Essential Interest-Bearing Money

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I examine a version of the Lagos and Wright (2005) monetary model where coercive lump-sum taxation is infeasible. Despite this restriction, I demonstrate that the first-best allocation remains implementable under an appropriately designed monetary policy; at least, if agents are sufficiently patient. The main conclusion is that any incentive-feasible monetary policy necessarily requires a strictly positive nominal interest rate to be paid on money balances.

1. INTRODUCTION

In a wide class of monetary models, deflating at the so-called Friedman rule constitutes an optimal policy. It is commonly understood that this policy prescription presumes that lump-sum taxation is feasible. It is perhaps less well-known that policies that pay interest on money–with at least a part of the aggregate interest expense financed via taxation–are also optimal, even if interest-bearing money policies are not essential. Absent lump-sum taxation, it is widely believed that the optimal policy is constrained to deliver a second-best allocation. This conclusion, however, ignores the possibility that agents might be induced in some manner to make voluntary tax contributions. I describe below how policies can be designed in this manner without violating or modifying any of the underlying assumptions imposed on an otherwise standard environment.

The environment I study is based on the quasi-linear model introduced by Lagos and Wright (2005), but absent search frictions. Agents lack commitment and it is impossible to monitor individual trading histories. There is a technology that allows for the creation of non-counterfeitable tokens (intrinsically useless objects that encode no personal information). These properties of the environment are known to generate an essential role for tokens as a means of payment. The supply of tokens is managed by society (a government) and society is limited in the manner by which it may penalize agents. In particular, coercive taxation is ruled out altogether. Trade among agents is assumed to occur in competitive spot markets.

I find that in this setup, an optimal policy can be designed that implements the first-best allocation; at least, assuming that agents are sufficiently patient. The optimal policy necessarily requires interest-bearing money, with at least a part of the aggregate interest expense financed by way of an individually-rational lump-sum

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tax. The way in which this can be accomplished is by issuing a second time-dated token that is offered in exchange to agents in return for their voluntary tax payment. The purpose of this second token is to serve as a verifiable record of an earlier tax contribution. Agents can be motivated to collect this record if it entitles them to earn interest on money in a high inflation environment. That is, the failure to pay taxes disentitles the agent from collecting interest and exposes him to the welfare cost of holding zero-interest money in an inflationary environment.

There is a sense in which this second token resembles a nominal government bond, but this is not quite right. In particular, this second token entitles the bearer to earn interest on his money holdings; the token itself more closely resembles a tax receipt than a bond. In fact, in the environment considered here, there is no role for a second token in the form of an interest-bearing bond; see, Berentsen, Camera, and Waller (2007). As in this latter paper, paying interest on money is essential. The manner in which these authors make this possible is by introducing a limited record-keeping and enforcement technology giving rise to "banks" that pay interest on cash deposits. The analysis in my paper describes how the same thing can be achieved in the absence of this modification to the basic environment.

2. THE ENVIRONMENT

There is a continuum of ex ante identical agents $i \in [0, 1]$. Time is discrete and the horizon is infinite. Each period is divided into two subperiods, labelled day and night. Agents have preferences defined over stochastic sequences $\{x_t(i), c_t(i) : t \geq 0\}$; where x_t denotes consumption in the day and c_t denotes consumption at night, at date t. These preferences are represented by a utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t(i) + \omega_t(i) u(c_t(i)) \right];$$

where $0 < \beta < 1$, and $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing and strictly concave. The parameter $\omega_t(i)$ is a preference shock that, for simplicity, I assume takes on only one of two values; i.e., $\omega_t(i) \in \{\omega_l, \omega_h\}$, with $0 < \omega_l < \omega_h < \infty$ (the analysis generalizes to a continuum of types). Define $\eta \equiv \omega_h/\omega_l > 1$. Assume that these preference shocks are *i.i.d.* across agents and across time; and assume $\Pr[\omega_t(i) = \omega_h] = 1/2$.

All agents are endowed with a non-storable endowment $y > 0$ at night. There are two resource constraints,

$$\begin{aligned} \int x_t(i)di &= 0; \\ \int c_t(i)di &= y. \end{aligned}$$

Note that $x_t(i)$ is unbounded from above and below (negative consumption during the day is allowed).

Consider the problem faced by a planner that wishes to maximize ex ante utility. Owing to quasi-linearity, any lottery $\{x_t(i)\}$ satisfying $E_t[x_t(i)] = 0$ will be feasible and efficient. Without loss then, assume $x_t(i) = 0$ for all i and t.

As this environment is stationary, consider allocations of the form $\{c(\omega)\}$. Ex ante utility is given by

$$W = \left(\frac{0.5}{1-\beta} \right) \left[\omega_l u(c(\omega_l)) + \omega_h u(c(\omega_h)) \right].$$

An efficient (first-best) allocation satisfies

$$u'(c^*(\omega_l)) = \eta u'(c^*(\omega_h)) \quad (1)$$

and

$$c^*(\omega_l) + c^*(\omega_h) = 2y. \quad (2)$$

Clearly, $c^*(\omega_l) < y < c^*(\omega_h)$.

