Structural changes in the US economy. Bad Luck or Bad Policy?

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Abstract

This paper investigates the relationship between changes in the output and inflation process and monetary policy in the US over the last 25 years. It estimates a structural Bayesian TVC-VAR with MCMC methods, where sign restrictions are used to identify monetary policy shocks, and analyzes the transmission of two types of disturbances: those to the non-systematic and those to the systematic component of monetary policy. Impulse responses are calculated as the difference between two conditional expectations, differing for a shock in the conditioning sets, of a vector of future time series. We find structural variations in both the coefficients of the model and the variance of the structural shocks but only the latters appear to be synchronized across equations. The transmission of monetary policy disturbances has hardly changed over the last 25 years and changes in the systematic component of policy have negligible effects on the dynamic of the system. Changes in inflation persistence are small and disconnection with the dynamics of monetary policy. Results are robust to a number of alterations in the auxiliary assumptions of the model.

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1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last couple of decades. In particular, several authors have noted a marked decline in the variance of real activity and inflation since the early 1980s (see e.g. Blanchard and Simon (2000), McConnell and Perez Quiroz (2001) and Stock and Watson (2003)). What are the reasons behind such a decline? A stream of literature attributes these changes to alterations in the mechanisms through which exogenous shocks spread across sectors and propagate over time. Since the transmission mechanism depends on the structure of the economy, the main implication of this viewpoint is that the underlying characteristics of the economy have changed over time. Several factors could be responsible for this evolution, for instance, changes in the behavior of consumers and firms or changes in preferences of policymakers. The recent literature has paid particular attention to monetary policy. Several studies, including Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2003), Boivin and Giannoni, (2002a), (2002b), have argued that monetary policy was "loose" in fighting inflation in the 1970s but become more aggressive since the early 1980s and see in this change of attitude the reason for the observed reduction of inflation and output volatility. This view, however, is far from unanimous. For example, Bernanke and Mihov (1998), Orphanides (2001), Leeper and Zha (2003) find little evidence of significant changes in the policy rule used over the last 25 years and in the propagation of monetary shocks to the economy while Hanson (2001) claims that the dynamics of output and inflation change because of alterations taking place in the rest of the economy. In addition, Sims (2001) and Sims and Zha (2002) claim that changes in the variance of exogenous shocks (and, in particular, of monetary policy innovations) are probably responsible for the observed changes in output and inflation variance.

This controversy is not new. In the past rational expectations econometricians (e.g. Sargent (1984)) have argued that policy changes involving regime switches, dramatically alter private behavior decisions and, as a consequence, the dynamics of the macroeconomic variables and searched for historical episodes supporting this view (see e.g. Sargent (1999)). VAR econometricians, on the other hand, often denied the empirical relevance of this argument suggesting that the systematic portion of monetary policy has rarely been altered, at least in the US, and that policy changes are better characterized as random draws for the non-systematic part (Sims (1982)). This long standing debate now has been cast into the dual framework of "bad policy" (failure to respond to inflationary pressure) vs. " bad luck" (shocks are drawn from a distribution whose moments vary over time) and new ground has been broken thanks to the development of tools which explicitly allow the examination of time variations in the structure of the economy and in the variance of the exogenous processes and the investigation of the timing of the changes. Overall, and despite the recent contributions, the role that monetary policy has in shaping the observed changes in the US economy is still open.

This paper provides novel evidence on the contribution of monetary policy to the structural changes occurred in the US economy over the last three decades. Our basic framework of analysis is a time varying coefficients VAR model (TVC-VAR), similar to the one employed by Cogley and Sargent (2001), where the coefficients evolve according to a nonlinear transition equation which puts zero probability on paths associated with explosive VAR roots. Cogley and Sargent (2003) add to this basic structure a stochastic volatility model for the reduced form innovation. We do not follow their approach here for three reasons. First, as shown in Canova (1993), time variations in the coefficients automatically produce an heteroschedatic representation for the forecast error of the reduced form model and a variety of non-linearities and non-normalities that the literature has found to be empirically important when modelling macro and financial time series. Second, the structure that time variations impose on the forecast errors allows us to sort out variations due to changes the coefficients from variations due to changes the variances of the shocks. Third, our setup retains a conditional linear structure which facilitates the derivation of posterior estimates and substantially reduces the computational costs.

We use Markov Chain Monte Carlo (MCMC) methods to estimate posterior distributions of the quantities of interest. Contrary to the existing literature, we explicitly conduct a structural analysis and consider both short run and long run relationships. Moreover, in addition to studying the timing of the changes in various equations, we also explicitly measure the variations in the propagation mechanism of monetary policy disturbances and the potential effect that changes in preference of the Fed would have had in particular historical episodes.

The structural setup we employ is particularly suited to study the main issues in the debate. In fact, we are able to separate structural changes in the output and inflation equations produced by i) changes in the systematic component of policy, ii) changes in the propagation of policy shocks, iii) changes in the variance of the monetary policy and other shocks and iv) changes in the rest of the economy. The distinction between i) and ii) is important and both reduced form time varying approaches or structural but constant coefficient approaches are unable to separate their relative contribution in accounting for the observed changes in the output and inflation process.

A structural model is obtained from the reduced form TVC-VAR, identifying structural disturbances by means of sign restrictions. While our focus is on monetary policy disturbances, and therefore arbitrarily orthogonalize the other shocks, the methodology can be employed to jointly identify multiple sources of structural disturbances if needed (see e.g. Canova and De Nicolo' (2002)). We choose to work with sign restrictions for two reasons. First, the contemporaneous zero restrictions conventionally used to identify VARs are often absent in those theoretical (DSGE) models economists like to use to guide the interpretation of the results. Second, standard decompositions impose strong restrictions, on the structure of time variations in our structural model which have no a-priori justification. In the last section of the paper, we show that the our conclusions are robust to the identification procedure and to a number of other changes in the auxiliary assumptions we used.

Because time variations in the coefficients induce important non-linearities in the dynamics of the model, standard statistics summarizing the dynamics in response to shocks impulse are inappropriate. For example, since at each point in time the coefficient vector is perturbed by a shock, assuming that between $T + 1$ and $T + \tau$ no shocks other than the monetary policy disturbance hit the system is unappealing and can give misleading conclusions. To study the evolution the economy in response to structural shocks when coefficients vary over time, we therefore employ a different concept which shares similarities with those used in Koop, Pesaran and Potter (1996), Koop (1996), and Gallant, Rossi and Tauchen (1996). In particular, impulse response functions are defined as the difference between two conditional expectations, which differ in the arguments of their conditioning sets.

Several important results emerge from our investigation. First, we find changes in the estimated structural relationships, but these changes are localized in time and only involve particular coefficients in certain equations. In particular, in agreement with Bernanke and Mihov (1998), Orphanides (2001) and Leeper and Zha (2003), we show that excluding the Volker experiment of the beginning of the 1980s, the monetary policy rule has been quite stable over time. Interestingly, the posterior mean of the (sum of) inflation coefficients fails to satisfy the so-called Taylor principle, not only in the 1970s but also in the 1990s. Hence, both in terms of timing of the changes and of aggressiveness in response to inflation movements, variations in the systematic component of monetary policy are unlikely to have driven the changes in the observed output and inflation process. Second, as in Sims and Zha (2002), we find evidence of a decrease in the uncertainty surrounding the structural disturbances of the system, including those of monetary policy shocks. Furthermore, the timing of the changes roughly coincides with the timing of the changes observed in the output and inflation equations. Taken together, these results suggest that the "bad luck" hypothesis has considerable more posterior support than the "bad policy" hypothesis in accounting for the observed dynamics of the US economy. Third, consistent with the subsample analysis of Hanson (2001), we show that also the transmission of monetary policy shocks has been very stable over time. In fact, the shape and the persistence of the responses are very similar and the quantitative differences in the posterior mean response are statistically negligible at all horizons for all years from 1979 to 2002.

Fourth, despite the presence of changes in the structural coefficients of the inflation equation, we find that inflation persistence (measured by the height of the zero frequency of the spectrum) has not statistically changed over time. We show that both monetary and non-monetary factors account for the magnitude of inflation persistence and that, although the relative contribution of monetary policy fluctuates over time, it is increasing since 1981.

We investigate whether a more aggressive response to inflation would have made a difference in the dynamics of output and inflation at the beginning of the 1980s. We show that such a stance would have reduced inflationary pressures in short and medium run in 1979 but not in any other date after that. Hence, while the Fed had some room to improve economic performance at the end of the 1970s, the considerable decline in the variance of the shocks hitting the economy since the beginning of the 1980s appears to be responsible for the improved macroeconomic performance experienced by the US economy since then. Finally, we show that our basic conclusions are robust to a number of changes in the auxiliary assumptions used. In particular, we show that our results do not depend on the identification scheme used, on the treatment of trends in the variables and the variables included in the VAR.

The rest of the paper is organized as follows. Section 2 presents the reduced form model, describes our identification scheme and the computational approach used to obtain posterior distributions of the structural coefficients of the model. Section 3 defines impulse response functions which are valid in our TVC-VAR model and describes how to compute dynamics to shocks in the non-systematic and the systematic component of the model. Section 4 presents the results and Section 5 concludes. A series of appendices describes the technical details involved in the computation of posterior distributions and of impulse responses.

2 The Reduced form Model

Let y_t be a $n \times 1$ vector of time series with the representation

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + ... + A_{p,t}y_{t-p} + \varepsilon_t \tag{1}$$

where $A_{0,t}$ is a $n \times 1$ vector, $A_{i,t}$, for $i = 1, ..., p$ are $n \times n$ matrices of coefficients and ε_t is a $n \times 1$ Gaussian white noise process with zero mean and covariance Σ_t. Let $A_t = [A_{0,t}, A_{1,t}...A_{p,t}]$, $x_t' = [1_n, y_{t-1}'...y_{t-p}']$, where 1_n is a row vector of ones of length n, let $vec(\cdot)$ denote the stacking column operator and let $\theta_t = vec(A_t')$. Then 1 can be written as

$$y_t = X_t'\theta_t + \varepsilon_t \tag{2}$$

where $X_t' = (I_n \bigotimes x_t')$ is a $n \times (np + 1)n$ matrix, I_n is a $n \times n$ identity matrix, and θ_t is a $(np + 1)n \times 1$ vector. If we treat θ_t as a hidden state vector, equation (2) represents the observation equation of a state space model. We assume that θ_t evolves according to the following nonlinear transition equation

$$p(\theta_{t+1}|\theta_t, \Omega_t) \propto \mathcal{I}(\theta_{t+1})f(\theta_{t+1}|\theta_t, \Omega_t) \tag{3}$$

where $\mathcal{I}(\theta_{t+1})$ is an indicator function discarding explosive paths of y_t. Such an indicator is necessary to make dynamic analysis sensible and, as we will see below, it is easy to implement numerically.

