Exchange Rate Pass-Through and Disinflation in the Presence of Asymmetric Sector Specific Shocks

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Abstract

We lay out a sticky price dynamic general equilibrium model in order to study the nominal exchange rate — tradable price pass-through, the tradable – non-tradable inflation gap, and the interconnection of these phenomena.

We show that if prices are sticky, then asymmetric productivity shocks are not the only factors which can influence the inflation gap. But still the key determinant of the large long-run inflation gap observable in Hungary is the productivity difference of the two sectors. Furthermore we show that the productivity shocks which generate the inflation gap are partly responsible for the slow exchange rate pass-through in Hungary.

1 Introduction

The National Bank of Hungary (NBH) introduced inflation targeting in 2001. The efficient conduction of this regime needs clear understanding exchange rate pass-through and the determinants of tradable – non-tradable inflation difference. Macroeconomic theory explains the long run evolution of these phenomena by *purchasing power parity* (PPP) and the *Balassa-Samuelson effect.* That is, in the long run domestic tradable prices are determined by foreign tradable prices and nominal exchange rate, an tradable – non-tradable inflation gap by asymmetric sectoral productivity shocks.

On the other, hand these traditional theories cannot provide consistent explanation for the short run behavior of these variables. They contradict empirical evidences on exchange rate pass-through, e.g. Darvas (2001) and Hornok et al. documented that the Hungarian exchange rate pass-through is slow, while PPP implies a perfect, infinitely quick pass-through. Furthermore, both PPP and the Balassa-Samuelson mechanism preclude demand as an explanatory factor. According to these theories neither government spending, nor foreign demand shocks has any effects on inflation.

For this reason the studies prepared in the NBH follow more pragmatic approaches. E.g. Benczúr et al. (2002) or Hornok et al. (2002) share the view that domestic tradable prices are determined solely by foreign prices and nominal exchange rate, but they refuse perfect exchange rate pass-through. Furthermore in their models demand has a role in determination of non-tradable prices.

But there are some problems with this pragmatic view: Firstly, if somebody refuse PPP and perfect exchange rate pass-through then implicitly assume that foreign and domestic tradables are not perfect substitutes and price formation is sticky. But this implies that domestic tradable prices are determined not simply by an arbitrage process but by the interaction of demand an supply. Hence nominal exchange rate and foreign prices cannot be the only determinants of these prices. They have their role, but there are other factors, as well.

Secondly, the paper of Hornok et al. uses the model of Milesi-Ferretti (2000) in order to give a role to demand factors in the explanation of tradable – non-tradable inflation gap. But that model is based on the assumption that the two sectors do not use common inputs. Since this assumption contradicts the Balassa-Samuelson type explanations their approach moved to the other extreme. But according to empirical studies, e.g. Kovács (2002), this is not a useful view.

The purpose of this paper is to study in a consistent *new open economy macroeconomics* (NOEM) framework[1] the exchange rate pass-through, the tradable – non-tradable inflation gap, and their interactions. Our model is based on the Christiano et al. (2000) modification of the sticky price model of Calvo (1983) and of the sticky wage model of Erceg et al. (2000). The sectoral asymmetries are treated similarly as in Woodford (2002) and we used key elements of the small open economy model of Gali and Monacelli (2002).

In our model both price and wage formation are sticky. Prices of each sector are determined by real marginal cost, which depends on productivity, input prices and real wage. Nominal exchange rate influences real marginal cost through the prices of imported inputs, and aggregate demand through real wages. Foreign demand is influenced by nominal exchange rate through terms of trade, hence nominal exchange rate has a demand and a supply side effect on the prices.

In traditional models or according to the pragmatic approach, the nominal exchange rate – tradable price pass-through and the tradable – non-tradable inflation gap are independent phenomena. But in this model they are not: the inflation gap observable in Hungary cannot be explained without real shocks, but this shocks disturbs the demand for and the supply of home made tradables, hence the price formation and indirectly the exchange rate pass-through.

[1] This approach started by the seminal paper of Obstfeld and Rogoff (1995), their novelty was the joint assumption of the non-perfect substitutability of foreign and home made tradables and sticky prices.

This paper tries to help to better understand the inflation process in Hungary in two ways. Firstly, in our model the exchange rate pass-through depends on microeconomic parameters, hence it can utilize the information contained in microeconomic empirical studies (e.g. Rátfai (2001), Tóth and Vincze (1999)). Secondly, the consistent framework can help to integrate the different submodules of the NBH's inflation forecasting system, since its econometric module and its module on government spending and wage formation is not yet fully compatible to each other.

The structure of the paper is organized as follows. Section 2 presents the structure of the model. In Section 3 we discuss the simulation results. Firstly, in Section 3.1 we explain the determinants of the tradable – non-tradable inflation, then in Section 3.2 we review the results on exchange rate pass-through, and discuss its connection to the inflation gap and asymmetric productivity shocks, finally in Section 3.3 we study the effects of some other shocks on prices.

2 Structure of the model

2.1 Households

The model contains a unit mass of uniform households, who maximize the following utility function,

$$\mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(U(c_t) - V(l_t) \right) \right], \tag{1}$$

where c_t is aggregate consumption, l_t is labor supply, $U(c) = c^{1-\sigma}/(1-\sigma)$ and $V(l) = l^{1+\varphi}/(1+\varphi)$. The consumption and labor supply of individual households can be different, hence it could be useful labelling the variables by indices, but to keep the notation simple and tractable we omit them. Since in equilibrium the economic activity of each household will be the same this simplification will not cause any confusion. Aggregate consumption can be decomposed into consumption of tradable, c_t^T, and non-tradable goods, c_t^N. Their relationship is defined by the following CES function,

$$c_t = \left[\left(a^T \chi_t^T \right)^{1-\eta} \left(c_t^T \right)^{\eta} + \left(a^N \chi_t^N \right)^{1-\eta} \left(c_t^N \right)^{\eta} \right]^{\frac{1}{\eta}},$$

where $1 > \eta > -\infty$, and χ_t^N, χ_t^N represent stochastic taste shocks which modify the structure of aggregate consumption, furthermore

$$a^T \chi_t^T + a^N \chi_t^N = 1. \tag{2}$$

Home made and foreign tradable goods are not perfect substitutes. (Hence their prices measured in the same currency can be different.) Aggregate tradable consumption is defined by the following CES aggregator,

$$c_t^T = \left[\left(a^{TF} \right)^{1-\rho} \left(c_t^{TF} \right)^{\rho} + \left(a^{TH} \right)^{1-\rho} \left(c_t^{TH} \right)^{\rho} \right]^{\frac{1}{\rho}}, \tag{3}$$

where c_t^{TF} is the consumption of foreign and c_t^{TH} is that of the home made tradable goods, $1 > \rho > -\infty$ and $a^{TF} + a^{TH} = 1$. Furthermore we assume that c_t^k, $k = TF$, TH, N are also created by aggregation of individual goods,

$$c_t^k = \left(\int_0^1 \left(c(i)_t^k\right)^{\varrho} \, di \right)^{\frac{1}{\varrho}},$$

where $1 > \varrho > 0$. Households maximize (1) subject to a sequence of intertemporal budget constraints,

$$\begin{aligned} & \int_0^1 \left[P(i)_t^{TF} c(i)_t^{TF} + P(i)_t^{TH} c(i)_t^{TH} + P(i)_t^N c(i)_t^N \right] \, di + D_{t+1} \\ \leq \quad & R_{t+1} D_t + W_t l_t + \Pi_t + T_t, \end{aligned} \tag{4}$$

where $P(i)_t^{TF} = e_t P(i)_t^{TF*}$, D_t is the nominal portfolio bought at the beginning of period t, and R_{t+1} is the stochastic return of the portfolio. Furthermore, we assume that financial markets are complete. Π_t denotes the profit of domestic firms and T_t is a tax/transfer variable.

Define for all $k = TF$, TH, N goods the following price indices,

$$P_t^k = \left(\int_0^1 \left(P(i)_t^k\right)^{\frac{\varrho}{\varrho-1}} \, di \right)^{\frac{\varrho-1}{\varrho}},$$

the price index of tradables,

$$P_t^T = \left[a^{TF} \left(P_t^{TF}\right)^{\frac{\rho}{\rho-1}} + a^{TH} \left(P_t^{TH}\right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}},$$

and the consumer price index, CPI,

$$P_t = \left[\chi^T a^T \left(P^T F_t\right)^{\frac{\eta}{\eta-1}} + \chi^N a^N \left(P_t^N\right)^{\frac{\eta}{\eta-1}} \right]^{\frac{\eta-1}{\eta}}.$$

One can demonstrate that using the above defined price indices the households' optimization problem can be simplified by using the following aggregated budget constraint sequence,

$$P_t C_t + D_{t+1} \leq R_{t+1} D_t + W_t l_t + T_t + \Pi_t. \tag{5}$$

This approach makes it possible to solve the optimization problem in four subsequent distinct steps. Firstly, one can solve the intertemporal decision problem for aggregate quantities. Then for given aggregate variables one can find optimal tradable and non-tradable consumption. Thirdly, for given tradable consumption it is possible to divide it into home made and foreign tradables. Finally, one can find the demand for individual goods.

Aggregate consumption

Assuming complete international financial markets the solution of the households' optimization problem provides the following first order condition,

$$\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)=\frac{1}{R_{t+1}}. \tag{6}$$

The assumption of international completeness implies, that foreign households' have to satisfy a similar condition,

$$\beta\left(\frac{c^*_{t+1}}{c^*_t}\right)^{-\sigma}\left(\frac{P^{F*}_t}{P^{F*}_{t+1}}\right)\left(\frac{e_t}{e_{t+1}}\right)=\frac{1}{R_{t+1}}, \tag{7}$$

where c^*_t is aggregate consumption of foreign consumers and P^{F*}_t is the foreign CPI, in foreign currency terms. Combining eq. (6) and (7) and iterating them yields,

$$c_t=\vartheta c^*_t\left(\frac{e_tP^{F*}_t}{P_t}\right)^{\frac{1}{\sigma}}, \tag{8}$$

where ϑ is a constant, which depends on initial conditions (details are in the Appendix). Since the size of the domestic economy is negligible to the world economy, foreign consumption is equal to foreign output, i.e. $c^*_t=y^*_t$. Loglinearizing (8) yields

$$\tilde{c}_t=\tilde{y}^*_t+\frac{\tilde{e}_t+\tilde{P}^{F*}_t-\tilde{P}_t}{\sigma}. \tag{9}$$

Since domestic economy is negligible, domestic goods have zero weight in the foreign CPI, and we assume that foreign tradables and non-tradables have the same price hence $P^{F*}_t=P^{TF*}_t$.

Let us define the terms of trade, $S_t=(e_tP^{TF*}_t)/P^{TH}_t$. Its loglinearized version is,

$$\tilde{S}_t=\tilde{P}^{F*}_t+\tilde{e}_t-\tilde{P}^{TH}_t. \tag{10}$$

Define the non-tradable – tradable relative price, $P^g_t=P^N_t/P^{TH}_t$, hence

$$\tilde{P}^g_t=\tilde{P}^N_t-\tilde{P}^{TH}_t.$$

It is useful to express the relative price in (9) alternatively. Since[2]

$$\tilde{P}_t=s^N\tilde{P}^g_t+\tilde{P}^{TH}+s^Ta^{TF}\tilde{S}_t. \tag{11}$$

[2]
$$\begin{aligned}\tilde{P}_t &= s^N\tilde{P}^N_t+s^Ta^{TH}\tilde{P}^{TH}_t+s^Ta^{TF}\left(\tilde{P}^{F*}_t+\tilde{e}_t\right)\\ &= s^N\tilde{P}^g_t+\left(s^N+s^Ta^{TH}\right)\tilde{P}^{TH}+s^Ta^{TF}\left(\tilde{P}^{F*}_t+\tilde{e}_t\right),\end{aligned}$$

and $1-s^Ta^{TF}=s^N+s^Ta^{TH}$.

the following identity will be true

$$\begin{aligned}\tilde{e}_t + \tilde{P}_t^{F*} - \tilde{P}_t &= (\tilde{e}_t + \tilde{P}_t^{F*} - \tilde{P}_t^{TH}) - s^N \tilde{P}_t^g - s^T a^{TF} \tilde{S}_t \\ &= (1 - s^T a^{TF}) \tilde{S}_t - s^N \tilde{P}_t^g, \end{aligned} \tag{12}$$

where s^T is the steady state share of tradable goods and $s^N = 1 - s^T$ is that of the non-tradables.[3] Combining (9) and (12) yields the following equation for aggregate consumption,

$$\tilde{c}_t = \tilde{y}_t^* + \frac{(1 - s^T a^{TF})\tilde{S}_t - s^N \tilde{P}_t^g}{\sigma}. \tag{13}$$

Demand for tradable and non-tradable goods

If the path of aggregate consumption is known, then one can calculate that of the tradable and non-tradable consumption,

$$\begin{aligned} c_t^T &= a^T \chi_t^T \left(\frac{P_t}{P_t^T} \right)^{\frac{1}{1-\eta}} c_t. \\ c_t^N &= a^N \chi_t^N \left(\frac{P_t}{P_t^N} \right)^{\frac{1}{1-\eta}} c_t. \end{aligned}$$

Loglinearizing these equations, and using[4]

$$\tilde{P}_t - \tilde{P}_t^T = \frac{s^N}{s^T} \left[(1 - s^N) \tilde{P}_t^g - s^T a^{TF} \tilde{S}_t \right],$$

and using, the fact that the loglinearized version of eq. (2) implies

$$\tilde{\chi}_t^N = -\frac{a^N}{a^T} \tilde{\chi}_t^N, \tag{14}$$

we get

$$\tilde{c}_t^T = \tilde{\chi}_t^T + \frac{s^N}{(1-\eta)s^T} \left(s^T \tilde{P}_t^g - s^T a^{TF} \tilde{S}_t \right) + \tilde{c}_t. \tag{15}$$

Similarly, loglinearizing the equation for c_t^N, and using

$$\tilde{P}_t - \tilde{P}_t^N = s^T a^{TF} \tilde{S}_t - (1 - s^N) \tilde{P}_t^g$$

yields

$$\tilde{c}_t^N = \tilde{\chi}_t^N + \frac{s^T a^{TF} \tilde{S}_t - (1 - s^N) \tilde{P}_t^g}{1 - \eta} + \tilde{c}_t. \tag{16}$$

[3] Since in steady state $P^T \neq P^N$, the loglinearized $\tilde{P}_t$ price index cannot be weigheted by a^T and a^N.