I make the following assumptions concerning the nature of the environment. First, agents lack commitment. Second, it is impossible to monitor individual trading histories; that is, there is an absence of societal memory. Third, society can create physical tokens that inscribe non-personal information (e.g., date of issue). Fourth, society is limited in the manner by which it may penalize agents. In particular, coercive taxation is ruled out altogether. Finally, I assume that trade among agents, if it is to occur at all, does so in a competitive manner (in particular, on a sequence of competitive spot markets).[2]

Together, these assumptions imply that fiat money (in the form of a supply of tokens managed by society) is necessary to facilitate trade. The restriction to competitive markets implies that these trades will occur on a sequence of spot markets where agents trade money for goods. The inability to tax rules out the use of the Friedman rule (a deflation induced by contracting the money supply via lump-sum taxation) as a feasible policy. These are restrictions that are commonly believed to rule out first-best implementation. The main purpose of my paper is to demonstrate that this is not necessarily the case. In particular, I show that for an appropriately designed monetary policy (one that does not violate any of the restrictions placed on the environment above), the first-best allocation remains implementable; at least, if agents are sufficiently patient.

3. A MONETARY ECONOMY

3.1. Timing and Budget Constraints

In the initial period, society creates a given quantity of physical tokens that are distributed evenly across the population. These tokens, which are to serve as money, are perfectly durable, divisible, and non-counterfeitable. Let (v_d, v_n) denote the value of money in the day and night, respectively. I use a superscript "+" to denote future (next period) values; e.g., (v_d^+, v_n^+). Let m_d denote the nominal value of money held by an agent at the beginning of the day and let $z \equiv v_d m_d$ denote the real value of money m_d in the day.

[2] One may also assume that preference types are private information to rule out the possibility of type-contingent transfers. As it turns out, type-contingent transfers have no real consequence in this environment so that this possibility can be ignored; see Berentsen, Camera, and Waller (2007).

The timing of events is as follows. First, agents have an opportunity to trade money m_d for goods x in the day-market. At the same time, they have the *option* of paying a lump-sum transfer of money to the government. Let τ denote the real value of this payment and let $\chi \in \{0, 1\}$ denote whether the option to pay τ is exercised or not. If the option is exercised ($\chi = 1$), then the agent receives in return for this payment a time-dated token (distinct from the money token) that will effectively serve as a tax receipt.[3]

Let $m_n \geq 0$ denote the money held by an agent following these day trades (hence, m_n is the money with which the agent enters the night-market). Let $q \equiv v_n m_n$ denote the real value of money m_n at night. An agent entering the day with real money z faces the following budget constraint,

$$x = z - \chi\tau - \phi q; \tag{3}$$

where $\phi \equiv v_d/v_n$. Note that $x < 0$ is possible here (interpret this as working for money).

At night, individuals can trade money m_n for goods $(y - c)$. Note that if $y > c$, agents are saving (selling their excess output for money). On the other hand, if $y < c$, then agents are dissaving (drawing down their money balances to purchase output in excess of their endowment). Because agents cannot borrow or create money, their night trades are constrained in the following manner,

$$m_n + v_n^{-1}(y - c) \geq 0;$$

or, expressed in real terms,

$$q + y - c \geq 0. \tag{4}$$

In what follows, I refer to (4) as a cash constraint.

Following these night trades, an agent is left with nominal money balances $\left[m_n + v_n^{-1}(y - c)\right] \geq 0$. Assume that at this point (at the end of the night), the government expands an individual's money balances by the factor R^χ, so that

$$m_d^+ = R^\chi \left[m_n + v_n^{-1}(y - c)\right] \geq 0;$$

or, expressed in real terms,

$$z^+ = (v_d^+/v_d)\phi R^\chi \left[q + y - c\right] \geq 0. \tag{5}$$

Here, one can interpret $R \geq 1$ as a gross nominal interest rate that is paid only in the event that $\chi = 1$ (which denotes that the individual made the contribution τ in the day). Note that as there is no record-keeping, some form of tangible evidence must be provided by an individual to the government reflecting the fact that τ was in fact paid. This tangible evidence exists in the form of the second time-dated token that could have been acquired during the day in any date t. This token entitles the individual to an interest rate R on his money balances at the end of date t if and only if the individual presents a date t token to the government for redemption. If an agent fails to present this second token for redemption, he will in effect earn a zero nominal interest rate on his money.

[3] By assumption, this tax receipt cannot be personalized, but this is not important here. It is important, however, that this second token, like the money token, cannot be counterfeited.

It remains to describe the government's budget constraint. Assume that the government expands the money supply at some constant rate, so that $M^{+} = \mu M$, where M denotes the aggregate supply of money at the beginning of a day. Hence, a government policy in this model is described by the triplet (R, μ, τ).

