International transmission of transitory and persistent monetary shocks under imperfect information

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Abstract

We analyze the transmission of monetary shocks in a new open-economy macroeconomics model with one-period nominal contracts and imperfect information. Shocks may have transitory and persistent components that can be disentangled only through the accumulation of information over time. As a consequence, the responses to shocks are significantly altered compared with the case of full information. There are persistent effects on international relative prices, and delayed exchange-rate overshooting is possible following a persistent shock. In some cases, there are (ex post) excess returns as a positive interest rate spread is accompanied by an

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appreciating currency (or vice versa). Lastly, it is demonstrated that staggering reinforces persistence.
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1. Introduction

Information flows are essential for financial markets, and a great deal of resources are absorbed by analyzing the developments in asset prices and exchange rates. Market participants and the media exert much effort to interpret current market signals in an attempt to infer information of relevance for the future to answer the pertinent question whether a given change is purely temporary (noise) or lasting. An obvious market where information plays a crucial role is the foreign-exchange market, since exchange rates can change instantaneously in response to new information. A recent example is the development in the dollar–euro exchange rate, where the issue of distinguishing transitory from persistent changes has taken center stage.

Despite considerable resources invested in information processing, market expectations of future exchange rates deviate systematically from realized exchange rates. As is evident from Fig. 1, where actual U.S. dollar exchange rates are depicted along the Consensus forecasts based on information available 4 months ahead. Ex post market prediction errors are highly persistent, suggesting fundamental information problems.

The purpose of this paper is twofold. First, we show that persistent deviations between expected and realized exchange rates do not necessarily reflect market anomalies, but may arise as a consequence of an inability to distinguish between transitory and persistent shocks. Second, we explore how these (rational) prediction errors affect the international transmission of monetary shocks, with special focus on the response of exchange rates, interest rates, and the terms of trade.

To address the international transmission of monetary shocks, we need an explicit intertemporal general equilibrium model. The specific structure builds on the new open-economy macroeconomics launched by Obstfeld and Rogoff (1995), which has proved to be a useful framework (for a survey, see Lane, 2001). In the specific application here, we introduce the minimum assumption needed to create nonneutralities by assuming one-period nominal (wage) contracts. Since the assumed nominal contracting setup does not, in itself, contribute to generate interesting dynamics, we are able to focus on the dynamic implications of imperfect information.\(^1\) We present an explicit analytical solution of the model, with the advantage that we can identify the mechanisms through which information problems affect market reactions under rational expectations.

Although the problem of distinguishing between transitory and persistent influences can arise from a whole range of sources (see below), we focus on monetary shocks. It is a simple

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\(^1\) Absent imperfect information nominal shocks have real effects in one period.
way to illustrate the gist of imperfect information: Current signals may later turn out to have conveyed irrelevant noise, whereas other pieces of information may turn out to be useful but have been given insufficient weight. Ex ante, there is a nontrivial problem in disentangling the two. Furthermore, it is well documented that nominal shocks play a significant role in explaining open-economy variables, like exchange rates (e.g., Canova and Nicolò, 2002), and focusing on nominal shocks allows us to address the issue why nominal exchange-rate changes can have persistent real effects even when nominal rigidities are short lived (Rogoff, 1996). Specifically, we model shocks to the money stock as being either transitory or persistent, and while all market participants observe the current and past money stocks, they do not directly observe the transitory and persistent components.

Several puzzles are unexplained in open-economy macroeconomics. We demonstrate that the informational problem of disentangling transitory and persistent changes, combined with one-period contracts, can possibly explain three of these puzzles: persistent effects on international relative prices, delayed overshooting, and the well-known observation that a depreciating (appreciating) currency is accompanied by a negative (positive) interest rate spread. First and foremost, learning gives an intuitive account of the persistent effects of

Fig. 1. Expected (4-month horizon) and actual U.S. dollar exchange rates. The solid line (left-hand axis) is the U.S. dollar exchange rate against the euro, Japanese yen (JPY), British Pound (GBP), and Canadian dollar (CAD) end-of-month. The dashed line (left-hand axis) is the consensus forecast on a 4-month horizon. The bars (right-hand axis) are the forecast error in percentage terms. Sources—actual exchange rates: EcoWin (code: 19005); expected exchange rates: Consensus Economics.
nominal shocks on international relative prices (terms of trade and real exchange rate; 
Rogoff, 1996). When a given shock hits the economy, the agents need time to pin down the 
nature of the shock, and during this learning process, there will be real effects.

Furthermore, we show that a persistent shock is capable of generating delayed nominal 
exchange-rate overshooting (Eichenbaum and Evans, 1995). Following a positive 
(persistent) monetary shock, which only gradually dies out, the nominal exchange rate 
depreciates on impact; but rather than settling on an appreciating path as the shock dies 
out, the nominal exchange rate keeps depreciating further for some time, before 
appreciating towards its preshock value. During the periods with depreciation, the interest 
rate spread is negative (Frankel and Rose, 1995), which shows that a depreciating 
(appreciating) currency can be accompanied by a negative (positive) interest rate spread. If 
market participants cannot readily distinguish noise (transitory shocks) from fundamental 
changes in market conditions (persistent shocks), they tend to react too strongly to the 
former and too little to the latter. As a consequence, sensitivity to noise and fundamentals 
is increased and decreased, respectively. The net effect in this rational expectations setting 
is that exchange-rate volatility is decreased. Thus, excess volatility in exchange rates is 
still most likely best explained by irrationalities (e.g., noise traders, see Devereux and 
Engel, 2002). Lastly, we show that, if the informational problem of shock confusion is 
combined with overlapping wage contracts, the two propagation mechanisms interact 
nontrivially. Persistence is increased, and, in the cases with nominal delayed overshooting, 
hump-shaped responses of the real variables are possible as well.

This open-economy paper is related to a vast and growing literature documenting and 
exploring the macroeconomic implications of information problems primarily in closed 
economies. Persistent errors in expectations are not isolated to foreign-exchange markets. 
Evans and Wachtel (1993) is a classic reference on inflation expectations, which has also 
been explored recently by Carroll (2003). In his model, agents only occasionally update 
expectations from news wires, and this creates inertia in expectations. Mankiw and Reis 
(2002) analyze the dynamic implications of inertia in information dissemination and show 
that a model with sticky information can account better for business-cycle facts than the 
standard sticky-price model does.

