Monetary Policy and Investment Decisions –

A Stylized Treatment of the Uncertainty Trap

by

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Summary

We use the option value of waiting approach to describe why a change of central bank interest rates may not be effective in impacting economic activity when revenue uncertainty is high. Against the background of sunk investment costs, our model provides important implications for monetary policy: (1) under a high uncertainty regime reducing interest rates might be an ineffective policy; (2) cutting rates may kill the option value of waiting and reduce the effectiveness of policy in future periods; and (3) a central bank that operates frequent interest rate changes might induce additional uncertainty and, hence, hamper firms’ investment decisions.

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Pressure on the European Central Bank to cut interest rates has been growing as short-term growth prospects for the euro area deteriorate. The arguments for even lower rates seem to be compelling. Inflation is now wandering around the ECB’s upper ceiling. Low growth is expected to cause downward pressure on the price level and ongoing uncertainty is assumed to dampen economic activity even further. However, a closer look at the economic implication of uncertainty suggests that monetary policy easing might be in fact a poor strategy under this scenario. This is because in times of uncertainty, as we will show, the effectiveness of monetary policy decreases greatly (Greenspan, 2004, Jenkins, Longworth, 2002).

Where does uncertainty typically come from? One conjecture would be to trace uncertainty of revenues back to the events of September 11th, 2001, and ensuing war against terrorism which have shaken the hitherto prevailing geopolitical order. In addition, high uncertainty could also stem from certain macro-economic disequilibria such as, for instance, the US current account situation, the strong increase in corporate debt, corporate malfeasance etc. Finally, piecemeal reforms are made responsible for an environment uncertain for investors.

To deal with the influence of uncertainty on economic decisions, economists have developed the concept of the "option value of waiting". This formalises a common-sense rule: if a decision involves some sunk costs, or any other element of irreversibility, it makes sense to wait until the uncertainty has been resolved. The convenience to postpone investment decisions is particularly strong when the uncertainty is likely to be resolved in the near future. There exists an extensive literature on the role, the conduct and the efficacy of monetary policy which we cannot completely review here. There are several papers which document
possible scenarios of policy effectiveness. But our attempt to link the theory of the “option value of waiting” to monetary policy is a novel contribution according to our knowledge. In contrast to other contributions in the field, our approach assumes that revenue uncertainty instead of interest rate uncertainty creates the “option value of waiting”. In other words, we investigate which level of the risk-free policy rate triggers investment if investors have to take real option values into account. For reasons of simplicity, we model interest rate expectations in a deterministic fashion but let a stochastic process determine future revenues.

One can easily imagine investors assessing various investment projects. Some would be slightly profitable under the prevailing degree of uncertainty, but they would be even more profitable if uncertainty were favourably resolved, and would cause a loss if not. In such a situation, investors would lose little (in terms of forgone profits) if they postpone investment decisions: Once the uncertainty had been resolved, it would still have the option to proceed if that was to its advantage. An analogous argument applies to the consumers which might delay their decisions to buy a durable consumer good in times of uncertainty. According to the simple models, uncertainty which cannot be hedged raises the variability of revenues and induces the investors to apply a higher discount rate on (expected) future revenues. Dixit (1989) introduces an additional motive why uncertainty should hamper investment: if investments bear an irreversible sunk cost character, there is an incentive to wait until the uncertainty has resolved; this is the "option value of waiting".

A brief case study might be helpful to convey the spirit of the argument. For this purpose, we have a brief look at the ECB. Some time ago, it was widely believed that a war in Iraq would not have any appreciable direct consequences for the European economy due to its low degree of openness towards the Gulf region. However, the indirect effects could be substantial if the

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war lasted longer than expected, or if it led to a disruption of oil supplies and wider regional instability and geo-political frictions. Such an outcome could not be ruled out. This uncertainty was likely to be resolved soon, perhaps not in a matter of weeks, as the US administration maintained at that time, but certainly in a matter of months. However, while it remained, one expected demand – especially investment demand – to remain quite weak in the near future.

Shouldn’t the ECB have tried to stimulate demand with an interest rate cut under this scenario? A first counter-argument would have been that the concept of the "option value of waiting" applied to the ECB just as much as it applies to everyone else. At that time, it was not clear whether a war might be averted, or it might be short and have little effect on oil prices. Hence, if the ECB would have cut policy rates, it would have risked having to reverse its decision almost immediately. The ECB should have cut its rates only if it was convinced that such a cut would make sense even if the uncertainty would be favourably resolved.

Let us add that we have no judgment to offer at all on whether the ECB had the right rate level or levels during 2001-2005. There is nothing in our paper that addresses this question. Since the same logic applies to (less benefits of) policy rate increases under an inflationary scenario, we are certainly not arguing in this paper for rate cuts or against them as a policy matter. However, in the context of the post-September 2001 depression which we use for illustration purposes, a cut as an insurance against a bad outcome does not make sense, since

(1) cutting interest rates is not effective if uncertainty is large,

(2) a central bank itself disposes of an option value of waiting with interest rate cuts. If, for instance the ECB cuts today, it kills this option to cut in the future (although this option might be very valuable in times of high uncertainty even if the interest rates are not zero).

(3) frequent interest rate changes by a central bank induce additional uncertainty which tends to aggravate the weakness of investment and consumer goods demand.
The models of decision-making under uncertainty also have further important implications for monetary policy. All economic decisions involve some transaction costs – whether they are about investment, or about hiring and firing. These last are especially important in Europe. This implies that businesses facing only a small change in prices may not respond (immediately). There is always a price range within which it does not pay to change course. The size of this range grows as uncertainty increases.

