Job Competition Over the Business Cycle

Implications for Labor Productivity and Unemployment Rates by Skill

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Abstract

In this paper I develop a matching model with endogenous skill requirements of jobs and heterogeneous workers. I study the impact of skill-mismatches between workers and jobs on the cyclical behavior of labor productivity and the cyclical behavior of unemployment rates across skill groups. I focus on the skill-mismatch that takes the form of high-skill workers accepting transitorily jobs below their skill level, thereby influencing the labor market prospects of lower skill groups, and continue searching while employed for more suitable jobs. The model explains why the burden of unemployment falls disproportionably on the lower skill groups and why the quality of worker-job matches is lower in recessions.

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1 Introduction

Previous theories that recessions "cleanse" the economy from low productivity jobs, conflict with evidence that jobs created in recessions are likely to be temporary and low paying.[1] The literature on matching models with heterogeneous agents stresses the importance of accounting not only for the cleansing effect of recessions, but also for their impact on the quality of worker-job matches that survive cleansing. To reconcile theory with evidence, the notion formalized in Barlevy (2002), which is labeled as the "sullying effect of recessions, is that workers are less likely to abandon bad matches in recessions, thus downturns exacerbate skill-mismatch.

However, observed differences in the cyclical behavior of different skill groups, have been overlooked when analyzing the cyclical properties of match quality. The standard assumption in matching models with heterogeneous agents is a common matching probability for all workers and vacancies regardless of their skill type. The underlying assumption is that all workers can perform any type of job. But this implies the same cyclical dynamics in mismatching and unemployment rates for all skill groups, which is empirically inconsistent: typically, the unemployment rate of the lower skilled is relatively higher and increases relatively more in downturns.[2] Moreover, unskilled workers have relatively lower exit rates from unemployment and lower propensity to search on the job.[3] If the propensity to search for another job is increasing in the workers' dissatisfaction with their current job, then this would suggest relatively lower mismatch rates for the unskilled.[4] Apart from not being empirically consistent, the assumption of identical matching probabilities fails to capture the search externalities and across-skill spillover effects that arise when workers of different skill compete for the same type of jobs.

The question I ask in this paper, is whether the observed procyclical match quality and differences in the labor market outcomes of different skill groups, can be explained by cyclical pattern in the matching behavior of high-skill workers, i.e., of downgrading to lower job levels to avoid unemployment, and upgrading to higher job levels by on-the-job search.[5] High skill workers can perform many of the jobs undertaken by the lower skilled, but not the other way around. This asymmetry, which has been previously overlooked, has two implications. First, since high-skill workers qualify for a wider range of job types, they are relatively more capable in finding transitory jobs, as opposed to remain unemployed until a suitable job offer comes along. Second, by accepting jobs below their skill level, high-skill workers hurt the labor market prospects of the lower skill groups, who can only perform a narrow range of job types. To explain why in recessions the burden of unemployment falls disproportionably on the lower skilled, I analyze the extent to which downturns exacerbate this type of skill-mismatch, which I call overqualification henceforth, and the resulting negative search externalities and spillover effects at the lower segment of the labor market.

Moreover, by characterizing the evolution of overqualification over the business cycle,

[1] Studies that formalize the cleansing view of recessions are Hall (1991, 2000), Mortensen and Pissarides (1994), Caballero and Hammour (1994, 1996) and Gomes, Greenwood, and Rebelo (1999). Evidence that job quality is procyclical can be found for example in Bowlus (1995), Davis, Haltiwanger, and Schuh (1996), Bils (1985), Shin (1994), Bowlus, Liu and Robinson (2002), and Liu (2003).

[2] see e.g., Topel (1993) and Juhn, Merphy, and Topel (2002) in the case of the US, van Ours and Ridder (1995) in the case of , Moscarini (1996) in the case of Manacorda and Petrongolo (1999) find that the increase in unemployment is concentrated mainly among the unskilled in the case of the US and the EU countries: Britain, France, Germany, Italy, the Netherlands, and Spain.

[3] e.g., Blau and Robins (1990); Pissarides and Wadsworth (1994); Belzil (1996); Beach and Kaliski (1987).

[4] The finding of Bowlus (1995) that the quality of matches falls during recessions more evidently in white collar than blue collar activities gives additional support to this view.

[5] For evidence on over-education phenomena and the "crowding out" of lower educated workers see e.g. Teulings and Koopmanschap (1989), Bewley (1995, 1999).

I portray the cyclical properties of average match productivity, as an approximation of match quality. Workers are most productive on the jobs they are best suited for. If workers' productivity is reflected in wages, then overqualified workers should earn less than similar workers who are suitably matched.[6] In addition, overqualified workers are more likely to quit to a more suitable job.[7] Hence, jobs occupied by overqualified workers have lower duration than jobs filled by suitable workers. Therefore, I investigate whether rising overqualification rates in recessions can explain evidence that jobs created in recessions have lower duration and offer lower wages.

In the economy I examine firms open vacancies for either high-productivity jobs, which have high skill requirements, or low-productivity jobs, with lower skill requirements. High-skill workers are best suited for high-skill jobs, but they also qualify for low-skill jobs, whereas, low-skill workers qualify only for low-skill jobs. Some unemployed high-skill workers accept transitorily low-skill jobs and continue searching for high-skill jobs. Therefore, job seekers can be either unemployed, or overqualified high-skill workers. High-skill workers weaken the labor marker conditions of low-skill workers in two ways. First, by congesting the lower segment of the labor market, they reduce the chances low-skill workers find jobs. Second, since overqualified workers are likely to abandon low-skill jobs sooner than low-skill workers, they lower the profits of low-skill jobs, and discourage firms from opening low-skill vacancies.

I study the consequences of two types of temporary shocks: a proportional reduction in the productivity of all matches, and an increase in the rate at which matches are exogenously destroyed. The latter is motivated by previous theories that view higher job destruction rates in recessions as facilitating the creation of new more productive jobs, by increasing the arrival rate of unemployed workers to vacant jobs. Given the asymmetric nature of the matching technology, changes in the labor market conditions, due to both types of shocks, shift the relative value of opening high- and low-skill vacancies, thus altering the skill mix of vacancies opened. The chances job seekers find high- and low-skill jobs, change accordingly, causing the composition of job seekers to change. In turn, the changes in the composition of job seekers, induce firms to either downgrade or upgrade the skill mix of vacancies. For instance, a rise in the fraction of overqualified job seekers raises the profits of high-skill vacancies, but lowers the profits of low-skill vacancies, encouraging firms to open relatively more high-skill vacancies. Consequently, the same circle of across-skill interactions is repeated, as changes in the skill mix of vacancies, subsequently affect the composition of job seekers.

[6] In the data there are wage differentials among identical workers in different industries and occupations (e.g. Krueger and Summers (1987, 1988)). Evidence on wage inequality both across and within skill groups can be found in Bound and Johnson (1992) and Katz and Murphy (1992) for the US and Machin (1996) for the UK. The evidence on the earnings differences between overqualified workers and non-overqualified workers are mixed and depend on the whether earnings are compared across jobs or workers of the same skill type. For example, Hartog and Oosterbeek (1988), Gautier et al (2002), and Sicherman (1991) find that overqualified workers earn more than correctly matched workers, but Groot (1993, 1996), Vedugo and Verdugo (1989) and Alba-Ramirez and Blazquez (2003) find the opposite. The returns to overqualification are lower when compared overqualifed workers with observationally identical workers that are suitably matched. For instance, Verdugo and Verdugo (1989) find that the return to a year of overeducation is -8.0% in the US, and Groot (1993) that the average return of a year of education for correctly educated workers is higher than the under- or overeducated workers.

[7] Hersch (1991), gives evidence that overqualified workers are less satisfied with their jobs and are more likely to quit. Sicherman (1991) finds that overeducated workers have higher rates of firm and occupational mobility and are more likely to move to a higher-level occupation. Likewise, Alba-Ramirez and Blazquez (...) obtain results that suggest that overqualified workers are more likely to have shorter tenure than adequately educated workers. There are also evidence that high-wage jobs have lower quit rates (e.g. Krueger and Summers (1998)). If high-wage jobs are jobs that are suitably matched with high-skill workers, then this also suggests higher quit rates for overqualified workers.

Consistent with evidence the model predicts that downturns hurt low-skill employability relatively more. At times of low aggregate productivity both types of workers suffer reductions in their chances of finding jobs, as firms open fewer vacancies of each type. However, low-skill workers suffer in addition from shifts in the skill mix of vacancies. Provided that the dispersion in the productivities of high- and low-skill jobs is higher at lower levels of aggregate productivity, on impact of a negative productivity shock firms respond by upgrading the skill mix of vacancies. As a consequence, at the onset of a recession low-skill workers suffer a higher decline in their chances of finding jobs than high-skill workers. Moreover, the fraction of overqualified high-skill workers declines at first, since workers are more likely to find a high- as opposed to a low-skill vacancy at the beginning of the recession. Subsequently, the rising unemployment, encourages firms to downgrade the skill mix of vacancies to take advantage of the increased supply of unemployed job seekers. Still, the downgrading in the vacancy mix does not improve the labor market conditions of low-skill workers relative to high-skill workers. In contrast, when low-skill vacancies become relatively more abundant, a higher number of unemployed high-skill workers is misallocated into low-skill jobs, while the flow out of them declines. Therefore, the fraction of overqualified high-skill workers increases. While high-skill workers escape unemployment by accepting transitorily low-skill jobs, low-skill workers suffer in addition the negative consequences of intense job competition from high-skill workers.

The declining overqualification rate at the onset of the recession, but rising overqualification rate afterwards, implies that downturns entail two countervailing effects on average match productivity. Which of the two effects dominates, depends on the dispersion between the productivity of high- and low-skill jobs. For sufficiently small dispersion the second effect dominates, and overqualification rises in downturns, in line with evidence that match quality is lower in recessions. Hence, the model suggests that the procyclical match quality can be the consequence of high-skill workers moving transitorily into "mediocre" types of jobs as opposed to very simple jobs.

A novel result, previously overlooked by focusing on symmetric frameworks is that in periods of high job turnover and flows into unemployment, firms open relatively more low-skill vacancies. Intuitively, in periods of high job turnover, firms are relatively better off creating low-surplus jobs that generate lower loss when destroyed, and have lower skill requirements so that they benefit relatively more from the greater supply of unemployed job seekers. Therefore, the model predicts that if recessions are characterized by high rates of job destruction, a higher fraction of high-skill workers is misallocated into jobs below their skill level, lowering average match productivity and in addition, hurting low-skill employability. While previous theories associate higher job destruction rates in recessions with cleansing effects, this model puts more emphasis on the sullying effects of job destruction.

Several modifications to the equilibrium matching model have been made, mainly by allowing for heterogeneity in agents' skills, to explain long run uneven developments in the unemployment rates of different skill groups. Mortensen and Pissarides (1999) examine the consequences of skill-biased technological change on the skill mix of vacancies and thus on unemployment rates by skill. However, they assume ex post perfectly segmented labor markets where skill-mismatches and thus interactions across skill groups never occur. Acemoglu (1999) is the first to introduce terms such as "overskilled" workers into the equilibrium matching model. The skill composition of jobs is endogenous and the unemployment rates of high- and low-skill workers change endogenously depending on the vacancy creation strategy of firms. In particular, firms find it profitable either to create simple jobs that can be performed by both low- and high-skill workers, or to create both simple and complex jobs and search for the appropriate candidates. In the first case, some workers are overskilled for the jobs, thus markets are not ex post segmented. However, by assuming a constant contact rate between unemployed workers and vacancies independent of labor market conditions or skill, Acemoglu does not deal with across-skill spillover effects and search externalities.

The more recent contributions by Albrecht and Vroman (2002), Gautier (2002), and Dolado *et al.* (2003), extend this type of model to include job competition at the lower segment of the labor market. Similar to Acemoglu (1999), in Albrecht and Vroman (2002) high- and low-skill submarkets can endogenously segregate or merge depending on the productivity differential of high- and low-skill jobs. It may be worthwhile for unemployed high-skill workers to accept low-skill jobs, in which case vacancy creation and unemployment in the high-skill market affects job creation and unemployment in the low-skill market. In Gautier (2002) and Dolado *et al.* (2003) overqualified high-skill workers search on the job. Thus, it is optimal for high-skill workers to accept a low-skill job as long as it pays more than their flow income while unemployed. In this regard, both models are similar to the model in this paper, but Gautier (2002) adopts in addition two atypical assumptions. First, high-skill workers are assumed to be more productivity on low-skill jobs than low-skill workers. As a result if high-skill workers are sufficiently more productive than low-skill workers, the latter may benefit from job competition from high-skill workers. Second, high- and low-skill jobs are offered in different markets. Therefore, by searching in the low-skill market high-skill workers congest the low-skill workers, but not the other way around.

The main difference between this paper and the studies mentioned above, is that it adopts a dynamic framework and focuses on the effects of business cycles on mismatching and unemployment across skill. Although the studies above lead the way in addressing the impact of job competition at the lower segment of they labor market, they are limited in that they only perform comparative static exercises on the steady-state equilibrium. Their focus is to explain long-term uneven developments in unemployment rates or wages of different skill groups, in response to either skill biased technological shocks or exogenous increases in the supply of skill. Therefore, they do not deal with across-skill differences in the cyclical patterns of unemployment or match quality and their connection to job competition and overqualification phenomena.

With regard to the cyclical properties of match quality, closer to this paper is the approach of Barlevy (2002), who formalizes the sullying effect of recessions via a business cycle matching model with two-sided skill heterogeneity and on-the-job search.[8] In order to make the role of aggregate shocks more transparent Barlevy's model adopts a symmetric framework and focuses only on symmetric equilibria in which firms create equal numbers of each type of vacancy. However, a symmetric framework cannot account for observed uneven cyclical fluctuations in unemployment rates of different skill groups. More importantly it overlooks the impact of business cycles on the skill mix of vacancies opened, which in turn has important consequences on how efficiently workers are allocated across jobs.