[4] We proove this identity in the Appendix.

Knowing the consumption of tradables we can derive the demand of home made and foreign tradables.

$$c_t^{TH} = a^{TH} \left(\frac{P_t^T}{P_t^{TH}} \right)^{\frac{1}{1-\rho}} c_t^T.$$

Loglinearization yields

$$\tilde{c}_t^{TH} = \frac{1}{1-\rho} \left(\tilde{P}_t^T - \tilde{P}_t^{TH} \right) + \tilde{c}_t^T.$$

The loglinear price index of tradables is

$$\tilde{P}_t^T = a^{TF}(\tilde{P}_t^{F*} + \tilde{e}_t) + (1 - a^{TF})\tilde{P}_t^{TH}.$$

Hence

$$\begin{aligned} \tilde{c}_t^{TH} &= \frac{a^{TF}}{1-\rho} \left(\tilde{P}_t^{F*} + \tilde{e}_t - \tilde{P}_t^{TH} \right) + \tilde{c}_t^T \\ &= \frac{a^{TF}}{1-\rho} \tilde{S}_t + \tilde{c}_t^T, \end{aligned} \tag{17}$$

the demand for c^{TF} can be found similarly.

Demand for individual goods

For all $k = TF$, TH, N types of goods

$$c(i)_t^k = \left(\frac{P(i)_t}{P_t^k} \right)^{\frac{1}{1-\varrho}} c_t. \tag{18}$$

Foreign demand

Similarly to domestic residents, we assume that foreigners has the following demand for home made goods,

$$c_t^{TH*} = a^{TH*} \left(\frac{e_t P_t^{F*}}{P_t^{TH}} \right)^{\frac{1}{1-\rho}} y_t^*$$

Using $P_t^{F*} = P_t^{TF*}$ and loglinearizing the previous demand equation yields

$$\tilde{c}^{TH*} = \frac{1}{1-\rho} \tilde{S}_t + \tilde{y}_t^*. \tag{19}$$

Aggregate demand

Define domestic tradable and non-tradable output as

$$\begin{aligned} y_t^T &= c_t^{TH} + c_t^{TH*}, \\ y_t^N &= c_t^N, \end{aligned}$$

and aggregate output is their sum. This summation seems incorrect since we sum different types of goods. But we show that in our case both in the steady state and the loglinearized version the variables can be expressed by the same measures, hence summation is meaningful. Since our analysis is based on the loglinearized model this is sufficient for us. Let us start with the steady state,

$$y = y^N + y^T = c^N + c^{TH} + c^{TH*}.$$

Express domestic tradable consumption alternatively: Assume that the steady state trade balance is in equilibrium and $P^{TH} = eP^{TF*}$. Then $P^{TH}c^{TH*} = eP^{TF}c^{TF} + eP^z z$ where z is and imported input good and P^{z*} its prices in foreign currency terms. Define $\bar{P^z} = eP^{z*}/P^{TH}$, then $c^{TH*} = c^{TF} + \bar{P^z}z$, and $y^T = c^{TH} + c^{TF} + \bar{P^z}z$. $P^{TH} = eP^{TF*}$ implies, that $c^{TH} = (1 - a^{TF})c^T$ and $c^{TF} = a^{TF}c^T$, hence

$$y^T = c^T + \bar{P^z}z.$$

Define

$$\begin{aligned}
\bar{a}^N &= a^N \left(\frac{P}{P^N}\right)^{\frac{1}{1-\eta}}, \\
\bar{a}^T &= a^T \left(\frac{P}{P^T}\right)^{\frac{1}{1-\eta}}.
\end{aligned}$$

Then the demand functions imply $c^N/\bar{a}^N = c$ and $c^T/\bar{a}^T = c$. Hence non-tradable output can be measured in tradable good terms,

$$y^N = c^N = \frac{\bar{a}^N}{\bar{a}^T}c^T.$$

Hence the tradables output is

$$y = y^N + y^T = \frac{\bar{a}^N}{\bar{a}^T}c^T + c^T + \bar{P^z}z.$$

Since $c^T = \bar{a}^T c$, the previous equation can be expressed in the following way,

$$y = \bar{a}c + \bar{P^z}z,$$

where $\bar{a} = \bar{a}^N + \bar{a}^T$. Define $\hat{a}^j = \bar{a}^j/\bar{a}$, $j = N, T$, $\bar{z} = (\bar{P^z}z)/(\bar{a}c)$ és $x^z = 1/(1+\bar{z})$. Then one can show, that

$$\frac{c^N}{y} = x^z\hat{a}^N, \quad \frac{c^T}{y} = x^z\hat{a}^T, \quad \frac{\bar{P^z}z}{y} = x^z\bar{z}. \tag{20}$$

Loglinearizing the aggregate output yields

$$\tilde{y}_t = \left(\frac{c^N}{y}\right)\tilde{c}_t^N + \left(\frac{c^T}{y}\right)\left(\frac{c^{TH}}{c^T}\right)\tilde{c}_t^{TH} + \left(\frac{c^{TH*}}{y}\right)\tilde{c}_t^{TH*}.$$

Since $c^{TH}/c^{T} = a^{TH}$ and $c^{TH*}/y = a^{TF}(c^{T}/y) + (\bar{P^z z})/y$, using eq. (20) the previous equation can be expressed as

$$ty_t = x^z \hat{a}^N \tilde{c}_t^N + x^z \hat{a}^T a^{TH} \tilde{c}_t^{TH} + x^z(\hat{a}^T a^{TF} + \bar{z})\tilde{c}_t^{TH*}.$$

If we consider eq. (14), (15), (13), (16), (17) and (19), then one can se that all $\tilde{c}_t^N$, $\tilde{c}_t^{TH}$, and $\tilde{c}^{TH*}$ are linear combinations of $\tilde{S}_t$, $\tilde{P}_t^g$, $\tilde{y}_t^*$ and $\tilde{\chi}_t^N$, hence it is possible to sum them. Substituting the previous equations into the loglinearized version of aggregate output yields

$$\tilde{y}_t = \frac{\omega_s}{\sigma}\tilde{S}_t - \omega_g \tilde{P}_t^g + \tilde{y}_t^* + \omega_\chi \tilde{\chi}_t^N, \tag{21}$$

where the expressions for the coefficients are in the Appendix. Since

$$\begin{aligned} \tilde{y}_t &= x^z \hat{a}^N \tilde{y}_t^N + (1 - x^z \hat{a}^N)\tilde{y}_t^T, \qquad \tilde{y}_t^N = \tilde{c}_t^N, \\ \tilde{y}_t^T &= \frac{x^z \hat{a}^T (1 - a^{TF})}{1 - x^z \hat{a}^N}\tilde{c}_t^{TH} + \frac{x^z \left(\hat{a}^T a^{TF} + \bar{z}\right)}{1 - x^z \hat{a}^N}\tilde{c}_t^{TH*}, \end{aligned}$$

the non-tradable and tradable output can be expressed as the linear combination of the same variables.

$$\tilde{y}_t^N = \lambda_s^N \frac{\omega_s}{\sigma}\tilde{S}_t - \lambda_g^N \omega_g \tilde{P}_t^g + \tilde{y}_t^* + \lambda_\chi^N \omega_\chi \tilde{\chi}_t^N, \tag{22}$$

$$\tilde{y}_t^T = \lambda_s^T \frac{\omega_s}{\sigma}\tilde{S}_t - \lambda_g^T \omega_g \tilde{P}_t^g + \tilde{y}_t^* + \lambda_\chi^T \omega_\chi \tilde{\chi}_t^N, \tag{23}$$

where

$$\begin{aligned} \lambda_j^N &= \frac{\omega_j^N}{\omega_j}, \quad \lambda_j^T = \frac{\omega_j^T}{\omega_j} \\ \omega_j &= x^z \hat{a}^N \omega_j^N + (1 - x^z \hat{a}^N)\omega_j^T. \end{aligned}$$

2.2 Price and wage setting

We already mentioned that both tradable and non-tradable goods are composite of many individual goods, which are not perfect substitutes We assume that each individual good is produced by a distinct individual firm. Eq. (18) implies that demand for the products of the ith non-tradable producer, and the jth tradable producer is

$$y(i)_t^N = \left(\frac{P^N(i)_t}{P_t^N}\right)^{\frac{1}{1-\varrho}} y_t^N, \quad y(j)_t^T = \left(\frac{P^T(j)_t}{P_t^T}\right)^{\frac{1}{1-\varrho}} y_t^T,$$

where $P(i)_t^N$ and $P(j)_t^T$ the individual prices, $y(i)_t^N$ and $y(j)_t^T$ are individual output levels, and $0 < \varrho < 1$. Hence firms are not price takers, but monopolistic competitors.

Following Calvo (1983) we assume that both the non-tradable and tradable producers in a given period independently of their past actions reset their prices by probability $1-\gamma^N$, $1-\gamma^{TH}$, respectively. Since we assume uncountably many producers in each time period $1-\gamma^N$ and $1-\gamma^{TH}$ fraction of the firms set new prices. Each entrepreneur who resets its prices does in an optimal forward looking manner.

Christiano et al. (2001) extended this model. They assume that in every period ν^N or ν^{TH} portion of those firms, which do not reset their prices optimally in the given period will reset their prices in a backward looking non-optimal manner. More exactly they rise their prices by the inflation rate of the previous period.

In the Appendix we show that these assumptions imply the following price setting equations,

$$\begin{aligned} \bar{\pi}_t^{TH} &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^{TH}\right] + \xi^{TH} \tilde{mc}^T, \\ \bar{\pi}_t^{N} &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^{N}\right] + \xi^{N} \tilde{mc}^N, \end{aligned}$$

where

$$\begin{aligned} \bar{\pi}_t^{TH} &= \pi_t^{TH} - \nu^{TH}\pi_{t-1}^{TH}, \quad \bar{\pi}_t^N = \bar{\pi}_t^N - \nu^N \bar{\pi}_{t-1}^N. && (24) \\ \pi_t^{TH} &= \tilde{P}_t^{TH} - \tilde{P}_{t-1}^{TH}, \qquad \pi_t^N = \tilde{P}_t^N - \tilde{P}_{t-1}^N, && (25) \end{aligned}$$

where $\tilde{mc}^T$, $\tilde{mc}^N$ are loglinearized real marginal cost of the two sectors. We omitted indices i and j, since in a given sector individual firms have the same technology and choose the same prices in equilibrium.