We assume that $f(\theta_{t+1}|\theta_t, \Omega_t)$ can be represented as

$$\theta_{t+1} = \theta_t + u_{t+1} \tag{4}$$

where u_t is a $(np + 1)n \times 1$ Gaussian white noise process with zero mean and covariance Ω_t. We select this simple specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that $\Sigma_t = \Sigma \;\; \forall t$; that $corr(u_t, \varepsilon_t) = 0$, and that Ω_t is diagonal. At first sight, these assumptions may appear to be restrictive but they are not. For example, the first assumption does not imply that the forecast errors of the model are homoschedastic. In fact, substituting (4) into (2) we have that $y_t = X_t'\theta_{t-1} + e_t$ where $e_t = \varepsilon_t + X_t'u_t$. Hence, one step ahead forecast errors have a time varying heteroschedastic structure even without assuming that Σ_t or Ω_t vary over time. The assumed structure is appealing since it is coefficient variation that impart heteroschedastic movements in the variance of the forecast errors (see Sims and Zha (2002) or Cogley and Sargent (2003) for alternative specifications). The second assumption is standard but somewhat stronger and implies that the dynamics of the model are conditionally linear. [1]. Sargent and Hansen (1998) showed how to relax this assumption by equivalently letting the innovations of the measurement equation to be serially correlated. Since in our setup ε_t is, by construction, a white noise process, the loss of information caused by imposing uncorrelation between the shocks is likely to be small. The third assumption implies that each element of θ_t evolves independently but it is irrelevant for the outcomes since structural coefficients will be allowed to evolve in a correlated manner.

Let S be the square root of Σ, i.e., $\Sigma = SS'$. Since Σ is time invariant also S is time invariant. Let H_t be an orthonormal matrix, independent of ε_t, such that $H_t H_t' = I$ and let $K_t^{-1} = H_t' S^{-1}$. K_t is a particular decomposition of Σ which transforms (2) in two ways: it produced uncorrelated innovations; it gives a structural interpretation to the equations of the system . Premultiplying y_t by K_t^{-1} we obtain

$$K_t^{-1} y_t = K_t^{-1} A_{0,t} + \sum_j K_t^{-1} A_{j,t} y_{t-j} + e_t \tag{5}$$

where $e_t = K_t^{-1}\varepsilon_t$ satisfies $E(e_t) = 0$, $E(e_t e_t') = I_n$. Equation (5) represents the class of "structural" representations of y_t we are interested in. For example, a standard Choleski representation can be obtained setting S to be lower triangular and $H_t = I_n$ and more general patterns by choosing S to be non-triangular and $H_t = I_n$. Here S is obtained via eigenvalue-eigenvector decompositions and H_t implements particular theory-based economic restrictions.

Letting $C_t = [K_t^{-1} A_{1t} ... K_t^{-1} A_{pt}]$, and $\gamma_t = vec(C_t')$, (5) becomes

$$K_t^{-1} y_t = X_t' \gamma_t + e_t \tag{6}$$

As in standard fixed coefficient VARs there is a mapping between γ_t and θ_t since $\gamma_t = (K_t^{-1} \bigotimes I_{np})\theta_t$ where I_{np} is a $(np+1) \times (np+1)$ identity matrix. Whenever $\mathcal{I}(\theta_{t+1}) = 1$, we also have

$$\gamma_{t+1} = \gamma_t + \eta_{t+1} \tag{7}$$

where $\eta_t = (K_t^{-1} \bigotimes I_{np}) u_t$, the vector of shocks to structural parameters, satisfies $E(\eta_t) = 0$, $E(\eta_t \eta_t') = E((K_t^{-1} \bigotimes I_{np}) u_t u_t' (K_t^{-1} \bigotimes I_{np})')$.

Hence, the vector of structural shocks $\xi_t' = [e_t', \eta_t']'$ is a white noise process with zero mean and covariance matrix $E\xi_t\xi_t' = \Xi = \begin{bmatrix} I_n & 0 \\ 0 & E((K_t^{-1} \bigotimes I_{np}) u_t u_t' (K_t^{-1} \bigotimes I_{np})') \end{bmatrix}$. Note that since each element of γ_t depends on several u_{it} via K_t, shocks to structural parameters are no longer independent

The structural model (6)-(7) contains two types of shocks: disturbances to the observations equations, e_t, and disturbances to structural parameters, η_t. While the former have the same interpretation as those in a fixed coefficients VARs, the second are

[1]This means, for instance, that we can not study whether shocks of different sign or of different magnitude have different dyanmic effects on the system.

new. To understand their meaning, suppose that the $n-th$ equation of (6) is the monetary policy equation and suppose we split it into a systematic component, summarized by $\tilde{\gamma}_t = [\gamma_{(n-1)(np+1),t}, ..., \gamma_{n(np+1),t}]'$, describing say, how interest rates respond to the developments in the economy, and the non-systematic component, summarized by the policy shock $e_{n,t}$. Then, innovations $\tilde{\gamma}_t$ represent changes in the preferences of the monetary authorities with respect to developments in the rest of the economy.

In our setup, identifying structural shocks is equivalent to choosing a matrix H_t. Here as in Faust (1998), Uhlig (2001), and Canova and De Nicoló (2002), we select H_t so that the sign of the impulse response functions at $t+j, j = 1, 2, \ldots, J$ matches some theoretical restriction. In particular, we assume that a contractionary monetary policy shock must generate a non-positive effects on output, inflation and nominal balances and a non-negative effect on the interest rate for two quarters after the shock.

We choose sign restrictions to identify shocks to the observation equation, since more standard identification schemes use restrictions very loosely connected to economic theory and have an undesirable property. Take, for example, a Choleski decomposition. Since Σ is time invariant, also its Choleski factor S is time invariant. Hence, since $H_t = I$, the contemporaneous effects of a monetary policy shock are time-invariant. That is, contemporaneous impulse responses will be constant no matter what point in time we choose to compute them and time varying responses will be obtained only if there are variations in the lagged coefficients of the reduced form model. Such a restriction is hard to justify and unduly restricts the pattern of time variations allowed in the structural coefficients.

Our identification approach allows for time variations in both contemporaneous and lagged effects. Furthermore, by restricting the sign of the impulse responses for at least two periods we make H_t depend on the conditional distribution of the states one period ahead [2]. For sensitivity analysis we report, in the last section of the paper, responses obtained identifying policy shocks as the third element of a Choleski system, i.e. we let monetary policy reacts to output and inflation movements but assume that it has no effects within a quarter on these variables. Since this paper is interested in recovering the systematic and non-systematic part of monetary policy and in analyzing how the economy respond their changes over time, we arbitrarily diagonalize the remaining disturbances without giving them any structural interpretation [3]

[2] If the parameters are time-varying, the conditional means of the coefficients will also be time-varying and therefore H_t must vary over time therefore making the contemporaneous effects K_t vary over time

[3] We follow Marsiglia (1972) in implementing sign restrictions. In particular, let h_t be the first column of H_t and assume that h_t is uniformely distributed independently of ε_t on a unit sphere S_n. Let $\mathsf{M}_{t,j}$ be the set of impulse response functions satisfying the restriction for period j. Since this set is dense, there exists several h_t over the unit sphere generating impulse response functions satisfying the restrictions. Call this set of points H_t Then, all the h_t belonging to H_t generate representations which are consistent with the definition of monetary policy shock - there may be more than one since nothing insures that H_t is a singleton.

3 Impulse Responses

One important question we would like to address in our study is whether the transmission of monetary policy shocks has changed over time. In a fixed coefficient model, impulse response functions provide information on how the variables of the system react to policy shocks. They are typically computed as the difference between two realizations of $y_{i,t+\tau}, \tau = 1, 2, \ldots$: these realizations are identical up to time t but one assumes that between $t+1$ and $t+\tau$ a shock in e_j has occurred at time $t+1$ and the other that no shocks will take place between $t+1$ and $t+\tau$.

In a TVC model, responses computed this way are inappropriate since they disregard the fact that between $t+1$ and $t+\tau$ the coefficients of the system can also change. Hence, meaningful impulse response functions ought to measure the effects of a shock in $e_{j,t+1}$ on $y_{it+\tau}$, allowing future coefficients shocks to be non-zero. For this reason, our impulse responses are obtained as the difference of two conditional expectations as of t of $y_{t+\tau}$. In both we condition on the history $y_1, \ldots, y_t$, the states θ_t up to t and the structural parameters of the transition equation (which are function of K_t). However, in the first case we condition on a draw for the current and all the future shocks, while in the second we condition on current and all future shocks being zero [4].