The remainder of the paper proceeds as follows. Section 2 introduces the baseline model. In section 3, we consider how the policy rate decisions of the collective agent, the monetary policy maker, may affect the investment entry and exit decisions of the principals without the option of waiting (certainty equivalent). In section 4, the model under one-off uncertainty and the possibility of waiting is analysed. In section 5, we give some illustrations of the impact of uncertainty on the effectiveness of monetary policy by means of numerical simulations. Section 6 finally concludes.

2. The baseline model

In the following, we focus on the micro level and disregard aggregation issues. Investments are typically characterised by large set-up costs which are often highly irreversible. These set-up costs consist of investment expenditures which cannot be resold (e.g., firm-specific investment) and the hiring and training costs for needed staff. In order to make an investment profitable, the revenues stemming from this investment project have to cover these costs.

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2 For an extensive formal treatment of the latter see Belke, Göcke (2001, 2001a).
The gross profit of an investment project, without consideration of these instalment costs, is:

\[ R_{a,t} = e_t \]  
\[ \text{if active} \],  
\[ R_{p,t} = 0 \]  
\[ \text{if passive} \].

with:  
\[ t \] : time index, and  
\[ e_t \] : present gross revenues if the investment project is executed  
(variable costs subtracted, i.e. the contribution margin)

It is assumed that the sunk investment/hiring costs \( H \) (with \( H > 0 \)) must be spent at the moment the investment is executed.\(^4\) It has to be noted that the parameter \( H \) can also be interpreted as anticipated scrapping / firing costs. In case of a one-time non-utilisation, we assume immediate depreciation. If the firm is inactive for only one period, the investment / staff must be completely re-set up and the hiring / investment costs must be paid anew. Since switching the state of activity leads to a complete depreciation of hiring costs, \( H \) have ex post to be regarded as sunk costs (Dixit, Pindyck, 1994, p 8; Bentolila, Bertola, 1990; Dornbusch, 1987, pp 7 ff.).\(^5\) Specific investments in new employees close to the production process may partly be irreversible because of market regulation and institutional arrangements.\(^6\)

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\(^3\) For a related trade model see Baldwin, Krugman (1989), p 638, and Göcke (1994). In the current paper we analyse only a single firm. However, heterogeneity effects are especially important for aggregation; see Belke, Göcke (2001, 2001a). For empirical evidence of heterogeneity for Italian manufacturing firms see Guiso, Parigi (1997).

\(^4\) Investment in employment that takes 'time to build' (i.e. implementing a lead) magnifies effects of uncertainty. See Pindyck (1988), p 973, Dixit, Pindyck (1994), pp 46 ff.

\(^5\) We abstract from additional uncertainty about \( H \).

\(^6\) However, one has to distinguish between specific investment as analysed in this paper and general investment, which enables the firm to cope with different situations in the future. Thus the latter type is often claimed to be positively correlated with uncertainty about revenues. See e.g. Gros (1987).
The decision as to whether or not the firm should invest/hire is reached by a comparison of the expected present values of the investments with or without being active in the decision period $t$. In addition to the state of activity in the preceding period, the present revenues and expenditures as well as the influence of the current activity decision on the future returns must be taken into account.

Let us start with some important definitions relevant for an assessment of the profitability of an investment. The variable $i$ denotes the *short-term* interest rate. It is valid until the next period and, hence, represents the current "control variable" of monetary policy. The *expected* level of *long-term* interest rate $i_1$ is assumed to be determined as follows:

$$i_1 = r + \alpha \cdot (i - r)$$  \hspace{1cm} \text{(expected long-term interest rate $i_1$).}$$

The corresponding discount factors are defined as:

$$\delta_0 \equiv \frac{1}{1 + i} \quad \text{and} \quad \delta_1 \equiv \frac{1}{1 + i_1} \quad \text{(with: $i, i_1 > 0 \Leftrightarrow \delta_0, \delta_1 < 1$).}$$

with:

- $\delta_0$ as the discount factor until the next period (based on $i$),
- $i_1$ as the *expected* long-term interest rate,
- $\delta_1$ as the corresponding long-term discount factor based on $i_1$, and
- $r$ as the "base value" for the expected long-term interest rate.

The coefficient $\alpha$ represents the "expectation pass-through parameter" from the short-term interest rate $i$ to the expectation with respect to the long-term interest rate $i_1$. The variable $r$ could, for instance, be defined as the real marginal productivity of capital plus a (credible) inflation target of, e.g., 2 percent.\footnote{Hence, the credibility of the inflation target is important in our context. Expansionary monetary policy with lower short-term rates might even undermine this credibility and imply higher expectations of $i_1$. Note that we do not claim to explain the term structure of interest rates in this paper in order to present our simple argument} Within our model of the option value of waiting, we focus
on uncertainty with respect to the general revenue performance e. However, uncertainty with respect to future interest rates is not explicitly included in our model. An interesting special case analysed by Belke, Göcke (1999) emerges for \( \alpha = 1 : \ i = i_1 \) and \( \delta_0 = \delta_1 \).

3. Decision without the option of waiting (certainty equivalent)

Let us now develop the model without the option value of waiting and regard the expected values as equivalents to certainty (i.e. we assume risk-neutrality). Motivated by the current scenario of low inflation in the euro area and in the US accompanied by high unemployment at least in the euro area we limit ourselves to the analysis of only one of the two logically possible status quo situations, namely the case of an firm being "passive in the preceding period". Hence, we illustrate the main aim of an expansionary monetary policy, i.e. of creating a stimulus for investment and employment by lowering financing costs.

3.1 Scenario "passive (unemployed) in the preceding period"

A previously non-active firm has two possibilities to act. Either it remains passive or it starts the investment project in period \( t \). If it stays passive, it earns neither current nor future profits (i.e. no present value of future revenues has to be calculated).