The technology of a given sector is the following:

$$y_t^k = A_t^k h_t^{\alpha^k},$$

where A_t^k is a stochastic productivity variable, $0 < \alpha^k \leq 1$ and $k = T, N$. $\alpha^k = 1$ means constant return-to scale, $\alpha^j < 1$ means decreasing return-to scale. h_t input variable is a Leontieff aggregate of labor, l_t, and imported inputs, z_t. Formally

$$h_t = \min\left[a^z z_t,\ (1-a^z) l_t\right],$$

where $0 < a^z < 1$. Now we can express the real marginal cost of each sector.

$$\begin{aligned} \tilde{mc}^T &= \tilde{w}_t^z + \tilde{P}_t - \tilde{P}^{TH} - \bar{\alpha}^T \tilde{y}_t^T - \frac{\tilde{A}_t^T}{\alpha^T}, \\ \tilde{mc}^N &= \tilde{w}_t^z + \tilde{P}_t - \tilde{P}^{N} - \bar{\alpha}^N \tilde{y}_t^T - \frac{\tilde{A}_t^N}{\alpha^N}, \end{aligned}$$

where $\bar{\alpha}^j = (1-\alpha^j)/\alpha^j$ and

$$\tilde{w}_t^z = (1-\bar{a}^z)\tilde{w}_t + \bar{a}^z(\tilde{P}_t^{z*} + \tilde{e}_t - \tilde{P}_t),$$

where w_t is real wage, $(1-\bar{a}^z)$ is the steady state portion of w in w^z. Knowing $\bar{\alpha}^T$ and $\bar{\alpha}^N$ we can express ξ^{TH} and ξ^N coefficients of the price setting equations,

$$\xi^{TH} = \frac{(1-\gamma^{TH})(1-\beta\gamma^{TH})}{\gamma^{TH}\left(1+\frac{\bar{\alpha}^T}{1-\varrho}\right)}, \qquad \xi^{N} = \frac{(1-\gamma^{N})(1-\beta\gamma^{N})}{\gamma^{N}\left(1+\frac{\bar{\alpha}^N}{1-\varrho}\right)}.$$

Let us define the inflation gap,

$$\pi_t^g = \tilde{P}_t^g - \tilde{P}_{t-1}^g, \tag{26}$$

and the following variable,

$$\bar{\pi}_t^g = \pi_t^g - \nu^N \pi_{t-1}^g. \tag{27}$$

To express the inflation gap we rearrange the price setting equation of the tradable sector,

$$\begin{aligned}
&\pi_t^{TH} - \nu^N \pi_{t-1}^{TH} - (\nu^{TH} - \nu^N)\pi_{t-1}^{TH} = \\
&\beta \left(\mathrm{E}_t \left[\pi_{t+1}^{TH} \right] - \nu^N \pi_t^{TH} - (\nu^{TH} - \nu^N)\pi_t^{TH} \right) + \xi^N \tilde{mc}_t^T + \left(\xi^{TH} - \xi^N \right) \tilde{mc}_t^T,
\end{aligned}$$

and that of the non-tradable sector,

$$\pi_t^N - \nu^N \pi_{t-1}^N = \beta \left(\mathrm{E}_t \left[\pi^N \right]_{t-1} - \nu^N \pi_t^N \right) + \xi^N \tilde{mc}_t^N.$$

Subtracting the first from the second yields,

$$\begin{aligned}
\bar{\pi}_t^g &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^g \right] + \xi^N \left(\tilde{mc}_t^N - \tilde{mc}_t^T \right) \\
&- \left(\xi^{TH} - \xi^N \right) \tilde{mc}_t^T + \left(\nu^{TH} - \nu^N \right) \left(\beta \pi_t^{TH} - \pi_{t-1}^{TH} \right).
\end{aligned}$$

Since

$$\tilde{mc}_t^T = \frac{\bar{\pi}_t^{TH} - \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^{TH} \right]}{\xi^{TH}},$$

and

$$\tilde{mc}_t^N - \tilde{mc}_t^T = \frac{\tilde{A}_t^T}{\alpha^T} - \frac{\tilde{A}_t^N}{\alpha^N} - \tilde{P}_t^g + \bar{\alpha}^N \tilde{y}_t^N - \bar{\alpha}^T \tilde{y}_t^T,$$

the inflation gap equation is

$$\begin{aligned}
\bar{\pi}_t^g &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^g \right] + \xi^N \left[\frac{\tilde{A}_t^T}{\alpha^T} - \frac{\tilde{A}_t^N}{\alpha^N} - \tilde{P}_t^g + \bar{\alpha}^N \tilde{y}_t^N - \bar{\alpha}^T \tilde{y}_t^T \right] \\
&- \left(1 - \xi^N / \xi^{TH} \right) \left(\bar{\pi}_t^{TH} - \beta \bar{\pi}_{t+1}^{TH} \right) - \left(\nu^N - \nu^{TH} \right) \left(\beta \pi_t^{TH} - \pi_{t+1}^{TH} \right).
\end{aligned} \tag{28}$$

Let us express alternatively the price setting equation of the tradables. Using eq. (11) the real marginal cost can be expressed as follows,

$$\begin{aligned}
\tilde{mc}_t^T &= (1 - \bar{a}^z)\tilde{w}_t + \bar{a}^z(\tilde{P}_t^{z*} + \tilde{e}_t - \tilde{P}_t) + \bar{\alpha}^T \tilde{y}_t^T - \frac{\tilde{A}_t^T}{\alpha^T} + \tilde{P}_t - \tilde{P}_t^{TH} \\
&= (1 - \bar{a}^z)\tilde{w}_t + \bar{a}^z(\tilde{P}_t^{z*} + \tilde{e}_t) + \bar{\alpha}^T \tilde{y}_t^T - \frac{\tilde{A}_t^T}{\alpha^T} \\
&+ (1 - \bar{a}^z) s^N \tilde{P}_t^g + (1 - \bar{a}^z) s^T a^{TF} \tilde{S}_t - \bar{a}^z \tilde{P}_t^{TH}.
\end{aligned}$$

Substituting this into the pricing equation of the tradable sector yields,

$$\begin{aligned} \bar{\pi}_t^{TH} &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^{TH} \right] + \xi^{TH} \Big[(1-\bar{a}^z) \tilde{w}_t + \bar{a}^z \tilde{P}_t^{z*} + \bar{a}^z \tilde{e}_t + \bar{\alpha}^T \tilde{y}_t^T \\ &+ (1-\bar{a}^z) s^N \tilde{P}_t^g + (1-\bar{a}^z) s^T a^{TF} \tilde{S}_t - \bar{a}^z \tilde{P}_t^{TH} - \frac{\tilde{A}_t^T}{\alpha^T} \Big]. \end{aligned} \tag{29}$$

Let us apply the sticky wage model of Erceg et al. (2002) combining it with the extension of Christiano et al. (2001). Suppose that labor inputs supplied by the individual households are not perfect substitutes. On the other hand, each firm use composite labor, l_t, defined by the following CES aggregator,

$$l_t = \left(\int_0^1 l_t(i)^{\varrho^w} \, di \right)^{\frac{1}{\varrho^w}},$$

where $l(i)$ denotes the individual labor inputs. The aggregate wage index is

$$W_t = \left(\int_0^1 W_t(i)^{\frac{\varrho^w}{\varrho^w - 1}} \, di \right)^{\frac{\varrho^w - 1}{\varrho^w}},$$

where $W(_t i)$ is the individual wage. The demand for a given type of labor input is

$$l_t(i) = \left(\frac{w_t}{w_t(i)} \right)^{\frac{1}{1-\rho^w}} l.$$

Since the individual labor suppliers are monopolistic competitors sticky wage formation can occur. They change their wages optimally by probability $1-\gamma^w$, and ν^w fraction of the rest change the wages in a backward looking manner. In the Appendix we show that if π^w is the nominal wage inflation, then

$$\bar{\pi}_t^w = \pi_t^w - \nu^w \pi_{t-1}^w \tag{30}$$

is determined in the following way,

$$\bar{\pi}_t^w = \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^w \right] + \xi^w \left[m\tilde{r}s_t - \tilde{w}_t \right],$$

where $m\tilde{r}s_t$ is the loglinearized MRS between leisure and labor, and

$$\xi^w = \frac{(1-\gamma^w)(1-\beta\gamma^w)}{\gamma^w \left(1 + \varphi \frac{1}{1-\varrho^w} \right)}.$$

If wages are flexible, i.e. $\gamma^w = 0$ and $1/\xi^w = 0$, then we have the usual $\tilde{w}_t = m\tilde{r}s_t$ labor supply formula. In the Appendix we show that the labor supply formula can be expressed in the following alternative way,

$$\begin{aligned} \bar{\pi}_t^w &= \beta \mathrm{E}_t \left[\bar{\pi}_{t+1}^w \right] \\ &+ \xi^w \left[\varphi \left(\frac{\bar{y}^T}{\alpha^T} \tilde{y}_t^T + \frac{\bar{y}^N}{\alpha^N} \tilde{y}_t^N - \frac{\bar{y}^T}{\alpha^T} \tilde{A}_t^T - \frac{\bar{y}^N}{\alpha^N} \tilde{A}_t^N \right) + \sigma \tilde{c}_t - \tilde{w}_t \right]. \end{aligned} \tag{31}$$

To determined the wage inflation we need the following identity, as well,

$$\pi_t^w = \tilde{w}_t - \tilde{w}_{t-1} + s^N \pi_t^g + \pi_t^{TH} + s^T a^{TF}(\tilde{S}_t - \tilde{S}_{t-1}), \tag{32}$$

where we used formula (11) for the CPI.

2.3 Equilibrium

The equilibrium of the previously presented model is determined by the following 13 equations: Eq. (21) represents aggregate demand, the demand for tradables and non-tradables is captured by eq. (22) and (23). The aggregate supply side contains eq. (29), which is the pricing formula of tradables, eq. (29), which describes the inflation gap, and eq. (31) sticky wage equation. The following definitions and identities close the model: eq. (10), (24), (27), (30), (25), (26) and (32). This system determines the path of the following 13 variables: $\tilde{y}_t$, $\tilde{y}_t^N$, $\tilde{y}_t^T$, $\tilde{w}_t$, $\tilde{S}_t$, $\tilde{P}_t^{TH}$, $\tilde{P}_t^g$, π_t^{TH}, π_t^g, π_t^w, $\bar{\pi}_t^{TH}$, $\bar{\pi}_t^g$, $\bar{\pi}_t^w$.

We can further simplify the system. Firstly, let us express variable $\tilde{S}_t$ by eq. (21) Then substitute it into eq. (22), (23) and (13). By the elimination of this variable we get expressions independent of $\tilde{S}_t$.

$$\begin{aligned}
\tilde{y}_t^N &= \lambda_s^N \tilde{y}_t + (\lambda_g^N - \lambda_s^N)\omega_g \tilde{P}_t^g + (1-\lambda_s^N)\tilde{y}_t^* + (\lambda_\chi^N - \lambda_s^N)\omega_\chi \tilde{\chi}_t^N, &(33)\\
\tilde{y}_t^T &= \lambda_s^T \tilde{y}_t + (\lambda_g^T - \lambda_s^T)\omega_g \tilde{P}_t^g + (1-\lambda_s^T)\tilde{y}_t^* + (\lambda_\chi^T - \lambda_s^T)\omega_\chi \tilde{\chi}_t^N, &(34)\\
\tilde{c}_t &= (1-\Phi)\tilde{y}_t^* + \Phi\tilde{y}_t + (\Phi\omega_g - s^N/\sigma)\tilde{P}_t^g - \Phi\omega_\chi\tilde{\chi}_t^N, &(35)
\end{aligned}$$

Substituting the definition of terms of trade, eq. (10), into eq. (21), and substituting eq, (33), (34), (35) into the price and wage setting equations, i.e. (28), (29) and (31), and then combining them by (24), (27) and (30) identities yiedls the following four equations:

$$\tilde{y}_t = \frac{\omega_s}{\sigma}\left(\tilde{P}_t^{F*} + \tilde{e}_t - \tilde{P}_t^{TH}\right) - \omega_g \tilde{P}_t^g + \tilde{y}_t^* + \omega_\chi \tilde{\chi}_t^N, \tag{36}$$

this represents aggregate demand.

$$\begin{aligned}
(1+\beta\nu^{TH})\pi_t^{TH} &= \beta \mathrm{E}_t\left[\pi_{t+1}^{TH}\right] + \nu^{TH}\pi_{t-1}^{TH} \qquad (37)\\
&+\xi^{TH}\left[(1-\bar{a}^z)\tilde{w}_t + \bar{a}^z\tilde{P}_t^{z*} + \bar{a}^z\tilde{e}_t - \bar{a}^z\tilde{P}_t^{TH} - \frac{\tilde{A}_t^T}{\alpha^T}\right]\\
&+\xi^{TH}\left[X_y^T\tilde{y}_t + X_g^T\tilde{P}_t^g - X_{y*}^T\tilde{y}_t^* + X_\chi^T\tilde{\chi}_t^N\right],
\end{aligned}$$

this is the price setting equation of the tradable sector.

$$\begin{aligned}
(1+\beta\nu^N)\pi_t^g &= \beta\mathrm{E}_t\left[\pi_{t+1}^g\right] + \nu^N\pi_{t-1}^g \qquad (38)\\
&+\xi^N\left[\frac{\tilde{A}_t^T}{\alpha^T} - \frac{\tilde{A}_t^N}{\alpha^N} - (X_g^g+1)\tilde{P}_t^g + X_y^g\tilde{y}_t + X_{y*}^g\tilde{y}_t^* + X_\chi^g\tilde{\chi}_t^N\right]\\
&+\beta\left(1-\xi^N/\xi^{TH}\right)\pi_{t+1}^{TH} + \left[\left(1-\xi^N/\xi^{TH}\right)\nu^{TH} + (\nu^N-\nu^{TH})\right]\pi_{t-1}^{TH}\\
&+\left[\left(1-\xi^N/\xi^{TH}\right)(1+\beta\nu^{TH}) + (\nu^N-\nu^{TH})\beta\right]\pi_t^{TH},
\end{aligned}$$

this determines the tradable – non-tradable inflation gap.

$$
\begin{aligned}
&(1+\beta\nu^w)\pi_t^w = \beta \mathrm{E}_t\left[\pi_{t+1}^w\right] + \nu^w \pi_{t-1}^g \\
&+\xi^w\left[\varphi X_y^h + \sigma\Phi\right]\tilde{y}_t + \xi^w\left[\varphi X_{y*}^h + \sigma(1-\Phi)\right]\tilde{y}_t^* \\
&+\xi^w\left[\varphi X_g^h + \sigma\Phi\omega_g - s^N\right]\tilde{P}_t^g + \xi^w\left[\varphi X_g^h - \sigma\Phi\omega_\chi\right]\tilde{\chi}_t^N \\
&-\xi^w\varphi\left(\frac{\bar{y}^T}{\alpha^T}\tilde{A}_t^T + \frac{\bar{y}^N}{\alpha^N}\tilde{A}_t^N\right) - \xi^w\tilde{w}_t,
\end{aligned} \tag{39}
$$

this describes wage setting. The coefficients of the above four equations are defined in the Appendix.