[8] Moscarini (2001) also investigates the quality of worker-job matches in depressed labor markets and finds that a higher cost of waiting for jobs due to low job creation and intense competition for jobs raises workers' willingness to accept offers that do not provide them with the highest possible value in the market. However, in his model, workers with more specialized skills search more selectively and contact fewer vacancies. Hence, "noisier" allocations across jobs are more likely to occur among the less specialized workers, especially when the labor market conditions worsen. This follows from the assumption that firms observe the characteristics of the applicants and hire only the applicant that "best fits the job", namely the more specialized workers who produce more profits. Unlike the model in this paper, in Moscarini firms always prefer more specialized to less specialized workers by assumption. Consequently, given that more specialized workers have higher chances of beating competing applicants, they are less likely to search randomly for jobs. As a result, less specialized workers are more likely to move randomly across jobs.

2 The Model

2.1 Main Assumptions

The labor force is composed by a fraction δ of low-skill (l) workers, and $(1-\delta)$ of high-skill (h) workers. Similarly, vacancies can be either high-skill (h) or low-skill (l), but the mix is determined endogenously. High-skill workers qualify for both types of vacancies, whereas low-skill workers can only perform low-skill jobs. Accordingly, a low-skill worker can be either employed and producing in a low-skill job or unemployed and searching, while a high-skill worker can be in any if the following three states: employed and producing in a high-skill job, unemployed and searching, and employed and producing in a low-skill job, but simultaneously searching for another job. I label a worker in the latter state as overqualified job seeker. I further assume that workers are risk neutral and the interest rate r is constant.

High-skill workers are more productive in high- than low-skill jobs, but as productive as low-skill workers in low-skill jobs. Hence, firms with low-skill vacancies are better off hiring low- instead of high-skill workers, since both types are equally productive, but the latter is likely to quit to a high-skill job.[9] The productivity of each match is assumed to be the product of an aggregate component y, and a match specific component α. More formally, let $y\alpha_{ij}$ denote the flow of output of a job of type $i=(h,l)$ that is filled by a worker of type $j=(h,l)$. Then, the production technology assumptions can be summarized as $y\alpha_{hh} > y\alpha_{lh} = y\alpha_{ll} > y\alpha_{hl} = 0$. For simplicity, hereafter I denote the specific productivity of low-skill jobs as α_l and of high-skill jobs as α_h. When unemployed, a worker produces a flow output b.[10] Wages are chosen so as workers and employers share the surplus of a job in fixed proportions at all times. A share β goes to the workers, while a share $1-\beta$ goes to the firm.

The condition that ensures a match is formed in equilibrium is simply that workers are more productive when employed than when unemployed, i.e., $y\alpha_{ij} > b$, $\forall\, i,j$, which as shown below ensures that the surplus each match generates is positive. As long as $y\alpha_{lh} > b$ it is optimal for unemployed high-skill workers to accept low-skill jobs, since they retain their chances of finding a high-skill job by continuing to search while employed. I assume that the rate at which workers meet vacancies is the same regardless of whether the workers is employed or not.

Each firm has at most one job which can be either vacant and searching for candidates or filled and producing. The mass of each type of vacancy is determined endogenously by a free-entry condition. The choice of the type of vacancy is irreversible. The exogenous component of job destruction follows a Poisson process with arrival rate s. Although s is common to both types of jobs, the effective job destruction rate of low-skill jobs is higher due to on-the-job search by overqualified workers. Whenever a match is destroyed the job becomes vacant and bears a maintenance cost c.

In sections 3 and 4 below I investigate separately the impact of aggregate productivity and job destruction fluctuations by allowing y and s to follow a Markov process. Aggregate productivity takes the value of y^0 in recessions and $y^1 > y^0$ in booms, while the job destruction rate takes the value s^0 in recessions and $s^1 < s^0$ in booms. Both variables switch

[9] I assume that high- and low-skill workers are equally productive in low-skill jobs for convenience. This assumption ensures that overqualified workers have a negative impact on the profits of low-skill jobs, without any additional restrictions on the parameter space. Assuming instead that overqualified workers are more productive than low-skill workers on low-skill jobs, requires additional assumptions to ensure that the negative quit effect dominates the positive productivity effect.

[10] Since there is no government or any form of taxation in the model, I refrain from naming b as unemployment benefit.

between the two levels with a transition probability p. At every point in time the current values of y and s are common knowledge to all agents.

2.2 Matching and Timing

Firms and workers meet each other via a matching technology $m[v^a, u + e_{hl}(1 - s^a)]$, where $v = v_h^a + v_l^a$ denotes the mass of high- and low-skill vacancies, $u = u_h + u_l$ is the mass of high- and low-skill unemployed workers, and $e_{hl}(1 - s^a)$ the number of high-skill workers in low-skill jobs that survive exogenous job destruction and the superscript $a \in [0, 1]$ denotes the realization of aggregate state. The function $m[\cdot, \cdot]$ is strictly increasing in its arguments, and exhibits constant returns to scale. This allows me to write the flow rate at which workers meet vacancies as $m(\theta^a)$, where $\theta^a = \frac{v_h^a + v_l^a}{u_h + u_l + e_{hl}(1 - s^a)}$ captures the degree of labor market tightness. I assume that workers cannot distinguish the vacancy type before applying, therefore cannot decide for which type of job to apply. Consequently, low-skill workers encounter low-skill vacancies with probability per unit of time that is proportional to the fraction of low-skill vacancies. Similarly, high-skill workers encounter low- and high-skill vacancies with a probability per unit of time that is proportional to the fraction of low- and high-skill vacancies, respectively. Assuming that $\eta^a = \frac{v_l^a}{v_l^a + v_h^a}$, the effective matching rate of low-skill workers is $\eta^a m(\theta^a)$, while overqualified workers find high-skill vacancies with a probability $(1 - \eta^a)m(\theta^a)$. Unemployed high-skill workers accept both high- and low-skill jobs, thus their effective matching rate is $m(\theta^a)$.

I assume that time is discrete. Let $e = \{e_{hh}, e_{hl}, e_{ll}\}$ be the distribution of employed workers across types of matches, at the beginning of period t. The timing within a period is as follows. At the beginning of period t the realizations of aggregate state become common knowledge to all agents. After the aggregate state is determined, agents produce. Subsequently, some of the existing matches are exogenously destroyed and vacancies are posted by firms to ensure zero profits. Finally, search takes place, leading to the following distribution of employed workers at the beginning of period $t + 1$

$$
\begin{aligned}
e'_{ll} &= e_{ll}(1 - s^a) + \eta^a m(\theta^a) \left[\delta - e_{ll}(1 - s^a)\right] \\
e'_{hh} &= e_{hh}(1 - s^a) + (1 - \eta^a)m(\theta^a) \left[(1 - \delta) - e_{hh}(1 - s^a)\right] \\
e'_{hl} &= e_{hl}(1 - s^a) + \eta^a m(\theta^a) \left[1 - \delta - (e_{hl} + e_{hh})(1 - s^a)\right] \\
&\quad -(1 - \eta^a)m(\theta^a)e_{hl}(1 - s^a) \qquad (1)
\end{aligned}
$$

where $\theta^a = \frac{v_l^a + v_h^a}{u_l + u_h + e_{hl}(1 - s^a)}$. Period $t + 1$ starts with $e' = \{e'_{hh}, e'_{lh}, e'_{ll}\}$.

The rate at which a firm meets a job seeker of any type is equal to $q(\theta^a) = m(1, \frac{1}{\theta^a})$, which is decreasing in θ^a and exhibits the standard properties.[11] To specify the matching rates of vacancies, it is convenient to define the fraction of low-skill unemployed and the fraction of unemployed job seekers as

$$
\begin{aligned}
\varphi &= \frac{u_l}{u_l + u_h} \\
\psi &= \frac{u_l + u_h}{u_l + u_h + e_{hl}(1 - s^a)}
\end{aligned}
$$

Low-skill vacancies match only with unemployed job seekers. An overqualified worker has no incentive to change employer unless the new employer offers a high-skill job. Accordingly,

[11] What is standardly assumed is that $\lim_{\theta^a \to 0} q(\theta^a) = \lim_{\theta^a \to \infty} \theta^a q(\theta^a) = \infty$ and $\lim_{\theta^a \to \infty} q(\theta^a) = \lim_{\theta^a \to 0} \theta^a q(\theta^a) = 0$.

some firms with low-skill vacancies meet overqualified workers who refuse to match. Therefore, the effective matching rate of a low-skill vacancy with a low-skill worker is given by $\psi\varphi q(\theta^a)$, while the corresponding rate with a high-skill worker is $\psi(1-\varphi)\, q(\theta^a)$. Likewise, employers with high-skill vacancies do not hire the low-skill workers they meet. Consequently, high-skill vacancies match only with either overqualified or unemployed high-skill workers, and thus, their effective matching rate can be written as $(1-\psi\varphi)\, q(\theta^a)$.

2.3 Value Functions

To describe the value functions, I adopt the following notation: $U_j^a(e)$ is the value of unemployment to a worker of type j, for a given employment distribution e, $V_j^a(e)$ denotes the value of a vacant job of type j, $W_{ij}^a(e)$ denotes the value of employment for a worker of type i in a job of type j, and finally, $J_{ij}^a(e)$ denotes the value to the firm of filling a job of type j with a worker of type i. The value of unemployment to a low-skill worker satisfies

$$U_l^a(e) = b + \frac{1}{(1+r)} \left[\eta^a(e'')m\left(\theta^a(e'')\right) EW_{ll}(e') + \left(1-\eta^a(e'')m(\theta^a(e''))\right) EU_l(e')\right] \tag{2}$$

where $e'' = (1-s^a)e$ and e' the distribution of employment at the beginning of next period as determined by the flow equations (1). This expression states that the value of unemployment to a low-skill worker is equal to the flow output while unemployed b, plus the present value of the expected value next period. The latter is given by the probability the worker finds a job $\eta^a m(\theta^a(e''))$, times the expected value of employment in a low-skill job $EW_{ll}(e')$, plus the probability the worker remains unemployed $(1-\eta^a m(\theta^a(e'')))$, times the corresponding expected value $EU_l(e')$. In turn, the expected values depend on the transition probability of the aggregate state. For example, $EW_{ll}(e') = pW_{ll}^0(e') + (1-p)W_{ll}^1(e')$. Similarly, given that high-skill workers accept both types of jobs, the value of unemployment to a high-skill worker satisfies

$$U_h^a(e) = b + \frac{1}{(1+r)} \left[\begin{array}{c} m(\theta^a(e''))[\eta^a(e'')EW_{hl}(e') + (1-\eta^a(e''))EW_{hh}] \\ +(1-m(\theta^a(e'')))EU_h(e') \end{array}\right] \tag{3}$$

The rest of the value functions take a similar form. Each one is equal to the flow output or cost of the corresponding state, and the present value of the expected value next period. The values to high- and low-skill workers of being employed in high- and low-skill jobs, respectively, satisfy

$$W_{hh}^a(e) = w_{hh}^a(e) + \frac{1}{(1+r)} \left[s^a EU_h(e') + (1-s^a)EW_{hh}(e')\right] \tag{4}$$

$$W_{ll}^a(e) = w_{ll}^a(e) + \frac{1}{(1+r)} \left[s^a EU_l(e') + (1-s^a)EW_{ll}(e')\right] \tag{5}$$

while the value to a high-skill workers of being employed in a low-skill job is give by

$$W_{hl}^a(e) = w_{hl}^a(e) + \frac{1}{(1+r)} \left[\begin{array}{c} s^a EU_h(e') + (1-s^a)EW_{hh}(e') \\ (1-s^a)(1-\eta^a(e''))m(\theta^a(e''))\left[EW_{hh}(e') - EW_{hl}(e')\right] \end{array}\right] \tag{6}$$

where $w_{ij}^a(e)$ denotes the corresponding wage in each case. The value of being overqualified incorporates in addition, the expected gain from on-the-job search. This is given by the term $(1-s^a)(1-\eta^a(e''))m(\theta^a(e''))\left[EW_{hh}(e') - EW_{hl}(e')\right]$ which is interpreted as follows: given that the match survives job destruction with a probability $(1-s^a)$, the worker meets a high-skill vacancy with a probability $(1-\eta^a(e''))m(\theta^a(e''))$, and obtain a surplus $[EW_{hh}(e') - W_{hl}(e')]$ from switching jobs.