The information problem arising from errors in preliminary data on which agents react 
has been analyzed by Bomfim (2001), focusing on the implications for business cycle 
fluctuations, while Faust et al. (2003) show that exchange-rate models, in general, perform 
much better when evaluated on the basis of preliminary rather than final data.

The interaction between policy and expectations formation when policy responses are 
based on imperfect knowledge of the true state of the economy or when the private sector 
holds incomplete knowledge on policy objectives has been addressed by Erceg and Levin 
(2003), Lansing (2000), Orphanides (2001), Romer and Romer (2000), and Rudebusch 

In a related paper, Gourinchas and Tornell (2002) analyze the interest-rate spread and 
exchange-rate dynamics when agents misperceive the true underlying process generating

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2 The importance of distinguishing between transitory and persistent shocks goes back to Muth’s (1960) 
discussion of the optimality of adaptive expectations (see also Sargent, 1982). On the role of informational 
problems for macroeconomic adjustment, see also Andersen (1994).
shocks. They find that misperception can explain the forward premium puzzle, as well as delayed overshooting.

Although our crude modeling of transitory and persistent shocks can be given a narrow motivation by errors in preliminary money stock data (Bomfim, 2001; Mankiw et al., 1984) and builds on the monetary approach to exchange-rate determination, we think that the insights provided go beyond this specific way of modelling foreign-exchange markets. The point that determinants of exchange and interest rates have both transitory and persistent components is generic to any model of exchange-rate determination. Thus, our model can be given a wider interpretation along the lines of those given in the literature (cf. above). There is also a parallel between this paper and the so-called microstructure model of financial markets (e.g., O’Hara, 1995) in the sense of stressing the fact that market participants face nontrivial information problems and that it is crucial for market behavior how new information enters the market. The present paper asks whether these information problems have any interesting macroeconomic implications and focuses on the dynamic adjustment process addressing stylized facts, which is hard to reconcile with standard open macromodels.

The paper is organized as follows. The two-country model with a flexible exchange rate is set up in Section 2. The stochastic process for money and the information structure are defined in Section 3. Section 4 describes the equilibrium. Section 5 considers the dynamics of nominal shocks and includes numerical illustrations of the main findings. Section 6 extends the model with staggering, and discussion and concluding remarks are presented in Section 7.

2. A stochastic two-country model

We consider a symmetric two-country model with a flexible exchange rate and specialized production (see also Obstfeld and Rogoff, 1995, 2000). There are two equally sized countries and two goods, one produced by Home and one produced by Foreign firms. There are two assets in the economy: money and a real bond, where the latter is traded in a perfect international capital market. There is no real capital and no internationally mobile labor.

Workers are organized in (monopoly) unions, and each union represents a (small) subset of workers supplying labor to a given group of firms. Each union is utilitarian and chooses a wage for period $t$ given all available information in period $t-1$ to maximize the expected utility of workers, which, in turn, depends on the wage income (consumption) and the disutility of work. Employment is determined by firms given the wage set by the union (right-to-manage structure). All other prices are determined in competitive markets.\(^4\)

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\(^3\) We assume that workers are willing to participate in the sense that for any labor demand, the marginal consumption value of the real wage is larger than the marginal disutility of effort (Corsetti and Pesenti, 2001).

\(^4\) All proofs and derivations are relegated to Andersen and Beier (2004).
2.1. Firms, consumers, and the government

All firms are price takers in both product and labor markets. Home firms demand labor \( N_t \) and produce the Home good \( Y^h_t \) subject to a decreasing returns technology

\[
Y^h_t = N^\gamma_t, \quad 0<\gamma<1.
\]

In the following superscripts, \( h \) (\( f \)), refer to traded variables originating in Home (Foreign), and Foreign variables are denoted by an asterisk. Profits are distributed to households. Profit maximization yields the following labor demand and output supply (in logarithms)

\[
n_t = \eta_{nv}(p^h_t - w_t), \quad \eta_{nv} = (1 - \gamma)^{-1}, \quad (1)
\]

\[
y^h_t = \eta_{yw}(p^h_t - w_t), \quad \eta_{yw} = \gamma(1 - \gamma)^{-1}. \quad (2)
\]

The countries are inhabited by consumers who consume goods \( C_t \), supply labor \( N_t \), and hold money \( M_t \) as well as bonds. Let \( E_t \) be the expectations operator conditional on period \( t \) information (the information structure is defined below), and \( P \) the consumer price index, then the representative consumer’s objective function is

\[
U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[ \frac{\sigma}{\sigma - 1} C_{t+j}^{e-\lambda} + \frac{\lambda}{1 - \beta} \left( \frac{M_{t+j}}{P_{t+j}} \right)^{1-\beta} - \frac{\kappa}{1 + \mu} N_{t+j}^{1+\mu} \right], \quad (3)
\]

\( \sigma>0, \ \lambda>0, \ \beta>0, \ \kappa>0, \ \mu>0, \ 0<\delta \leq 1. \)

\[
C_t = \left( C^h_t \right)^{\frac{1}{2}} \left( C^f_t \right)^{\frac{1}{2}}.
\]

Thus, the real consumption index aggregates across the consumption of the Home \( (C^h_t) \) and the Foreign goods \( (C^f_t) \), where the elasticity of substitution between the two goods is assumed to be one. Our results apply to a more general CES specification, but the unitary elasticity of substitution simplifies the analysis significantly. The price index corresponding to composite consumption is also Cobb-Douglas

\[
P_t = 2(P^h_t)^{\frac{1}{2}}(P^f_t)^{\frac{1}{2}},
\]

where \( P^h_t \) (\( P^h_t^* \)) is the price of the Home good in Home (Foreign) currency, and \( P^f_t \) (\( P^f_t^* \)) is the price of the Foreign good in Home (Foreign) currency. As our focus will be on nominal wage rigidity, we assume that the law of one price holds for both goods; that is, \( P^h_t = S_t P^h_t^* \) and \( P^f_t = S_t P^f_t^* \). \( S \) is the nominal exchange rate, defined as the Home price of Foreign currency. A direct implication of the law of one price is that PPP holds as well; that is, \( P_t = S_t P^*_t \). As a consequence, the subsequent analysis will focus on how nominal shocks affect the terms of trade. It can be shown in a setting including nontradables (Hau, 2000) that the movements in the real exchange rate are qualitatively equivalent to the movements in the terms of trade; hence, our results can be directly related to the PPP puzzle.\(^5\)

