A flexible price theory of equilibrium real exchange rates and output*

Peter Benczur†and Istvan Konya‡

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Abstract

This paper develops a flexible price, two-sector nominal growth model, in order to study the nominal effects of capital accumulation (convergence). We adopt a classical model of a small open economy with traded and nontraded goods, and enrich its structure with gradual investment and a preference for real money holdings. The modeling framework gives the following results: (1) the level of the exchange rate can have a medium-run impact on nominal and real variables but no long-run effect on real variables; (2) along the real equilibrium path (which can be implemented by flexible exchange rates), capital accumulation implies an increase in the price of nontradables (a real appreciation); (3) by comparing the nominal and the real path of the economy, we define a notion of under- and overvaluation; (4) and analyze its impact on real and nominal variables.

JEL Classification Numbers: F32, F41, F43

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1 Introduction

The nominal exchange rate is one of the most important prices for a small open economy, influencing its structure and performance in the short-run. There are strong linkages among permanent or temporary exchange rate movements, the external position, the growth rate and fluctuations of the economy, the latter often showing sectoral asymmetries as well.

The nominal exchange rate can also influence the intertemporal behavior of a small open economy. As suggested by consumption smoothing, converging economies should be borrowing

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†Magyar Nemzeti Bank and Central European University. Corresponding author, email: benczurp@mnb.hu

‡Magyar Nemzeti Bank and Central European University; email: konya@mnb.hu

against their future income. Besides showing such a general tendency, these economies also accumulate domestic assets. These assets are usually denominated in local currency, while liabilities are in foreign currency. This implies that the evolution of nominal variables, and particularly the nominal exchange rate, will influence this process. Our objective is to develop a simple but sufficiently rich framework, which is capable of addressing the aggregate and sectoral features of such a nominal convergence, and also enables an interpretation of over- and undervaluation along the nominal growth path.

The structure of the model is the following. We consider a small open economy, with a traded and a nontraded sector. Both sectors use labor and capital, but not necessarily with the same intensity. Factors are perfectly mobile between the two sectors, but their international mobility is restricted. In particular, labor is immobile between countries, while international capital flows are hampered by adjustment costs. We adopt the now classic *Tobin-q* approach to capture gradual capital flows.

The source of growth is capital accumulation.[1] We assume that the initial capital stock is below the steady state level, so the country experiences capital accumulation and excess growth along its convergence path. For simplicity, we assume that the entire capital stock is owned by foreigners.

The nominal side of the growth process is represented by the well-known "money-in-the-utility" framework, which assumes that households derive utility directly from holding (real) money balances.[2] As the income of consumers grows, they want to consume more and also to hold more money. By having access to an international bond market, they can borrow against their future income, thus being able to consume more *and* hold more money. To prevent complete consumption smoothing, we utilize the standard assumption that there is an endogenous risk premium (one that is decreasing in the country's average asset *holdings*). Together with gradual investment, these intertemporal elements are already sufficient to produce a lasting effect of one-period nominal shocks.

Cash and fixed income securities are *inherently* sticky with respect to nominal exchange rate movements, their value in foreign currency changes one-to-one. In this sense, their presence can be viewed as an "original stickiness". By neglecting price and wage setting frictions, we want to show that nominal exchange rates can have systematic medium-run real effects even under

[1] One could extend the model to allow for exogenous TFP growth, either symmetric or asymmetric across sectors. In the case of an exogenous TFP growth in tradables, almost all of our results remain identical in terms of effective (normalized) variables, but the expressions and derivations are substantially more complicated. For this reason, we stick to the case of constant TFP.

[2] An alternative would be to consider a cash-in-advance economy, which assumes that certain transactions require the appropriate cash at hand. Both assumptions are largely ad hoc and lead to similar conclusions. As we want to relate our results to current neokeynesian models, which usually employ money-in-the-utility, we stick to that assumption. Moreover, one can also interpret the money stock that consumers hold as a buffer stock.

flexible prices and wages.[3] Another consideration is the simplicity of the modeling framework.

After setting up the model we turn to the analysis of the nominal growth process. We first show that in case of flexible exchange rates, the nominal economy behaves identically to the real economy, as the gradual increase in money holdings is implemented by an appreciating nominal exchange rate. Equivalently, even with exchange rates fixed, the right amount of money creation by the central bank can implement the real path.

The nominal and the real paths differ, however, when the exchange rate is fixed and money growth is exogenous. This is the case, for example, when the country operates a currency board economy (zero money growth), or chooses the euro conversion rate (joining a monetary union). Historically, the gold standard shared the same features. Under these assumptions any increase in the domestic money stock must come from abroad. This necessitates either a trade surplus or foreign borrowing. Since borrowing is costly (debtors face a positive risk premium), the nominal economy features an extra saving motif, the accumulation of nominal wealth. Consequently, the exchange rate becomes misaligned and the growth path differs from that of an economy where money plays no role. An important application of our model is thus the choice of the euro conversion rate for EMU aspirants.

When the nominal and real paths diverge, the exchange rate is not automatically at its "equilibrium" level. Thus we can define the under- and overvaluation of the currency, with a suitable benchmark choice. We discuss the possible benchmarks - the medium-run equilibrium real and nominal exchange rate - after setting up the model. We also analyze the symptoms and consequences of over- and undervaluation.

The paper is organized as follows. The next section puts the model into context. Section 3 describes the model. Section 4 explains its mechanics for the flexible exchange rate case, while Section 5 discusses the currency board regime (the medium-run equilibrium concept and impulse responses). Section 6 offers some quantitative policy simulations, and Section 7 concludes. The Appendix contains all the detailed calculations.

2 The context of the model

2.1 Theory

Usual explanations for nominal shocks having lasting real effects usually build on staggered price or wage contracts. An early example is Taylor (1980). Recently, state- or time-dependent pricing models constitute as the workhorse for analyzing nominal scenarios (see chapter 3 of

[3] Tille (2005) is another example when exchange rate movements can lead to persistent real effects without pricing or wage setting frictions.

Woodford (2003) for a general discussion). Instead of pricing problems, we focus on nominal wealth accumulation (captured by money-in-the-utility), which is also influenced by nominal shocks.

The major building blocks of our model are *money-in-the-utility* (a nominal effect), a debt-dependent interest rate, gradual investment (a real friction) and sectoral technology differences (capital-labor intensities). These are already sufficient to produce real effects of a nominal shock.[4] There is also a positive correlation between domestic savings and investment (like the Feldstein-Horioka (1980) puzzle), although investment is not financed from domestic savings at all. The link comes from a "crowding out" effect of nominal expenditures on investment, which is due to the general equilibrium developments of relative prices.

The nominal effect comes from the gradual adjustment of expenditures to income, or in other words, from intertemporal consumer optimization. This can be also viewed as some sort of a nominal rigidity (illusion), which ensures that nominal shocks have an impact effect on spending. Such a behavior can be microfounded by an explicit intertemporal maximization of a utility function containing real money balances.[5]

In a more general setting, one can think of the role of money here as a precautionary buffer stock. As the economy grows, consumers want to increase these asset holdings. The fact that the assets are nominal (financial) gives the notion of nominal convergence. Moreover, nominal shocks can revalue this stock (as argued by Lane and Milesi-Ferretti (2004), or Gourinchas and Rey (2004)), which in turn changes consumer behavior. Tille (2005) also analyses the real effects of such a revaluation.

It is well-known that having access to an international bond market where the world-wide interest rate is constant (and equal to the domestic discount rate) would lead to complete consumption smoothing, implying unrealistic levels of foreign indebtedness. Moreover, such open economy models could not pin down the steady state level of foreign debt. Schmitt-Grohe and Uribe (2003) offers various ways of closing such open economy models, one being a debt-dependent interest rate. That assumption uniquely determines the level of debt in steady state, and also slows down consumption smoothing.

The presence of a traded and a nontraded sector allows us to merge trade theory insights with a monetary framework: for example, the presence of nontraded goods means that a redistribution

[4] Benigno (2003) and part 3.2.5 of Woodford (2003) also highlight the role of sectoral asymmetries, though not in the context of traded versus nontraded goods.

[5] Classical real exchange rate (trade theory) models often use the relationship $E = VH$, nominal expenditures being proportional to money holdings, to allow for nominal shocks. Examples include part 3 of Dornbusch (1980) and Krugman (1987). Dornbusch and Mussa (1975) show that under certain conditions (power-Cobb-Douglas utility and constant inflation), the intertemporal optimization problem with money-in-the-utility implies a saddle path with $E = VH$. This is one simplification adopted by Benczúr (2003).

of income between countries will affect their relative wages (the classical transfer problem, like in Krugman (1987)), or the Stolper-Samuelson theorem, linking changes in goods prices with movements in factor rewards.

Many current papers point to the importance of gradual investment in shaping business cycle properties, inflation or real exchange rate behavior. Eichenbaum and Fisher (2004) argue that the empirical fit of a Calvo-style sticky price model substantially improves with firm-specific capital (and a nonconstant demand elasticity). Christiano et al (2001) present a model in which moderate amounts of nominal rigidities are sufficient to account for observed output and inflation persistence, after introducing variable capital utilization, habit formation and capital adjustment costs. Chapter 4 of the Obstfeld and Rogoff (1996) textbook contains an exposition of a two-sector growth model (the standard Balassa-Samuelson framework), with gradual investment in some of the sectors. We depart from these approaches by dropping staggered price setting, but – unlike Obstfeld and Rogoff – still allowing for a nominal side of the economy.

