Monetary Policy and the evolution of US economy:
1948-2002

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Abstract

This paper investigates the relationship between monetary policy and the changes experienced in the US economy using a small scale New-Keynesian model. We estimate the model with Bayesian techniques and examine the stability of policy parameter estimates and the transmission mechanism of policy shocks. The model fits well the data and produce forecasts comparable to those of unrestricted alternatives. The parameters of the policy rule, the variance and the transmission mechanism of policy shocks have been remarkably stable. Instability in the Phillips curve trade-off is caused by instability in the elasticity of the labor supply parameter. Posterior estimates imply that a low and stable amount of price stickiness suffices.

JEL classification no: E52, E47, C53

Key words: New Keynesian model, Bayesian methods, monetary policy, stability.

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1 Introduction

Many researchers have noted the US economy has undertaken significant changes in the last 30 years. Blanchard and Simon (2000), McConnell and Perez Quiroz (2000) and Stock and Watson (2002) have documented a marked decline in the variance of real activity and inflation and in the persistence of inflation.

Some authors, in particular Taylor (1998), Sargent (1999) and Clarida, Gali and Gertler (1999), have attributed these changes to a permanent alteration in the relative weight that output and inflation have in the objective function of the monetary authority. The popular version of the story runs as follows: the run-up of inflation in the 1970s occurred because the authorities believed that there was an exploitable trade-off between inflation and output. Since output was low following the two oil shocks, the temptation to inflate to bring output back, or above its potential level, was strong. Between keeping inflation low (and output low) or inflation high (and output high), monetary authorities systematically choose the latter option. Hence, inflation in the long run turned out to be higher while output simply settled to its potential level. Since the 1980s, the perception of the output-inflation trade-off has changed. The Fed has learned that it was not exploitable and concentrated on the objective of fighting inflation. A low inflation regime ensued and the larger predictability of monetary policy contributed to make the macroeconomic environment more stable.

While prevalent, this view underscoring the power of monetary policy in influencing the economy is not fully shared in the profession. In fact, several researchers claim that monetary policy has not experienced any permanent regime switch since the late 1970s; that the same policy rule characterizes most of the post WWII experience; that monetary policy has little influence on output fluctuations; and that good luck, as opposed to good policies, is responsible for the observed outcome (see e.g. Bernanke and Mihov (1998), Leeper, Sims and Zha (1998), Hanson (2001), Leeper and Zha (2003)). Others, have proposed "real" reasons to explain the observed changes in inflation and output dynamics (see e.g. Ireland (1999) or McConnell and Perez Quiroz (2000)).

Recently, important progress has been made in the investigation of these issues using models where coefficients are explicitly allowed to vary. Sargent and Cogley (2001) and (2003), who were the first to use a reduced form version of a time varying coefficient model, find evidence that supports the causation story running from monetary policy changes to changes in the rest of the economy. Canova and Gambetti (2004) and Sims and Zha (2004), who estimate structural time varying coefficients VAR models, find little posterior evidence supporting this hypothesis. Since these two papers only use a minimal amount of the restrictions implied by the current generation of DSGE models when deriving structural relationships, one may wonder how truly structural is the monetary policy reaction function they estimate and whether the stability found was really not the result of a gross misspecification of crucial relationships.

Ireland (2001) and Boivin and Giannoni (2002), who explicitly condition their analyses on a particular estimated small scale DSGE model, find evidence of instability in many reduced form relationships and attribute this instability to monetary policy, but limit their
comparison to arbitrarily chosen subsamples. Because output growth (inflation) display a U shape (inverted U shape) pattern over the last 30 years, the conclusions one draws depend on the selected break point. Hence, the evidence these authors provide is not fully convincing.

This paper attempts to provide structural evidence on the role of monetary policy in shaping the changes observed in the US economy by recursively estimating a small scale DSGE model with Bayesian techniques. Bayesian methods, which have become a popular tool to bring DSGE models to the data, thanks to the work of Schorfheide (2001), Smets and Wouters (2003), Schorfheide and Del Negro (2003) and Rabanal and Rubio (2003), have a major advantage over traditional maximum likelihood techniques: they work well even when the model is a "false" description of the data generating process. This is important when dealing with DSGE models since, despite recent attempts to make them more realistic, they are still highly stylized; many important relationships are modeled with black-box frictions; and measurement errors or ad-hoc shocks are used to dynamically span the probabilistic space of the data. In fact, parameter estimates obtained with maximum likelihood are often unreasonable or on the boundary of the parameter space and tricks of several sorts (e.g. fixing parameters which are hard to estimate, or arbitrarily constraining the search for the maximum) are used to produce economically sensible estimates. None of these tricks is necessary when Bayesian methods are employed. In fact, we will estimate a highly stylized version of a New-Keynesian model, use a relatively loose prior specifications and still be able to draw useful conclusions on the issues at stake. A Bayesian framework is also preferable to an indirect inference estimation approach (which e.g. finds structural parameters matching impulse responses) in two respects: all the information of the model is efficiently used; the trade-off between identifiability and nonlinearities is dealt with in a more transparent and informative way.

The model we consider is basic and does not feature any of the standard frictions typically included to produce a good match with the data. Nevertheless, we show that when the priors are appropriately chosen, and the policy rule schrewdly specified, the statistical fit is satisfactory, the economic fit reasonable and the out-of-sample forecasting performance comparable to the one obtained with more densely parametrized, unrestricted VAR models.

We estimate the model over 19 different samples, most of which of the same length, spanning the period 1948-2002 and analyze the evolution of the posterior distributions of the structural parameters and of interesting economic functions of them. Our analysis is geared to shed light on three main issues. First, we would like to know if the posterior distribution of the coefficients of the monetary policy rule has significantly and permanently changed over time making, e.g., the reaction of interest rates to inflation stronger than the one to output. Second, we would like to know whether there are time variations in the posterior distribution of the responses to policy shocks and/or the variance of the unsystematic component of interest rates. Intuitively, even if while the reaction function of the Fed has been stable, policy shocks may have had very different effects over time because of structural changes in the rest of the economy. Third, we are interested in investigating the features of the posterior distribution of the Phillips curve trade-off over samples and in analyzing
whether the observed evolution is due to variations in the "deep" parameters of household preferences, or to changes in the sluggishness of price movements over samples.