\(^5\) For alternative assumptions concerning price setting and discussion see, e.g., Betts and Devereux (2000) and Obstfeld (2001).
We assume that there is one internationally traded real bond denoted in the composite consumption good $C$. Let $r_t$ be the consumption-based real interest rate between dates $t$ and $t+1$. The consumers’ budget constraint for any period $t$ is given by

$$P_tB_t + M_t + P_tC_t = (1 + r_{t-1})P_tB_{t-1} + M_{t-1} + W_tN_t + \Pi_t + P_t\tau_t. \quad (4)$$

The right-hand side gives available resources as the sum of the gross return on bond holdings $(1+r_{t-1})P_tB_{t-1}$, initial money holdings $M_{t-1}$, labor income $W_tN_t$, nominal profit income $\Pi_t$, and transfers from the government $P_t\tau_t$. Resources are allocated to consumption $P_tC_t$, nominal money holdings $M_t$, and bond holdings $P_tB_t$.

Given the constant elasticity consumption index, Home consumers’ demands for the Home good and the Foreign good are

$$D^h_t = \frac{1}{2} \left( \frac{p^h_t}{P_t} \right)^{-1} C_t, \quad D^f_t = \frac{1}{2} \left( \frac{p^f_t}{P_t} \right)^{-1} C_t,$$

respectively, and mutatis mutandis for the demands by Foreign consumers. Aggregating, we find the total demand for the Home good to be (and similarly for the Foreign good)

$$D_t = D^h_t + D^f_t = \frac{1}{2} \left( \frac{p^h_t}{P_t} \right)^{-1} (C_t + C^*_t), \quad (5)$$

The household chooses consumption ($C$), money demand ($M$), bond demand ($B$), and wages ($W$) to maximize Eq. (3) subject to the sequence of the budget constraints given in Eq. (4). In determining the wage, it is taken into account that employment is (demand) determined. Written in log-linear form,\(^6\) the first-order conditions are given by the Euler equation, money demand, and the wage rate:

$$E_t c_{t+1} = c_t + \sigma \log(1 + r_t), \quad (6)$$

$$m_t - p_t = \eta_{mc} c_t - \eta^1_{mc} E_t c_{t+1} + \eta_{mp} (p_t - E_t p_{t+1}), \quad (7)$$

$$w_t = E_{t-1} [\eta_{mp} p^h_t + (1 - \eta_{mp}) (s_t + p^*_f) + \eta_{wc} c_t], \quad \eta_{mc} = \sigma (1 - \delta) \beta^{-1}, \quad (8)$$

$$\eta^1_{mc} = \delta \eta_{mc}, \quad \eta_{mp} = \delta (1 - \delta) \beta^{-1}, \quad \eta_{wc} = \sigma (1 + \mu \eta_{nw})^{-1}. \quad (9)$$

Lower-case variables denote the log-deviations from a symmetric steady state of the corresponding upper-case variables, and all constants are neglected since our primary interest is the adjustment process to shocks.\(^7\) The wage Eq. (8) satisfies the basic homogeneity property generic to any microfounded wage-setting model and implies that nominal wages and, thus, prices depend on expected exchange rates. This captures a channel through which exchange rates affect the real side of the economy. Note that the

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\(^6\) The model is specified to yield a log-linear model. However, log-linearizations are needed for money demand and the budget constraint.

\(^7\) These constant terms include conditional variance terms, which are constant under the stochastic process considered.
log-linearized version of the Home price index is \( p_t = \frac{1}{2}(p_t^h + s_t + p_t^f) \), and the terms of trade are defined as \( q_t = \frac{p_t^h}{C_0} \).

We assume that the government balances its budget each period; that is, \( M_t - M_{t-1} = P_t \). In other words, the only role of the government is to issue money. Money is transferred to Home consumers in a lump-sum fashion. The stochastic process governing money supply, along with the assumptions on the information structure, is described in detail in the next section. We end the description of the model by noting that Foreign is completely symmetric and that an (symmetric) equilibrium exists (see Andersen and Beier, 2004), in which money is neutral absent nominal rigidities.

3. Information structure and money supply

In a flexible exchange-rate regime, changes in supply and demand translate immediately into changes in the exchange rate. It follows that changes in exchange rates may originate from various forms of shocks arising on either the demand or the supply side. These shocks could be real or monetary in nature and leave a nontrivial problem of separating transitory from persistent changes in the exchange rate. Building this problem into a fully specified general equilibrium model is by no means trivial, since it requires not only a specification of shocks with transitory and persistent components, but also an account of how these shocks affect the agents (preferences, endowments, technology, etc.).

To simplify, we focus on nominal shocks. Thereby, we also address the more difficult problem of explaining the persistent effects of nominal shocks. Since the information problem of interpreting changes is essential to our story, we exploit the model simplification that can be achieved by considering changes in the money stock, which are either transitory or persistent. It is assumed that all current information is freely available, but agents face the problem of making inferences about its implications for the future.

3.1. The money-supply process

A straightforward way to introduce the problem of distinguishing between transitory and persistent influences is to assume that the relative money-supply process is:

\[
\begin{align*}
    m_t - m_t^* &= z_t + u_t, \\
    z_t &= \theta z_{t-1} + \varepsilon_t, \quad 0 < \theta < 1,
\end{align*}
\]

where \( u \) and \( \varepsilon \) are independent and normally distributed mean-zero shocks with variances \( \sigma_u^2 \) and \( \sigma_\varepsilon^2 \), respectively.