Formally speaking, let $y^t = [y_1', \ldots y_t']'$ be a history for y_{t+1}; $\theta^t = [\theta_1', \ldots \theta_t']'$ be a trajectory for the states up to t. Also, let $y_{t+1}^{t+\tau} = [y_{t+1}', \ldots y_{t+\tau}']'$ denote a collection of future observations and $\theta_{t+1}^{t+\tau} = [\theta_{t+1}', \ldots \theta_{t+\tau}']'$ a future trajectory of states from $t+1$ up to time $t+\tau$. Let $V = (\Sigma, \Omega)$; recall that $\xi_t' = [e_t', \eta_t']'$ and let $\zeta_t = [\varepsilon_t', u_t']'$. Let $\xi_{i,t+1}^{\delta}$ be a particular realization of size δ of $\xi_{i,t+1}$. Then an impulse response function to a shock $\xi_{i,t+1}^{\delta}$, $i = 1, \ldots, n$ for each $\tau = 1, \ldots, T$ is defined by:

$$IR_y(t,\tau) = E(y_{t+\tau}|y^t, \theta^t, V, K_t, \xi_{t+1}^{\delta}, \zeta_{t+2}^{t+\tau}) - E(y_{t+\tau}|y^t, \theta^t, V, K_t, \xi_{t+1}^{0}, \zeta_{t+2}^{t+\tau}) \qquad (8)$$

(8) resembles the impulse response function suggested by Gallant et al. (1996), Koop et al. (1996) and Koop (1996). Three important differences however need to be noted. First, rather than treating histories as random variables, we condition on a particular realization. Since we analyze how responses vary over time, we want impulse responses to be history dependent. Second, our impulse responses (to equations shocks) are independent of the sign and the size of the shocks (as it is in a fixed coefficient case). This is the result of the uncorrelation between shocks to the observation and the transition equations uncorrelated. Third, since the parameters of the transition equation appear in the conditioning set, we do not condition on a particular realization of θ_t (for example, its conditional mean) but instead treat θ_t as a random variable and integrate it out when calculating impulse responses. This implies, among other things, that $IR_y(t,\tau)$ are random variables as well. Integrating θ_t out of impulse responses allows us to concentrate on time difference which depend on the history of y_t but not on the size of the sample. Finally, note that $IR_y(t,\tau)$ can be also made state dependent if we condition on a particular stretch of a history (a boom or a recession)

[4] An alternative definition of impulse responses could be obtained by averaging out future shocks so as to exclude them from the conditioning set from both expectations.

and that when coefficients are constant, (8) coincides with the impulse response obtained with a standard definition.

Since there are two types of shocks in our system, we describe next how to trace out the dynamics effects of each of the shocks separately. Let $\xi_{i,t+1} = e_{i,t+1}$. Then $IR_y(t,\tau)$ is given by

$$\begin{aligned} IR_y(1,\tau) &= K_t^{-1,i} e_{i,t+1} \\ IR_y(t,\tau) &= \Psi^i_{t+\tau,\tau-1} e_{i,t+1} \text{for} \quad \tau = 2, ...T. \end{aligned} \tag{9}$$

(10)

where $\Psi_{t+\tau,\tau}$ is a nonlinear combination of the structural parameters of the model (precisely defined in Appendix A) and $\Psi^i_{t+\tau,\tau}$ is the column of Ψ corresponding to the ith shock. Note that (9) collapses to $IR_y(t,\tau) = \Psi^i_\tau$ when coefficients are constant, where Ψ_τ is the coefficient on $e_{t-\tau}$ in the MA representation of the constant coefficient VAR. Clearly, IR_y depends on the identifying matrix K_t and is not explosive, since $\Psi_{t+\tau,\tau}$ is the product of matrices whose eigenvalues are non-explosive.

When $\xi_{i,t+1} = \eta_{j,t+1}$ for $j = (n-1)(np+1), ..., n(np+1)$, appendix A shows that $IR_y(t,1)$ is

$$IR_y(1,1) = \left[E(\mathbf{A}_{t+1,1}|y^t, \theta^t, V, \xi^\delta_{-i,t+1}, \zeta^{t+\tau}_{t+2}) - E(\mathbf{A}_{t+1,1}|y^t, \theta^t, V, \xi^0_{t+1}, \zeta^{t+\tau}_{t+2})\right] y_t \tag{11}$$

and $IR_y(t,\tau)$ $\tau = 2, ..., T$ is

$$\begin{aligned} IR_y(t,\tau) &= \left[E(\tilde{\mathbf{\Phi}}_{t+\tau,\tau}|y^t, \theta^t, V, \xi^\delta_{t+1}, \zeta^{t+\tau}_{t+2}) - E(\tilde{\mathbf{\Phi}}_{t+\tau,\tau}|y^t, \theta^t, V, \xi^0_{t+1}, \zeta^{t+\tau}_{t+2})\right] y_t \\ &+ E\left(\sum_{i=1}^{k} \Phi_{t+\tau,j} A_{0,t+\tau-i}|y^t, \theta^t, V, \xi^\delta_{t+1}, \zeta^{t+\tau}_{t+2}\right) \\ &- E\left(\sum_{i=1}^{k} \Phi_{t+\tau,j} A_{0,t+\tau-i}|y^t, \theta^t, V, \xi^0_{t+1}, \zeta^{t+\tau}_{t+2}\right) \\ &+ E\left(\sum_{j=0}^{k-1} \Phi_{t+k,j}|y^t, \theta^t, V, \xi^\delta_{t+1}, \zeta^{t+\tau}_{t+2}\right) \varepsilon_{t+k,j} \\ &- E\left(\sum_{j=0}^{k-1} \Phi_{t+k,j}|y^t, \theta^t, V, \xi^0_{t+1}, \zeta^{t+\tau}_{t+2}\right) \varepsilon_{t+k,j} \end{aligned} \tag{12}$$

where $\mathbf{A}$ is the companion matrix of the VAR and Φ is defined in the appendix. There are three components in the responses to shocks in the structural coefficients: the first line in equation (11) shows how the shock spreads through the system via lags of y_t; the second and the third line how it spreads through the constant term and the last two lines how spreads through future shocks to the structural equations. Note that when a shock hit the systematic component of policy, IR_y depends on K_t only because $u_t = (K_t^{-1} \bigotimes I_{np})^{-1}\eta_t)$. Note that, also in this case, IR_y are non-explosive.

4 Estimation

The model (6)-(7) is estimated using Bayesian methods. That is, we specify prior distributions for θ^t, V and H^t and use data up to t to compute posterior estimates of the structural parameters and of continuous functions of them. Since our sample covers quarterly data from 1960:1 to 2002:4, we initially estimate the model for the sample 1960:1-1978:3 and then reestimate it 97 times moving the terminal date by one quarter from 1978:4 to 2002:4.

Posterior distributions for the structural parameters are not available in a closed form. MCMC methods are used to simulate sequences from the posterior distributions obtained with the information up to time t. Estimation of reduced form TVC-VAR models with or without time variations in the variance of the shocks to the transition equation is now standard (see e.g. Cogley and Sargent (2001)): it requires treating parameters which are time varying parameters as a block in a Gibbs sampling algorithm. Therefore, at each t and in each cycle of the Gibbs sampler, one runs the Kalman filter and the Kalman smoother, conditional on the draw of the other time invariant parameters. In our setup the calculations are complicated by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model. This means that we need to apply at each step the identification scheme, discarding paths which are explosive and paths which do not satisfy the restrictions we impose. While this complicates the Gibbs sampler loop, and dramatically reduces the number of draws available for inference, it is straightforward to implement and relatively cheap in terms of computing time.

Because of the heavy notation and the technicalities involved with the construction of posterior distributions we defer the details of the estimation to appendix B.

5 The Results

All the data we use is taken from the FREDII data base of the Federal Reserve Bank of San Louis. In our basic exercise we use the log of (linearly) detrended (linear) real GDP, the log of first difference of GDP deflator, the log of (linearly) detrendend M1 and the federal funds rate in that order. Systems containing the growth rate of GDP and M1 or the (linearly) detrendend unemployment rate and nonborrowed reserves are analyzed in the next section. Estimates of the distribution of structural parameters and of interesting functions of them are obtained with the amount of information available up to that point in time. That is, to produce posterior estimates for, say, 1981:1 we use (revised) data from 1960:1 up to 1981:1 and to produced posterior estimates for 1981:2 we use (revised data) from 1960:1 up to 1981:2.

We divide the presentation of the results around four general themes: (i) Do reduced form coefficients display significant changes? (ii) Are there synchronized changes in the structural coefficients of different equations and/or in the structural variances of the model? (iii) Are there changes in the propagation of monetary policy disturbances in the short and the long run? (iv) Would it have made a difference in terms of macroeconomic performance if policy had been more aggressive against inflation at the end of the 1970's?

5.1 The evolution of reduced form coefficients

Figure 1 plots the evolution of the mean of the posterior distribution of the reduced form coefficients in each of the four equations (top panel) and their change (bottom panel). The first date corresponds to estimates obtained with the information available up to time 1978:3, the last one to estimates obtained with the information up to time 2002:4.

Several interesting aspects of the figure deserve some comments. First, consistent with the evidence of Sargent and Cogley (2001) and (2003) all equations display some coefficient variation. In terms of size, the money and interest rate equations are those which display the largest changes, while for the other two equations time variations are smaller. Interestingly, variations in the coefficients of the inflation equation are the smallest of all. Second, while variations appear to be stationary in nature, there are few coefficients which display a clear trend over time. For example, in output equation the coefficient on the first lag of money is drifting downward from 0.6 in 1979 to essentially zero at the end of the sample; while in the interest rate equation, the first lagged money coefficient is drifting upward from roughly zero in 1978 to about 0.9 in 2002. In general, and excluding for the 1979-1986 period, coefficients drift is smooth and relatively slow. Perhaps more importantly, we find little evidence in favor of once-and-for-all structural break in the coefficients of the output or the inflation equations (i.e. coefficients that jumps at some date and stays there afterward). Third, the majority of the changes appear to be concentrated at the beginning of the sample. The period 1979-1982 is the one which displays the most radical changes; there is some coefficient drift up to 1986, and after that date variations appear to be random in nature and small. Furthermore, these changes seem to involve primarily the coefficients on the first lag of money (this is the case in three equations) or of interest rates (one equation). Finally, centered 68% posterior bands for the coefficients at the beginning (1978:3) and at the end of the sample (2002:4) overlap in many cases. Therefore, barring few relevant exceptions, instabilities appear to be associated with the Volker (1979-1982) experiment and the adjustments following it, and are temporary and mean reverting in nature.

To go beyond the simple documentation of patterns of time variations in reduced form coefficients and study whether there are structural alterations in the economy and whether monetary policy is responsible for the changes, we next examine structural time variations and the causal links in the structural model.