I next turn to the employers' values. The values of high- and low-skill jobs filled by the suitable worker satisfy

$$J^a_{hh}(e) = y^a\alpha_h - w^a_{hh}(e) + \frac{1}{(1+r)}\left[s^a EV_h(e') + (1-s^a)EJ_{hh}(e')\right] \tag{7}$$

$$J^a_{ll}(e) = y^a\alpha_l - w^a_{ll}(e) + \frac{1}{(1+r)}\left[s^a EV_l(e') + (1-s^a)EJ_{ll}(e')\right] \tag{8}$$

while the value of a low-skill job filled by a high-skill worker is

$$J^a_{hl} = y^a\alpha_l - w^a_{hl}(e) + \frac{1}{(1+r)}\left[\begin{array}{c} s^a EV_l(e') + (1-s^a)EJ_{hl}(e') \\ -(1-s^a)(1-\eta^a(e''))m(\theta^a(e''))[EJ_{hl}(e') - EV_l(e')] \end{array}\right] \tag{9}$$

where the term $(1-s^a)(1-\eta^a(e''))m(\theta^a(e''))[EJ_{hl}(e')-EV_l(e')]$ corresponds to the expected loss from employing an overqualified worker, because of on-the-job search. Finally, the values of opening high- and low-skill vacancies are given by

$$V^a_h(e) = -c + \frac{1}{(1+r)}\left[(1-\psi\varphi)q(\theta^a(e''))(EJ_{hh}(e') - EV_h(e')) + EV_h(e')\right] \tag{10}$$

$$V^a_l(e) = -c + \frac{1}{(1+r)}\left[\begin{array}{c} \psi\varphi q(\theta^a(e''))[EJ_{ll}(e') - EV_l(e')] \\ +\psi(1-\varphi)q(\theta^a(e''))[EJ_{hl}(e') - EV_l(e')] + EV_l(e') \end{array}\right] \tag{11}$$

2.4 Equilibrium

The surplus of each type of job can be expressed as

$$S^a_{ij}(e) = W^a_{ij}(e) + J^a_{ij}(e) - U^a_i(e) - V^a_j(e) \tag{12}$$

Given that worker and firms share the surplus in fixed proportions with β being the worker's share, the wage $w^a ij(e)$ satisfies $W^a_{ij} = \beta S^a_{ij}$ and $J^a_{ij} = (1-\beta)S^a_{ij}$. Moreover, we can write $EW_{ij}(e') = \beta ES_{ij}(e')$ and $EJ_{ij}(e') = (1-\beta)ES_{ij}(e')$. Substituting these expressions and the value functions into the surplus expression above, yields

$$S^a_{ll}(e) = y^a\alpha_l - b + \frac{1}{(1+r)}[(1-s^a)ES_{ll}(e') - \beta\eta^a(e'')m(\theta^a(e''))ES_{ll}(e')] \tag{13}$$

$$S^a_{hh}(e) = y^a\alpha_h - b + \frac{1}{(1+r)}\left[\begin{array}{c} (1-s^a)ES_{hh}(e') - \beta(1-\eta^a(e''))m(\theta^a(e''))ES_{hh}(e') \\ -\beta\eta^a(e'')m(\theta^a(e''))ES_{hl}(e') \end{array}\right] \tag{14}$$

$$S^a_{hl}(e) = y^a\alpha_l - b + \frac{1}{(1+r)}\left[\begin{array}{c} (1-s^a)ES_{hl}(e') - (1-s^a)(1-\eta^a(e''))m(\theta^a(e''))ES_{hl}(e') \\ -\beta\eta^a(e'')m(\theta^a(e''))ES_{hl}(e') \\ -s^a\beta(1-\eta^a(e''))m(\theta^a(e''))ES_{hh}(e') \end{array}\right] \tag{15}$$

The surplus of a low-skill job filled by a low-skill worker S_{ll} takes the standard form. The first term, gives the net productivity of the match; the second term gives the present value of expected surplus next period; and the third term captures the loss to the worker for giving up the opportunity to search for a new job.[12] Once employed, in addition to the flow output from unemployment b, the worker gives up the opportunity to match with a low-skill vacancy with a probability $\eta^a(e'')m(\theta^a(e''))$ and gain a share β of the resulting surplus

[12] Notice from (12) that the value of unemployment, which reflects the value of worker's options outside the match is subtracted from the surplus of the match.

$ES_{ll}(e')$. The surplus of a high-skill job S_{hh}, takes a similar form. The only difference is that high-skill workers lose their opportunity to search for both high- and low-skill jobs once employed. Therefore, the surplus changes accordingly. The surplus of a low-skill job filled by a high-skill worker S_{hl} changes to incorporate in addition the loss due to on-the-job search from overqualified workers. This is captured by the second term in (15). Given that the match survives to the next period with a probability $(1 - s^a)$, overqualified job seekers can find a high-skill job with a probability $(1 - \eta^a(e''))m(\theta^a(e''))$, in which case, $ES_{hl}(e')$ is lost. The third term reflects the worker's forgone opportunity to match with a low-skill vacancy. Finally, since overqualified workers search on the job for high-skill jobs, they lose their opportunity to find a high-skill vacancy while overqualified, only if the job is exogenously destroyed with a probability s. Accordingly, only a fraction s^a of the corresponding value is lost, and this is captured by the forth term in (15).

In equilibrium firms open vacancies until $V_i^a(e) = 0$. Therefore, $EV_i(e) = 0$ must also hold in equilibrium. It follows that the free entry conditions for low- and high-skill vacancies are given by

$$\frac{(1-\beta)\left[\psi\varphi ES_{ll}(e') + \psi(1-\varphi)ES_{hl}(e')\right]}{(1+r)} = \frac{c}{q(\theta^a(e''))} \tag{16}$$

$$\frac{(1-\beta)(1-\psi\varphi)ES_{hh}(e')}{(1+r)} = \frac{c}{q(\theta^a(e''))} \tag{17}$$

The left hand side captures the expected profit of filling the vacancy and the right hand side the cost of keeping the vacancy unfilled. The free entry conditions state that in equilibrium the expected profits should equal the costs, and implicitly define θ^a and η^a for each realization of aggregate state a and distribution of employment e. More formally, the equilibrium is given by a vector $\{v_l^a, v_h^a\}$ that satisfies the following: (i) the three types of matches are formed voluntarily, i.e., $y\alpha_h > b$ and $y\alpha_l > b$; (ii) the two free entry conditions in (16) and (17) are satisfied so that the values of maintaining low- and high-skill vacancies are zero; and (iii) the state variables e_{hh}, e_{hl}, and e_{ll} are determined by the set of flow equations (1).

Notice that other things equal, a uniform increase in the expected profits of all jobs requires an increase in θ^a, while unequal changes in the expected surpluses of high- and low-skill jobs require adjustments in the equilibrium value of η^a to keep the values of both types of vacancies equal to zero. Observe also that changes in the composition of job seekers affect the two equilibrium conditions in a different way. Since high and low-skill workers are equally productive in low-skill jobs, but the former are more likely to quit, it follows that $S_{ll}^a(e') - S_{hl}^a(e') > 0$, which implies that $ES_{ll}(e') - ES_{hl}(e') > 0$. Therefore, as can be verified in (16) and (17), an increase in the fraction of unemployed job seekers ψ raises the expected surplus of low-skill jobs, but it lowers the expected surplus of high-skill jobs. It follows that an increase in the fraction of overqualified job seekers (i.e., a reduction in ψ) induces firms to open relatively fewer low-skill vacancies (i.e., reduces η^a), making it more difficult for low-skill workers to find jobs. Likewise, an increase in the fraction of low-skill unemployed φ, raises the expected surplus of low-skill jobs, while lowering the expected surplus of high-skill jobs. Therefore, a rise in φ, increases η^a.

3 Comparative Static Results

In this section I first solve for a unique steady state equilibrium, and then I illustrate the comparative static effects of changes in aggregate productivity y and job destruction rate s. The proofs of the results presented in this section are given in the Appendix. The purpose of this analytic exercise is to provide a more rigorous intuition for the results of the numerical analysis that follow. Evidently, this exercise is limited, because it does not provide insights into the dynamics associated with shocks. Moreover, the focus on steady states only cannot be justified in this model. The presence of asymmetric matching and on-the-job search requires that the endogenous variables depend on the distribution of employment across types of matches in a complicated non-monotonic way, making it difficult to establish that the model exhibits global asymptotic stability. In particular, it cannot be guaranteed that for any initial distribution the economy converges to a unique steady state. Nevertheless, in the numerical simulations that follow the endogenous variables always converge to the same values, suggesting that the system is globally stable.

Assuming continuous time, the steady state free entry conditions along which the value of opening a vacancy is equal to zero, are given by the set of equations below

$$(1-\beta)q(\theta)[\psi\varphi S_{ll} + \psi(1-\varphi)S_{hl}] = c \quad (18)$$

$$(1-\beta)q(\theta)(1-\psi\varphi)S_{hh} = c \quad (19)$$

where

$$S_{ll} = \frac{ya_l - b}{\lambda_1} \quad (20)$$

$$S_{hl} = \frac{ya_l - b}{\lambda_2} \quad (21)$$

$$S_{hh} = \frac{(ya_h - b)\lambda_2 - \beta\eta m(\theta^a)(ya_l - b)}{\lambda_2\lambda 3} \quad (22)$$

and $\lambda_1 = (r+s+\beta\eta m(\theta))$, $\lambda_2 = (r+s+\beta\eta m(\theta)+(1-\eta)m(\theta))$, and $\lambda_3 = (r+s+\beta(1-\eta)m(\theta))$.

Sufficient conditions to ensure the steady state equilibrium is unique are: *i*) $\frac{(1-\delta)}{\delta} \leq \frac{(y\alpha_l - b)}{(y\alpha_h - b)}$; *ii*) $\beta \geq \frac{1}{2}$; *iii*) $\frac{(y\alpha_l - b)}{(y\alpha_h - b)} \leq \beta$.[13] The first condition requires that the fraction of low-skill workers in the labor force is sufficiently high so that total net flow of output they can produce exceeds the net flow of output high-skill workers can produce. This condition ensures that the fraction of unemployed low-skill job seekers decreases when there are more vacancies available per job seeker, and is sufficient to establish that the value of opening low-skill vacancies declines with θ. Conditions *ii*) and *iii*) ensure that a higher η increases the value of low-skill vacancies, but lowers the value of high-skill vacancies.[14] Under these conditions, the free entry conditions (18) and (19) have opposite slopes in the $[\eta, \theta]$ plane. The equilibrium is characterized by the intersection of the two loci, as shown in Figure 1

I next turn to the impact of a reduction in aggregate productivity y on the equilibrium values of η and θ. It is straightforward to verify that a reduction in y lowers the values of both types of vacancies by lowering the surpluses of jobs. Therefore, both loci shift down in response to a rise in y, and the equilibrium value of θ declines. Intuitively, when aggregate productivity is low, each job is proportionally less productive, thus firms post fewer vacancies per job seeker. The impact on η depends on which of the two loci shifts down by more. To determine this, I evaluate the impact of a rise in y on the ratio of the equilibrium value of a low-skill vacancy to the equilibrium value of a high-skill vacancy, while keeping θ fixed. Given that the free-entry conditions have the same right hand side, we can write

$$R = \frac{\psi\varphi}{(1-\psi\varphi)}\frac{S_{ll}}{S_{hh}} + \frac{\psi(1-\varphi)}{(1-\psi\varphi)}\frac{S_{hl}}{S_{hh}} \quad (23)$$

[13] Dolado *et al.* (2003) also derive a condition that rules out the corner solution in which firms open only low-skill vacancies. The condition can be written as $V_h > V_l = 0$ when $\eta = 1$ and gives $(y\alpha_h - b) > \left[1 + \frac{\delta(r+s)}{(1-\delta)(r+s+\beta m(\theta^*))}\right](y\alpha_l - b)$, where θ^* is the solution to $V_l = 0$ when $\eta = 1$.

[14] An increase in η lowers U_h and increases U_l. Since the workers' value of unemployment is subtracted from the surplus of jobs, a rise in η increases S_{hl} and lowers S_{ll}. Condition *ii*) ensures that the decline in S_{ll} dominates the increase in S_{hl}, so that V_l declines. Condition *iii*) ensures that positive impact of the fall in U_h dominates the negative impact of the rise in S_{hl} on S_{hh}, so that V_h increases.

which increases with y. Hence, a reduction in aggregate productivity has a stronger negative impact on the value of low-skill vacancies. As illustrated in Figure 2 the free entry condition for low-skill vacancies shifts down relatively more so that a decline in y lowers the equilibrium value of η. The reason is that when y falls, the percentage gap between the productivity of the job ($y\alpha_{ij}$) and the flow output from unemployment (b) declines more for low- than for high-skill jobs, pushing the relative net productivity, and therefore, the relative surplus of high-skill jobs up.[15] In other words, given that high-skill jobs are more productive than low-skill jobs, the dispersion in the net productivities of high- and low-skill jobs is higher at lower values of y. Accordingly, and as can be verified in the appendix, the decline in R, and thus in η is higher the higher the difference between α_h and α_l.

Consequently, the burden of unemployment due to a permanent reduction in aggregate productivity, falls more heavily on low-skill workers. The reduction in θ implies that high-skill workers have more difficulty finding vacancies, because $m(\theta)$ declines. However, in addition to the reduction in $m(\theta)$, low-skill workers bear the shift in the vacancy mix towards high-skill vacancies, i.e., the reduction in η. Hence, they suffer a higher reduction in their job finding probability relatively to high-skill workers, implying a relatively higher increase in their unemployment rate.

A conclusion regarding the impact of a fall in y on overqualification cannot be reached based on comparative static results alone. A fall in y implies that high-skill workers encounter low-skill vacancies less frequently, as $\eta m(\theta)$ declines. However, if the rise in high-skill unemployment due to the fall in $m(\theta)$ is sufficiently high, then the number of overqualified workers may still rise. Since the comparative static results do not gauge quantitatively the impact of a rise in s on high-skill unemployment, the characterization of the evolution of overqualification is taken up in the next section by numerical simulations.

I next turn to the impact of a rise in job destruction rate s. Both the surplus of high-skill jobs (S_{hh}) and the fraction of high-skill job seekers $(1-\psi\varphi)$ decline with s. Therefore, an increase in s lowers the value of opening high-skill vacancies, shifting the high-skill locus down. But the impact on the value of opening low-skill vacancies is more cumbersome to determine. On the one hand, a higher job destruction rate lowers the surplus of low-skill jobs (S_{hl} and S_{ll}). On the other hand, the composition of job seekers shifts towards more unemployed and fewer overqualified job seekers (($\psi\varphi$ and $\psi(1-\varphi)$ increase), raising the value of opening low-skill vacancies. Whether the low-skill locus shifts down or up depends on which of the two effects dominates. Assuming that the composition effect dominates, then the low-skill locus shifts up. Hence, η definitely increases, but the impact on θ cannot be determined. If the surplus effect dominates the low-skill locus shifts down. In this case, θ definitely declines, but the impact on η depends on which of the two loci shifts down relatively more. In the simulations that follow the transition to the new steady state always involves a downgrading in the vacancy mix, which as discussed below, is what drives the rise in overqualification rates following an increase in the rate at which jobs are exogenously destroyed.