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8 Since information flows continuously and the wage contracts are assumed to be fixed for a given period of time, it follows that some aggregation of information has already implicitly taken place in transforming data to match the contract length.
The money-supply process captures that some changes are transitory (\(u\)) and some are persistent (\(z\)). Agents cannot readily disentangle one type of shock from the other since they only observe the sum of the two components. Agents know the current and past realizations of relative money supplies and learn over time as they accumulate information. Hence, an agent’s information set, \(I_t\), is given as \(\{m_t-m^*_t, m_{t-1}-m^*_{t-1}, \ldots\} \subseteq I_t\).

The specification (Eq. (9)) can be given several interpretations. Empirically, it can be motivated by noise in preliminary announcements of money stock data (Bomfim, 2001; Mankiw et al., 1984), where \(u\) represents the measurement error or noise. Eq. (9) can also be interpreted literally as reflecting that money-supply (monetary policy) changes may be either transitory or persistent. This may arise if the policy maker reacts to changes in variables, which, in turn, are affected by shocks that may be either transitory or persistent in nature. Another reason may be that the policy maker operates under imperfect information or that market participants have different information than the policy maker does, or that there may be imperfect knowledge about the objectives of the policy maker.\(^9\)\(^10\) However, the specific formulation adopted here may also capture more general information problems. It is natural to interpret the transitory component (the \(u\)-part) as reflecting noise and the persistent part (the \(z\)-part) as fundamentals, since the former is not helpful in predicting the future, while the latter does affect future market conditions. In a money-supply setting, this would apply if the liquidity created in the financial system is not one-to-one related to the money base (which can be observed with high precision); that is, the money multiplier varies, and there is imperfect information on the causes. The formulation can also reflect that the aggregate information set available in the market is not sufficiently detailed to allow a precise identification of market fundamentals of importance for future market developments (Figlewski, 1982).

3.2. Expectations formation

Given the assumptions made above on the money supply and the information structure, we can turn to expectations formation. Predicting the future money supply is a question of predicting its persistent component, i.e.,

\[
E_t(m_{t+1} - m^*_t) = E_t(z_{t+1}),
\]

where \(E_t\) is shorthand for the mathematical expectation conditional on the information set \(I_t\). Information on future changes in the relative money supply arrives via

\(^9\) Cukierman and Meltzer (1986) show why the policy maker, for strategic reasons, may disseminate imprecise information.

\(^10\) The contemporaneous debate on transparency in monetary policy making can be interpreted as a way to minimize the noise component and thereby provide more information on the fundamentals underlying monetary policy.
observations of the relative money supply, and it can be shown by use of Kalman-filter techniques that the conditional expectation can be written as (Hamilton, 1994)

\[ E_t(m_{t+1} - m^*_{t+1}) = \theta E_{t-1}(m_t - m^*_t) + \theta h[m_t - m^*_t - E_{t-1}(m_t - m^*_t)], \]

where \( h = \frac{1 + \Delta - (1 - \theta^2)\eta}{1 + \Delta + (1 + \theta^2)\eta} \epsilon(0,1), \quad \Delta^2 = [(1 - \theta^2)\eta - 1]^2 + 4\eta, \]

\[ \eta = \frac{\sigma_u^2}{\sigma_v^2}, \quad \frac{\partial h}{\partial \eta} < 0, \quad \frac{\partial \epsilon}{\partial \sigma_u^2} < 0, \quad \frac{\partial \epsilon}{\partial \sigma_v^2} > 0. \]

Expectations of tomorrow’s relative money supply are given as a weighted sum of yesterday’s expectations of today’s relative money supply and the information obtained by observation of today’s relative money supply. The latter is the difference between the actual period, \( t \), money supply and its expected value (conditional on period \( t-1 \) information) times \( h \), since a fraction \( h \) of this is perceived to be persistent, of which a fraction \( h \) carries forward to the next period. The coefficient \( h \) is crucial for the updating of expectations because it determines the weight given to new information. It is decreasing in the noise-to-signal ratio \( \eta \). That is, if all shocks are transitory (\( \eta \to \infty \)), we have \( h=0 \), and the information content of the signal is nil, whereas if all shocks are persistent (\( \eta \to 0 \)), we have \( h=1 \), reflecting that current signals contain all information of relevance for predicting future money supplies. Interestingly, when the noise-to-signal ratio is one, \( h > 0 \) is greater than one-half. This reflects the learning aspect involved in expectation formation. Not only does a given surprise reflect information about new shocks, it also contains information about last period’s surprise as well, i.e., learning. A positive surprise last period followed by another positive surprise this period indicates that a larger part of last period’s surprise was due to a persistent shock than was originally expected.

If we interpret the transitory shock \( (u) \) as noise and the persistent part \( (z) \) as fundamentals, the updating formula has a very intuitive interpretation. The more noise (larger \( \sigma_u^2 \), smaller \( h \)), the less weight is put on the current observation of money supply since agents know that current movements tend to reflect noise; current signals have a low information content.

Since the updating of expectations to shocks is crucial to the results of this paper, it is useful to consider the learning process in some detail. The following tracks the adaptation of expectations to given shocks. The nature of shocks is unknown to the agents but known to the analyst.

Fig. 2 describes the adjustment path for the actual and (un)expected money stock to transitory and persistent shocks, respectively. In both cases, we consider a 1% positive shock to the relative money stock.\(^{11}\)

\(^{11}\) We consider expectations under the assumption that they have been zero up to date 1, where a one-time 1% positive monetary shock hits the economy. No shock occurs after that, which is unknown to the agents. For instance, in the case of a transitory shock, the agents believe a part of the initial shock to be persistent and a part of the negative surprise the period after is seen as being due to a new shock.
For a transitory shock, we see that, although the relative money supply is only affected in one period, it takes several periods for the agents to learn that the shock was transitory. Accordingly, a positive transitory shock will imply that the money stock is unanticipatedly low in subsequent periods until agents eventually learn the type of the shock.