5.2 Structural time variations

As figure 1, the upper panel of figure 2 presents the evolution of the posterior mean, obtained by the Gibbs sampler and our identification scheme, of lagged structural coefficients of each equation and the bottom panel their changes at each date in the sample. The first date corresponds again to estimates obtained with the information up to time 1978:3, the last one to estimates obtained with the information up to time 2002:4. Figure 3 presents the evolutions of the coefficients of the monetary policy equation (which is normalized to be the last one of the system). Contemporaneous coefficients appear in the top panel and lagged coefficients in the bottom panel.

Figure 2 contains two interesting features. First, changes in the structural coefficients are typically larger and more generalized than those in the reduced form coefficients. The output and the monetary policy equations are those displaying the largest absolute coefficient changes - these are up to 4 times as large as the largest absolute changes present in the other two equations - while the coefficients of the structural inflation equation are the most stable ones. Second, except for the money (demand) equation, most the variations are concentrated in the first part of the sample, are large in size, statistically and often economically significant. Consistent with the conventional wisdom the money (demand) equation displays trending coefficients (its own first lagged one goes from -0.6 in 1979 to 0.7 in 2002) and large swings in the output and interest rate coefficients from 1991 on. More interestingly from our point of view, there is a pattern in the structure of time variations. The output equation displays two regimes of coefficient variations (one with high variations up to 1986 and one with low variations thereafter) and, within the high volatility regime, the largest coefficient variations occur in 1986. The inflation equation shows the largest coefficient changes up to 1982 and, barring few exceptions, a more stable pattern has resulted since then. Finally, our identified monetary policy equation displays large and erratic lagged coefficient changes up to 1986 and coefficients variation is considerably reduced after that. Since the timing of the variations in the structural coefficients of the output and inflation equations are somewhat asynchronous with those observed in our monetary policy equation, figure 2 casts some doubts on a causal interpretation of the observed changes running from changes in the policy rule to changes in the dynamics of output and inflation.

Figure 3 examines in more detail the pattern of time variations present in the monetary policy rule and allows to understand better the relationship with the structural changes in the rest of the system. Three facts stand out. First, posterior mean estimate of all contemporaneous coefficients are humped shaped: they significantly increase from 1979 to 1982 and smoothly decline afterwards. Second, although all contemporaneous coefficients are higher at the end than at the beginning of the sample, they are typically lower than the conventional wisdom would suggest. In particular, the contemporaneous inflation coefficient peaks at about 1.2 in 1982 and then declines to a low 0.3, on average, in the 1990s. This pattern is shared by the two lagged inflation coefficients: they both peak in 1982 and smoothly decline afterward. In other words, except for the beginning of 1980s, the sum of coefficients on current and lagged inflation in the monetary policy equation fails to satisfy the so-called Taylor principle. In this sense, Greenspan was only marginally more effective than Burns in insuring price stability: interest rate responses to inflation movements were barely more aggressive in the 1990s than they were in the 1970s. Note also that, again excluding the beginning of the 1980's, the estimated monetary policy rule displayed considerable stability, in line with the evidence presented, e.g. by Bernanke and Mihov (1998). Hence, the fact that the macroeconomic performance was considerably different in the two periods seems to suggest that the size and characteristics of the shocks hitting the US economy in the two periods were different. We will elaborate on this issue later on.

Our estimated policy rule displays a six fold-increase in all contemporaneous and first lagged coefficients from 1979 to 1982. Interestingly, this increase is not limited to the inflation coefficients, but also involve output and the money coefficients. The high responsiveness of interest rates to economic conditions is consistent with the idea that by targeting monetary aggregates the Fed forced interest rates to jump to equilibrate a "fixed" money supply with a largely varying money demand (the period was characterized by a number of important financial innovations). The pervasive instability of this period and the subsequent three years adjustments contrasts with the substantial stability of the coefficients of the monetary policy rule in the rest of the sample. Hence, excluding the "Volker experiment", the systematic component of monetary policy has hardly changed over time and if, any change must be noted, it is more toward a decline in the responsiveness of interest rates to economic conditions. This "business as usual but with less active management" view is consistent with the conclusions of Leeper and Zha (2003) and the estimated time profile of the policy rule obtained estimating a small scale DSGE model with Bayesian methods (see e.g. Canova (2004)).

The evidence we have so far collected seems to give little credence to the "bad policy" hypothesis. If policy mistakes were made in the late 1970's, they seemed to have been repeated in the 1990's. Still, output and inflation dynamics and volatility were quite different. The "back luck" hypothesis suggests that policy has little to do with the observed changes and that instead changes in the distribution of the shocks hitting the economy that is responsible for the improved macroeconomic outcome.

Figure 4 presents some evidence on this issue. In the top panel we report the time evolution of the posterior mean estimate of the variance of the forecast errors of the structural model and, in the bottom panel, the variations produced by its heteroschedastic component, i.e the variations induced by product of the estimated innovations in the coefficient times the regressors of the model. Four features of the figure are of interest. First, forecast error variances in three of the four equations are humped shaped: they show a significant increase from 1979 to 1982 followed by a smooth decline. As it happened with structural coefficients, the posterior mean estimate of the variance of the shocks in 2002 is roughly similar in magnitude to the posterior mean estimate obtained in 1978. Second, the time profile of the changes in the forecast error variances of the output and the inflation equations are sufficiently well synchronized with the variations in the forecast error variance of our estimated monetary policy equation. In particular, since 1981, the variance of the forecast error of these three equations displayed a common and significantly declining trend. Third, the time variations in the posterior mean of the forecast error variance of the output and inflation equations due to changes in the coefficients are much larger than in the other two equations up to 1982. Since that date, the heteroschedastic component of the forecast error variance of all equations has common pattern and similar size. Finally, the proportion of the variance of the forecast error attributable to the variance of the shocks of the model has generally declined after 1982. The decline is stronger in the inflation equation, but somewhat cyclical in the output equation. Note shocks to the model contribute most to the variability of the forecast error between 1979 and 1981: in the output and inflation equation they account for about 50% of the variance. In sum, there is evidence of declines in the variances of the forecast errors, in the heteroschedastic component and in the contribution of shocks to the model to the variance of the forecast errors. Since the decline is, to a large extent, simultaneous the probability distribution from which shocks to the model and to the coefficients are drawn has become less dispersed over time.

Since the systematic component of monetary policy appears to have been remarkably stable over time and since the contribution of the non-systematic component of policy to the variance of forecast errors is roughly constant, we next examine whether changes in the propagation of monetary policy disturbances over time are present and whether can be related in any meaningful way to the changes in structural coefficients.

5.3 Changes in the propagation of monetary policy disturbances?

Figure 5 reports the posterior mean responses of output and inflation to identified monetary policy shocks for each date in the sample for horizons from 1 to 12 quarters. We do not report interest rate responses because they are similar across time, quite standard in shape and magnitude: after the initial impulse, the initial increase dissipates quite quickly and responses become insignificantly different from zero after the 3th quarter for each date in the sample. Figure 6 present 68% posterior confidence bands at selected horizons for the hypothesis that the 1982 response of output and 1978 response of inflation differ from the responses at all the other dates.

Four features of these figures are striking. First, the shape of both output and inflation responses is quite similar over time. Output responses are U-shaped, with a though response after about 3 quarters and a smooth convergence to zero. Inflation responses are also slightly U-shaped, the effect at one quarter is typically the largest, and responses smoothly converge toward zero after the second quarter. Note that, by construction, monetary shocks produce non-positive output and inflation effects for two quarters. Since, in the case of output responses, the effect is persistent and significant for about 5-6 quarters in every date of the sample, the timing of output responses is consistent with a-priori expectations.

Second, while there appear to be some differences in the mean posterior responses of output and inflation over time, they are quantitatively very small. In the case of output, the posterior mean of the instantaneous response is always centered around -0.15 while some differences emerge in the size of the through responses at lag 3 (the range is from -0.20 to -0.05). For the case of inflation, minor magnitude difference occur at lag one (posterior mean varies between -0.07 to -0.16) while inflation responses are slightly more persistent at horizons from 3 to 8 in 1978.

Third, differences in inflation responses at all horizons are both statistically and economically small. In fact, as shown in figure 6, the posterior 68% confidence band for the largest discrepancy (the one at lag 1) includes zero at all horizons and if we exclude the initial three years, the time path of inflation responses is unchanged over time. The posterior 68% confidence band for the largest discrepancy (the one at lag 3) in the output responses does at times exclude the zero. In particular, the trough response in 1982 appear to be significantly deeper than the though response in 1978 and 1979 and at most dates after 1992. However, differences between the 1990s and the end of 1970s responses are quite small. Note that the differences are also economically insignificant: over the 12 quarters horizon, the maximum difference in the cumulative output multiplier is only 0.5%. In other words, a one percent increase in interest rates produced output responses which differ over time on average by 0.04% points at each horizon.

Finally, the ability of monetary policy to affect the economy has not significantly changed over time: responses in the end of the 1990's look very similar in shape and size to those of the end of the 1970's.

5.4 Inflation Dynamics and Monetary Policy

Sargent and Cogley (2001) and (2003) have examined measures of core inflation to establish their claim that monetary policy is responsible for the observed changes in the inflation dynamics. They define core inflation as the persistent component of inflation, statistically measured by the zero frequency of the spectrum (that is, by the sum of all autocovariances of estimated inflation process), and show i) persistence has substantially declined over time and ii) there is synchronicity between the changes in persistence and a narrative account of monetary policy changes. Pivetta and Reis (2004), using univariate conventional classical methods, dispute the first claim showing that differences over time in two measures of inflation persistence and in the half-life of inflation responses are statistically insignificant. Since our examination of the changes in the relationship between monetary policy and inflation has so far concentrated on short/medium run frequencies, we next turn to investigate whether their longer run relationship alter the conclusions we have reached so far. In particular, we are curious as to whether different frequencies of the spectrum carry different information regarding the relative merit of the two hypotheses under investigation.