Our results are clear cut and broadly agree with the evidence recently reported in structural time varying coefficient VAR analyses. We find that the posterior distribution of the policy parameters is relatively stable over samples and no evidence suggesting a permanent regime shift, from lax to tough anti-inflation stance, in the 1980's or at any other date in the sample. We also find a remarkable stability in the features of the transmission mechanism of monetary policy disturbances and no posterior evidence that the variance of the policy shocks has systematically decreased over time. These similarities stand in sharp contrast with the significant variations obtained in the posterior distribution of the parameters regulating private sector behavior. However, because of the highly non-linear nature of impulse responses, changes in the posterior distribution of private sector structural parameters do not necessarily translate in measurable changes in the shape or the sign of the responses to monetary shocks. We also find that the posterior distribution of the Phillips curve trade-off varies dramatically in location over time; that variations of the posterior distribution of price stickiness are of minor importance and that the posterior distribution of some of the parameters of agents’ preference move significantly over samples, both economically and statistically. Overall, it appears that the role of monetary policy in shaping the observed changes in the US economy has been overemphasized and that investigations attempting to understand the reasons behind the movements in the parameters of agents’ preferences have the potential to shed important light on the dynamics of the post WWII US economy.

The rest of the paper is organized as follows. Section 2 presents the model, describes the estimation technique, sets up the prior and discusses economic, statistical and forecasting tests used to evaluate the quality of the approximation of the model to the data. Section 3 presents the estimation results. Section 4 verifies various hypotheses about the role of monetary policy. Section 5 concludes. Technical details concerning the estimation appear in the appendix.

2 The Model

The model we consider in this paper is a standard New-Keynesian three equation model, composed of a log-linearized Euler equation, a forward looking Phillips curve and a monetary policy rule which expresses the policy instrument (interest rate) as a function of the lagged interest rate, lagged output gap and lagged inflation. Each equation is driven by an idiosyncratic shock: the one attached to the Euler equation is interpreted as a demand shock (shock to preferences or random government expenditure if this enters the utility of the agents); the one attached to the Phillips curve is interpreted as a cost push shock or as an error in measuring marginal costs; finally, the one attached to the policy rule is interpreted as a monetary policy shock.

The system in log-linear form is:

\[ x_t = E_t(x_{t+1}) - \frac{1}{\varphi} (i_t - E_t(\pi_{t+1})) + e_{1t} \]  \hspace{1cm} (1)
\[ \pi_t = \beta E_t \pi_{t+1} + (\varphi + \vartheta_n)(1 - \zeta_p)(1 - \beta \zeta_p) x_t + e_{2t} \quad (2) \]

\[ i_t = \phi_r i_{t-1} + (1 - \phi_r)(\phi_{\pi} \pi_{t-1} + \phi_x x_{t-1}) + e_{3t} \quad (3) \]

where \( \zeta_p \) measures the degree of price stickiness (in a Calvo staggered price setting), \( \beta \) is the discount factor, \( \varphi \) is the parameter of constant relative risk aversion, \( \vartheta_n^{-1} \) is the elasticity of labor supply, and \( (\phi_r, \phi_{\pi}, \phi_x) \) are the parameters of the monetary policy rule. Here \( x_t \) is the output gap (deviation of output from its flexible price potential), \( \pi_t \) is the inflation rate and \( i_t \) is the nominal interest rate. While the model we use is driven by three shocks, one could also extend the specification to consider a fourth source of uncertainty (technology disturbances) by explicitly modelling the economic relationships entering the output gap variable. We assume that both \( e_{1t} \) and \( e_{2t} \) are AR(1) processes with persistence \( \rho_1, \rho_2 \) and standard errors \( \sigma_1, \sigma_2 \), respectively, while \( e_{3t} \) is assumed to be iid with standard error \( \sigma_3 \).

A system of equations like (1)-(3) can be obtained in a standard dynamics stochastic general equilibrium model with sticky prices, monopolistic competition and preferences which are additive in consumption and leisure when the only productive factor is labor (see e.g. Clarida, Gali and Gertler (1999)). The specification of the monetary policy rule is consistent with the idea that the monetary authority has available only lagged values of the output gap and of inflation when deciding the current interest rate. Such a specification differs from the typical Taylor rule employed in the literature, where the nominal interest rate is allowed to contemporaneously react to the output gap and inflation. We choose this specification for two reasons. First, given existing informational lags, it seems reasonable to assume that the central bank takes one period to react to the development in the private side of the economy. Furthermore, when estimating the contemporaneous coefficients of a standard Taylor rule in a VAR, these turn out to be typically small and, at times, insignificant. Second, a specification which makes interest rates react contemporaneously to output and inflation is statistically unsatisfactory. In fact, in our simple model such a specification for the policy rule forces the smoothness parameters \( \phi_r \) to capture all the missing dynamics and results in estimates of \( \phi_{\pi} \) which is close to and statistically indistinguishable from 1 (see also e.g. Ireland (2004)). Alternatively, if policy smoothness is kept within a reasonable bound, the specification counterintuitively leaves a great deal of serial correlation (and cross disturbance correlation) in the policy shock \( e_{3t} \).

Although the AR(1) assumption on \( e_{1t} \) and \( e_{2t} \) is standard and can account for omitted variables, it is somewhat arbitrary in our context. At a preliminary stage of this project we have also experimented with a Phillips’ curve which also allows for backward looking dynamics and with an Euler equation featuring habit persistence in consumption, while maintaining the AR(1) assumption on the disturbances. It turns out that point estimates of the backward looking parameters in the two equations are small and roughly of the same magnitude as in Ireland (2004). Furthermore, except for the serial correlation parameters, which decrease, estimates of the two specifications are indistinguishable. Consequently, we prefer the simpler version which, although more parsimonious, still allows us to draw important conclusions from the investigation. The only drawback is that the dynamics of the cost push and demand disturbances may not be completely structural.
Orphanides (2001) has emphasized that the output gap variable may be corrupted with measurement error and that such error could be considerably reduced if the growth rate of output is used. We examine whether our conclusions are sensitive to the measurement of the output gap in section 4. To anticipate, none of our results depends on this choice.

