How Can Governments Borrow so Much?

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Motivation

Many papers have attempted to reproduce prevailing country debt/GDP ratios.

These papers have made two central assumptions:

(i) Governments have infinite horizon.
(ii) Governments default strategically when outstanding debt exceeds the present value of the costs of default to the country.

Neither assumption is entirely natural, at least for modern democracies.

Assumption (i) contrasts with the widely-held view that a government's horizon extends only as far as its expected term in office.

Assumption (ii) is contradicted by governments' demonstrated reluctance to default (Levy Yeyati and Panizza, 2011).

Perhaps this explains why, despite their numerous achievements, existing papers generate debt/GDP ratios that are too low.

Cohen and Villemot (2013) report debt/GDP ratios that range from 1.5% to 30%, which their own work extends to 47%.

Actual debt ratios are markedly higher for the majority of countries.

Debt/GDP Ratios, Selected Countries

Country	Avg. Debt/GDP, 1980-2011, %	Debt/GDP, 2011, %
Argentina	73	45
Australia	20	24
Austria	64	72
Ecuador	34	18
France	50	86
Germany	61	80
Greece	85	165
Hungary	65	80
Italy	108	120
Japan	117	229
Russia	29	12
Spain	47	69
Sweden	55	38
Switzerland	50	47
Turkey	53	39
Ukraine	32	36
U. Kingdom	45	82
U. States	65	103
Uruguay	73	55

Main Idea and Result

We replace Assumptions (i) and (ii) by their more or less exact opposites:

(i) Governments' horizons are limited to their expected terms in office

(ii) (Democratic) governments (almost always) do their utmost not to default.

Assumption (i) is consistent with the self-interest Public Choice Theory attributes to government's motives and behavior.

Assumption (ii) is consistent with governments' demonstrated reluctance to default (Levy Yeyati and Panizza, 2011).

Governments are reluctant to default because:

- They often lose power after default (Borensztein and Panizza, 2009; Malone, 2011): domestic bondholders are also voters; even the bonds sold initially to foreign bondholders may ultimately accrue to domestic bondholders through trading in secondary markets (Boner, Martin, and Ventura, 2010).
- They are wary of impeding the functioning of the banking system, which relies on government bonds as collateral for interbank loans (Bolton and Jeanne, 2011).
- They do not wish to be seen as engaging in inexcusable default (Grossman and Van Huyck, 1988): unlike excusable default, inexcusable default is generally punished by lenders through the exclusion from debt markets hypothesized by Eaton and Gersovitz (Tomz, 2007).

A government whose horizon is limited to its expected term in office naturally neglects possibly negative consequences of borrowing that occur beyond that term. The government therefore can be expected to borrow more.

Investors who do not fear strategic default are, up to a point, willing to accommodate such borrowing as they recognize that the limit to lending stems from the government's ability rather than willingness to service debt.

There is therefore higher debt than in existing models.

We provide a simple way of computing a country's maximum sustainable debt as a function of the world risk-free interest rate and three country-specific variables:

(1) the mean level of GDP growth
(2) the volatility of GDP growth
(3) the ratio of government income to GDP

Knowledge of a country's maximum sustainable debt allows us to compute the country's probability of default as a function of its current debt.

Our main finding is that maximum sustainable debt differs markedly across countries.

We thereby provide an explanation for different countries' differing debt (in)tolerance (Reinhart, Rogoff, and Savastano, 2003; Catão and Kapur, 2004): some countries default at very low debt/GDP ratios, other sustain very high ratios for a long time.

Literature Review

Eaton and Gersovitz (1981): governments borrow to insure their countries against output shocks; they repay for fear of losing access to debt markets.

Bulow and Rogoff (1989a, 1989b): debt otherwise to be repaid can be used by a defaulting government to buy an insurance contract that provides the same benefits as access to debt markets; exclusion from debt markets alone is unlikely to deter strategic default; direct sanctions are necessary.

Aguiar and Gopinath (2006) and Arellano (2008): study interactions of default risk, output, consumption, the trade balance, interest rates, and foreign debt in setting of small open economy.

Mendoza and Yue (2012): endogenize output and collapse of output in default.

Cuadra and Sapriza (2008): consider the role of political risk.

Yue (2009) and Benjamin and Wright (2009): consider the role of renegotiation in default.

Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012): consider the role of debt maturity.

Fink and Scholl (2011): consider the role of conditionality.

Cohen and Villemot (2013): develop a model in which the cost of default is borne 'in advance.'