Huffman and Wynne (1999) develop a multisector real model with investment frictions (sector-specific investment goods and costs of adjusting the product mix in the investment sector). Their objective is, however, to match the *closed economy comovements* of real activity across sectors (consumption and investment). In our model, the two sectors have a completely different nature (traded and nontraded). These two sectors do not necessarily move together, as indicated by the countercyclicality or acyclicality of net exports (see Fiorito and Kollintzas (1994) for G7 countries, Aguiar and Gopinath (2004) for emerging economies). Aguiar and Gopinath (2004) also construct a *one-sector real model* to explain the countercyclicality of net exports and the excess volatility of consumption. Balsam and Eckstein (2001) develop a real model with traded and nontraded goods, aimed at explaining the procyclicality of Israel's net exports and excess consumption volatility.

The growth literature also employs multisector models, but the two sectors there differ in the investment good they produce (physical versus human capital). Examples include Rebelo (1991) and Lucas (1988). Ventura (1997) is an example of a multisector growth model with an explicit trade framework. His model of growth in interdependent economies clearly illustrates the importance of merging trade and growth theory. The implications of a nontraded sector, however, are not addressed by that paper. None of the existing models, up to our knowledge, share all the distinctive features of our model: a flexible price, nominal, open-economy, two-sector model with investment frictions, giving a lasting real effect of nominal disturbances.[6]

There is also an important similarity between the dynamic equations of the model and differ-

[6]In fact, the general equilibrium tax incidence analysis of Harberger (1962) has very similar features: in his analysis, taxation plays a related role to the nominal exchange rate in our model.

ent equilibrium real exchange rate concepts (like the NATREX approach of Stein (1994), or the BEER, FEER approaches). In this sense, the model can be viewed as an explicit optimization-based, flexible price theory of medium- and long-run equilibrium real exchange rates. More precisely, the long-term equilibrium concept is the steady state of our model (when both capital and money holdings are at their steady state level). The medium-term equilibrium concept is defined by the trade balance condition. Loosely speaking, it corresponds to the nonmonetary version of the model (when the adjustment of money holdings is much faster than that of capital, thus the income and expenditure of consumers are always equal to each other, and money does not influence any real variables). In other words, flows (the trade balance) are in equilibrium *given* the current level of stock variables. There is also a corresponding value of GDP, where *both the relative price and the relative size of the two sectors* equal the medium-run equilibrium value. In this context, misalignment means a deviation from the medium-run equilibrium.

The medium-run equilibrium determines the nominal side of the economy through the trade balance condition. If one keeps money growth constant, this implies an equilibrium path of the nominal exchange rate. As long as the economy accumulates capital and the nontraded sector is more labor intensive, there is an equilibrium *real and nominal* appreciation.

2.2 Stylized facts

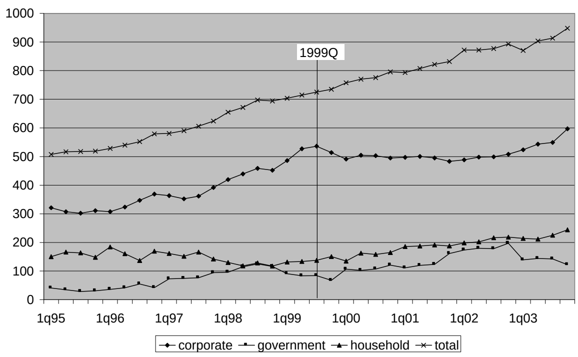
Besides its theoretical aspects, the model gives important predictions about employment, price and wage dynamics after nominal exchange rate shocks. By replacing the word "increase" with "higher than the medium-term equilibrium value", we get the symptoms of an overvalued economy. In particular, a nominal appreciation leads to (1) an increase in wages ("too high wages"); (2) a reallocation of labor from manufacturing to services; (3) a fall in the rental rate; (4) a halt in investment with a marked sectoral asymmetry: increase in service sector investments, fall in manufacturing; (5) an increase in the nontraded-traded relative price; (6) an overall consumption boom, accompanied by a deteriorating trade balance; (7) a temporary increase in real GDP. A depreciation would produce exactly the opposite of these effects.

This is in line with the performance of exchange-rate based disinflations, and its reverse conclusions are relevant to price and wage developments after large devaluations. Rebelo and Végh (1995) find the following main stylized facts of exchange rate based stabilization programs: (1) high economic growth, (2) which is dominantly fueled by consumption, (3) slow price adjustment, (4) deteriorating trade balance.[7] They also show some indicative evidence of a superior nontradable performance for Uruguay, Mexico, and cite Bufman and Leiderman (1995) as ev-

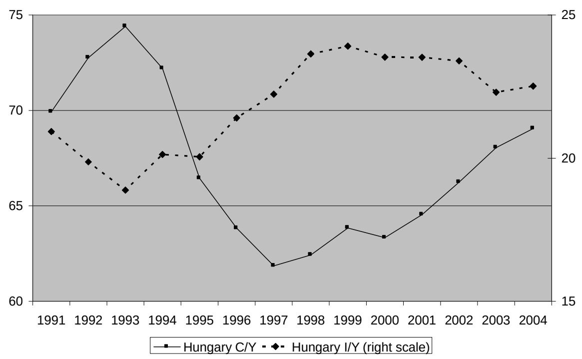
[7] Hamann (2001) argues that many of these features are not restricted to exchange-rate based stabilizations. Given that we are interested in the response to a nominal exchange rate shock, it is irrelevant whether these features are shared by other types of stabilizations.

idence for Israel. Burstein et al (2002) analyze large *devaluation* episodes, and find that (1) inflation is low relative to the depreciation, (2) the relative price of nontradables fall, (3) export and import prices (goods that are truly traded and not just tradable) track more closely with the exchange rate than the full CPI, (4) real GDP growth declines, and (5) there is a rise in the trade surplus.

To provide a specific example to the symptoms discussed above, we present some recent evidence from Hungary, in particular the experience of the late 90's and early 2000's. Looking at Hungarian data between 1999-2003, we find the following:[8] (1) a drop in real corporate investment around 1999, and a flattening of the total investment to GDP ratio (Panels A and B of Figure 1); (2) a strong increase in the consumption to GDP ratio since 2000 (Panel B); (3) a strong comovement of corporate investment and the stock market index – the 1999 episode is mixed here with the Russian crisis, but from 2000, the U-shaped pattern of investment and the stock market is common (Panel C); (4) massive real wage growth episodes around 1999, 2000, partly driven by public sector wages (Panel D); (5) a general increase in the nontraded-traded relative price, with historical highs since 2000-2001 (Panel E); (6) a shift of (total) investment from industry towards services and real estate (Panel F);[9] (7) a tilt of employment towards the service sector (Panel G); (8) and an overall high current account deficit, particularly deteriorating since 1998, with a temporary reversal in 2001 and 2002 (Panel H).



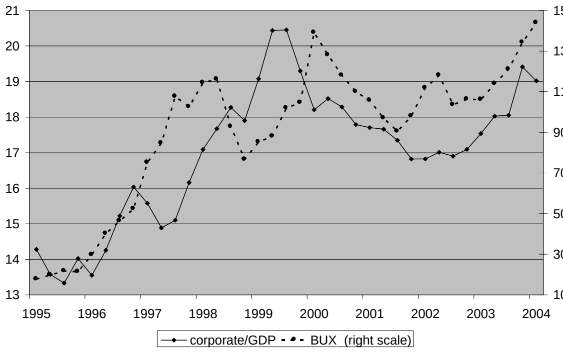
Panel A: Investment (real)



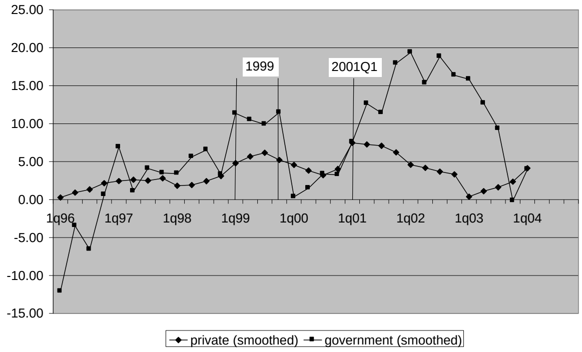
Panel B: Investment and consumption ratios

[8] There was no apparent extra GDP growth – but the fact that there was no slowdown among the international stagnation of the 2000s can be interpreted in such a way. By 2003, GDP growth indeed declined.

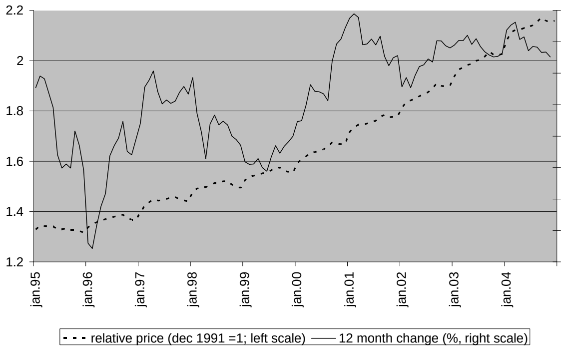
[9] This change in total investment shares is mostly driven by a constant industry share within corporate investment, and an overall increase in public investment (dominantly services) and household investment (dominantly real estate).