In deriving the government's budget constraint, assume that $\chi = 1$ is individually-rational. The aggregate money supply M at the beginning of the day is reduced by the amount $v_d^{-1}\tau$ by the end of the day. The quantity $M - v_d^{-1}\tau$ is then augmented by new money $(R-1)\left[M - v_d^{-1}\tau\right]$ at then end of the night, so that $M^{+} = R\left[M - v_d^{-1}\tau\right]$. Defining $Q \equiv v_d M$, this latter expression may alternatively be expressed as

$$\tau = \left[1 - \frac{\mu}{R}\right] Q. \tag{6}$$

A feasible policy (R, μ, τ) is one that satisfies (6).

3.2. Individual Decision-Making

In what follows, I assume for the moment that $\chi = 1$ is individually-rational. The restrictions that are necessary to make this so are discussed in due course.

Let $V(q;\omega)$ denote the value of entering the night-market with real money balances q and having realized type $\omega \in \{\omega_l, \omega_h\}$. Assume, for the moment, that V is increasing and weakly concave in q. Let $W(z)$ denote the value of entering the day with real money balances z. Then, in recursive form, the choice problem of an agent during the day can be written as,

$$W(z) \equiv \max_{q \geq 0} \left[z - \tau - \phi q\right] + 0.5\left[V(q;\omega_l) + V(q;\omega_h)\right]; \tag{7}$$

where here, I have made use of (3). The FONC characterizing the demand for money $0 < \hat{q} < \infty$ is given by

$$\phi = 0.5\left[V_1(\hat{q};\omega_l) + V_1(\hat{q};\omega_h)\right]; \tag{8}$$

where V_j denotes the derivative of V with respect to argument j.

Technically, condition (8) determines a unique value for $\hat{q}$ only in the event that V is strictly concave in q. As it turns out, V will turn out to be strictly concave only when we are away from the first-best allocation. At the first-best, V turns out to be linear in q, so that there is some indeterminacy in $\hat{q}$. Nevertheless, even in this case, condition (8) will continue to hold in equilibrium; a point that I will return to later. In the meantime, note that by the envelope theorem,

$$W_1(z) = 1. \tag{9}$$

With $\hat{q}$ determined in some manner during the day, let us now turn to decision-making at night. Here, the choice problem can be written recursively as follows,

$$V(\hat{q};\omega) \equiv \max_{c, z^{+} \geq 0} \omega u(c) + \beta W(z^{+}); \tag{10}$$

where z^{+} is given by (5). If the constraint $z^{+} \geq 0$ remains slack, then desired consumption is determined by,

$$\omega u'(\hat{c}(\omega)) = (v_d^{+}/v_d)\phi R\beta. \tag{11}$$

If the constraint $z^+ \geq 0$ binds, then desired consumption is determined by,

$$\hat{c}(\omega) = \hat{q} + y. \tag{12}$$

In either case, the envelope theorem implies

$$V_1(\hat{q};\omega) = \omega u'(\hat{c}(\omega)). \tag{13}$$

Note that this latter condition implies that $V_1(\hat{q};\omega) = (v_d^+/v_d)\phi R\beta$ in case (11); that is, V is linear in $\hat{q}$. On the other hand, if the cash-constraint binds, then $V_1(\hat{q};\omega) = \omega u'(\hat{q}+y)$; so that V is strictly concave in $\hat{q}$ in case (12). Letting E_ω denote an expectations operator over ω, it follows that $E_\omega[V(\hat{q};\omega)]$ is strictly concave in $\hat{q}$ if and only if the cash-constraint binds for at least one type $\omega \in \{\omega_l, \omega_h\}$. If no cash-constraint binds tightly, then $E_\omega[V(\hat{q};\omega)]$ is linear in $\hat{q}$.

Now, combining (13) with (8), we have $0 < \hat{q} < \infty$ satisfying,

$$\phi = 0.5\left[\omega_l u'(\hat{c}(\omega_l)) + \omega_h u'(\hat{c}(\omega_h))\right]. \tag{14}$$

Note that if case (11) applies to both $\omega \in \{\omega_l, \omega_h\}$, then the condition above reduces to $1 = (v_1^+/v_1)R\beta$, so that $\hat{q}$ is indeterminate in the sense that any $\hat{q} \geq \hat{c}(\omega_h) - y$ is consistent with optimal behavior. One may therefore, without loss of generality, assume $\hat{q} = \hat{c}(\omega_h) - y$ in this case. In other words, $\hat{q}$ is determined by the minimum amount of real money balances necessary to ensure that the agent does not "run out of cash" in the state where he needs its the most (the impatient state). In this case, the cash-constraint binds weakly in state ω_h and remains slack in state ω_l.

3.3. Monetary Equilibrium Assuming $\chi = 1$

In describing equilibria, I restrict attention to stationary allocations. One implication of stationarity is that $v_d M = v_d^+ M^+ = Q$; which implies

$$\left(\frac{v_d^+}{v_d}\right) = \left(\frac{1}{\mu}\right). \tag{15}$$

That is, the equilibrium inflation rate from one day to the next is determined entirely by policy.