Acharya and Rajan (2012) and Rochet (2006) have preceded us in examining the implications of short government horizon. Acharya and Rajan assume strategic rather than excusable default.

A feature of our model is that it is not subject to the Bulow-Rogoff critique (1989a, 1989b). A government that has excusably defaulted has no income with which to buy the insurance contract put forward by Bulow and Rogoff.

Model

For simplicity, we let the length of a government's term in office equal to one period and consider one period debt.

We denote

- y_t government disposable income at date t. Government disposable income is can be thought of as the maximum primary surplus that can be achieved after all non-essential spending has been cut.
- $B_t = b_t y_t$ government borrowing at date t.
- $D_{t+1} = d_t y_t$ government debt outstanding at date $t+1$ but raised at date t.
- R_t the gross interest rate, $R_t = D_{t+1} / B_t$.
- g_t the growth in disposable government income, $g_t = y_{t+1} / y_t$.

A government that 'inherits' debt repayment $D_t = d_{t-1} y_{t-1}$ and borrows $B_t = b_t y_t$ can engage in non-debt related spending $y_t + B_t - D_t = y_t + b_t y_t - d_{t-1} y_{t-1}$.

We assume a constant risk-free rate r.

We further assume that the log of the growth rate in government income is i.i.d. We denote $F(.)$ the distribution of g_t.

In competitive capital markets, if the loss given default (LGD) were 100%, we would have

$$b_t y_t = \frac{(1 - PD_t) d_t y_t}{1 + r}$$

where PD_t denotes the default probability

$$PD_t \equiv Pr[y_{t+1} + b_{t+1} y_{t+1} < d_t y_t] = F\left(\frac{d_t}{1 + b_{t+1}}\right)$$

The gross interest R_t rate would be

$$R_t \equiv \frac{D_{t+1}}{B_t} = \frac{d_t}{b_t} = \frac{1 + r}{1 - PD_t} \approx 1 + r + PD_t$$

In practice, the LGD is not 100%.

Consistently with our assumption that governments do their utmost not to default, we assume that governments cede all disposable income to lenders in case of default.

Consistently with the evidence of 'sudden stops,' we assume that there is no further lending in default.

The relation between debt and borrowing becomes

$$b_t y_t = \frac{1}{1+r}\left[Pr\left[\left(1+b_{t+1}\right)y_{t+1} > d_t y_t\right]d_t y_t + \int_0^{d_t y_t/\left(1+b_{t+1}\right)} y_{t+1} dF\left(y_{t+1}\right)\right]$$

Note that the amount lenders are willing to provide in period t depends on the amount they expect to be provided in period $t+1$.

As is to be expected, the gross interest rate and the default probability remain positively related

$$\frac{1}{R_t} = \frac{b_t}{d_t} = \frac{1}{1+r}\left[1 - PD_t + \frac{1}{d_t}\int_0^{d_t/\left(1+b_{t+1}\right)} g dF(g)\right]$$

Maximum Sustainable Debt and Debt Intolerance

We define maximum sustainable debt to be the maximum bounded debt path; we define maximum sustainable borrowing similarly.

As noted above, maximum sustainable debt may be viewed as a measure of debt (in)tolerance (Reinhart, Rogoff, and Savastano, 2003; Catão and Kapur, 2004).

We motivate these maximums by assuming a myopic government, whose unique concern is its current term in office.

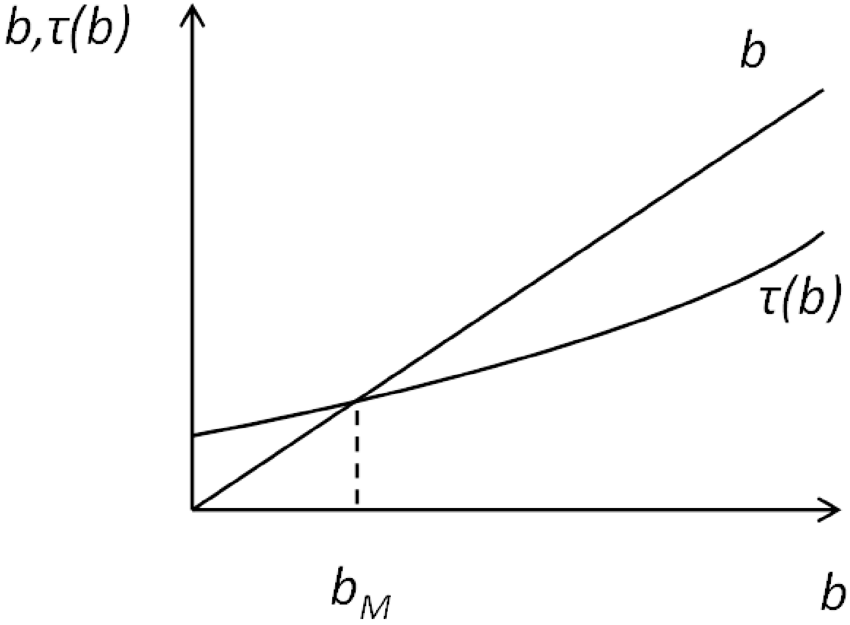
A myopic government naturally maximizes borrowing, because it reaps the benefits of borrowing in the current period but escapes the costs of debt repayment in the next period, which is of no concern to the government.