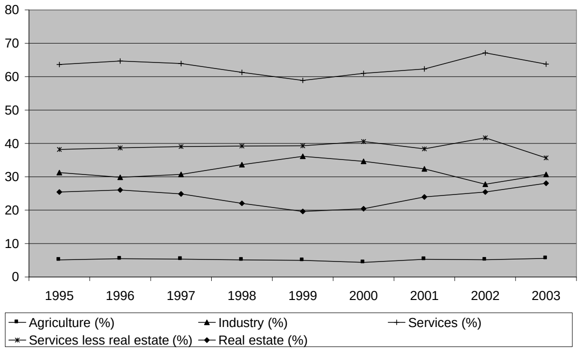
Panel C: Investment and the stock index



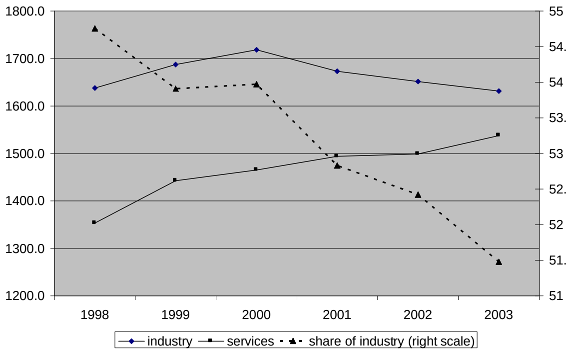
Panel D: Real wages



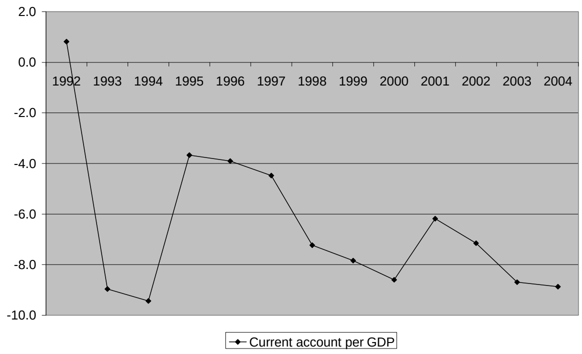
Panel E: The nontraded-traded relative price



Panel F: Investment by sectors



Panel G: Employment in the private sector



Panel H: The current account

Figure 1: Hungary in the late nineties

While there were many different impulses coming from both monetary and fiscal policy, most of these impulses point in the same direction. In the language of the model, most changes were shocks to nominal wealth. Since our model has the same predictions for any such shock, it is not important (and also not feasible) to separate out the impact of nominal appreciation.[10] Thus

[10] The policy environment can be summarized as (1) a correction in the public versus private sector wage ratio,

while the exact contribution of each shock is unclear, we feel confident that the final picture is consistent with the model's predictions about an overvalued economy.

3 The model

3.1 Consumers

Consumers solve the following problem:

$$\begin{aligned} \max U_t &= \int_t^{\infty} e^{-\rho(s-t)} \left[\log C(s) + \gamma \log \frac{H(s)}{P(s)} \right] ds \\ \text{s.t. } \dot{A} &= W - PC + T + i(a-h)(A-H), \end{aligned}$$

where W is aggregate labor income (supplied inelastically), T is a government transfer,

$$C = C_T^{\lambda} C_{NT}^{1-\lambda}$$

is the intratemporal utility of consumption. Consumers consume a mix of tradable and non-tradable goods. P is the ideal price index associated with C (see below) and ρ is the worldwide discount rate and also the rate of interest abroad. Changes in nominal wealth (A) come either from the government (T) or from abroad. The latter requires households to be net savers (relative to the rest of the world).

Part of wealth is held as money, and the rest is invested (or borrowed) in foreign bonds ($B = A - H$). To ensure the long-run existence of a well-defined steady state, we assume a debt-dependent bond rate $i(a-h)$, as in Schmitt-Grohe and Uribe (2003). In fact, this assumption is also crucial for money to play a non-negligible role: without it, we would observe full consumption-smoothing and constant money holdings. Here the lowercase letters correspond to amounts measured in foreign currency (euro). The foreign and the local nominal interest rates are linked with an uncovered interest parity condition:

$$i(a-h) = \rho + d(a-h) + \frac{\dot{e}_t}{e_t}, \tag{1}$$

where $d(\cdot)$ is a risk premium which is decreasing in its argument(recall that $a-h$ is the negative

around the beginning of 1999; (2) a large increase in minimum wage legislation, around the beginning of 2001; (3) investment subsidies to SMEs and (4) subsidized real estate loans, from around 1999; (5) a large nominal appreciation (monetary restriction), in the form of widening the exchange rate band in May 2001, (6) followed by a massive fiscal expansion, partly in the form of public sector wage increases (end of 2002). The exact timing of this latter fiscal expansion is somewhat unclear: the rise in public sector wages unambiguously came after the monetary contraction, but the fiscal stance before and after the monetary developments is subject to heated political debates in Hungary.

of debt), and $d(\bar{b}) = 0$. We assume that individual households do not internalize the effect of their borrowing or lending on $i(\cdot)$, i.e. the debt premium depends on average (country level) bond holdings.

The form of the utility function allows a sequential solution of the consumer problem: we first calculate the share of tradables and nontradables given current nominal expenditures (intratemporal step), and then we determine the optimal evolution of expenditures (intertemporal step).

The usual intratemporal optimization conditions imply that:

$$PC = eC_T + p_{NT}C_{NT} \tag{2}$$

$$\frac{eC_T}{p_{NT}C_{NT}} = \frac{\lambda}{1-\lambda} \tag{3}$$

$$P = \lambda^{-\lambda} (1-\lambda)^{\lambda-1} e^{\lambda} p_{NT}^{1-\lambda}. \tag{4}$$

The intertemporal problem is solved by writing down the current-value Hamiltonian. The objective function becomes

$$\mathfrak{H} = \log C + \gamma \log \frac{H}{P} + \theta \left[W - PC + T + i(A-H)(A-H)\right],$$

and the first-order conditions are given by

$$\frac{1}{C} = \theta P \tag{5}$$

$$\frac{\gamma}{H} = i(A-H)\theta \tag{6}$$

$$\dot{\theta} = \left[\rho - i(a-h)\right]\theta \tag{7}$$

$$\dot{A} = W - PC + T + i(a-h)(A-H). \tag{8}$$

Dornbusch and Mussa (1975) use a similar framework to give a microfoundation of the $X = VH$ relationship (nominal spending being proportional to money holdings): with a power Cobb-Douglas aggregate ($C^{\alpha}(H/P)^{\beta}$), constant inflation and no bond markets, they show that X/H is indeed constant along the saddle path of the intertemporal optimization, as long as inflation is constant. In our work, however, inflation is changing through time. Given that the proportionality of X and H is no longer true, we decided to use the more standard logarithmic Cobb-Douglas felicity function. This gives a less direct role of money in the consumption decision (the marginal utilities are separable), and it is also the standard choice of new-keynesian intertemporal models (see Woodford (2003), chapter 2.3.4 for consequences of nonseparable utility functions).

3.2 Producers

Production functions are given by

$$\begin{aligned} Y_T &= L_T^{\beta} K_T^{1-\beta} \\ Y_{NT} &= L_{NT}^{\alpha} K_{NT}^{1-\alpha}, \end{aligned}$$

and profit maximization implies

$$W = e\beta L_T^{\beta-1} K_T^{1-\beta} = p_{NT} \alpha L_{NT}^{\alpha-1} K_{NT}^{1-\alpha} \tag{9}$$

$$R = e\left(1-\beta\right) L_T^{\beta} K_T^{-\beta} = p_{NT}\left(1-\alpha\right) L_{NT}^{\alpha} K_{NT}^{-\alpha}. \tag{10}$$

Notice that we assume the indifference of both factors *between the two sectors*, so $W_T = W_{NT} = W$, $R_T = R_{NT} = R$. This does not automatically imply full international mobility of capital.

We would not argue that the labor mobility assumption is fully realistic. One could also set up a model with slow labor adjustment. This would, however, excessively complicate the model, while the other two adjustments are vital to our analysis (for a real effect of nominal shocks, we need to have slow adjustment of nominal spending; and slow capital adjustment is necessary to analyze investment behavior). The other crucial assumption is that capital is indifferent between the *two domestic sectors*, but not necessarily between home and foreign. If the initial difference in sectoral returns of capital is not "too large", their equalization is feasible entirely through new investment. It is possible that a large shock necessitates disinvestment in one of the sectors: then one needs to assume that capital is mobile between sectors up to this degree. A further alternative would be to consider two separate q-theories in the two sectors, like Balsam and Eckstein (2001).

A third, hidden assumption is the immediate and full pass-through of the nominal exchange rate into tradable prices (but not necessarily into nontradables and factor prices). It is well-documented that the pass-through of exchange rate movements into tradable prices is far from full and immediate. Our focus, however, is essentially on the adjustment of the economy to a change in tradable prices. For this reason, similarly to most of the open economy macro literature, we work with a perfect pass-through into tradable prices.

3.3 Investment

One of the cornerstones of the "standard", "long-run" Balassa-Samuelson model (the one advocated by chapter 4 of the Obstfeld-Rogoff textbook) is the full mobility of capital. It implies that the rental rate at home equals the international rental rate. However, this implies a very fast and also mechanical capital accumulation and adjustment process. If we add the standard labor flexibility assumption ($w_T = w_{NT}$), the real exchange rate (traded-nontraded relative price) is fully supply-determined. The transformation curve is linear, and nominal variables (or preferences) have no effect on relative prices, only on quantities. For this reason, we assume that investment is subject to adjustment costs, which makes its response gradual:

$$\begin{aligned}\max V_t &= \int_t^{\infty} e^{-\rho(s-t)} \left[\frac{R(s)\,K(s)}{e} - I(s) - \frac{\delta}{2}\frac{I(s)^2}{K(s)} \right] ds \\ s.t.\ \dot{K} &= I.\end{aligned}$$

This is the standard q problem, and the first-order conditions are

$$\begin{aligned} q &= 1 + \delta \frac{I}{K} \\ \dot{q} &= \rho q - \frac{R}{e} - \frac{\delta}{2}\left(\frac{I}{K}\right)^2. \end{aligned}$$