Several authors, including Smets and Wouters (2003), Rabanal and Rubio (2003) and others, have specified more complicated and realistic structures which allow for wage and price indexation and additional shocks and frictions. We do not follow this route for two reasons. First, as shown below, the model captures sufficiently well the dynamics of output gap, inflation and interest rates observed in the US without these features. Second, since the scope of this paper is to examine the contribution of monetary policy to the observed changes in the output and inflation process in the US, the most stripped down specification suffices.

The model contains 12 parameters, 7 structural ones $\alpha_1 = (\beta, \varphi, \theta, \zeta, \phi, \phi_x, \phi_x')$ and 5 auxiliary ones, $\alpha_2 = (\rho_1, \rho_2, \sigma_1, \sigma_2, \sigma_3)$. Our exercise is geared to obtain posterior distributions of $\alpha_T = (\alpha_1, \alpha_2)$ over different samples $T$ and to compare the time series properties of the posterior distributions of a subset of the parameters and of interesting economic functions of them.

The system (1)-(3) can be rewritten as a VAR(1): $Gy_{t+1} = Hy_t + J\epsilon_t$ where $y_{t+1} = (\pi_t, x_t, i_t, \pi_{t+1}, x_{t+1})$ and $v_t = [0,0,\epsilon_3, \epsilon_1, \epsilon_2]$ and can be solved using standard log-linear methods (e.g. Blanchard and Kahn (1980)). Its solution has a state space format

\[
\begin{align*}
y_{1t+1} &= A_1(\alpha)y_{1t} + A_2(\alpha)\epsilon_t \\
y_{2t} &= A_3(\alpha)y_{1t}
\end{align*}
\]

where $y_{2t} = [\pi_t, x_t, i_t], y_{1t} = [\pi_{t-1}, x_{t-1}, i_{t-1}, \epsilon_1, \epsilon_2, \epsilon_3]$ and the matrices $A_i(\alpha), i = 1, 2, 3$ are complicated nonlinear functions of the structural parameters $\alpha$.

Bayesian estimation of (4) and (5) is relatively simple: given some $\alpha$, we compute the likelihood of the model, denoted by $f(y_T|\alpha)$, by means of the Kalman filter and the prediction error decomposition. Then, for any specification of the prior distribution, denoted by $g(\alpha)$, the posterior distribution for the parameters of the model is $g(\alpha|y_T) = g(\alpha)f(y_T|\alpha)$. The analytical computation of this function is impossible in our setup since the denominator of the expression, $f(y_T)$, can be obtained only integrating $g(\alpha)f(y_T|\alpha)$ with respect to $\alpha$, which is a 12 dimensional vector. To obtain numerically a sequence from this unknown posterior distribution, we employ the Metropolis-Hastings algorithm. Roughly speaking, given some $\alpha_0$ and a transition function satisfying appropriate regularity conditions, we can produce a sequence for the unknown posterior, iterating on this transition function after discarding an initial burn-in period of draws. Once the sequence is obtained, we use kernel methods to estimate the posterior density (coordinate by coordinate). We then compare the posterior we obtain with an estimate of the prior density, also obtained with kernel methods from a similar sequence of draws. Details on the algorithm, on the selected transition function, on the criteria used to check convergence and on other choices made are in the appendix.
Table 1: Prior Moments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Standard error</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
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<td>$\beta$</td>
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<td>0.9801</td>
<td>0.0139</td>
<td>0.8969</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\varphi$</td>
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<td>1.9989</td>
<td>0.7481</td>
<td>-0.0378</td>
<td>4.4815</td>
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<tr>
<td>$\vartheta_n$</td>
<td>4.0411</td>
<td>4.0262</td>
<td>1.2527</td>
<td>-1.022</td>
<td>8.2345</td>
</tr>
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<td>$\zeta_p$</td>
<td>0.6873</td>
<td>0.6683</td>
<td>0.1782</td>
<td>0.0544</td>
<td>0.9932</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>0.7400</td>
<td>0.7192</td>
<td>0.1560</td>
<td>0.1193</td>
<td>0.9957</td>
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<tr>
<td>$\phi_\pi$</td>
<td>1.6929</td>
<td>1.6912</td>
<td>0.4999</td>
<td>-0.0012</td>
<td>3.5896</td>
</tr>
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<td>$\phi_x$</td>
<td>0.9950</td>
<td>0.9957</td>
<td>0.2005</td>
<td>0.1674</td>
<td>1.7499</td>
</tr>
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<td>$\rho_1$</td>
<td>0.7582</td>
<td>0.7516</td>
<td>0.0937</td>
<td>0.3615</td>
<td>0.9796</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.7571</td>
<td>0.7497</td>
<td>0.0947</td>
<td>0.3144</td>
<td>0.9745</td>
</tr>
<tr>
<td>$\sigma_2^1$</td>
<td>0.0171</td>
<td>0.0202</td>
<td>0.0014</td>
<td>0.0001</td>
<td>0.1228</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>0.0164</td>
<td>0.0198</td>
<td>0.0014</td>
<td>0.0001</td>
<td>0.1228</td>
</tr>
<tr>
<td>$\sigma_2^3$</td>
<td>0.0166</td>
<td>0.0197</td>
<td>0.0013</td>
<td>0.0002</td>
<td>0.1231</td>
</tr>
</tbody>
</table>

We assume that the prior distribution can be factored as $g(\alpha) = \prod_{i=1}^{12} g(\alpha_i)$, and assume that $eta \sim Beta(98,2)$, $\varphi \sim N(2.0,0.75^2)$, $\vartheta_n \sim N(4.1,25)$, $\zeta_p \sim Beta(4,2)$, $\phi_r \sim Beta(5,2)$, $\phi_\pi \sim N(1.7,0.35^2)$, $\phi_x \sim N(1.0,0.15^2)$, $\rho_1 \sim Beta(17,3)$, $\rho_2 \sim Beta(17,3)$, $\sigma_2^1 \sim Gamma(2,0.001)$, $\sigma_2^2 \sim Gamma(2,0.001)$, $\sigma_2^3 \sim Gamma(2,0.001)$. A summary of the properties of these priors is in table 1, where we report the mean, the median, the standard error, the minimum and the maximum values obtained from a sample of 5000 observations.