However, a firm which enters / invests will gain the period \( t \) gross revenue \( e_t \). To simplify matters, we assume an infinite horizon of investors. Since, we assume that an investor expects the same contribution margin for the whole infinite future \( (e_{t+i} = e_t = e) \), the present value of annuity due of future gross revenues under activity from period \( t+1 \) to the infinite future has to be calculated. In period \( t+1 \), the firm receives, if it is running the investment project, an expected present value of annuity due \( V_{a,t+1} \):

\[
V_{a,t+1} = \frac{e}{1 - \delta_1}.
\]
(1 − δ₁) = δ₁ · i₁ is the rate of interest costs in case of the annuity due (i.e., we apply a simple formula for present value of annuity due). Remember that e (without index t) is the certainty equivalent gross revenue without consideration of the interest/financing costs of sunk costs, i.e. the contribution margin per period before financing the sunk costs.

If the firm invests, it has to pay for the sunk instalment costs H to be able to earn current and future profits (present value in current period t of annuity due of future revenues under activity from period t+1 on, applying the short-term interest rate i in period t: δ₀ · Vₐ,t+1) using equation (3):

\[
Vₐ,t = -H + e + \delta₀ · Vₐ,t+1 = -H + e + \frac{\delta₀ · e}{1 - δ₁} = -H + e + \frac{e}{1 + i} \cdot \frac{1}{1 - \frac{1}{1 + i₁}}.
\]

In order to calculate the entry-trigger revenue under certainty, we have to proceed as follows.\(^8\) The firm is indifferent between remaining passive or entering if the present value of continuing non-activity (i.e., 0) equals the present value \(Vₐ,t\) of an instantaneous investment ("entry"):

\[
0 = Vₐ,t \iff 0 = -H + e + \frac{\delta₀ · e}{1 - δ₁}.
\] (indifference).

Application of the long term interest expectation (2) on the indifference condition yields:

\[
0 = -H + e + \frac{e}{(1 + i) \cdot \left(1 + r + α(i - r)\right)}.
\]

Solving (6) or (6') for \(e\) results in the contribution margin \(e^e_{\text{entry}}\) which triggers an entry:

\(^8\) The calculation is the same for a case with certainty and for a situation with uncertainty and risk neutrality, but without the option to wait. In this case, the corresponding present value has to be interpreted as expected values.
The firm enters if the contribution margin $e$ exceeds $e_{\text{entry}}$. The entry decision becomes favourable if $e$ covers at least the interest costs on sunk investment costs. Interest costs of entry become relevant as they have to be interpreted as an opportunity gain of staying passive. Due to the sunk hiring costs, the necessary revenue after subtracting variable costs is larger than null. So the required surplus over variable costs, i.e. the contribution margin $e$, will be the larger the higher the sunk costs are. Entry will happen, as soon as $e$ covers the interest costs (i.e. approximately interest rate $i_1$ times $H$).

We now ask how the central bank can impact the profitability calculations of investors. Hence, we have to calculate the short-term interest rate $i$, which makes investment just worthwhile. If the indifference condition results according to eq. (6) is solved for the short-term interest rate $i$, the interest rate which triggers investment can be derived.

$$i_{\text{entry}}^c = \frac{2e + (e/i_1) - H}{H - e} \quad (\text{entry if } i < i_{\text{entry}}^c).$$

However, in our model we have to differentiate between two effects of the short-term interest rate $i$: (1) a short-term interest payment effect during the current period $t$ (i.e. between the start of period $t$ and the start of period $t+1$) and (2) an impact on expectations of the long-term interest rate $i_1$ according to parameter $\alpha$ and, by this, on the present value of annuity due. The same is valid for a monetary authority which uses the interest rate $i$ as a control variable. Thus, if – under application long term interest expectation (2) – the indifference condition according to eq. (6') is solved for the short-term interest rate $i$, we observe:

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9 The long-term interest rate is placed in the denominator of the formula of the present value of the annuity due and, thus, determines the realised present value (if the long-term interest rate moves to zero, the present value converges towards infinity).
At this stage of analysis, equation (9) might not be accessible to a straightforward economic interpretation. Hence, we leave this task for section 3.2, where we consider two special cases of $\alpha=0$ and $\alpha=1$ which admits a more simple interpretation. In this section, we illustrate the main aim of an expansionary monetary policy, i.e., rendering a stimulus for investment and employment by lowering financing costs. However, our analysis is not complete in all respects. For example we only regard the financing costs of the sunk investment costs. We do not explicitly consider the need for financing also those fixed capital costs of the whole investment project which are not sunk. Implicitly, this could be taken into account in our model by an increase of $e$ (the residual revenue before financing sunk costs). Instead, we feel justified to assume in a simplifying fashion that all investment costs are sunk due to, e.g., irreversibility. The reason is that investments are to a large extent firm-specific and thus have to be considered as sunk from an ex post–perspective.10

\[ \begin{align*}
\text{(9)} \quad \bar{i}^c_{\text{entry}} &= \frac{1}{2} \left( 2 e \alpha + e r \alpha - e \alpha r - H \alpha - H r + H \alpha r + (H^2 \alpha^2 - 6 e \alpha^2 H r + 6 e \alpha H r + 4 e r^2 H \alpha - 2 e \alpha^3 r H - 2 e r^2 H + 2 H^2 \alpha^2 r - 2 H^2 \alpha r - 2 H^2 r^2 \alpha + H^2 \alpha^2 r^2 - 4 e \alpha^3 H + H^2 r^2 - 4 e^2 \alpha + e^2 r^2 + 4 e^2 \alpha^2 + 4 H \alpha e - 4 e^2 \alpha r + e^2 \alpha^2 r^2 + 4 e^2 \alpha^2 r - 2 e^2 r^2 \alpha \right)^{1/2} \left( H \alpha - e \alpha \right)
\end{align*} \]

3.2 Special cases: No and/or complete pass-through of the short-term interest rate on the expected long-term interest rate

We have to consider the following special cases which are highly relevant in our monetary policy effectiveness context:

(A) The first special case consists of the assumption of $\alpha=1$. This parameter restriction implies static expectations, i.e. a complete identity of the short-term interest rate and the

\[ \text{10} \quad \text{The model was augmented by us to take account of this effect. The results become a little weaker. However, the pattern of the results stays the same.} \]
expected long-term interest rate. This exactly corresponds to scenario investigated by Belke, Göcke (1999):

(10) for $\alpha = 1$ ($\Rightarrow i_1 = i$): 
$$ic_{\alpha=1} = \frac{e}{H-e} > 0$$
with $H > e$.

