Inefficient employment decisions, entry costs, and the cost of fluctuations

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Abstract

Fluctuations in firms' revenues reduce firms' viability and are costly from a social welfare point of view even when agents are risk neutral if (i) the decision to continue operating a firm is not efficient at the margin so that fluctuations shorten firms' life expectancy (because they increase the chance revenue levels are such that discontinuation is unavoidable) and (ii) the shortening of the life expectancy reduces entry. Welfare consequences are large, even for moderate fluctuations: Implied estimates for the per period costs of business cycles can easily be equal to several percentage points of GDP. These estimates are based on a direct measurement of cyclical changes in the value added generated by workers that recently were not employed. This extensive margin measure of the cyclical change in output is of independent interest.

Keywords: business cycles, frictions

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1 Introduction

Fluctuations are a fact of life. They come in many varieties such as idiosyncratic, sectoral, regional, and aggregate. This paper documents that even modest fluctuations, like business cycles, are quite costly in a very simple framework with risk neutral agents and the following quite standard features. First, a fixed entry cost is required to create a project. Second, the decision to start or continue operating an existing project is subject to inefficiencies, that is, "frictions" prevent some profitable projects from producing. Third, the fluctuations affect the severity of the inefficiency, either positively or negatively. Using this framework, we show that fluctuations are costly because they deter entry and lower the *average* level of output produced. Whereas it has been a difficult challenge to develop a model in which moderate fluctuations have non-negligible per capita costs, the costs of business cycles in our framework can easily correspond to a permanent drop in output that exceeds several percentage points, that is, they are substantially larger than those reported in the classic Lucas (1987) paper.

Before providing intuition for the mechanism, we motivate the key underlying features of our framework. Starting a "project", whether it is a company, a plant, or a job, is almost never costless and entry costs are part of many economic models. Regarding the inefficient decision to start or continue operating a project, one can think of the inability to obtain financing,¹ the inability to motivate workers or avoid them from shirking,² or the inability to write contracts that prevent the employer from exploiting the employee.³ Finally, it is a natural feature of models with inefficiencies that the impact of the inefficiency depends on a time-varying state variable affecting the agents' decisions. For example, firms with lower net worth levels may be less likely to obtain credit, because they are more likely to exploit the convexity in the payoffs due to limited liability and increase the amount of risk undertaken. Similarly, the ability for financial intermediaries to channel funds from savers to firms may very well be weakened during a recession.

¹As for example in Townsend (1979), Bernanke and Gertler (1989) Kiyotaki and Moore (1997), Carlstrom and Fuerst (1998), and Bernanke, Gertler, and Gilchrist (1999).

²See Shapiro and Stiglitz (1984).

³See Ramey and Watson (1997).

The reason why fluctuations are costly in our framework is quite intuitive. Consider projects whose characteristics are such that they are at risk of making an inefficient operating decision and suppose that these firms are affected by a stochastic variable Φ_p ; positive movements in Φ_p decrease the number of inefficient decisions not to operate and negative movements in Φ_p increase the number of inefficient decisions. If there are no entry costs, then there is no robust reason why the positive effects of an increase in Φ_p would not offset the negative effects of a decrease in Φ_p . With entry costs, however, this is no longer true, because fluctuations in Φ_p reduce the lifetime of projects. The costliest consequence of fluctuations in Φ_p is that some projects no longer enter because of the reduction in the expected lifetime of the project. If the decision to operate the project is inefficient, then this reduction in entry involves projects that have positive value from a social welfare point of view.

The framework used is simple and contains only a small set of structural parameters and for most it is not difficult to consider a set of plausible values. One important ingredient in our quantitative assessment is the mass of projects that choose not to enter in a world with business cycles, but would choose to do so in a world without. Since these projects are not observed in the actual world with business cycles, we have to find a way to estimate this mass. Our identification procedure consists of two elements. First, economic theory pins down exactly which type of projects would be created, even in the not observed world without business cycles. Second, in a way that will be made more precise below, we basically assume that there are no sudden changes in how different types of projects are distributed in the relevant area.

Following the classic Lucas (1987) paper, there have been numerous attempts to develop models in which business cycles are costly.⁴ One strand of the literature considers preferences in which fluctuations are more harmful to the agent.⁵ But if agents are truly highly risk averse, then—as pointed out in Lucas (2003)—the question arises why high risk aversion does not show up in, for example, the diversification of individual portfolios, the level of insurance deductibles, or the wage premiums of jobs with high earnings risk.

⁴See Lucas (2003) for a summary.

⁵Examples of this line of research are Alvarez and Jermann (2005) and Tallarini (2000).

A second strand of the literature considers the possibility that risk is not spread evenly across agents. When idiosyncratic risk is persistent, then this line of research generates estimates for the cost of business cycles that are an order of magnitude larger than those found by Lucas. For risk aversion parameters equal to 1 or less, however, the estimates in the literature do not exceed 1%, whereas we can generate larger numbers with risk neutral agents.⁶

Our paper fits into a line of research that investigates the effect of uncertainty on the level or growth rate of output, which Lucas (2003) refers to as "... a promising frontier on which there is much to be done". Gali, Gertler, and Lopez-Salido (2007) consider a simple New-Keynesian model in which the efficiency losses due to mispricing in a recession are not offset by the efficiency gains in a boom, so that business cycles are welfare reducing, but the effects are quantitatively small. Ramey and Ramey (1991), Jones, Manuelli, and Stacchetti (2000), Epaulard and Pommeret (2003), Barlevy (2004), Mertens (2008) are other examples in the literature in which volatility affects the growth rate and/or the level of output. Besides the assumption of linear utility, our framework differs from these papers in that we focus on different characteristics to generate the relationship between volatility and the level of real activity, namely entry costs and an inefficiency in the decision to operate a project, two simple features often found in the literature.

The rest of this paper is organized as follows. In Section 3, we develop our framework. In Section 4, we derive an analytical expression for the cost of fluctuations. In Section 5, we discuss how the parameters are calibrated and in Section 6 we provide a numerical assessment of the cost of business cycle fluctuations. Our framework is quite abstract and allows for several different interpretations. To simplify the exposition, we give a standard, but very specific interpretation to the variables throughout the main text. In Section 7, we give alternative interpretations.

⁶See Storesletten, Telmer, and Yaron (2001), Krebs (2007), and Krusell, Mukoyama, Sahin, and Smith (2009).

2 Empirical motivation

Our framework predicts that fluctuations are costly, because they reduce entry and consequently lower the average level of output produced. In this section, we provide some rudimentary empirical support for this prediction. There are some papers that document a negative relationship between volatility and growth.⁷ We document that there also is a negative effect of volatility on the level of GDP even if one controls for the growth rate.

We use the logarithm of per capita GDP data, converted into PPP equivalents, for 27 OECD countries from the Penn World Tables.⁸ For each country, we calculate the average per capita output level relative to the US level. We then regress this average on the following control variables: average investment share in GDP, openness, the growth rate, and the share of services.⁹ Each country's regression residual is then compared with the standard deviation of this country's HP-filtered output.¹⁰ Figure 1 plots the results, together with the fitted regression line.¹¹ The negative relationship is significant at the 0.04% level. An increase in observed volatility from the lowest observed value (for France) to the highest observed value (for Poland) implies a reduction in per capita income equal to 5.6 percentage points, a non-trivial reduction.

3 Model

In this section, we present a model that includes all the features essential for our argument. The model is less general than it could be. For example, we choose a very simple friction to ensure that some firms make inefficient decisions. Moreover, to ease the exposition,

⁷See Burnside and Tabova (2009), Martin and Rogers (2000), and Ramey and Ramey (1995).

⁸The sample period starts in 1970 and ends in 2004. The Czech Republic, Korea, and Slovakia are excluded because of data limitations.

⁹The average investment share and openness are standard control variables in this literature. We add the growth rate, to establish that the relationship found is not driven by the growth rate and we include the service share, because as countries get richer they tend to specialize more in the less volatile services sector. We tried several alternative specifications and found similar results.

¹⁰Following Ravn and Uhlig (2002), we set the smoothing parameter for annual data equal to 6.23.

¹¹The UK is an outlier; given its relatively small investment share, it has a high per capita income level.

we will interpret the variables in a familiar but specific way. The simplicity of the model and the specific choices made are helpful in explaining why fluctuations are costly. The argument itself, however, carries over to more general settings. For example, what matters is that there are inefficiencies in the decision to operate a project, not what generates the inefficiency. In Section A.1 of the appendix, we show that the ad hoc inefficiency imposed here is identical to the one that comes out of a model in which the contractual fragility framework of Ramey and Watson (1997) is used to explicitly introduce an agency problem and in Section A.2 we show that a model with a standard financial friction leads to a very similar specification of the inefficiency that in fact would lead to even higher costs of business cycles.

In Section 3.1, we describe the agents in the economy and what their choices are. In Section 3.2, we describe the social planner's solution and in Section 3.3 we describe the competitive equilibrium.

3.1 Environment

This section describes the agents in the economy, the choices they can make, and the friction they are confronted with.

Workers and projects. Producing output requires a worker and a project. There is a continuum of projects and a continuum of workers. Project *i* is characterized by an entry cost, $\phi_c(i)$, and a productivity level, $\phi_p(i)$; both idiosyncratic variables are assumed to be constant through time. For simplicity, we assume that the heterogeneity across workers matches the heterogeneity across projects and that a worker of type *i* can only work in a project of type *i*.¹² Actual production of project *i*, $y_t(i)$, is given by

$$y_t(i) = \phi_p(i)\Phi_{p,t},\tag{1}$$

¹²This is not important for the analysis, but simplifies the formulas considerably, because the only possible outside options for the workers are (i) not operating the project and (ii) operating an *identical* project.

where $\Phi_{p,t}$ is aggregate productivity. From now on, we suppress the *i* index, but the reader should keep in mind that ϕ_p and ϕ_c vary across projects and workers and $\Phi_{p,t}$ does not.

Two different assumptions about $\Phi_{p,t}$ are considered. Under the first assumption, $\Phi_{p,t}$ is constant through time and equal to 1. In this case, the economic agents are heterogeneous, but face an unchanging economic environment. Under the second assumption, $\Phi_{p,t}$ is a stochastic variable that varies across time with an unconditional mean equal to 1. The most common interpretation of $\Phi_{p,t}$ is that it is an aggregate shock that is common to all agents. In this case, fluctuations in $\Phi_{p,t}$ correspond to business cycle fluctuations. For simplicity, we assume that $\Phi_{p,t}$ can take on only two values, Φ_+ in a boom and Φ_- in a recession. The transition probability of leaving a boom, $1 - \pi$, is equal to the transition probability of leaving a recession. This implies that (*i*) the expected durations of staying in a boom and a recession are equal to each other and that (*ii*) $\Phi_+ - 1 = 1 - \Phi_-$ since $E[\Phi_{p,t}] = 1$.

Outside option. When a worker is not involved in operating a project, then he receives μ^* in private benefits. From a social planner's point of view these benefits are equal to μ . Typically, $\mu < \mu^*$, because μ^* include transfers like unemployment benefits and μ does not.

Entry. Two decisions have to be made before production can take place. First, the decision has to be made whether to *create* the project by paying the entry cost ϕ_c . Second, the decision has to be made whether to *operate* the project.

Projects can be created instantaneously by paying a start-up cost, ϕ_c . When the project has been idle for some time, then ϕ_c would have to be paid once more to restart it. That is, one cannot simply mothball the project and restart it as if there had been no interruption. For example, it may take some time and effort before the project is operating at its potential productivity of ϕ_p again. For some projects, the value of ϕ_c may be very low. For example, in the US car industry it is not uncommon to leave capital idle or underutilized for some time and recall former workers when economic conditions improve. In our benchmark calibration, we set the lower bound of the distribution of ϕ_c equal to zero, which clearly would accommodate the possibility of low (re)starting costs.

In the competitive equilibrium, entry takes place if

$$N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_{p,t}) - \phi_c \ge \mu^* + \beta E_t \left[N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right], \tag{2}$$

where $N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_{p,t})$ is the discounted value of the worker's current and future earnings when the entry costs have been paid, $N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is the discounted value of the worker's earnings when entry costs have not been paid, and E_t is the expectation conditional on the period-*t* information set.¹³ When the entry costs are not paid, then the worker receives μ^* in the current period, but has the option to start the project at a later date.

In this model, the worker creates his own project. In Section 3.5, we extend the model and adopt the standard formulation used in the literature in which an entrepreneur creates the project. This allows for the possibility of inefficient entry. This turns out to be important for the magnitude of the costs of fluctuations, but not for the presence of these costs. Therefore, we first focus on a version of the model in which there are no entrepreneurs and the entry decision is by construction privately efficient.

Inefficient operating decision. The friction imposed on projects is that they are only allowed to operate if

$$\chi \le \phi_p \Phi_{p,t}.\tag{3}$$

For this constraint to be relevant, we assume that

$$\mu^* < \chi. \tag{4}$$

The easiest interpretation of (3) is that government regulation requires a minimum level of efficiency. In Section A.1, we show that we can use the contractual fragility framework of Ramey and Watson (1997) to derive Equation (3) and that a similar condition, which in fact leads to even higher costs of business cycles, can be derived from a model in which a financial friction prevents some firms from operating the project. We want to stress,

¹³That is, $N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is equal to $N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_{p,t}) - \phi_c$ when entry is optimal and is equal to $\mu^* + \beta \mathcal{E}_t N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1})$ when entry is not optimal.

however, that our argument depends on their being inefficiencies in the operating decision, not on the particular reason behind the inefficient operating decision. To keep the model as simple as possible, we simply impose the friction at this point.

If the friction imposed in Equation (3) is excluded from the model, then projects for which (i) $\phi_p \Phi_{p,t} > \mu^*$ and (ii) the value of ϕ_c is low enough, would be created and always produce. If the friction imposed in Equation (3) is included in the specification of the model, then projects for which $\mu^* < \phi_p \Phi_{p,t} < \chi$ will never be created, no matter how low the value of ϕ_c .

3.2 Social planner's solution

Figure 2 plots the social planner's solution when there are no fluctuations in $\Phi_{p,t}$. Projects for which ϕ_p is less than μ will never operate. Projects for which ϕ_p is bigger than μ will operate if the entry cost is low enough relative to the surplus $\phi_p - \mu$. Thus, projects under the cut-off level for ϕ_c , $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, are created and will operate until they are hit by the exogenous destruction shock. Projects that are above the cut-off will never operate.

Now consider the case in which there are business cycles. Suppose for the moment that the social planner makes entry decisions as if there are no business cycles. Then business cycles would have no welfare consequences, since agents are assumed to be risk neutral: Projects that start out in a recession loose a bit and projects that start out in a boom gain a bit. Not surprisingly, if the social planner chooses his decisions optimally, then he can do better. Now suppose that the social planner is allowed more freedom in making his choices, but that his choices are still restricted in the sense that if a project is created it must be operated until hit by an exogenous severance shock.

To focus the discussion, let $\phi_p > \mu/\Phi_-$, so that entry is attractive as long as ϕ_c is low enough. Since it is more attractive to start out in a boom, the cut-off level for entry shifts up in a boom and similarly it shifts down in a recession. That is,

$$\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) < \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p) < \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+).$$
(5)

If ϕ_c is sufficiently high, i.e., above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, then business cycles have no impact on the entry decision, because the project will not be created, not even if it can start in a boom. Similarly, if ϕ_c is sufficiently low, i.e., below $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$, then business cycles have no impact, because the project will always be immediately created, even in a recession. For projects with a value of ϕ_c in between $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$ and $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, however, the entry decision depends on whether the economy is in a boom or a recession; entry is worthwhile in a boom, but in a recession it is better to postpone entry. We will refer to this as the "timed-entry" strategy. That business cycles are welfare increasing is obvious for projects with a value of ϕ_c above $\tilde{\phi}_{c,no-bc}(\phi_p)$ and below $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$. Having the option to start such a project has no value in a world without business cycles, but has strictly positive value in a world with business cycles.¹⁴ But the option to wait also increases the value of projects with a value of ϕ_c below $\tilde{\phi}_{c,no-bc}(\phi_p)$ and above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$.¹⁵

The top panel of Figure 3 plots the shifts in the cut-off level of ϕ_c when the social planner is forced to keep a created project running until hit by an exogenous destruction shock. As documented in the graph, the change in the cut-off levels is not symmetric for booms and recessions. That is, for a given value of ϕ_c , the distance between $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-)$ and $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ is larger than the distance between $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ and $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. This is easiest to understand when ϕ_c is equal to 0. A project for which $\phi_p = \mu/\Phi_-$ and $\phi_c = 0$ is right at the cut-off during a recession, because entering immediately or waiting and entering when the boom starts are equivalent; under both strategies the revenues are μ until the boom occurs and paying the entry costs later does not add value if the entry costs are zero. Now consider a project with a value of ϕ_p equal to μ/Φ_+ , that is, the mirror image. This project will definitely not be created, not even in a boom. Under the imposed restriction that the worker is tied to the project until exogenous severance, the

¹⁴For $\phi_c = \tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, we have that $N(\phi_p, \phi_c, 0, \Phi_+) = N(\phi_p, \phi_c, 1, \Phi_+) - \phi_c = \mu + E_t \left[N(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right]$, since entry and no entry give by definition the same level of benefits. We also know that $N(\phi_p, \phi_c, 0, \Phi_-) = \mu + \beta E_t \left[N(\phi_p, \phi_c, 1, \Phi_{p,t}) \right]$. Thus, $N(\phi_p, \phi_c, 0, \Phi_-) = N(\phi_p, \phi_c, 0, \Phi_+) = \mu/(1-\beta)$ when $\phi_c = \tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$. This means that the option to start a project has no value when $\phi_c = \tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, but has strictly positive value (also in a recession) for projects with a lower value of ϕ_c . Thus, if $\tilde{\phi}_{c,no-bc}(\phi_p) < \phi_c < \tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ business cycles create value.

¹⁵ If the social planner always immediately enters, that is, independent of the value of $\Phi_{p,t}$, then average welfare is not affected for a project with a value of ϕ_c equal to $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$, but for firms with a higher value of ϕ_c the option to postpone paying ϕ_c is adding value.

project's revenues are equal to μ in a boom and less than μ in a recession.

The analysis above showed that business cycles are beneficial, even if the social planner is restricted to continue with once created projects. If we lift this restriction, then business cycles are even more beneficial. This is simple to see for the project with $\phi_c = 0$ and with the lowest possible value for ϕ_p such that entry during a boom occurs when the restriction is still in place. At this minimum value of ϕ_p , it must be the case that $\phi_p \Phi_+ > \mu$, as discussed in the last paragraph. That is, even when entry costs are zero, there must initially be a positive surplus, because the project cannot be terminated during a recession when the surplus turns negative. For this marginal project, ϕ_p is such that business cycles have just no welfare consequences when the restriction on discontinuation is imposed. If the restriction is lifted, then business cycles are beneficial for this project, because it becomes possible to enter during a boom and to end the project in a recession. We refer to this as the "cyclical" strategy. The cyclical strategy exploits that there is a lower bound on earnings, namely μ and this leads to a convex payoff structure. For the cyclical strategy, we can also calculate the cut-off level for entry costs. These projects have a shorter duration than other projects. Consequently, the cut-off level of ϕ_c under the cyclical strategy increases less with ϕ_p than the cut-off level of ϕ_c under the timed-entry strategy.