Formally, a myopic government maximizes current borrowing b_t given lenders' expectation of future borrowing b_{t+1}

$$b_t = \underset{d_t}{Max} \frac{1}{1+r}\left[d_t \left[1 - F\left(\frac{d_t}{1+b_{t+1}} \right) \right] + \int_0^{d_t/(1+b_{t+1})} g dF(g) \right] \equiv \tau(b_{t+1})$$

We assume that $F(.)$ is lognormal: $ln(g_t) \sim N(\mu, \sigma^2)$.

Proposition 1: If $E(g) < 1+r$, $\tau(b)$ is a contraction mapping and has a unique fixed point b_M which defines maximum sustainable borrowing.



Maximum Sustainable Borrowing b_M

Given maximum sustainable borrowing b_M, actual borrowing in the current period b and debt outstanding in the next period d are related by the relation

$$b = B(d) \equiv \frac{1}{1+r}\left[\left[1-F\left(\frac{d}{1+b_M}\right)\right]d + \int_0^{d/(1+b_M)} g\,dF(g)\right]$$

The corresponding interest rate is

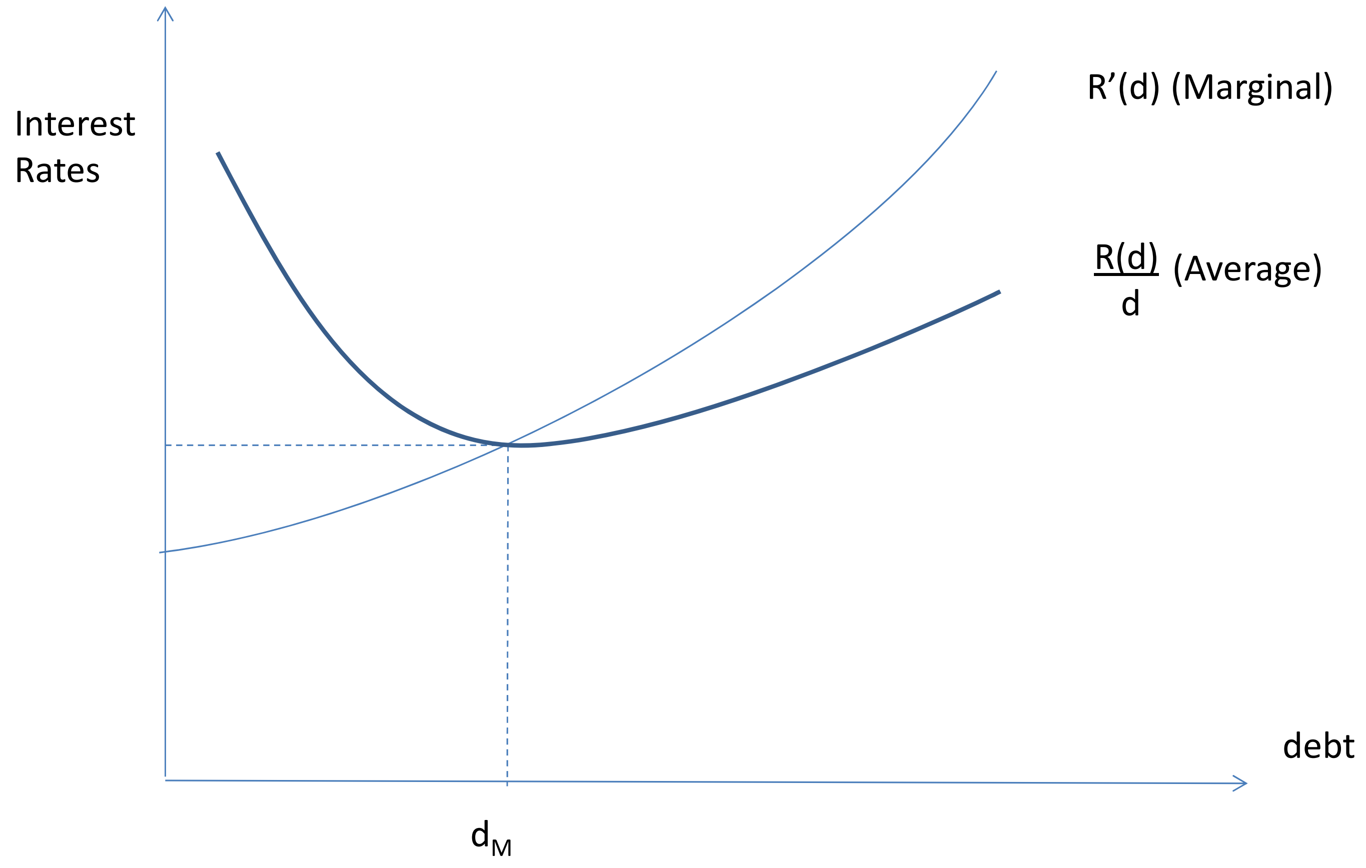
$$R(d) \equiv \frac{d}{B(d)}$$

Maximum sustainable debt d_M is that which maximizes $B(d)$: $d_M = \arg\max_d B(d)$. We can show

$$R'(d_M) = \frac{R(d_M)}{d_M}$$

Maximum sustainable debt equates the marginal and 'average' interest rates.

The 'average' interest rate reaches its minimum at maximum sustainable debt.



Maximum Sustainable Debt d_M

Proposition 2: Maximum sustainable borrowing b_M is increasing in the mean growth rate μ and decreasing in growth rate volatility σ (for a probability of default less than $1/2$) and in the risk-free rate r.

Proposition 3: For a given level of debt d, the probability of default is decreasing in the mean growth rate μ and increasing in growth rate volatility σ (for a probability of default less than $1/2$) and in the risk-free rate r.

$$PD(d) = F\left(\frac{d}{1+b_M}\right) = \Phi\left(\frac{\ln(d) - \ln(1+b_M) - \mu}{\sigma}\right)$$