The prior mean for each coefficient is typically located around standard calibrated values. Furthermore, the densities we have selected, although proper, are sufficiently non-informative. For example, the risk aversion parameter $\varphi$ has an a-priori range of $[0,4.5]$, the smoothness parameter $\phi_r$ a range $[0.11, 0.99]$ while the two policy parameters, $\phi_\pi$ and $\phi_x$, can assume values in the range $[0, 3.5]$ and $[0.16, 1.75]$, respectively. The prior range for the stickiness parameter $\zeta_p$ is also large and values from 0.05 to 0.99 have a-priori positive probability. We have selected "loose" priors in order to minimize subjective information - here limited to produce bounds on the priors consistent with theoretical and empirical considerations - and to allow the posterior to move away from the prior if the data is sufficiently informative. Since we maintain the same prior in every sample, differences in the location and in the shape of the posterior distribution will indicate that there is substantially different information in different samples. Note that, contrary to what is typically done in the literature, we do not preliminary calibrate any of the "difficult" parameters: instead, we let the data tell us which parameter is identifiable and which is not.

The data we use covers quarterly observations on the output gap (here proxied by GDP in deviation from a linear trend), CPI inflation and the Federal funds rate for the period 1948:Q1-2002:Q1. The source of the data is the FREDII databank of the Federal Reserve...
We have checked the quality of the model’s approximation to the data in several ways. First, we have conducted a simple forecasting exercise, comparing the fit of the model, measured here by the marginal likelihood, to the fit obtained with a three variable VAR and a three variable BVAR, endowed with a Minnesota prior. Since there are 12 parameters in the model, we specify VARs and BVARs with one and two lags, for a total of 15 (nine autoregressive and 6 covariance) and 24 (18 autoregressive and 6 covariance) parameters, for comparison. Second, we have visually examined the fit of the interest rate equation, plotting the actual nominal interest rate path and the 68 percent posterior band for the interest rate path predicted by the model. Third, we have checked for violations of the Euler condition. That is, we have examined whether lagged values of the output gap, of inflation, or of the real interest rate comove with the quasi-differenced residual of the Euler equation, given the posterior draws for the parameters. Since these three statistics examine different statistical and economic aspects, they provide useful information on the properties and the fit of the model to the data.

3 Full sample estimation

We present prior and posterior estimates of the densities of parameters for the 1948:1-2002:1 sample in figure 1. Dotted lines correspond to prior distributions and solid lines to posterior distributions. Few features of the figure deserve comments. First, the data appears to be informative. In fact, for 10 of the 12 parameters the posterior distribution has a spread which is smaller than the prior spread. The only two parameters for which this is not the case are those regulating price stickiness ($\zeta_p$), and policy smoothness ($\phi_r$). In a few instances, the location of the distribution also changes. For example, the output gap parameter in the policy rule has a posterior distribution whose central tendency is somewhat higher than the one of the prior while the opposite is true for the policy smoothness parameter. Interestingly, the posterior distribution of $\phi_\pi$ is centered around 2 and there is little mass in the area below 1, suggesting that the case for indeterminacy in the sample is relatively small. In fact, only in 0.12 percent of the simulations $\phi_\pi$ falls below one.

The posterior distribution of the two autoregressive parameters is centered at around 0.85 and there is little posterior mass on the area above 0.96. That is, the model has some internal propagation mechanism so that to match the unit root-like dynamics of output and inflation, no unit-root-like exogenous processes are needed (contrary, e.g. to Smets and Wouters (2003)). The data also imply that, a-posteriori, labor supply is sufficiently inelastic (the posterior mean of $\vartheta_n$ is 3.74 with standard deviation equal to 0.27) and that agents have mild aversion toward risk (the posterior mean of $\varphi$ is 1.72 with standard deviation equal to 0.30).

The data does not appear to be very informative about the price stickiness parameter $\zeta_p$: in fact, prior and posterior distributions overlap almost entirely. This could be due to lack of information in the data or to the fact that the prior is too much data based - a great deal of data information has gone into building the prior moments so that the prior and
Figure 1: Prior (dotted) and Posterior (solid) densities
the likelihood coincide. The sensitivity analysis we conduct below allows us to distinguish these two possibilities. Finally, the shocks to the three equations have similar posterior variances. Taken at face value these distributions imply that monetary, real demand and supply impulses have similar magnitude in the sample, a result which agrees with the structural VAR estimates of Canova and De Nicolo (2002), but contrasts, for example, with both the common wisdom that monetary disturbances have been a minor source of cyclical fluctuations in the US economy and the maximum likelihood estimates of Ireland (2004). However, if some of the variables are measured with a large error, this result could simply reflect the fact that measurement errors dominate in size and variability structural errors.

The forecasting performance of the estimated model is reasonable although less astonishing than the one reported in Smets and Wouters (2004). In fact, the marginal likelihood of the model is -2.0145, those of a VAR(1) and of a VAR(2) are -1.7645 and -1.6723, respectively, while those of a BVAR(1) and of a BVAR(2) are -1.7214 and -1.5164. To put these numbers into perspective, note that our model is at most, 30 percent worse than the best, densely parametrized alternative specification we consider. The model is inferior to the VAR models primarily because the lagged nominal interest rate, which is missing from (2), enters the inflation equation of the VAR with a significant negative sign.

The model fits reasonably well the Euler equation and only in 0.5 percent of the draws we find that the residuals of the equation violate orthogonality conditions. In these few cases, the information contained in the past output gap appears to be important in explaining deviations from the null. The model also fits reasonably well the policy equation. Figure 2 presents the predicted and actual interest rate path from 1950 to 2002. The actual interest rate path is always inside the posterior 68% band for the interest rate predicted by the model and, except for the 1965-1975 period, where the upper part of the band is significantly above the actual path, posterior 68% bands are reasonably tight and follow the ups and downs of the actual nominal interest rate. Notice also that the model predicts remarkably well the drastic fall in interest rates occurred in 2001.