If we add to the system eq. (36)-(39) the definition of π_t^{TH}, i.e. (25, that of π_t^g, i.e (26), and if we combine eq. (32) and (10), then we have a 7 equation system for the following 7 variables, $\tilde{y}_t$, π_t^{TH}, $\tilde{P}_t^{TH}$, π_t^g, $\tilde{P}_t^g$, π_t^w, $\tilde{w}_t$ path. This dynamic linear system can be solved by *the undetermined coefficients* method, see Uhlig (1999).

2.4 Special cases

It is worth looking over some special cases of the model. Suppose that the domestic and foreign tradables are perfect substitutes, that is $\rho = 1$, price and wage setting are flexible , that is $\gamma^{TH} = \gamma^N = \gamma^W = 0$, furthermore, constant return-to scale technologies, that is $\alpha^N = \alpha^T = 0$. Then $\sigma/\omega_s = 0$, hence eq. (36) becomes $\tilde{P}_t^{TH} = \tilde{P}_t^{F*} + \tilde{e}_t$. In this case the domestic tradable prices are determined solely by nominal exchange rate and foreign prices, and the exchange rate pass-through is perfect. Flexible price and wage setting imply that $1/\xi^{TH} = 1/\xi^N = 1/\xi^w = 0$, and constant return-to scale implies that $X_g^g = X_y^g = X_{y*}^g = X_\chi^g = 0$. Substituting these values into eq. (38) it becomes $\tilde{P}_t^g = \tilde{A}_t^T - \tilde{A}_t^N$, that is the tradable–non-tradable relative price is solely determined by productivity factors. Furthermore eq. (36) and (37) implies that the real wage is independent of demand. Hence we get a version of our model which practically equivalent to the Balassa-Samuelson model presented in chapter 4 of Obstfeld and Rogoff (1996).

For some parameter values our model becomes the same as that of Gali and Monacelli (2002). Let us assume, that there is no imported input, i.e. $\bar{z} = \bar{a}^z = 0$ és $x^z = 1$, and there is no non-tradable sector, i.e. $a^N = \hat{a}^N = s^N = \bar{y}^N = 0$, and $a^T = \hat{a}^T = s^T = \bar{y}^T = 1$. Furthermore wages are flexible, i.e. $1/\xi^w = 0$ and there is no backward looking pricing, i.e. $\nu^{TH} = 0$, and there is constant return-to scale, i.e. $\alpha^T = 1$. Then $\omega_g = \omega_\chi = 0$ and eq. (36) becomes

$$
\tilde{y}_t = \frac{\omega_s}{\sigma}\tilde{S}_t + \tilde{y}_t^*,
$$

where $\omega_s = 1 + a^{TF}(2 - a^{TF})[\sigma/(1-\rho) - 1]$. This is the first fundamental equation of Gali and Monacelli. Of course in this case eq. (38) becomes irrelevant, and eq. (39) simply implies that real wage is equal to MRS. In our case since

$\Phi = (1 - a^{TF})/\omega_s$ and $X_y^h = 1$ és $X_{y*}^h = X_g^h = X_\chi^h = 0$, it is

$$\tilde{w}_t = \left(\varphi + \frac{\sigma(1 - a^{TF})}{\omega_s}\right)\tilde{y}_t + \frac{\sigma(\omega_s - 1 + a^{TF})}{\omega_s}\tilde{y}_t^* - \varphi \tilde{A}_t^T.$$

In eq. (37) $X_g^T = X_\chi^T = 0$ and $X_y^T = a^{TF}\sigma/\omega_s$, and $X_{y*}^T = a^{TF}\sigma/\omega_s$. Hence in eq. (37) the term, which represents the real marginal cost becomes

$$\tilde{w}_t + a^{TF}\frac{\sigma}{\omega_s}\tilde{y}_t - a^{TF}\frac{\sigma}{\omega_s}\tilde{y}_t^* - \tilde{A}_t^T.$$

Combining this with the previous expression for $\tilde{w}_t$, and substituting for the real marginal cost in eq. (37) yields the second fundamental equation of Gali and Monacelli,

$$\pi_t^{TH} = \beta \mathrm{E}_t\left[\pi_{t+1}^{TH}\right] + \xi^{TH}\left[\left(\frac{\sigma}{\omega_s} + \varphi\right)\tilde{y}_t + \sigma\left(1 - \frac{1}{\omega_s}\right)\tilde{y}_t^* - (1+\varphi)\tilde{A}_t^T\right].$$

To close the model we need the definition of terms of trade, $\tilde{S}_t$, eq. (10). The authors use this three-equation system to study, for example domestic inflation targeting, when π_t^{TH} is exogenous and the endogenous variables are $\tilde{e}_t$, $\tilde{S}_t$ and $\tilde{y}_t$.

3 Simulation results

3.1 Determinants of tradable – non-tradable inflation gap

In this subsection we study (38) to understand the determinants of the inflation gap. This equation contains π_t^g and $\tilde{P}_t^g$, and four exogenous shocks $\tilde{A}_t^T$, $\tilde{A}_t^N$, $\tilde{y}_t^*$ and $\tilde{\chi}_t^N$, and two more endogenous variables π_t^{TH} and $\tilde{y}_t$. Since we study eq. (38) separately in this subsection we treat this two endogenous variables like exogenous shocks. We solve the system of eq. (38) and the identity $\pi_t^g = \tilde{P}_t^g - \tilde{P}_{t-1}^g$ by Uhlig's (1999) algorithm. It provides as a solution for π_t^g the following loglinear function,

$$\begin{aligned} \pi_t^g &= \mu^\pi \pi_{t-1}^g + \mu^P \tilde{P}_{t-1}^g + \theta^{A^T}\tilde{A}_t^T + \theta_1^{A^T}\tilde{A}_{t-1}^T + \theta^{A^N}\tilde{A}_t^N + \theta_1^{A^N}\tilde{A}_{t-1}^N \\ &+ \theta^{\chi^N}\tilde{\chi}_t^N + \theta_1^{\chi^N}\tilde{\chi}_{t-1}^N + \theta^{y*}\tilde{y}_t^* + \theta_1^{y*}\tilde{y}_{t-1}^* \\ &+ \theta^y\tilde{y}_t + \theta_1^y\tilde{y}_{t-1} + \theta^{\pi^{TH}}\pi_t^{TH} + \theta_1^{\pi^{TH}}\pi_{t-1}^{TH}. \end{aligned} \tag{40}$$

Since we assume that the shocks follow second order autoregessive processes, the solution contains lags of shock variables.[5]

[5] The shocks are determined by the following equations,

$$\begin{aligned} \tilde{A}_t^k &= 1.8\tilde{A}_{t-1}^k - 0.081\tilde{A}_{t-2}^k + \varepsilon_t^{A^k}, \quad \tilde{\chi}_t^N = 1.8\tilde{\chi}_{t-1}^N - 0.081\tilde{\chi}_{t-2}^N + \varepsilon^\chi \\ \tilde{y}_t^* &= 1.8\tilde{\chi}_{t-1}^N - 0.081\tilde{y}_{t-2}^* + \varepsilon^{y*}, \quad \tilde{y}_t = 1.8\tilde{\chi}_{t-1}^N - 0.081\tilde{y}_{t-2} + \varepsilon^y, \\ \pi_t^{TH} &= 8\pi_{t-1}^{TH} + \varepsilon^\pi. \end{aligned}$$

It is worth considering the special cases of eq. (38). Suppose the price setting behavior of both sectors are the same, i.e. $\xi^{TH} = \xi^N$ and $\nu^{TH} = \nu^N$. Furthermore assume constant return-to scale technologies, i.e. $\alpha^N = \alpha^T$. This latter assumption implies that X^g_g, X^g_y, X^g_{y*} coefficients are equal to zero. That is, (38) is reduced to the following special form,

$$(1 + \beta\nu^N)\pi^g_t = \beta \mathrm{E}_t\left[\pi^g_{t+1}\right] + \nu^N \pi^g_{t-1} + \xi^N\left[\tilde{A}^T_t - \tilde{A}^N_t - \tilde{P}^g_t\right].$$

One can observe that the inflation difference is determined by solely by productivity difference of the sectors. If tradable pricing is flexible, i.e $1/\xi^N = 0$, then $\tilde{P}^g_t = \tilde{A}^T_t - \tilde{A}^N_t$, as we have seen in the previous sector. If price setting is sticky then the previous relationship will be more complex, but still be determined only by productivity factors.

If we relax the assumption of constant return-to scale, then

$$\begin{aligned}(1 + \beta\nu^N)\pi^g_t &= \beta \mathrm{E}_t\left[\pi^g_{t+1}\right] + \nu^N \pi^g_{t-1} \\ &+\xi^N\left[\frac{\tilde{A}^T_t}{\alpha^T} - \frac{\tilde{A}^N_t}{\alpha^N} - (X^g_g + 1)\tilde{P}^g_t + X^g_y \tilde{y}_t + X^g_{y*}\tilde{y}^*_t + X^g_\chi \tilde{\chi}^N_t\right].\end{aligned}$$

In this case beyond productivity two more exogenous shocks, $\tilde{y}^*_t$ and $\tilde{\chi}^N_t$, have influence on inflation gap. These two variables are independent form monetary policy. But in this case aggregate output $\tilde{y}_t$, which depends on monetary policy, also has influence.

In the general case ξ^{TH} and ξ^N can be different, ν^{TH} and ν^N also. Then the inflation rate of tradable goods π^{TH}_t also can influence the inflation gap, see eq. (38), hence this is another channel of monetary policy.

We started the simulations with the assumption of uniform pricing and technologies. We studied flexible and 1.5, 3, 4 and 6 quarter pricing (i.e. $\gamma^{TH} = \gamma^N = 0,\ 1/3,\ 1/2,\ 3/4,\ 5/6$) We compared the coefficients that measured the contemporary effects of the shocks, i.e. θ^{A^T}, θ^{A^N}, θ^{χ^N}, θ^y, θ^{y*}. With constant return to-scale only the productivity shocks matter: In case of flexible pricing $\theta^{A^T} = 1$ and $\theta^{A^N} = -1$. As prices become more sticky these coefficients will decrease, see panel 1 of **Graph 1**

As return-to scale becomes decreasing demand shocks will matter, as well. But their effect is negligible compared to productivity shocks. The strongest among them χ^N relative demand shock (its 1 % increase accompanies with a 1 % rise of non-tradable consumption) But if $\alpha^T = \alpha^N = 0,8$ its effect is still around 20% of the productivity shocks, see panel 2 and 3 in **Graph 1**. **Graphs 3** presents the impulse responses of $\tilde{P}^g_t$ and π^g_t if $\alpha^T = \alpha^N = 0.8$, $\nu^{TH} = \nu^N = 0.8$ and $\gamma^{TH} = \gamma^N = 1/3$.

Summarizing: With uniform pricing and technologies only the productivity shocks have the strongest effects, with decreasing return-to scale demand factors have some impact, but it is negligible relative to that of the productivity shocks.

In the next phase we allowed asymmetric technologies. Panel 1 of **Graph 3** shows the $\alpha^T = 0.9$, $\alpha^N = 1$ case, panel 2 the $\alpha^T = 0.8$, $\alpha^N = 1$ and panel 3 the $\alpha^T = 0.8$, $\alpha^N = 0,9$ ($\nu^{TH} = \nu^N = 0.8$), i.e. in all three cases the tradable sector has more decreasing return-to scale technology. It is clear, that the effect of the two productivity shocks are not the same any more, but they are still closer to each other, than the other shocks. The role of the output gap, $\tilde{y}_t$, becomes more important. Its effect is around 25% of that of $\tilde{A}_t^N$. The same can be observed on the impulse responses of **Graph 4**. (Be careful in the graph we displayed $-\tilde{y}_t$.)

Graph 5 and **6** shows $\alpha^N = 0,9$, $\alpha^T = 1$; $\alpha^N = 0,8$, $\alpha^T = 1$ and $\alpha^N = 0,8$, $\alpha^T = 0,9$ cases. The impact of $\tilde{A}_t^N$ becomes stronger than $\tilde{A}_t^T$ The influence of $\tilde{y}_t$ becomes negligible, the impact of $\tilde{y}_t^*$ remains roughly the same as in the uniform technology case, and that of χ_t^N a bit weaker.