Guided by this result, I focus on the conditions under which η increases in response to a rise in s. Even if an increase in s lowers the value of low-skill vacancies, the equilibrium value of η still increases, if it lowers the value of high-skill vacancies by relatively more. This would be the case if $\frac{\partial R}{\partial s} \geq 0$, which implies that the high-skill locus shifts down more than the low-skill locus, as illustrated in Figure 3. Hence, I examine the conditions under

[15] It is important to point out that an additive aggregate productivity shock (i.e. $y + a_{ij}$ instead of $y\alpha_{ij}$) would imply an even higher increase in the relative surplus of high-skill vacancies and thus an even higher increase in η. Moreover, this result is not sensitive to the assumption that b is the same for both types of workers. Assuming that high-skill workers generate b_h while unemployed and low-skill workers generate b_l while unemployed, the same result would still hold as long as the net productivity of high-skill jobs ($y\alpha_h - b_h$) is greater than the net productivity of low-skill jobs ($y\alpha_l - b_l$).

which this derivative is positive. Given that $\psi\varphi$ and $\psi(1-\varphi)$ increase with s, then $\frac{\psi\varphi}{(1-\psi\varphi)}$ and $\frac{\psi(1-\varphi)}{(1-\psi\varphi)}$ also increase with s. Moreover, $\frac{S_{hl}}{S_{hh}}$ rises with s. That is, an increase in the rate at which jobs are exogenously destroyed lowers the surplus of high-skill jobs more than the surplus of low-skill jobs filled by high-skill workers.[16] Intuitively, an increase in the probability of job destruction hurts firms with low-skill jobs less than firms with high-skill jobs, because a low-skill vacancy can be re-filled relatively faster in case the match splits up and the job remains vacant.[17] In addition, when filled by an overqualified worker low-skill jobs generate lower surplus than high-skill jobs. Therefore, the loss from loosing a high-skill worker is lower for firms with low-skill jobs. If it could also be established analytically that $\frac{S_{ll}}{S_{hh}}$ rises with s, then the proof of $\frac{\partial R}{\partial s} \geq 0$ would have been completed. However, this cannot be proved analytically because when filled by low-skill workers, low-skill jobs can generate either higher or lower surplus than high-skill jobs, depending on the labor market conditions. The former are of lower productivity, but the value of unemployment is higher for high- than for low-skill workers, since the former can take both types of jobs.[18]

Even if it is difficult to prove that the steady state value of η is always higher for higher values of s, some important insights emerge from this comparative static exercise. The most important is that in response to a rise in the job destruction rate the composition of job seekers shifts in favor of low-skill vacancies. Intuitively, in periods of high job turnover, firms are relatively better off creating jobs with low skill requirements so that they benefit relatively more from higher arrival rates of unemployed job seekers. If in addition low-skill jobs are of lower surplus and thus generate lower loss when destroyed, then this result is reinforced. Irrespective of whether η eventually converges to a higher or lower level once the model settles to the new steady state, the transition always involves a downgrading in the vacancy mix as firms take advantage the increased supply of unemployed job seekers. As will be shown in the next section, this drives the dynamic response of unemployment and overqualification rates following a rise in job destruction.

4 Simulations

The purpose of this section is to gauge qualitatively the effects of business cycle fluctuations on unemployment rates, overqualification rate, and average match productivity. I consider shocks to aggregate productivity and job destruction rate separately in order to illustrate their individual effects, and I turn to numerical techniques to analyze the model.[19] I use the free entry conditions given by equations (16) and (17) to find the

[16] Notice from the surplus expressions (20)-(22) that an increase in s, is similar to an increase in the discounting factor, r. Both changes lower the present value of the expected surplus.

[17] I assume for simplicity that the cost of keeping vacancy unfilled c is the same for both types of vacancies. It should be clarified however, that if the cost of keeping a high-skill vacancy open was higher than the cost of keeping a low-skill vacancy open this result would be stronger. Firms would benefit even more from opening low- instead of high-skill vacancies at times job are destroyed more frequently.

[18] One sufficient condition derived in the appendix that ensures $\frac{\partial R}{\partial s} > 0$ is: $\frac{(\alpha_h - b)}{(\alpha_l - b)} \geq \frac{\beta}{(1-\varphi)}$. This condition states that the net productivity of high-skill jobs, should be sufficiently high relative to the net productivity of low-skill jobs, and the required productivity differential increases the higher the fraction of low-skill unemployed φ. Notice from (18) that the higher φ is, the higher the expected surplus from filling a low-skill vacancy, because the chances of hiring an overqualified worker are lower. Therefore, the higher φ is, the higher the net productivity of high-skill jobs needs to be relative to the net productivity of low-skill jobs to ensure that high-skill jobs generate higher surplus.

[19] One could argue that in reality the fluctuations in the rate at which jobs are destroyed are driven by fluctuations in aggregate productivity. Therefore, allowing for a co-movement between the aggregate productivity and job destruction process, instead of considering them separately would be more realistic. However, the results of such an exercise would depend on the assumed covariance between the two processes, making it difficult to isolate their individual consequences. Ideally, the model would allow for endogenous job destruction as in Mortensen and Pissarides (1994) for instance. But the nature of the model requires

state-contingent market tightness $\theta^a(e)$ and fraction of low-skill vacancies $\eta^a(e)$. I then simulate the model as follows: first, I generate a sequence of aggregate state (either s or y) realizations; then, starting with the first realization of aggregate state, and an initial distribution of employment $e = \{e_{hh}, e_{hl}, e_{ll}\}$, I use the flow equations in (1) to compute the new distribution of employment at the beginning of the next period; and then I repeat. At the end of each period, I record the, employment rates, unemployment rates, and average match productivity series along a sequence of aggregate state realizations.

The exogenous variables are set at the following values: $\beta = .65$, $r = .01$, $c = .4$, $b = .1$, $\delta = .75$, $a_{hh} = .9$ and $a_{ll} = a_{lh} = .7$. The matching function $m[\cdot,\cdot]$ is a Cobb Douglas function in which job seekers and vacancies are assumed to have equal elasticities of 0.5. I set $p = .05$. If a period is taken to be a quarter, then a given realization of aggregate state persists for 5 years on average, consistent with business cycle frequencies. To illustrate the various effects I compute impulse responses to each shock. I simulate the model assuming that once the shock arrives it follows a sample path in which it persists for 20 periods.[20]

4.1 Aggregate Productivity Fluctuations

In this section I demonstrate the consequences of a temporary fall in aggregate productivity. I normalize the high value of aggregate productivity to $y_1 = 1$ and set the low value equal to $y_0 = .35$, while keeping the job destruction rate fixed at $s = .05$. These parameter choices result in an average equilibrium value of $m(\theta)$ and η of 0.6 and 0.74 respectively. The unemployment rates for high- an low-skill workers average 8.5% and 11% respectively, yielding an overall unemployment rate of approximately 10%. Although this is higher than the US average, it is close to the average unemployment rate of EU countries, where overqualification phenomena received greater attention in the empirical literature.[21]

The insights from the comparative static exercise above, carry over when aggregate productivity is allowed to fluctuate over time. In response to a negative productivity shock firms open fewer vacancies of both types, but relatively fewer low-skill vacancies. As can be verified in Figures 4 and 5 on impact of a negative productivity shock, $m(\theta)$ and η decline. The meeting rate somewhat rises afterwards, as rising unemployment increases the arrival rate of workers to vacant firms, but still remains low. Likewise, the fraction of low-skill vacancies subsequently rises, but never reaches its initial level. The paths of the probabilities of meeting low- and high-skill vacancies, $\eta m(\theta)$ and $(1-\eta)m(\theta)$, respectively, are illustrated in Figure 6. Evidently, the reduction in aggregate productivity has a large negative effect on the probability of meeting low-skill vacancies, but only a moderate negative effect on the probability of meeting high-skill vacancies. The former, in addition to the decline in $m(\theta)$ suffers the reduction in η, while the latter evens out some of the decrease in $m(\theta)$ by the increase in $(1-\eta)$. The reason for the partial upturn in η following the initial decline, is the change in the composition of job seekers, brought about by the initial fall in job finding probabilities. In particular, as shown in Figure 7, the composition of job seekers shifts towards more unemployed and fewer overqualified ($\psi\varphi$ and $\psi(1-\varphi)$ increase). Moreover, the fraction of high-skill job seekers $(1-\psi\varphi)$, traced in Figure 8, declines. These changes

the endogenous variables to depend on the distribution of workers across types of matches, making it hard not only to solve analytically, but also computationally. Although incorporating temporary shocks to aggregate productivity in a model with asymmetric matching is a significant step towards the introduction of endogenous job destruction, it is still a difficult, if not an impossible, task to be accomplished.

[20] Of course, the value functions used in the simulations assume that agents believe that the shock persists only with a probability p each period, as specified in section 2.1 above.

[21] Given that the purpose of this section is to illustrate the qualitative, but not the quantitative effects of temporary shocks, the parameters' values where chosen in a rather ad hoc manner.

entail a higher arrival rate of job seekers to low-skill firms, but a lower arrival rate of workers to high-skill firms, and thus encourage firms to downgrade the skill mix of vacancies. Hence, the probability of meeting a low-skill vacancy moderately increases after the initial decline, reflecting the upturn in $m(\theta)$ and η.

Figure 9 illustrates the evolution of high- and low-skill unemployment rates and the evolution of the overqualification rate (i.e., the fraction of high-skill workers who are overqualified). As expected, independent of the level of aggregate productivity, the low-skill unemployment rate is higher than the high-skill unemployment rate, because high-skill workers accept both types of jobs. A negative productivity shock raises both unemployment rates, but the low-skill unemployment rate increases disproportionably by more. Initially, the low-skill unemployment rate rises relatively more, because of the shift in the vacancy mix towards high-skill vacancies, which as mentioned above, lowers the probability of finding low-skill vacancies relatively more. Moreover, the subsequent upturn in the probability of finding low-skill vacancies does not improve low-skill workers' labor market conditions relative to high-skill workers. In contrast, it eases the misallocation unemployed high-skill workers into low-skill jobs, thus hurting low-skill employability even more. Although the overqualification rate declines on impact, reflecting the sizable fall in the probability of meeting low-skill vacancies at the onset of the recession, it rises afterwards. Following the rise in high-skill unemployment and the moderate improvement in the probability of finding low-skill vacancies, the overqualification rate begins rising and eventually converges to a higher level eclipsing any of the initial decline. While a higher fraction of high-skill workers escapes unemployment by becoming overqualified, the unemployed low-skill workers suffer additional congestion and the negative job competition externalities on low-skill vacancy creation. The impact negative impact of rising overqualification rates on low-skill vacancy creation is reflected in the skill composition of vacancies. As can be verified in Figure 5 after the initial decline and partial upturn, the fraction of low-skill vacancies gradually declines. Hence, once the overqualification rate begins rising, the high-skill unemployment rate starts converging and settles to a level only 4.5 p.p. higher than the original. The low-skill unemployment rate, on the other hand, continues to rise, and converges to a level 6.5 p.p. higher than the original.

Given that the rise in the overqualification eclipses any of the initial decline, the model confirms surveys that workers are more likely to report underutilized during recessions(e.g., Akerlof, Rose and Yellen (1988), and Acemoglu (1999)) and evidence that in downturns the quality of job-worker matches is lower. Following Barlevy (2002) I consider average match productivity, measured by

$$\frac{(e_{ll}+e_{hl})\alpha_l+e_{hh}\alpha_h}{e_{ll}+e_{hl}+e_{hh}} \tag{24}$$

as an approximation of average match quality. The evolution of average match productivity reflects the evolution of the overqualification rate. At the onset of the recession, a cleansing effect arises due to the initial decline in the overqualification rate. Subsequently, a sullying effect arises as the overqualification rate rises. As shown in Figure 10 average match productivity increases initially, but declines later on, and converges to a level lower than the original. The simultaneous rise in the overqualification rate and unemployment rate of high-skill workers implies that in downturns a lower fraction of them is suitably matched with high-skill jobs. Jobs occupied by overqualified workers have shorter duration and generate lower surplus, thus offer lower wages than both high- and low-skill jobs filled with the suitable workers. Accordingly, the model suggests that jobs created in recession have lower average duration and offer lower wages, consistent with the evidence. Moreover, the probability of finding high-skill vacancies is lower in recessions implying that overqualified workers are less likely to upgrade to higher job levels in downturns, in line with well established evidence that job to job transitions are procyclical.[22]

However, if the dispersion in the productivity of jobs is sufficiently high, the cleansing effect at the onset of the recession dominates the sullying effect. As suggested also by the comparative static results, the initial decline in the fraction of low-skill vacancies and thus, the magnitude of the initial reduction in overqualification rate, is greater the greater the dispersion in job productivities. Therefore, as can be confirmed in Figure 11, when high-skill jobs are sufficiently more productive than low-skill jobs, the rise in the overqualification rate is not sufficient to eclipse the initial decline. As a consequence, average match productivity can actually remain higher in downturns, as shown in Figure 12.

Clearly, the finding that for sufficiently high dispersion in job productivities a reduction in aggregate productivity improves average match productivity, does not square with empirical findings that match quality is procyclical. However, if overqualification phenomena are more likely to occur in "mediocre" jobs instead of very simple jobs, which are only marginally productive, then the this finding is not necessarily inconsistent with evidence. By taking temporarily jobs they are overqualified for, workers may have to tolerate costs that are unknown to the employer, and thus are not internalized into the surplus of the job, but significant enough to induce workers to be more selective in the jobs they accept. Such costs are for instance scarring effects on future career prospects, or dissatisfaction from performing unsuitable tasks. If this is the case then workers are willing to accept transitorily only jobs in which they can be sufficiently productive to even out these costs. Hence, the dispersion between high- and low-skill jobs' productivities should be sufficiently small for a cross-skill matching equilibrium to exist.