The model is relatively poor in matching inflation dynamics. For example, the posterior mean of the coefficient on the output gap is only 0.5 with a standard deviation equal to 0.32, implying that the dynamics of inflation are represented by a near-random walk. Also, we find that the residuals of the equation are generally correlated with lagged values of the nominal rate. These observations confirm results obtained with other estimation techniques (see Gali and Gertler (1999) or Linde (2002)), and suggest that the New-Keynesian Phillips curve, where the output gap proxies for marginal costs, has hard time to account for the dynamics of inflation. It is worthwhile to stress that adding a backward looking component to the equation (for example, assuming inflation indexation) will improve the dynamic fit decreasing residual serial correlation (see e.g. Rabanal and Rubio (2003)) but will not alter the basic conclusion that posterior mean estimates implies minimal effects from marginal costs to inflation. In other words, the specification does not fit the data well not because the dynamics are backward (as opposed to forward) looking but because estimates imply that inflation only weakly responds to the endogenous movements in the output gap, induced by changes in the marginal costs. However, as we will see below, there are subsamples where
estimates imply that the matching is both statistically and economically adequate.

We have checked the robustness of our posterior estimates to changes in the prior distribution. This exercise is important for two reasons. First, since there are posterior distributions which lie on top of the priors, we can distinguish if this occurs because the prior is too much in agreement with the data or because the likelihood is uninformative. Second, since the priors have subjectively large dispersions, it is important to know how the posterior distributions change if we are less uncertain about the prior range of values the parameters of the model must take. Table 2 reports the mean and the standard deviation of the posterior in the baseline case and in two alternative specifications, obtained making the prior progressively more informative. We have done this using priors which maintain the location fixed and rescale the probability densities after reducing the prior ranges by 10 and 20 percents. Note that in the limit, when priors are very tight, sample information plays no role in determining the posterior distribution. Therefore, the degenerate posteriors one obtains in this case, trivially corresponds to those produced calibrating the parameters to a single value. Following Geweke (1998), posterior draws from the new distribution are obtained reweighting the posterior draws obtained in the baseline case with \( w(\alpha) = \frac{g^i(\alpha)}{g^B(\alpha)} \) where \( g^i(\alpha) \) is the new prior and \( g^B(\alpha) \) is the baseline prior.

Table 2 indicates that the posterior results are reasonably invariant to changes in the prior specification. When the spread of the prior is decreased by 10 percent, the posterior means are broadly unchanged except for those of \( \phi_r \) and \( \phi_{\pi} \). In general, posterior estimates become much more precise when the prior spread is decreased by 10 percent. Decreasing the prior spread by 20 percent significantly changes the location of \( \theta_{n} \) and significantly increases the posterior mean of the smoothness parameters \( \phi_r \). The standard deviations
<table>
<thead>
<tr>
<th></th>
<th>Basic</th>
<th>90 percent Spread</th>
<th>80 percent Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Standard dev.</td>
<td>Mean Standard dev.</td>
<td>Mean Standard dev.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9828 4.1035e-03</td>
<td>0.9817 2.6541e-04</td>
<td>0.9837 4.7859e-04</td>
</tr>
<tr>
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<td>0.7312 5.7920e-03</td>
<td>0.6781 0.0104</td>
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<td>2.7931 0.1359</td>
<td>3.9144 0.1118</td>
</tr>
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<td>0.6994 0.0196</td>
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<td>$\phi_\pi$</td>
<td>3.2173 0.4319</td>
<td>3.7229 7.5880e-03</td>
<td>3.7035 0.0220</td>
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<tr>
<td>$\phi_x$</td>
<td>0.6605 0.0592</td>
<td>0.7359 1.3357e-03</td>
<td>0.7288 3.4131e-03</td>
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<tr>
<td>$\rho_1$</td>
<td>0.8574 0.0385</td>
<td>0.8490 4.0643e-03</td>
<td>0.8618 0.0126</td>
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<tr>
<td>$\rho_2$</td>
<td>0.8509 0.0379</td>
<td>0.8494 5.2429e-03</td>
<td>0.8401 9.0964e-03</td>
</tr>
<tr>
<td>$\sigma^2_1$</td>
<td>0.0169 7.3052e-03</td>
<td>0.0183 8.3136e-04</td>
<td>0.0200 1.3348e-03</td>
</tr>
<tr>
<td>$\sigma^2_2$</td>
<td>0.0158 7.5132e-03</td>
<td>0.0165 5.2746e-04</td>
<td>0.0162 1.8502e-03</td>
</tr>
<tr>
<td>$\sigma^2_3$</td>
<td>0.0151 8.3042e-03</td>
<td>0.0137 7.4628e-04</td>
<td>0.0157 2.7194e-03</td>
</tr>
</tbody>
</table>

Table 2: Posterior Moments, Different Priors

are also affected but, surprisingly, variations are not monotonic. Note also that both the location and the spread of the posterior of $\zeta_p$ change with the prior. Therefore, prior and posterior largely overlap because the prior already contained information present in the likelihood. Finally, the posterior distribution of the nuisance parameters (AR parameters and the variance of the shocks) is roughly independent of the prior spread. To summarize, the general features of the posterior distributions we have constructed are robust and none of the conclusions we derive in the next section hinges on the spread of the prior.

4 Recursive Analysis

As mentioned, there is substantial controversy in the literature regarding the role that monetary policy had in shaping the dynamics of US output and inflation over the last 30 years. While common wisdom suggests that changes in monetary policy caused the changes in the variability and serial correlation properties of output and inflation, several authors have raised serious doubts about such an interpretation. In particular, the recent work of Sims and Zha (2004), who used a time varying Markov switching specifications for a structural VAR, and Canova and Gambetti (2004), who used continuously time varying coefficients in a structural VAR, provide strong evidence against this conventional view.

In the context of the DSGE model we consider, we can address four important questions which can shed important light on the variations taking place in the autocovariance function of output and inflation over the last 30 years. First, are there significant changes in the systematic component of monetary policy? That is, does the posterior distribution of the policy parameters shift significantly (and permanently) over samples? Second, is there a change in the transmission of monetary policy shocks to the economy? Third, is there any evidence that the size of the variance of the monetary policy innovations has been
permanently reduced? Fourth, is there any evidence that the magnitude of the Phillips curve trade-off has been altered significantly over time and, if this is the case, what has caused the shift?