Summarizing: Technological asymmetry makes stronger the effect of some types of demand shocks, but productivity factors remain the dominant ones.

Now let us study the effect of asymmetric pricing. In this case π^{TH} can influence inflation gap. We considered two cases. In the first case the price setting of the non-tradable sector is stickier, $\gamma^N = 1/2$ and $\gamma^{TH} = 1/3$, see the left column of **Graph 7.** On the other hand the right column presents the opposite case, i.e. $\gamma^N = 1/3$ and $\gamma^{TH} = 1/2$. The graph clearly reveals that tradable inflation can induce movements of the inflation gap, but its effect is not persistent, and the inflation gap changes sign quickly, i.e. its movement is cyclical. On the other hand the initial impact can be quite strong if there is significant asymmetry between ν^{TH} and ν^N.

Let us summarize the findings of this subsection from two points of view. The first question is how we can explain the large, 5-8 %, and persistent inflation gap in Hungary, the second is what can deviate this gap temporarily from its long run level.

Our answer for the first question is the following: The argument based on the Balassa-Samuelson effect is basically correct. Without significant and persistent asymmetric productivity shocks it is hard to explain the observable long run inflation gap. On the other hand, the short run fluctuation of the inflation gap can be influenced by other factors, even by monetary policy (through $\tilde{y}_t$ and π_t^{TH}), but these can be significant if there are technological or price setting asymmetries.

3.2 Nominal exchange rate – tradable price pass-through

In this subsection we consider how the parameters of the model influence the nominal exchange rate – tradable price pass-through. The most important parameters α^T and α^N technological coefficients, ρ the measure of substitution in eq. (3) ($\rho = 1$ means perfect substitution, $\rho = -\infty$ perfect complements) and γ^k, ν^k, ($k = TH$, N, w) price setting parameters.

We set α^T and α^N in [0.8, 1], $\rho = 0.5$, this latter means that foreign and home made tradables are close but not perfect substitutes.

The most important are the price and wage setting parameters There are two empirical studies on price setting in Hungary, which based on micro data, Tóth and Vincze (1999), and Rátfai (2000). Both contain information on the frequency of price revision in Hungary, but they do not say much on the method of these revisions. The Calvo-Christiano et al. model we use needs information on optimal forward looking and boundedly rational backward looking pricing, but the two above mentioned papers do not separates this two types of behavior. For example in the Calvo-Christiano et al. model both $\gamma^k = 0$, i.e. flexible prices , and γ^k close to 1, $\nu^k = 1$, i.e. very sticky prices, can be consistent with quarterly pricing, which is reported by the two above mentioned empirical paper.

Hence we considered a wide parameter range for γ^k: [1/3, 3/4], and we set ν^k in [0.8, 1]. We compared these results with the pass-through estimation of Hornok et al. (2002), which reports a quite slow passthrough, 40% in the first year and 60% for the second.

Graph 8 shows the case when wage setting is flexible only price setting is sticky. But in this case even the stickiest case produce too quick pass-through, (80% in the firs year).

In **Graph 9** we consider cases when both prices and wages are sticky. Panel 4 shows the case when optimal forward looking revision of the prices occurs annually, and backward looking revision quarterly. In this case the pass-through is 40% in the first year as in Hornok et al. (2002), but in the second year our simulation produces faster pass-through than it is reported in the empirical study.

As we mentioned in the Introduction it is an important property of this model, that domestic tradable prices determined by not only foreign tradable prices and nominal exchange rate, but by other factors, as well. On the other hand, if in an empirical study domestic tradable prices are explained only by these above mentioned two factors, then it is possible that the nominal exchange rate pass-through estimation incorporate the effects of some other omitted variables. Hence the measured slow exchange rate pass-through in Hungary may be the result of some other not considered shocks.

That is why, we assume that a negative persistent non-tradable productivity shock, $\tilde{A}_t^N$, occurs in the economy (6 periods before the exchange rate shock) and this generates a persistent large tradable – non-tradable inflation gap. We present in **Graph 10** the joint impulse responses of the nominal exchange rate and productivity shock (we used the same parameters as in **Graph 9**). The last panel reveals that in the second year the pass-through becomes smaller than in the case without productivity shock, hence it replicates the findings of the mentioned empirical papers.

The intuitive explanation of this phenomenon is the following: A negative $\tilde{A}_t^N$ shock reduces the supply of non-tradables, hence its relative price increases. Since tradables and non-tradables are not imperfect substitutes the relative price of home made tradables will also rise, although less than the non-tradable prices. As a consequence, S_t terms of trade variable will decrease. This implies by eq. (10), that the absolute value of $\tilde{e}_t - \tilde{P}_t^{TH}$ will increase. In other words, this means that if an appreciation occurs domestic tradable prices will follow the nominal exchange rate movement slower than otherwise. That is, the price adjustment will be more rigid downward. This is important, because with long and good quality time series data it is possible to separate the exchange rate and the productivity factor, but with the recent available Hungarian data set this is not necessarily possible.

If a positive persistent $\tilde{A}_t^T$ shock occurs in the tradable sectors it produces the same inflation gap as in the previous case, and by similar reasoning one can show that it accelerates downward movements of tradable prices. But according to our simulations this acceleration is unrelaistically large, hence we do not consider further this option.

It is worth considering the interpretation of a negative productivity shock of non-tradable sector. It does not mean that productivity decreases in the non-tradable sector. It is better to assume that in foreign tradable, domestic tradable and non-tradable sector productivity has the same long run growth rate, and when a negative productivity shock occurs in the non-tradable sector, than its productivity growth will be temporarily lower then in the other two sectors.

3.3 Other shocks

See **Graph 11**.

4 Conclusion

In this paper we laid out a sticky price dynamic general equilibrium model in order to study the nominal exchange rate – tradable price pass-through, the tradable – non-tradable inflation gap, and the interconnection of these phenomena.

We showed that if technologies has decreasing return-to scale or the price setting behavior of the tradable and non-tradable sector are different, then not only asymmetric productivity shocks, but demand factors and monetary policy can influence the inflation gap. On the other hand according to our simulations the key determinant of the large long-run inflation gap which can be observed in Hungary is the productivity difference of the two sectors.

Since we assumed that foreign and home made tradables are imperfect substitutes the domestic tradable prices has other determinants than foreign tradable prices and nominal exchange rate, among these determinants one can find those factors which determines the tradable – non-tradable inflation gap. We demonstrated by our simulations that the asymmetric productivity shocks which can generate the inflation gap observable in Hungary significantly can influence tradable prices.

If we generate the inflation difference by a negative shock of the non-tradable sector, then aggregate supply decreases, which increases the domestic relative prices. This implies real depreciation, and a consequence of this phenomenon is that the domestic nominal tradable price level will increase relative to the nominal exchange rate. Hence if the monetary authority wants to decreases the prices by a nominal appreciation, then the tradable prices will follow the nominal exchange rate path slower than otherwise. In other words, the pass-through will be slower.

On the other hand, if the inflation gap is generated by a positive productivity shock of the tradable sector, then by the same reasoning the pass-through will be quicker if appreciation occurs. But our simulations demonstrated that the tradable productivity shock needed to replicate the inflation gap in Hungary causes an unrealistic path of the tradable prices.

Hence our conclusion is that a slower than trend growth of the tradable productivity generates the long-run inflation gap in Hungary, which is partly responsible for the slow empirically observable exchange rate pass-through.

A Appendix

A.1 Steady state

A.2 Price and wage setting

A.3 Coefficients of the model

$$\begin{aligned}
\omega_s^T &= \frac{\frac{\sigma}{1-\rho}\left\{\left[(1-a^{TF})\left(1-s^N\frac{1-\rho}{1-\eta}\right)+1\right]a^{TF}\hat{a}^T+\bar{z}\right\}}{\hat{a}^T+\bar{z}} \\
&+ \frac{\hat{a}^T(1-a^{TF})(1-s^Ta^{TF})}{\hat{a}^T+\bar{z}}, \\
\omega_s^N &= \frac{s^Ta^{TF}\sigma}{1-\eta}+(1-s^Ta^{TF}), \\
\omega_s &= x^z\hat{a}^N\omega_s^N+(1-x^z\hat{a}^N)\omega_s^T.
\end{aligned}$$

$$\begin{aligned}
\omega_g^T &= -\frac{\hat{a}^T(1-a^{TF})s^N\left[\frac{1}{1-\eta}-\frac{1}{\sigma}\right]}{\hat{a}^T+\bar{z}}, \\
\omega_g^N &= \frac{1-s^N}{1-\eta}+\frac{s^N}{\sigma}, \\
\omega_g &= x^z\left\{\frac{\hat{a}^Ns^T-\hat{a}^Ts^N(1-a^{TF})}{1-\eta}+\frac{s^N(1-\hat{a}^Ta^{TF})}{\sigma}\right\}.
\end{aligned}$$

$$\begin{aligned}
\omega_\chi^T &= -\frac{\hat{a}^T(1-a^{TF})\frac{a^T}{a^N}}{\hat{a}^T+\bar{z}}, \quad \omega_\chi^N = 1, \\
\omega_\chi &= x^za^N\left[1-(1-a^{TF})\frac{\hat{a}^T}{\hat{a}^N}\frac{a^N}{a^T}\right].
\end{aligned}$$

$$
\begin{aligned}
X_y^T &= \bar{\alpha}^T \lambda_s^T + (1-\bar{a}^z) s^T a^{TF} \frac{\sigma}{\omega_s}, \\
X_{y*}^T &= -\bar{\alpha}^T (1-\lambda_s^T) + (1-\bar{a}^z) s^T a^{TF} \frac{\sigma}{\omega_s}, \\
X_g^T &= (1-\bar{a}^z) s^N - \bar{\alpha}^T (\lambda_g^T - \lambda_s^T) \omega_s + (1-\bar{a}^z) s^T a^{TF} \sigma \frac{\omega_g}{\omega_s}, \\
X_\chi^T &= -\bar{\alpha}^T (\lambda_\chi^T - \lambda_s^T) \omega_\chi + (1-\bar{a}^z) s^T a^{TF} \sigma \frac{\omega_\chi}{\omega_s}.
\end{aligned}
$$

$$
\begin{aligned}
X_y^g &= \bar{\alpha}^N \lambda_s^N - \bar{\alpha}^T \lambda_s^T, \\
X_{y*}^g &= \bar{\alpha}^N (1-\lambda_s^N) - \bar{\alpha}^T (1-\lambda_s^T), \\
X_g^g &= \left[\bar{\alpha}^N (\lambda_g^N - \lambda_s^N) - \bar{\alpha}^T (\lambda_g^T - \lambda_s^T) \right] \omega_g, \\
X_\chi^g &= \left[\bar{\alpha}^N (\lambda_\chi^N - \lambda_s^N) - \bar{\alpha}^T (\lambda_\chi^T - \lambda_s^T) \right] \omega_\chi.
\end{aligned}
$$

$$
\begin{aligned}
X_y^h &= \frac{\bar{y}^T}{\alpha^T} \lambda_s^T + \frac{\bar{y}^N}{\alpha^N} \lambda_s^N, \\
X_{y*}^h &= \frac{\bar{y}^T}{\alpha^T} (1-\lambda_s^T) + \frac{\bar{y}^N}{\alpha^N} (1-\lambda_s^N), \\
X_g^h &= \left[\frac{\bar{y}^T}{\alpha^T} (\lambda_g^T - \lambda_s^T) + \frac{\bar{y}^N}{\alpha^N} (\lambda_g^N - \lambda_s^N) \right] \omega_g, \\
X_\chi^h &= \left[\frac{\bar{y}^T}{\alpha^T} (\lambda_\chi^T - \lambda_s^T) + \frac{\bar{y}^N}{\alpha^N} (\lambda_\chi^N - \lambda_s^N) \right] \omega_\chi.
\end{aligned}
$$

References

[1] Benczúr, P., A. Simon and V. Várpalotai, 2002, Dezinflációs számítások kisméretű makromodellel, NBH working papers 2002/4.

[2] Calvo, G., 1983, Staggered Price Setting in a Utility Maximizing Framework, *Journal of Monetary Economics* 12, 383-398.

[3] Chari, V.V., P.J. Kehoe és E.R. McGrattan, 2000, Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the persistence Problem?, *Econometrica* 68, 1151-1180.

[4] Christiano et al., 2001, .

[5] Darvas, Zs., 2001, Árfolyam-begyűrűzés és reálárfolyam EU aspiráns országokban, NBH manuscript.

[6] Erceg, J.C., D.W. Henderson and A.T. Levin, 2000, Optimal Monetary Policy with Staggered Wage and Price Contracts, *Journal of Monetary Economics* 46, 281-313.

[7] Gali, J. and T. Monacelli, 2002, Monetary Policy and Exchange Rate Volatility in a Small Open Economy, CEPR Discussion Papers 3346.

[8] Hornok, C., M. Z. Jakab , Z. Reppa and K. Villányi, 2002, Inflation forecasting at the National Bank of Hungary, NBH manuscript.