According to eq. (10), the interest rate has to be smaller than the "internal rate of return" of the investment project. The "internal rate of return" can be defined as the gross revenue $e$ divided by irreversible investment costs $H$ minus the instantaneous revenues from the first period which instantaneously partly cover the investment costs.

(B) The second scenario is $\alpha = 0$, i.e. the current performance of short-term interest rates is meaningless for the expectation of long-term interest rates. In other words, market participants expect a "mean reversion" towards the base value $r$ after the central bank has "shocked" the money market rate (at least on average in the long-run, like for instance in an error-correction model).\(^\text{11}\) Corresponding to eq. (8) the expression of the interest trigger now melts down to:

(11) for $\alpha = 0$ ($\Rightarrow i_1 = r$): 
$$ic_{\alpha=0} = \frac{2e + (e/r) - H}{H-e} = ic_{\alpha=1} + \frac{e + (e/r) - H}{H-e}$$
with $H > e$: $ic_{\alpha=0} < ic_{\alpha=1}$ if $[e + (e/r)] > H$.

\(^{11}\) Note again that we do not have modeled a stochastic process driving the interest rate but only the stochastic process driving uncertainty about the revenues $e$. With respect to the interest rate we limited ourselves to descriptively model the transmission (mean reversion) from short-term to long-term interest rates by means of the parameter $\alpha$. In section 2, we already mentioned that we do not claim to explain the term structure of interest rates in our simple model.
Uncertainty about future revenues typically generates an option value of waiting, and therefore introduces a bias in favour of a "wait-and-see"-strategy. Since the firm's investment / employment decision can be understood as irreversible, we follow a real option approach. The firm's investment / employment opportunity corresponds to a call option that gives the firm the right to invest and employ, sunk investment / hiring costs being the exercise price of the option, and to obtain a 'project'. The option itself is valuable, and exercising the investment "kills" the option.

We analyse the effects of an expected future stochastic one-time shock. However, assuming a risk-neutral firm, we abstract from risk-aversion. Focusing on the impacts of uncertainty on the effectiveness of monetary policy, we further develop an idea originally proposed by Dornbusch (1987), pp 8 f., Dixit (1989), p 624, fn. 3, Bentolila, Bertola (1990), and Pindyck (1991), p 1111. Option price effects are modelled in a technically sophisticated way in these references. However, based on the model by Belke, Göcke (1999), we model uncertainty effects as simple as possible, since the basic pattern of the effects of uncertainty are left unchanged.\textsuperscript{12}

We assume a non-recurring single stochastic change in the gross revenues, which can be either positive (+\(\varepsilon\)) or negative (−\(\varepsilon\)) [with \(\varepsilon\geq0\), mean preserving spread]. This kind of binomial stochastic process was introduced into the theory of option pricing by Cox, Ross and Rubinstein (1979). Both realisations of the change \(\varepsilon\) are presumed to have the same probability of \(\frac{1}{2}\) : \(e_{t+1}=e_t\pm\varepsilon\) and \(E_t(e_{t+1})=e_t\). From period \(t+1\) on, the potential investor

\textsuperscript{12} Guender (2003) appends an instrument rule to a simple stochastic macroeconomic model and examines the optimal setting of the policy parameter under inflation targeting. He shows that that the size of the policy parameter depends on the sources of uncertainty, the policymaker’s preferences, and both parameters of the model.
will be able to decide under certainty again. The stochastic change between t and t+1 leads to an increase in the gross revenue trigger in our one-shot model. If the latter is passed, investment becomes worthwhile. Moreover and even more important in our context, the interest rate i which makes an investment worthwhile becomes lower than in the base scenario without option value effects.

Under certainty, the relevant alternative strategies are to invest immediately or not. Under uncertainty and the feasibility to delay an investment, a third alternative has to be taken into account: the option to wait and to make the respective investment decision in the future. The option to invest in the future is valuable because the future value of the 'asset' obtained by the investment is uncertain. If its value will decrease, the firm will not need to invest and will only lose what it will have spent to keep the investment opportunity. This limits the risk downwards and with this generates the inherent value of the option.

A previously inactive firm has to decide whether to invest now or to stay passive, including the option to invest later. The firm anticipates the possibility of internalising future gains by an investment in t+1 if the future revenue turns out to be favourable (+ε). Besides, the firm foresees that it can avoid future losses if the revenue change will be negative (–ε) by staying passive. Waiting and staying inactive implies zero profits in t. Conditional on a, the firm will use its option to invest in t+1 causing discounted sunk investment / hiring costs $\delta_0 \cdot H$, and gaining an annuity value of $\delta_0 \cdot (e + \epsilon)/(1 - \delta_1)$. Thus, the present value in the case of a (+ε)-realisation is:

$$V_{t}^{wait \ if \ +\epsilon} = - \delta_0 \cdot H + \frac{\delta_0 \cdot (e + \epsilon)}{1 - \delta_1}.$$  

For a (–ε)-realisation the firm will remain passive with a present value of inactivity being $V_{t}^{wait \ if \ -\epsilon} = 0$. Consequently, the expected present value of the wait-and-see strategy is given by
The expected present value of the wait-and-see strategy in period $t$ is defined as the probability-weighted average of the present values of both $\pm \varepsilon$-realisations:

$$E_t(V_{\text{wait}}^t) = \frac{1}{2} \left( -\delta_0 \cdot H + \frac{\delta_0 \cdot (e_t + \varepsilon)}{1 - \delta_1} \right).$$