In the case of a persistent shock, it also takes several periods before expectations and de facto money converge; money is unexpectedly high for a number of periods. Furthermore, there may be delayed overshooting. Initially, expectations rise, and only several periods later do they begin to fall. Delayed overshooting occurs when \( E_t(m_{t+1}-m^*_t) > E_{t+1}(m_{t+2}-m^*_{t+2}) \), and this is ensured if \( 1 - 2\theta + 3h < 0 \). This condition will turn up later when we consider exchange-rate dynamics. Lastly, sensitivity analysis with respect to the noise-to-signal ratio, \( \eta \), shows that the more noise (\( \eta \) large), the longer it takes for agents to learn that the shock, in fact, was persistent.

Fig. 2 clearly illustrates that conditional on a particular shock, the learning process implies systematic expectations errors. This is essential if a given business cycle (of a duration of some years) is interpreted as a realization of a particular string of shocks; that is, a business cycle episode is represented by a small sample of observations from an underlying stochastic process with properties assumed known to the agents. Hence, when
relating our analysis to actual observations like those reported in Fig. 1, we take the perspective that they should be interpreted in terms of theoretical results that consider the adjustment process contingent on a particular string of shocks.\textsuperscript{12} In our case, agents are rational and know the properties of the stochastic process (Eq. (9)), and to simplify, we consider monetary shocks only.

4. Equilibrium

Characterizing the equilibrium analytically is complicated not only by the presence of nominal contracts and the information problem but also by the intertemporal structure linking current and future decisions via expectations. We demonstrate in Andersen and Beier (2004) how to find an analytical solution so that we can explicitly characterize the processes for the endogenous variables.

Since Obstfeld and Rogoff (1995), the qualitative working of this type of model (with a one-period nominal contract) has been well understood; a positive monetary shock leads to an exchange-rate depreciation (terms-of-trade deterioration), leading to a switch in demand towards the Home good, and thus Home production increases. Wealth reallocations are ruled out by the assumption of a unitary demand elasticity, and therefore, relative consumption between Home and Foreign is invariant to all types of shocks. These basic effects are not changed, but richer dynamics arise under imperfect information. The assumption on demand elasticity allows us to demonstrate the main points in the least technical way. The results carry over to the general case, where demand elasticity is different from one.

Consider as a benchmark for the subsequent analysis the case where agents have perfect information, allowing them to identify transitory and persistent shocks to the relative money supply; that is, the underlying process generating relative money supplies (Eq. (9)) is unchanged, and there are still surprises. The information set is now \{\(z_t, \frac{z_t}{C_0}, \ldots, u_t, \frac{u_t}{C_0}, \ldots\}\ \subseteq \hat{I}_t\ (\text{compare with } I_t)\. In this case, the nominal exchange rate can be written

\[
s_t = \phi_{sz} z_t + \phi_{su} u_t, \tag{11}
\]

\[
0 < \phi_{su} = (1 + \eta_{mp})^{-1} < (1 + \eta_{mp} - \theta \eta_{mp})^{-1} = \phi_{sz} < 1.
\]

Transitory shocks have a smaller effect on the exchange rate than persistent shocks do, since the latter also affect future money supplies. The terms of trade are given as

\[
q_t = \phi_{qz} z_t + \phi_{qu} u_t, \tag{12}
\]

\[-1 < \phi_{qz} = -\gamma \phi_{sz} < -\gamma \phi_{su} = \phi_{qu} < 0.\]

In the absence of information problems, the real adjustment to unanticipated shocks is ended after a period of time equal to the contract length. The dynamics is trivial, and the impulse-response functions are implausible.

\textsuperscript{12} The reason is that, in a large sample, under the assumption of rational expectations, there would obviously be no systematic expectations error. Agents would be right on average. In a short sample, however, this would not necessarily be the case, and this motivates the perspective taken here.
4.1. Imperfect information

With informational imperfections, we have

\[ s_t = \theta (1 - h)s_{t-1} + \phi_{sm}(m_t - m_t^*) + \phi_{sm}^1(m_{t-1} - m_{t-1}^*), \quad (13) \]

\[ \phi_{sm} = \frac{1 + \eta_{mp} - \theta (1 - h)\eta_{mp}}{(1 + \eta_{mp})(1 + \eta_{mp} - \theta \eta_{mp})} > 0, \]

\[ \phi_{sm}^1 = \theta (h - 1)\phi_{sm} + \theta h \frac{(1 - h)\eta_{mp}}{(1 + \eta_{mp})(1 + \eta_{mp} - \theta \eta_{mp})}. \]

Similarly, the terms of trade can be written as\(^{13}\)

\[ q_t = \theta (1 - h)q_{t-1} + \phi_{qm}(m_t - m_t^*) + \phi_{qm}^1(m_{t-1} - m_{t-1}^*), \quad (14) \]

\[ \phi_{qm} = - \gamma \phi_{sm}, \quad \phi_{qm}^1 = - \theta \phi_{qm}. \]

The implications for the adjustment of nominal interest rates \((i_t)\) can easily be worked out by noting that the real asset available to households implies that it is possible to construct a nominal asset for which the return is given by uncovered interest rate parity\(^{14}\) \((R_t = \log[1 + i_t])\)

\[ R_t - R_t^* = E_s s_{t+1} - s_t. \quad (15) \]

Using the equilibrium value for the exchange rate, it follows that the interest rate spread can be written

\[ R_t - R_t^* = \theta (1 - h)(R_{t-1} - R_{t-1}^*) + \phi_{im}(m_t - m_t^*) + \phi_{im}^1(m_{t-1} - m_{t-1}^*), \quad (16) \]

\[ \phi_{im} = \theta h (1 + \eta_{mp} - \theta \eta_{mp})^{-1} - \phi_{sm} < 0, \quad \phi_{im}^1 = - \phi_{sm}^1. \]

Relative production is inversely related to the terms of trade, \(y_t^b - y_t^f = - q_t\), hence, we focus in the following on the process for the nominal exchange rate, the terms of trade, and the interest rate spread, noting that the real effects for production and, thus, employment can easily be inferred from the behavior of the terms of trade. It is clear that imperfect information generates a richer adjustment path for both the nominal exchange rate and the terms of trade. In particular, there are nontrivial dynamics running beyond the length of nominal contracts (one period).