Here q is the dynamic multiplicator (co-state variable). Rearranging the conditions yields

$$\frac{\dot{K}}{K} = \frac{q-1}{\delta} - g \tag{11}$$

$$\dot{q} = \rho q - R/e - \frac{(q-1)^2}{2\delta}. \tag{12}$$

3.4 Equilibrium

Let us introduce the term $X = PC$, which is – as can be seen from (8) – nominal expenditure. From (5) and (7) we get

$$\dot{X} = \frac{d}{dt}(PC) = -\frac{\dot{\theta}}{\theta^2} = -\frac{\rho - i(a-h)}{\theta} = [i(a-h) - \rho]\,X.$$

The other equilibrium conditions are

$$\begin{aligned} \frac{\dot{K}}{K} &= \frac{q-1}{\delta} \\ \dot{q} &= \rho q - \frac{R}{e} - \frac{(q-1)^2}{2\delta} \\ \dot{A} &= W - X + T + i(a-h)(A-H). \end{aligned}$$

The equations for K and q are in foreign currency, which means that the nominal exchange rate e does not directly enter those expressions. Let us transform all of the equilibrium conditions into foreign currency. Introducing $x = X/e, h = H/e, a = A/e, r = R/e, w = W/e, \tau = T/e$

yields

$$\begin{aligned}
\frac{d}{dt}\left(\frac{X}{e}\right) &= \frac{\dot{X}}{e} - \frac{X}{e}\frac{\dot{e}}{e} = \left(i\left(a-h\right)-\rho\right)x - x\frac{\dot{e}}{e} = d(a-h)x \\
\frac{d}{dt}\left(\frac{A}{e}\right) &= \frac{\dot{A}}{e} - \frac{A}{e}\frac{\dot{e}}{e} = w - x + \tau + i\left(a-h\right)\left(a-h\right) - a\frac{\dot{e}}{e} \\
&= w - x + \tau + i\left(a-h\right)\left(a-h\right) - h\frac{\dot{e}}{e}.
\end{aligned}$$

From here on, let us work entirely in foreign currency; the dynamics are summarized by[11]

$$\frac{\dot{k}}{k} = \frac{q-1}{\delta} \tag{13}$$

$$\dot{q} = \rho q - r - \frac{(q-1)^2}{2\delta} \tag{14}$$

$$\dot{a} = w - x + \tau + \left(\rho + d\left(a-h\right)\right)\left(a-h\right) - h\frac{\dot{e}}{e} \tag{15}$$

$$\frac{\dot{x}}{x} = d(a-h). \tag{16}$$

(13) - (16) is a system of four equations for five variables: k, q, a, x and e (the other variables k_T, h, w, r and c are functions of these five). A fifth equations is given by monetary policy. One assumption is that the change in the nominal exchange rate is constant, i.e. $\frac{\dot{e}}{e} = \varepsilon$. Under fixed exchange rates or a crawling peg, we have four equations with four endogenous variables, and three forcing variables: τ, ε and e, which could be viewed as vehicles of monetary and fiscal policy. For a steady state to exist, monetary policy must satisfy $\tau = h\varepsilon$ in the long run. In case of a fixed exchange rate regime (currency board), this implies zero long-run money growth. In general, any exchange rate level and rate of devaluation are consistent with the long-run steady state, with an appropriate money growth process.

The steady state conditions are

$$\bar{q} = 1 \tag{17}$$

$$\bar{r} = \rho \tag{18}$$

$$\bar{w} = \bar{x} - \rho\bar{b} \tag{19}$$

$$\bar{h} = \gamma\frac{\bar{x}}{\rho + \varepsilon} \tag{20}$$

$$\bar{a} = \bar{b} + \bar{h}. \tag{21}$$

Notice that the exchange rate does not influence $\bar{r}$. Consequently, all the technology-determined variables are independent of the path of the nominal exchange rate. Thus $\bar{w}$ and $\bar{x}$ are also

[11] For notational clarity, we will use k for the capital stock. Since K is a real variable, $k = K$.

independent from ε, it is only the steady state level of wealth (money and bond holdings) that depends (inversely) on the rate of devaluation.[12]

One needs to choose a long-run wealth level – $\bar{b}$ or $\bar{a}$. One possibility would be to assume $\bar{b} = 0$ – an alternative is to set $\bar{a} = 0$, meaning that consumers do not want to hold wealth, so they completely leverage their buffer stock of money ($\bar{h} = -\bar{b}$). Within this paper, we will work with the assumption of $\bar{b} = 0$, but in any case, we avoid log-differentiating a or b.

In what follows, we consider two alternative policy regimes: flexible and fixed exchange rates. The next section develops the flexible exchange rate regime in details and shows that the path of real variables is identical to a model where money has no role ($\gamma = 0$). For the fixed exchange rate, we set $\varepsilon = 0$. In terms of monetary policy (τ), we assume that the fixed exchange economy satisfies the steady state condition $\bar{\tau} = \varepsilon\bar{h} = 0$ in every period. In other words, the government simply does not print money, and any increase in money demand must be financed through a money inflow from the rest of the world. It can happen through borrowing or a trade surplus. As we will demonstrate, this leads to deviations from the real model, which is not the case for a flexible exchange rate.

These assumptions are characteristic of the gold standard system, or currency board regimes. The relevance of these frameworks is due to the fact that a monetary union is essentially a currency board regime. Our results can thus address the real effects of the choice of the euro conversion rate.

4 Flexible exchange rates

Let us assume that foreigners are unwilling to hold domestic currency. Under flexible exchange rates, the central bank is not committed to any exchange rate behavior, which implies that it is unwilling to take an open position in the local currency either. Under these assumptions, a flexible exchange rate regime implies a constant (exogenous) money stock. The regime with constant money could be labeled as "money growth targeting", while a constant exchange rate (with the appropriate money growth) is "exchange rate targeting". We will focus on the case when money is constant, but as we will see, both regimes would implement the nonmonetary (real) version of the model.

[12] Notice that $\bar{h}$ must be nonnegative, so there is an upper bound on the rate of devaluation which is still consistent with a steady state.

Setting $\tau = 0$ and $\dot{H} = 0$ in (13)-(16), the dynamic system becomes

$$\frac{\dot{k}}{k} = \frac{q-1}{\delta} \tag{22}$$

$$\dot{q} = \rho q - r - \frac{(q-1)^2}{2\delta} \tag{23}$$

$$\dot{b} = \dot{a} - \dot{h} = \dot{a} - \frac{\dot{H}}{e} + h\frac{\dot{e}}{e} = w - x + (\rho + d(b))\, b \tag{24}$$

$$\frac{\dot{x}}{x} = d(b), \tag{25}$$

while the steady state conditions become

$$\begin{aligned} \bar{q} &= 1 \\ \bar{r} &= \rho \\ \bar{w} &= \bar{x} + \rho\bar{b} \\ \bar{b} &= 0. \end{aligned}$$

Notice that (22)-(25) describe an entirely real system (this would not hold under a currency board, where $\dot{H} \neq 0$). This is the same as the nonmonetary version of the model: consumers solve

$$\begin{aligned} \max U_t &= \int_t^{\infty} e^{-\rho(s-t)} \log c(s)\, ds \\ s.t.\ \dot{b} &= w - pc + [\rho + d(b)]\, b, \end{aligned}$$

where all variables are measured in consumption units, i.e. we normalize the (real) price of consumption to unity. This is the same as measuring everything in foreign units, so the bond rate is indeed $i(b)$. The intertemporal problem is now represented by the Hamiltonian

$$\mathfrak{H} = \log c + \theta\, [w - c + (\rho + d(b))\, b]\,.$$

The first-order conditions are

$$\begin{aligned} \frac{1}{pc} &= \theta \\ \dot{\theta} &= -d(b)\theta \\ \dot{b} &= w - pc + [\rho + d(b)]\, b. \end{aligned} \tag{26}$$

The production and investment side remains the same as in the nominal case. Rewriting (26):

$$\dot{x} = \frac{d}{dt}(pc) = -\frac{\dot{\theta}}{\theta^2} = -\frac{-d(b)}{\theta} = d(b)\,x.$$

As all the other static and dynamic equations remain the same, the flexible exchange rate economy indeed implements the real version of the model.

To determine the evolution of e under flexible exchange rates, remember that

$$\frac{\gamma}{h} = \frac{i(a-h)}{x},$$

thus

$$h = \frac{\gamma x}{\rho + d(b) + \frac{\dot{e}}{e}}.$$

This is the equation that determines the development of the nominal exchange rate:

$$\frac{\dot{e}}{e} = \frac{\gamma x}{h} - \rho - d(b) = \frac{\gamma x}{H/e} - \rho - d(b)\,. \tag{27}$$

Given e, x and b, this indeed gives the law of motion for e.

Under exchange rate targeting, $\dot{e} = 0$ and

$$h = \frac{\gamma x}{\rho + d(b)}$$

describes the evolution of the money stock.

The Appendix shows that the loglinearized system becomes

$$\begin{aligned}
\frac{d}{dt}\hat{q} &= \hat{q}\rho + \rho\frac{\beta}{\alpha-\beta}\hat{p} \\
\frac{d}{dt}\hat{x} &= -\psi_2 db \\
\frac{d}{dt}\hat{k} &= \hat{q}\frac{1}{\delta} \\
\frac{d}{dt}db &= \frac{1-\beta}{\alpha-\beta}\bar{w}\hat{p} - \bar{x}\hat{x} + \left(-\psi_2\bar{b} + \rho\right)db,
\end{aligned}$$

where $\hat{p} = \frac{1}{A}\hat{x} - \frac{B}{A}\hat{k}$, $A = \frac{1-\beta-B}{\alpha-\beta}$, and $B = 1 + \frac{1-\beta}{(1-\lambda)(\beta-\alpha)}$.

It is interesting to note that without access to foreign borrowing, the flexible exchange rate (and the real) economy would involve a balanced trade all the time. That would imply that the $k-q$ block is independent from x and a (h in that case), so once we solve the real model, its nominal implementation follows easily. With borrowing, however, the real model remains four dimensional (two after substituting in the saddle path). Both k_0 and b_0 are needed to pin down the equilibrium. Given k and b, however, we can compare the evolution of the real and the nominal economy from then on.