The bottom panel of Figure 3 combines the timed-entry and the cyclical strategy. Note that for projects with a low value of ϕ_p the cut-off for ϕ_c according to the cyclical strategy is above the cut-off according to the timed-entry strategy. The downward sloping line in the graph corresponds to projects for which the two strategies have the same NPV. One obvious project for which the two strategies have the same NPV is the one for which $\phi_c = 0$ and $\phi_p = \mu/\Phi_-$. Under both strategies it earns $\mu\Phi_+/\Phi_-$ in a boom and μ in a recession. If ϕ_c increases and ϕ_p is kept equal to μ/Φ_- , then the cyclical strategy becomes less attractive, because the revenues are unchanged, but one has to pay the entry cost every time the economy enters a boom. If ϕ_p decreases then the cyclical strategy becomes more attractive, because it would increase $\mu - \phi_p \Phi_-$, i.e., the difference between the recession payoffs under the cyclical and the timed-entry strategy.

NPV is downward sloping.

If fluctuations are beneficial if the social planner is restricted to using only the timedentry strategy, fluctuations are definitely beneficial if the social planner can also use the cyclical strategy. In contrast, fluctuations typically have negative welfare consequences in the competitive equilibrium. This will be discussed next.

3.3 Competitive equilibrium

Section 3.3.1 discusses the competitive equilibrium when the friction specified in Equation (3) is *not* part of the model and the next two sections when it is. Section 3.3.2 discusses the case without and 3.3.3 discusses the case with fluctuations.

3.3.1 Competitive equilibrium - no friction

If the friction specified in Equation (3) is not part of the model, then μ would have to be replaced by μ^* in Figure 3, but there would be no other changes to the graph. In terms of welfare analysis, however, there is a difference. To see this most clearly, suppose that μ is sufficiently below μ^* , so that a social planner would like all workers following the cyclical and the timed-entry strategy to always immediately create their project and operate it. Relative to the case without fluctuations, there would be a welfare enhancing increase in the number of operating projects during a boom and a welfare reducing reduction in the number of operating projects in a recession. It is possible that the difference between μ and μ^* is large enough and the distribution of ϕ_p and ϕ_c is such that the gains in a boom would not offset the losses in a recession. This is not the route followed in this paper. When the friction specified in Equation (3) is taken into account, then the result that business cycles are costly is much more robust.

3.3.2 Competitive equilibrium - without fluctuations

If there are no fluctuations and the friction is imposed, then only projects with a production level above χ can operate. This means that projects with a value of ϕ_p below χ will not be created no matter how low the entry costs are. Figure 4 describes the possibilities. The feature of this figure that is key to understand the analysis below is that projects with a marginal productivity level, that is, projects for which $\phi_p = \chi$, have a strictly positive private surplus, that is, the production level of the project is strictly more than what the worker could earn outside the relationship. In the competitive equilibrium discussed in Section 3.3.1, in which the friction of Equation (3) is not present, this is not the case. In that version of the model, marginal projects are those with $\phi_p = \mu^*$ and their surplus is equal to zero.

3.3.3 Competitive equilibrium - with fluctuations

Now consider the case with fluctuations. The top panel in Figure 5 illustrates the consequences of business cycles for the question whether the inefficiency is binding. If Φ_p takes on the high value, the constraint is relaxed and projects with a lower value of ϕ_p could overcome the inefficiency and produce. If Φ_p takes on the low value the opposite happens. If the density is increasing—a not implausible assumption in the lower tail of the distribution—the negative effects during a recession would dominate the positive effects during a boom, but this story is not interesting enough to write a paper about and is unlikely to be quantitatively important.

The two remaining panels of this figure illustrate the consequences of business cycles on entry. The middle panel documents the effect on entry for those projects for which productivity is so high that they are never affected by the friction, not even in the recession, that is, projects for which $\phi_p \geq \chi/\Phi_-$. The presence of fluctuations does not affect the entry and operating decision for these projects when the entry costs are either sufficiently high or sufficiently low, that is, when

either
$$\phi_c \le \phi_{c,\mathrm{bc}}(\phi_p, \Phi_-)$$
 or $\phi_c > \phi_{c,\mathrm{bc}}(\phi_p, \Phi_+)$. (6)

Projects with a value of ϕ_c such that

$$\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) < \phi_c \le \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) \tag{7}$$

follow a timed-entry strategy identical to the one discussed above for the social planner's version of the model and these projects are, thus, affected by business cycles.

Among the projects that follow the timed-entry strategy there are two types. Projects for which

$$\widetilde{\phi}_{c,\text{no-bc}}(\phi_p) < \phi_c \le \widetilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$$
(8)

would never enter in the world without business cycles, but would enter in a boom in a world with business cycles. We refer to these as the TE-Gain projects. Projects for which

$$\widetilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) < \phi_c \le \widetilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)$$
(9)

would always enter in a world without business cycles, but would only enter in a boom in a world with business cycles. We refer to these as the TE-Loss projects. Thus,

$$\begin{split} \text{TE-Gain:} \quad \phi_p \geq \tilde{\phi}_{p,\text{bc}}(\Phi_-) & \& \quad \tilde{\phi}_{c,\text{no-bc}}(\phi_p) < \phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) \\ \text{TE-Loss:} \quad \phi_p \geq \tilde{\phi}_{p,\text{bc}}(\Phi_-) & \& \quad \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p) \end{split}$$

Quantitatively, these two areas turn out not to be important at all.

The top two figures consider the effects on the operating and the entry decision in isolation. They both suggest that fluctuations cannot be important for risk neutral agents, since booms and recessions simply change the outcomes in opposite directions. The story is quite different, however, when we consider the interaction between the entry and the operating decision. In particular, consider the projects with a value of ϕ_p such that the value of $\Phi_{p,t}$ determines whether or not they can overcome the friction, i.e., projects for which

$$\frac{\chi}{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+)} \le \phi_p < \frac{\chi}{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)}.$$
(10)

If there is no friction, then these projects' cut off levels for ϕ_c would simply shift up and down with $\Phi_{p,t}$ as happens for projects with a value of ϕ_p high enough to never be affected by the friction. The friction forces these projects to stop operating when the recession starts. This unavoidable shortening of the project's lifetime is the reason fluctuations are costly. Since projects for which $\chi/\tilde{\phi}_{p,bc}(\Phi_+) \leq \phi_p < \chi/\tilde{\phi}_{p,bc}(\Phi_-)$ have a shorter expected life span than projects for which $\phi_p \geq \chi/\tilde{\phi}_{p,bc}(\Phi_-)$, their cut-off level for ϕ_c will be lower. In fact, since these projects enter in a boom and are destroyed in a recession, they follow a cyclical strategy. It is the unavoidable inefficiency in the operating decision that forces these workers to follow the cyclical strategy and from a social welfare point of view it is not optimal that workers choose to follow a cyclical strategy when $\chi/\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p < \chi/\tilde{\phi}_{p,\mathrm{bc}}(\Phi_-).$

Figure 6 puts everything together and identifies the three different groups that follow a cyclical strategy. Those are

$$\begin{array}{lll} \text{C-TEMP-Gain:} & \frac{\chi}{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \leq \phi_{p} < \frac{\chi}{\tilde{\phi}_{p,\text{no-bc}}} & \& & \phi_{c} \leq \tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+}) \\ \text{C-TEMP-Loss:} & \frac{\chi}{\tilde{\phi}_{p,\text{no-bc}}} \leq \phi_{p} < \frac{\chi}{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} & \& & \phi_{c} \leq \tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+}) \\ \text{C-PERM-Loss:} & \frac{\chi}{\tilde{\phi}_{p,\text{no-bc}}} \leq \phi_{p} < \frac{\chi}{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} & \& & \tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+}) < \phi_{c} \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_{p}) \end{array}$$

In the remainder of this subsection, we discuss these three different groups.

Temporary-gain projects: $\frac{\chi}{\tilde{\phi}_{p,\mathbf{bc}}(\Phi_+)} \leq \phi_p < \frac{\chi}{\tilde{\phi}_{p,\mathbf{no-bc}}}$ and $\phi_c \leq \tilde{\phi}_{c,\mathbf{bc}}(\phi_p, \Phi_+)$. Business cycles are beneficial for C-TEMP-Gain projects. During a recession the inefficiency cannot be surmounted, but the inefficiency also prevents these projects from producing in a world without business cycles. That is, in both cases the worker "produces" μ . In a boom, however, the revenues are high enough to overcome the inefficiency and the worker produces more than he does in a world without business cycles.

Temporary-loss projects: $\frac{\chi}{\tilde{\phi}_{p,\text{no-bc}}} \leq \phi_p < \frac{\chi}{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}$ and $\phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$. Business cycles are harmful for C-TEMP-Loss projects for two reasons. The first reason is the opposite of why business cycles are beneficial for the C-TEMP-Gain projects. The C-TEMP-Loss projects operate in a boom and not in a recession, whereas in a world without business cycles they always produce. The second reason is that they have to pay entry costs more frequently. In a world without business cycles, they only pay the entry cost after they have been exogenously destroyed, but in a world with business cycles these projects also pay entry costs when they are restarted at the beginning of an economic expansion.

Temporary gains and losses. Do the gains of the C-TEMP-Gain projects and the losses of the C-TEMP-Loss projects offset each other? Suppose that the distribution

is symmetric around $\tilde{\phi}_{p,\text{no-bc}}$ and suppose that we ignore that the output levels of the C-TEMP-Gain projects are slightly higher than the output levels of the C-TEMP-Loss projects. Even then it is true that the losses are bigger than the gains. Under this assumption, the output gains of the C-TEMP-Gain projects would offset the output losses of the C-TEMP-Loss projects. Business cycles are still costly, however, because in a world without business cycles, only the C-TEMP-Loss projects pay entry costs and they only pay them after an exogenous destruction, whereas in a world with business cycles both pay them and overall they are paid more often.

Permanent Loss: $\frac{\chi}{\tilde{\phi}_{p,\text{no-bc}}} \leq \phi_p < \frac{\chi}{\tilde{\phi}_{p,\text{bc}}(\Phi_-)}$ and $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. The most important reason for costly fluctuations is the presence of this group of C-PERM-Loss projects. In a world without business cycles, these projects are productive enough to overcome the inefficiency introduced by Equation (3) and their entry costs are low enough to enter. In a world with business cycles, these projects are still productive enough to overcome the inefficiency during a boom, but not during a recession. Moreover, for these projects the entry costs are so high that—given the shortening in duration—entry is no longer worth while.

The C-TEMP-Gain and the C-TEMP-Loss projects also exist in the competitive equilibrium when workers do not face the operating inefficiency. In contrast, C-PERM-Loss projects only exist when the inefficiency is part of the model, that is, when the operating decision is not always privately efficient.

3.4 Necessary ingredients

In this subsection, we explain why both entry costs *and* inefficient operating decisions are needed for business cycles to be costly.

Why are entry costs essential? Suppose that entry costs are equal to zero for all projects. This would mean that in Figure 6 the whole graph would collapse onto the x-axis. Recall that projects in the C-PERM-Loss area are such that they are not created in the presence of business cycles, because fluctuations shorten the expected duration of these

projects and their entry costs are too high given this shorter duration. But if there are no entry costs, then entry costs cannot be "too high" and this type of project does not exist. Now consider the C-TEMP-Gain and the C-TEMP-Loss area. Above we pointed out that in terms of output gains and losses these projects would roughly offset each other. For these two types of projects, the total gains and losses do not off set each other in general, because the amount paid on entry costs is higher in a world with business cycles. Of course, this cannot happen if entry costs are zero.

Why are inefficient operating decisions essential? There are three aspects of the inefficient operating decision that are important. First, the inefficiency makes it impossible to compensate worsened conditions during a recession with improved conditions during a boom. That is, the inefficiency specified in Equation (3) has to hold at each point in time, not just on average. Second, in the presence of inefficient operating decisions, marginal projects have a positive surplus when defined relative to the true outside option. This means that it is costly from a social welfare point of view that the shortening of the duration prevents some projects from being created. Third, the friction must lead to *private* inefficiencies. If Equation (3) is not part of the model, then there are no projects that are permanently prevented from operating by the presence of fluctuations,¹⁶ while this is the most important reason why fluctuations are costly in the model with the inefficiency imposed.

3.5 Extension: workers, entrepreneurs and inefficient entry

In the model described so far, workers create their own project, but it is more common and more realistic to let an entrepreneur create the project. An interesting feature of the extension with entrepreneurs is that it allows for another inefficiency, namely inefficient entry, since the entrepreneur pays for the entry costs, but has to share the revenues with the worker.

¹⁶Given that the unconditional mean of $\Phi_{p,t}$ is not affected by fluctuations and agents are risk neutral, it does not make sense to completely stop operating a project if there are no privately inefficient decisions.

The discussion of the model without inefficient entry made clear that inefficient entry is not necessary for business cycles to be costly. For cyclical projects, inefficient entry does not affect the elements of our story; it only makes our channel quantitatively more important. For the timed-entry projects, however, the efficiency of the entry decision does matter for the question whether business cycles increase or decrease welfare. For these projects, business cycles are always welfare enhancing with efficient entry, but could be welfare reducing with inefficient entry. It will be shown below, that the magnitude of the effects are always small.

Environment. In the extended model, there is a unit mass of entrepreneurs and an entrepreneur of type i can create a project of type i. Workers of type i can still only work in projects of type i. An entrepreneur that has created a project can find a worker of type i instantaneously, that is, we abstract from matching frictions. Finally, in contrast to the worker, an entrepreneur does not generate anything useful if he is not associated with a project.

Sharing rule. If the project operates, the entrepreneur receives

$$y_{e,t} = \omega_e(\phi_p \Phi_{p,t} - \mu) \tag{11}$$

and the worker receives $\phi_p \Phi_{p,t} - y_{e,t}$. This way of writing the sharing rule turns out to be convenient for several of our calculations and for thinking about the efficiency of the entry decision. In particular, entry is efficient if ω_e is equal to 1 in which case workers only receive what they could generate outside the relationship in terms of home production plus the extra utility from not working, i.e., μ . This sharing rule is not very restrictive, since ω_e is a free parameter that we will calibrate. It is only restrictive in the sense that we do not allow ω_e to vary with the business cycle, but we do not think that this matters for our results. **Entry decision.** The entry decision is very similar to the one given in Equation (2), except that only the payoffs for the entrepreneur matter. Thus,

$$N_{e,bc}(\phi_p, \phi_c, 1, \Phi_{p,t}) - \phi_c \ge \beta \mathcal{E}_t \left[N_{e,bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right],$$
(12)

where $N_{e,bc}(\phi_p, \phi_c, 1, \Phi_{p,t})$ is the discounted value of current and future benefits for the entrepreneur that has created the project and $N_{e,bc}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is the discounted value when the project has not yet been created. Even though the entrepreneur cannot produce anything of value outside the relationship, the value of $N_{e,bc}(\phi_p, \phi_c, 0, \Phi_{p,t})$ is positive unless the value of ϕ_p is such that the entrepreneur can never overcome the friction of Equation (3) or the value of ϕ_c is so high that he could never recover his investment.

Why does inefficient entry only affect our main channel quantitatively? In this model, in which there are no matching frictions and the entrepreneur has to pay the entry costs, a value of ω_e equal to 1 induces the entrepreneur to make socially efficient entry decisions. Whether ω_e takes on this efficient value or not is inconsequential for our key qualitative results. In particular, Figure 6 looks identical for different values of ω_e , because both $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$ and $\tilde{\phi}_{c,no-bc}(\phi_p)$ are proportional to ω_e . This suggests that the value of ω_e also does not affect our quantitative results, but this is not true. For each value of ϕ_p , a lower value of ω_e implies that—averaged across all projects that enter—the entry costs *relative* to ϕ_p are lower. One positive aspect of the disappearance of the C-PERM-Loss projects is that their entry costs no longer have to be paid. But this positive aspect is quantitatively less important when ω_e (and, thus, the average level of entry costs) is smaller, which means that for cyclical projects the costs of business cycles are larger for smaller values of ω_e .

Inefficient entry and the effects of business cycles on the timed-entry projects.

The value of ω_e does have a qualitative effect on the area with the timed-entry projects. When entry is efficient, i.e., when $\omega_e = 1$, then business cycles have a positive effect, because business cycles introduce the worthwhile option to postpone entry. Quantitatively, this effect turns out to be minuscule. For smaller values of ω_e this small gain of business cycles could turn into a cost. The reason is the following. The option to wait still has positive value for the entrepreneur. The problem is that the entrepreneur does not take into account that by postponing entry he also postpones the worker getting $(1-\omega_e)(\phi_p \Phi_{p,t}-\mu)+\mu$, which exceeds μ if entry is inefficient, i.e., when $\omega_e < 1$. The value of ω_e , thus, determines whether business cycles are beneficial for timed-entry projects or not. Quantitatively, however, the impact of business cycles on timed-entry projects is always small.

4 Measuring the costs of business cycles

In this section, we discuss the metric used to measure the costs of business cycles, state assumptions, and derive an analytical expression for the costs of business cycles.

4.1 Metric used to measure costs of business cycles

To calculate the cost of business cycles, we carry out the following thought experiment. Consider a population consisting of worker-entrepreneur pairs characterized by values of ϕ_c and ϕ_p . For each pair we calculate the expected utility when they are placed in a world without business cycles and when they are placed in a world with business cycles. Subtracting the two NPVs and integrating over ϕ_c and ϕ_p gives us the net gain for society.

In the case without business cycles, the net present value of current and future joint earnings of a worker-entrepreneur pair is denoted by $N_{\text{no-bc}}(\phi_p, \phi_c, e)$, where e = 1 when the entry costs have been paid and e = 0 when they have not. In the case with business cycles, the net present value is given by $N_{\text{bc}}(\phi_p, \phi_c, e, \Phi_p)$, which also depends on the aggregate state.

The difference in NPVs, averaged across the population is given by $U_{\rm no-bc} - U_{\rm bc}$, where

$$U_{\text{no-bc}} = \int \int \left[N_{\text{no-bc}}(\phi_p, \phi_c, 0) \right] f(\phi_p, \phi_c) d\phi_p d\phi_c \text{ and}$$
(13)

$$U_{\text{bc}} = \int \int \left[0.5 N_{\text{bc}}(\phi_p, \phi_c, 0, \Phi_+) + 0.5 N_{\text{bc}}(\phi_p, \phi_c, 0, \Phi_-) \right] f(\phi_p, \phi_c) d\phi_p d\phi_c.$$

Both $U_{\text{no-bc}}$ and U_{bc} weigh the different NPVs with the density over ϕ_p and ϕ_c . U_{bc} is also

an average of the NPV when starting out in a boom and the NPV when starting out in a recession, using the unconditional probabilities.

Since agents are assumed to be risk neutral and business cycles do not affect the unconditional expectation of $\Phi_{p,t}$, fluctuations only affect agents' welfare if they affect entry and/or operating decisions. Consequently, fluctuations do not affect projects with values of ϕ_c and ϕ_p such that either entry never occurs or entry as well as production always take place.

To be able to interpret the difference between the two NPVs in Equation (13), we calculate the difference in the two NPVs as a fraction of per capita GDP, that is, we calculate the value of Ω defined as

$$U_{\rm no-bc} - U_{\rm bc} = \Omega Y, \tag{14}$$

where Y is the average per capita output level in the world with business cycles. If agents in the world with business cycles would be given $(1-\beta)\Omega Y$ each period, then the expected utility *averaged* across agents would be equal in the world with and without business cycles. The value of $(1-\beta)\Omega$ is our metric for the cost of business cycles. For example, if $(1-\beta)\Omega$ is equal to 0.01, then agents in a world with business cycles would be on average as well of as agents in a world without business cycles if *each* of them receives a transfer equal to 1% of per capita output.¹⁷

This measure completely abstracts from distributional consequences of business cycles. As pointed out above, business cycles have no effect at all on the welfare of agents associated with projects whose entry and operating decisions are not affected. Among the agents whose decisions are affected, i.e., those with cyclical and timed-entry projects, there are winners and losers. Our welfare measure, $(1 - \beta)\Omega Y$, is the average amount of compensation required. The costs of business cycles are obviously larger for the losers.