To address these questions we have estimated the model over several samples. We started from the sample [1948:1, 1978:1] and then repeated the estimation moving the starting date by one year while keeping the size of the sample constant to 30 years. Keeping a fixed window size is important in order to minimize differences produced by different precision of the estimates. The last subsample is [1962:1-2002:1], which means that we produce 15 posterior distribution for the parameters. We also produce posterior distributions for 3 complementary samples, [1978:2-2002:1], [1982:2-2002:1], [1984:2-2002:1], which allows us to compare our results with those present in the literature where the sample is arbitrarily split at one of these dates. The final sample we consider, [1986:2-2002:1], corresponds to Greenspan’s tenure and therefore permits us to compare policies in the 1990’s with those of the 1970’s and infer to what extent the reaction function of the Fed has turned from weak to aggressive in fighting inflation.

4.1 The systematic component of policy

Figure 3 presents the evolution of the posterior 68 percent band for the coefficients of the policy rule over different samples. For the sake of legibility, the figure reports bands only for selected samples (listed on the horizontal axis of the graph). Since for intermediate samples, posterior bands monotonically connect posterior bands obtained at the reported dates, there is no loss of information in concentrating on what we report.

Several important features emerge from figure 3. First, there is no posterior evidence that any of the three coefficients has permanently shifted over time. In fact, it is easy to check that the envelope of the posterior 68 percent bands, constructed so that the coverage is at least 68 percent in each sample, includes the median of the posterior for each of the samples. Second, our posterior distribution analysis fails to support the idea that in the pre-1980 period monetary policy was weak in fighting inflation: the whole shape of the distribution is roughly similar during Greenspan and Burns tenures. In fact, the posterior median of the inflation coefficient in the 1948-1978 sample (2.24) is higher than the posterior median in the 1986-2002 sample (1.89). Given that the dispersion of posterior estimates is comparable and the posterior distribution of the policy smoothness parameter ($\phi_r$) is broadly unchanged, one must conclude that the two tenures are characterized by similar regimes.

Remarkable stability is also present in the posterior distribution of the other two policy parameters. For example, the median value of the posterior distribution of $\phi_x$ oscillates between a minimum of 1.01 and a maximum of 1.19 and the posterior distribution of the differences between these two estimates is centered around zero and sufficiently symmetric. The smoothness parameter has a posterior median which is in the neighbor of 0.70 and the posterior standard deviation is of the order of 0.20 in every sample we consider. Interestingly, the estimate we obtain imply that the median estimate of the long run response of interest rates to output and inflation over the full sample is strong and about 6 for the former and
of 3 for the latter. Estimates of long run interest rate responses in different samples are of similar magnitude. In fact, all estimates fall into the posterior 68 percent bands obtained for the full sample.

In conclusion, as in Sims and Zha (2004) and Canova and Gambetti (2004), we fail to detect permanent variations in the posterior distribution of the policy parameters which would justify the claim that the monetary policy regime has changed. We also fail to find posterior evidence that the response of interest rates to inflation was weak in the 1970’s and strong in the 1990’s. In this respect, our analysis confirms Leeper and Zha’s (2003) conclusion that policy has been very much as usual over the majority of the sample under consideration, and agrees with Bernanke and Mihov’s (1998) result that a relatively stable interest rule characterized the behavior of monetary policy in the US over most of the last 50 years.

4.2 The transmission properties of monetary policy shocks

While the systematic component of monetary policy appears to be stable, it is clearly possible that changes in the structure of the economy (in our case, coming from the Euler equation or the Phillips curve) altered the transmission of policy disturbances. That is to say, while the systematic component of policy does not show any permanent shift, unsystematic shocks to the policy equation may have had different dynamic effects over different
Figure 4: Impulse responses to policy shocks, selected samples

samples. To examine this possibility we present impulse responses to monetary policy disturbances in selected samples in figure 4. To make the comparison meaningful, the plots are scaled so that the impulse to the policy equation is the same in each episode (and equal to one standard deviation).

Figure 4 indicates that the dynamics following monetary policy shocks have been qualitatively similar in the various samples. In particular, a monetary policy shock temporarily increases interest rates and produces a negative response of output and inflation. The immediate effect on inflation is much larger than the one on output but, in general, less persistent. In fact, the inflation effects of an interest rate shock die out within 3 quarters of the impulse while output effects last one quarter longer. The lack of inflation persistence generated by monetary shocks in this model is a well known fact and the addition of standard friction do not necessarily increase persistence in response to monetary shocks.
Output is more persistent here than in comparable models because the policy equation is backward looking. In general, however, it is still the case that the model does not strongly propagate policy shocks to the rest of the economy. Interestingly, the largest response of output and inflation is always contemporaneous. Therefore, the presence of monopolistic competition and price stickiness does not imply that output and inflation satisfy the zero restrictions typically used to identify monetary policy disturbances in standard VARs.

On the qualitative side, some differences across samples emerge. For example, the posterior bands in the 1948-2002 sample appear to be significantly smaller than in any other sample, probably as a result of more precise parameter estimates. Also, output responses appear to be stronger in the 1948-1978 and 1986-2002 samples, suggesting that the contribution of monetary disturbances to output fluctuations may have changed over time. Finally, the response of inflation to policy shocks is smaller in size in the 1948-2002 sample than to any other sample we consider.

In sum, the transmission of policy disturbances is qualitatively unchanged over time. Although our analysis detects instabilities in the Euler and Phillips curves relationships (see below), in particular in the samples which include the end of the 1970’s, these instabilities do not translate in changes in the shape or in the sign of the responses of output and inflation to policy shocks. Taken together, the results we have presented in these two subsections indicate that monetary policy does not have much to do with the changes in output and inflation observed over the last 30 years. The policy rule has been very similar over samples and impulses to the policy equation in different samples would have produced similar responses in the economy if they were of similar size. While the evidence seems conclusive, there is still one caveat one would like to consider. It is, in fact, possible that the variance of the policy shocks may have changed over time. That is, even though the impact of unitary shocks is the same, the absolute magnitude of the effects could have been significantly different in various samples. To examine this possibility, figure 5 presents the time path of the posterior 68 percent band for the variance of the policy shock. While small variations are present, the bands look very much the same in every sample we consider. In particular, we find no posterior evidence of a permanent reduction in the variance of the policy shocks since the mid 1980s, nor that policy shocks under Greenspan’s tenure were significantly smaller than in any other period in the post WWII US history.