The expected present value of an immediate investment (without re-exit) is $E_t(V_{\text{entry}}^t)$:

$$E_t(V_{\text{entry}}^t) = -H + e_t + \frac{\delta_0 \cdot e_t}{1 - \delta_1} \quad \text{since} \quad E_t(e_{t+1}) = e_t.$$

The option value of having the flexibility to make the investment decision in the next period rather than to invest either now or never, can easily be calculated as the difference between the two expected net present values: $OV(e_t, \varepsilon) = E_t(V_{\text{wait}}^t) - E_t(V_{\text{entry}}^t)$, with $\partial OV/\partial e_t < 0$, $\partial OV/\partial \varepsilon > 0$. An increase in uncertainty enlarges the value of the option to invest later. The reason is that it enlarges the potential payoff of the option, leaving the downside payoff unchanged, since the firm will not exercise the option if the revenue falls. The firm is indifferent between investment in $t$ and wait-and-see if

$$E_t(V_{\text{wait}}^t) = E_t(V_{\text{entry}}^t) \quad \text{i.e. indifference if} \quad OV = 0.$$

The revenue entry trigger under uncertainty follows as:

$$e_{\text{entry}}^u = \frac{2H - 2\delta_1 \cdot H - \delta_0 \cdot H + \delta_0 \cdot \delta_1 \cdot H + \delta_0 \cdot \varepsilon}{2 + \delta_0 - 2\delta_1} \quad \text{(investment if} \quad e_t > e_{\text{entry}}^u).$$

From this equation, it becomes obvious that uncertainty increases the probability that a firm stays passive; since $\varepsilon$ enter the expression in a positive way.

However, some words (and calculations) of caution seem to be justified at this stage of analysis. Our assumptions with respect to entry and exit for $(+\varepsilon)$ respectively $(-\varepsilon)$ are of course only valid, if investors really enter the market in period $t+1$, if $(+\varepsilon)$ is realized and if
there is really no entry in t+1 in cases of realisation of (–ε). A firm's entry in period t+1 happens only if the trigger under certainty $e_{\text{entry}}^{c,t+1}$ is passed. Since the calculation of the option has to be based on assumptions which are dynamically consistent, an additional condition for the size of the shock ε [see eq. (18) below] becomes necessary. Mathematically, the necessary condition for this can be calculated as follows (assumption):

$$0 = -H + e_{\text{entry}}^{c,t+1} + \delta_1 \cdot \frac{e_{\text{entry}}^{c,t+1}}{1 - \delta_1} \iff e_{\text{entry}}^{c,t+1} = (1 - \delta_1) \cdot H. \quad (17)$$

$$\varepsilon > \varepsilon_{\text{min}} \quad \text{with} \quad \varepsilon_{\text{min}} = \frac{(1 - \delta_1) \cdot (\delta_0 - \delta_1) \cdot H}{1 - \delta_1 + \delta_0}. \quad (18)$$

This result implies that normally, ε has to be a little bit larger than zero (in fact by not too much, since the difference $(\delta_1 - \delta_0)$ in the numerator is not too large). In the case of the large uncertainty analysed in this paper (see, for instance, the introduction for September 11th and the Iraq conflict) this assumption should be valid anyway. Approximately this condition implies that ε has to be larger than zero. In the special case $\alpha = 1$ (i.e. $\delta_0 = \delta_1$) the following relation holds exactly:

$$\varepsilon_{\text{min}} = 0 \quad \iff \quad \varepsilon > 0. \quad (19)$$

In this case ($\alpha = 1$) the profit trigger $e_{\text{entry}}^{u}$ under uncertainty and the option of waiting converges to the trigger calculated for the case without the option of waiting $e_{\text{entry}}^{c}$ ("c-trigger") if the size of the shock ε converges to zero.\footnote{If $\alpha$ is not equal to one, this is valid only approximately. This is due to the following. If we calculate the option value, in the formula of the present value in case of activity (realisation of $+\varepsilon$) only $\delta_1$ is used for discounting, whereas in the case of immediate entry under certainty in the first period t the discount factor $\delta_0$ (i.e., the short-term interest rate $i$) has to be applied. If $\alpha$ is smaller than one, there is a difference between $i$ and $i_1$. However, these considerations are not decisive, if $\varepsilon$ reaches the minimum level calculated before.} Insofar as assumption (18) of a
minimum realisation of $\varepsilon$ is valid, the following relation holds (which can be shown mathematically, proof is available on request):

(20) If $\varepsilon > \varepsilon_{\text{min}}$ then $e_{\text{entry}}^u > e_{\text{entry}}^c$.

Hence, uncertainty leads to a higher revenue entry trigger which by itself causes a more resistant investment behaviour which the central bank has to take into account when measuring out its intended interest rate changes. However, the final aim of our calculations is to identify the interest rate entry trigger which is of central importance for the central banks as a benchmark for interest rate setting in times of uncertainty.