¹⁷In our model, consumption consists of home and market production minus entry costs. We express welfare costs as a fraction of output, because our assumptions do not allow us to calculate aggregate consumption. Costs of business cycles are typically expressed as a fraction of aggregate (market) consumption. Note that these estimates would be lower if they—like our measure—are expressed relative to GDP, since aggregate consumption is less than GDP.

4.2 Useful variables and formulas

To calculate the costs of business cycles, we need the formulas for the NPVs and the formulas for the cut-off levels for ϕ_c . Although the formulas are straightforward, several of them are quite cumbersome. Therefore, we only give the formulas for the C-PERM-Loss area, which is also the area we highlight in our derivation of the analytical expression for the cost of business cycles.

Value generated when not employed. For the unemployed workers associated with cyclical and timed-entry projects, we need to specify what they generate in terms of home production and possibly net utility of not working.¹⁸ For all other workers it does not matter what unemployment benefits are, because they would cancel out when calculating $N_{\text{no-bc}}(\phi_p, \phi_c, 0) - \mathbb{E}[N_{\text{bc}}(\phi_p, \phi_c, 0, \Phi_p)]$. In the discussion above, the generated value when not employed was assumed to be constant and equal to μ . For the cyclical and timed-entry projects, they are assumed to be proportional to ϕ_p , that is, $\mu = \mu_{\phi_p} \phi_p$. This assumption simplifies the formulas considerably, but the variation of μ with ϕ_p , or the lack of variation with ϕ_p , is of only minor importance.¹⁹

NPV formulas. Projects in the C-PERM-Loss area are characterized by values of ϕ_c and ϕ_p that satisfy

$$\tilde{\phi}_{p,\text{no-bc}} \leq \phi_p < \tilde{\phi}_{p,\text{bc}} \left(\Phi_{-}\right) \text{ and } \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_{+}) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p).$$
(15)

¹⁸Unemployment transfers should be exluded, since the cost of business cycles are calculated from society's point of view.

¹⁹One reason is that one can interpret μ as the average value of $\mu_{\phi_p}\phi_p$ across the affected jobs. The other reason is that for those projects for which the costs of business cycles are most severe, i.e., the cyclical projects, the variation in μ is not that large to begin with; the values of μ would vary from $\mu_{\phi_p}\chi/\Phi_+$ to $\mu_{\phi_p}\chi/\Phi_-$.

By definition, these projects are always able to produce in a world without business cycles and never do so in a world with business cycles. If Equation (15) is satisfied, then

$$N_{\text{no-bc}}(\phi_p, \phi_c, 0) = -\phi_c + N_{\text{no-bc}}(\phi_p, \phi_c, 1)$$

$$= -\phi_c + \phi_p + \beta \begin{pmatrix} \rho N_{\text{no-bc}}(\phi_p, \phi_c, 1) + \\ (1 - \rho) N_{\text{no-bc}}(\phi_p, \phi_c, 0) \end{pmatrix}$$

$$= \frac{\phi_p - (1 - \beta \rho)\phi_c}{1 - \beta} \text{ and}$$

$$N_{\text{bc}}(\phi_p, \phi_c, 0, \Phi_p) = \frac{\mu_{\phi_p}\phi_p}{1 - \beta},$$

$$(16)$$

and the two corresponding cut-off levels are given by

$$\widetilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\omega_e \phi_p (1 - \mu_{\phi_p})}{1 - \beta \rho} \text{ and}$$
(18)

$$\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) = \frac{\omega_e \phi_p(\Phi_+ - \mu_{\phi_p})}{1 - \beta \pi \rho}.$$
(19)

Lower bound on ϕ_c . In our calculations, we start by setting the lower bound of ϕ_c equal to zero. The lower this lower bound, the lower the costs of business cycles turn out to be, so setting the lower bound equal to zero is a conservative choice. However, the assumption that some projects can be created for free may be too good to be true. Therefore, we also consider alternative values for the lower bound. To be able to interpret the values of the lower bounds considered, we focus on the average amount of entry costs a firm would pay if it would continue operating during a recession. The firm would then on average invest, $(1 - \rho) \phi_c$. Let $i(\phi_c, \phi_p)$ be equal to this average amount of entry costs expressed as a fraction of average firm output. Thus,

$$i(\phi_c, \phi_p) = \frac{(1-\rho)\phi_c}{\phi_p} \text{ or } \phi_c = \frac{i(\phi_c, \phi_p)}{1-\rho}\phi_p.$$
 (20)

In our calibration exercise, we consider different values for the lower bound for $i(\phi_c, \phi_p)$. If $i(\phi_c, \phi_p)$ is bounded from below by \underline{i} , then the lowest possible value of ϕ_c , $\underline{\phi_c}(\phi_p)$, is a linear function of ϕ_p . That is,

$$\phi_c \ge \underline{\phi_c}(\phi_p) = \frac{\underline{i}}{1-\rho}\phi_p = \underline{i^*}\phi_p.$$

Mass in C-PERM-Loss relative to mass in C-TEMP-Loss. The ratio of the mass of projects in the C-PERM-Loss area relative to the mass in the C-TEMP-Loss area turns out to be important for our quantitative results. In our calibration, we relate this ratio of probabilities to the theoretical ratio of the lengths of the corresponding intervals. The idea is that if the length of a particular interval gets smaller, then the mass in this interval is getting smaller as well. Since the horizontal lengths of the C-PERM-Loss and C-TEMP-Loss areas are equal to each other, it is the vertical length that matters, that is, the range of values of ϕ_c . Let

$$R^*_{\mathsf{C-probabilities}}(\phi_p) \equiv \frac{\operatorname{prob}\left\{\phi_c : \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) < \phi_c \le \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p) | \phi_p\right\}}{\operatorname{prob}\left\{\phi_c : \underline{i^*}\phi_p \le \phi_c \le \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) | \phi_p\right\}}, \quad (21)$$

$$R^*_{\mathsf{C-lengths}}(\phi_p) \equiv \frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p) - \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) - \underline{i^*}\phi_p}.$$
(22)

 $R^*_{C-probabilities}(\phi_p)$ indicates the ratio of the mass in C-PERM-Loss relative to the mass in C-TEMP-Loss for a given value of ϕ_p and $R^*_{C-lengths}(\phi_p)$ is the ratio of the corresponding relative lengths according to our theory. Combining these two definitions, we get that the mass of the projects we are interested in, namely the mass of the C-PERM-Loss projects, is equal to

$$\operatorname{prob}\left\{\phi_{c}: \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) < \phi_{c} \leq \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) | \phi_{p}\right\} \equiv$$

$$\left(\frac{R_{\mathsf{C-probabilities}}^{*}(\phi_{p})}{R_{\mathsf{C-lengths}}^{*}(\phi_{p})}\right) \left(\frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) - \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) - \underline{i^{*}}\phi_{p}}\right) \times$$

$$\operatorname{prob}\left\{\phi_{c}: \underline{i^{*}}\phi_{p} \leq \phi_{c} \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) | \phi_{p}\right\}.$$

$$(23)$$

The idea of the calibration is to consider different values for the ratio of $R^*_{C-probabilities}(\phi_p)$ to $R^*_{C-lengths}(\phi_p)$. For example, if this ratio is equal to 1, then the mass in C-PERM- Loss relative to the mass in C-TEMP-Loss is equal to the relative lengths of the vertical distances of the two regions. This would be consistent with, for example, a uniform distribution.

One slight complication is that this ratio could vary with ϕ_p . Therefore, we focus on the lowest value of the ratio in the interval $\left[\tilde{\phi}_{p,\text{no-bc}}(\phi_p), \tilde{\phi}_{p,\text{bc}}(\phi_p, \Phi_-)\right]$, that is,

$$R_{\rm C} = \min_{\tilde{\phi}_{p,\rm no-bc} \le \phi_p \le \tilde{\phi}_{p,\rm bc}(\Phi_-)} \frac{R^*_{\rm C-probabilities}(\phi_p)}{R^*_{\rm C-lengths}(\phi_p)}.$$
 (24)

By focusing on the lowest value of the ratio, we obtain a conservative estimate of the mass in C-PERM-Loss. The length of the interval $\left[\tilde{\phi}_{p,\text{no-bc}}(\phi_p), \tilde{\phi}_{p,\text{bc}}(\phi_p, \Phi_-)\right]$ over which we are maximizing is small, because it is determined by the magnitude of the variations in the aggregate shock, Φ_p . Consequently, we can expect the ratio of $R^*_{\text{C-probabilities}}(\phi_p)$ to $R^*_{\text{C-lengths}}(\phi_p)$ not to vary that much.

4.3 Cyclical projects

In Section 4.3.1, we state the assumptions we make about cyclical projects and in Section 4.3.2 we derive an analytical expression for the cost of business cycles.

4.3.1 Assumptions

One set of assumptions with which our results can be derived consists of the following:

• If the values of ϕ_p and ϕ_c are such that

$$\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-) \text{ and } \underline{i^*}\phi_p \leq \phi_c \leq \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)$$

then

$$\begin{split} \phi_p &\sim U(\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+), \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)); \\ \mathrm{if} & \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) \leq \phi_c \leq \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p) \quad \mathrm{then} \quad \phi_c \sim U(\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+), \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)); \\ \mathrm{and} \\ \mathrm{if} & \underline{i^*}\phi_p \leq \phi_c \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) \quad \mathrm{then} \quad \phi_c \sim U(\underline{i^*}\phi_p, \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)). \end{split}$$

That is, in the area of cyclical projects the distribution of ϕ_p is uniform and the distribution of ϕ_c , conditional on the value of ϕ_p , is uniform both above and below $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$. The assumption that the distribution is uniform is stronger than is needed to derive our results. In the appendix, we give a weaker set of assumptions. To improve the readability of the paper, we rely on this stronger set of assumptions in the main text.

In our benchmark calibration, we set $R_{\rm C}$ equal to 1 and \underline{i} equal to 0. In this case, the last two parts of this assumption reduce to the assumption that ϕ_c is distributed uniformly in the interval $[0, \tilde{\phi}_{c,\text{no-bc}}(\phi_p)]$ for each value of ϕ_p in the interval $\left[\tilde{\phi}_{p,\text{bc}}(\Phi_+), \tilde{\phi}_{p,\text{bc}}(\Phi_-)\right]$.

4.3.2 Welfare losses for cyclical projects

 A_{ϕ_c}

Getting analytical expressions for $N_{\text{no-bc}}(\phi_p, \phi_c, 0) - \mathbb{E}[N_{\text{no}}(\phi_p, \phi_c, 0, \Phi_p)]$ for the different types of projects is straightforward. Unfortunately, they are quite tedious, except for those projects for which business cycles do not matter. Therefore, we only derive the expression for the C-PERM-Loss area in the main text and discuss the derivations for the others in the appendix.

The welfare loss due to the C-PERM-Loss area, can be written as

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p})} \begin{bmatrix} N_{\text{no-bc}}(\phi_{p},\phi_{c},0) \\ -E\left[N_{\text{bc}}(\phi_{p},\phi_{c},0,\Phi_{p})\right] \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{c}d\phi_{p} \qquad (25)$$

$$= \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p})} \begin{bmatrix} \frac{\phi_{p}-(1-\beta\rho)\phi_{c}}{1-\beta} - \frac{\mu_{\phi_{p}}\phi_{p}}{1-\beta} \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{c}d\phi_{p} \\
= \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p})} \begin{bmatrix} -(1-\beta\rho)\phi_{c} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\phi_{p} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \\
= \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p})} \begin{bmatrix} \left(-(1-\beta\rho)\right)A\phi_{c} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\phi_{p} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \\
= \left(\frac{-(1-\beta\rho)}{1-\beta}A\phi_{c} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right)\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{no-bc}}(\phi_{p},\Phi_{+})} [\phi_{p}\right]f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \\
= \left(\frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p}) + \tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}{2\phi_{p}}\right).$$

The second expression is obtained using the formulas for the NPVs given in equations (16) and (17). In the third expression, the joint density is rewritten as the product of the marginal and the conditional density. To obtain the fourth expression, the assumptions of Section 4.3.1 are used to calculate the mean of ϕ_c . Rewriting the fourth expression gives the last term.

We have now obtained an expression for the welfare costs of business cycles for the C-PERM-Loss area as a function of the integral of ϕ_p in this area, that is, as a function of the output generated by these projects (when $\Phi_p = 1$). This is of limited value, because these projects are never observed in the actual world in which business cycles are a fact of life. The key step in our quantitative analysis is to relate the output generated in the C-PERM-Loss area to the output generated in the C-TEMP-Loss area, as shown in the following equations:

$$\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{hc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p})} \begin{bmatrix} N_{\text{no-bc}}(\phi_{p},\phi_{c},0) \\ -E\left[N_{\text{bc}}(\phi_{p},\phi_{c},0,\Phi_{p})\right] \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{c}d\phi_{p} \qquad (26)$$

$$= \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right) \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{hc}}(\Phi_{-})} \left[\left(\int_{\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})}^{\tilde{\phi}_{c,\text{hc}}(\phi_{p})} f_{\phi_{c}}|\phi_{c}\rangle d\phi_{c}\right)\phi_{p} \right] f_{\phi_{p}}(\phi_{p})d\phi_{p}$$

$$= \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right) \times \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{hc}}(\Phi_{-})} \left[\operatorname{prob}\left\{\phi_{c}:\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+}) < \phi_{c} \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_{p})|\phi_{p}\right\}\phi_{p} \right] f_{\phi_{p}}(\phi_{p})d\phi_{p}$$

$$\geq \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right) R_{C} \frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p}) - \tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})}{\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+}) - \frac{i^{*}\phi_{p}}{\phi_{p}}}$$

$$\times \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{hc}}(\Phi_{-})} \left[\operatorname{prob}\left\{\phi_{c}:\frac{i^{*}}{\delta}\phi_{p} \leq \phi_{c} \leq \tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})|\phi_{p}\right\}\phi_{p} \right] f_{\phi_{p}}(\phi_{p})d\phi_{p}$$

$$= \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right) R_{C} \frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p}) - \tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})}{\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+}) - \frac{i^{*}\phi_{p}}{\phi_{p}}}$$

$$\times \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{hc}}(\Phi_{-})} \left[\operatorname{prob}\left\{\phi_{c}:\frac{i^{*}}{\delta}\phi_{p} \leq \phi_{c} \leq \tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})|\phi_{p}\right\}\phi_{p} \right] f_{\phi_{p}}(\phi_{p})d\phi_{p}$$

$$= \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right) R_{C} \frac{\tilde{\phi}_{c,\text{hc}}(\phi_{p}) - \tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+})}{\tilde{\phi}_{c,\text{hc}}(\phi_{p},\Phi_{+}) - \frac{i^{*}\phi_{p}}{\phi_{p}}}$$

$$= \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_{c}} + \frac{1-\mu_{\phi_{p}}}{1-\beta}\right)\frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_{p}) - \tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+}) - \underline{i^{*}}\phi_{p}}\frac{R_{C}}{\Phi_{+}} \\ \times \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})}\int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}\left[\phi_{p}\Phi_{+}\right]f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \\ = \Omega_{C-\text{PERM-Loss}}\int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})}\int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})}\left[\phi_{p}\Phi_{+}\right]f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \\ = \Omega_{C-\text{PERM-Loss}}Y_{C-\text{TEMP-Loss}}(\Phi_{+}),$$

where

$$\Omega_{\text{C-PERM-Loss}} = \left(\frac{-(1-\beta\rho)}{1-\beta}A_{\phi_c} + \frac{1-\mu_{\phi_p}}{1-\beta}\right)\frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_p) - \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)}{\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) - \underline{i^*}\phi_p}\frac{R_{\text{C}}}{\Phi_+}$$

The second expression is a slightly rewritten version of the last expression in Equation (25). The third expression is obtained by rewriting the integral over the density of ϕ_c using notation related to our calibration. The next step is the crucial one. It relates the mass in between $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ and $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, i.e., the C-PERM-Loss area, to the mass in between $\underline{i}^*\phi_p$ and $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$, i.e., the C-TEMP-Loss area, using the relative theoretical lengths of the two intervals and a scaling factor $R_{\rm C}$, for which we consider a range of values in the calibration. In the remaining steps, simple algebra is used to get an expression in terms of the output generated by projects in the C-TEMP-Loss area.

Although the formula looks a bit messy, it is straightforward to calculate the outcome. There are three parts to the formula. First, the output generated by projects in the C-TEMP-Loss area, for which we will obtain an empirical estimate. Second, a set of structural parameters for which one can choose quite easily reasonable values. Those are β , ρ , μ_{ϕ_p} , π , and Φ_+ . Third, the values for $R_{\rm C}$ and \underline{i} , which are harder to calibrate, but for which we consider a range of values.

For the C-TEMP-Gain and the C-TEMP-Loss areas, we can obtain similar expressions, with the only difference that these cost measures are directly expressed relative to the output generated in the area itself, that is, we get²⁰

 $\Omega_{\text{C-TEMP-Gain}} Y_{\text{C-TEMP-Gain}}(\Phi_+) + \Omega_{\text{C-TEMP-Loss}} Y_{\text{C-TEMP-Loss}}(\Phi_+).$

4.4 Total cost of business cycles.

Since agents are risk neutral and business cycles only affect the choices made for the cyclical and the timed-entry projects, these are the only projects we have to consider. In Appendix B, we show that business cycles have very small welfare consequences for the timed-entry projects. These costs are, therefore, ignored in the main text, except that we provide some intuition for this finding in Section 6.1.

The relevant formula for the cost of business cycles is given by

$$U_{\rm no-bc} - U_{\rm bc} = \tag{28}$$

 $\Omega_{\text{C-TEMP-Gain}} Y_{\text{C-TEMP-Gain}}(\Phi_+) + (\Omega_{\text{C-TEMP-Loss}} + \Omega_{\text{C-PERM-Loss}}) Y_{\text{C-TEMP-Loss}}(\Phi_+),$

where the Ω coefficients are known functions of β , ρ , μ_{ϕ_p} , π , Φ_+ , \underline{i} , and $R_{\rm C}$. Our assumptions imply that conditional on ϕ_c the value of ϕ_p is distributed uniformly around $\tilde{\phi}_{p,\text{no-bc}}$, which in turn implies that

$$Y_{\text{C-TEMP-Gain}}(\Phi_+) \le Y_{\text{C-TEMP-Loss}}(\Phi_+), \tag{29}$$

since the value of ϕ_p is smaller below than above $\tilde{\phi}_{p,\text{no-bc}}$. This and the fact that $\Omega_{\text{C-TEMP-Gain}} <$

²⁰The formulas for the terms on the right-hand side are given in the appendix.