In a stationary equilibrium, the level of consumption in each day will be determined by the type that was realized in the previous night; i.e., $\hat{x}(\omega^-)$ for $\omega^- \in \{\omega_l, \omega_h\}$. Hence, market-clearing at each day and night require, respectively,

$$0.5\hat{x}(\omega_l) + 0.5\hat{x}(\omega_h) = 0; \tag{16}$$
$$0.5\hat{c}(\omega_l) + 0.5\hat{c}(\omega_h) = y. \tag{17}$$

In a non-autarkic equilibrium ($\hat{q} > 0$), it must be the case that the cash-constraint for at least one type of agent (patient) remains slack. Hence, condition (14), together with (15), implies,

$$2\phi = \left[\phi\left(\frac{R\beta}{\mu}\right) + \omega_h u'(\hat{c}(\omega_h))\right].$$

Making use of (11), the expression above may alternatively be expressed as,

$$\left[2\left(\frac{R\beta}{\mu}\right)-1\right]u'(\hat{c}(\omega_l))=\eta u'(\hat{c}(\omega_h)). \tag{18}$$

For a given policy (R,μ), conditions (17) and (18) characterize the equilibrium allocation at night $\hat{c}(\omega)$. Comparing these latter two restrictions with (1) and (2), we have the following result:

Lemma 1 *The competitive monetary equilibrium allocation $\hat{c}(\omega)$ corresponds to the first-best allocation $c^*(\omega)$ if and only if policy satisfies $\mu = R\beta$.*

In what follows then, let me assume that policy satisfies the restriction $\mu = R\beta$. In this case, $\hat{c}(\omega) = c^*(\omega)$; and, in particular, note that $q^* = c^*(\omega_h) - y > 0$, with $\hat{z}^+(\omega_h) = 0$ (the cash-constraint for the impatient agent binds just weakly). Moreover, from (11) and (15),

$$\phi^* = \omega u'(c^*(\omega)). \tag{19}$$

Now, at the end of the day, nominal money demand must equal the nominal supply of money available; i.e., $\hat{m}_n = M - v_d^{-1}\tau$, or expressed in real terms,

$$\phi^* q^* = Q - \tau. \tag{20}$$

The policy $\mu = R\beta$ together with the government budget constraint (6) implies $\tau = (1-\beta)Q$; which, when combined with (20) yields an expression for the equilibrium real value of the money supply,

$$Q^* = \beta^{-1}\phi^* q^*. \tag{21}$$

With Q^* determined in this manner, the equilibrium lump-sum tax is then given by,

$$\tau^* = (1-\beta)Q^*. \tag{22}$$

The equilibrium prices (v_d^*, v_n^*) are determined by,

$$v_d^* = \left(\frac{Q^*}{M}\right);\ v_n^* = \left(\frac{v_1^*}{\phi^*}\right) \tag{23}$$

Finally, the distribution of real money balances at the beginning of each day (apart from the initial date) is given by,

$$z^*(\omega^-) \equiv \phi^*\beta^{-1}\left[q^* + y - c^*(\omega^-)\right] \geq 0, \tag{24}$$

where $\omega^- \in \{\omega_l, \omega_h\}$ denotes the previous night's preference shock.

By the budget constraint (3),

$$\begin{aligned} x^*(\omega^-) &= z^*(\omega^-) - \phi^* q^* - \tau^*; \\ &= z^*(\omega^-) - Q^*, \end{aligned}$$

where this latter derivation makes use of (21) and (22). Combining this with (24),

$$\begin{aligned} x^*(\omega_l) &= \phi^*\beta^{-1}\left[q^* + y - c^*(\omega_l)\right] - Q^*; \\ x^*(\omega_h) &= \phi^*\beta^{-1}\left[q^* + y - c^*(\omega_h)\right] - Q^*; \end{aligned}$$

or, by employing (21),

$$\begin{aligned} x^*(\omega_l) &= \phi^*\beta^{-1}\left(y - c^*(\omega_l)\right) > 0; \\ x^*(\omega_h) &= \phi^*\beta^{-1}\left(y - c^*(\omega_h)\right) < 0. \end{aligned} \tag{25}$$

Note that $0.5x^*(\omega_l)+0.5x^*(\omega_h) = 0$; which is consistent with (16) and the first-best restriction that $E_t\left[x_t^*(i)\right] = 0$.

Proposition 1. *If $\chi \equiv 1$, then any policy (R, μ, τ^*) satisfying $\mu = R\beta$ and (22) will implement the first-best allocation as a competitive monetary equilibrium.*

The proposition above states a result that is familiar for this class of models; in particular, if lump-sum taxation is feasible ($\chi \equiv 1$), then the Friedman rule policy $(R, \mu) = (1, \beta)$ is consistent with first-best implementation. In fact, the result is more general than this; in particular, that there are many policies with the property $R > 1$ and $\mu > \beta$ that can also implement the first-best allocation (each of these policies require the same level of taxation τ^*). One such policy includes holding the money supply constant and paying interest $R = 1/\beta$. As long as $\mu = R\beta$, money earns the real rate of return $1/\beta$ and the sequence of competitive money-goods markets substitute perfectly for the missing private debt market. But when lump-sum taxation is feasible, interest-bearing money is not essential.