[9] Huang, K.X.D. and Zheng Liu., 2002, Staggered Price Setting, Staggered Wage Setting, and Business Cycle Persistence, *Journal of Monetary Economics* 49, 405-433.

[10] Kovács, M. A., 2002, Balassa-Samuelson hatás 5 közép-kelet európai országban, NBH working papers 2002/5.

[11] Lane, P. and G.M. Miless-Ferretti, 2000, The Transfer Problem Revisited: Net Foreign Assets and Real Exchange Rate, CEPR Discussion Papers 2511.

[12] Mankiw, N.G., 2001, The Inexorable and Mysterious Trade Off between Inflation and Unemployment, *The Economic Journal* 111, C45-C61.

[13] Monacelli, T., 1999a, Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy, http://www2.bc.edu/~monacelt/.

[14] Mussa, M., 1986, Nominal Exchange Regimes and the Behavior of Real Exchange Rates: Evidence and Implications, *Carnegie-Rochester Conference Series on Public Policy*, 25,117-214 .

[15] Obstfeld, M. and K. Rogoff, 1995, Exchange Rate Dynamics Redux, *Journal of Political Economy* 103, 624-660.

[16] Obstfeld, M. and K. Rogoff, 1996, *Foundations of International Macroeconomics*, Cambridge, MA: MIT Press.

[17] Rátfai, A., 2000, The Size, Frequency and Synchronization of Price Adjustment: Microeconomic Evidence, Discussion Paper, University of Southampton, http://www/soton.ac.uk./~econweb/dp/dp00.html.

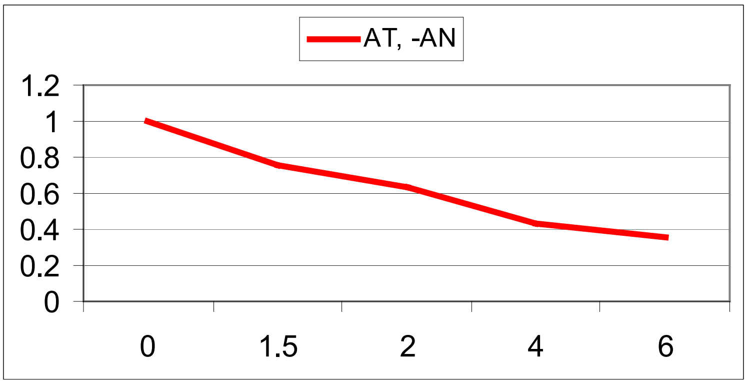
[18] Uhlig, H., 1999, A Toolkit for Analyzing Dynamic Stochastic Models Easily, in *Computational Methods for the Study of Dynamic Economies*, eds. R. Marimon and A. Scott, Oxford: Oxford University Press.

[19] Tóth, I.J. and J. Vincze, 1999, Magyar Vállalatok Árképzési Gyakorlata, *Bankszemle* .

[20] Woodford, M., 2002, Interest and Prices, http://www.princeton.edu/~woodford/.

Graph 1

$\alpha^T=\alpha^N=1 \qquad \nu^{TH}=\nu^N=\nu^w=0.8$



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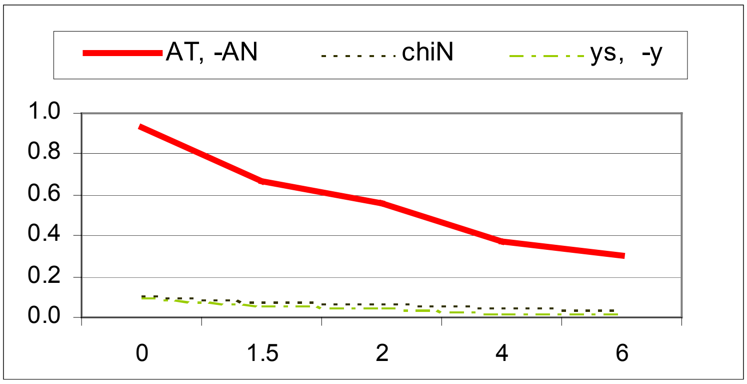
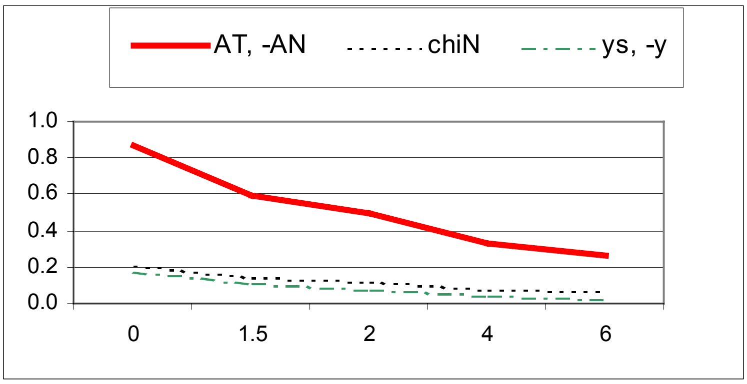
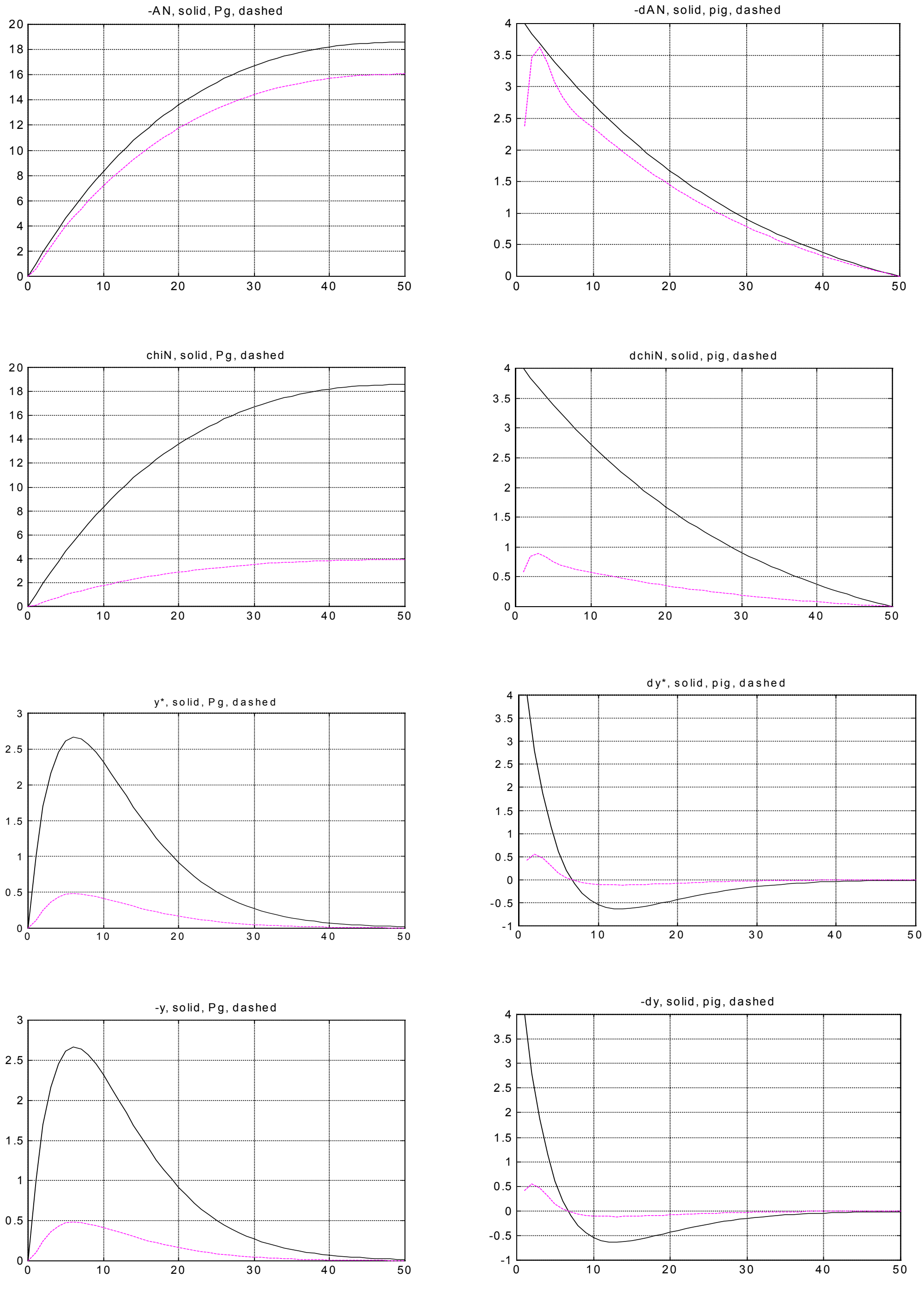


$\alpha^T=\alpha^N=0.9 \qquad \nu^{TH}=\nu^N=\nu^w=0.8$



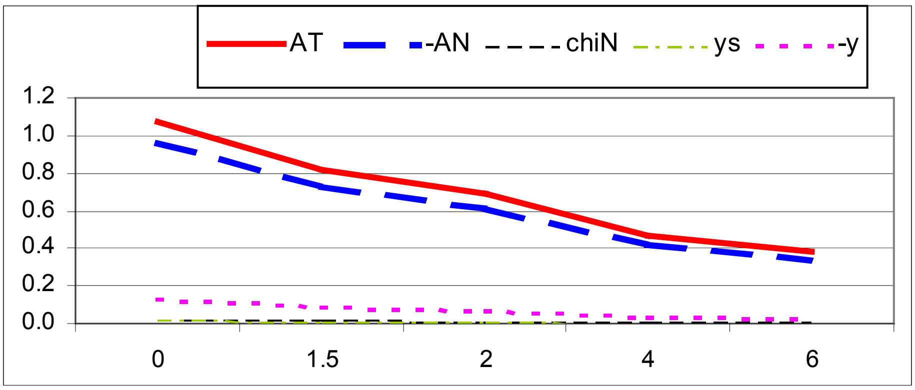
Graph 2

$$\alpha^T=\alpha^N=0.8 \qquad \nu^{TH}=\nu^N=\nu^w=0.8 \qquad \gamma^{TH}=\gamma^N=1/3$$

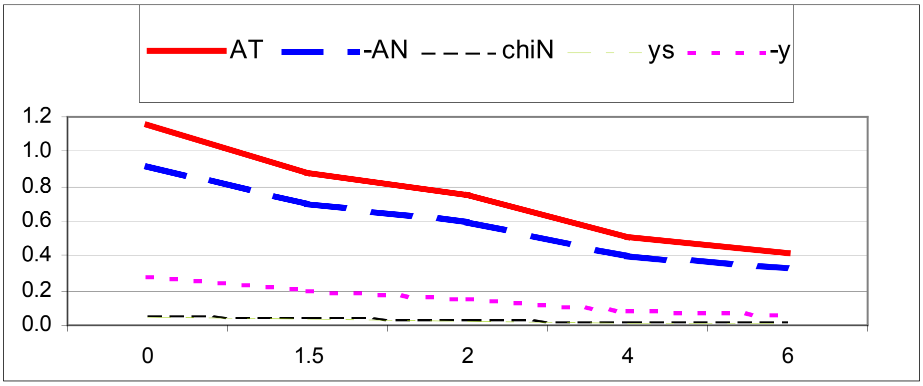


Graph 3

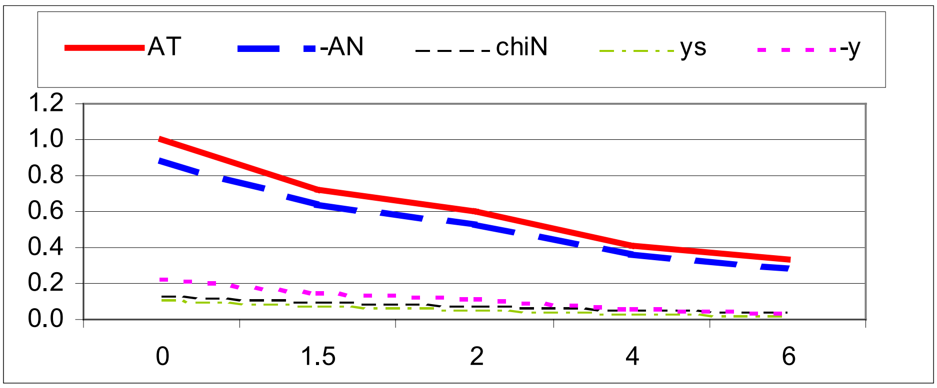
α^T=0.9 α^N=1 $\nu^{TH}=\nu^N=\nu^w$=0.8