0, i.e., for these projects business cycles are beneficial, implies that

$$U_{\text{no-bc}} - U_{\text{bc}}$$

$$\geq (\Omega_{\text{C-TEMP-Gain}} + \Omega_{\text{C-TEMP-Loss}} + \Omega_{\text{C-PERM-Loss}}) \frac{Y_{\text{C-TEMP}}(\Phi_+)}{2}$$

$$= \Omega Y.$$
(30)

where

$$\begin{split} Y_{\text{C-TEMP}}(\Phi_{+}) &= Y_{\text{C-TEMP-Gain}}(\Phi_{+}) + Y_{\text{C-TEMP-Loss}}(\Phi_{+}),\\ \Omega &= \frac{(\Omega_{\text{C-TEMP-Gain}} + \Omega_{\text{C-TEMP-Loss}} + \Omega_{\text{C-PERM-Loss}})Y_{\text{C-TEMP}}(\Phi_{+})}{2Y}, \end{split}$$

and Y is average per capita output. To express this measure as an average per-period compensation, we have to multiply it with $(1 - \beta)$.

5 Calibration

The costs of business cycles are equal to

$$\left(\frac{\left(\Omega_{\text{C-TEMP-Gain}} + \Omega_{\text{C-TEMP-Loss}} + \Omega_{\text{C-PERM-Loss}}\right)}{2}\right) \left(\frac{Y_{\text{C-TEMP}}(\Phi_+)}{Y}\right)$$

where $\Omega_{\text{C-TEMP-Gain}}$ and $\Omega_{\text{C-TEMP-Loss}}$ are functions of the structural parameters, i.e., β , ρ , μ_{ϕ_p} , ω_e , π , \underline{i} , and Φ_+ and $\Omega_{\text{C-PERM-Loss}}$ is a function of these same parameters and R_{C} . This section consists of three parts. First, we show how to obtain an estimate for $Y_{\text{C-TEMP-Loss}}(\Phi_+)/Y$. This is the output generated by cyclical projects during a boom, expressed as a fraction of average GDP, Second, we discuss our choices for the structural parameters β , ρ , μ_{ϕ_p} , ω_e , π , $\underline{i^*}$, and Φ_+ . Third, we motivate our choice for R_{C} , the scaling parameter.

5.1 Cyclical changes through the extensive margin

The key piece of information needed to calculate the cost of aggregate fluctuations is $Y_{\text{C-TEMP}}(\Phi_+)/Y$, that is, the output generated by cyclical projects during a boom, relative to average output. If the value added of these projects is small, then business cycles cannot be costly for society as a whole. In our model, the workers associated with the cyclical

projects are the least productive ones, but this would not have to be true in a more general version of the model. For example, χ does not have to be the same across projects; one could have a model with different types of projects, each with its own value for χ , so that the least productive projects of each type form the group of cyclical projects.

Although, we obviously have good measures on how much output fluctuates over the business cycle, it is much harder to determine which part of the fluctuations is due to the *intensive* margin, i.e., the change in output produced by existing projects, and which part is due to the *extensive* margin, i.e., the change in output through the creation of new projects. $Y_{\text{C-TEMP}}(\Phi_+)/Y$ is an estimate of the latter. The value of $Y_{\text{C-TEMP}}(\Phi_+)/Y$ is bounded from above by the total percentage difference between the level of output in a recession and the level in a boom. We first calculate an estimate for this difference.

Using US data from 1947Q1 to 2009Q1, we find an estimate for the standard deviation of HP-filtered output equal to 1.66%. This is based on a smoothing coefficient of 1,600, which restricts fluctuations to have a period of less than eight years (roughly). When we use a smoothing coefficient equal to 10^5 , the value commonly used in the labor-macro literature, the standard deviation increases to 2.55%. For German data from 1975Q1 to 2004Q4, the standard deviations of HP-filtered output are equal to 1.17% and 2.89% for smoothing parameters equal to 1,600 and 10^5 , respectively. Obviously, one can easily find higher numbers when the pre-war period is included or by considering other countries.

In the empirical measure constructed below, we associate the extensive margin with worker flows. This may lead to an underestimate of the actual change in output through the extensive margin and, thus, to an underestimate of the cost of business cycles. For example, consider an existing firm that starts a new project and operates it by letting an existing worker work more or more efficiently. If this project is subject to entry costs and prevented from operating if aggregate productivity is low, then—according to our model—it should be counted to the extensive not the intensive margin, whereas in our empirical work it is part of the intensive margin.

Using individual wage data to measure the extensive margin of value added. Obtaining micro-level data for the total value added of different "projects" is obviously difficult if not impossible, but data on individual wages, i.e., the largest part of total value added, is available. In particular, we use the IAB monthly employment panel, a 2% representative subsample from the German social security and unemployment records. It is described in more detail in Jung and Kuhn (2009). The data set excludes self-employed and civil servants, but nevertheless covers 80% of the West German labor force. In Appendix D, we document that aggregated wage data according to this panel data set follow true aggregate wages very closely.

We start by constructing the number of workers that recently had a non-employment spell. To do this, we first determine whether the worker was employed T months ago for T equal to 24 and 36 months.²¹ If he was employed T months ago and the total number of days the worker was "not employed" during the last T months was less than 30 days, then we do not include him.²² This means that short periods in between jobs are not counted as non-employment spells.

Figure 7 plots the cyclical component of the fraction of total wages generated by workers with a recent non-employment spell together with the cyclical components of the unemployment rate for the two different values of T considered.²³ One would expect the fraction earned by workers with a recent non-employment spell to increase when the unemployment rate decreases. If the wages of these workers are very low relative to average wages, however, then this fraction would not increase by much. This is not what we find at all. The graph makes clear that there is a strong negative correlation between the unemployment rate and the fraction of wages earned by workers with a recent non-employment rate drops, the fraction of wages earned by workers with a recent non-employment spell. As the unemployment spell displays a substantial increase indicating that the value added of these additional workers is non-trivial.

Figure 8 is the equivalent graph, but with the unemployment rate replaced with the

 $^{^{21}}$ To be precise, we check whether he was employed in the reference week T months ago.

²²The data set keeps track of an experience variable that counts the total number of days a worker has worked. By looking at the increase in this variable, it is easy to check which fraction of a particular period a worker was actually working.

 $^{^{23}}$ The graphs in the main text are based on a smoothing coefficient of the HP filter equal to 10^5 . Graphs based on a smoothing coefficient of 1,600 are given in Appendix D.

cyclical component of the total deflated wage bill according to this data set. The graph makes clear that the share of total value added earned by workers with a recent nonemployment spell is also strongly and positively correlated with total wages.

Let n_+ (n_-) be the average number of workers during an expansion (recession) that had a non-employment spell in the last T months, and let y_+ (y_-) the corresponding average wage. In our theory, cyclical workers are not employed in a recession and are employed during a boom. That is, we would have $n_- = 0$. In reality, there are of course always workers that recently were non-employed, for example, new entrants to the labor force. Consequently, we think of $n_+ - n_-$ as the number of cyclical workers.

Ideally, we would use data on the number of projects and the value added by project, but we will use changes in the number of workers as an estimate for changes in the number of projects and changes in wages as an estimate for changes in total value added generated. The increase in output generated by the increase in n is equal to

$$\Delta = y_+ (n_+ - n_-) = f_+ Y_+ - f_- Y_- - n_- (y_+ - y_-),$$

where

$$f_+ = \frac{y_+ n_+}{Y_+}$$
 and $f_- = \frac{y_- n_-}{Y_-}$

and Y_+ (Y_-) is total aggregate output generated in a boom (recession). Let

$$1 + 2g_f = \frac{f_+}{f_-}$$

and similarly for the other variables. This means that

$$\Delta = [f_+(1+2g_Y) - f_- - 2g_y f_-] Y_-.$$

Thus, the fraction of the total increase in output, $Y_+ - Y_-$, generated by the cyclical workers, r, is equal to

$$r = \frac{[f_{+}(1+2g_{Y}) - f_{-} - 2g_{y}f_{-}]Y_{-}}{Y_{+} - Y_{-}}$$

$$= \frac{[f_{+}(1+2g_{Y}) - f_{-} - 2g_{y}f_{-}]}{2g_{Y}}$$

$$\geq \frac{[f_{+}(1+2g_{Y}) - f_{-} - 2g_{Y}f_{-}]}{2g_{Y}},$$
(31)

where the inequality follows from the assumption that the growth rate of the value added generated by workers with a recent non-employment spell, y, is less than the growth rate of total output Y.

$$\frac{Y_{\text{C-TEMP}}}{Y} = r \frac{Y_{+} - Y_{-}}{Y}.$$
(32)

The results are reported in Table 1. To understand our calculations consider the case when T is equal to 36, the smoothing coefficient of the HP filter is equal to 10^5 , and German data are used. Since the mean of f is equal to 20.22% and the standard deviation of the HP-filtered series equal to 1.26, we get that $f_+ = 0.2022 + 0.0126$ and $f_- = 0.2022 - 0.0126$ and since the standard deviation of HP-filtered output is equal to 2.89% we get that $g_y = 0.0289$.²⁴ This means that r is equal to 45%, that is roughly half of the increase in total value added is due to the value added generated by the $n_+ - n_-$ cyclical workers. The resultant estimate of $Y_{\text{C-TEMP}}/Y$ is equal to 2.62%.

For the US, we do not have estimates for f_+ and f_- , and, thus, not for r. To obtain estimates for the US, we either assume that the values for f_+ and f_- are equal to those of Germany or that the value of r is equal to the German value. When the smoothing coefficient is equal to 1,600, then the US estimates are somewhat above those of Germany and when the smoothing coefficient is equal to 10^5 , then they are somewhat lower.

The estimates for $Y_{\text{C-TEMP}}/Y$ vary from 1.22% to 2.62%. As our preferred estimate we take the midpoint, 1.92%, but we also consider the two extreme values.²⁵

One advantage of using West-German data is that wages turn out to be very flexible. Jung and Kuhn (2009) document that the median wage in this data set moves basically one-to-one with GDP. Moreover, they show that in this data set the wages of the new jobs are somewhat *less* cyclical, which means that our estimate may even be an underestimate.

5.2 Structural parameters

The value of β is set equal to the standard value of 0.99. In setting π , we follow Krusell and Smith (1998) and set π equal to 0.875, which means that the expected duration of a

²⁴If a variable can take on only two values that occur with equal probability, then a standard deviation

of Z% implies a difference between the boom and the recession value equal to $2 \times Z\%$.

²⁵In Appendix E, we use some very simple rule-of-thumb calculations and obtain very similar estimates.

boom and a recession is equal to 8 quarters.

The parameter μ_{ϕ_p} indicates how productive workers of cyclical projects are outside the relationship. Shimer (2005) uses a value of leisure that is equal to 40% of market production. But his measure refers to all benefits that an unemployed worker receives, whereas our measure has to exclude any type of transfer such as unemployment benefits; it should only include the value of home production and possibly the utility gain if a worker does not work. How productive workers are at home is likely to differ a lot across workers. Our measure applies to workers that are in principle willing to work, but are at times forced out of work because of inefficiencies. Because of the first characteristic, they are not likely to be the most skilled in tasks like child care and housekeeping. As our benchmark value, we assume that half of the number used by Shimer (2005) consists of actual net benefits generated by an unemployed worker, that is, $\mu_{\phi_p} = 0.2$. The value used by Shimer (2005) is considered to be too low by some.²⁶ Hall (2006) estimates the flow value of leisure forgone to be equal to 43%, and we consider this as an alternative estimate.²⁷

The parameter ω_e controls the revenues the entrepreneur receives. Using US data from 1948 to 2008 we find that proprietor's income was on average 10% of personal income.²⁸ In our analysis, it is convenient to define the revenues of the entrepreneur as a fraction of the surplus, with the surplus defined from the social planner's point of view. If the outside option is 20% of production and the entrepreneur receives 10% of production, then the entrepreneur receives 12.5% as a fraction of the surplus, which is our benchmark value for ω_e . In addition, we consider values for ω_e equal to 0.05, 0.20, and 1. A value of ω_e equal to 1 is obviously not realistic, but it is interesting to look at this value that implies efficient entry, also because the cost of business cycles are lowest for this value.

The parameter ρ controls the rate of exogenous destruction. That is, $1/(1-\rho)$ is the

²⁶See Mortensen and Nagypál (2007) for a discussion.

²⁷Personally we value our leisure a lot less, but maybe we are overestimating our market production levels.

²⁸Dividend income was on average 3.6%. If dividend income is only a return for providing capital to the firm, then it should not be included to obtain an estimate for ω_e , but if the firm founder is awarded shares for entrepreneurial activities, then it should be.

expected duration of the project if the duration is not affected by business cycle considerations. Obviously, there are many types of projects with different expected lifetimes. If the project is a company, a plant, a mine, or a ship, then the expected duration could easily exceed 10 years or 40 quarters, which would correspond to a value of ρ equal to 0.975. But there are also projects with much shorter durations. The values of ρ considered are equal to 0.875, 0.9167, and 0.975, which correspond with expected durations of respectively 2, 3, and 10 years.

Finally, we have to choose a value for Φ_+ (which by symmetry also pins down Φ_-).²⁹ The value of Φ_+ has almost no effect on the results. We set $\Phi_+ - 1$ (and thus $1 - \Phi_-$) equal to 0.007, which is a standard value in the literature.³⁰

5.3 R_{c} parameter

An important part of our calculations is to relate the mass of C-PERM-Loss projects, which we do not observe, to the mass of C-TEMP-Loss projects, which we do. By construction, the output levels are the same in these two areas, so the question is about the distribution of ϕ_c .

Economic theory. Economic theory pins down the boundaries of the two areas. In particular, the lower bound of ϕ_c in the C-TEMP-Loss area is equal to 0, since negative entry costs are not plausible, the upper bound of ϕ_c for C-TEMP-Loss, $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, is pinned down by the free-entry condition in the world *with* business cycles, and the upper bound of ϕ_c for C-PERM-Loss, $\tilde{\phi}_{c,no-bc}(\phi_p)$, is pinned down by the free-entry condition in the world *without* business cycles. Note that the lower bound of C-PERM-Loss is equal to the upper bound of C-TEMP-Loss. Although these boundaries impose restrictions, they are not the end of the story.

²⁹The magnitudes of Φ_+ and Φ_- are, together with the distribution of ϕ_c and ϕ_p , important in determining $Y_{\text{C-TEMP}}(\Phi_+)$, but in our calibration exercise we obtain a direct empirical estimate for $Y_{\text{C-TEMP}}(\Phi_+)$.

³⁰This implies that $(\Phi_+ - \Phi_-)/\Phi_- = 1.4\%$. The standard deviation of total output is roughly equal to $0.5(Y_{\text{C-TEMP}}(\Phi_+)/Y + (\Phi_+ - \Phi_-)/\Phi_-)$. If $(\Phi_+ - \Phi_-)/\Phi_-$ is equal to 1.4%, then this standard deviation varies between 1.7% and 2.7% for our range of estimates for $Y_{\text{C-TEMP}}(\Phi_+)/Y$, which matches closely the observed estimates of 1.66% and 2.55% discussed on page 30.
These restrictions imply that we do not have to consider points in the distribution of ϕ_c outside these bounds, but the boundaries of the distribution of ϕ_c could be *inside*. For example, the lower bound of ϕ_c could be above zero. By assuming that ϕ_c can take on values as low as zero, which we do in our benchmark calibration, we underestimate the costs of business cycles. Similarly, the upper bound of the distribution of ϕ_c could be below $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. If the upper bound of the distribution of ϕ_c does not reach $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, then the assumption that ϕ_c does take on values as high as $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ would imply that we are overestimating the costs of business cycles, . Therefore, this requires a bit more discussion.

How high can ϕ_c be? The question is whether cyclical projects, i.e., projects for which $\tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$ below , have values of ϕ_c above $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p,\Phi_+)$ and whether the theoretical upper bound of $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ limits any existing projects from entering. The difficulty is that projects with a value of ϕ_c in between $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$ and $\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)$ are never observed in a world with business cycles. To understand whether values of ϕ_c as high as $\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)$ are plausible, consider projects with a value of ϕ_p just above $\phi_{p,bc}(\Phi_{-})$, that is, projects that are just not cyclical. These projects cannot be that different in their characteristics from the projects that are cyclical, given that they have a similar ϕ_p value.³¹ Instead of answering the question whether projects with a value of ϕ_p below $\tilde{\phi}_{p,bc}(\Phi_-)$ have values of ϕ_c have values of ϕ_c as high as $\tilde{\phi}_{c,no-bc}(\phi_p)$, we first contemplate whether projects with a value of ϕ_p just above $\tilde{\phi}_{p,bc}(\Phi_-)$ have values of ϕ_c as high as $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. For these projects, the cut-off point for ϕ_c is equal to $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ in a boom with $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) > \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_p)$. If there are values of ϕ_c above $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$, then there are values of ϕ_c above $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ and if this is true for projects with values of ϕ_p just above $\phi_{p,bc}(\Phi_-)$, then it should be true for values of ϕ_p below $\phi_{p,bc}(\Phi_-)$ as well, unless there are sharp changes in the distribution.

Suppose that none of the projects with a value of ϕ_p just above $\tilde{\phi}_{p,bc}(\Phi_-)$ have a value of ϕ_c above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$. This would imply that for projects with marginal ϕ_p

³¹None of the cyclical projects has a value of ϕ_p that is much below $\tilde{\phi}_{p,bc}(\Phi_-)$, since fluctuations in Φ_p are small.

values, entry is always profitable. That is, there *never* is an entrepreneur that decides not to create the project, because its entry costs are too high even though the project's productivity is marginal. We would think that life cannot be this good. We hope to have convinced the reader by now that it is not implausible to assume that the distribution of ϕ_c reaches values as high as $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ for projects with a value of ϕ_p just above and just below $\tilde{\phi}_{p,\text{bc}}(\Phi_-)$.

Distribution of ϕ_c . Knowing that there may be some projects with values of ϕ_c as high as $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$, only provides limited information, however, about the distribution. The advantage of our calibration procedure is that we do not need to know the exact distribution of ϕ_c . We do, however, need some information. In particular, we need to know the fraction of the mass of projects with values of ϕ_c in between $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ and $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ to the mass of projects with values of ϕ_c below $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$. We argue that information about this ratio is provided by the ratio of the length of $[\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+), \tilde{\phi}_{c,\text{no-bc}}(\phi_p)]$ to the length of $[\underline{i^*}\phi_p, \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)]$, which is precisely pinned down by theory. The coefficient $R_{\rm C}$ relates this theoretical ratio to the ratio of probabilities we are interested in. Our benchmark value of $R_{\rm C}$ is equal to 1. This value would be implied by the assumption that the distribution of ϕ_c is uniform for values of ϕ_c below $\phi_{c,\text{no-bc}}(\phi_p)$, but other distributional assumptions can lead to the same value.

As a robustness check, we also consider a value of $R_{\rm C}$ equal to 0.5. This implies that above $\tilde{\phi}_{c,\rm bc}(\phi_p, \Phi_+)$ the mass of projects is spread more thinly than below $\tilde{\phi}_{c,\rm bc}(\phi_p, \Phi_+)$. Since the costs of business cycles are higher if there are more projects with values of $\phi_{c,\rm bc}(\phi_p, \Phi_+)$, i.e., the projects in the C-PERM-Loss area, a lower value of $R_{\rm C}$ reduces the estimate of the cost of business cycles.

6 Quantitative assessment.

Table 2 reports the first set of results. This table considers different values of ρ , which affect the expected lifetime of a project, different values of $Y_{\text{C-TEMP}}$, which is the output generated by cyclical projects, and different values of ω_e , the share of the surplus received

by the entrepreneur. The other parameter values are set equal to their benchmark values.