Moreover, the model performs better as regards the cyclical properties of average match productivity when recessions are characterized by high flows of workers into unemployment. As mentioned in an earlier footnote, a caveat of the model is that it does not allow for endogenous responses of job destruction to fluctuations in aggregate productivity. However, as will be explained in the next section, when recessions are characterized by high rates of job destruction, then the sullying effect captured in this model is actually stronger. The reason is the rise in unemployment and the high job destruction costs, associated with periods of high job turnover, which induce firms to open relatively more low-skill vacancies that can be filled more easily and generate lower losses when destroyed.

Before moving to the sullying effects of job destruction, a few remarks are in order about the model's prediction regarding the impact of reductions in aggregate productivity on match quality. First, in this model fewer vacancies per job seeker in recessions does not necessarily imply higher mismatch rates. Consequently, focusing on symmetric frameworks and thus, not accounting for changes in the skill mix of vacancies over the business cycle, may be misleading. Second, the evolution of average match productivity shown in Figure 10 is markedly similar to the evolution of match productivity in Barlevy (2002). In Barlevy's simulations, however, average match productivity rises at the onset of the recession because the parameter values were intentionally chosen in a way that the lowest productivity matches generate negative surplus when aggregate productivity falls, and thus cannot survive periods of low productivity. In contrast, the cleansing effect in this model is brought about by an endogenous decline in the probability of finding low-skill jobs at the cost of higher low-skill unemployment. Finally, the sullying effect in this model, arises due to rising unemployment in periods of low aggregate productivity, which encourages firms to downgrade the skill mix of vacancies. Hence, contrary to the cleansing view, high unemployment actually

[22] See, for example, Moscarini and Thomsson (2006) who find that the flow of workers in two consecutive months that report different 3-digit occupations in the US is procyclical, and that the latest two recessions had a very persistent negative impact on the job-to-job mobility. Likewise, Fallick and Fleischman (2004) find that in loose labor markets the employer-to-employer flows drop sharply.

exacerbates the underutilization of high-skill workers, as opposed to facilitating the creation of new high productivity jobs.

4.2 Job destruction Fluctuations

In this section, I illustrate the impact of a rise in the job destruction rate s. While keeping the aggregate productivity fixed, I assume that s switches from a low value $s_0 = .05$ to a higher value $s_1 = .1$ with probability $p = .05$. The rest of the parameters are as in the previous section.

I begin with the evolution of the skill mix of vacancies, because it lies at the heart of the results that follow. Figure 13, traces this evolution. On impact of a negative job destruction shock, the fraction of low-skill vacancies increases. The downgrading of the vacancy mix is due to a rise in the arrival rate of unemployed job seekers to low-skill vacancies and a reduction in the arrival rate of high-skill job seekers to high-skill vacancies. Figure 14 reports the fractions of high- and low-skill unemployed in the mass of job seekers. Consistent with the comparative static results discussed earlier, both fractions increase once the job destruction rate rises, while the fraction of high-skill job seekers decreases on impact. The meeting rate and job finding probabilities are reported in Figures 15 and 16. On impact, the meeting rate increases. This relatively modest increase reflects the sharp increase in unemployment, which implies higher arrival rates of workers to vacancies and encourages firms to open more vacancies. Subsequently, as unemployment increases, the meeting rate declines and finally converges to a lower level. Since both $m(\theta)$ and η increase on impact, the probability of finding low-skill jobs increases on impact as well. The probability of finding high-skill vacancies, on the other hand, decreases as the decline in $(1-\eta)$ dominates the modest increase in $m(\theta)$.

As can be verified in Figure 17, the rise in high-skill unemployment due to the rise in the job destruction rate, together with the rise in the probability of meeting low-skill vacancies bring a rise in the overqualification rate. A higher fraction of high-skill workers is misallocated into low-skill jobs, while the fall in the probability of finding high-skill vacancies implies that they remain overqualified for a longer period. The resulting negative job competition externalities on the profitability of low-skill jobs, account for the gradual upgrading in the vacancy mix (see Figure 13), and explain the overturn in job finding probabilities (see Figure 16). The probability of meeting low-skill vacancies eventually settles to a lower level. The probability of meeting high-skill vacancies, on the other hand, recovers most of the initial decline.

Despite the downgrading of the skill mix of vacancies once the job destruction rate rises, the burden of unemployment still falls more heavily on low-skill workers. The initial increase in the probability of meeting low-skill vacancies does not improve the relative position of low-skill workers in the labor market, since both types of workers take low-skill jobs while unemployed. In addition, the rise in the rate at which workers encounter low-skill vacancies comes at the cost of the strong negative job competition externalities that follow. As shown in Figure 17 the overqualification rates increases on impact and continues to increase until it settles to a higher level. The same figure reports the evolution of unemployment rates. The immediate impact of the shock is to raise both unemployment rates. However, the high-skill unemployment rate starts to converge as more unemployed high-skill workers become overqualified. The low-skill unemployment rate, on the other hand, continues to increase, because the negative job competition externalities on the profitability of low-skill jobs induce an upgrading in the vacancy mix, thus lowering their chances of finding jobs. Eventually, the low-skill unemployment rate converges to a level 8 p.p. above the original, while the high-skill unemployment rate converges to a level only 6.5 p.p. above the original.

Figure 18 draws the path of average match productivity. As the overqualification rate rises average match productivity decreases gradually and converges to a lower level. Hence, the model suggests that sullying effects are more prominent when small changes in aggregate productivity induce sharp increases in job destruction and flows into unemployment.[23] As mentioned earlier, the key result of the model, previously overlooked by focusing only on symmetric frameworks is that higher unemployment encourages firms to downgrade the skill mix of vacancies. While so far higher job destruction rates in recessions where associated with cleansing effects, this model puts more emphasis on the sullying effects of job destruction. One could argue that previous cleansing theories associate recessions with cleansing effects when job destruction is concentrated on low productivity jobs. That is, recessions kill marginally productive jobs, while higher productivity jobs are more likely to remain in operation. For this reason, I also investigated whether the results described above carry over when the job destruction rate of low-skill jobs increases more than the job destruction rate of high-skill jobs. As it turned out, the composition of job seekers changes even more in favor of firms with low-skill vacancies when job destruction is concentrated on low-skill jobs: the fraction of unemployed low-skill job seekers increases relatively more, while the fraction of overqualified job seekers declines relatively more. As a consequence, the fraction of low-skill vacancies rises more prominently, thus raising the chances high-skill workers find low- instead of high-skill vacancies further. It follows that when in periods of high job turnover job destruction is concentrated on low-skill jobs, the overqualification rate still rises.[24] In fact, it may rise even more notably if there is a sufficiently high increase in high-skill unemployment.

5 Conclusion

This paper has developed a matching model that explains why the burden of unemployment falls disproportionably on the lower skill groups and the quality of job-worker matches is lower in recessions. I demonstrate that these two pieces of evidence can be explained by a vertical type of skill mismatch that takes the form of workers downgrading to lower job levels (i.e., accepting jobs they are overqualified for) to avoid unemployment, and upgrading to higher job levels by on-the-job search. The model allows for the skill composition of jobs to change endogenously and relaxes the common assumption that all workers can perform any type of job independent of their skill level. High-skill workers qualify for both high- and low-skill jobs, whereas low-skill workers qualify only for low-skill jobs. Some high-skill workers accept transitorily low-skill jobs until an offer for a high-skill job comes along, thus worsening the labor market prospects of low-skill workers who are unable to compete with them for high-skill jobs. Firms respond to changes in aggregate conditions by altering the skill mix of vacancies to take advantage of changes in the relative profitability of jobs and changes in the composition of job seekers. Therefore, the model allows for aggregate shocks to affect the job finding probabilities of high- and low-skill workers unevenly.

[23] Although it is difficult to empirically assess the nature of the co-movement between aggregate productivity fluctuations and job destruction fluctuations, some findings indicate that small changes in labor productivity are accompanied by large changes in job destruction. Davis and Haltiwanger established that job destruction varies noticeably over the cycle, and spikes during recessions. Moreover, the countercyclical fluctuations in the deviations of the log separation rate series computed in Shimer (2005) using employment and unemployment data from are much larger than the procyclical fluctuations in the deviation of the log average labor productivity series. The constructed series does not distinguish involuntary from voluntary separations. However, given that empirical evidence suggest that voluntary separations are procyclical, the fluctuations should be larger when focusing only on involuntary separations.

[24] Obviously, when job destruction is concentrated on high-skill jobs, the overqualification rate rises, because both the fraction of low-skill vacancies and high-skill unemployment rise more prominently. The only case a cleansing effect may be stronger than the sullying effect is when job destruction in recessions is heavily concentrated on mismatches.

Moreover, by allowing for the job finding probabilities to vary by skill, the model captures the across-skill search externalities that arise when high- and low-skill workers compete for the same jobs.

Provided that the productivity dispersion between high- and low-skill jobs is sufficiently small, the model predicts higher rates of overqualified workers in recessions. The key result, previously overlooked by assuming a single matching probability for all skill types is that at times of high unemployment firms downgrade the skill mix of vacancies to benefit relatively more from the increased supply of unemployed job seekers. As a consequence, downturns facilitate the misallocation of high-skill unemployed workers into low-skill jobs, while impeding their transition out of them. Hence, periods of low aggregate productivity, are characterized by lower average quality of job-worker matches. It follows that this sullying effect is more prominent if recessions are characterized by high rates of job destruction and thus flows into unemployment. While previous theories associate higher job destruction rates with the cleansing of the economy from low productivity jobs, the model in this paper stresses the role of job destruction in exacerbating the underutilization of workers.

Consistent with the evidence, I show that the lower skill groups suffer disproportionably higher increase in their unemployment rate in downturns. The conventional result that recessions hurt the matching process as firms open fewer vacancies per job seeker is present in this model as well. Both types of workers have lower chances finding vacancies in recessions, but low-skill workers suffer in addition, from unfavorable shifts in the skill mix of vacancies and job competition from high-skill workers. An upgrading in the vacancy mix lowers the chances low-skill unemployed workers find jobs they can perform, while it does not influence the chances high-skill workers escape unemployment, but only the quality of the matches they form. Likewise, when firms downgrade the vacancy mix to take advantage of the rising unemployment, low-skill employability does not improve relative to high-skill employability. In contrast, when low-skill vacancies are relatively more abundant, and high-skill unemployment is higher, low-skill workers suffer the negative consequences of intensified job competition from high-skill workers.