α^T=0.8 α^N=1 $\nu^{TH}=\nu^N=\nu^w$=0.8

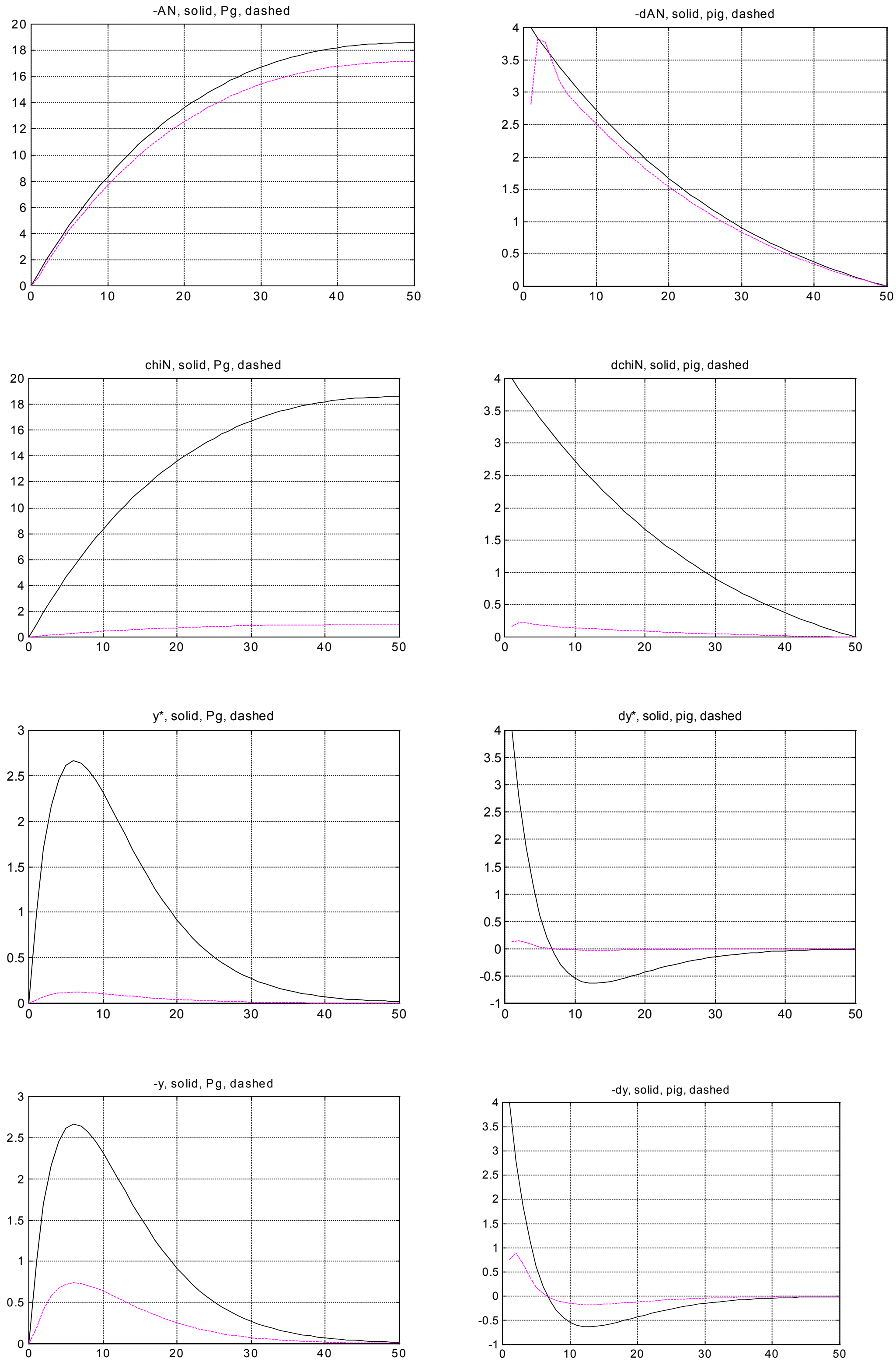


α^T=0.9 α^N=0.9 $\nu^{TH}=\nu^N=\nu^w$=0.8



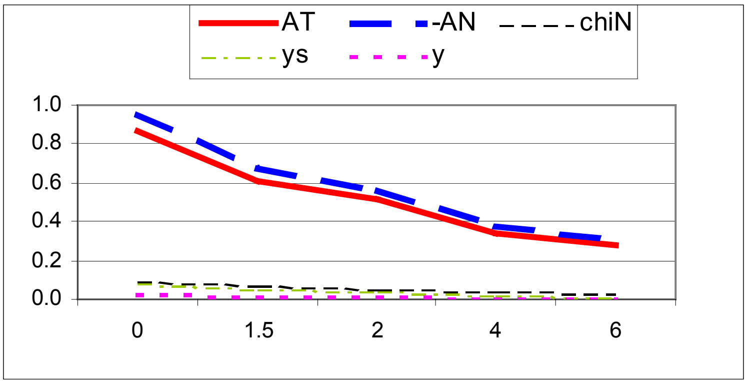
Graph 4

$\alpha^T=0.8 \;\; \alpha^N=1 \quad \nu^{TH}=\nu^N=\nu^w=0.8 \qquad \gamma^{TH}=\gamma^N=1/3$

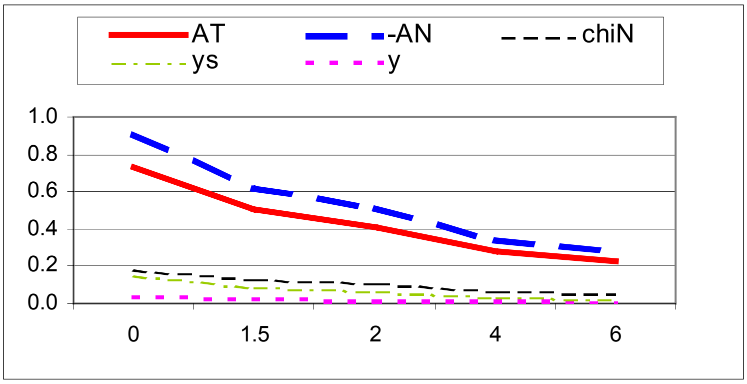


Graph 5

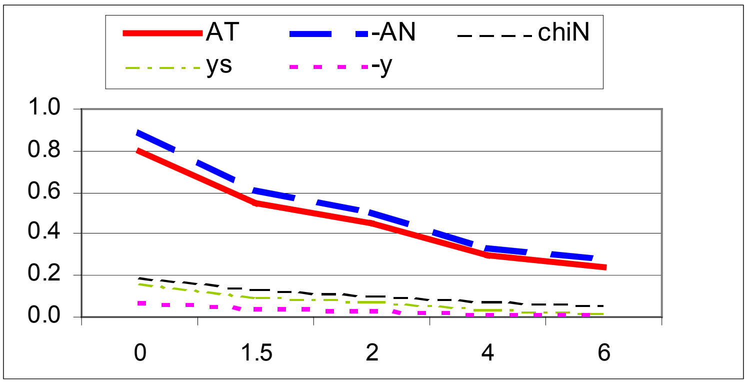
α^T=1 α^N=0.9 $v^{TH}=v^N=v^w$=0.8



α^T=1 α^N=0.8 $v^{TH}=v^N=v^w$=0.8

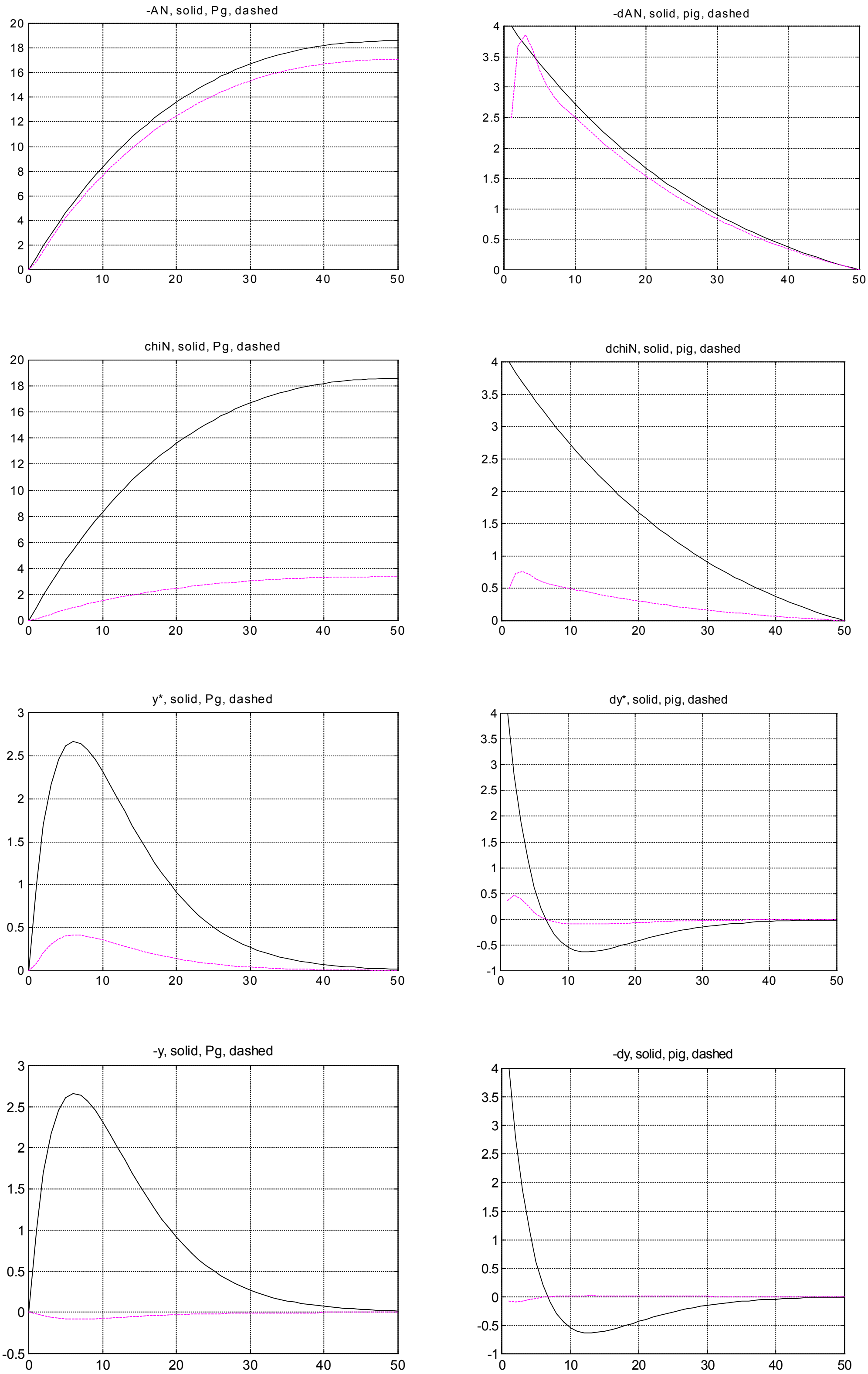


α^T=0.9 α^N=0.8 $v^{TH}=v^N=v^w$=0.8



Graph 6

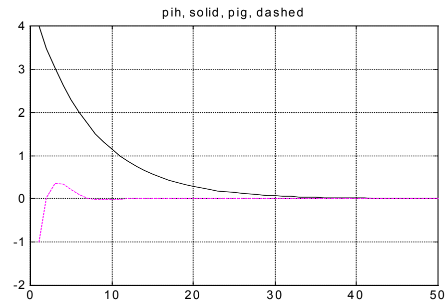
$$\alpha^T=1 \;\; \alpha^N=0.8 \quad \nu^{TH}=\nu^N=\nu^w=0.8 \qquad \gamma^{TH}=\gamma^N=1/3$$



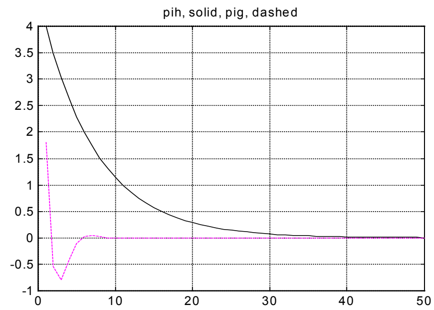
Graph 7

$\alpha^T=\alpha^N=1 \quad \nu^{TH}=\nu^N=\nu^w=0.8$

$\gamma^{TH}=1/3 \; \gamma^N=1/2$

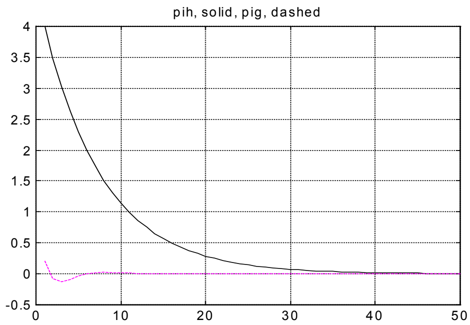


$\gamma^{TH}=1/2 \; \gamma^N=1/3$

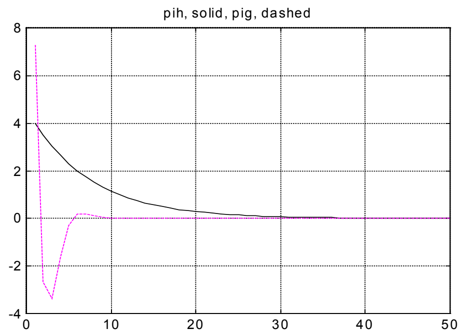


$\alpha^T=1 \; \alpha^N=1 \quad \nu^{TH}=1 \quad \nu^N=\nu^w=0.8$

$\gamma^{TH}=1/3 \; \gamma^N=1/2$



$\gamma^{TH}=1/2 \; \gamma^N=1/3$



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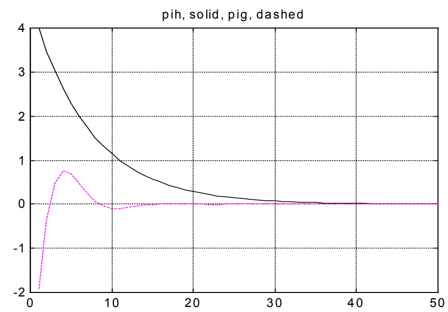
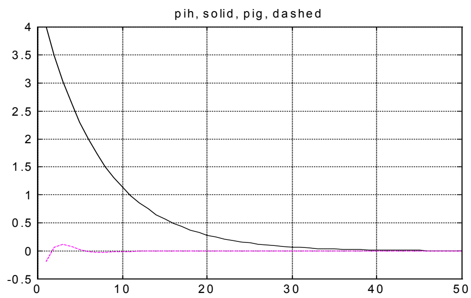
$\gamma^{TH}=1/3 \; \gamma^N=1/2$