When $Y_{\text{C-TEMP}}/Y$ is equal to our preferred value of 1.92%, then the costs of business cycles are equal to 0.56%, 0.86%, and 2.45% of output for values of ρ equal to 0.875, 0.9167, and 0.975, respectively. The expected lifetime of a project, i.e., $1/(1 - \rho)$, is very important, but even when the expected lifetime is only 2 years, then the costs of business cycles for our risk neutral agents are already equal to 0.56%, which is still substantially higher than the 0.05% that Lucas (2003) reports for the standard model when agents have log preferences. The reason the expected duration is so important is that in our framework business cycles shorten the expected duration of a project; if projects do not last very long anyway, then this is less important.

Changing the value of $Y_{\text{C-TEMP}}/Y$ simply changes the business cycles measures proportionally. If $Y_{\text{C-TEMP}}/Y$ is set equal to 2.62%, the highest value in our set of our empirical estimates, then the costs of business cycles range from 0.77% to 3.34%.

Relatively large changes in the value of ω_e have little effect on the costs of business cycles. For example, when ω_e increases from 0.125 to 0.2, then the cost of business cycles drop from 2.45% to 2.35% (when $Y_{\text{C-TEMP}}/Y = 1.92\%$ and $\rho = 0.975$).

For the three lower values of ω_e considered (all less than or equal to 0.2), business cycles are costly mainly through changing output levels not through changing the amount of entry costs paid. In fact, the loss of the C-TEMP-Loss projects is almost offset by the gain of the C-TEMP-Gain projects. In particular, when $Y_{\text{C-TEMP}}/Y = 1.92\%$, $\omega_e = 0.125$, and $\rho = 0.975$, then the benefit of business cycles for the C-TEMP-Gain projects is equal to 0.36% and the loss for the C-TEMP-Loss projects is equal to 0.39%. This means that the costs of business cycles is driven almost completely by the loss for the C-PERM-Loss area. Note that all costs are expressed as the cost per person when the cost is spread over the whole population. For the individual agents involved the costs (and benefits) are obviously much larger. As a fraction of their own average (market) productivity level, the gains are equal to 37.6% for the C-TEMP-Gain projects and the losses are 40.7% and 73.9% for the C-TEMP-Loss projects and the C-PERM-Loss projects, respectively.

The results in Table 2 are based on the assumption that $R_{\rm C}$ is equal to 1, which means

that the concentration of projects above the cut-off value for ϕ_c in the presence of business cycles is similar to the concentration of projects below this cut-off value. Table 3 reports the results when $R_{\rm C}$ is equal to 0.5, which implies that the concentration of projects above the cut-off value is substantially less than the concentration below the cut-off value. This would cut the cost of business cycles in half.

A more interesting variation is an increase in the lower bound on entry costs. The results in Table 2 are based on the assumption that the lower bound is equal to zero. Table 4 reports the results when \underline{i} is equal to 0.005, which means that on average firms would pay 0.5% of their output level on entry costs. This leads to a substantial increase in the costs of business cycles for our benchmark parameter values. For this value of \underline{i} , our preferred estimates range from 0.62% (when $\rho = 0.875$) to 3.54% (when $\rho = 0.975$).

Finally, Table 5 reports the results when the net benefits an unemployed worker generates for society are equal to 43% of his market productivity, which is based on the estimate of Hall (2006). This lowers the costs of business cycles, but they remain substantial. Our preferred estimates now range from 0.39% (when $\rho = 0.875$) to 1.73% (when $\rho = 0.975$).

If we take our preferred numbers in each of the tables, then we find that the costs of business cycles vary from 0.29% to 3.54%.

6.1 Timed-entry projects

If entry is efficient, i.e., when $\omega_e = 1$, then fluctuations have a positive effect on the timedentry projects, but we always find this effect to be quantitatively very small. If entry is not efficient, then fluctuations could have a negative net effect on the timed-entry projects. Since the formulas are tedious and its importance small, we only provide some intuition for these findings in the main text.³²

If $\omega_e = 1$ and $\phi_p \Phi_- > \chi$, then the entry decisions for the timed-entry projects in the competitive equilibrium are identical to those made by the social planner. Thus, business cycles are welfare improving for *all* timed-entry projects, as explained in Section 3.2. Although business cycles have a positive effect on the welfare of agents with a timed-entry

³²See Appendix B for a detailed discussion.

project, the effects are small because aggregate fluctuations are small and because these projects never generate a large surplus (given their high value of ϕ_c).

It may be in the entrepreneur's best interest to postpone entry during a recession for the same reason that it is for the social planner. When $\omega_e < 1$, then the entrepreneur ignores that by postponing entry he also postpones the worker getting his share of the surplus instead of unemployment benefits.

For $\omega_e = 0.125$ and $\rho = 0.9167$, we calculate the welfare gain of business cycles for agents in the TE-Gain area and the welfare loss for agents in the TE-Loss area. Expressed as the permanent percentage change of the agent's market production,³³ we find an average increase in welfare equal to 54.5% for the TE-Gain projects, and an average decrease in welfare equal to 15.6% for the TE-Loss projects. It makes sense that the cost of business cycles for an agent in the TE-Loss area is smaller than the gain for an agent in the TE-Gain area. The reason is that with business cycles the agent in the TE-Gain area creates valuable projects that are never created in the world without business cycles, whereas business cycles only causes worthwhile projects in the TE-Loss area to be postponed. Although the losses are smaller than the gains, there still could be a net loss for the TE area as a whole, because there are more projects in the TE-Loss than in the TE-Gain area. This is related to the fact that the timed-entry cut-off value of ϕ_c in a recession is more below the no-business-cycles cut-off value, than the timed-entry cut-off value in a boom is above it, as explained in detail in Section 3.2. In particular, $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ is 0.25% above $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$ and $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-)$ is 0.88% below $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. Although there are more TE-Loss projects than TE-Gain projects, the total gains still dominate the total losses, but by a very small amount. In particular, as a fraction of aggregate GDP we find that the gains of the TE-Gain projects are equal to 0.00139% and the losses of the TE-Loss projects are equal to 0.00138%.

³³To make the gains and the losses comparable we calculate them relative to the same benchmark, namely the agent's market production level, ϕ_p . But note that in a world without business cycles, unemployed agents in the TE-area only earn $\mu_{\phi_n}\phi_p$.

7 Alternative interpretations

The framework presented in Section 3 is very stylized and the variables have been interpreted in a very specific way. In this section, we want to make clear that the channel highlighted in this paper can be expected to operate in a wide class of models.

In the main text, we focused on business cycles, but our framework predicts that all fluctuations are costly, not just business cycles. That is, $\Phi_{p,t}$ could also be interpreted as a sectoral, as a regional, and even as an idiosyncratic shock. Whenever there are entry costs and inefficiencies, then one can expect fluctuations to be costly. Moreover, if the presence of business cycles goes together with the presence of say sectoral shocks, then the costs of business cycles are obviously larger than the ones presented here.

The friction we use in the theoretical framework is very simple and ad hoc. In Appendix A.1, we show that the contractual fragility framework of Ramey and Watson (1997) can be used to derive exactly this restriction and we show in Appendix A.2 that a friction on obtaining financing implies even larger costs of business cycles.

In our framework, we interpret ϕ_p as idiosyncratic and $\Phi_{p,t}$ as aggregate productivity. But the channel we put forward would also be present in a New-Keynesian model in which firms face limitations in adjusting their prices and business cycles are caused by aggregate demand instead of aggregate productivity shocks. Idiosyncratic productivity could be the same across firms; instead, heterogeneity could be introduced by differences in the value of χ or other firm characteristics that affect the firm's surplus level, such as, for example, the ability to adjust prices.

The friction we focus on is itself constant, but its impact is countercyclical, that is, it affects firms to a lesser extend in a boom. What matters is that the impact of the friction is time-varying, not that it is pro- or countercyclical. Fluctuations are costly because they shorten the duration of matches; it does not matter whether the inefficient breakup occurs in a boom or a recession. This is an important observation because some types of frictions are procyclical. For example, it may be easier to prevent a worker from shirking in a recession when his outside option is bad. Although our theoretical framework does not rely on the effect of the friction being pro- or countercyclical, our calibration procedure is based on the assumption that the effect of the friction is countercyclical.

A Theories consistent with our inefficiency

In the main text, we simply imposed the friction that firms can only operate if

$$\phi_p \Phi_{p,t} \ge \chi. \tag{33}$$

Thus, firms need to be able to generate enough revenues and this is easier when $\Phi_{p,t}$ is higher. In this section, we show that one can obtain such a requirement on the minimum production level in two very different theoretical frameworks. The first is the contractual fragility framework of Ramey and Watson (1997) and the second is a framework in which the entrepreneur has to borrow to finance the investment made and borrowing is subject to a standard agency problem. Section A.1 discusses the contractual fragility framework and Section A.2 discusses the framework with the financial friction. The contractual fragility specification results in a condition that is exactly equal to the one given in Equation (33). The financial friction leads to a slightly different specification in which one can expect the costs of business cycles to be even higher.

A.1 Contractual fragility of Ramey and Watson (1997)

In the model described in this section, the physical environment is identical to the one described in Section 3.5 *if* the agents cooperate, i.e., do not "cheat" on each other. In Ramey and Watson (1997) both participants have the option to cheat, but to make our point it is sufficient that just the entrepreneur has an option that may be privately attractive, but is inefficient from the relationship's point of view. For example, the entrepreneur may deviate from the original business plan and choose one that is riskier, but gives him personally more prestige. It is also possible that he diverts funds to himself or acquaintances. The total current-period private benefits the entrepreneur can obtain by cheating are equal to χ_e and these consist of the actual funds the entrepreneur receives, $\chi_{e,\$}$, plus any non-pecuniary benefits, $\chi_e - \chi_{e,\$}$. The entrepreneur obtains pecuniary benefits by extracting a larger share of the resources than agreed upon, for example, by not paying out overtime, by not paying out bonuses, or by not promoting workers. Increases in prestige or human capital and improvements of the entrepreneur's network are examples of non-pecuniary benefits. If the entrepreneur chooses the alternative business plan, then output is equal to

$$\phi_{\chi}\phi_p\Phi_{p,t},\tag{34}$$

and the extra disutility for the worker of working is equal to $\chi_w \ge 0.^{34}$ For the alternative choice to be inefficient, it must be the case that

$$(1 - \phi_{\chi})\phi_p \Phi_{p,t} > \left(\chi_e - \chi_{e,\$}\right) - \chi_w.$$
(35)

That is, the loss in the project's revenues are larger than the net utility gain (when $\phi_{\chi} < 1$) or the gain in revenues are smaller than the net utility loss (when $\phi_{\chi} > 1$).

An entrepreneur is only willing to choose the original business plan if

where w_t is the wage rate of the worker under the original business plan.³⁵ For simplicity, we assume that the entrepreneur's choice to cheat does not affect his continuation value.³⁶ Equation (36) can, then, be written as

$$\phi_p \Phi_{p,t} - w_t \ge \chi_e. \tag{37}$$

The current period benefits of the worker when he is not employed are equal to μ^* . A ³⁴The alternative business plan may not only require more effort from the worker, but may also force him to move and may even result in dismissal.

 $^{^{35}}$ Under the alternative business plan, the worker receives $\phi_{\chi}\phi_{p}\Phi_{p,t}-\chi_{e,\$}.$

³⁶In the unfair world we live in, it is probably not an unrealistic assumption that the employer can impair his workers' well being without this having little effect on the options available to him in the next period. The possibility that the entrepreneur's continuation value is affected is discussed at the end of this section.

worker is only willing to participate in a project if

$$w_{t} + \beta \mathbf{E}_{t} \left[\rho N_{w}(\phi_{p}, \phi_{c}, \Phi_{p,t+1}) + (1 - \rho) U_{w}(\phi_{p}, \phi_{c}, \Phi_{p,t+1}) \right]$$

$$\geq$$

$$\mu^{*} + \beta \mathbf{E}_{t} \left[U_{w}(\phi_{p}, \phi_{c}, \Phi_{p,t+1}) \right],$$

$$(38)$$

where w_t is the wage rate of the worker, $N_w(\phi_p, \phi_c, \Phi_{p,t+1})$ is the discounted value of current and future benefits that accrue to the worker when he starts next period in a relationship, $U_w(\phi_p, \phi_c, \Phi_{p,t+1})$ the discounted value of current and future benefits that accrue to the worker when he starts next period not being in a relationship. Since the matching probability is equal to 1, it does not matter whether you leave period t in a relationship or not; in period t+1 the worker still has the freedom to choose what is best. Consequently, $N_w(\phi_p, \phi_c, \Phi_{p,t+1}) = U_w(\phi_p, \phi_c, \Phi_{p,t+1})$ and the condition given in Equation (38) can simply be written as

$$w_t \ge \mu^*. \tag{39}$$

A necessary and sufficient condition to satisfy the participation condition of the worker and the no-cheating condition of the entrepreneur is given by^{37}

$$\phi_p \Phi_{p,t} \ge \chi_e + \mu^*. \tag{40}$$

If we let $\chi = \chi_e + \mu^*$, then we get exactly the condition in Equation (3) used in the main text to model the friction.

In the remainder of this section, we provide some more intuition on why contractual fragility makes production impossible when $\phi_p \Phi_{p,t}$ is less than χ . Consider the case when

$$\mu^* < \phi_p \Phi_{p,t} < \chi_e + \mu^*. \tag{41}$$

If an existing project does not operate when $\phi_p \Phi_{p,t} > \mu^*$, then this is not efficient. However, for these values of $\phi_p \Phi_{p,t}$ it is not possible to both pay the entrepreneur enough so that he will not choose the alternative business plan and pay the worker enough so

³⁷Since the entrepreneur can never earn any benefits outside of a relationship, there is no participation constraint for the entrepreneur.

that his wage exceeds μ^* . In this case, the project will not be operated. The idea of the contractual fragility of Ramey and Watson (1997) is that no credible contracts can be written that will prevent the entrepreneur from choosing the alternative business plan. The entrepreneur may promise to pay the worker a wage above μ^* and promise that he will not go for the alternative business plan and afflict the worker with the utility loss worth χ_w , but if $\phi_p \Phi_{p,t} < \chi_e + \mu^*$, the entrepreneur cannot both pay the worker more than μ^* and satisfy his own incentive compatibility condition. The worker knows that the entrepreneur will face this dilemma and chooses not to work for this entrepreneur.

The beauty of the contractual fragility framework of Ramey and Watson (1997) is that it allows for the possibility that it is inefficient not to operate, but nevertheless not possible to write contracts such that the project will operate. If the project is not productive, then the entrepreneur and the worker only generate μ^* in resources, which is less than what they could generate if they would operate the project. Moreover, from a social welfare point of view, they generate even less, because $\mu < \mu^*$.

The framework described here results in a restriction that is identical to the one used in the main text. There are, of course, more general specifications. For example, The values of χ_e and χ_w could depend on ϕ_c . A higher entry costs means larger investments in the firm and possibly more options for the entrepreneur to extract resources from the firm. The financial friction also predicts that χ depends positively on ϕ_c and we will argue that the costs of business cycles are higher when there is such positive dependence. But if the entrepreneur cannot continue operating his project after having exploited the worker and has to pay part of ϕ_c again, then this would dampen and possibly even overturn this effect.

A.2 Financial friction

There are many different models with financial frictions. In this section, we develop a model in which the friction leads to inefficient production decisions. The condition deviates somewhat from the ad hoc friction imposed in the main text, but in a way that increases the costs of business cycles. Suppose that the entrepreneur does not have any net worth and has to borrow to finance the entry costs. Also, we assume that the entrepreneur simply rolls over this debt every period until he stops producing. Let the interest rate be equal to r, which includes a premium for the fact that producers that stop producing default on the debt. For simplicity assume that all firms face the same interest rate, for example, because lenders cannot distinguish between different types of borrowers.³⁸ A standard financial friction is a limit on the amount that can be collateralized. In particular, assume that the entrepreneur can extract χ_e when he defaults on his loan. Examples of assets of the firm that cannot be collateralized are human capital, the value of the good will created, and the value of the networks build up.

The entrepreneur would not default on his loan if

$$\phi_p \Phi_{p,t} - w_t - r\phi_c + \beta \mathbf{E}_t \left[\rho N_e(\phi_p, \phi_c, 1, \Phi_{p,t+1}) + (1 - \rho) N_e(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right]$$

$$\geq \chi_e + \beta \mathbf{E}_t \left[N_e(\phi_p, \phi_c, 0, \Phi_{p,t+1}) \right]$$

$$(42)$$

But if the entrepreneur can borrow the funds to finance the entry costs, then the value of the project's revenues that accrue to the entrepreneur before or after entry costs have been paid are identical, that is, $N_e(\phi_p, \phi_c, 1, \Phi_{p,t+1}) = N_e(\phi_p, \phi_c, 0, \Phi_{p,t+1})$ and we can write the last equation as

$$\phi_p \Phi_{p,t} - w_t - r\phi_c \ge \chi_e. \tag{43}$$

Since $w_t \ge \mu^*$, it must be true that

$$\phi_p \Phi_{p,t} \ge \chi_e + \mu^* + r\phi_c \tag{44}$$

and if we let $\chi = \chi_e + \mu^*$, then we get

$$\phi_p \Phi_{p,t} \ge \chi + r \phi_c. \tag{45}$$

³⁸To ensure that even the demand borrowed does not reveal the type of the borrower, one could assume that one can scale each project. An entrepreneur with a high value for ϕ_c can then invest in say half a project and, thus, hide that he has a project with a high value for ϕ_c .

This condition differs from the one we used in the main text, because it implies an upward shaping cut-off curve in the (ϕ_p, ϕ_c) space, whereas the condition that $\phi_p \Phi_{p,t} \ge \chi$ implies a vertical cut-off for the production decision. There are plausible modifications of the model that would steepen the curve and that would get the prediction of this model closer to the vertical curve used in the main text. For example, it may be possible that after a default the entrepreneur may not be able to borrow the full entry cost.

More importantly, an upward sloping instead of a vertical curve actually reinforces our channel, that is, the costs of business cycles are actually *larger* when the operating cut-off curve is upward sloping. In all our calibrations, we make the assumption that the projects in the C-PERM-Loss area are as productive as the projects in the C-TEMP-LOSS area, but if the operating cut-off curve is upward sloping then the projects in the C-PERM-Loss area are *more* productive. If this is true, then we are actually underestimating the output loss due to the projects in the C-PERM-Loss area not being created. Of course, the calibration would be more difficult if the operating cut-off is upward sloping, but since the costs of business cycles are already shown to be substantial with a vertical operating cut-off, we have not attempted such a calibration exercise.

B TIMED-ENTRY projects

In Section B.1, we give the formulas for NPVs and cut-off levels as well as definitions used to state the assumptions. In Section B.2, we specify the assumptions to derive our costs of business cycles measure. In Section B.3, we derive an analytic expression for the cost of business cycles due to timed-entry projects. In Section B.4, we document that the welfare consequences of business cycles are very small for timed-entry projects.

B.1 Formulas for timed-entry projects

B.1.1 NPV formulas for timed-entry projects

Timed-entry projects have a productivity level that is high enough to overcome efficiency problems, even in a recession. Thus, an existing project, i.e., one with e = 1, will continue

to operate even if the economy gets into a recession. Projects in the TE-Gain area are always idle in a world without business cycles, because their value of ϕ_c is just above the cut-off value. In a world with business cycles, however, the entrepreneur would choose to enter during a boom. The reason is that starting out in a boom raises the expected discounted value of Φ_p , even though the unconditional expectation of $\Phi_{p,t}$ is equal to the constant value of aggregate productivity in the world without business cycles. Projects in the TE-Loss area enter in a world without business cycles, because the value of ϕ_c is just low enough. In a world with business cycles, the entrepreneur of such a project would obviously enter during a boom, but he chooses not to enter during a recession. Note that projects in both TE areas would continue to operate after they have entered and paid the entry cost, independent of the value of $\Phi_{p,t}$, i.e., projects in this area only stop operating when they are hit by an exogenous destruction shock.