The short-term interest rate threshold which makes investment worthwhile (and thus triggers off investment activity) under revenue uncertainty can from the indifference condition given in eq. (15) be calculated as follows:

(21) $i_{\text{entry}}^u = \frac{1}{2} (-H \alpha - 2 e \alpha r + 2 H \alpha r - 2 H r + 3 e \alpha - \alpha \varepsilon + 2 e r + (4 H^2 \alpha r^2 + 4 \varepsilon^2 - 8 \varepsilon^2 \alpha) \\
+ 12 \varepsilon^2 \alpha r - 12 \varepsilon^2 \alpha - 4 H^2 \alpha r + H^2 \alpha^2 - 4 H \alpha r \varepsilon - 4 e \alpha r \varepsilon + 4 e \alpha r \varepsilon + 4 H \alpha^2 r \varepsilon - 8 e r^2 H + 2 H \alpha^2 \varepsilon - 8 H \alpha e + \alpha^2 \varepsilon^2 - 8 \varepsilon^2 \alpha - 16 e \alpha^2 r H r + 16 e \alpha \varepsilon H r + 16 e r^2 H \alpha - 8 e \alpha^2 r H + 4 e \alpha^2 \varepsilon^2 + 6 e \alpha^2 \varepsilon + 4 H^2 \alpha^2 r^2 + 9 e^2 \alpha^2 - 6 e \alpha^2 H - 8 H^2 r^2 \alpha + 8 H \alpha e + 8 e \alpha e + 4 H^2 \alpha^2 r) \quad (2 H \alpha - 2 e \alpha)^{1/2}) / (2 H \alpha - 2 e \alpha)$

The calculation of the interest rate entry trigger is more simple in the special cases of the parameter restrictions $\alpha = 1$ and $\alpha = 0$. For $\alpha = 1$ (i.e. if the long term interest rate expectation $i_1$ is fully determined by the short term interest rate $i$) we obtain:

(22) for $\alpha = 1$ (\Rightarrow $i_1 = i$): $i_{\text{entry}}^{u, \alpha = 1} = \frac{3 e - H - \varepsilon + \sqrt{(H + e + \varepsilon)^2 - 8 H \cdot e}}{4 \cdot (H - e)}$.

Like in the case of the revenue trigger for $\alpha = 1$, this result converges towards the result under certainty if $\varepsilon$ moves towards zero. In order to yield positive real results for the trigger interest rates, the following conditions must simultaneously hold:
(23) for $\alpha = 1$:

1. $H > e$, i.e. the sunk investment $H$ must be "large" relative to annual profit $e$,

2. $(H+e+\varepsilon)^2 > 8 \cdot H \cdot e$, i.e. for real roots uncertainty $\varepsilon$ must be "small enough",

3. $\sqrt{(H+e+\varepsilon)^2 - 8 \cdot H \cdot e} > 3e - H - \varepsilon$.

The reaction of the interest rate trigger on changes of uncertainty is:

$$\frac{\partial i_{u,\alpha=1}^{\text{entry}}}{\partial \varepsilon} = \frac{e - 3H + \varepsilon - \sqrt{(H+e+\varepsilon)^2 - 8 \cdot H \cdot e}}{4(H-e) \cdot \sqrt{(H+e+\varepsilon)^2 - 8 \cdot H \cdot e}}.$$  

This impact of uncertainty on the interest trigger is negative if condition 4 holds

(25) for $\alpha = 1$:

$$\sqrt{(H+e+\varepsilon)^2 - 8 \cdot H \cdot e} > e - 3H + \varepsilon.$$  

Starting from a very small level of uncertainty (from $\varepsilon \to 0$) conditions 3 and 4 converge to:

(26) for $\alpha = 1$ and $\varepsilon \to 0$: 3. $H > e$ and 4. $H > 0$.

Summarizing, if the value of sunk investment costs $H$ is "large" compared to the annual gross profit $e$ we can expect the investment trigger interest rate $i_{u,\alpha=1}^{\text{entry}}$ to decrease if we change from a situation with low uncertainty $\varepsilon$ to a more uncertain situation.

If the long term interest rate is expected to be independent of the current short term interest rate level (i.e. for $\alpha = 0$) the trigger follows as:

(27) for $\alpha = 0$ ($\Rightarrow i_1 = r$): 

$$i_{u,\alpha=0}^{\text{entry}} = \frac{r \cdot (3e - H - \varepsilon) + e - \varepsilon}{2r \cdot (H-e)}.$$  

The reaction of the interest rate trigger on changes of uncertainty is:

$$\frac{\partial i_{u,\alpha=0}^{\text{entry}}}{\partial \varepsilon} = -\frac{1-r}{-2r \cdot (H-e)}.$$
Again, we have a negative impact of uncertainty on the interest rate trigger (if $H>e$ and $0<r<1$). Again, uncertainty leads not only to a higher revenue entry trigger $e_{\text{entry}}^{u}$ but also to a **lower** interest rate entry trigger. In this sense, in a situation with uncertainty monetary policy becomes less effective, as the probability that the investment triggers of many firms are passed by a reduction of short term interest rates is lowered. Hence, our model describes a kind of “uncertainty trap” (see also Aoki, Hoshikawa, 2003).

Unfortunately, we apply a very simple formal setting (with a simple discrete model and only one stochastic shock) in order to illustrate our intuition. Starting for instance with a model with two successive stochastic revenue changes [as conducted by Belke, Göcke (1999) for successive exchange rate changes], our analysis could be extended by adding more periods of uncertainty which induces the calculation of additional option value effects. This implies a repeated backward induction along the lines taken above, but this would be a hard way to walk. Another possibility is the transition to continuous time models with permanent uncertainty. However, we dispense with the use of the latter, since it implies the application of advanced mathematical tools (e.g. Ito's lemma) without leading to significant additional insights concerning our research purposes.

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14 In eq. (27) this will result, if total amount of the sunk investment is higher than one-period profits ($H>e$). The condition should be valid for a marginal investment project.