### 4.3 Phillips curve trade-off

The Phillips curve trade-off in our model is regulated by a (nonlinear) function of four different structural parameters: the coefficient of relative risk aversion \((\varphi)\), the intertemporal elasticity of labor supply \((\vartheta_{n}^{-1})\), the discount factor \((\beta)\) and the price stickiness parameter \((\zeta_{p})\). As we have mentioned, the posterior median of the trade-off is 0.5 for the full sample, and the posterior standard deviation is large, suggesting that marginal costs (here proxied by the output gap) exert a marginal effect on the dynamics of inflation.

Narrative evidence obtained plotting the output gap against inflation over time suggests that the slope of the relationship has changed magnitude and sometimes even sign. It is
therefore worth to study whether our structural analysis confirms the evidence contained in simple plots and, if this is the case, investigate which of the four structural parameters is responsible for the time variations we observe. Given our negative conclusion about the relationship between changes in monetary policy and changes in output and inflation processes over time, such an analysis may also shed some light on reasons behind the changes in the US economy and tells us whether the inability to detect changes is intrinsic to the estimation technique we use or specific to some equations of the model.

We present the posterior 68 percent band for the coefficient regulating the Phillips curve trade-off in figure 6 (solid lines), together with the prior 68 percent band (dotted lines). While we fail to find sign reversal over the various samples, it is clear that the posterior band for this coefficient is very unstable: the minimum value is in the 1978-2002 sample (the median is 0.03) and the maximum is in the sample 1956-1986 (the median is 29.96). Furthermore, although it is difficult to draw definite conclusions, it appears that samples which include the late 1970’s imply a strong feedback from marginal costs to inflation, while samples that exclude them display a much moderate or even small trade-off. Which structural parameter is responsible for this instability? Figure 7, which presents the posterior median and the posterior 68 percent bands for the parameters together with the 68 percent prior bands, suggests that the posterior distributions of all four parameters are moving over samples. However, given the non-linear relationship between the model parameters and the Phillips curve coefficient, it is difficult to link the changes observed in figure 6 with those present in figure 7. Relatively speaking, changes in the posterior distribution of the elasticity of labor supply are the largest, followed by changes in the risk aversion parameter. For example, the posterior distribution of $\vartheta_n$ varies from a median
value of 1.2 in the sample 1951-1981 to a median value of 7.43 in the 1982-2002, indicating a higher elasticity of labor supply in the first sample. Similarly, the coefficient of relative risk aversion fluctuates from a median value of 0.01 in the 1972-2002 sample to a median value of 2.42 in the 1984-2002 sample. By contrast, the parameter controlling price stickiness has been much more stable over subsamples and generally low: if we exclude the 1978-2002 and the 1948-2002 samples, the median value of the posterior distribution of this parameter is around 0.25 implying, at most, a one quarter price stickiness. Since the prior bands are centered around 0.7, the data strongly suggests that only a minimal amount of stickiness is necessary to fit the data. Note that this median estimate is of the same magnitude as the one obtained by Bills and Klenow (2002) despite the fact that the data and estimation techniques are completely different.

In sum, figure 7 suggests that both the Euler equation and the Phillips curve have been unstable and that the magnitude of the changes is comparable. While policy coefficients and the parameter controlling pricing decisions of firms appear to be relative similar across samples, the parameters describing private agents’ utility function have changed significantly. Hence, while we can exclude the possibility that monetary policy “caused” the observed changes in the output and inflation process, we can also tentatively suggest that modifications in the labor and goods markets, for example along the lines of those suggested by McConnell and Perez Quiroz (2000), have the potential to account for the observed changes in the US economy.
Figure 7: Evolution of private agents parameters: Posterior(solid) Prior (dotted)
The output gap measure we use in our exercise is probably subject to a large amount of measurement error. Consequently, estimates of the structural parameters and of the impulse responses may fail to move around across samples because of the constant and large amount of measurement error present in each sample. Similarly, we may fail to detect changes in the posterior distribution of the variance of the policy shock if the variables entering the policy rule are measured with error and such error contaminates the residuals of the equation. To study whether measurement error could affect our conclusions, we have repeated estimation using output growth in place of the output gap in all three equations of the model. Figure 8 reports a sample of the results: we present the evolution of the 68 percent posterior band for the policy parameters and for the variance of policy disturbance.

It is clear that none of the conclusions we have previously reached is altered when output growth is used. As a matter of fact, posterior distributions are even more stable across samples with this specification and this stability extends also to impulse responses and, to a lesser extent, the Phillips curve trade-off. Hence, measurement error is unlikely

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2 An appendix with the complete set of results obtained with this specification is available on request from the author.
to account for the pattern of results we obtain.

5 Conclusions

This paper recursively estimates a small scale DSGE model using US post-WWII data and Bayesian techniques. The model belongs to the class of New-Keynesian models that have been extensively used in the literature for welfare and other policy analyses. Bayesian techniques are preferable to standard likelihood methods or to indirect inference (impulse response matching) exercises to estimate the parameters, because the model we consider is clearly false and possibly misspecified. We show that the method delivers reasonable posterior distributions for the structural parameters when priors are broadly non-informative and the policy reaction function shrewdly chosen. We also show that the model tracks the ups and downs of the actual interest rate quite well; that parameter estimates do not imply violations of theoretical orthogonality conditions and that, in a forecasting sense, the model is competitive with more densely parametrized VAR and BVAR models.

We estimate the model 19 times, recursively, using a different starting date, to analyze the role that monetary policy had in shaping the observed changes in output and inflation over the last 30 years. We have geared the analysis to shed light on three general questions. First, we have studied whether the posterior distribution of the policy coefficients has significantly and permanently moved over samples in the direction of making the reaction of interest rates to inflation stronger. Second, we have investigated whether we can detect changes in the dynamic responses to policy shocks and/or in the variance of the policy shocks over samples. Third, we have examined whether the posterior distribution of the Phillips curve trade-off has changed over samples and analyzed which of the structural parameters entering the trade-off is responsible for the changes.