Our analysis differs from the above mentioned ones in two important respects: first, we use output in place of unemployment in the estimated system; second, we measure persistence using the estimated structural coefficients. While the first difference is minor, the second is not. In fact, thanks the orthogonality of the structural shocks and the orthogonality of the ordinates of the spectrum we can not only describe the evolution of the spectrum of inflation over time, but also measure of proportion of the spectral power at frequency zero due to monetary policy and describe its evolution over time. In fact, from the structural MA representation of the system we have that $\pi_t = \sum_{i=1}^{n} \phi_{it}(\ell) e_{it}$ where e_{it} is orthogonal to e_{jt}. Hence the spectrum at Fourier frequencies ω is $S_\pi(\omega) = \frac{1}{2\pi}\sum_{i=1}^{n} |\phi_{it}(\omega)|^2 \sigma_i^2$ and the component at frequency zero due to monetary policy shocks is $S_\pi^*(\omega = 0) = \frac{1}{2\pi}|\phi_{nt}(\omega = 0)|^2 \sigma_n^2$.

The top panel of figure 7 shows the evolution of the posterior mean of the spectrum of inflation. On the vertical axis we report the size of the spectral density, and on the two horizontal axis the frequency (on the side) and the date of the sample(in front). The spectrum at frequency zero displays an initial increase in 1979-1980 followed by a sharp decline the year after; since 1981 estimated posterior mean of the zero frequency of the spectrum has been very stable (with the exclusion of 1991). The initial four fold jump and the following ten fold decrease appear visually large and statistically significant. In fact, from the bottom panel we see that the 68% posterior band for difference the log spectrum between 1979 and 1996 does not include zero at the zero frequency, contrary to Pivetta and Reis's conclusion. Note that at all other frequencies, there are very few differences over time both in terms of size and shape of the spectrum. Hence, if we exclude frequency zero, the posterior distribution of the spectrum of inflation has been relatively stable over time.

Figure 8 provides visual evidence on the role that monetary policy had in shaping the inflation persistence. The top panel plots the evolution of $S_{\pi}(\omega = 0)$ and $S^{*}_{\pi=0}(\omega)$ over time and the bottom panel reports, at each date, the percentage of the inflation persistence at each date due to monetary policy shocks. Three important conclusions can be derived from this figure. First, the two graphs in the top panel track each other very well, in particular in the 1978-1982 period and in 1991, suggesting that, in at least in terms of timing variations in monetary policy accounts for variations in the point estimates of inflation dynamics at the frequency zero. Second, the contribution of monetary policy to inflation persistence varies over time: fluctuations are large and over the 1978-2002 sample the percentage explained ranges from about 20 to about 75 percent. Interestingly, there is a significant trend increase since 1981 and the percentage found in 2002 is roughly the same as it was in 1978. Third, there is a substantial portion of inflation persistence (roughly 50 percent on average over the 1981-2002 period) which has nothing to do with monetary policy. Determining what lies behind this large percentage is beyond the scope of this paper. Nevertheless, one can conjecture that the hump in the point estimate present between 1980 and 1982 occurs because of changes in the structure of the economy (possibly due to financial innovations and changes in the real side of the economy) and because the Volker experiment made interest rates very volatile in response to these changes. The subsequent decline is consistent with the reduction of interest rate volatility and with a substantially more stable macroeconomic environment and the return to the pre-1979 orthodoxy.

In conclusions, as in Sargent and Cogley we find visual evidence instabilities in the posterior mean of our measure of inflation persistence. We also find that these changes are statistically significant while, roughly speaking, the posterior distribution of the spectrum of inflation at other frequencies has been stable over time. Changes in the point estimates of the posterior of inflation persistence go hand in hand with changes in the contribution of monetary policy shocks but their quantitative contribution is varying significantly over time. Perhaps more importantly, we find that the contribution of monetary policy to the variations in the posterior means of inflation persistence is smaller than expected. Hence, the role of monetary policy to account to the observed changes in inflation persistence is probably limited.

5.5 What if monetary policy would have been more aggressive?

It is common in the literature to argue by mean of counterfactuals that monetary policy failed to perform an inflation stabilization role in the 1970s (see e.g Clarida, Gali and Gertler (2000) or Boivin and Giannoni (2002b)) and that, had it followed a more aggressive stance against inflation, dramatic changes in the economic performance would have resulted. While exercises of this type are meaningful only in dynamic models with clearly stated microfundations, our structural setup allows us to approximate the ideal type of exercise without falling into standard Lucas-critique type traps. In fact, to the extent that the monetary policy equation we have identified is structural we can examine what would have happened in the economy if, e.g., a particular shock to the preferences of the Fed would have made the coefficients on inflation significantly higher. Given the estimated distribution of coefficients, we interpret "significantly higher" as the posterior mean plus two standard deviations in the coefficients at that particular point time.

Figure 9 plots the resulting (unnormalized) changes in output and inflation which would have been produced if such a change have occurred at selected dates in the sample. Three important features of this figure are worth mention. First, inflation would have considerably decreased if such a shock therapy would have been applied in 1978 but at all the other dates in the sample, the effect would have been minor. Second, a more aggressive stance on inflation would have induced negative output effects. In fact, in four of the reported dates output falls and in 1978 the fall is the largest. Third, a tighter response to inflation in 2002:4 would have temporarily decreased inflation and then increases it. At the same time it would have produced the largest output effects in the sample. Hence, there appear to be a significant Phillips curve trade-off, but the slope of the trade-off changes over time. In general, stabilizing inflation is costly and if we exclude the puzzling behavior at the end of the sample, it is only in 1983 that inflation reduction activities would have produced beneficial output effects.

Overall, while there was room for improvement in the conduct of monetary policy at the beginning of the sample, it is not that clear that the economy would have benefitted from a tougher inflation stance except in 1983. Since then, and again excluding the last year of the sample, the US economy has become became more and more insulated from changes in the systematic component of monetary policy.

6 Robustness analysis

There are a number of choices we have made in our analysis which may be responsible for the results we obtain. In the section we analyze the sensitivity of the outcomes to three particular choices: the identifying restrictions, the time series properties of the variables, variables included in the VAR.

All figures we have presented so far have been produced identifying monetary policy shocks using sign restrictions on the dynamics of money, inflation and output. Would the pattern of time variations, the estimated policy rule and the time profile of impulse responses be altered if another identification scheme is used? Figure 10 shows the estimated structural coefficients, the estimated variances of the forecast error and the responses of output and inflation at selected dates [5] Note that since contemporaneous coefficients are constant over time, the evolution of structural coefficients reproduces the pattern of time variations present in the reduced form coefficients (they are simply multiplied by a constant over time). Therefore, all the discussion subsection 5.1 apply here without a change. In particular, the sum of the inflation coefficients in the interest rate equation less than one at every date in the sample and by a substantial amount (maximum value is 0.6). Note

[5] Monetary shocks are identified via a Choleski decomposition where the monetary policy equation is the third in the list.

also that the time evolution of the estimated forecast error variance matches to a large extent the one present in figure 4. Finally, while the pattern of impulse responses changes with identification scheme - output fall in response to interest rate increases but inflation increases for at least a year after the shocks - the main conclusion that the propagation of monetary shocks has not changed over time is still valid. In fact the maximum discrepancy in the output responses occurs at the third lag when we consider the 1982 and the 1978 responses, the gap is 0.10 and the differences are not significantly different. Similarly, the maximum discrepancy in the inflation responses occur at lag 2 between the 1978 and the 1996 responses, the gap is 0.19 and again is insignificantly different from zero.

Some may feel uncomfortable with dynamic exercises conducted in a system where linearly detrended output and linearly detrended money are used. The typical argument is that after these transformations these two variables are still close to be integrated and are not necessarily cointegrated. Hence, the dynamics we trace out may be spurious. While we have serious doubts that this argument has any importance since the residuals of our structural VAR are well behaved, we have rerun our VAR system using the growth rate of output and the growth rate of M1 in place of the detrended values of output and M1. We report our results in figure 11: first we plots the contemporaneous coefficients of the monetary policy rule and the variances of the forecasts errors of the model; second, we present the time profile of output and inflation responses to a monetary policy shock, identified using sign restrictions as in our benchmark case.

Four features of figure 11 are worth discussing. First, there is even less evidence of structural instability in the policy coefficients: if we exclude the spike occurred between 1981 and 1984 the coefficients appear to be very stable in the entire the sample. This pattern is also present in the lagged coefficients of the policy rule and in the coefficients of the other equations. Second, also with this specification we find that the variance of all four forecast errors declines significantly over time: for example, the variability of GDP and inflation forecast errors in the 1990's is about half what it was in the 1980's and 1970's. Third, it is still the case that the sum of coefficients on inflation in the policy rule is less than one, and now this occurs for the entire sample. Finally, the transmission of monetary policy shocks has been largely unchanged over time. The only important variation concerns the response of inflation in the medium run, which was stronger at the beginning of the sample than at the end. Overall, these results confirm that our basic results are robust to the treatment of trends in the variables in the system.

Finally, we have examined the sensitivity of our conclusions to changes in the variables of the VAR. It is well known that small scale VAR models are appropriate only to the extent that omitted variables exert no influence on the dynamics of the included ones. However, a-priori there is no way to know what variable is more important and to check if our system effectively marginalized the influence of all relevant variables. For this reason we repeated our exercise substituting unemployment rate to detrended output and non-borrowed reserves to money. Figure 14 report the time variations of the resulting monetary policy rule obtained in the two cases

(TO BE CONTINUED)

7 Conclusions

This paper provides novel evidence on the contribution of monetary policy to the structural changes in the US economy. Contrary to previous work we derive a time varying structural representation of the US economy which allows to evaluate the magnitude of the changes in the coefficients and in the variance of the forecast errors at each point in time, and to analyze whether timing of the changes in the output and inflation equations match those observed in the policy equation. Our framework of analysis also allow us to assess the magnitude of time variations in the propagation of policy shocks both in the short and in the long run and to run some counterfactuals, to understand whether changes in the systematic component of policy would have significantly altered macroeconomic performance at some crucial date.

We employ a TVC-VAR model to address issues of interest and obtain posterior distributions for the structural coefficients by means of MCMC methods. Structural disturbances are identified using sign restrictions, but the main trust of the results is independent of the identification scheme employed. Because time variations in the coefficients induce important non-linearities in the dynamics of the model, we provide and implement a new definition of impulse responses, which is valid when coefficients vary over time, based on the difference of two conditional expectations differing in the arguments of their conditioning set. This definition allows us to trace out the effects of structural shocks and of shocks to the coefficients of the model.