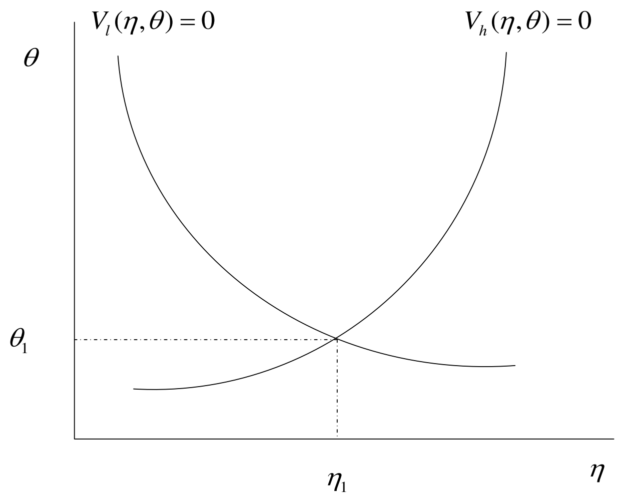

Fig. 1: Steady State Equilibrium

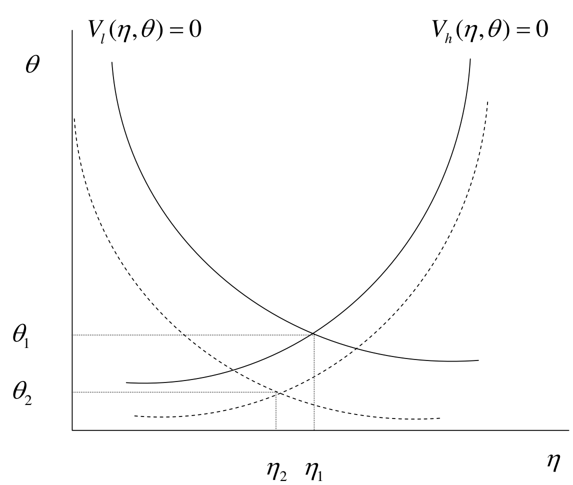

Fig. 2: Impact of a negative productivity shock on the steady state equilibrium

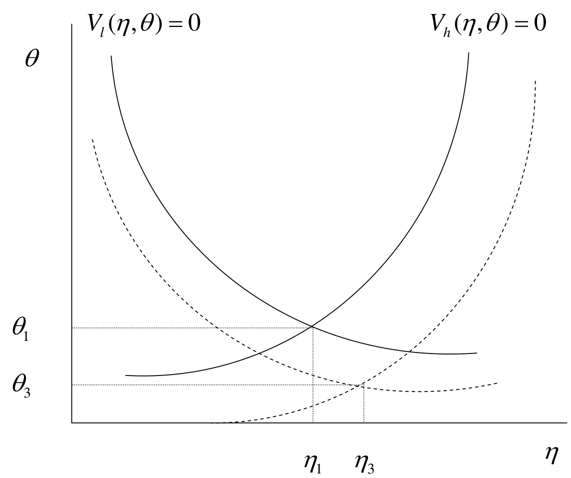

Fig. 3: Impact of a rise in job destruction rate on the steady state equilibrium

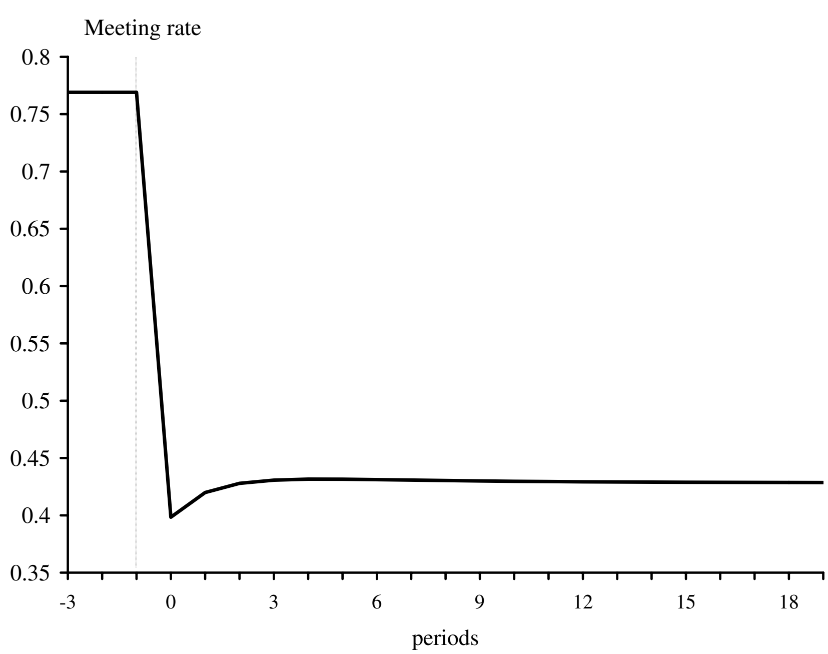

Fig. 4: Effect of a negative productivity shock on the meeting rate

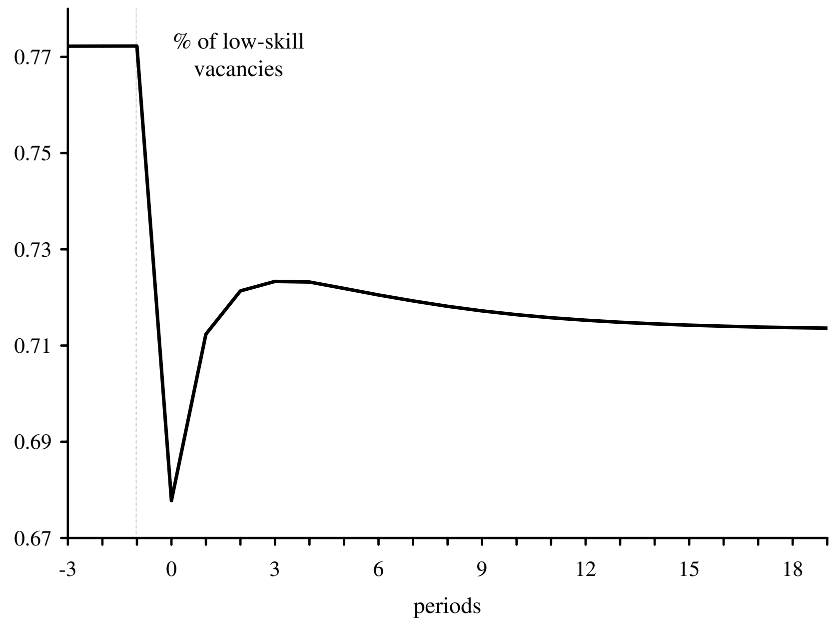

Fig. 5: Effect of a negative productivity shock on the skill mix of vacancies

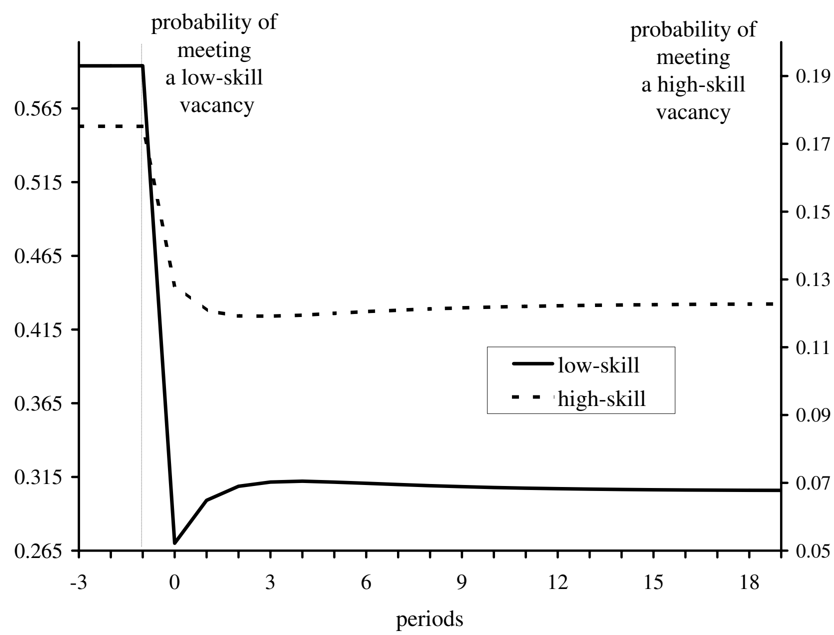

Fig. 6: Effect of a negative productivity shock on job finding probabilities

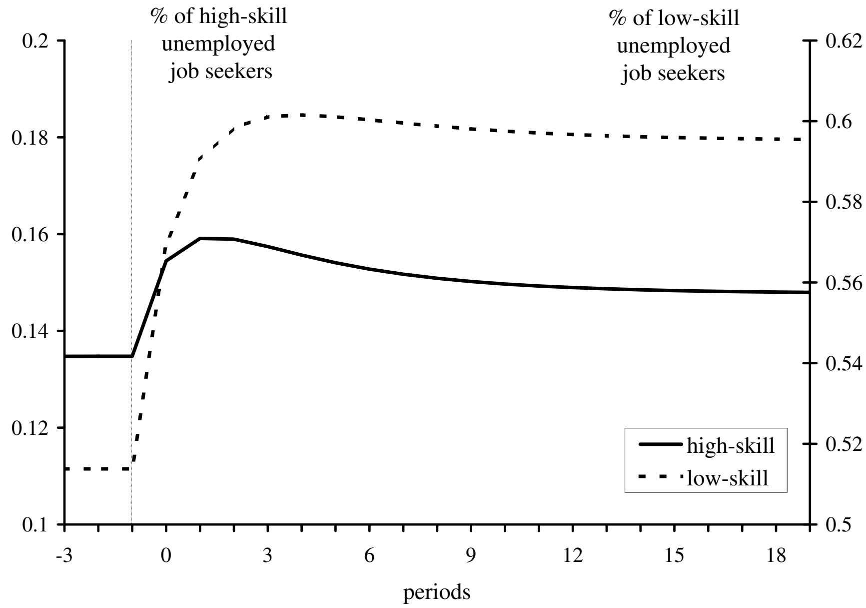

Fig. 7: Effect of a negative productivity shock on the fractions of unemployed job seekers

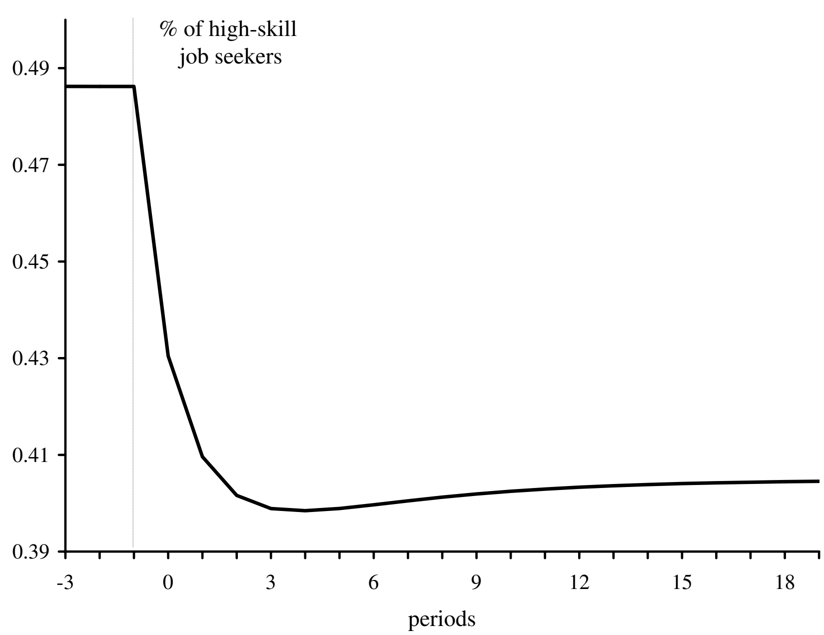

Fig. 8: Effect of a negative productivity shock on the fraction of high-skill job seekers

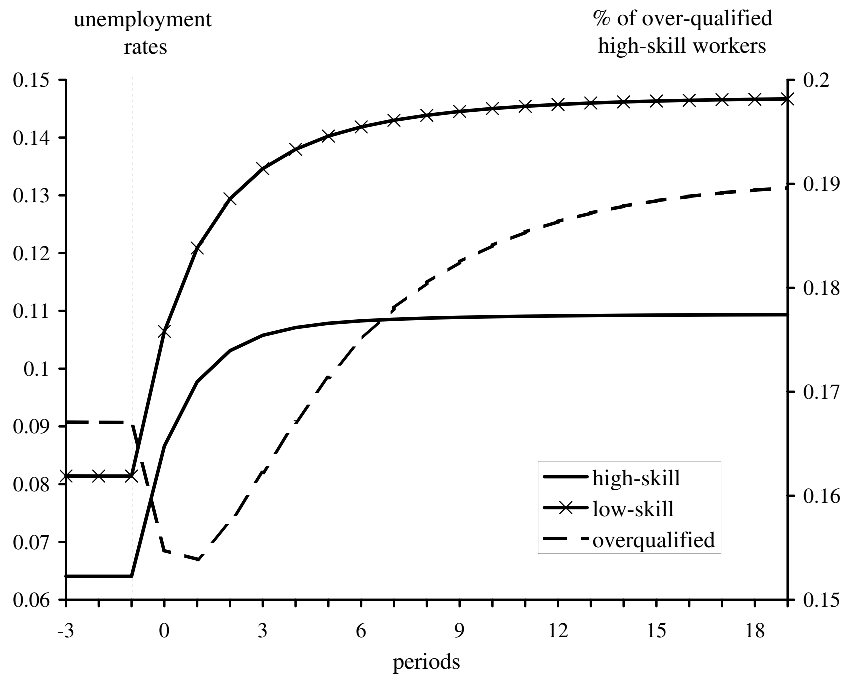

Fig. 9: Effect of a negative productivity shock on unemployment and overqualification rates