In a world without business cycles, projects in the TE-Gain area would never operate and projects in the TE-Loss area would always operate. Thus, we would get

$$N_{\text{no-bc}}(\phi_p, \phi_c, 0) = \frac{\mu_{\phi_p} \phi_p}{1 - \beta}$$
for $\phi_p > \tilde{\phi}_{p,\text{bc}}(\Phi_-)$ and $\tilde{\phi}_{c,\text{no-bc}}(\phi_p) < \phi_c \le \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$

$$(46)$$

and

$$N_{\text{no-bc}}(\phi_p, \phi_c, 0) = -\phi_c + N_{\text{no-bc}}(\phi_p, \phi_c, 1)$$

$$= -\phi_c + \phi_p + \beta \left(\rho N_{\text{no-bc}}(\phi_p, \phi_c, 1) + (1 - \rho) N_{\text{no-bc}}(\phi_p, \phi_c, 0)\right)$$

$$= \frac{\phi_p - (1 - \beta \rho)\phi_c}{1 - \beta}$$
for $\phi_p > \tilde{\phi}_{p,\text{bc}}(\Phi_-)$ and $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p).$

$$(47)$$

In a world with business cycles, the values of $N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+)$ and $N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-)$

can be solved from the following linear system:

$$N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) = -\phi_c + N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_+), \qquad (48a)$$

$$N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_+) = \phi_p \Phi_+ + \beta \begin{bmatrix} \pi \rho N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_+) \\ \pi (1-\rho) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) \\ (1-\pi) \rho N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_-) \\ (1-\pi) (1-\rho) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) \end{bmatrix}, \qquad (48b)$$

$$N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) = \mu_{\phi_p} \phi_p + \beta \begin{bmatrix} \pi N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) \\ (1 - \pi) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) \end{bmatrix}, \qquad (48c)$$

$$\begin{bmatrix} \pi \rho N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_-) \end{bmatrix}$$

$$N_{\rm bc}(\phi_{p},\phi_{c},1,\Phi_{-}) = \phi_{p}\Phi_{-} + \beta \begin{bmatrix} \pi(1-\rho)N_{\rm bc}(\phi_{p},\phi_{c},0,\Phi_{-}) \\ (1-\pi)\rho N_{\rm bc}(\phi_{p},\phi_{c},1,\Phi_{+}) \\ (1-\pi)(1-\rho)N_{\rm bc}(\phi_{p},\phi_{c},0,\Phi_{+}) \end{bmatrix}, \quad (48d)$$

for $\phi_{p} \geq \tilde{\phi}_{p,\rm bc}(\Phi_{-})$ and $\tilde{\phi}_{c,\rm bc}(\phi_{p},\Phi_{-}) < \phi_{c} \leq \tilde{\phi}_{c,\rm bc}(\phi_{p},\Phi_{+}).$

The first equation follows from the fact that timed-entry projects are created in a boom by definition, which means that an idle firm would pay the entry cost and become an operating firm. That is, if entry occurs, it is always the case that the difference in the value between an idle and an operating project is equal to the entry cost.

In a recession, entry does not occur, again by definition of a timed-entry job. If the value of ω_e is equal to 1, then the decision not to enter coincides with the one the social planner would make and for our NPVs, that are calculated from society's point of view, we have

$$N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_-) - \phi_c \ge \mu + \beta E N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1})$$
(49)

when entry occurs and

$$N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_-) - \phi_c \le \mu + \beta E N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_{p,t+1})$$
(50)

when entry does not occur. Note that $N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-)$ represents the current and future *joint* benefits based on the choice of not entering in a recession. We also consider the case when $\omega_e < 1$, in which case decisions not to enter do not necessarily coincide with the one the social planner would make, in which case the last inequality does not necessarily hold.

B.1.2 Cut-off levels for timed-entry projects

This is the area where $\phi_p \geq \tilde{\phi}_{p,bc}(\Phi_-)$ and entry depends on aggregate conditions. There are three cut-off levels we have to determine. The first is the cut-off level of ϕ_c when there are no business cycles and this is equal to the one given in Equation (18). The second cut-off level, $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$, specifies the highest value of ϕ_c so that entry always occurs, even in a recession. The value of $\tilde{\phi}_{c,bc}(\phi_p, \Phi_-)$ satisfies the condition that when $\Phi_p = \Phi_-$ the entrepreneur is indifferent between immediate entry and waiting with entry until $\Phi_p = \Phi_+$. That is, it can be solved from the following system.

$$N_{e}(\phi_{p}, \tilde{\phi}_{c, bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) = \beta \begin{bmatrix} \pi N_{e}(\phi_{p}, \tilde{\phi}_{c, bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) + \\ (1 - \pi) N_{e}(\phi_{p}, \tilde{\phi}_{c, bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{+}) \end{bmatrix}$$
(51a)

$$N_e(\phi_p, \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-), 0, \Phi_-) = -\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) + \omega_e \phi_p(\Phi_- - \mu_{\phi_p})$$
(51b)

$$+\beta \begin{bmatrix} \pi\rho \left(N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) + \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}) \right) + \\ \pi(1-\rho)N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) + \\ (1-\pi)\rho \left(N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{+}) + \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}) \right) + \\ (1-\pi)(1-\rho)N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{+}) \end{bmatrix}$$

$$N \left(\phi \quad \tilde{\phi} \quad (\phi \quad \Phi_{-}) \quad 0 \quad \Phi_{-} \right) = \tilde{\phi} \quad (\phi \quad \Phi_{-}) + \phi \quad (\Phi \quad \psi_{-}) \quad (51a)$$

$$N_{e}(\phi_{p}, \phi_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{+}) = -\phi_{c,bc}(\phi_{p}, \Phi_{-}) + \omega_{e}\phi_{p}(\Phi_{+} - \mu_{\phi_{p}})$$
(51c)
+
$$\beta \begin{bmatrix} \pi\rho \left(N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{+}) + \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}) \right) + \\ \pi(1 - \rho)N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}), 0, \Phi_{+}) + \\ (1 - \pi)\rho \left(N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) + \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}) \right) + \\ (1 - \pi)(1 - \rho)N_{e}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{-}), 0, \Phi_{-}) \end{bmatrix}$$

The cut-off value in a boom, $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$, can be solved from the condition that the entrepreneur is indifferent between creating and not creating the project. For $\phi_p \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-)$, those are the following:

$$\tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}) = \omega_{e}\phi_{p}(\Phi_{+} - \mu_{\phi_{p}})$$
(52a)
$$+\beta \begin{bmatrix} \pi \rho \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}) \\ \pi (1 - \rho) \times 0 \\ (1 - \pi)\rho N_{e,bc}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}), 1, \Phi_{-}) \\ (1 - \pi)(1 - \rho) \times 0 \end{bmatrix}$$

$$N_{e,bc}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}), 1, \Phi_{-}) = \omega_{e}\phi_{p}(\Phi_{-} - \mu_{\phi_{p}})$$
(52b)
$$+\beta \begin{bmatrix} \pi \rho N_{e,bc}(\phi_{p}, \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}), 1, \Phi_{-}) \\ \pi (1 - \rho) \times 0 \\ (1 - \pi)\rho \tilde{\phi}_{c,bc}(\phi_{p}, \Phi_{+}) \\ (1 - \pi)(1 - \rho) \times 0 \end{bmatrix}$$

B.1.3 Definitions for timed-entry projects

For the area with cyclical projects, we defined a variable $R_{\rm C}$ to relate the ratio of

 $\frac{\text{mass in the C-PERM-Loss area}}{\text{mass in the C-TEMP-Loss area}}$

 to

vertical length of C-PERM-Loss area vertical length of C-TEMP-Loss area

For the area with timed-entry projects, we define three variables that fulfill a similar role. The three variables correspond to the three timed-entry areas: the TE-Gain area, the TE-Loss area, and the combined area. The R^* variables relate the mass in a particular area to the length. The R variables are a ratio of R^* variables and, thus, relate the relative mass of two areas to the relative vertical lengths of the corresponding areas.

$$\begin{split} R_{\mathrm{TE}}(\phi_p) &= \frac{R_{\mathrm{TE}}^*(\phi_p)}{R^*(\phi_p)}, \\ R_{\mathrm{TE}}^*(\phi_p) &= \frac{\operatorname{prob}\left\{\phi_c: \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) | \phi_p\right\}}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) - \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-)}, \\ R^*(\phi_p) &= \frac{\operatorname{prob}\left\{\phi_c: \underline{i}^*\phi_p \leq \phi_c \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) | \phi_p\right\}}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_-) - \underline{i}^*\phi_p}, \end{split}$$

$$\begin{split} R_{\text{TE-Gain}}(\phi_p) &= \frac{R_{\text{TE-Gain}}^*(\phi_p)}{R^*(\phi_p)}, \\ R_{\text{TE-Gain}}^*(\phi_p) &= \frac{\text{prob}\left\{\phi_c: \tilde{\phi}_{c,\text{no-bc}}(\phi_p) < \phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) | \phi_p\right\}}{\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) - \tilde{\phi}_{c,\text{no-bc}}(\phi_p)}, \end{split}$$

$$\begin{aligned} R_{\text{TE-Loss}}(\phi_p) &= \frac{R_{\text{TE-Loss}}^*(\phi_p)}{R^*(\phi_p)}, \text{ and} \\ R_{\text{TE-Loss}}^*(\phi_p) &= \frac{\text{prob}\left\{\phi_c : \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-) < \phi_c \leq \tilde{\phi}_{c,\text{no-bc}}(\phi_p) | \phi_p\right\}}{\tilde{\phi}_{c,\text{no-bc}}(\phi_p) - \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_-)} \end{aligned}$$

Note that no optimization is used to define the $R_{\rm TE}$ variables, in contrast to the definition of $R_{\rm C}$. The reason is that in the TEMP2 area ϕ_p can take on a wider range of values. Therefore, the extremum could be completely unrepresentative for typical values over this interval in which case the lower bound on the cost of business cycles could be meaningless. Instead, we focus on averages of these values, as is made more precise in the next subsection.

B.2 Assumptions for timed-entry projects

In this section, we specify a set of conditions that makes it possible to obtain an estimate of the effect of business cycles for timed-entry projects.

Assumption TE.1

$$\begin{split} & \mathbf{E}_{t} \begin{bmatrix} \phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{-}) < \phi_{c} \leq \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) \\ \phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-}) \end{array} \right] & = \begin{array}{c} \frac{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{-}) + \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})}{2} \\ & \mathbf{E}_{t} \begin{bmatrix} \phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) < \phi_{c} \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ \phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-}) \end{array} \right] & = \begin{array}{c} \frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+})}{2} \\ & \mathbf{E}_{t} \begin{bmatrix} \phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) < \phi_{c} \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) = \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) + \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) = \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c} \tilde{\phi}_{c,\mathrm{b}}(\phi_{p}, \Phi_{+}) \\ & \mathbf{E}_{t} \left[\phi_{c} \middle| \begin{array}{c}$$

This is a weak assumption and the reason is that conditional on ϕ_p these are small intervals.

Assumption TE.2

$$\begin{split} & \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} R_{\mathrm{TE}}(\phi_{p}) \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p} \\ = & \mathrm{E} \left[R_{\mathrm{TE}}(\phi_{p}) | \phi_{p} > \tilde{\phi}_{p}(\Phi_{-}) \right] \\ & \times \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p} \end{split}$$

$$\int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} R_{\mathrm{TE-Gain}}(\phi_{p}) \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p}$$

$$= \mathbb{E} \left[R_{\mathrm{TE-Gain}}(\phi_{p}) | \phi_{p} > \tilde{\phi}_{p}(\Phi_{-}) \right]$$

$$\times \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p}$$

$$\int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} R_{\mathrm{TE-Loss}}(\phi_{p}) \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p}$$

$$= \mathbb{E} \left[R_{\mathrm{TE-Gain}} | \phi_{p} > \tilde{\phi}_{p}(\Phi_{-}) \right]$$

$$\times \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\infty} \phi_{p} f_{\phi_{p}}(\phi_{p}) d\phi_{p}$$

The functions $R_{\text{TE}}(\phi_p)$, $R_{\text{TE-Gain}}(\phi_p)$, and $R_{\text{TE-Loss}}(\phi_p)$ quantify how the ratio of the mass of timed-entry projects to the mass of PERM-Emp projects is related to the relative lengths.³⁹ For example, if $R_{\text{TE}}(\phi_p) = 1$ for a particular value of ϕ_p , then the ratio of the mass of the TE projects to the mass of the PERM-Emp projects is equal to the ratio of the lengths of the two intervals. Assumption TE.2 is trivially satisfied if the $R_{\text{TE}}(\phi_p)$ functions are constant. It may not be realistic, however, to assume that the functions are constant given that the functions are defined over a wide range of values for ϕ_p . The assumption allows the values of $R_{\text{TE}}(\phi_p)$ to vary with ϕ_p , but over the whole interval considered, $(\phi_{p,\text{bc}}(\Phi_{-}), \infty)$, the comovement cannot be systematic. For example, strictly monotone functions are excluded. The implication of this assumption is that we only have to specify the average value of the R_{TE} functions.

³⁹PERM-EMP projects are projects that always enter.

B.3 Cost of business cycles for timed-entry projects

Welfare cost for the TE-Gain projects. The welfare cost for the TE-Gain projects can be written as

$$\begin{split} & \int_{\tilde{\phi}_{p,bc}(\Phi_{-})}^{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{+})} \begin{bmatrix} \frac{\mu_{\phi_{p}}\phi_{p}}{1-\beta} \\ -E\left[N_{bc}(\phi_{p},\phi_{c},0,\Phi_{p})\right] \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \\ &= \int_{\tilde{\phi}_{p,bc}(\Phi_{-})}^{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{+})} \left[\eta_{c}\phi_{c}+\eta_{p}\phi_{p}\right] f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \end{aligned} \tag{53} \\ &= \int_{\tilde{\phi}_{p,bc}(\Phi_{-})}^{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{+})} \left[\eta_{c}\phi_{c}+\eta_{p}\phi_{p}\right] f_{\phi_{c}|\phi_{p}}(\phi_{c})d\phi_{c}f\phi_{p}(\phi_{p})d\phi_{p} \\ &= \left(\eta_{c}X_{\mathrm{TE-Gain}}+\eta_{p}\right)\int_{\tilde{\phi}_{p,bc}(\Phi_{-})}^{\tilde{\phi}_{c,hc}(\phi_{p},\Phi_{+})} \left[\int_{\tilde{\phi}_{c,no-bc}(\phi_{p})}^{\tilde{\phi}_{c,hc}(\phi_{p},\Phi_{+})} f_{\phi_{c}|\phi_{p}}(\phi_{c})d\phi_{c}\right]\phi_{p}f\phi_{p}(\phi_{p})d\phi_{p} \\ &= \eta\int_{\tilde{\phi}_{p,bc}(\Phi_{-})}^{\tilde{\phi}_{c,bc}(\Phi_{-})} \left[prob\left\{\phi_{c}:\tilde{\phi}_{c,no-bc}(\phi_{p}) \\ \phi_{p}\geq\tilde{\phi}_{p,bc}(\Phi_{-}) \right\} E\left[R_{\mathrm{TE-Gain}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,bc}(\Phi_{-})\right] \\ &\times \int_{\phi_{p}>\tilde{\phi}_{p}(\Phi_{-})}^{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{-})-\frac{i^{*}\phi_{p}}{2}\phi_{p}\leq\phi_{c}\leq\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{-})|\phi_{p}\right\}\phi_{p}\right]f(\phi_{p})d\phi_{p} \\ &= \eta\left(\frac{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{+})-\tilde{\phi}_{c,no-bc}(\phi_{p})}{\tilde{\phi}_{c,bc}(\phi_{p},\Phi_{-})-\frac{i^{*}\phi_{p}}{2}\phi_{p,bc}(\Phi_{-})}\right) E\left[R_{\mathrm{TE-Gain}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,bc}(\Phi_{-})\right] \\ &\times L_{t}\left[R_{\mathrm{TE-Gain}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p}(\Phi_{-})\right]Y_{\mathrm{PERM-Emp}(\Phi_{-}) \end{aligned}$$

where

$$\begin{split} X_{\text{TE-Gain}} &= \frac{\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) + \tilde{\phi}_{c,\text{no-bc}}(\phi_p)}{2\phi_p}, \\ \eta &= \left(\eta_c X_{\text{TE-Gain}} + \eta_p\right), \end{split}$$

and the values of η_c and η_p are found by substituting in the solutions for $N_{\rm bc}(\phi_c, \phi_p, 0, \Phi_-)$ and $N_{\rm bc}(\phi_c, \phi_p, 0, \Phi_+)$ from the system of equations given in Equation (48) and collecting terms.⁴⁰ This cost is negative, that is, business cycles are beneficial for these projects.

⁴⁰Note that the ratio $\left(\tilde{\phi}_{c,bc}(\phi_p, \Phi_+) - \tilde{\phi}_{c,no-bc}(\phi_p)\right) / \left(\tilde{\phi}_{c,bc}(\phi_p, \Phi_-) - \underline{i^*}\phi_p\right)$ does not vary with ϕ_p , except that it is different above and below $\tilde{\phi}_{p,bc}(\Phi_-)$, which explains why we condition on the value of ϕ_p .

Welfare cost for the TE-Loss projects. The welfare cost for the TE-Loss projects can be written as

$$\begin{split} &\int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \int_{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p})}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \begin{bmatrix} \left(\frac{\phi_{p}-(1-\beta\rho)\phi_{c}}{1-\beta}\right) \\ -\mathrm{E}\left[N_{\mathrm{bc}}(\phi_{p},\phi_{c},0,\Phi_{p})\right] \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \\ &= \left.\bar{\eta}\left(\frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})-\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})-\underline{i^{*}}\phi_{p}}\right|_{\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \right) \mathrm{E}\left[R_{\mathrm{TE-Loss}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})\right] \\ &\times \int_{\phi_{p}>\tilde{\phi}_{p}(\Phi_{-})} \left[\operatorname{prob}\left\{\phi_{c}:\underline{i^{*}}\phi_{p}\leq\phi_{c}\leq\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})|\phi_{p}\right\}\phi_{p}\right]f(\phi_{p})d\phi_{p} \\ &= \left.\bar{\eta}\left(\frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})-\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})-\underline{i^{*}}\phi_{p}}\right|_{\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \right) \\ &\times \mathrm{E}_{t}\left[R_{\mathrm{TE-Loss}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})\right]Y_{\mathrm{PERM-Emp}}(\Phi_{-}) \end{split}$$

where

$$\begin{split} \bar{\eta} &= \left(\bar{\eta}_{c} X_{\text{TE-Loss}} + \bar{\eta}_{p}\right), \\ X_{\text{TE-Loss}} &= \frac{\tilde{\phi}_{c,\text{bc}}(\phi_{p}, \Phi_{-}) + \tilde{\phi}_{c,\text{no-bc}}(\phi_{p})}{2\phi_{p}}, \end{split}$$

and the values of $\bar{\eta}_c$ and $\bar{\eta}_p$ are solved for in an analogous manner to η_c and η_p .