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\(^{13}\) Or \(q_t = \phi_{qm}(m_t - m_t^* - E_{t-1}(m_t^* - m_t^*));\) only unanticipated nominal shocks have real effects.

\(^{14}\) Follows by use of PPP. Constants are disregarded.
5. Dynamics under imperfect information

We now turn to a detailed analysis of the dynamic adjustment path under imperfect information. In particular, we lay out the economy’s response following an increase in relative money supply. Our strategy is to analyze how imperfect information alters the impulse-response functions to shocks, and whether the responses are consistent with the stylized facts in open-economy macroeconomics. In particular, can learning help explain puzzles like excess exchange-rate volatility (relative to fundamentals), persistent effects of nominal shocks on international relative prices, delayed nominal exchange-rate overshooting, and the fact that investors in countries with high interest rates at times also tend to reap the benefits of an appreciating currency?

The analytical results are supplemented by numerical illustrations based on the following parameter values: \( c = 0.67 \), \( \mu = 10 \), \( \sigma = 0.75 \), \( \beta = 9 \), \( \delta = 1/1.05 \), \( \theta = 0.9 \), and \( \eta = 1 \). In Fig. 3, we provide impulse-response functions to a 1% increase in Home (relative) money in period 1, which is either transitory or persistent under both full and imperfect information.

As a benchmark Fig. 3 provides the responses to the two types of shocks under full information. In both cases, there are only one-period dynamics in the terms of trade, and the real effects are increasing in the persistence of the shock. For the interest rate spread, the impact effect is decreasing. Since a fall in the interest rate is necessary to make agents hold the money over to the next period, it follows that the more persistent the shock, the less the interest rate is expected to rise in the future and, therefore, the less it has to fall on impact. The nominal exchange rate displays one-period dynamics for a transitory shock and multiperiod dynamics for a persistent shock.

Under imperfect information, the dynamic responses reflect the learning process when agents, over time, acquire more information, as illustrated in Fig. 2, displaying how expectations are updated. In particular, the terms of trade deteriorate on impact, but then improve given a transitory shock. This reflects that the monetary change disappears, but agents still expect the money shock to have increased, since it takes a while to accumulate sufficient information to infer that the shock is transitory. The subsequent terms-of-trade improvement is gradually worked out of the system. While persistent shocks also have persistent effects on nominal variables (exchange rate and interest rate spread) under full information, the dynamic adjustment process is different and includes persistent real effects under imperfect information.

15 A thorough quantitative investigation warrants the model to be augmented with capital, price, and wage staggering, as well as calibration of the informational parameters, \( \theta \) and \( h(q) \). What should be clear from our simple exercise, though, is the potential for persistence in international relative prices.

16 The productivity parameter \( \gamma \) is chosen to match the wage share of about 2/3, while \( \mu \) is chosen to imply a labor supply elasticity of 0.1. The next three coefficients correspond to those adopted in, e.g., Sutherland (1996). The last coefficient value is arbitrarily set at 1.
5.1. Excess sensitivity to noise and volatility

Under imperfect information, the impact effect of an expansion in (relative) Home money supply (regardless of the type of shock) is a depreciation of the nominal exchange rate, a terms-of-trade deterioration, and a fall in the interest rate spread, i.e.,

\[
\frac{\partial s_t}{\partial (m_t - m_t^*)} = \phi_{sm} > 0, \quad \frac{\partial q_t}{\partial (m_t - m_t^*)} = \phi_{qm} < 0, \quad \frac{\partial (R_t - R_t^*)}{\partial (m_t - m_t^*)} = \phi_{im} < 0.
\]

Consider first how markets react to news. The impact effect under imperfect information is the same whether the shock is transitory or persistent, since agents cannot immediately distinguish the two. It turns out that the impact response can be written as a weighted average of the impact effects of a transitory and a persistent shock under full information, respectively. More specifically, for the nominal exchange rate (similar reasoning follows straightforwardly for the other variables):

\[
\phi_{sm} = (1 - h)\phi_{su} + h\phi_{sz}.
\]  

(17)
The intuition for the weighting is that \((1-h)\) measures the weight attached to the shock being transitory, and \(h\) to it being persistent. Since

\[ \phi_{su} < \phi_{sm} < \phi_{sz} \]

it follows that there is an overreaction, or excess sensitivity, to transitory shocks or noise \((\phi_{su} < \phi_{sm})\), while there is an underreaction to persistent changes or fundamentals \((\phi_{sm} < \phi_{sz})\). The intuition is straightforward; part of a transitory shock is taken to be persistent and vice versa. It is an implication that the less informed the market (high \(\eta\), low \(h\)), the more cautious is the market adjustment \((\phi_{sm} \text{ lower})\). That is, if the information quality of observable signals is low, the response to persistent shocks tends to be muted, since a larger fraction of changes is taken to reflect noise.

Given the averaging in the response to shocks under imperfect information, it might be inferred that the average response or, more generally, volatility is unaffected by the information problem. This is not correct, as it can be shown\(^{17}\) that the volatility of the nominal exchange rate is lower under imperfect information. To see why, consider a case where \(\sigma_u^2 = 0\) and \(\sigma_z^2 > 0\); that is, there is no information problem. If \(\sigma_u^2\) increases (more noise), it will have the direct effect of increasing volatility. However, there is also an indirect effect since the information content of signals decreases, and this tends to reduce volatility. Under perfect information, only the direct effect will be present, and hence, volatility under perfect information is larger than under imperfect information case. It is an interesting corollary that an improvement in information does not reduce volatility (see also Bomfim, 2001). As \(\sigma_u^2 \rightarrow \infty\), there will again be de facto no information problem, and volatility under both full and imperfect information will be the same.