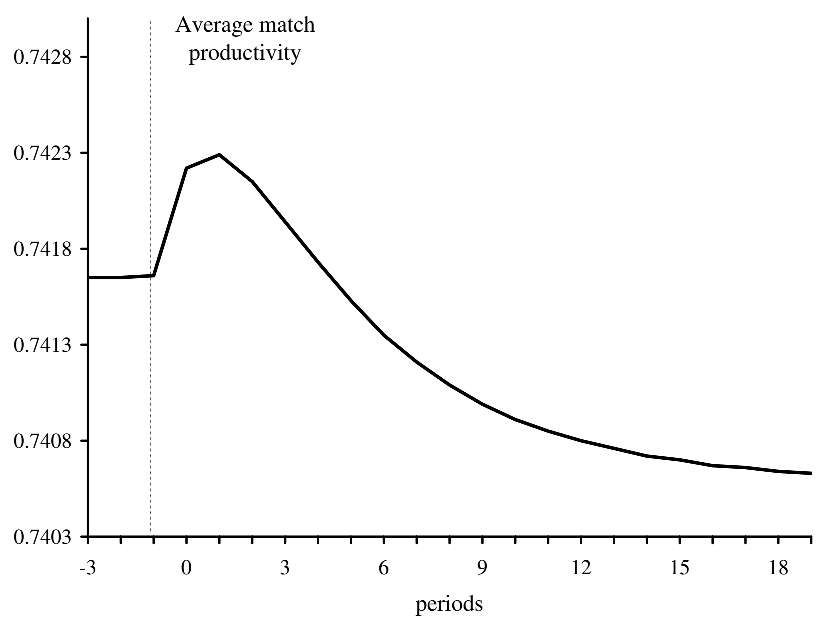

Fig. 10: Effect of a negative productivity shock on average match quality

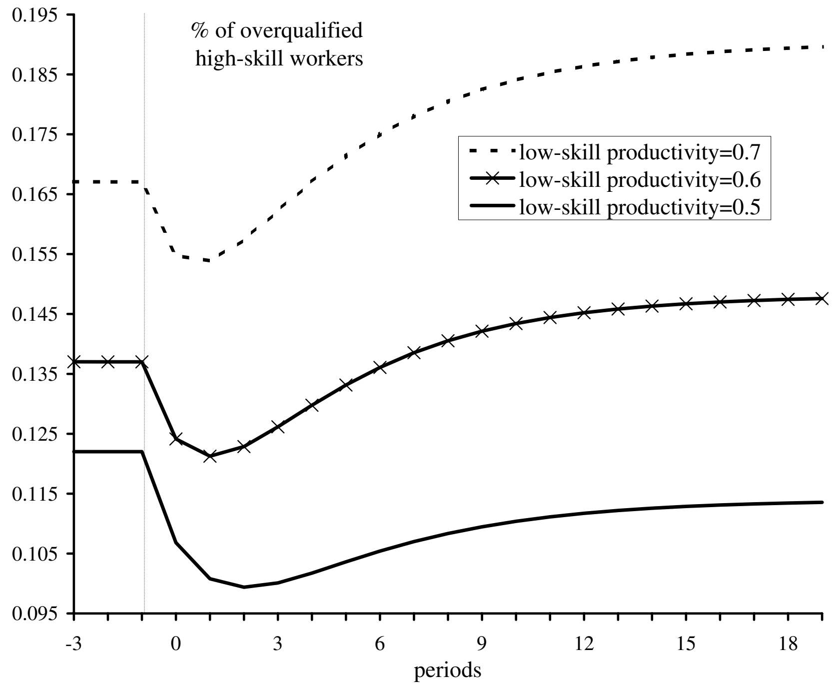

Fig. 11: Overqualification rates for different productivity dispersion

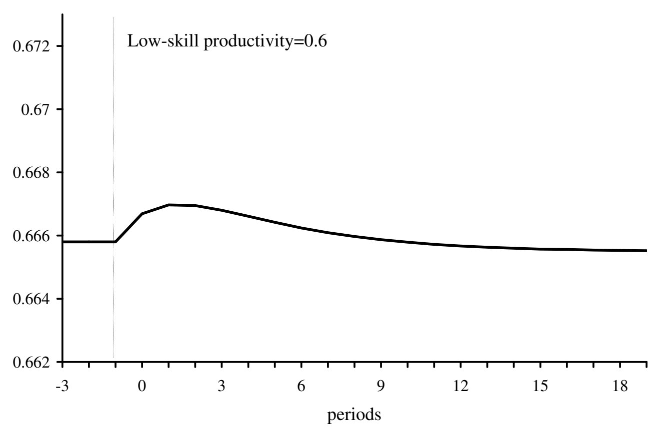

Fig. 12: Average match productivity for different productivity dispersion

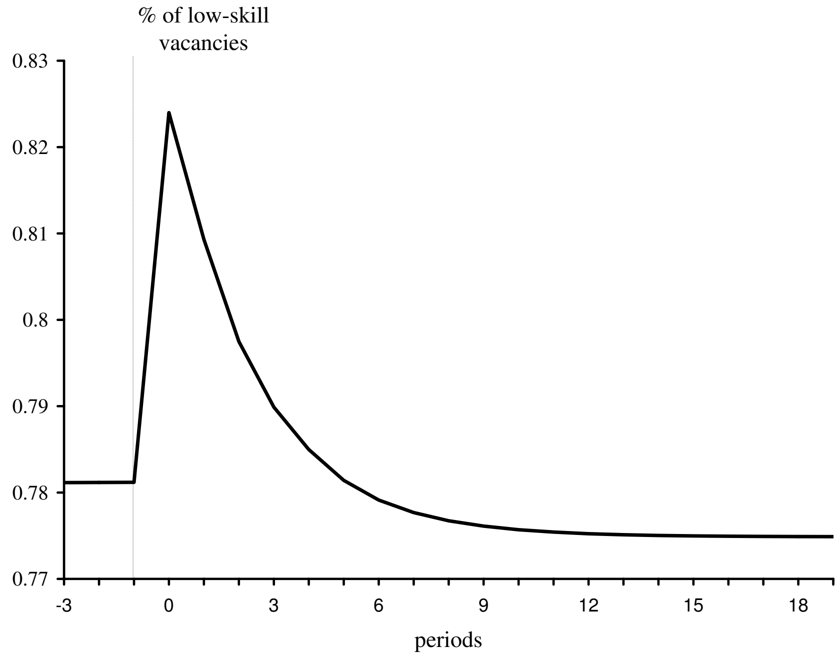

Fig. 13: Effect of a rise in the job destruction rate on the skill mix of vacancies

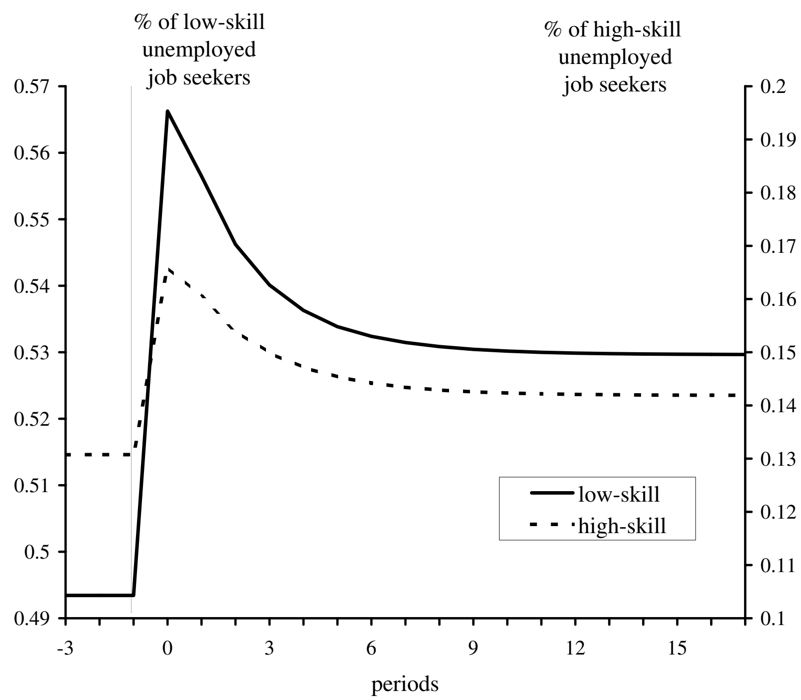

Fig. 14: Effect of a rise in the job destruction rate on the fractions of unemployed job seekers

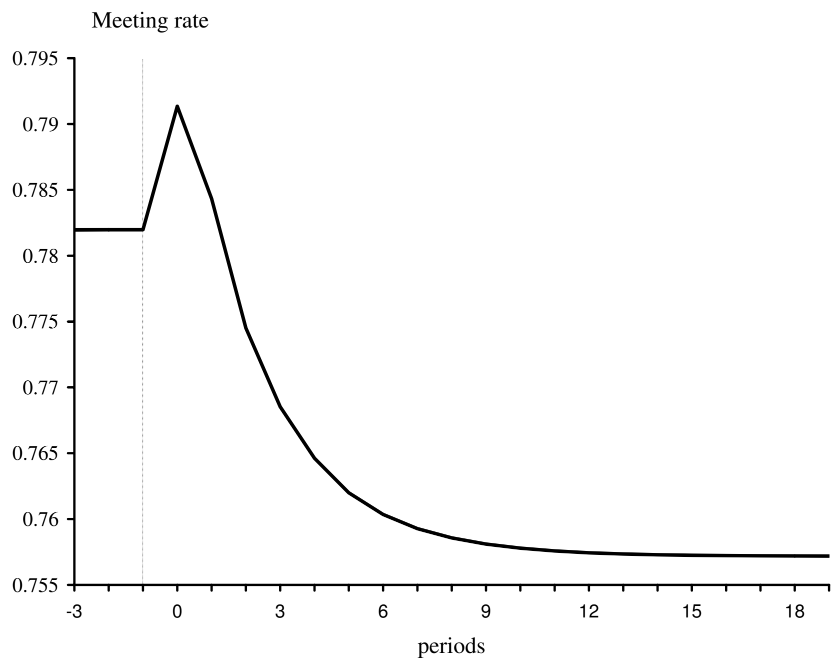

Fig. 15: Effect of a rise in the job destruction rate on the meeting rate

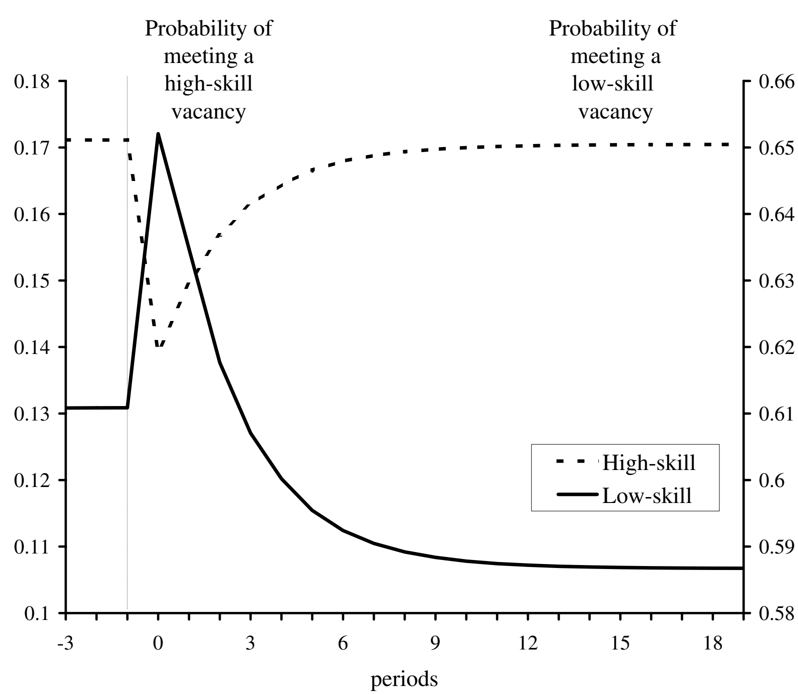

Fig. 16: Effect of a rise in the job destruction rate on job finding probabilities

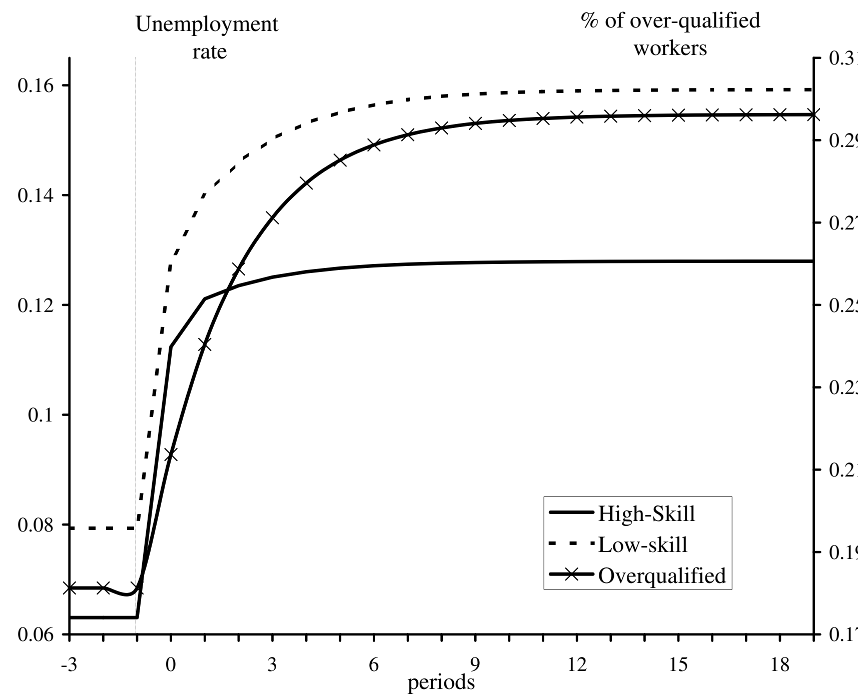

Fig. 17: Effect a rise in the job destruction rate on unemployment and overqualification rates

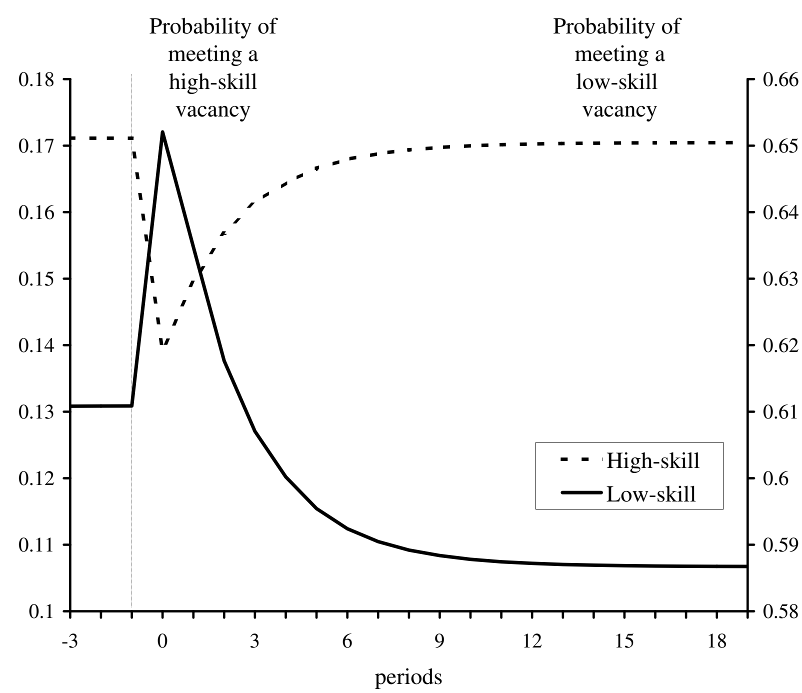

Fig. 18: Effect of a rise in the job destruction rate on average match productivity

APPENDIX

A A Unique Steady State Equilibrium

The steady state distribution $\{e_{ll}, e_{hl}, e_{hh}\}$ is constant over time. Therefore, by equating the flows in to the flows out the steady state values of $\psi\varphi$ and $\psi(1-\varphi)$ are uniquely determined for a given value of η and θ as follows

$$\psi\varphi = \frac{\delta(s+(1-\eta)m(\theta))}{\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))} \tag{25}$$

$$\psi(1-\varphi) = \frac{(1-\delta)(s+\eta m(\theta))(s+(1-\eta)m(\theta))}{[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))](s+m(\theta))} \tag{26}$$

Substituting these expressions together with the surplus expressions (20) to (22) into the free entry conditions (18) and (19) yields a set of equations in terms of the endogenous variables η and θ, which I denote as $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$, respectively. To ensure the existence and uniqueness of a steady state equilibrium I define the parameter conditions under which $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$ intersect only once.