We find that the posterior distributions of the policy parameters are relatively stable over samples and there is no posterior evidence in favor of a permanent regime shift from a lax to a tough anti-inflation stance. In particular, we find that the posterior distribution of the inflation coefficient in the policy equation is roughly similar in both the pre-1978 and in the post-1982 period in shape and location. Moreover, there is a remarkable stability in the features of the transmission mechanism of monetary policy disturbances and no posterior evidence that the variance of the policy shocks has systematically decreased. These similarities stand in sharp contrast with the substantial instability of the posterior distribution of the Phillips curve trade-off we estimate. We also show that variations over samples in the posterior distribution of the price stickiness parameter are minor; that the instability of the posterior distribution of the Phillips curve trade-off is largely due to the instability of the parameters of agents’ preference and that these changes are both economically and statistically relevant.

All in all, we conclude that the role that monetary policy had in shaping the observed changes in the US economy has been largely overemphasized and that understanding the reasons behind the movements in agents’ preferences over subsamples is likely to shed important light on the dynamics of output and inflation in the post WWII era.
Our conclusions agree to a large extent to those put forward by Sims and Zha (2004) and Canova and Gambetti (2004), who estimated structural VAR models with time varying (continuously or with Markov switches) coefficients. Relative to their analyses, we are able to go beyond the simple documentation of instabilities and pin down which of the structural parameters causes this instability. Our results are also consistent with the analyses of Bernanke and Mihov (1998) and Leeper and Zha (2003). As these authors we find that monetary policy was reasonably characterized by the same interest rate rule for the majority of post WWII sample and that, in many respects, the systematic component of policy of the 1990’s was very similar to the one of the 1970’s.

Our conclusions differ from those of Cogley and Sargent (2001), (2003) who use narrative evidence together with reduced form estimates to establish the causality from monetary policy to changes in the real economy, and in Ireland (2001) and Boivin and Giannoni (2002), who also estimate DSGE equilibrium models, but simply perform structural stability tests using arbitrarily chosen subsamples. In the presence of an inverted U-shape pattern for inflation, an arbitrarily chosen break date may give a misleading impression of the magnitude and the duration of the breaks.

To the extent that shocks driving the equations of the model are truly structural, our analysis also suggests that impulses causing business cycle fluctuations have been similar in size across shocks and over samples. While this does not necessarily imply that the contribution of various shocks to the variability of output and inflation has been stable, it hints at this possibility. Furthermore, given the relatively large magnitude of policy shocks, our results also hints to the fact that macroeconomic performance could have been significantly improved by tightening policymakers hands so as to eliminate non-systematic fluctuations in interest rates.

Finally, as a by-product of the analysis, we have shown that a model which fit reasonably well the actual interest rate path requires little price stickiness to match inflation dynamics. Since our low posterior point estimate agrees with those produced by Bills and Klenow (2002) with completely different techniques and data sets, it is likely that previous studies, requiring high stickiness to match the data, reflect misspecifications and inappropriate account of structural instabilities.
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Appendix

The Metropolis-Hasting Algorithm

In order to obtain draws from the unknown posterior distribution of the parameters we use the following algorithm:

1. Choose a $\alpha_0$. Evaluate $g(\alpha = \alpha_0)$ and use the Kalman filter to evaluate the likelihood of the model $L(y_t|\alpha_0)$.
2. For each $i = 1, \ldots, N$ set $\alpha_i = \alpha_{i-1}$ with probability $1 - p$ and $\alpha_i = \alpha_i^*$ with probability $p$, where $\alpha_i^* = \alpha_{i-1} + v_i$ and $v = [v_1, \ldots, v_N]$ follows a multivariate uniform distribution and $p = \min\{1, \frac{L(y_t|\alpha_{i}^*)g(\alpha_{i}^*)}{L(y_t|\alpha_{i-1})g(\alpha_{i-1})}\}$.
3. Repeat steps 1. and 2. $\bar{L} + L$ times and discard the first $\bar{L}$ draws.

An important issue concerns the convergence of simulated draws. In particular, it is very important to adjust the variance of the innovations $v_i$ (that is, the range of the uniform distribution) to get a reasonable acceptance rate. If the acceptance rate is "too small" the chain will not visit the parameter space in a reasonable number of iterations. If it is too high, the chain will have the tendency not to stay long enough in the high probability regions. It is typical to choose an acceptance rate of about 35-40%. In all our samples, the acceptance rate oscillates between 38% to 44%. We draw chains of 30000 elements each time the model is estimated. We check for convergence using the cumulative sum of the draws (CUMSUM) statistics. We found that convergence typically obtains within 20000 iterations or less. The parameter for which convergence is most difficult is the output coefficients in the policy rule. Some difficulties in converging was also experimented for the inflation coefficient in the policy rule in the 1951-1981 and 1956-1986 samples. We keep the last 5000 draws for inference and use one out of five to reduce the serial correlation of the draws. This means that posterior distributions and impulse responses are calculated using 1000 draws at each of the selected dates. We kept track of the draws which generated an explosive system. We found that the maximum number was 509 out 30000 in the 1966-1996 sample; in all the others samples their number is below 100.

The Marginal likelihood

In order to compare the forecasting performance of the DSGE model with VAR models we need to compute the marginal likelihood of each model. For each model $M_i$, we approximate $L(y_t|M_i)$ using $\frac{1}{P} \sum_{l=1}^{P} f(\alpha_i^l) \frac{L(y_t|\alpha_i^l,M_i)}{E_{y_t|\alpha_i^l,M_i}}^{-1}$ where $\alpha_i^l$ is the draw $l$ of the parameters $\alpha$ of model $i$ and $f$ is a truncated normal distribution with mean $\bar{\alpha}^i = \frac{1}{P} \sum_{l=1}^{P} \alpha_i^l$, variance $\Sigma^i = \frac{1}{P} \sum_{l=1}^{P} (\alpha_i^l - \bar{\alpha}^i)(\alpha_i^l - \bar{\alpha}^i)'$ and the truncation eliminates the region of the parameter space which exceeds a $\chi^2(k_i)$ where $k_i$ is the number of parameters in model $i$ (see Geweke (1998)). Therefore the marginal likelihood is computed using the harmonic mean of the draws with weights given by $f(\alpha_i^l)$. 

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