The comparison made here is between a case where agents can and cannot ex post distinguish between transitory and persistent shocks. In either case, transitory shocks are present. It is trivial to show that transitory shocks or noise create more volatility in exchange rates compared with a situation without noise. Although agents cannot observe the type of shock, we assume rational expectations, and under this assumption, while sensitivity to noise is increased, volatility is reduced. Devereux and Engel (2002) analyze a similar model with irrational expectations (among noise traders), and they demonstrate that this can generate exchange-rate volatility (see also Duarte and Stockman, 2001; Bacchetta and Wincoop, 2003).\(^{18}\)

### 5.2. Persistent terms-of-trade effects

A standing puzzle in open-economy macroeconomics is persistence in international relative prices (real exchange rate, terms of trade), which Rogoff (1996) coined the PPP puzzle.\(^{19}\) Although imperfect information under rational expectations does not

\[^{17}\text{We show this analytically in Andersen and Beier (2004). The unconditional exchange-rate volatility can be found by writing Eq. (13) in a Wold representation and applying standard techniques for finding the unconditional variance.} \]

\[^{18}\text{Devereux and Engel (2002) make other assumptions as well, e.g., local currency pricing and incomplete capital markets.} \]

\[^{19}\text{Our stripped-down model can be extended to cases with a non-constant real exchange rate by including local currency pricing or non-tradables.} \]
account for volatility in exchange rates (and thus in international relative prices), it
does give an intuitive explanation for the observed persistence; on impact, agents
cannot observe the type of shock, and it simply takes time to extract the relevant
information.

It is well known that intertemporal general equilibrium business-cycle models have
difficulties matching the observed strong persistence in the adjustment process. In the
new open-economy macroeconomics literature, there has been focus on persistence
generated by staggered contracts, and there is a growing consensus that the critical
determinants of persistence are marginal costs sensitivity to output and price sensitivity
to marginal costs (Lane, 2001). Researchers try to find conditions under which the
sensitivity is low in both cases. The present analysis suggests another source of
persistence, and interestingly, it works independently of properties of marginal costs and
prices over the business cycle.20

The equilibrium processes for the endogenous variables, given by Eqs. (13), (14)
and (16) and Fig. 3, reveal that informational problems result in a more complicated
dynamic adjustment path driven by the accumulation of information over time, i.e.,
learning. The interim dynamic process for the variables is seen to follow an
ARMA(1,2) process in the relative money supply, and the autoregressive part is the
same for all three variables. This indicates that persistence in the adjustment process
spreads to all variables. Furthermore, the autoregressive coefficient \( \theta[1-h] \) depends
only on the parameters characterizing the information structure \( (\theta, h) \), which brings
out that the information structure has a potentially important role for the dynamic
adjustment process, even if nominal contracts only have a duration of one period.
Specifically, we find that more persistence in the persistent part of the shocks (high \( h \))
and more information confusion (high \( \eta, \) low \( h \)) generate the strongest persistence in
the response of the variables following nominal shocks. A high \( \theta \) means that great
emphasis is put on changes being persistent and, therefore, on the last period’s ex-
pectations, and a low \( h \) means that little new information of relevance for predicting
future money supplies is obtained from the most recent observations, cf. Eq. (10).
While the moving average part of the process differs across variables, the auto-
regressive part works similar to all types of shocks, and this shows that there will be
persistence in the adjustment to shocks, irrespective of whether they are transitory or
persistent.

5.3. Delayed nominal exchange-rate overshooting

Eichenbaum and Evans (1995) present nominal exchange-rate impulse-response
functions to monetary shocks, which are hump shaped, or rather, display delayed

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20 This is basically a question of the type: Is the glass half empty or half full? Learning in our model can be
seen as driving a wedge between marginal costs and output or, in other words, making marginal costs less
dependent on output. When learning is slow (much noise), there are persistent effects on output without
(relative) wage adjustment. Imperfect information implicitly makes marginal cost less sensitive to output
changes.
overshooting. It turns out that persistent shocks can generate delayed overshooting (cf. Fig. 3), which requires (for an expansionary shock)

$$\frac{\partial s_t}{\partial e_t} < \frac{\partial s_{t+1}}{\partial e_t}, \quad \frac{\partial s_{t+j+1}}{\partial e_t} < \frac{\partial s_{t+j}}{\partial e_t}, \quad j \geq 1.$$  

A necessary condition for delayed overshooting is that $1 - 2\theta + \theta h < 0$, implying that a combination of a high degree of persistence in the shock (large $\theta$) and much noise (low information content of signals, i.e., small $h$) tend to create this phenomenon. This condition on $\theta$ and $h$ is sufficient for generating delayed overshooting in money expectations (see Section 3). Intuitively, this has to be fulfilled for delayed exchange-rate overshooting to occur, but it is not sufficient.

Fig. 4 illustrates the impulse-response functions for different values of the signal-to-noise ratio $\eta$ and, hence, the coefficient $h$. For large values of $\eta$, delayed overshooting disappears as learning becomes too slow, and similarly for small values as learning becomes too fast. Moreover, a larger $\eta$ implies smaller impact effects (vice versa for the interest rate spread) and more persistence as agents take longer to learn the exact nature of the shock.

It can be shown that unconditional delayed overshooting is not possible; that is, within the present rational expectations framework, delayed overshooting is shock conditional, given that it only arises for persistent shocks if there is sufficient confusion about whether it is actually a transitory shock. This implies that empirical studies, which find unconditional delayed overshooting, may suffer from a small sample problem in the sense that the particular sample has an overrepresentation of persistent shocks relative to

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21 See Andersen and Beier (2004). Gourinchas and Tornell (2002) find that unconditional delayed overshooting can arise if agents misperceive the process underlying the shocks.
the distribution underlying expectations formation (for a similar argument, see Faust et al., 2003, for model evaluations based on different information sets than those available to agents, or Lansing, 2000, for evaluations of policy rules not based on the information set available to policy makers). Alternatively, it is a rejection of the rational expectations hypothesis. Either way, the shock dependence in dynamic adjustment paths found here points to the problems of interpreting empirical analyses based on small samples. 22

5.4. Interest rate spread: excess returns

It is a well-established empirical fact that an appreciating (depreciating) exchange rate tends to be accompanied by a positive (negative) interest rate spread (Frankel and Rose, 1995), and this is sometimes interpreted as evidence against the joint hypothesis of uncovered interest rate parity and rational expectations (Eichenbaum and Evans, 1995). As seen in Fig. 3, the present framework explains that (ex post) excess returns are persistent due to the interplay between nominal rigidities and imperfect information (see also Gourinchas and Tornell, 2002). To put it differently, we find that expectational errors in a rational expectations setting can account for systematic interest rate spreads. 23 In particular, we find that conditional on a persistent shock (mistaken to be partially transitory), it is possible to observe a depreciating currency and a negative interest-rate spread. Interestingly, this arises under the same circumstance as delayed overshooting, both of which were found in, e.g., the Eichenbaum and Evans (1995) analysis, which supports the point made above on shock contingencies and small samples. It is also worth pointing out that the spread may be time varying, reflecting the type of shock hitting the economy and the learning problem.