15 For an application of continuous time models in a related context see Darby et. al. (1997), Calcagnini, Saltari (2000), Dixit (1989), pp 624 ff., Dixit, Pindyck (1994), pp 59 ff., and Pindyck (1991), pp 1118. Adding further periods of uncertainty will lead to a further widening of the band of inaction. However, these additional option value effects will be the smaller the more far in the future the uncertain shocks will occur, since the effects of the shock are discounted more and more. Thus, even in the case of a permanent uncertainty, the option value effect would not be infinitely large, but converges towards an upper bound. See e.g. Dixit (1989) for a model with permanent uncertainty and a limited width of the band of inaction.
5. Numerical examples

In order to convey an idea of the impacts of the underlying model and to illustrate our results, we calculate some simple numerical examples. In the first example we let the hiring and firing costs be quite large with an eye on the fact that in the euro area institutional rigidities may lead to such high realisations of $H$. We take the short-term interest rate given as $i = 2\%$ and the "base value" for the expected long-term interest rate as $r = 10\%$ per period. The parameter $\alpha$ is set to 0.5, i.e. the expected long-term interest rate corresponds to an arithmetic average of the short-term interest rate and the "base value" $r$ for the expected long-term interest rate.

Later on, we compare the results for $\alpha = 0.5$ with the special cases of $\alpha = 0$ (second example) and $\alpha = 1$ (third example).

**First scenario:** $\alpha = 0.5$ ; $H = 1$ (normalized) ; $r = 0.1$

\[
\begin{align*}
&\epsilon_{\text{entry}}^c = 0.054584374 ; \\
&\epsilon_{\text{entry}}^u = 0.052774019 + 0.89648173 \varepsilon ; \\
&\varepsilon_{\text{min}} = 0.0020193997 < \varepsilon
\end{align*}
\]

**Second scenario:** $\alpha = 0$ (other: see first scenario)

\[
\begin{align*}
&\epsilon_{\text{entry}}^c = 0.084858569 ; \\
&\epsilon_{\text{entry}}^u = 0.079754601 + 0.84355828 \varepsilon ; \\
&\varepsilon_{\text{min}} = 0.0060505219 < \varepsilon
\end{align*}
\]

**Third scenario:** $\alpha = 1$ (other: see first scenario)

\[
\begin{align*}
&\epsilon_{\text{entry}}^c = 0.019607843 ; \\
&\epsilon_{\text{entry}}^u = 0.019607843 + 0.96153846 \varepsilon ; \\
&\varepsilon_{\text{min}} = 0 < \varepsilon
\end{align*}
\]

From Fig. 1 we see that a higher level of uncertainty $\varepsilon$ results in a higher profit/revenue $\epsilon$ which is necessary for triggering an entry/investment. Fig. 2 shows that a higher short term interest rate $i$ as well results in a higher profit $\epsilon$ which is necessary for an entry / investment. However, the effect of short term interest rate on trigger profit under uncertainty is relatively weak. A summary of both effects on the profit trigger and their mutual amplification is illustrated in Fig. 3.

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16 For lower values of $H$ in the US case see Krugman (1989), p 57.
Fig. 1: Gross profit trigger $e_{\text{entry}}^u$ dependent on uncertainty / shock size $\varepsilon$

[first scenario ($\alpha = 0.5$), infinite time-horizon, $i = 0.02$]

Fig. 2: Entry trigger profit under uncertainty $e_{\text{entry}}^u$ dependent on short term interest rate $i$

[first scenario ($\alpha = 0.5$), and "uncertainty" $\varepsilon = 0.2$]
Fig. 4 illustrates that in a situation with a high uncertainty $\varepsilon$ and low gross profits /revenues $e$ the interest rate has to be very low in order to induce an entry/investment. In the grey shaded area we see combinations of $\varepsilon$ and $e$, where a non-negative short term interest rate is not compatible with an investment, i.e. monetary policy is not effective at all in such a situation with high revenue uncertainty and low profits. Fig. 5 illustrates that the higher the pass-through parameter $\alpha$ of short term interest rate to long term interest rate expectation, the lower is a necessary reduction of the short term interest rate $i$ resulting in an entry/investment. I.e. the higher $\alpha$, the more effective is an interest rate reduction. The same implication can be seen the next figures. Fig. 6 is based on the second scenario with $\alpha = 0$. In such a situation without any spill-over of short term interest rate $i$ on expected long term interest rate, the impact of short term interest rate changes on the triggering revenue levels is very weak (since the curves are nearly horizontal), i.e. monetary policy is very ineffective. In contrast, in a situation with a complete pass-through of short term rate to long term interest expectation (third scenario, see Fig. 7), we have a strong impact of $i$ on the entry-trigger profit. However, this effectiveness is again weakened by uncertainty in a situation with the option to wait. For a summarising illustration see Fig. 8: The effect of increasing uncertainty $\varepsilon$ on the interest rate investment trigger is negative for all levels of the expectation parameter $\alpha$. Moreover, the weaker the
relation between short term interest rate and long run expectation (i.e. the lower $\alpha$), the stronger is the negative effect of $\varepsilon$ on the interest trigger, i.e. the less effective in stimulating investments is monetary policy via interest rate cuts.

Fig. 4: Interest rate entry trigger $i^u_{\text{entry}}$ dependent on gross profit $e$ and uncertainty $\varepsilon$

[first scenario ($\alpha = 0.5$)]

Fig. 5: Interest rate entry trigger $i^u_{\text{entry}}$ dependent on pass-through parameter $\alpha$.