4. INDIVIDUAL RATIONALITY

Imagine now that agents cannot be forced to pay the tax τ^*. This would then seem to pose a problem, as the contribution τ^* is necessary either to finance the requisite deflation or the aggregate interest expense of interest-bearing money. For agents to be willing to pay the tax τ^*, it must be individually-rational to do so. In what follows, I refer to an incentive-feasible policy as one that simultaneously satisfies the government budget constraint and respects individual rationality.

To begin, assume that all agents play $\chi = 1$. In this case, the competitive equilibrium corresponds to the first-best; and the (ex ante) utility payoff associated with choosing $\chi = 1$ is

$$W(z) \equiv \left[z - \phi^* q^* - \tau^*\right] + 0.5\left[V(q^*; \omega_l) + V(q^*; \omega_h)\right]. \tag{26}$$

Here, it will be useful to note the following,

Lemma 2 *If $\chi = 1$ is individually-rational, and if policy is set optimally, then money is superneutral (nothing real depends on μ).*

The proof of Lemma 1 follows as a corollary to Proposition 1. Among other things, Lemma 1 implies that $W(z)$ does not depend on μ.

Next, consider the payoff associated with an individual defection; i.e., when one agent plays $\chi = 0$ expecting all others to continue playing $\chi = 1$. As agents belong to a continuum, an individual defection will have no aggregate consequences. Hence, I can evaluate the payoff associated with defection assuming that the equilibrium price-system remains unchanged.

Let $W^d(z)$ denote the value associated with defection in the day, and let $V^d(q;\omega)$ denote the value associated with defection at night. In this case, conditional on defection, the choice problem is given by

$$W^d(z) \equiv \max_{q \geq 0} [z - \phi^* q] + 0.5 \left[V^d(q;\omega_l) + V^d(q;\omega_h) \right]. \tag{27}$$

For the defector then, the demand for real money balances $\hat{q}$ is determined by

$$\begin{array}{lll} \phi^* = 0.5 \left[V_1^d(\hat{q};\omega_l) + V_1^d(\hat{q};\omega_h) \right] & \text{if} & \hat{q} > 0; \\ \phi^* > 0.5 \left[V_1^d(0;\omega_l) + V_1^d(0;\omega_h) \right] & \text{if} & \hat{q} = 0. \end{array} \tag{28}$$

Note that $W_1^d(z) = 1$.

Having defected during the day, the defector enters the night without the second time-dated token. This does not inhibit the ability to trade at night, but it does imply that money balances held at the end of the night do not earn interest. Hence, by condition (5) with $\chi = 0$,

$$z^+ = \left(\frac{\phi^*}{\mu} \right) [\hat{q} + y - c] \geq 0. \tag{29}$$

The choice problem at night can therefore be expressed as,

$$V^d(\hat{q};\omega) \equiv \max_{c, z^+ \geq 0} \omega u(c) + \beta \max \left\{ W^d(z^+), W(z^+) \right\}, \tag{30}$$

with z^+ given by (29).

Note that the choice problem at night embeds the possibility that the decision to defect one day is reversed the next. In fact, one can demonstrate that if defection is optimal at any date, it will remain optimal forever. It should be evident by the quasi-linear preference structure that,

Lemma 3 *The net gain from defection* $\left[W^d(z) - W(z) \right]$ *is independent of* z.

Lemma 3 implies that if $W^d(z) > W(z)$, then $W^d(z^+) > W(z^+)$, so that if defection is desirable one day, it will also be desirable the next; i.e.,

$$\max \left\{ W^d(z^+), W(z^+) \right\} = W^d(z^+).$$

Hence, if an agent finds it optimal to defect in the day, his choice problem at night can be stated more simply as,

$$V^d(\hat{q};\omega) \equiv \max_{c, z^+ \geq 0} \omega u(c) + \beta W^d(z^+). \tag{31}$$

The solution to this problem is characterized by,

$$\begin{array}{lll} \omega u'(\hat{c}(\omega)) = \phi^* \mu^{-1} \beta & \text{if} & \hat{z}^+(\omega) > 0; \\ \hat{c}(\omega) = \hat{q} + y & \text{if} & \hat{z}^+(\omega) = 0; \end{array} \tag{32}$$

for $\omega \in \{\omega_l, \omega_h\}$.

By the envelope theorem,

$$V_1^d(\hat{q};\omega) = \omega u'(\hat{c}(\omega)); \tag{33}$$

which, when combined with (28), implies

$$\begin{array}{ll} \phi^* = 0.5\left[\omega_l u'(\hat{c}(\omega_l)) + \omega_h u'(\hat{c}(\omega_h))\right] & \text{if} \quad \hat{q} > 0; \\ \phi^* > 0.5\left[\omega_l u'(\hat{c}(\omega_l)) + \omega_h u'(\hat{c}(\omega_h))\right] & \text{if} \quad \hat{q} = 0. \end{array} \tag{34}$$

At this point, it should be evident that the following is true.