Persistence in the interest rate spread does not leave any ex ante risk-free arbitrage possibilities, since the real rate of return is the same in both countries and the difference in nominal interest rates under uncovered interest rate parity reflects the changes in the nominal exchange rate expected by all market participants.

6. Staggered contracts and imperfect information

To clarify the role of learning as a propagation mechanism, the model presented above did not include other propagation mechanisms, but an interesting question is how various propagation mechanisms may interact. In this setting, it is obvious to include staggered or overlapping wage contracts, and we consider so-called Taylor contracts (see also Beier, 2001). 24 With an overlapping contract structure, wage setters take into account that their

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22 Note that, consistent with this view, other papers do not find delayed overshooting to be a robust fact (e.g., Kim and Roubini, 2000).

23 Recall that the “expectational errors” are rational errors in the sense that the expectations are rational, given the information set of the agents, and this information set contains the sum of the transitory and persistent components only.

24 Erceg and Levin (2003) assume staggered contracts, but their focus is not on the interaction of propagation mechanisms.
wage decision will interact both with existing and future contracts. This forward–
backward looking feature has the potential to interact with imperfect information since
each group of wage setters writes contracts at different points in time. The asynchronized
timing and the gradual accumulation of information over time imply that each group will
write contracts based on different information sets and, thus, different expectations about
future macrovariables.

To formalize this idea, unions were partitioned into two equally sized groups, where
unions in a given group determine nominal wages in an overlapping fashion; one half
setting the wage for two periods in even periods, and the other half in odd periods. The
unions are utilitarian and take into account that employment is demand determined given
the wage set. For the group of unions setting the wage at the end of period \( t \), applying
to periods \( t \) and \( t+1 \), it is easy to show that the expression of one-period contracts (consult
Eq. (8)) can be generalized to (see Andersen and Beier, 2003, 2004)

\[
   w_t = \frac{1}{1 + \varnothing} \left[ \eta_{wp} t^h + (1 - \eta_{wp}) (s_t + p^r_t) + \eta_{wc} c_t \right] + \frac{\delta}{1 + \varnothing} E_{t-1}
   \times \left[ \eta_{wp} t^{h+1} + (1 - \eta_{wp}) (s_{t+1} + p^{r+1}) + \eta_{wc} c_{t+1} \right],
\]

and similarly for the group setting the wage as of periods \( t-2 \), \( t \), \( t+2 \).

Fig. 5 shows the economy’s response to a persistent expansionary monetary shock both
with one-period contracts and staggered contracts under imperfect information. The two
propagation mechanisms interact to make the impulse-response qualitatively and
quantitatively (for the parameters chosen) different. The terms of trade follow a
(conditional) hump-shaped response, and the adjustment process displays more persistence.

The interaction is underlined by the fact that both imperfect information and staggering
are needed to generate the smooth hump shape in Fig. 5, although a hump shape does not
arise for all parameter values. A necessary condition is nominal delayed overshooting, but

Fig. 5. Response to a persistent shock with staggered contracts. The reaction of \( q \) to a one-unit increase in relative
money in period 1. Drawn for \( \eta=1 \), \( \theta=0.95 \), and \( \gamma=0.95 \).
to get a hump shape for real variables, the internal propagation mechanism generated by staggering alone must be sufficiently strong (for a detailed analysis, see Andersen and Beier, 2003). The intuition is that, with strong propagation generated by staggering, wages (and thus prices) adjust slowly, and this is needed for the subsequent further depreciation of the nominal exchange rate to be transmitted into increased real effects. This line of reasoning also stresses that neither imperfect information nor a strong internal propagation mechanism from staggering viewed in isolation can generate the hump shape. Learning is needed to generate nominal delayed overshooting, and a strong internal propagation mechanism is needed to have sufficient sluggishness in the adjustment of nominal wages (and prices).

7. Concluding remarks

The problem of distinguishing between transitory and persistent monetary changes has been considered in an explicit intertemporal general equilibrium model focusing on the adjustment of exchange and interest rates. A major finding is that imperfect information can have significant implications for the dynamic adjustment path to shocks, and that the learning process involved in disentangling the nature of shocks adds persistence to the adjustment process. Our model has been stripped down to allow for clear-cut analytical and intuitive results, and this opens for interesting further work.

We have modelled the information problem in a very stylized way, including a time invariant information structure. In reality, the relative variances of shocks vary over time, as do the market’s perceptions of these. All business cycles are different (IMF, 2002), and even ex post, it can be difficult to obtain consensus regarding the causes of particular cycles. In this paper, we have explored a minimal deviation from perfect information, but maintained the hypothesis of rational expectations, and shown that this has significant effects for the dynamic adjustment path. Ultimately, it is an empirical question how shocks are distributed and how agents perceive the distribution, as well as how agents actually update expectations (Carroll, 2003).

Additionally, we demonstrated that learning interacted with staggering in a nontrivial way. The two interact to strengthen persistence in the adjustment process and to enrich the dynamics, e.g., by causing a persistent monetary shock to induce a hump-shaped response. This indicates that the interplay of transmission mechanisms can possibly strengthen the internal propagation. Another candidate in this respect is capital formation. Furthermore, other shocks might be subject to transitory-persistent confusion (technology shocks), and it would be interesting to quantify the effects of imperfect information in a fully fledged business-cycle model.

References


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