5 The currency board

To understand the mechanics of the currency board regime, recall that the change in consumer wealth (measured in domestic currency) is given by

$$\begin{aligned} \dot{A} &= W - PC + T + i\,(a-h)\,(A-H) \\ &= Y_T + p_{NT}Y_{NT} - RK - C_T - p_{NT}C_{NT} + T + i\,(a-h)\,(A-H) \\ &= (Y_T - C_T) + i\,(a-h)\,(A-H) - RK + T. \end{aligned} \tag{28}$$

This is purely an accumulation equation (identity): the change in assets is equal to GNP minus expenditure, plus government transfers. GNP is the sum of traded and nontraded production (GDP), plus the interest income flow on NFA holdings, minus capital rents (that belongs to foreigners). Since the nontraded sector is in equilibrium, the value of nontraded production must equal the value of nontraded consumption. Using that $A = B + H$, we get

$$\dot{H} = -\dot{B} + (Y_T - C_T) + i\,(b)\,B - RK + T. \tag{29}$$

Change in money holdings thus equals the change in foreign assets, plus the excess production of tradables, plus the income from NFA holdings, minus capital rents, plus the exogenous term T.

Under the currency board arrangement, the government is prohibited from printing money, so $T = 0$, and naturally, e is fixed. Just like in the flexible exchange rate case, assume that foreigners cannot use the local currency for their transactions, so they do not accept it at all. How can consumers still increase the domestic money stock? They receive foreign currency (say, euros) for their trade surplus and foreign investment income (the current account balance), which they take to their own central bank. The central bank takes the euros, adds them to its foreign reserves, and issues domestic money in return. An alternative is to borrow from the rest of the world ($-\dot{B}$) in euros and again, exchange it to domestic money through the central bank. In both ways the rest of the world does not need to take any positions in the currency board country's local currency.

Realizing that H equals the foreign reserves of the central bank, one can reinterpret A as the net foreign asset position of the economy. Then (28) is simply the equality of the current and the financial account (including changes in reserves).

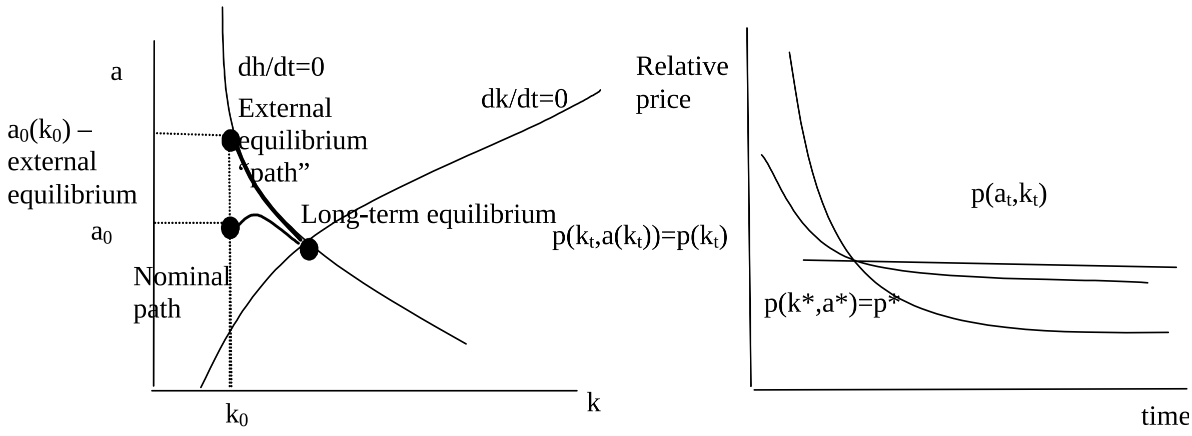


Figure 2: Long-term equilibrium, medium-term equilibrium and observed behavior

5.1 The medium-run equilibrium

Equation (29) offers a direct interpretation of the dual concept of equilibrium exchange rates (relative price). Notice first that as long as e is constant, the evolution of foreign versus domestic currency denominated variables are identical. We can thus switch to foreign currency terms again (lowercase variables).

The system has two state variables, a and k. In the long-term equilibrium, both variables are constant. There is a corresponding relative price p, which implies a long-term path for the relative price. The medium-term equilibrium allows for $k \neq \bar{k}$, but requires money holdings to be constant at every moment (the economy is always on the $\frac{d}{dt}h = 0$ locus). In other words, the medium-run equilibrium is temporarily "nonmonetary". In ”reality”, there is a nontrivial money adjustment, so the economy may be out of its medium-term equilibrium. In every moment (i.e., for every k_t), one can still define the corresponding medium-term equilibrium value of the real exchange rate $p\left(a\left(k_t\right), k_t\right)$, and the misalignment of the real exchange rate $p\left(a\left(k_t\right), k_t\right) - p\left(a_t, k_t\right)$.

More generally, the long-term equilibrium concept is the balanced growth path of a model with many state variables. The medium-term equilibrium corresponds to a transition path where some of the state variables (or their combinations) adjust immediately, the corresponding flow variables are in equilibrium, and only a subset of the laws of motion drives the dynamics. The nominal equilibrium (realized behavior of the economy) is then described by the full model, where all state variables adjust slowly. This is illustrated on Figure 2.

There are in fact two ways to introduce the medium-term equilibrium. The *real equilibrium* is the path along which monetary policy adjusts in a way that money is kept constant. This real path can be implemented through floating the exchange rate (as discussed in the previous section), or by an appropriate rate of money growth. In either case, the evolution of real variables coincides with that of a model without money.

The *external equilibrium* is a more restrictive concept. In this case the change in money reserves is temporarily zero, but the economy is not necessarily on the real path. If an economy is along a nominal path different from the real path, it would converge to the external equilibrium only in the long run, but we can still define the money stock or wealth (depending on the capital stock) that would implement the external equilibrium *momentarily.*

The difference between the two concepts is due to the fact that our full model is four dimensional, but two variables are pinned down by an asymptotic (transversality) condition. Both equilibrium concepts are defined by the condition $\frac{d}{dt}h = 0$, which translates into a relationship between the current capital stock k and nominal expenditures x. This latter variable, however, is forward looking, thus it is influenced by the full dynamics of capital accumulation. It means that there is a different value of a which yields the prescribed x under the real equilibrium capital accumulation and under the nominal process. In other words, investors in the real model take into account that $\frac{d}{dt}h = 0$ would hold in the entire future, which translates into a different future path of capital, thus a different current level of a being consistent with $\frac{d}{dt}h = 0$.

Though the real equilibrium concept is more appealing as being labeled "natural", it substantially complicates the comparison of medium- and short-run equilibrium trajectories. Suppose that wealth is above the natural level. Then one would expect an overvalued economy, with lower than equilibrium investment and savings, for example. Higher money holdings, however, do not unambiguously translate into higher nominal spending, lower q and s – since all of these variables are forward looking and depend on the evolution of capital in the future as well. Perhaps surprisingly, the real model may give slower wealth and capital accumulation: in the nominal model, there is a saving motive for consumers, namely, to build up their money stock. When we want to implement the real model within the nominal framework, the required increase in money is achieved by exogenously printed money. Hence consumers can spend more, which then crowds out capital (decreases q) and savings. There are offsetting dynamic effects: lower capital and wealth stocks increase the savings and investment of the economy in the future. As we shall see in our numerical example, the ranking of the nominal and the real economy may switch.

For this reason, we will also discuss the external equilibrium (the $\frac{d}{dt}h = 0$ curve) as a medium-run equilibrium concept.[13] Notice, however, that this concept does not correspond to an entire time path of these variables: the economy cannot go from this period's external equilibrium to next period's external equilibrium using the nominal system's dynamics, since

[13] It is also important to note that the real equilibrium cannot be implemented within a currency board, so the real path is not a feasible benchmark in that case.

wealth and capital accumulation would necessitate a change in money, but that is ruled out by the $\frac{d}{dt}h = 0$ condition. The virtue of this concept is that one can show that the GDP of an overvalued economy (where the relative price is higher than the equilibrium value, which is the same as having more money) is higher than the external equilibrium level of GDP as long as $\alpha > \beta$, and the opposite holds if $\alpha < \beta$. This statement is true both for the fixed price GDP (using the steady state relative price) and for real GDP (using the current relative price, but still measuring GDP in foreign currency). Higher GDP thus also matches an excessive current account deficit[14] (as $\frac{d}{dt}a < 0$). If $a > \beta$, higher than equilibrium GDP implies a smaller than equilibrium value of Tobin's q, thus lower investment and capital inflows. Section 5.3 explains these comparisons in greater details. These results are not true for the real equilibrium path.

5.2 Loglinearization

The Appendix contains the full details of the loglinearization. Here we only collect the results. The following system of differential equations describe the evolution of the four dynamic variables (their log-derivative, or in other words, relative deviation from steady state):

$$\begin{aligned}
\frac{d}{dt}\hat{q} &= \rho\hat{q} + \rho\frac{\beta}{\alpha-\beta}\frac{1}{A}\hat{x} - \rho\frac{\beta}{\alpha-\beta}\frac{B}{A}\hat{k} \\
\frac{d}{dt}\hat{x} &= \psi_2\bar{h}C\hat{x} + 0\cdot\hat{k} - \psi_2\left(1+\bar{h}D\right)da \\
\frac{d}{dt}\hat{k} &= \frac{1}{\delta}\cdot\hat{q} \\
\frac{d}{dt}da &= 0\cdot\hat{q} + \left[\frac{1-\beta}{\alpha-\beta}\bar{w}\frac{1}{A} - \bar{x} - \bar{h}C\left(-\psi_2\left(\bar{a}-\bar{h}\right)+\rho\right)\right]\hat{x} \\
&\quad -\frac{1-\beta}{\alpha-\beta}\bar{w}\frac{B}{A}\hat{k} + \left[-\psi_2\left(\bar{a}-\bar{h}\right)+\rho\right]\left(1+\bar{h}D\right)\cdot da,
\end{aligned}$$

where $A = \frac{1-\beta-B}{\alpha-\beta}$, $B = 1 + \frac{1-\beta}{(1-\lambda)(\beta-\alpha)}$, $C = \frac{\rho}{\rho+\psi_2\bar{h}}$ and $D = \frac{\psi_2}{\rho+\psi_2\bar{h}}$. From here, on, we concentrate on the case when $\bar{b} = 0$, which simplifies the algebra substantially.