PROPOSITION 2. *The "Friedman rule" policy* $(R, \mu, \tau) = (1, \beta, \tau^*)$ *is not individually-rational.*

Proof. Under the Friedman rule, non-defectors pay a tax τ^* and earn zero interest, while defectors avoid paying the tax τ^* and also earn zero interest. Hence, $(\hat{c}(\omega), \hat{q}) = (c^*(\omega), q^*)$ is both desirable and feasible for the defector and yields the same expected utility in night-market exchanges as non-defectors. However, as the defector avoids paying the tax τ^* in the day, it must therefore be the case that $W^d(z) > W(z)$. ∎

One way to understand this result is as follows. Given the nature of this environment, fiat money is the only asset with which agents can self-insure. Hence, money is an instrument created by society to serve a desirable social objective; in this sense, fiat money is like a public good. Efficient risk-sharing requires that money earn an adequate real rate of return. When $R = 1$, the only way to increase the real rate of return on money is through deflation. This deflation requires that some part of the money supply be destroyed in every period. As the government cannot simply force people to hand over their money, they must be willing to hand it over, if the public good is to be served. But at the individual level, agents can "free-ride" on this public good provision by not paying their taxes. When $R = 1$, there is no individual cost associated with not collecting a tax receipt. As all agents can be expected to behave in the same manner, the Friedman rule policy is not incentive-feasible.[4]

4.1. Incentive-Feasible Monetary Policy

Intuition suggests that defectors can be punished here indirectly by creating an inflation (increasing μ). By Lemma 1, such an inflation will have no effect on non-defectors; that is, the nominal interest rate will simply adjust in proportion to the inflation rate. But as defectors cannot protect themselves against inflation (they are not entitled to earn interest), an inflation will hamper their ability to self insure. The welfare cost associated with this reduced level of insurance may be sufficiently high to induce them to pay their taxes.

To investigate this possibility, let us first consider the choice problem facing a defector during the day. Condition (34) characterizes the defector's desired quantity of real money balances $\hat{q}$, to be carried into the night-market. Clearly, under the

[4] This interpretation was suggested to me by Todd Keister.

Friedman rule policy, neither cash-constraint will bind tightly for the defector; so that in this case, $\hat{q} = q^*$. But for any inflation rate $\mu > \beta$, it must be the case that at least one cash-constraint binds tightly for the defector.

Lemma 4 *If $\mu > \beta$, then the defector's cash-constraints cannot both remain slack.*

Proof. If both cash-constraints remain slack, then (32) implies $\omega u'(\hat{c}_2(\omega)) = \phi^* \beta \mu^{-1}$ for $\omega \in \{\omega_l, \omega_h\}$. Combining this with (34) (when $\hat{q} > 0$) implies $\mu = \beta$; which is a contradiction. ∎

Imagine then that only one cash-constraint binds for the defector; clearly, this will occur for the impatient type. Utilizing (32), the allocation for the defector at night is characterized by

$$\omega_l u'(\hat{c}(\omega_l)) = \phi^* \left(\frac{\beta}{\mu} \right); \tag{35}$$

$$\hat{c}(\omega_h) = \hat{q} + y; \tag{36}$$

with $\hat{q}$ determined by (34); i.e.,

$$\phi^* \left(2 - \frac{\beta}{\mu} \right) = \omega_h u'(\hat{q} + y); \tag{37}$$

at least, as long as $\hat{q} > 0$.

Hence, when the allocation associated with defection is characterized by (35), (36), and (37), the effect of increasing μ away from β is to increase $\hat{c}(\omega_l)$ and to reduce both $\hat{q}(\mu)$ and $\hat{c}(\omega_h) = \hat{q}(\mu) + y$. At the same time, $\hat{z}^+(\omega_l) = 0$ remains invariant to μ; while $\hat{z}^+(\omega_l) = (\phi^*/\mu)\left[\hat{q}(\mu) + y - \hat{c}(\omega_l)\right]$ declines monotonically in μ. These properties imply that, at some point, this second cash-constraint will have to bind as inflation is increased. Let μ_0 define the minimum inflation rate for which $\hat{z}^+(\omega_l) = 0$.

Note that it is conceivable that the constraint $q \geq 0$ binds for some $\mu \leq \mu_0$; whether this is true or not is likely to depend on parameters and, in particular, on the curvature properties of u. In the event that $\hat{q} = 0$ when $\mu \leq \mu_0$, then the punishment associated with defection is autarky. On the other hand, it is also conceivable that the constraint $q \geq 0$ does not bind at $\mu = \mu_0$. In this case, condition (34)) implies that $\hat{q}_0 \geq 0$ is determined by

$$2\phi^* = (\omega_l + \omega_h) u'(\hat{q}_0 + y). \tag{38}$$

One can verify that for the class of utility functions with $u'(c) = c^{-\sigma}$, $\sigma > 0$, that $\hat{q}_0 > 0$ for $\sigma > 1$; and that $\hat{q}_0 = 0$ for $\sigma \leq 1$.