$\gamma^{TH}=1/2 \; \gamma^N=1/3$



Graph 8

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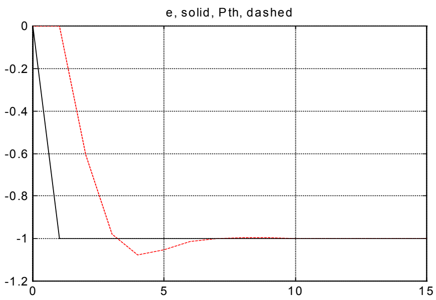

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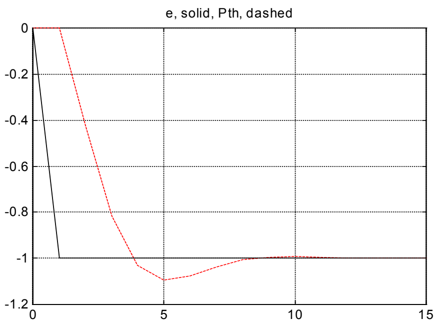

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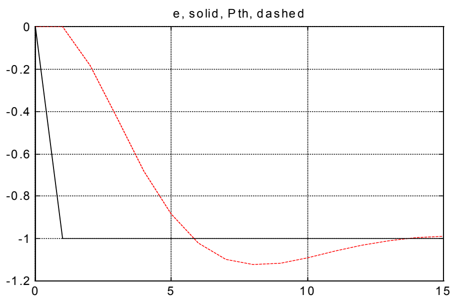
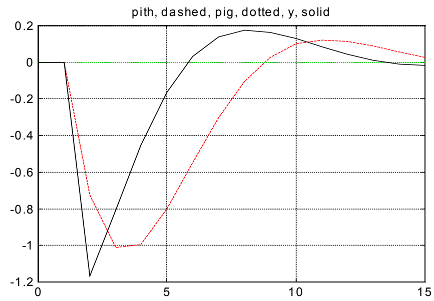
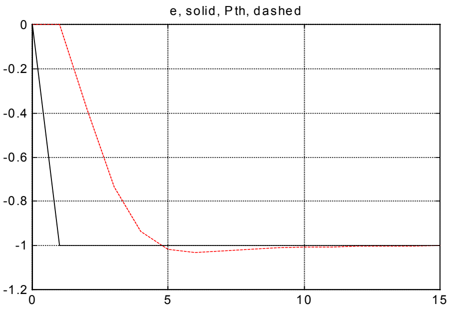
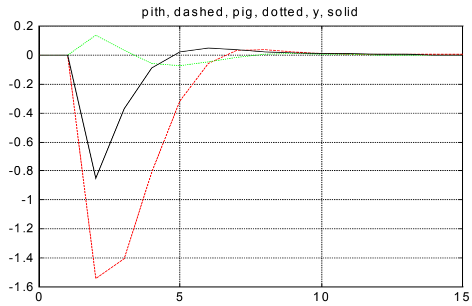




Graph 9

$\alpha^T=\alpha^N=0.8$ $\nu^{TH}=\nu^N=\nu^w=0.8$ $\gamma^{TH}=1/3\ \gamma^N=1/3\ \ \gamma^w=1/2$





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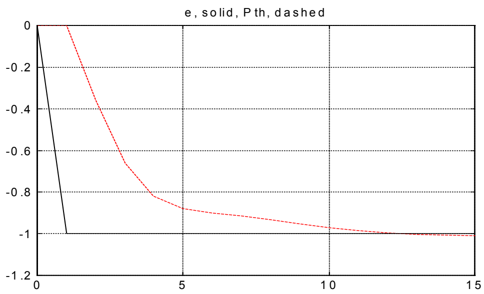

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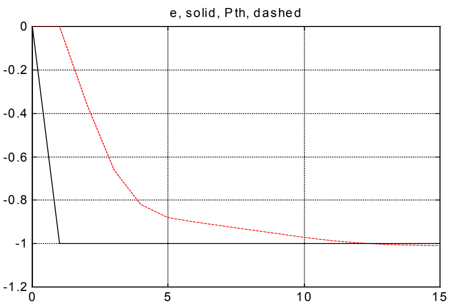

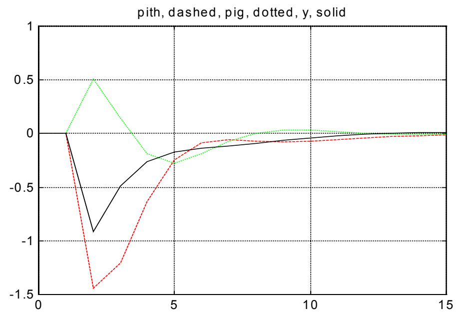

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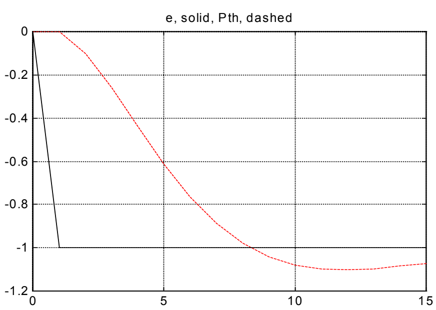

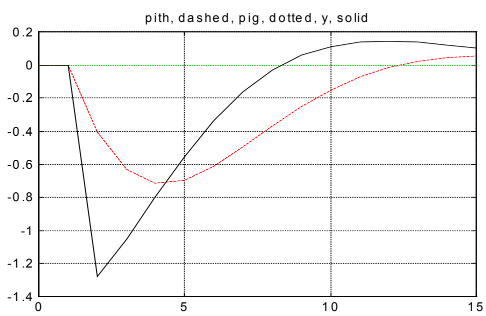

Graph 10

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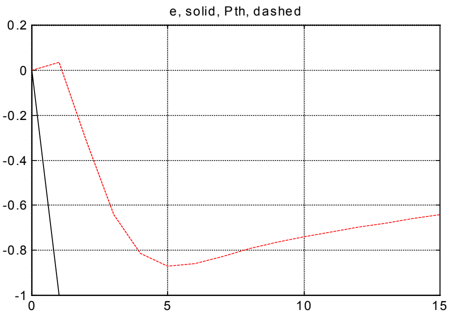

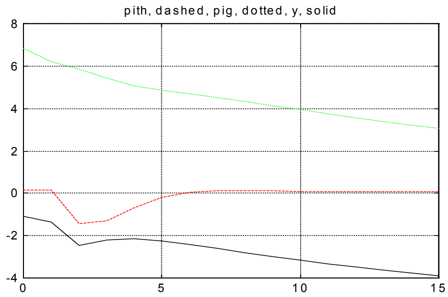

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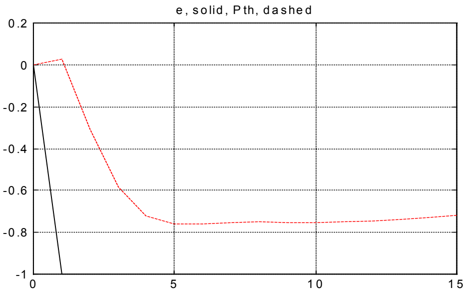

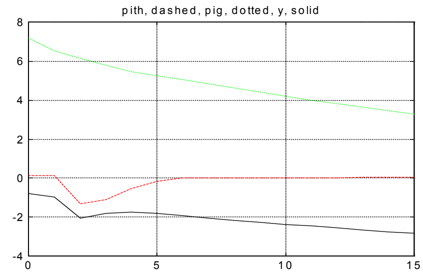

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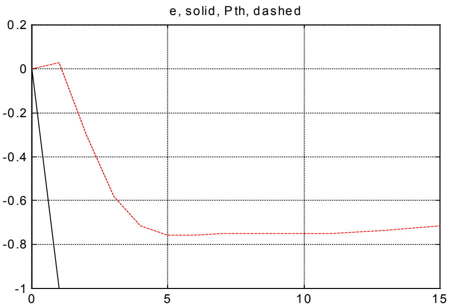
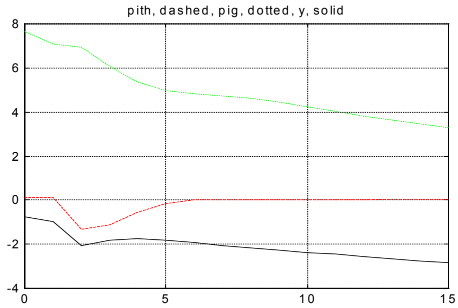
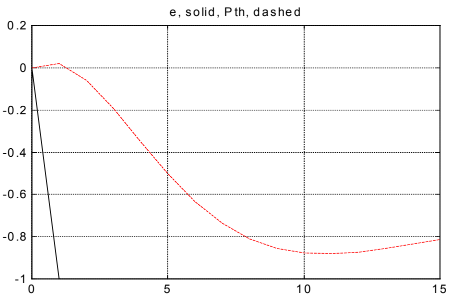
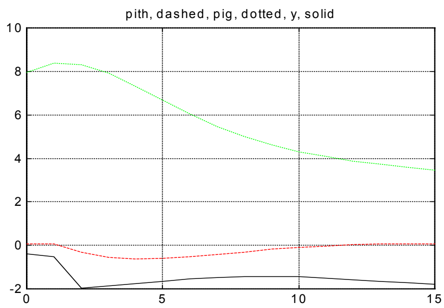




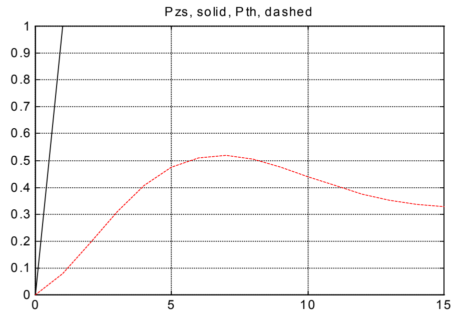
$\alpha^T=\alpha^N=1$ $\nu^{TH}=\nu^N=\nu^w==1$ $\gamma^{TH}=3/4$ $\gamma^N=3/4$ $\gamma^w=3/4$

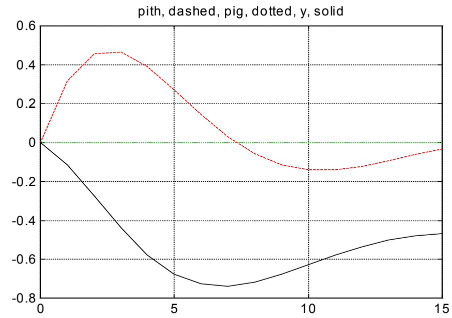




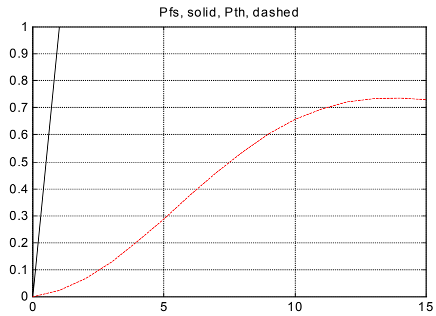
Graph 11

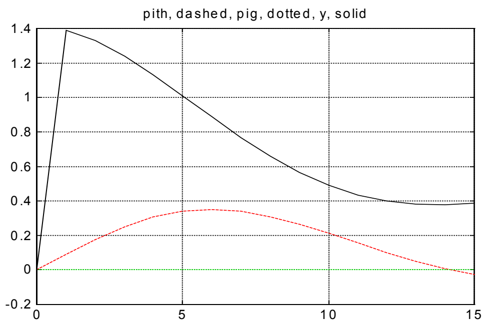
$P^{z}*$ sokk





$P^{F}*$ sokk





$y*$ sokk