The external equilibrium wealth stock is defined by the condition $\frac{d}{dt}\hat{h} = 0$. Using $\hat{h} = \frac{\rho}{\rho+\psi_2\bar{h}}\hat{x} + \frac{\psi_2}{\rho+\psi_2\bar{h}}da$ (equation (38) in the Appendix), we get

$$\begin{aligned}
0 &= \rho\frac{d}{dt}\hat{x} + \psi_2\frac{d}{dt}da = \rho\left(-\psi_2 da + \psi_2\bar{h}\hat{h}\right) + \psi_2\left[\bar{w}\hat{w} - \bar{x}\hat{x} + \rho\left(da - \bar{h}\hat{h}\right)\right] \\
&= \psi_2\left(\bar{w}\hat{w} - \bar{x}\hat{x}\right).
\end{aligned}$$

Since $\bar{w} = \bar{x}$, $\frac{d}{dt}\hat{h} = 0$ is equivalent to $\hat{w} = \hat{x}$. Using (33) and (41), we get $\hat{x} = (1-\beta)\hat{k}$. Along the nominal capital accumulation path (this is where the external equilibrium departs from the

[14] Darvas and Simon (2000) establishes an empirical link between potential output and foreign trade, though they focus on inflationary pressures from excess imports through a weakening of the currency.

real equilibrium), we have $\hat{x} = c_0 \cdot da + c_1 \cdot \hat{k}$, so

$$da^{ext} = \frac{1 - \beta - c_1}{c_0} \hat{k}. \tag{30}$$

5.3 Signing impulse responses and the external equilibrium

The transition matrix (both in the nominal and the real case) must have two convergent and two divergent eigenvalues, since the system is pinned down by two initial conditions (for capital and money) and two terminal conditions (coming from the transversality conditions of consumer and investor optimization). Denote the two eigenvectors corresponding to the convergent roots by $\mathbf{v}_1$ and $\mathbf{v}_2$. Then

$$\hat{k}, da, \tilde{q}, \hat{x} = F\mathbf{v}_1 e^{\lambda_1 t} + G\mathbf{v}_2 e^{\lambda_2 t}.$$

Coefficients F and G are set by the two initial conditions, so they can be expressed as linear combinations of $\hat{k}_0$ and da_0. Then $\hat{q}_0$ and $\hat{x}_0$ are also linear combinations, so

$$\hat{x}_0 = c_0 \cdot da_0 + c_1 \cdot \hat{k}_0,$$

where c_0 and c_1 are (known) functions of the two eigenvectors. One can check whether c_0 is positive, using $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$. We have done this in a simpler version of the model (when there is no access to foreign borrowing or lending), but here we resort only to numerical exercises.[15] In this section, we briefly summarize the impact effect of changing k_0 or a_0 on all relevant variables (i.e., $\frac{dy_0}{dk_0}$ and $\frac{dy_0}{da_0}$).

	wealth shock		capital shock	
	nominal	real	nominal	real
investment	-	-	-	-
expenditure	+	+	+	+
relative price	+	+	+	+
wages	+	+	+	+
rental rates	-	-	-	-
capital-labor ratios	+	+	+	+
NT employment	+	+	-	-
household savings	-	-	+	+

Table 1: Signing impulse responses

[15] We are working on the analytical proofs as well. Basically, the crucial step is to get the impulse responses of the jumping variables x (and q). Once we have the response of x, all other signs follow easily from the loglinearization results of the Appendix.

Table 1 presents the signs of the initial responses to shocks in the state variables, k and a for both the nominal and real models. There is no difference in the directions of the changes between the real and nominal models, although the magnitudes may differ (see the numerical results). The signs are sensible and accord with economic logic. For example, an increase in (nominal) wealth leads to lower investment (q), higher wages and spending (w, x), a reallocation towards non-tradables (p, k_T), increased capital intensity in both sectors (k_T, k_{NT} - not shown), and to lower savings. The results for a capital shock are mostly similar, with the exception of labor allocation and savings. The difference in the behavior of the savings rate is that in the case of a wealth shock households slowly decumulate their access wealth, so the country can run a current account deficit. For a capital shock, households want to accumulate nominal assets to smooth consumption, so they run an initial current account surplus.

The effects of a wealth shock directly apply to a nominal appreciation, and to symptoms of over- and undervaluation. For example, an overvalued economy will have lower investment, higher nominal spending etc., *relative to the benchmark level of nominal wealth.* As we discussed above, the choice of the benchmark is not obvious, so in the next section we present simulations where the comparison points are the real model and the external equilibrium point.

6 Policy exercises

6.1 Choice of parameters

For illustrative purposes, let us fix all the parameters:

$\alpha = 0.8$ – labor intensity of the nontraded sector.

$\beta = 0.5$ – labor intensity of the traded sector. All this starting assumption does is to assume that $\alpha > \beta$, which is a standard choice, though it might not hold in certain countries.

$\lambda = 1/3$ expenditure share on tradables; this is not an unreasonable assumption, particularly if we take into account that traded prices also have large service components.

$\rho(= r^*) = 0.05$ – required real rate of return on capital (assuming that one year is a unit time interval, then it means 5% annually).

$\delta = 3$ – this parameter can be chosen to match a priori expectations about the speed of adjustment. Our choice means that the half-life of an innovation to the capital stock in the real model ($\hat{k} < 0$, $db = 0$) is 10 years.

$\psi_2 = 0.007$. This parameter is higher than the choice (0.000742) of Schmitt-Grohe and Uribe (2003). In case of an emerging economy, it is not unreasonable to assume a risk premium that is more responsive to foreign debt than in an industrial economy. Under our parameter choices, $\bar{h} = 2$, so if we assume that a typical level of foreign debt is $b = 0.5$, then the risk-adjusted interest rate becomes $\rho + \psi_2 \left(e^{0.5} - 1\right) \approx 0.05 + 0.0045 = 0.0545$. The contribution of the risk premium is overall quite reasonable. The most important consequence of choosing ψ_2 is the speed of adjustment following a wealth shock. In the real model with exogenous wages ($\hat{w} = 0$), the wealth-expenditure block becomes a saddle-path stable system with an eigenvalue of $-.1699468010$ (a half-life of 4 years). Notice that a lower ψ_2 would imply a much slower adjustment process.

$\gamma = 0.02$ – based on the steady state relationship $\frac{\bar{h}}{\bar{w}} = \frac{\gamma}{\rho} = 0.4$, our parameters mean that steady state money holdings are equal to 40% of annual income. The choice of γ also influences the speed of adjustment following a wealth shock in the nominal model. Again with exogenous wages, the half-life of a wealth shock becomes 4.8 years. This is slightly higher than for the real model, but the overall contribution of the nominal friction is reasonably low.

6.2 Real and nominal convergence paths

Let us start with results corresponding to the real equilibrium path. Convergence implies an appreciating real exchange rate, if the nontraded sector is more labor-intensive. If labor intensities are equal across sectors, then capital accumulation has no impact on the equilibrium real appreciation, while if the nontraded sector is less labor-intensive, we observe a real depreciation.

All these are fully consistent with international trade theory: as long as capital is scarce, it has a high factor price. In the flexible model, an increase in world interest rates increases the relative price of that sector which uses capital more intensively (inverse Stolper-Samuelson theorem). For high rental rates the NT relative price starts from a low relative price, thus it must increase. It means a positive but vanishing excess inflation (real appreciation)

Figure 3 shows the evolution of capital, the nontraded relative price, bond holdings, and the equilibrium nominal exchange rate (under the assumption of constant money stock). The paths correspond to $\hat{k}_0 = -50\%$, the initial effective capital stock per worker being half of its long-run value,[16] while initial bond holdings are equal to the corresponding external equilibrium wealth level (≈ -1.22).[17] As shown before, there is an extra increase of the relative price: under our choice of parameters, there is an 18% initial price gap due to the low stock of capital. If money is fixed, the required increase in real money holdings can be implemented by a gradual strengthening of the nominal exchange rate (a total of 22%). Though the economy starts from

[16] Clearly such a large deviation from steady state is inconsistent with the loglinear approximation. Given that the numerical solution of the exact system is problematic (due to its saddle nature), we still believe that our numerical exercises are good illustrations of the theoretical results.

[17] As a comparison: the long-run money stock in the nominal economy is 2.