Condition (38) implies that there is an upper bound on the punishment that can be inflicted by way of inflation on the defector. This upper bound is either given by autarky, or the allocation associated with $\hat{q}_0 > 0$ (which does not depend on inflation). In this latter case, the defector is cash-constrained in both states and consumption at night is equalized across states; $\hat{c}(\omega) = \hat{q}_0 + y$. Evidently, if preferences at night are sufficiently linear (e.g., if u less than log concave), then the defector will still desire to accumulate cash each day and spend it at night.

Lemma 5 *If $\mu \geq \mu_0$, then $\hat{c}(\omega)$ and $\hat{q}_0$ are independent of β.*

If $\mu = R\beta$ is incentive-feasible, then the equilibrium allocation and price system does not depend on β. If policy is further restricted such that $\mu \geq \mu_0$, then Lemma 5 asserts that the allocation associated with defection is also independent of β. The only equilibrium variable that depends on β is the real value of the lump-sum tax, which by (21) and (22) is given by

$$\tau^*(\beta) = \left(\frac{1-\beta}{\beta}\right)\phi^* q^*. \tag{39}$$

Assume then that policy is such that $\mu = R\beta$ for some $\mu \geq \mu_0$. In this case, the night allocation associated with defection is $\hat{c}(\omega) = \hat{q}_0 + y$. Apart from the first day in which the defection occurs, all future day allocations for the defector (both actual and expected) is given by $E_\omega \hat{x}(\omega) = -\phi^* \hat{q}_0 \leq 0$. Hence, the expected utility payoff associated with defection from the first night onward is given by

$$E_\omega V^d(\hat{q}_0; \omega) = \left[\frac{A(\beta)}{1-\beta}\right];$$

where

$$A(\beta) \equiv 0.5(\omega_l + \omega_h)u(\hat{q}_0 + y) - \beta\phi^* \hat{q}_0.$$

The corresponding value for the non-defector is given by

$$E_\omega V(q^*; \omega) = \left[\frac{B}{1-\beta}\right];$$

where

$$B \equiv 0.5\left[\omega_l u(c^*(\omega_l)) + \omega_h u(c^*(\omega_h))\right].$$

Lemma 6 $g(\beta) \equiv B - A(\beta) > 0$ *for all* $\beta \in [0, 1)$.

Appendix A provides a proof of Lemma 6; but the result is readily apparent for the case in which $\hat{q}_0 = 0$ (so that the value $A(\beta)$ corresponds to the flow payoff associated with a future of autarky).

Lemma 7 *Define* $G(\beta) \equiv (1-\beta)^{-1} g(\beta) > 0$. *Then* $G'(\beta), G''(\beta) > 0$ *for all* $\beta \in [0, 1)$, *and* $\lim_{\beta \to 1} G(\beta) = +\infty$.

Appendix A also provides a proof of Lemma 7. The function G measures the net benefit associated with not defecting beginning with the first night onward. The lemma asserts that G is monotonically increasing in β and approaches infinity as the discount factor approaches unity.

Finally, consider the day in which the defection initially occurs. An individual who begins the day with real money balances z and chooses $\chi = 1$ will consume $[z - \phi^* q^* - \tau^*]$ in the day. The same individual who chooses $\chi = 0$ will consume $[z - \phi^* \hat{q}_0]$. Hence, the immediate net benefit to not defecting is given by

$$\phi^*(\hat{q}_0 - q^*) - \tau^*(\beta) < 0.$$

That is, there is an immediate direct cost associated with paying the tax. Using (39), this latter expression may be rewritten as

$$\phi^* \left[\hat{q}_0 - \left(\frac{1}{\beta} \right) q^* \right] < 0.$$

Combining the terms derived above, the net benefit associated with not defecting can be expressed as,

$$W(z) - W^d(z) = \phi^* \left[\hat{q}_0 - \left(\frac{1}{\beta} \right) q^* \right] + G(\beta).$$

Clearly, $W(z) - W^d(z) \geq 0$ requires,

$$G(\beta) \geq \phi^* \left[\left(\frac{1}{\beta} \right) q^* - \hat{q}_0 \right]. \tag{40}$$

Lemma 7 establishes the properties of $G(\beta)$. The right-hand-side of (40) is monotonically decreasing in β, approaching $+\infty$ as $\beta \to 0$ and approaching $-\phi^* \hat{q}_0 \leq 0$ as $\beta \to 1$. Hence, we have the following result,

PROPOSITION 3. *The first-best allocation is implementable as a competitive monetary equilibrium for any $\beta \in [\beta^*, 1)$. The optimal policy necessarily has the property that $R > 1$ and $\mu > \beta$.*

5. CONCLUSION

Most discussions concerning the optimality of the Friedman rule assume that lump-sum taxation is feasible. Absent the ability to collect lump-sum taxes, the general presumption appears to be that only second-best allocations are feasible in monetary economies. The analysis above, however, suggests that even in societies for which coercive taxation is difficult (or even impossible) to implement, policy might nevertheless be designed in a manner that elicits voluntary "tax" payments that support a first-best allocation.