Our results indicate that while there are changes in the estimated structural relationships, they tend to be localized in time and involve particular coefficients in certain equations. We show that, if we exclude the beginning of the 1980s, the monetary policy rule has been quite stable over time and that the posterior mean of the (sum of) inflation coefficients fails to satisfy the so-called Taylor principle, not only in the 1970s but also in the 1990s. Hence, both in terms of timing of the changes and in terms of aggressiveness to inflation movements, changes in the systematic component of monetary policy are unlikely to drive the changes in the output and inflation process. We also find evidence of a generalized decrease in the uncertainty surrounding the US economy over time and that the timing of the changes in the variance of the forecast error in the output and inflation equations roughly coincides with the timing of the changes observed in the variance of the monetary policy equation. Taken together, these facts suggest that the "bad luck" hypothesis has considerable more posterior support than the "bad policy" hypothesis in accounting for the observed structural changes in the US economy.

We present evidence suggesting that not only the monetary policy rule is unchanged, but also that the transmission of monetary policy shocks to output and inflation has also been very stable over time. Despite the presence of coefficient changes in the structural inflation equation, we find that inflation persistence (as measured by the height of the zero frequency of the spectrum) have not statistically changed over time. Both monetary and non-monetary factors account for the magnitude of inflation persistence at each point in time and while the relative contribution of monetary policy fluctuates over time, it shows an increasing role since 1981.

Finally, we investigate the claim that a more aggressive stance on inflation would have made a difference on output and inflation dynamics at the beginning of the 1980s. We find that such a policy it would have reduced inflationary pressures in short and medium run in 1979 but not afterwards but also that the output costs of such a policy would not have been negligible. Hence, while the Fed had some room to improve economic performance at the end of the 1970s by choosing a more aggressive stance on inflation, the considerable decline in the variance of the shocks hitting the economy since the beginning of the 1980s appear to be responsible for the improved macroeconomic performance experienced by the US economy since then.

Our results differ to some extent from those present in the literature. We can explain the differences in several ways. First, we are the first to conduct a time varying structural analysis of the causal link between monetary policy the dynamics of output and inflation. Previous studies which used the same level of econometric sophistication we use to construct posterior distributions (such as Sargent and Cogley (2001) (2003)) have concentrated on reduced form analysis and were forced to use the timing of the observed changes to infer the contribution of monetary policy to changes in output and inflation variances and dynamics. Our approach still allows us to use informal tests based on timing of the changes but, in addition, permits us to quantify the contribution of monetary policy changes to observed changes in output and inflation dynamics. Relative to earlier studies such as Bernanke and Mihov (1998), Hanson (2001) or Leeper and Zha (2003), which use subsample analyses to analyze the changes over time in structural VAR coefficients and in monetary policy equations, we are able to precisely track the evolution of the coefficients over time and produce a more complete and reliable picture of the insignificance of the observed monetary policy changes.

Our results broadly agree with those obtained recursively estimating a DSGE model with Bayesian methods (see Canova (2004)) and contrast with those of Boivin and Giannoni (2002b) who use an indirect inference principle to estimate the parameters of a DSGE model over two subsamples. We conjecture the differences are due to the properties of the two estimation methods when the model is a false representation of the data. Finally, our results agree with those of Sims and Zha (2002), despite the different methodologies employed. We add to their analysis the conclusion that factors other than monetary policy (for example, those emphasized in McConnell and Perez Quiroz (2001)) could be more important in explaining the structural changes that the US economy has witnessed over the last 25 years.

This paper leaves open several avenues of research which would be interesting to investigate in the future. First of all, one would like to know if there are reasons, other than the Volker experiment and the decreased incidence of shocks over time, which account for the pattern of structural time variations we have presented in this paper. Since inflation persistence appears to be driven by factors other than monetary policy, and since real factors seem to be important, such an enterprise could provide important evidence on the sources of structural variations in the US economy over the last 25 years. Second, we have repeatedly mentioned that the monetary policy rule failed to satisfy the Taylor principle in the 1990s. Why is it that inflation did not follow the same pattern as in the 1970? What is the contribution of technological changes to this improved macroeconomic framework? Clearly such an investigation would nicely complement the analysis conducted here. Third, it would be interesting to investigate the economic reasons behind the changes in the volatility of the forecast errors of the system we have documented. Analysis along the lines of McConnell and Perez Quiroz (2001) could shed important light on such an issue. Finally, as we mentioned, the 1979-1982 period was different from the rest of the sample. One would like to know if the resulting instability is due to the choice of monetary policy regime or whether it would have been present anyway and was simply enhanced by the choice of monetary targeting. We leave such investigations to future work.

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Appendix

A. Impulse Response Functions

Substituting recursively into the system for ks we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{0,t+k} + \sum_{j=1}^{k-1} \left(\prod_{i=0}^{j-1} \mathbf{A}_{t+k-j} \right) \mathbf{A}_{0,t+k-j} + \prod_{j=0}^{k-1} \mathbf{A}_{t+k-j} \mathbf{y}_t + \sum_{j=1}^{k-1} \left(\prod_{i=0}^{j-1} \mathbf{A}_{t+k-i} \right) {}^2{}_{t+k-j} + {}^2{}_{t+k}$$

Rewrite the above representation as

$$\mathbf{y}_{t+k} = \mathbf{A}_{0,t+k} + \sum_{j=1}^{k-1} \mathbf{\Phi}_{t+k,j} \mathbf{A}_{0,t+k-j} + \mathbf{\Phi}_{t+k,k-1} \mathbf{y}_t + \sum_{j=0}^{k-1} \mathbf{\Phi}_{t+k,j} {}^2{}_{t+k-j}$$

where $\mathbf{\Phi}_{t+k,j} = \prod_{i=0}^{j-1} \mathbf{A}_{t+k-j}$ for $j = 1, 2, ...,$ $\mathbf{\Phi}_{t+k,0} = I$. Let $ext_{(h,k)}(X)$ be an extractor, a function which extract the h-rows and the k-columns of the matrix X. Since $y_t = ext_{(n,1)}(\mathbf{y}_t)$ we have

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k-1} \mathbf{y}_t + \sum_{j=0}^{k-1} \Phi_{t+k,j} S H_{t+k-j} e_{t+k-j}$$

where $\tilde{A}_{0,t+k} = A_{0,t+k} + \sum_{j=1}^{k-1} \Phi_{t+k,j} A_{0,t+k-j}$ for $k > 1$ and $\tilde{A}_{0,t+1} = A_{0,t+1}$, $\tilde{\mathbf{\Phi}}_{t+k,k-1} = ext_{(n,n^2p)}(\mathbf{\Phi}_{t+k,k-1})$ and $\Phi_{t+k,j} = ext_{(n,n)}(\mathbf{\Phi}_{t+k,j})$. Thus

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k-1} \mathbf{y}_t + \sum_{j=0}^{k-1} \Psi_{t+k,j} e_{t+k-j} \tag{13}$$

where $\Psi_{t+k,j} = \Phi_{t+k,j} S H_{t+k-j}$ $\Psi_{t+k,0} = \Phi_{t+k,0} S H_t = S H_t$ and . Let us now consider a partitioned version of equation (). Let $e_t = (e_{i,t} | e_{-i,t})$ where $e_{i,t}$ is an element of e_t and $e_{-i,t}$ is the vector containing the other $n-1$ elements of e_t. Let $H_t = (h_t | h_t^{-i})$, where h_t is a column of H_t corresponding to $e_{i,t}$ and h_t^{-i} is the matrix formed by remaining $n-1$ columns of H_t. Then we can rewrite equation (13) as

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k-1} \mathbf{y}_t + \sum_{j=0}^{k-1} \Phi_{t+k,j} S h_t e_{i,t+k-j} + \sum_{j=0}^{k-1} \Phi_{t+k,j} S h_t^{-1} e_{-i,t+k-j}$$

or

$$y_{t+k} = \tilde{A}_{0,t+k} + \tilde{\mathbf{\Phi}}_{t+k,k-1} \mathbf{y}_t + \sum_{j=0}^{k-1} \Psi^{i}_{t+k,j} e_{i,t+k-j} + \sum_{j=0}^{k-1} \Psi^{-i}_{t+k,j} e_{-i,t+k-j} \tag{14}$$

where $\Psi^{i}_{t+k,j} = \Phi_{t+k,j} S h_t$ and $\Psi^{-i}_{t+k,j} = \Phi_{t+k,j} S h_t^{-i}$.

Time-Varying Coefficients VAR: Non-Systematic Component

Let us consider equation (3n). Fix $e_{i,T+1}$ to be a monetary policy shock to the non-systematic component occurring at time $T+1$. Define the sets $I_1 = \{y^t, \theta^t, V, K_t, \xi^{\delta}_{i,t+1}, \xi_{-i,t+1}, \xi^{t+\tau}_{t+2}\}$

and $I_2 = \{y^t, \theta^t, V, K_t, \xi_{-i,t+1}, \xi_{t+2}^{t+\tau}\}$. Taking expectations conditional to the two sets we obtain

$$\begin{aligned} E\left(y_{T+\tau}|I_1\right) &= E(\tilde{A}_{0,T+\tau}|I_1) + E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}\mathbf{y}_T|I_1\right) + \\ &+E\left(\sum_{j=0}^{\tau-1}\Psi^i_{T+\tau,j}e_{i,T+\tau-j}|I_1\right) \\ &+E\left(\sum_{j=0}^{\tau-1}\Psi^{-i}_{T+\tau,j}e_{-i,T+\tau-j}|I_1\right) \end{aligned} \tag{15}$$

and

$$\begin{aligned} E\left(y_{T+\tau}|I_2\right) &= E(\tilde{A}_{0,T+\tau}|I_2) + E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}\mathbf{y}_T|I_2\right) + \\ &+E\left(\sum_{j=0}^{\tau-1}\Psi^i_{T+\tau,j}e_{i,T+\tau-j}|I_2\right) \\ &+E\left(\sum_{j=0}^{\tau-1}\Psi^{-i}_{T+\tau,j}e_{-i,T+\tau-j}|I_2\right) \end{aligned} \tag{16}$$

Using the orthogonality assumption between e_t and η_t and taking the difference between the two conditional expectations we have

$$\begin{aligned} E\left(y_{T+\tau}|I_1\right) - E\left(y_{T+\tau}|I_2\right) &= E(\Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau}|I_1) - E\left(\Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau}|I_2\right) \\ &= \Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau} - \Psi^i_{T+k,k}E(e_{i,T+k-k}|I_2) \\ &= \Psi^i_{T+\tau,\tau}e_{i,T+\tau-\tau} \end{aligned} \tag{17}$$

Time-Varying Coefficients VAR: Systematic Component

Let $\eta_{i,T+1}$ be shock in the systematic component of the monetary policy. Taking expectation conditional to I_j with $j = 1, 2$we have

$$\begin{aligned} E\left(y_{T+\tau}|I_j\right) &= E(\tilde{A}_{0,T+\tau}|I_j) + E\left(\tilde{\mathbf{\Phi}}_{T+\tau,\tau}|I_1\right)\mathbf{y}_T + \\ &+E\left(\sum_{j=0}^{\tau-1}\Psi_{T+\tau,j}|I_j\right)e_{T+\tau-j} \end{aligned} \tag{18}$$

And taking the difference between conditional expectations we obtain equation (11).