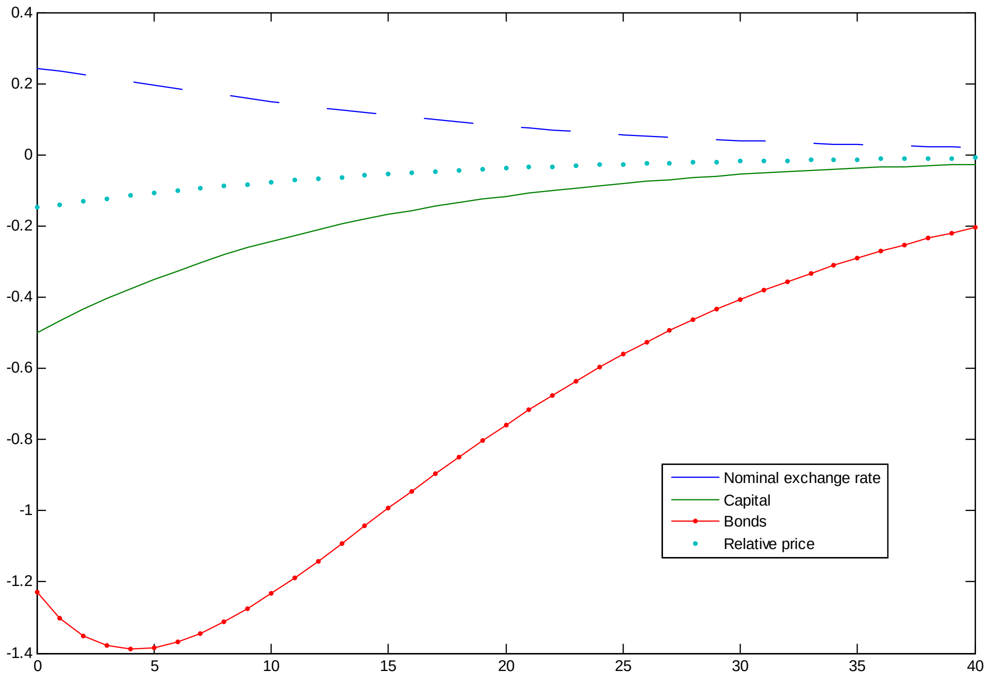


Figure 3: The real convergence process: deviation from steady state

debt, consumers still borrow more, and they start repayments after 4-5 periods. An even lower level of initial wealth would eliminate the initial increase in foreign debt.

Next we show the results of the nominal case (under a currency board regime), and compare them to the real equilibrium path and the pointwise external equilibrium. In both cases, we choose $\hat{k}_0 = 50\%$. The initial value of da is set such that $e = 1$ yields $\hat{p}(t_0) = \hat{p}^{ext}(t_0)$. In other words, if $e = 1$, then the nominal economy initially has the external equilibrium money stock and relative prices. Numerically, it means that $da_0 \approx -1.22$.

Figure 4 depicts the evolution of various variables under the nominal scenario. The curves show the percentage difference from the real path and the pointwise external equilibrium values. Panels 1 and 7 are exceptions: the pointwise external equilibrium has the same capital stock as the nominal equilibrium by definition (panel 1); while there is no money in the real model (panel 7).

The first panel shows the evolution of capital. We can see that the nominal convergence process exhibits faster initial capital accumulation, which reverts later on. This is confirmed by the panel plotting the evolution of Tobin's q, which is proportional to investment. Relative to the external equilibrium, q is undervalued (the deviation is positive). When compared to the real path, q is initially undervalued (higher), and then it switches to an overvaluation.

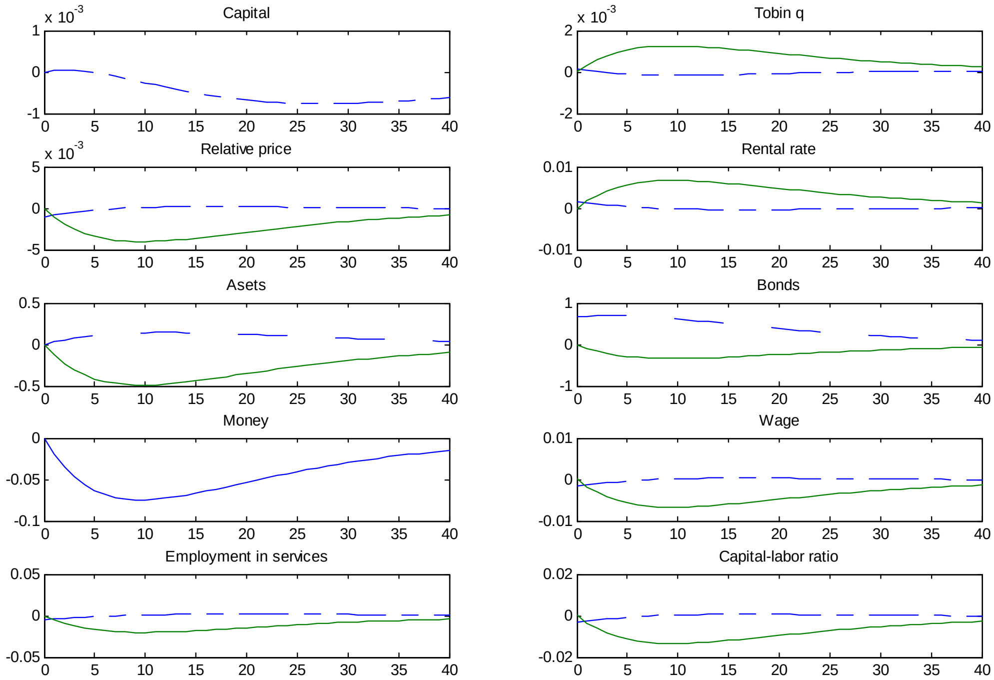


Figure 4: The deviation of the nominal path from the medium-run equilibrium solid line: deviation from the external equilibrium; dashed line: deviation from the real equilibrium

In general, the comparison to the external equilibrium repeats the theoretical discussions about the signs of the impulse responses of these variables to wealth: starting from an external equilibrium position, the economy becomes undervalued for the rest of its convergence path.[18] This translates into higher rental rates, lower asset, foreign bond and money holdings, lower wages, nontraded employment and capital-labor ratios (in both sectors).

When compared to the real economy, we have already seen that there is some extra initial investment in the nominal model. It is also matched by some initial extra savings: in the first 9-10 periods, the nominal economy accumulates more wealth than the real economy. This is the consequence of the extra saving motif in the nominal economy: consumers want to reach the same steady state wealth in both cases. Both economies tend to go into debt for a while, to smooth consumption. In the nominal economy, however, consumers want to consume and accumulate money. This could be achieved by going even more into debt, but that drives the borrowing rate up, working against smoothing. The rest of the variables (the relative price,

[18] The external equilibrium corresponds to the $\frac{d}{dt}\hat{h}\left(\hat{k}\right) = 0$ curve. As $\hat{k}$ grows, this leads to an increase in da and $\hat{h}$ as well. The nominal economy cannot satisfy $\frac{d}{dt}\hat{h}\left(\hat{k}\right) = 0$ and still produce an increase in money holdings, unless there is an extra exogenous increase in $\hat{h}$. If the exchange rate is fixed and the foreign currency value of money is constant, the nominal economy must deviate from the real equilibrium.

wages, nontraded employment and capital-labor ratios) show little difference overall.

7 Some concluding comments

This paper presents a simple theoretical model that addresses the growth process of a small trading economy with a traded and a nontraded sector. Besides presenting a flexible price, intertemporal optimization-based theory of equilibrium real exchange rates and output, the modeling framework is capable of addressing structural properties of a nominal growth process. The model also gives rise to a lasting real effect of nominal exchange rate shocks without price or wage setting frictions.

It is essentially a standard flexible price, two-sector (traded and nontraded), two-factor small open economy growth model with an endogenous risk premium, enriched with money-in-the-utility. Overall, the model highlights that real exchange rate developments and capital accumulation have deep two-sector, two-factor, open-economy determinants – in particular, adding money-in-the-utility and q-theory to a standard two-sector, two-factor open economy model with an endogenous risk premium is enough for short-run non-neutrality of money.

Another notable result is the comovement of investment and savings after a nominal shock, even though investment is financed exclusively form the world capital market. The crucial step is that the nominal exchange rate influences traded prices, while money (or more generally, fixed income securities) are fixed in local currency. In a sense, these assets can be viewed as an "original nominal stickiness".

The results are particularly relevant for understanding the effects of nominal exchange rate movements (in levels or in the rate of change), the impact of the exchange rate regime on the growth process, or the choice of the euro conversion rates for EMU candidates. The framework can also be utilized in assessing the price level implications of fiscal or income shocks. From a theory point of view, it also embeds a Balassa-Samuelson-type effect with a nominal side and gradual capital movements, thus a temporary role for demand. The model also enables the introduction of over- and undervaluation, ant their consequences for nominal and real variables.

Finally, our results show that a multisector model with money-in-the-utility, endogenous risk premium and any real friction that makes the short-run transformation curve nonlinear already implies short- and medium-run non-neutrality of monetary policy and nominal exchange rate shocks. Adding price or wage setting frictions definitely increases the realism, fit and persistence of such a model, but one has to be careful in evaluating the role of price and wage setting in the results.

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Appendix

Loglinearization

From firm-level profit maximization (9)-(10):

$$\begin{aligned} r = (1-\beta)\, k_T^{-\beta} &\implies \hat{r} = -\beta \hat{k}_T \\ w = \beta k_T^{1-\beta} &\implies \hat{w} = (1-\beta)\, \hat{k}_T \\ k_{NT} = \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta} k_T &\implies \hat{k}_{NT} = \hat{k}_T \\ p_{NT} = \frac{\beta}{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)^{\alpha-1} k_T^{\alpha-\beta} &\implies \hat{p}_{NT} = (\alpha-\beta)\, \hat{k}_T. \end{aligned}$$

One can thus express everything in terms of $\hat{p}$:

$$\hat{k}_T = \frac{1}{\alpha-\beta}\hat{p} \tag{31}$$

$$\hat{k}_{NT} = \frac{1}{\alpha-\beta}\hat{p} \tag{32}$$

$$\hat{w} = \frac{(1-\beta)}{\alpha-\beta}\hat{p} \tag{33}$$

$$\hat{r} = \frac{-\beta}{\alpha-\beta}\hat{p}. \tag{34}$$

Notice that $\hat{p}$ can also be interpreted as the misalignment of the real exchange rate (the relative price). Equations (31)-(34) thus express the current capital intensities and factor prices (the technology side of the economy) as a function of the real exchange rate's misalignment.