Welfare cost for all TE projects. Combining the two terms of the two types of TE projects gives that the total effect of business cycles on projects in this area as a fraction of average output, Y, is equal to

$$\begin{pmatrix} \eta \left(\frac{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{+}) - \tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-}) - \underline{i^{*}}\phi_{p}} \Big|_{\phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \right) \mathrm{E} \left[R_{\mathrm{TE-Gain}}(\phi_{p}) |\phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-}) \right] \\ + \bar{\eta} \left(\frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p}) - \tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-}) - \underline{i^{*}}\phi_{p}} \Big|_{\phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \right) \mathrm{E} \left[R_{\mathrm{TE-Loss}}(\phi_{p}) |\phi_{p} \geq \tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-}) \right] \\ \times (Y_{\mathrm{PERM-Emp}}(\Phi_{-})/Y)$$

$$\leq \qquad (54)$$

$$\left(\begin{array}{c} \eta\left(\frac{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{+})-\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})-\underline{i^{*}}\phi_{p}}\Big|_{\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}\right) \mathrm{E}\left[R_{\mathrm{TE-Gain}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})\right] \\ +\bar{\eta}\left(\frac{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})-\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})}{\tilde{\phi}_{c,\mathrm{bc}}(\phi_{p},\Phi_{-})-\underline{i^{*}}\phi_{p}}\Big|_{\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}\right) \mathrm{E}\left[R_{\mathrm{TE-Loss}}(\phi_{p})|\phi_{p}\geq\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})\right] \end{array}\right)$$

B.4 Quantitative evaluation of the TE area

In this section, we document that quantitatively the timed-entry area is not important at all. First, consider the case when we set the parameters equal to their benchmark values. Expressed as a fraction of aggregate output, the gains for the TE-Gain area are equal to 0.00172%, 0.00139%, and 0.00069% for ρ equal to 0.875, 0.9167, and 0.975, respectively. The losses for the TE-Loss area are equal to 0.00169%, 0.00138%, and 0.00069% for the same three values of ρ . Adding up the gains and the losses results in a tiny negligible gain.

The importance of the timed-entry area increases when the lower bound on the entry costs, \underline{i} , increases. Suppose that \underline{i} is equal to 0.05 and $\omega_e = 0.125$. For entrepreneurs with the *lowest* entry costs, this means that each period the total amount of entry costs paid on restarting projects is equal to half of the total revenues received by entrepreneurs, a substantial sum. This lower bound is so high that it would wipe out all the cyclical projects in the world with business cycles. For such a high value of \underline{i} , the area with timed-entry projects becomes relatively more important, but the total gains for the timed-entry area are still only equal to 0.00005%, 0.00004%, and 0.00002%, for ρ equal to 0.875, 0.9167, and 0.975 respectively.

C Cyclical projects

In Section C.1, we give the formulas for NPVs and cut-off levels. In Section C.2, we provide a weaker set of assumptions under which our derived formulas are valid. In the main text, we derived an analytic expression for the cost of business cycles due to projects in the C-PERM-Loss area. In Section C.3, we derive an analytic expression for the cost of business cycles for the other two types of cyclical projects.

C.1 Formulas for cyclical projects

C.1.1 NPV formulas for cyclical projects

In the main text, we only presented the NPV formula for the C-PERM-Loss area. In this subsection, we present the NPV formulas for the other two types of cyclical projects.

C-TEMP-Loss and C-TEMP-Gain areas. In a world without business cycles, workerfirm pairs in the C-TEMP-Gain area have a productivity level that is never high enough to overcome the inefficiency. Since they never operate, it does not make sense to pay the entry cost. Thus, the relevant NPV for the C-TEMP-Gain area is the NPV of the projects that never operate, thus,⁴¹

$$N_{\text{no-bc}}(\phi_p, \phi_c, 0) = \frac{\mu_{\phi_p} \phi_p}{1 - \beta}$$
for $\tilde{\phi}_{p,\text{bc}}(\Phi_+) \le \phi_p < \tilde{\phi}_{p,\text{no-bc}}$ and $\underline{i^*} \phi_p \le \phi_c \le \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+).$
(55)

In contrast, pairs in the C-TEMP-Loss area would always operate in a world without business cycles, so the relevant NPV for the C-TEMP-Loss area is the NPV of the projects that always enter, that is,

$$N_{\text{no-bc}}(\phi_p, \phi_c, 0) = -\phi_c + N_{\text{no-bc}}(\phi_p, \phi_c, 1)$$

$$= -\phi_c + \phi_p + \beta \left(\rho N_{\text{no-bc}}(\phi_p, \phi_c, 1) + (1 - \rho) N_{\text{no-bc}}(\phi_p, \phi_c, 0) \right)$$

$$= \frac{\phi_p - (1 - \beta \rho) \phi_c}{1 - \beta}$$
for $\tilde{\phi}_{p,\text{no-bc}} \leq \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-)$ and $\underline{i^*}\phi_p \leq \phi_c \leq \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+).$
(56)

In a world with business cycles, the value of Φ_p determines whether these cyclical projects can overcome the inefficiencies or not; they can when $\Phi_p = \Phi_+$ and they cannot when $\Phi_p = \Phi_-$. When the economy is in a boom, we have

$$N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) = -\phi_c + N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_+)$$
(57)
$$= -\phi_c + \phi_p \Phi_+ + \beta \left(\begin{array}{c} \pi \left(\rho N_{\rm bc}(\phi_p, \phi_c, 1, \Phi_+) + (1-\rho) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) \right) \\ + (1-\pi) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) \end{array} \right)$$
for $\tilde{\phi}_{p, \rm bc}(\Phi_+) \le \phi_p < \tilde{\phi}_{p, \rm bc}(\Phi_-)$ and $\underline{i^*}\phi_p \le \phi_c \le \tilde{\phi}_{c, \rm bc}(\phi_p, \Phi_+)$

⁴¹Even if these jobs would have paid the entry costs, then the expected benefits would have been the same, that is, $N_{\text{no-bc}}(\phi_p, \phi_c, 0) = N_{\text{no-bc}}(\phi_p, \phi_c, 1)$.

and in a recession it would be

$$N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) =$$

$$\mu_{\phi_p} \phi_p + \beta \left(\pi N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_-) + (1 - \pi) N_{\rm bc}(\phi_p, \phi_c, 0, \Phi_+) \right)$$
for $\tilde{\phi}_{p,\rm bc}(\Phi_+) \le \phi_p < \tilde{\phi}_{p,\rm bc}(\Phi_-)$ and $\underline{i^*}\phi_p \le \phi_c \le \tilde{\phi}_{c,\rm bc}(\phi_p, \Phi_+).$
(58)

It turns out to be the case, that to calculate the cost of fluctuations, we only have to know the average of these two values and this is equal to

$$\frac{N_{\rm bc}(\phi_p,\phi_c,0,\Phi_+) + N_{\rm bc}(\phi_p,\phi_c,0,\Phi_-)}{2}$$

$$= -0.5\phi_c + 0.5\frac{\phi_p\Phi_+ + \mu_{\phi_p}\phi_p - \beta(1-\pi\rho)\phi_c}{1-\beta}$$

$$= 0.5\frac{\phi_p\Phi_+ + \mu_{\phi_p}\phi_p - (1-\beta\rho\pi)\phi_c}{1-\beta}$$
for $\tilde{\phi}_{p,\rm bc}(\Phi_+) \leq \phi_p < \tilde{\phi}_{p,\rm bc}(\Phi_-)$ and $\underline{i}^*\phi_p \leq \phi_c \leq \tilde{\phi}_{c,\rm bc}(\phi_p,\Phi_+).$
(59)

C.1.2 Cut-off levels for cyclical projects

Entry is decided by the entrepreneur and entry will happen if the current and discounted future benefits that accrue to the entrepreneur exceed the entry costs. The NPV for an entrepreneur is denoted by $N_{e,bc}(\phi_p, \phi_c, e, \Phi_p)$ when there are and by $N_{e,no-bc}(\phi_p, \phi_c, e)$ when there are no business cycles. To obtain the formula for $N_e(\cdot)$, one can use the formula for $N(\cdot)$, first set μ_{ϕ_p} equal to zero and then replace ϕ_p with $\omega_e \phi_p (\Phi_p - \mu_{\phi_p})$.

C-TEMP-Gain and C-TEMP-Loss areas. The upper bound in this area is determined by the condition that the benefits the entrepreneur receives when he enters in a boom just offset the entry costs, that is,

$$N_{e,bc}(\phi_p, \phi_{c,bc}(\phi_p, \Phi_+), 0, \Phi_+) = 0.$$
(60)

From this equation, we get

$$\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) = \frac{\omega_e \phi_p(\Phi_+ - \mu_{\phi_p})}{1 - \beta \pi \rho} \text{ for } \tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) \le \phi_p < \tilde{\phi}_{p,\mathrm{bc}}(\Phi_-).$$
(61)

C-PERM-Loss area. When there are no business cycles, then the cut-off level of ϕ_c is determined by

$$N_{e,\text{no-bc}}(\phi_p, \tilde{\phi}_{c,\text{no-bc}}(\phi_p), 0) = 0,$$
(62)

which gives

$$\tilde{\phi}_{c,\text{no-bc}}(\phi_p) = \frac{\omega_e \phi_p (1 - \mu_{\phi_p})}{1 - \beta \rho} \text{ for } \tilde{\phi}_{p,\text{no-bc}} \le \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-).$$
(63)

The lower bound of the C-PERM-Loss area is the upper bound of the C-TEMP-Loss area defined above.

C.2 Alternative assumptions for cyclical projects

According to the assumptions of Section 4.3.1, the distribution of ϕ_p and the distribution of ϕ_c conditional on ϕ_p are uniform. This assumption is stronger than needed. In this section, we give a weaker set of assumptions under which the formulas used are also correct.

Assumption C.1

$$\int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{-})} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{no-bc}}} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{no-bc}}} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{no-bc}}} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{c,\mathrm{no-bc}}(\phi_{p})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p} \leq \int_{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \int_{\underline{i^{*}}\phi_{p}}^{\tilde{\phi}_{p,\mathrm{bc}}(\Phi_{+})} \phi_{p}f(\phi_{p},\phi_{c})d\phi_{p}$$

This condition specifies that the amount produced by firms in the C-TEMP-Gain area, is not less than the amount produced by firms in the C-TEMP-Loss area. This is a weak assumption. It would be satisfied, for example, if the density $f(\phi_p|\phi_c)$ is weakly increasing in this lower tail of the distribution and the cut-off levels $\tilde{\phi}_{p,\text{bc}}(\Phi_-)$ and $\tilde{\phi}_{p,\text{bc}}(\Phi_+)$ are symmetric around $\tilde{\phi}_{p,\text{no-bc}}$, just like Φ_- and Φ_+ are symmetric around 1. By making this assumption, we make sure that output produced by firms in the C-TEMP-Gain area does not dominate the output produced by firms in the C-TEMP-Loss area, which seems a reasonable assumption given that values of ϕ_p in the C-TEMP-Loss area exceed the values of ϕ_p in C-TEMP-Gain area and the density is more likely to be increasing than decreasing in the lower tail of the distribution. **Assumption C.2** For three particular intervals, in the area with cyclical projects, we assume that the average entry costs is equal to the average of the two end points of the interval considered. This given the following three assumptions.

Assumption C.2.1

$$E_t \left[\phi_c \left| \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) \le \phi_c \le \tilde{\phi}_{c,\text{no-bc}}(\phi_p) \right| = \frac{\tilde{\phi}_{c,\text{no-bc}}(\phi_p) + \tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)}{2} \right]$$

for $\tilde{\phi}_{p,\text{no-bc}} \le \phi_p < \tilde{\phi}_{p,\text{bc}}(\Phi_-).$

This assumption states that in the C-PERM-Loss areas we can approximate the average value of ϕ_c with the average value of the end points of the interval. The upper bound of the distribution of ϕ_c could be below $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. Nevertheless we will use as the end point the highest possible upper bound that theory allows, which is $\tilde{\phi}_{c,\text{no-bc}}(\phi_p)$. If we overestimate the entry cost in this area, then we underestimate the cost of business cycles, since in the presence of business cycles these projects do not pay entry costs and these costs saved are counted as a benefit of having business cycles.

Assumption C.2.2

$$\begin{split} \mathbf{E}_t \left[\phi_c | \underline{i^*} \phi_p \leq \phi_c \leq \tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+) \right] &= \frac{\underline{i^*} \phi_p + \phi_{c,\mathrm{bc}}(\phi_p, \Phi_+)}{2} \\ & \text{for } \tilde{\phi}_{p,\mathrm{bc}}(\Phi_+) &\leq \phi_p \leq \tilde{\phi}_{p,\mathrm{no-bc}}. \end{split}$$

In those cases when we set $\underline{i^*} = 0$, we are most likely *under* estimating the entry costs in the C-TEMP-Gain area. In the presence of business cycles entry increases, so underestimating the entry costs results in an underestimate of the true cost of business cycles.

Assumption C.2.3

$$\begin{split} E_t \left[\phi_c | \underline{i^*} \phi_p \le \phi_c \le \tilde{\phi}_{c, \mathrm{bc}}(\phi_p, \Phi_+) \right] &= \frac{\underline{i^*} \phi_p + \phi_{c, \mathrm{bc}}(\phi_p, \Phi_+)}{2} \\ & \text{for } \tilde{\phi}_{p, \mathrm{no-bc}} &\le \phi_p < \tilde{\phi}_{p, \mathrm{bc}}(\Phi_-). \end{split}$$

Entry costs have a negative effect on the NPV formulas. For the C-TEMP-Loss area it is not clear whether entry costs have a *larger* negative effect on the NPV for the case with business cycles than for the case without. In the world without business cycles, all projects in this area would enter and pay entry costs. After initial entry, firms do not pay entry costs again, however, until after an exogenous severance. In the world with business cycles, only half of the projects in this area would pay the entry costs initially, namely those that start in a boom. Firms would expect to pay them again if a full business cycle occurs, a possibility that does not exist in the world without business cycles. Moreover, even those projects that start in a recession will still pay entry costs when the economy gets out of the recession. For some of the parameter values we consider, entry costs have a larger negative effect on the NPV for the case with business cycles.⁴² In this case, Assumption B.3.3 would lead to an underestimate of the cost of business cycles. However, for equally plausible parameter values the opposite is true and we overestimate the cost of business cycles. The projects in the C-TEMP-Loss area are very similar to projects in the C-TEMP-Gain area. The first have a value of ϕ_p just above $\tilde{\phi}_{p,\text{no-bc}}$ and the latter have a value of ϕ_p just below $\tilde{\phi}_{p,\text{no-bc}}$. Consequently, if our estimate of the average entry costs is too low in the C-TEMP-Loss area, then it is almost certainly too low in the C-TEMP-Gain area as well. Any possible overestimation of the cost of business cycles due to underestimation of entry costs in the C-TEMP-Loss area will be dominated by an underestimation due to entry costs in the C-TEMP-Gain area. The reason is that, because of the offsetting effects, entry costs have a much smaller effect on the difference between $U_{\rm no-bc}$ and $U_{\rm bc}$ in the C-TEMP-Loss area than in the C-TEMP-Gain area.

C.3 Costs of business cycles for cyclical projects

Welfare cost for the C-TEMP-Gain projects. This term is related to those cyclical projects for which business cycles are welfare improving (i.e., a negative cost), because in a world without business cycles they would never be able to overcome the inefficiency, while in a world with business cycles they can overcome the inefficiency during a boom.

⁴²In particular, this would occur if $0.5(1 - \beta \rho \pi) > (1 - \beta \rho)$.

The welfare gain is equal to the following expression:

$$\begin{split} & \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{e},\mathrm{hc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \frac{\mu_{\phi_{p}}\phi_{p}}{1-\beta} \\ -0.5 \frac{(1+\mu_{\phi_{p}}/\Phi_{+})\phi_{p}\Phi_{+}-(1-\beta\pi\rho)\phi_{c}}{1-\beta} \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \end{split} (64) \\ &= \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{e},\mathrm{hc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} -0.5 \frac{(1-\mu_{\phi_{p}}/\Phi_{+})\phi_{p}\Phi_{+} - (1-\beta\pi\rho)\phi_{c}}{1-\beta} \\ 1-\beta \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \end{aligned} \\ &= 0.5 \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{e},\mathrm{hc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \frac{\mu_{\phi_{p}}/\Phi_{+}-1}{1-\beta}\phi_{p}\Phi_{+} \\ +\frac{1-\beta\pi\rho}{1-\beta}\phi_{c} \end{bmatrix} f\phi_{c}|\phi_{p}(\phi_{c})f\phi_{p}(\phi_{p})d\phi_{c}d\phi_{p} \end{aligned} \\ &= 0.5 \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{e},\mathrm{hc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \frac{\mu_{\phi_{p}}/\Phi_{+}-1}{1-\beta}\phi_{p}\Phi_{+} \\ +\frac{1-\beta\pi\rho}{1-\beta}\phi_{c}\Phi_{e}(\Phi_{+})} +\frac{1-\beta\pi\rho}{1-\beta}\phi_{p}\Phi_{+} \end{bmatrix} f\phi_{c}|\phi_{p}(\phi_{c})f\phi_{p}(\phi_{p})d\phi_{c}d\phi_{p} \end{aligned} \\ &= 0.5 \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{p},\mathrm{hc}} \begin{bmatrix} \frac{\mu_{\phi_{p}}/\Phi_{+}-1}{1-\beta}} \\ +\frac{1-\beta\pi\rho}{1-\beta}X_{\mathrm{C},\mathrm{TEMP}} \end{bmatrix} \phi_{p}\Phi_{+} \end{bmatrix} f\phi_{c}|\phi_{p}(\phi_{c})f\phi_{p}(\phi_{p})d\phi_{c}d\phi_{p} \end{aligned} \\ &= 0.5 \begin{pmatrix} \frac{\mu_{\phi_{p}}/\Phi_{+}-1}{1-\beta} + \frac{1-\beta\pi\rho}{1-\beta}X_{\mathrm{C},\mathrm{TEMP}} \\ \times \int_{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})}^{\bar{\phi}_{p,\mathrm{hc}}(\Phi_{+})} \int_{\underline{i}^{+}\phi_{p}}^{\bar{\phi}_{p},\mathrm{hc}} \phi_{p}(\phi_{c})d\phi_{c} \end{pmatrix} \phi_{p}\Phi_{+} \end{bmatrix} f\phi_{p}(\phi_{p})d\phi_{p} \end{aligned}$$

The value of $X_{\text{C-TEMP}}$ satisfies

$$X_{\text{C-TEMP}} = \frac{\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+) + \underline{i^*}\phi_p}{2\phi_p\Phi_+}$$
(65)

and $\tilde{\phi}_{c,\mathrm{bc}}(\phi_p, \Phi_+)$ is given by Equation (61). Note that $X_{\mathrm{C-TEMP}}$ does not depend on ϕ_p , which means it can be taken out of the integral. In the third step of the derivation, Assumption C.2.2 is used to calculate the mean of ϕ_c conditional on ϕ_p . The presence of business cycles is beneficial for projects in this area.⁴³

⁴³Using the definition of $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$, it can be shown that the expression in Equation (64) is negative, i.e., business cycles are beneficial, as long as $\mu + \omega_e(\Phi_+ - \mu) < \Phi_+$. But this condition says that the

This welfare measure is expressed in terms of the output that is produced by firms in the C-TEMP-Gain area during a boom, $Y_{C-TEMP-Gain}(\Phi_+)$.