Fixed Coefficients VAR

Consider equation (?) and assume that the coefficients are constant. Thus the time varying coefficients become $\Phi_{t+k,k} = \Phi_k = \mathbf{A}^k$ and $\Psi_k = \mathbf{A}^k SH$ for all k which correspond to traditional impulse response function for structural shocks in fixed coefficients VARs. Thus IR for shock i will be Ψ^i_k.

B. Estimation

Priors

We choose prior specifications for the unknowns which gives us analytic expressions for the conditional posteriors of subvectors of the unknowns. Let T be the end of the estimation sample and let $\bar{k}$ be the periods for which the identifying restrictions must be satisfied. Let $H_t = \rho(\omega_t)$, where ω_t is a vector in R^6 whose elements are $U[0, 1]$, be a rotation matrix (see...) whose columns represents orthogonal points in the hypershere. Let $\mathcal{M}_t$ the set of impulse response functions at time t satisfying the restrictions and let $F(\mathcal{M}_t)$ be an indicator function which assumes value one if the identifying restrictions are satisfied, that is it is one if $(\Psi^i_{t+1,1}, ..., \Psi^i_{t+K,K}) \in \mathcal{M}_t$ and zero otherwise. Let the joint prior for $\theta^{T+\bar{k}}$, V and ω_T be proportional to the joint prior of $\theta^{T+\bar{k}}$ and V whenever the identifying restrictions are satisfied, that is

$$p(\theta^{T+K}, V, \omega_T) = p(\theta^{T+K}, V)F(\mathcal{M}_T)p(\omega_T) \tag{19}$$

where the first term, the joint prior for states and variances, can be factored as $p(\theta^{T+K}, V) = p(\theta^{T+K}|V)p(V)$. Here $p(\theta^{T+K}|V)$ is the conditional density of θ^{T+K} which is given by $p(\theta^{T+K}|V) \propto I(\theta^{T+K})f(\theta^{T+K}|V)$ where $f(\theta^{T+K}|V) = f(\theta_0|V)\prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, V)$ and $I(\theta^{T+K}) = \prod_{t=0}^{T+K} I(\theta_t)$. Hence, the conditional density of θ^{T+K} is normal times the indicator function.

We assume that the covariance matrices Σ and Ω are inverse-Wishart distributed with scale matrix Σ_0^{-1}, Ω_0^{-1} and degree of freedom T_{01} and T_{02}. We also assume that the prior for the initial state is a Gaussian truncated random variable independent of Σ and Ω, i.e. $p(\theta_0) \propto I(\theta_0)N(\bar{\theta}, \bar{P})$. Such a prior specification is similar to the one used by Uhlig (2001) in a fixed coefficients VAR. The uniform prior is justified by the fact that all the trajectories satisfying the restrictions are a-priori equally likely.

Collecting the pieces the joint prior is:

$$p(\theta^{T+K}, V, \omega_T) \propto I(\theta^{T+K})F(\mathcal{M}_T)f(\theta_0)\prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, V)p(\Sigma)p(\Omega) \tag{20}$$

Note that for a Choleski scheme $H_t = I_n$, and the priors for θ^{T+K}, V remain unchanged. Thus, the prior reduces to

$$p(\theta^{T+K}, V, h_T) = I(\theta^{T+K})f(\theta_0)\prod_{t=0}^{T+K-1} f(\theta_{t+1}|\theta_t, V)p(\Sigma)p(\Omega) \tag{21}$$

We calibrate the prior by estimating a fixed coefficients VAR using data from 1960:I up to 1969:I. We set $\bar{\theta}$ equal to the point estimates of the coefficients and $\bar{P}$ to the estimated covariance matrix. We set Σ_0 equal to the VAR innovations covariance matrix and $\Omega_0 = \varrho\bar{P}$. The parameter ϱ measure how much the time variation is allowed in coefficients. Although as t grows likelihood tends to dominate the prior, results are somewhat sensitive to the choice of ϱ. In particular, specifying a small ϱ we restrict the time variation in the coefficients while specifying a large ϱ reduces the probability of finding draws which generate non-explosive paths. We end up choosing ϱ on the basis of the sample size i.e. for the sample 1969:I-1978:III $\varrho = 0.0025$, 1969:I-1981:III $\varrho = 0.003$, 1969:I-1983:III $\varrho = 0.0035$, for 1969:I-1987:III $\varrho = 0.004$, 1969:I-1992:III $\varrho = 0.007$, 1969:I-1996:I $\varrho = 0.008$, 1969:I-2002:IV $\varrho = 0.01$. The range of values of ϱ implies a quiet conservative prior coefficient variations: in fact, time variation accounts from a 0.35 and a 1 percent of the total coefficients standard deviation.

Our primary goal is to compute impulse response functions, which depend on $\Phi_{t+k,k}$'s, the square factor S and the matrix H_t. Therefore, we characterize first the posterior distributions of these parameters and then describe a sampling approach from these posteriors to construct a draw for the impulse responses. Note that in the sign restriction case H_t is a random variable (while under the Choleski identification scheme H_t is a matrix of constants) Thus in the second case, need only to characterize the posterior distribution of θ^{T+K} and V (no need to worry about h_T).

Posteriors

To draw the relevant quantities we need to obtain $p(h_{T+1}^{T+K}, \omega_T, \theta_{T+1}^{T+K}, \theta^T, V|y^T)$, which is analytically intractable. However, it is can be decomposed into simpler components. First, note that such a distribution is proportional to the unrestricted posterior predictive distribution times the two indicator variables. In fact

$$\begin{aligned} p(\omega_T, \theta_{T+1}^{T+K}, \theta^T, V|y^T) &= p(\omega_T, \theta^{T+K}, V|y^T) \\ &\propto p(y^T|\omega_T, \theta^{T+K}, V)p(\omega_T, \theta^{T+K}, V) \end{aligned} \tag{22}$$

the first term of the right hand side in the second line is the likelihood and is invariant for any orthogonal rotation thus $p(y^T|\omega^T, \theta^{T+K}, V) = p(y^T|\theta^{T+K}, V)$. The second term is the joint prior $p(\omega_T, \theta^{T+K}, V) = p(\theta^{T+K}, V)F(\mathcal{M}_T)p(\omega_T)$. Thus we have

$$p(\omega_T, \theta^{T+K}, V|y^T) \propto p(\theta^{T+K}, V|y^T)F(\mathcal{M}_T)p(\omega_T) \tag{23}$$

where $p(\theta^{T+K}, V|y^T) \propto p(y^T|\theta^{T+K}, V)p(\theta^{T+K}, V)$ is the posterior distribution for θ^{T+K} and V (i.e. the posterior for reduced form parameters). Such a posterior can be factored as

$$\begin{aligned} p(\theta^{T+K}, V|y^T) &= p(\theta_{T+1}^{T+K}, \theta^T, V|y^T) \\ &= p(\theta_{T+1}^{T+K}|y^T, \theta^T, V)p(\theta^T, V|y^T) \end{aligned} \tag{24}$$

where the first term of the right hand side represents beliefs about the future and the second term represents the posterior density for states and hyperparameters. First notice that $p(\theta_{T+1}^{T+K}|y^T, \theta^T, V) = p(\theta_{T+1}^{T+K}|\theta^T, V)$ and because of the Markov assumptions future states can be factored as

$$p(\theta_{T+1}^{T+K}|\theta^T, V) = \prod_{k=1}^{K} p(\theta_{T+k}|\theta_{T+k-1}, V) \tag{25}$$

and θ_{T+k} is conditionally normal with mean θ_{T+k-1} and variance Ω times the indicator variable. Therefore we can write

$$\begin{aligned} p(\theta_{T+1}^{T+K}|\theta^T, V) &= I(\theta_{T+1}^{T+K}) \prod_{k=1}^{K} f(\theta_{T+k}|\theta_{T+k-1}, V) \\ &= I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K}|\theta_T, V) \end{aligned} \tag{26}$$

The posterior density for the hyperparameters and the states can be factored as

$$p(\theta^T, V|y^T) \propto p(y^T|\theta^T, V)p(\theta^T, V) \tag{27}$$

The first term is the likelihood function which, given the states, has Gaussian innovations and then $p(y^T|\theta^T, V) = f(y^T|\theta^T, V)$. The second term is the joint prior for states and hyperparameters. This second term can be factored into a conditional for the states and a marginal for the hyperparameters

$$p(\theta^T, V|y^T) \propto f(y^T|\theta^T, V)p(\theta^T|V)p(V) \tag{28}$$

The conditional density for the states can be written as $p(\theta^T|V) \propto I(\theta^T)f(\theta^T|V)$ where $f(\theta^T|V) = f(\theta_0|V)\prod_{t=1}^{T} f(\theta_t|\theta_{t-1}, V)$ and $I(\theta^T) = \prod_{t=0}^{T} I(\theta_t)$, thus we obtain

$$p(\theta^T, V|y^T) \propto I(\theta^T)f(y^T|\theta^T, V)f(\theta^T|V)p(V) \tag{29}$$

But $f(y^T|\theta^T, V)f(\theta^T|V)p(V)$ is the posterior density resulting if no restrictions were imposed, $p_u(\theta_T, V|y^T)$. Thus we have

$$p(\theta^T, V|y^T) \propto I(\theta^T)p_u(\theta^T, V|y^T) \tag{30}$$

and

$$I(\theta^T) = \prod_{t=0}^{T} I(\theta_t) \tag{31}$$

Collecting the pieces the posterior predictive distribution is

$$p(\omega_T, \theta_{T+1}^{T+K}, \theta^T, V|y^T) \quad \propto \quad F(\mathcal{M}_T)\left[I(\theta^{T+K})f(\theta_{T+1}^{T+K}|\theta^T, V)p_u(\theta^T, V|y^T)\right]p(\omega_T) \tag{32}$$

Note that for a Choleski identification

$$p(\theta_{T+1}^{T+K}, \theta^T, V|y^T) = I(\theta^{T+K})f(\theta_{T+1}^{T+K}|\theta^T, V)p_u(\theta^T, V|y^T) \tag{33}$$

Drawing from the Posterior of structural parameters

To draw structural parameters we proceed as follows.