In monetary economies, there appears to be a natural manner in which punishment can be dispensed on individuals noncompliant in tax obligations necessary to promote the public good. That is, an appropriate policy requires three basic elements: [1] an inflation (away from the Friedman rule); [2] a strictly positive nominal interest rate on money balances; and [3] the issuance of non-counterfeitable tax receipts that entitle the bearer to collect interest. Failing to pay taxes in a high-inflation/high-interest-rate environment reduces the ongoing value associated with monetary exchange. If individuals are sufficiently patient, they may find it individually-rational to make their contribution to society. Doing so entitles them to earn nominal interest on their money balances, which mitigates (or even eliminates) the otherwise harmful effects of inflation.

Appendix A

Lemma 6 asserts that

$$g(\beta) \equiv B - A(\beta) > 0,$$

where

$$\begin{aligned} A(\beta) &\equiv 0.5(\omega_l + \omega_h)u(\hat{q}_0 + y) - \beta\phi^*\hat{q}_0; \\ B &\equiv 0.5[\omega_l u(c^*(\omega_l)) + \omega_h u(c^*(\omega_h))]. \end{aligned}$$

There are two cases to consider; one in which $\hat{q}_0 = 0$ (autarky) and in which $\hat{q}_0 > 0$ (both cash-constraints bind, but the agent accumulates cash in the day and spends it all at night).

Consider the following program,

$$\max \{0.5[\omega_l u(c(\omega_l)) + \omega_h u(c(\omega_h))] : y \geq 0.5 [c(\omega_l) + c(\omega_h)]\};$$

and the associated Lagrangian,

$$L(c(\omega_l), c(\omega_h), \lambda) = 0.5[\omega_l u(c(\omega_l)) + \omega_h u(c(\omega_h))] + \lambda [y - 0.5c(\omega_l) - 0.5c(\omega_h)]. \tag{41}$$

The solution to this program is the first-best allocation $c^*(\omega)$ with an associated multiplier $\lambda^* = \phi^* > 0$, and value $L(c^*(\omega_l), c^*(\omega_h), \lambda^*) = B$.

For the case in which $\hat{q}_0 = 0$, the value $A(\beta)$ corresponds to the payoff associated with autarky (and is independent of β). In this case, it is clear that $g(\beta) = B - A(\beta) > 0$ and that $g(\beta)$ is independent of β.

The case in which $\hat{q}_0 > 0$ is not as straightforward. That is, while both cash-constraints bind in this case, it is also true that the defector enjoys a higher average level of consumption at night (this is offset to some extent by a lower average level of consumption in the day).

To proceed, consider next the following problem,

$$\max \{0.5[\omega_l u(c(\omega_l)) + \omega_h u(c(\omega_h))] - \beta\phi^* q : q + y \geq 0.5 [c(\omega_l) + c(\omega_h)]\}.$$

Substituting the constraint into the objective, one may rewrite this problem as,

$$A^1(\beta) \equiv \max \{0.5[\omega_l u(c(\omega_l)) + \omega_h u(c(\omega_h))] + \beta\phi^* [y - 0.5c(\omega_l) - 0.5c(\omega_h)]\}. \tag{42}$$

This problem is equivalent to maximizing the Lagrangian in (41), but subject to the added constraint $\lambda = \beta\phi^*$. Note that the solution here corresponds to $(c^*(\omega_l), c^*(\omega_h), \lambda^*)$ when $\beta = 1$, so that $A^1(1) = B$.

Next, impose another constraint on the problem (42) by assuming that $c(\omega_l) = c(\omega_h) = c$. In this case, (42) may be expressed as,

$$A^2(\beta) \equiv \max \{0.5 (\omega_l + \omega_h) u(c) + \beta\phi^* [y - c]\}.$$

Note that the solution here corresponds to $\hat{q}_0$ when $\beta = 1$ (with $\hat{c}_0 = \hat{q}_0 - y$). Clearly, $A^2(1) < A^1(1) = B$.

Finally, observe that $A(1) = A^2(1)$. Moreover, as $\hat{q}_0$ is independent of β, it follows that $A(\beta)$ is decreasing in β. Hence, $A(\beta) \leq A^2(1) < B$ for all β. But then, this implies that $g(\beta) > 0$ for all β.

Next, consider Lemma 7. Again, there are two cases to consider. In the event that $\hat{q}_0 = 0$, $g(\beta) > 0$ is independent of β, so that $G(\beta)$ is monotonically increasing in β with $\lim_{\beta\to 1} G(\beta) = +\infty$.

Consider now the case in which $\hat{q}_0 > 0$. Observe that

$$G'(\beta) = \frac{T(\beta)}{(1-\beta)^2};$$

where $T(\beta) \equiv g(\beta) - (1-\beta)\phi^* \hat{q}_0$. This latter expression reduces to $T(\beta) = g(1) > 0$; which implies that $G'(\beta) > 0$. Moreover, note that $T(\beta)$ is independent of β; so that $G''(\beta) > 0$. In short, G is monotonically increasing in β, with $\lim_{\beta\to 1} G(\beta) = +\infty$.

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