Next, capital accumulation and the evolution of Tobin's q are driven by

$$\frac{\dot{k}}{k} = \frac{q-1}{\delta} \Longrightarrow \frac{d}{dt}\hat{k} = \hat{q}\frac{1}{\delta} \tag{35}$$

$$\dot{q} = \rho q - r - \frac{(q-1)^2}{2\delta} \Longrightarrow \frac{d}{dt}\hat{q} = \frac{\dot{q}}{\bar{q}} = \rho\hat{q} - \rho\hat{r} = \hat{q}\rho + \rho\frac{\beta}{\alpha-\beta}\hat{p}. \tag{36}$$

Since the stance of fiscal policy is described by $\tau = 0$, wealth accumulation is governed by

$$\dot{a} = w - x + i\,(a-h)\,(a-h)\,.$$

Then we can loglinearize the wealth accumulation equation:

$$\begin{aligned}\dot{a} &= \frac{w}{\bar{w}}\bar{w} - \bar{x}\frac{x}{\bar{x}} + i\,(a-h)\left(da + \bar{a} - \hat{h}\bar{h} - \bar{h}\right) \\ &= \hat{w}\bar{w} - \bar{x}\hat{x} + [i\,(a-h) - \rho]\left(da + \bar{a} - \hat{h}\bar{h} - \bar{h}\right) + \rho\left(da - \hat{h}\bar{h}\right).\end{aligned}$$

Let us work with the same choice of $i\,(a-h)$ as Schmitt-Grohe and Uribe (2003):

$$\rho + \psi_2\left(e^{-\left(b-\bar{b}\right)} - 1\right).$$

Then

$$\frac{d}{dt}da = \dot{a} = \frac{1-\beta}{\alpha-\beta}\bar{w}\hat{p} - \bar{x}\hat{x} - \psi_2\left(\bar{a}-\bar{h}\right)\left(da - \bar{h}\hat{h}\right) + \rho\left(da - \hat{h}\bar{h}\right). \tag{37}$$

We still need to obtain an expression for $\frac{d}{dt}\hat{x}$, plus express $\hat{p}$ and $\hat{h}$ in terms of the other hat variables (it will be a function of $\hat{x}$ and $\hat{k}$). Then we have the loglinearization of entire system, with 2 state and 2 jumping variables: k and a are state variables (initial conditions), while x and q are jumping variables. The first corresponds to effective nominal expenditures. The reason for changing variables (from $\hat{p}$ to $\hat{x}$) is that the law of motion for $\hat{p}$ is too complicated, while we get an additional zero element in the transition matrix with $\hat{x}$.

Loglinearizing (16):

$$\begin{aligned}\dot{x} &= \psi_2 x\left(e^{-\left(b-\bar{b}\right)} - 1\right) = -\psi_2\bar{x}\,(\hat{x}+1)\left(b-\bar{b}\right) = -\psi_2\bar{x}\left(b-\bar{b}\right) = -\psi_2\bar{x}\left(da - \bar{h}\hat{h}\right) \\ \frac{d}{dt}\hat{x} &= \frac{\dot{x}}{\bar{x}} = -\psi_2\left(da - \bar{h}\hat{h}\right).\end{aligned}$$

Loglinearizing (6):

$$\begin{aligned}h &= \frac{\gamma x}{i\,(a-h)} \\ \hat{h} &= \hat{x} - \hat{\imath} = \hat{x} - \frac{-\psi_2}{\rho}db = \hat{x} + \frac{\psi_2}{\rho}\left(da - \bar{h}\hat{h}\right) \\ \hat{h} &= \underbrace{\frac{\rho}{\rho+\psi_2\bar{h}}}_{C}\hat{x} + \underbrace{\frac{\psi_2}{\rho+\psi_2\bar{h}}}_{D}da.\end{aligned} \tag{38}$$

The very last thing is to get $\hat{p}$. Loglinearize the definition of x :

$$\hat{x} = \hat{c} + (1-\lambda)\,\hat{p}.$$

Using the definition of c and the consumption optimality condition (3), we get

$$\begin{aligned} c &= c_T^{\lambda} c_{NT}^{1-\lambda} = \left(\frac{\lambda}{1-\lambda}\right)^{\lambda} p^{\lambda} c_{NT} \\ \hat{x} &= \hat{p} + \hat{c}_{NT}. \end{aligned}$$

From market clearing in nontraded goods:

$$\begin{aligned} c_{NT} &= l k_{NT}^{1-\alpha} \\ \hat{c}_{NT} &= \hat{l} + \frac{1-\alpha}{\alpha-\beta}\hat{p} \\ \hat{x} &= \frac{1-\beta}{\alpha-\beta}\hat{p} + \hat{l}. \end{aligned}$$

From capital market clearing with producer maximization, we get

$$l = \frac{k - k_T}{k_{NT} - k_T} = \frac{1}{1 - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}\left(\frac{k}{k_N} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right).$$

Loglinearization then yields

$$\begin{aligned} \bar{l} + dl &= \frac{1}{1 - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}\left[\frac{\bar{k}}{\bar{k}_{NT}}\left(1 + \hat{k} - \hat{k}_{NT}\right) - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right] \\ \hat{l} &= \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}\left(\hat{k} - \hat{k}_{NT}\right) = \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}\left(\hat{k} - \frac{1}{\alpha-\beta}\hat{p}\right), \end{aligned} \tag{39}$$

and finally,

$$\hat{x} = \frac{1-\beta}{\alpha-\beta}\hat{p} + \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}\left(\hat{k} - \frac{1}{\alpha-\beta}\hat{p}\right).$$

In steady state:

$$\begin{aligned} \bar{k}_T &= \left(\frac{1-\beta}{\rho}\right)^{1/\beta} \\ \bar{k}_{NT} &= \bar{k}_T \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta} \\ \bar{p} &= \frac{\beta}{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)^{\alpha-1}\left(\frac{1-\beta}{\rho}\right)^{\frac{\alpha-\beta}{\beta}} \\ \bar{c} &= \left(\frac{\lambda}{1-\lambda}\right)^{\lambda}(\bar{p})^{\lambda}\,\bar{l}\bar{k}_{NT}^{1-\alpha} \\ \bar{w} &= \beta\bar{k}_T^{1-\beta} = \chi\bar{p}^{1-\lambda}\bar{c}/\gamma \end{aligned}$$

Plugging everything into this last expression yields

$$\alpha(1-\lambda)=\bar{l},$$

which implies

$$\frac{k}{k_{NT}}=\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\frac{1+\alpha\lambda-\beta\lambda-\alpha}{1-\beta}, \tag{40}$$

so the expression for $\hat{x}$ is

$$\begin{aligned}\hat{x} &= \frac{1-\beta}{\alpha-\beta}\hat{p}+\left[1+\frac{1-\beta}{(1-\lambda)(\beta-\alpha)}\right]\left(\hat{k}-\frac{1}{\alpha-\beta}\hat{p}\right) \\ \hat{x} &= A\hat{p}+B\hat{k}\end{aligned} \tag{41}$$

which can be inverted to

$$\hat{p}=\frac{1}{A}\hat{x}-\frac{B}{A}\hat{k}.$$

The log-linearized dynamic system is therefore

$$\begin{aligned}\frac{d}{dt}\hat{q} &= \rho\hat{q}+\rho\frac{\beta}{\alpha-\beta}\frac{1}{A}\hat{x}-\rho\frac{\beta}{\alpha-\beta}\frac{B}{A}\hat{k}+0\cdot da \\ \frac{d}{dt}\hat{x} &= 0\cdot\hat{q}+\psi_2\bar{h}C\hat{x}+0\cdot\hat{k}-\psi_2\left(1+\bar{h}D\right)da \\ \frac{d}{dt}\hat{k} &= \frac{1}{\delta}\cdot\hat{q}+0\cdot\hat{x}+0\cdot\hat{k}+0\cdot da \\ \frac{d}{dt}da &= 0\cdot\hat{q}+\left[\frac{1-\beta}{\alpha-\beta}\bar{w}\frac{1}{A}-\bar{x}-\bar{h}C\left(-\psi_2\left(\bar{a}-\bar{h}\right)+\rho\right)\right]\hat{x} \\ &\quad -\frac{1-\beta}{\alpha-\beta}\bar{w}\frac{B}{A}\hat{k}+\left[-\psi_2\left(\bar{a}-\bar{h}\right)+\rho\left(1+\bar{h}D\right)\right]da.\end{aligned}$$

The stability of the system is determined by the signs of (the real part of its) eigenvalues, while general solutions can be obtained as linear combinations of its eigenvectors. Given that the investment and consumption optimization problem is also subject to a transversality condition, two initial conditions (on h and k) pin down the system. This means that we must have two stable (with a positive real part) and two unstable eigenvalues.

Turning to the real model, the loglinearization of $\hat{q}$ and $\hat{k}$ remains unchanged. Loglinearizing (16):

$$\frac{d}{dt}\hat{x}=\frac{\dot{x}}{\bar{x}}=\psi_2\frac{x}{\bar{x}}\left[e^{-\left(b-\bar{b}\right)}-1\right]=-\psi_2\left(\hat{x}+1\right)\left(b-\bar{b}\right)=-\psi_2 db,$$

while

$$\frac{d}{dt}db=\frac{d}{dt}da=\dot{a}=\frac{1-\beta}{\alpha-\beta}\bar{w}\hat{p}-\bar{x}\hat{x}-\psi_2\bar{b}db+\rho db.$$