ for those in work and $\frac{N}{u\delta}$ for others.

The seignorage difference between the two groups $S(w) - \tilde{S}(u)$, is $\frac{N}{\delta}[\ln w - \ln u]$. Carrying out the integrations prescribed by (16), and repeating the previous task of evaluating the sign of $\frac{\partial \psi}{\partial N}$ at $N \sim 0$ with $\frac{\partial \psi}{\partial u} = 0$, establishes, once again, that a positive welfare weight on minimum utility is necessary and sufficient for taxing money at negligible unemployment. Again, a strict Benthamite would never tax money; and a higher welfare weight on the poorest will raise the likelihood of inflation becoming optimal. One difference, however, is that the conditions for taxing money in the strict Rawlsian case ($\alpha = 1$) become much more stringent. The unemployment threshold falls to about 3%, substantially lower than when shopping time was quadratic.

Repeating these experiments with the isoelastic money demand function reveals, again, that $\alpha > 0$ is necessary and sufficient for any departure from the Friedman rule in the neighbourhood of negligible (and optimal) unemployment. When welfare is strictly Rawlsian, the unemployment threshold at which money starts to be worth taxing is, in this case, sensitive to the parameter ε , which here represents the (absolute value of the) nominal-interest-elasticity of money demand. At $\varepsilon = 1/2$, the unemployment threshold is about one-ninth. With $\varepsilon = 1/4$, it rises to one-seventh. The intuition here is that the less interest-elastic money demand is, the more appealing taxes on money become from a Ramsey-efficiency standpoint.

What inferences can we draw from this analysis? First, whether money should be taxed at all is indeed an open question. We have found no support for taxing money when welfare is strictly Utilitarian, even at levels of vanishingly low levels of unemployment, occasioned by enormous given requirements for expenditure on public goods.

The greater the welfare weight on minimum utility, however, the greater the likelihood that money should be taxed. When there is even some welfare weight on minimum utility, over and above that implied by a utilitarian welfare conception, money must be taxed in the absence of unemployment and will continue to be worth taxing unless government spending needs to shrink to a certain threshold. The position of this threshold depends on the social welfare function weightings between minimum and mean utility. It also depends negatively upon the interest elasticity of money demand.

It is interesting to compare these findings with what happens in the simpler case where the marginal rate of income tax is just a constant, t , faced by all. With the rest of our assumptions unchanged, the now constant income tax rate t will be

$$t = 2(g + u + \int_u S(w)dw - (1-u)S(u))/(1+u^2)$$

All utilities include the additive term $\ln(1-t)$. So the Roberts social welfare function can be expressed in terms of u and n alone. It transpires that the range of values of u for

which money *should* be taxed is considerably larger, now, than with the parabolic benefit function that individuals optimize for themselves, which has been explored above. This is true for all shopping time functions. The intuition for this is that the quadratic form for benefits and tax gives an additional degree of freedom to income taxation, which is unavailable when the marginal rate is constant and common. One advantage this leads to is a reduction in the efficiency losses with income tax, since the marginal rate is lowest where it counts most – for the ablest. So it should be little surprise that taxing money (a blunt instrument that, unlike income tax, has qualitatively similar effects on all) should become less attractive.

Some Qualifications

Several qualifications must be declared. The individual's choice of the tax-benefit regime that suits him best, and his money demand function, have been derived from a general utility function. But the analysis of the paper has specialised utility to one where the marginal utility of consumption is the reciprocal of consumption, and the marginal utility of leisure is constant and unitary.

Experiments with other utility functions have emerged with some interesting contrasts with the quasi-linear logarithmic form studied here: a symmetric Cobb-Douglas function, logarithmic in consumption and leisure, finds that Friedman's Optimum Quantity of Money Rule must be followed in all circumstances, even when social welfare is

exclusively *Rawlsian*. But a quasi-linear function where the elasticity of the marginal utility of consumption is 1.5 (as against unity in the logarithmic case) permits positive n for very high government spending (corresponding to unemployment of about 2%) under purely *Benthamite* social welfare.

Three different forms of the shopping time function have been explored, all of them special, in order to obtain clear results. But each of them has given consumption no direct role in shopping time. And the shopping time approach is only one of several. Its rivals include money-in-the-utility-function, and cash-in-advance. These can, however, be criticised on the grounds of arbitrariness, and the cash-in-advance approach is well known for reinforcing the case for Friedman's rule because it imposes a unitary consumption – elasticity of money demand. Within a shopping-time approach extended to include consumption directly, a representative agent model reveals that money should never be taxed when that elasticity is unitary (or higher). The reason for this is essentially that taxing currency involves double-taxation on consumption, which is already being squeezed by income taxation.

Moreover, the parabolic benefit function, which has formed the basis of the income tax regime studied in this paper, is only one of several possibilities. It carries with it the implication that marginal income tax rates decline linearly with earnings plus benefit less seignorage. Other shapes are of course possible.

Further, the government's direct spending on public goods has been taken as given, despite that fact that it, too, will in general form part of a still broader optimisation problem. The final qualifications that merit emphasis here relate to the absence of dynamics, stochastics and banks offering a substitute for currency. Dynamics can be incorporated straightforwardly; the analysis of the present paper can be treated as the steady state of an infinite-horizon optimising model⁸.

Stochastics may be inferred from the isomorphism of deterministic heterogeneous-agent, average-utility maximising (under the Benthamite specialisation of the Roberts social welfare function studied here), with expected-utility maximising by a representative agent. Uncertainty aversion in the latter context has its counterpart in the weight, α , placed on minimum utility in the Roberts welfare function.

Imperfect competition in product markets, and menu costs, are among other absentees from the cast list in this paper. Inflation has additional effects that operate when these are present. Menu costs by themselves would argue in favour of a zero general rate of inflation, since this would minimize them. Menu costs in combination with monopoly suggest a somewhat faster rate of inflation, because the average real price charged by a profit-maximizing monopolist fall with inflation (a point due to Diamond (1993)). Then there are impacts from inflation upon the size and numbers of firms in a monopolistic competition setup, recently studied by Wu and Zhang (2002).

⁸ In other settings, dynamics may pose further challenges, however. Jha et al (2002) find that an endogenous growth and

The introduction of banks⁹, and in particular imperfectly competitive banks¹⁰, calls for a much more complex set-up, which would take us too far from the confines of this paper.

Key conclusion

When government has to rely on distortionary taxes to meet spending obligations and arrange transfers, taxing money is indeed sometimes justified. But it is noticeably *less* likely to be justified when the marginal rate of income tax is non-uniform, and, instead, along with transfer receipts, declines linearly with income. This conclusion emerges clearly when utility is linear in leisure and logarithmic in consumption. In this special case, a Benthamite social welfare function, identified with average utility, will never permit the taxation of money in all the shopping-time examples we have explored, no matter how high the government's direct spending financing requirements. So non-linear income taxation appears to strengthen the case for Friedman's Optimum Quantity of Money Rule.

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money model may be unstable in the vicinity of $n=0$ (and elsewhere too).

⁹ Interest bearing bank deposits are explored in a perfect competition banking setup by Simonsen and Cysne (2001).

¹⁰ Mullineux and Sinclair (2002) offer a study of optimum taxation of income and money in this environment.

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