[except for $\alpha$: first scenario and revenue $e = 0.25$, uncertainty $\varepsilon = 0.2$]
Fig. 6: Entry trigger profit under uncertainty $e_{\text{entry}}^u$ dependent on short term interest rate $i$

[second scenario ($\alpha = 0$), and uncertainty $\varepsilon = 0.2$]

![Entry profit vs short term interest rate](image1)

Fig. 7: Entry trigger profit under uncertainty $e_{\text{entry}}^u$ dependent on short term interest rate $i$

[third scenario ($\alpha = 1$), and $\varepsilon = 0.2$]

![Entry profit vs short term interest rate](image2)
Taking into account the option values induced by revenue uncertainty implies an amplification of areas of low reaction/hysteresis effects. Our theoretical results are compatible with recent empirical studies, which show that option values can be large. Hence, monetary policy actions which rely on investment rules that do not take the latter into account can be very misguided.17

6. Conclusions

In this paper we study the impact of uncertainty on the effectiveness of monetary policy. We base our modelling approach on the theory of the “option value of waiting”. The model is very simple and. Uncertainty of future revenues, current and expected interest rates are the forces which drive investment decisions. Under uncertainty and with sunk costs a firm is faced with the option of investing at date t or delay the investment decision to the future date t+1 when the uncertainty has been resolved. In this scenario, the central bank monetary policy

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17 See e.g. the studies cited by Dixit, Pindyck (1994), p 7.
may affect the investment entry decision of the firm via its controlling of the short-term interest rate. We show that high uncertainty leads to a higher revenue entry trigger and to a lower short-term interest entry trigger. This 'weak' relationship between investment / employment and the interest rate is augmented by revenue uncertainty. As a result of option value effects, the relationship between the interest rate and the investment is strongly weakened by uncertainty (as numerical examples demonstrate). Thus, monetary policy gets into a kind of uncertainty trap and may be very ineffective in an uncertain economic environment.

The model proposed before was based on a risk-neutral single-unit investment / employment decision under revenue uncertainty induced by revenue (step) volatility and fixed sunk (i.e. irreversible) investment and hiring costs. In principle, it can be compared to other models were an irreversible investment decision is analysed. In contrast to similar work in that area we did not rely on the asymmetry of adjustment costs (Caballero, 1991) and on scrapping values (Darby et. al., 1997), since we analysed also 'investments' in employment and did not focus only on real capital investments. Additionally, the degree of competition in the output market and economies of scale (Caballero, 1991) did not play a predominant role since we analyse a single-unit decision.

Nevertheless, the highly stylised model developed here as a novel starting point for such kind of considerations may provide three important implications for monetary policy in the euro area against the background of the uncertainty prevailing from the perspective of the ECB: (1) in a high uncertainty regime reducing interest rates might be an ineffective policy; (2) cutting or (increasing) rates under high uncertainty may “kill” the option value of waiting and therefore reduce the effectiveness of policy in future periods; and (3) a central bank that operates frequent interest rate changes induces additional uncertainty in the economy and in so doing it impedes firms’ investment decisions. Note, however, that we have no judgment to offer at all on whether the ECB had the right rate level or levels from 2001 on because there
really is nothing in our paper that addresses this question. Hence, we are certainly not arguing for rate cuts or against them as a policy matter.

One might feel inclined to ascribe real impacts of revenue volatility solely to times of excessively high uncertainty, i.e. to crashing events like September 11th. However, since uncertainty $\varepsilon$ was included additively in the revenue function it was straightforward to interpret $\varepsilon$ as an all comprising expression of uncertain revenues like, e.g., disequilibria of the US economy since the turn-of-year 2000/01 (current account, consumer financial position, over-investment). Moreover, the relation (including the 'weak reaction' characteristic) between investment / employment and all its determinants (not only interest rates but also e.g. the wages and the oil prices) was affected by uncertainty. Thus, the impacts implied by sunk costs and uncertainty are manifold. We only calculated interest rate triggers, holding other determinants of investment / employment constant. Summarising, compared to the prediction of the majority of models of monetary policy transmission, real world investment / employment may appear less sensitive to changes in the interest rate, due to uncertainty.

Of course it is true that when risk premiums are high, a given change in the riskless interest rate produces less of a proportional change in the user cost and required rate of return on capital than when the risk premium is low. If one assumes iso-elastic (log-log) investment response functions, one would deduce that changes in the riskless rate are less effective when uncertainty is high. However, with heterogeneous agents and projects, there are always some projects near the margin that respond to changes in the required rate of return at any level of uncertainty. Hence our “range of inaction” at first glance appears to be a discontinuity due to overly “representative” modeling, not to anything “real” that should concern monetary policy. However, in order to derive macroeconomic implications which are empirically testable Belke, Göcke (2001, 2001a) deal with the aggregation of the approach proposed in this paper. They assume that the firms have different exit ("disinvestment") and entry ("investment")
triggers. Special attention is paid to the problem of aggregation under uncertainty. It is shown that under uncertainty 'areas of weak reaction' have to be considered even at the macroeconomic level. Due to the similarities of the macro relations under uncertainty to the micro behaviour derived in this contribution our micro-approach can serve as a first base for empirical tests.

How do our formal considerations fit with the monetary policy strategy of the ECB in reality? According to its two-pillar strategy, the rationale for the ECB for taking investment / employment demand functions into account when deciding on interest rate cuts (or increases) is to support general economic policy in times of low (high) inflation. Moreover, empirical evidence as a stylised fact comes up with the result that Taylor-rule type monetary policy reaction functions describe the actual behaviour of the ECB quite well. The ECB will be confronted with an unusually highly uncertain environment still for some time for several reasons. First, many of the underlying causes of world wide uncertainty do not seem to be resolved, although the Iraq conflict itself was terminated unexpectedly early. Second, experience has shown that the effects of the quick termination of war actions in the Gulf region were more than compensated by an increase in uncertainty with respect to the shape of the post-war world order. As long as uncertainty stays relevant for consumers and investors, the approach of the option value of waiting should be relevant for the monetary policy of the world’s leading central banks.

In the light of the results of the paper, the remarks on the ineffectiveness of monetary policy made in the introduction are corroborated in a subtle sense. Under the presumption of a net reduction of revenue uncertainty, the investment / employment impacts of a lower interest rate level continue to be twofold. A reduction of uncertainty, e.g. after the end of the Iraq conflict, leads to a contraction of the area of weak reaction. Hence, the interest rates triggering investments need not to be as low as before. Hence, the effectiveness of expansionary
monetary policy via cutting interest rates is increased (lowered) by a low (high) degree of uncertainty.
References