Welfare cost for the C-TEMP-Loss projects The welfare cost for the C-TEMP-Loss projects can be written as

$$\begin{split} & \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\underline{i}^{*} \phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \frac{\phi_{p}-(1-\beta\rho)\phi_{c}}{1-\beta} \\ -0.5 \frac{(1+\mu\phi_{p}/\Phi_{+})\phi_{p}\Phi_{+}-(1-\beta\rho\pi)\phi_{c}}{1-\beta} \end{bmatrix} f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \end{split}$$
(66)

$$&= 0.5 \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\underline{i}^{*} \phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \left(\frac{2/\Phi_{+}-1-\mu\phi_{p}/\Phi_{+}}{1-\beta}\right)\phi_{p}\Phi_{+} \\ -\frac{1-\beta\rho(2-\pi)}{1-\beta}\phi_{c} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \end{bmatrix} \\ &= 0.5 \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\underline{i}^{*} \phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \left(\frac{2/\Phi_{+}-1-\mu\phi_{p}/\Phi_{+}}{1-\beta}\right)\phi_{p}\Phi_{+} \\ -\frac{1-\beta\rho(2-\pi)}{1-\beta}\phi_{c} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \end{bmatrix} \\ &= 0.5 \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\underline{i}^{*} \phi_{p}}^{\tilde{\phi}_{c,\text{bc}}(\phi_{p},\Phi_{+})} \begin{bmatrix} \left(\frac{2/\Phi_{+}-1-\mu\phi_{p}/\Phi_{+}}{1-\beta}\right)\phi_{p}\Phi_{+} \\ -\frac{1-\beta\rho(2-\pi)}{1-\beta}X_{\text{C-TEMP}} \right)\phi_{p}\Phi_{+} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \end{bmatrix} \\ &= 0.5 \left(\frac{2/\Phi_{+}-1-\mu\phi_{p}/\Phi_{+}}{1-\beta} - \frac{1-\beta\rho(2-\pi)}{1-\beta}X_{\text{C-TEMP}} \right)\phi_{p}\Phi_{+} \end{bmatrix} f_{\phi_{c}|\phi_{p}}(\phi_{c})f_{\phi_{p}}(\phi_{p})d\phi_{c}d\phi_{p} \end{bmatrix} \\ &\times \int_{\tilde{\phi}_{p,\text{no-bc}}}^{\tilde{\phi}_{p,\text{bc}}(\Phi_{-})} \int_{\underline{i}^{*} \phi_{p}}^{\tilde{\phi}_{p}}\phi_{p}\Phi_{+}f(\phi_{p},\phi_{c})d\phi_{p}d\phi_{c} \end{bmatrix} Y_{\text{C-TEMP-Loss}}(\Phi_{+})$$

D More on the panel data set

Figure A.1 compares the aggregate wage data (total compensation) with the sum of individual wages in our panel. Since our panel only has data for West Germany and true aggregate wage data are after 1991 only available for all of Germany, we restrict our comparison to the years up to 1991. The top (bottom) panel plots the HP-filtered components of the two series, when the smoothing parameter is equal to 1,600 (10⁵). The figure makes clear that there is a close match. There are some episodes during which there are some productivity outside the relationship, μ , plus the transfer to the entrepreneur has to be less than total revenues, i.e., the wage rate plus the transfer to the entrepreneur and this is always satisfied. high frequency fluctuations in the true aggregate wage series that are not present in the aggregate series obtained from the panel. The most striking difference between the two series is observed in 1984. One would think this is measurement error, but an identical but dampened version of the same pattern is observed in GDP data. The way individual wage data is constructed in the panel data set automatically smooths out some high frequency observations.⁴⁴

Figures A.2 and A.3 document the robustness of Figures 7 and 8 to using a smoothing coefficient equal to 1,600 instead of 10^5 .

E Simple measure of extensive margin of value added

In the main text, we used a German panel data set to construct a measure of the cyclical change in output through the extensive margin. Here we provide support for this measure using some simple rule-of-thumb calculations based on estimates of the extensive margin of U.S. employment.

Using the total number of non-farm employed persons, we find that the standard deviations of cyclical employment are equal to 1.4% and 2.2%, for HP-detrended data with a smoothing coefficient of 1,600 and 10⁵, respectively. In a framework, in which there is only a boom and a recession level of employment, E_b and E_r , and the unconditional probabilities are fifty-fifty, these estimates would imply values for $(E_b - E_r)/E = (E_b - E_r)/(0.5(E_b + E_r))$ equal to 2.8% and 4.4%. Now suppose that the $E_b - E_r$ workers that become employed (unemployed) during a boom (recession) are all like those without a college degree and suppose that the E_r workers that are not affected by business cycles are all like college graduates. This means that the cyclical workers are roughly half as productive as the other workers.⁴⁵ Combining this productivity difference with the estimates for $(E_b - E_r)/E$ would imply estimates for Y_{C-TEMP}/Y equal to 1.4% and 2.2%. Since the corresponding estimates for $(Y_b - Y_r)/Y_r$ are equal to 3.32% (HP coefficient equal to

⁴⁴The wage in a particular month is equal to the total wage sum earned by this individual in this job in this year, divided by the length of the time spent in this job during the year.

⁴⁵See, for example, Goldin and Katz (2008).

1,600) and 5.1% (HP coefficient equal to 10^5), this means that estimates of the fraction of cyclical fluctuations in output that are due to the extensive margin range are a bit less than 50%, i.e., very similar to those based on the German wage data.

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Figure 1: Unexplained per capita GDP and volatility

Notes: This graph plots a country's residual from a cross-country regression explaining the level of per capita GDP (relative to the US level) versus the country's standard deviation of cyclical GDP.

Figure 2: No business cycles - no friction



Notes: This graph plots the social planner's solution when there are no business cycles. Projects under the cut-off would be created and would always produce.

Figure 3: With business cycles - no friction



Notes: The top panel plots the social planner's solution when it is restricted in the sense that once a project is created it has to continue until hit by an exogenous destruction shock. The bottom panel gives the optimal strategy when the social planner can also follow a cyclical strategy, i.e., end projects in a recession and restart them in a boom.

Figure 4: No business cycles - with friction



Notes: This graph plots the outcome in the competitive equilibrium. Only projects to the right and below the solid line are created. Created projects always produce.
Figure 5: With business cycles - with friction - building up the story



Notes: Panel A illustrates the effect of business cycles on the inefficiency, Panel B the effect of business cycles on the entry decision *if* a project could continue until exogenous destruction occurs, and Panel C gives the effect of business cycles on entry when business cycles shorten the project's expected duration.



Figure 6: With business cycles - with friction - Complete overview

Notes: This graph indicates the fives areas for which business cycles affect the choices made. The cyclical projects are the C-TEMP-Gain (C-G), the C-TEMP-Loss (C-L), and the C-PERM-Loss (C-PL) projects. The timed-entry projects are the TE-TEMP-Gain (TE-G) and the TE-TEMP-Loss (TE-L) projects.



Figure 7: Fraction wages earned by recently non-employed and unemployment rate recently not employed = "non-employment spell during the last 24 months

Notes: This graph plots the German unemployment rate (left-side axis) together with the fraction of total wages earned by workers that recently had a non-employment spell (right-side axis). Both series are quarterly averages of monthly series. The series are the HP-filtered series using a smoothing coefficient equal to 10^5 plus the mean.



Figure 8: Fraction wages earned by recently non-employed and total wages

Notes: This graph plots the log of total wages (left-side axis) together with the fraction of total wages earned by workers that recently had a non-employment spell (right-side axis). Both series are quarterly averages of monthly series. The series are the HP-filtered series using a smoothing coefficient equal to 10^5 . The mean is added for the fraction earned by workers with a recent non-employment spell.



Figure A1: True aggregate wage data versus aggregated wages in panel

Notes: This graph plots the cyclical component of total deflated wages using true aggregate wages obtained from the statistics office together with the cyclical component of the deflated series obtained by explicitly aggregating wages in the panel in each period. The HP-filter is used to obtain the cyclical component and the value of the smoothing coefficient is indicated above each graph.



Figure A2: Fraction wages earned by recently non-employed and unemployment rate recently not employed = "non-employment spell during the last 24 months

Notes: This graph plots the German unemployment rate (left-side axis) together with the fraction of total wages earned by workers that recently had a non-employment spell (right-side axis). Both series are quarterly averages of monthly series. The series are the HP-filtered series using a smoothing coefficient equal to 1,600 plus the mean.



Figure A3: Fraction wages earned by recently non-employed and total wages

Notes: This graph plots the log of total wages (left-side axis) together with the fraction of total wages earned by workers that recently had a non-employment spell (right-side axis). Both series are quarterly averages of monthly series. The series are the HP-filtered series using a smoothing coefficient equal to 1,600. The mean is added for the fraction earned by workers with a recent non-employment spell.

Table 1: Implied estimates for $Y_{\text{C-TEMP}}/Y$

		-		
	HP = 1,600		HP =	$= 10^5$
	T = 24	T = 36	T = 24	T = 36
Germany	1.22%	1.38%	1.75%	2.62%
US				
f_+ and f as in Germany	1.23%	1.39%	1.74%	2.61%
r as in Germany	1.73%	1.96%	1.55%	2.32%

Notes: \hat{f}_+ (\hat{f}_-) is the average fraction of wages earned by workers that had a nonemployment spell during the last T months when the economy is in an expansion (downturn) and \hat{r} is the fraction of the total increase in wages due to an increase in the number of workers that had a non-employment spell in the last T months.

Table 2: Benchmark case: $R_C = 1$, $\underline{i}=0$, $\mu = 0.2$

	$\rho = 0.875$	$\rho = 0.9167$	$\rho = 0.975$
$Y_{C-Temp}/Y = 1.22\%$			
$\omega_e = 0.05$	0.37%	0.57%	1.62%
$\omega_e = 0.125$	0.36%	0.54%	1.56%
$\omega_e = 0.2$	0.34%	0.52%	1.49%
$\omega_e = 1$	0.19%	0.29%	0.83%
$Y_{C-Temp}/Y = 1.92\%$			
$\omega_e = 0.05$	0.58%	0.89%	2.55%
$\omega_e = 0.125$	0.56%	$\mathbf{0.86\%}$	$\mathbf{2.45\%}$
$\omega_e = 0.20$	0.54%	0.82%	2.35%
$\omega_e = 1$	0.30%	0.46%	1.31%
$Y_{C-Temp}/Y = 2.62\%$			
$\omega_e = 0.05$	0.80%	1.22%	3.47%
$\omega_e = 0.125$	0.77%	1.17%	3.34%
$\omega_e = 0.20$	0.74%	1.12%	3.21%
$\omega_e = 1$	0.41%	0.62%	1.78%

Notes: This table reports the costs of business cycles as a fraction of GDP. $R_C = 1$ means that the concentration of values of ϕ_c above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ is identical to the concentration below $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ per unit interval, $\underline{i} = 0$ means that the lowest possible value at which a project can be created is equal to zero, and $\mu = 0.2$ means that workers associated with cyclical projects can produce (from society's point of view) 20% of their market production level outside the market. Y_{C-Temp}/Y is the fraction of GDP earned by cyclical workers in a boom, the expected duration of a project is equal to $1/(1-\rho)$, and ω_e is the fraction of the surplus the entrepreneur receives. The other parameter values are set equal to their benchmark value. Our preferred values are in bold.

	0 / -	- /1	
	$\rho {=} 0.875$	$\rho {=} 0.9167$	$\rho = 0.975$
$Y_{C-Temp} = 1.22\%$			
$\omega_e = 0.05$	0.19%	0.28%	0.81%
$\omega_e = 0.125$	0.18%	0.28%	0.79%
$\omega_e = 0.20$	0.18%	0.27%	0.76%
$\omega_e = 1$	0.15%	0.21%	0.51%
$Y_{C-Temp} = 1.92\%$			
$\omega_e = 0.05$	0.29%	0.45%	1.28%
$\omega_e = 0.125$	0.29%	0.44%	1.24%
$\omega_e = 0.20$	0.28%	0.43%	1.20%
$\omega_e = 1$	0.23%	0.33%	0.80%
$Y_{C-Temp} = 2.62\%$			
$\omega_e = 0.05$	0.40%	0.61%	1.74%
$\omega_e = 0.125$	0.39%	0.60%	1.69%
$\omega_e = 0.20$	0.39%	0.59%	1.64%
$\omega_e = 1$	0.32%	0.45%	1.09%

Table 3: Lower $R_{\rm C}$: $R_{\rm C} = 0.5, \, \underline{i} = 0, \, \mu = 0.2$

Notes: This table reports the costs of business cycles as a fraction of GDP when R_C is not set equal to its benchmark value, but is such that projects with higher entry costs are less likely. $R_C = 0.5$ means that the concentration of values of ϕ_c above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ is half the concentration below $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ per unit interval, $\underline{i} = 0$ means that the lowest possible value at which a project can be created is equal to zero, and $\mu = 0.2$ means that workers associated with cyclical projects can produce (from society's point of view) 20% of their market production level outside the market. Y_{C-Temp}/Y is the fraction of GDP earned by cyclical workers in a boom, the expected duration of a project is equal to $1/(1 - \rho)$, and ω_e is the fraction of the surplus the entrepreneur receives. The other parameter values are set equal to their benchmark value. Our preferred values are in bold.

	$\rho = 0.875$	$\rho = 0.9167$	$\rho = 0.975$
$Y_{C-Temp} = 1.22\%$			
$\omega_e = 0.05$	0.49%	0.82%	7.03%
$\omega_e = 0.125$	0.39%	0.62%	2.25%
$\omega_e = 0.20$	0.36%	0.57%	1.85%
$\omega_e = 1$	0.19%	0.30%	0.86%
$Y_{C-Temp} = 1.92\%$			
$\omega_e = 0.05$	0.77%	1.29%	11.07%
$\omega_e = 0.125$	$\mathbf{0.62\%}$	$\mathbf{0.98\%}$	$\mathbf{3.54\%}$
$\omega_e = 0.20$	0.57%	0.89%	2.91%
$\omega_e = 1$	0.30%	0.46%	1.36%
$Y_{C-Temp} = 2.62\%$			
$\omega_e = 0.05$	1.05%	1.76%	15.11%
$\omega_e = 0.125$	0.85%	1.33%	4.83%
$\omega_e = 0.20$	0.78%	1.22%	3.97%
$\omega_e = 1$	0.41%	0.63%	1.85%

Table 4: Higher lower bound for ϕ_c : $R_c = 1, \, \underline{i} = 0.005, \, \mu = 0.2$

Notes: This table reports the costs of business cycles as a fraction of GDP when the lower bound on the entry costs is not equal to zero, but is such that on average 0.5% of production is paid on entry costs. $R_C = 1$ means that the concentration of values of $\phi_{c,\text{bc}}(\phi_p, \Phi_+)$ is identical to the concentration below $\tilde{\phi}_{c,\text{bc}}(\phi_p, \Phi_+)$ per unit interval and $\mu = 0.2$ means that workers associated with cyclical projects can produce (from society's point of view) 20% of their market production level outside the market. Y_{C-Temp}/Y is the fraction of GDP earned by cyclical workers in a boom, the expected duration of a project is equal to $1/(1 - \rho)$, and ω_e is the fraction of the surplus the entrepreneur receives. The other parameter values are set equal to the benchmark value. Our preferred values are in bold.

Table 5:	Higher	value	of leis	sure:	$R_c = 1$	$, \underline{i} =$	$0, \mu =$	0.43
			<i>ρ</i> =0.8	375	$\rho = 0.91$	67	$\rho = 0.97$	5

$Y_{C-Temp} = 1.22\%$			
$\omega_e = 0.05$	0.26%	0.40%	1.15%
$\omega_e = 0.125$	0.25%	0.38%	1.10%
$\omega_e = 0.20$	0.24%	0.37%	1.06%
$\omega_e = 1$	0.13%	0.21%	0.59%
$Y_{C-Temp} = 1.92\%$			
$\omega_e = 0.05$	0.41%	0.63%	1.80%
$\omega_e = 0.125$	$\mathbf{0.39\%}$	0.60%	1.73%
$\omega_e = 0.125$ $\omega_e = 0.20$	0.39% 0.38%	0.60% 0.58%	1.73% 1.67%
$\omega_e = 0.125$ $\omega_e = 0.20$ $\omega_e = 1$	0.39% 0.38% 0.21%	0.60% 0.58% 0.32%	1.73% 1.67% 0.93%
$\omega_e = 0.125$ $\omega_e = 0.20$ $\omega_e = 1$	0.39% 0.38% 0.21%	0.60% 0.58% 0.32%	1.73% 1.67% 0.93%
$\omega_e = 0.125$ $\omega_e = 0.20$ $\omega_e = 1$ $Y_{C-Temp} = 2.62\%$	0.39% 0.38% 0.21%	0.60% 0.58% 0.32%	1.73% 1.67% 0.93%
$\omega_e = 0.125$ $\omega_e = 0.20$ $\omega_e = 1$ $Y_{C-Temp} = 2.62\%$ $\omega_e = 0.05$	$\begin{array}{c} \textbf{0.39\%} \\ 0.38\% \\ 0.21\% \\ 0.56\% \end{array}$	0.60% 0.58% 0.32% 0.86%	$1.73\% \\ 1.67\% \\ 0.93\% \\ 2.46\%$
$\begin{split} & \omega_e = 0.125 \\ & \omega_e = 0.20 \\ & \omega_e = 1 \end{split} \\ & Y_{C-Temp} = 2.62\% \\ & \omega_e = 0.05 \\ & \omega_e = 0.125 \end{split}$	$\begin{array}{c} \textbf{0.39\%} \\ 0.38\% \\ 0.21\% \\ 0.56\% \\ 0.54\% \end{array}$	$\begin{array}{c} \textbf{0.60\%} \\ 0.58\% \\ 0.32\% \\ \end{array}$ $\begin{array}{c} 0.86\% \\ 0.83\% \end{array}$	$1.73\% \\ 1.67\% \\ 0.93\% \\ 2.46\% \\ 2.37\%$
$\omega_e = 0.125$ $\omega_e = 0.20$ $\omega_e = 1$ $Y_{C-Temp} = 2.62\%$ $\omega_e = 0.05$ $\omega_e = 0.125$ $\omega_e = 0.20$	$\begin{array}{c} \textbf{0.39\%} \\ 0.38\% \\ 0.21\% \\ \end{array}$ $\begin{array}{c} 0.56\% \\ 0.54\% \\ 0.52\% \end{array}$	$\begin{array}{c} \textbf{0.60\%} \\ 0.58\% \\ 0.32\% \\ 0.86\% \\ 0.83\% \\ 0.79\% \end{array}$	1.73% $1.67%$ $0.93%$ $2.46%$ $2.37%$ $2.27%$

Notes: This table reports the costs of business cycles as a fraction of GDP when the value of not working (from sociey's point of view) is 43% of the value of working instead of 20%. $R_C = 1$ means that the concentration of values of ϕ_c above $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ is identical to the concentration below $\tilde{\phi}_{c,bc}(\phi_p, \Phi_+)$ per unit interval and $\underline{i} = 0$ means that the lowest possible value at which a project can be created is equal to zero. Y_{C-Temp}/Y is the fraction of GDP earned by cyclical workers in a boom, the expected duration of a project is equal to $1/(1 - \rho)$, and ω_e is the fraction of the surplus the entrepreneur receives. The other parameter values are set equal to the benchmark value. Our preferred values are in bold.