1. Draw from the unrestricted posterior, $p_u(\cdot)$, computed with the Gibbs sampler (see below), a vector of θ^T. Apply the filter $I(\theta^T)$ and discard explosive paths.

2. Draw a sequence of future states θ_{T+1}^{T+K}, i.e. obtain N draws of u_t from $N(0, \Omega)$ and iterate in $\theta_{T+i} = \theta_{T+i-1} + u_{T+i}$ N times (θ_{T+i} is conditionally normal with mean θ_{T+i-1} and variance Q). Apply the filter $I(\theta^{T+K})$ and discard explosive paths.

3. Draw $\omega_{i,T}$ for $i = 1, ..., 6$ from a $U[0, 1]$. Construct $H_t = \rho(\omega_t)$

4. Draw R from the unrestricted posterior. Compute the triangular matrix S, such that $R = SS'$. Construct K_t^{-1}.

5. Compute $(\Psi_{T+1,1}^{i,\ell}, ..., \Psi_{T+K,K}^{i,\ell})$ and $F(\mathcal{M}_T)^\ell$ and apply the filter $F(\mathcal{M}_T)$, if the draw satisfies identification restrictions keep the draw γ_t otherwise discard it.

Computing Posteriors of reduced form parameters: the Gibbs Sampler

The Gibbs Sampler we use iterate on two steps. The implementation we use is identical to Cogley and Sargent (2001)

• Step 1: States given hyperparameters

Conditional on hyperparameters and the data, the unrestricted posterior of the states is normal and $p_u(\theta^T|y^T, V) = f(\theta_T|y^T, V) \prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1}, y^t, V)$ All the density in the right end side are Gaussian they their conditional means and variances can be computed using the backward recursion of the Kalman filter. Define

$$\begin{aligned}
\theta_{t|t} &\equiv E(\theta_t|y^t, V) \\
P_{t|t-1} &\equiv Var(\theta_t|y^{t-1}, V) \\
P_{t|t} &\equiv Var(\theta_t|y^t, V)
\end{aligned}$$

Given some initial $P_{0|0}$, $\theta_{0|0}$, Ω and Σ, we compute forward Kalman filter recursions

$$\begin{aligned}
P_{t|t-1} &= P_{t-1|t-1} + \Sigma \\
K &= (P_{t|t-1} X_t)(X_t' P_{t|t-1} X_t + \Omega)^{-1} \\
\theta_{t|t} &= \theta_{t-1|t-1} + K_t(y_t - X_t' \theta_{t-1|t-1}) \\
P_{t|t} &= P_{t|t-1} - K_t(X_t' P_{t|t-1})
\end{aligned}$$

The last time iteration gives $\theta_{T|T}$ and $P_{T|T}$ which are the conditional means and variance of $f(\theta_t|y^T, V)$. Hence $f(\theta_T|y^T, V) = N(\theta_{T|T}, P_{T|T})$. The other $T-1$ densities can be computed using the backward Kalman filter recursions, i.e

$$\theta_{t|t+1} = \theta_{t|t} + P_{t|t} P_{t|t+1}^{-1} (\theta_{t+1} - \theta_{t|t-1}) \tag{34}$$

$$P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t} \tag{35}$$

where $\theta_{t|t+1} \equiv E(\theta_t|\theta_{t+1}, y^t, V)$ and $P_{t|t+1} \equiv Var(\theta_t|\theta_{t+1}, y^t, V)$ are the conditional means and variances of the remaining terms in $p_u(\theta^T|y^T, V)$. Thus $f(\theta_t|\theta_{t+1}, y^t, V) = N(\theta_{t|t+1}, P_{t|t+1})$

Therefore, to sample θ^T from the conditional posterior we proceed backward, sampling θ^T from $N(\theta_{T|T}, P_{T|T})$ and θ^t from $N(\theta_{t|t+1}, P_{t|t+1})$ for all $t < T$.

• Step 2: Hyperparameters given states

To sample V, notice that we can sample separately Σ and Ω because of the independence assumption. Conditional on the states and the data ε_t and u_t are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results is an inverse-Wishart posterior, so that

$$\begin{aligned} p(\Sigma|\theta^T, y^T) &= IW(\Sigma_1^{-1}, T_{11}) \\ p(\Omega|\theta^T, y^T) &= IW(\Omega_1^{-1}, T_{12}) \end{aligned} \tag{36}$$

where $\Sigma_1 = \Sigma_0 + \Sigma_T$, $\Omega_1 = \Omega_0 + \Omega_T$ and $T_{11} = T_{01} + T$ $T_{12} = T_{02} + T$ and Σ_T and Ω_T are proportional to the covariance estimator

$$\begin{aligned} \frac{1}{T}\Sigma_T &= \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t' \\ \frac{1}{T}\Omega_T &= \frac{1}{T}\sum_{t=1}^{T} u_t u_t' \end{aligned}$$

To draw sample from inverse-Wishart distributions, we draw T n-dimensional vectors $\tilde{\varepsilon}_t$ from $N(0, \Sigma_1)$ and construct, e.g., $\Sigma^{-1} = \sum_{t=1}^{T} \tilde{\varepsilon}_t \tilde{\varepsilon}_t'$. Σ^{-1} is Wishart distributed and therefore Σ is inverse-Wishart.

Under some regularity conditions (see Tierney (1994)) and after a burn-in period, iterations on these two steps produce draw from $p_u(\theta^T, V|y^T)$.

For each date we are interested in computing impulse responses, 10000 iteration of the Gibbs sampler are made. We have constructed CUMSUM graphs to check for convergence and found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are always well behaved and none is multimodal. We keeping one every two of the remaining draws and discard all the draws generating explosive paths. The autocorrelation function of draws which are left is somewhat persistent and autocorrelations twenty draws apart are still significantly different from zero. This is somewhat of a problem. We could reduce the autocorrelation taking draws more largely spaced (say, one every 5) but this come at the price of reducing the number of draws which satisfy the identification restrictions and therefore substantially reduce the precision of the exercise. With our approach we ended up having approximately 300 draws for each date to conduct structural inference. All these properties are very similar at each date in the sample. we consider.

Computation of IR

• Shocks to the Non-Systematic Component

Steps 1-5 discussed for drawing structural parameters.

1. Steps 1-5 discussed for drawing structural parameters.

2. Construct the matrix product $\Psi_{t+k,k}$.

• Shocks to the Systematic Component

In order to compute a draw for the IR to the systematic component of monetary policy we proceed as follows.

1. Draw θ_T and V from the posterior distribution.
2. Compute S and draw a sequence of ε_{t+1}^{T+K} and u_{t+2}^{T+K}.
3. Draw a H_t as previously described.
4. Fix $\eta_{i,t+1}^{\delta}$ and draw $\eta_{-i,t+1}$ from the conditional distribution of $\eta_{-i,t+1}|\eta_{i,t+1} = \delta$ and form the vector η_{t+1}^{δ}.
5. Compute $u_{t+1}^{\delta} = (K_t^{-1} \bigotimes I_{np+1})^{-1}\eta_{t+1}^{\delta}$ and define $u_{t+1}^{\delta,t+k} = \{u_{t+1}^{\delta}, u_{t+2}^{T+K}\}$
6. Using $u_{t+1}^{\delta,t+k}$, ε_{t+1}^{T+K} and θ_t compute $\tilde{\theta}_{T+1}^{T+K}$ compute $\tilde{\Phi}_{T+k,i}$ and $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j}\tilde{A}_{0,T+k-i}$ $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j}\varepsilon_{t+k,j}$ for $k = 1, ..., K$ and $i = 1, ...K - 1$. Using these draws compute y_{t+k}^{δ}. This is a draw for $E(y_{t+k}|I_1)$..
7. Fix $\eta_{i,t+1}^{0}$ and draw $\eta_{-i,t+1}$ from the conditional distribution of $\eta_{-i,t+1}|\eta_{i,t+1} = 0$ and form the vector η_{t+1}^{0}.
8. Compute $u_{t+1}^{0} = (K_t^{-1} \bigotimes I_{np+1})^{-1}\eta_{t+1}^{0}$ and define $u_{t+1}^{0,t+k} = \{u_{t+1}^{0}, u_{t+2}^{T+K}\}$
9. Using $u_{t+1}^{0,t+k}$, ε_{t+1}^{T+K} and θ_t compute $\tilde{\theta}_{T+1}^{T+K}$ compute $\tilde{\Phi}_{T+k,i}$ and $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j}\tilde{A}_{0,T+k-i}$ $\sum_{i=1}^{k} \tilde{\Phi}_{T+k,j}\varepsilon_{t+k,j}$ for $k = 1, ..., K$ and $i = 1, ...K - 1$. Using these draws compute y_{t+k}^{0}. This is a draw for $E(y_{t+k}|I_2)$..
10. Take the difference of the two realizations of the conditional expectations.