First, I show that for a given η, an increase in θ lowers both loci. The partial derivatives with respect to θ are given by

$$\frac{\partial F_l}{\partial\theta} = \frac{\partial(q(\theta)\psi\varphi)}{\partial\theta}S_{ll} + q(\theta)\psi\varphi\frac{\partial S_{ll}}{\partial\theta} + \frac{\partial(q(\theta)\psi(1-\varphi))}{\partial\theta}S_{hl} + q(\theta)\psi(1-\varphi)\frac{\partial S_{hl}}{\partial\theta} \tag{27}$$

$$\frac{\partial F_h}{\partial\theta} = \frac{\partial(q(\theta)(1-\psi\varphi))}{\partial\theta}S_{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S_{hh}}{\partial\theta} \tag{28}$$

The second and last terms in (27) are negative, because $\frac{\partial S_{hl}}{\partial\theta} < 0$ and $\frac{\partial S_{ll}}{\partial\theta} < 0$. Moreover, we can write

$$\frac{\partial\psi(1-\varphi)}{\partial\theta} = -\frac{\partial m(\theta)}{\partial\theta}\frac{s(1-\delta)[\delta(1-\eta)(s+(1-\eta)m(\theta))^2+(1-\delta)\eta(s+\eta m(\theta))^s]}{(s+m(\theta)^2[\delta(s+(1-\eta)m(\theta))+(1-\delta)(s+\eta m(\theta))]^2} < 0 \tag{29}$$

Therefore, given that $q'(\theta) < 0$, the second term is also negative. To complete the proof of $\frac{\partial F_l}{\partial\theta} < 0$, I need to show that $\frac{\partial\psi\varphi}{\partial\theta} \leq 0$. This is given by

$$\frac{\partial\psi\varphi}{\partial\theta} = \frac{\partial m(\theta)}{\partial\theta}\frac{\delta(1-\delta)s(1-2\eta)}{[(1-\delta)(s+\eta m(\theta))+\delta(s+(1-\eta)m(\theta))]^2} \tag{30}$$

which is negative as long as $\eta \geq \frac{1}{2}$. I claim that this is true when $\delta(ya_l-b) \geq (1-\delta)(ya_h-b)$. Taking the ratio of $F_l(\eta,\theta)$ to $F_h(\eta,\theta)$ yields,

$$\frac{\delta(ya_l-b)\lambda_1(s+(1-\eta)m(\theta))}{(1-\delta)(ya_h-b)\lambda_2(s+\eta m(\theta))} + \frac{(ya_l-b)\lambda_1(s+(1-\eta)m(\theta))}{(ya_h-b)\lambda_3(s+m(\theta))} + \frac{\beta\eta m(\theta)(ya_l-b)}{\lambda_3(ya_h-b)} = 1 \tag{31}$$

which must be satisfied in equilibrium. The left hand side of this condition declines both with η and θ. Evaluating it at $\eta = \frac{1}{2}$, and the maximum value $m(\theta)$ can take, i.e., at $m(\theta) = 1$ yields

$$\frac{\delta(ya_l-b)}{(1-\delta)(ya_h-b)} + \frac{\lambda_1}{\lambda_3}\frac{(ya_l-b)}{(ya_h-b)}\frac{(s+\frac{1}{2})}{(s+1)} + \frac{\beta}{2\lambda_3}\frac{(ya_l-b)}{(ya_h-b)} \tag{32}$$

which is greater or equal to 1 as long as $\delta(ya_l-b) \geq (1-\delta)(ya_h-b)$. Hence, under this condition, equation (31) is satisfied in equilibrium only if η is greater than $\frac{1}{2}$.

Given that $\frac{\partial S_{hh}}{\partial \theta} < 0$, the second term in (28) is negative, but to establish that the first term is also negative requires an additional restriction on the matching technology. Namely, that $\frac{\partial q(\theta)(1-\psi\varphi)}{\partial \theta} < 0$. As Dolado *et al.* (2003) also mention, compared to the standard matching model, this is the only additional restriction that needs to be imposed on the matching technology. The standard assumption (see e.g., Mortensen and Pissarides, 1994) is that the elasticity of $q(\theta)$ with respect to θ is between -1 and 0. The assumption $\frac{\partial q(\theta)(1-\psi\varphi)}{\partial \theta} < 0$ imposes a tighter restriction on the elasticity, but still, numerical simulations show that this derivative is always positive for values of $\delta \geq \frac{1}{2}$.

I next show that $\frac{\partial F_h}{\partial \eta} \geq 0$ whereas $\frac{\partial F_l}{\partial \eta} \leq 0$, so that the two loci have opposite slopes in the $[\theta,\eta]$ plane. By taking the derivative of $F_l(\eta,\theta)$ and $F_h(\eta,\theta)$ with respect to η we can write

$$\frac{\partial F_l}{\partial \eta} = q(\theta)\frac{\partial \psi\varphi}{\partial \eta}(S_{ll}-S_{hl}) + q(\theta)\frac{\partial \psi}{\partial \eta}S_{hl} + q(\theta)\psi\varphi\frac{\partial (S_{ll}-S_{hl})}{\partial \eta} + q(\theta)\psi\frac{\partial S_{hl}}{\partial \eta} \tag{33}$$

$$\frac{\partial F_h}{\partial \eta} = q(\theta)\frac{\partial (1-\psi\varphi)}{\partial \eta}S_{hh} + q(\theta)(1-\psi\varphi)\frac{\partial S_{hh}}{\partial \eta} \tag{34}$$

The first, second, and third terms in (33) are negative since,

$$\begin{aligned}
\frac{\partial \psi\varphi}{\partial \eta} &= -\frac{\delta(1-\delta)m(\theta)(2s+m(\theta))}{\chi^2} < 0 \\
S_{ll}-S_{hl} &= \frac{(1-\eta)m(\theta)}{\lambda_2\lambda_3} > 0 \\
\frac{\partial \psi}{\partial \eta} &= -\frac{(1-\delta)m(\theta)}{\chi^2}\left[\begin{array}{c} (s+(1-\eta)m(\theta))[3\delta(1-\eta)+3(1-\delta)\eta+\delta\eta] \\ +\delta(s+\eta m(\theta))\eta m(\theta) \end{array}\right] < 0 \\
\frac{\partial (S_{ll}-S_{hl})}{\partial \eta} &= -\frac{(y\alpha_l-b)m(\theta)}{(\lambda_2\lambda_3)^2}\left[\begin{array}{c} \lambda_2(r+s)+\lambda_3(1-\beta(1-\eta))m(\theta) \\ +\beta\eta m(\theta)^2+(1-\beta)(1-\eta)m(\theta)^2 \end{array}\right] < 0
\end{aligned}$$

where $\chi = [\delta(s+(1-\eta)m(\theta)) + (1-\delta)(s+\eta m(\theta))]$, but the last term is positive, because $\frac{\partial S_{hl}}{\partial \eta} \geq 0$. Nevertheless, combining the third and last terms yields a negative term as long as $\beta \geq \frac{1}{2}$ and $\varphi \geq \frac{1}{2}$. Since the probability of separation is the same for both types of workers, whereas low-skilled workers have lower job probability of finding a suitable job, they must be over-represented in the pool of unemployed. Therefore, $\varphi \geq \delta$ must hold. The assumption that $\delta(ya_l - b) \geq (1-\delta)(ya_h - b)$, implies that $\delta \geq \frac{1}{2}$ given that $(ya_h - b) \geq (ya_l - b)$, ensuring also that $\varphi \geq \frac{1}{2}$. Hence, the only additional assumption sufficient to ensure $\frac{\partial F_l}{\partial \eta} \leq 0$ is $\beta \geq \frac{1}{2}$.

Since $\frac{\partial \psi\psi}{\partial \eta} \leq 0$, the first term in expression (34) is positive. Moreover, sufficient but not necessary condition for

$$\frac{\partial S_{hh}}{\partial \eta} = \frac{1}{\lambda_2^2\lambda_3^2} \times \left[\begin{array}{c} (y\alpha_h - y\alpha_l)[\beta m(\theta)\lambda_2\lambda_3 + \beta^3\eta^2 m(\theta)^3 + \beta(1-\beta)\eta(1-\eta)m(\theta)^3] \\ +(y\alpha_h - b)\beta(1-\beta)(1-\eta)m(\theta)^2\lambda_2 + (\beta y\alpha_h - y\alpha_l)\beta\eta m(\theta)^2\lambda_3 \end{array}\right] \tag{35}$$

to be positive is $\frac{(y\alpha_l - b)}{(y\alpha_h - b)} \leq \beta$. Therefore, when this condition is satisfied,$\frac{\partial F_h}{\partial \eta} < 0$.

B Proofs of Comparative Static Results

Here I derive the comparative static results discussed in section 3. I begin by showing that $\frac{\partial R}{\partial y} \geq 0$, where R is as defined in equation (23). After substituting in the surplus expressions (20)-(22) the derivative of R with respect to y is given by

$$\frac{\partial R}{\partial y} = \left[\frac{b\lambda_3\lambda_1(\psi\varphi\lambda_3 + \psi(1-\varphi)\lambda_2)}{(1-\psi\varphi)\lambda_2[(y\alpha_h - b)\lambda_3 - \beta\eta m(\theta)(y\alpha_l - b)]^2}\right](\alpha_h - \alpha_l) \tag{36}$$

which is always positive, because by assumption $(a_{hh} - a_{ll}) \geq 0$.

I then turn to the impact of a rise in s on the $F_h(\eta, \theta)$ locus. It is straightforward that $\frac{\partial S_{ll}}{\partial s} < 0$. Moreover,

$$\frac{\partial \psi\varphi}{\partial s} = \frac{\delta(1-\delta)m(\theta)(2\eta - 1)}{\chi^2} \tag{37}$$

is greater than zero, when $\delta(ya_l - b) \geq (1-\delta)(ya_h - b)$ holds, which ensures $\eta \geq \frac{1}{2}$. Therefore, $\frac{\partial(1-\psi\varphi)}{\partial s} < 0$. This completes that proof of $\frac{\partial F_h(\eta,\theta)}{\partial s} < 0$.

As already mentioned in section 3, is difficult to determine analytically the impact of a rise in s on $F_l(\eta, \theta)$. On the one hand, a higher s lowers the surplus of low-skill jobs, but on the other hand, it changes the composition of job seekers in favor of firms with low-skill vacancies. The first effect can be easily verified from the surplus expressions, which yield $\frac{\partial S_{hh}}{\partial s} < 0$ and $\frac{\partial S_{hl}}{\partial s} < 0$. The composition of job seekers changes in favor of low-skill vacancies, because, as shown in (37), $\frac{\partial \psi\varphi}{\partial s} > 0$, and in addition,

$$\frac{\partial \psi(1-\varphi)}{\partial s} = \frac{(1-\delta)(s + (1-\eta)m(\theta))(s + \eta m(\theta))}{\chi^2(s + m(\theta))^2} \times \begin{bmatrix} \frac{\delta(1-\eta)m(\theta)(s+(1-\eta)m(\theta))}{(s+\eta m(\theta))} \\ +\frac{(1-\delta)\eta m(\theta)(s+\eta m(\theta))}{(s+(1-\eta)m(\theta))} \\ +(1-\delta)(s + \eta m(\theta)) \end{bmatrix} \tag{38}$$

is greater than zero. Therefore, $\frac{\partial F_l(\eta,\theta)}{\partial s}$ can be either positive or negative depending which of the two effects dominates.

I next show that when $\frac{(\alpha_h - b)}{(\alpha_l - b)} \geq \frac{\beta}{(1-\varphi)}$, R increases with s. The derivative of R with respect to s is given by

$$\frac{\partial R}{\partial s} = \begin{array}{c} \frac{\partial(\frac{\psi\varphi}{(1-\psi\varphi)})}{\partial s} \frac{(1-\eta)m(\theta)S^{hl}}{\lambda_2 S^{hh}} + \frac{\partial(\frac{\psi}{(1-\psi\varphi)})}{\partial s} \frac{S^{hl}}{S^{hh}} + \frac{\psi}{(1-\psi\varphi)} \frac{\partial(\frac{S^{hl}}{S^{hh}})}{\partial s} \\ + \frac{\psi\varphi(1-\eta)m(\theta)}{(1-\psi\varphi)\lambda_2} \frac{\partial(\frac{S^{hl}}{S^{hh}})}{\partial s} - \frac{\psi\varphi(1-\eta)m(\theta)S^{hl}}{(1-\psi\varphi)\lambda_2^2 S^{hh}} \end{array} \tag{39}$$

Under the parameter restrictions imposed above to ensure a unique steady state equilibrium, all terms but the last in the above expression are positive. In particular, $\frac{\partial(\frac{\psi\varphi}{(1-\psi\varphi)})}{\partial s} = \left(\frac{\delta}{1-\delta}\right)\left(\frac{(2\eta-1)m(\theta)}{(s+\eta m(\theta))^2}\right)$, is greater than zero as long as $\eta \geq \frac{1}{2}$. Moreover, it follows that $\frac{\partial(\frac{\psi}{(1-\psi\varphi)})}{\partial s} = \frac{\partial(\frac{\psi\varphi}{(1-\psi\varphi)})}{\partial s} + \frac{\eta m(\theta)}{(s+m(\theta))^2} > 0$. The sign of third and forth terms is determined by the sigh of $\frac{\partial(\frac{S^{hl}}{S^{hh}})}{\partial s} = \frac{S_{hl}}{S_{hh}} [\beta\eta m(\theta) y(\alpha_h - \alpha_l) + (1-\beta)(1-\eta)m(\theta)]$, which given that $(\alpha_h - \alpha_l) > 0$, is positive. Finally, combining the third, forth and last terms yields

$$\frac{\psi\varphi S_{hl}}{(1-\psi\varphi)S_{hh}} \left[\frac{(y\alpha_h - b)(\lambda_3 - \lambda_2) - \beta\eta m(\theta)(y\alpha_l - b)}{(y\alpha_h - b)\lambda_3 - \beta\eta m(\theta)(y\alpha_l - b)} \frac{1}{\lambda_2} \left(\frac{(1-\eta)m(\theta)}{\lambda_1} + \frac{1}{\varphi} \right) - \frac{(1-\eta)m(\theta)}{\lambda_1^2} \right] \tag{40}$$

which is greater than zero when $\frac{(\alpha_h - b)}{(\alpha_l - b)} \geq \frac{\beta}{(1-\varphi)}$.

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