Proposition 4: Maximum sustainable debt d_M is increasing in the mean growth rate μ and decreasing in the risk-free rate r (for a probability of default less than $1/2$). The interest rate $R_M \equiv d_M / b_M$ is increasing in the risk-free interest rate r (for a probability of default less than $1/2$).

Calibration: Maximum Sustainable Debt

We estimate μ and σ from IMF data over the period 1980-2011 and set $r = 6.9\%$, the average US long bond rate over that period.

Recalling that d denotes the debt to government disposable income ratio, we transform d_M into its corresponding debt/GDP ratio by multiplying it by the product of the ratio of government income to GDP and the ratio of government disposable income to government income.

$$\frac{Debt}{GDP} = \frac{Debt}{Gvt.Disp.Inc.} \times \frac{Gvt.Disp.Inc.}{Gvt.Inc.} \times \frac{Gvt.Inc.}{GDP}$$
$$\equiv d_M \times \alpha \times \frac{Gvt.Inc.}{GDP}$$

where α is there ratio of government disposable income to government income.

Note that the Debt/GDP ratio is increasing in the ratio of government income to GDP.

We obtain $Debt / GDP$ and $Gvt.Inc. / GDP$ from IMF data and set $\alpha = 40\%$.

Debt/GDP, Max.Debt/GDP, Max.PD, Selected Countries

Country	D, 1980-2011, %	D, 2011, %	d_M, %	PD_M, %
Argentina	73	45	61	3.01
Australia	20	24	166	0.62
Austria	64	72	224	0.58
Ecuador	34	18	79	1.5
France	50	86	222	0.53
Germany	61	80	172	0.82
Greece	85	165	107	1.28
Hungary	65	80	122	1.65
Italy	108	120	171	0.73
Japan	117	229	106	1.06
Russia	29	12	72	3.69
Spain	47	69	150	0.84
Sweden	55	38	216	0.91
Switzerland	50	47	140	0.66
Turkey	53	39	97	2.04
Ukraine	32	36	53	6.81
U. Kingdom	45	82	148	0.89
U. States	65	103	135	0.81
Uruguay	73	55	80	2.18

Different countries clearly have different d_M as well as PD_M.

Lower d_M and/or higher PD_M countries are those that are more debt intolerant (Reinhart, Rogoff, and Savastano, 2003).

Debt intolerance tends to decrease in the ratio of government income to GDP (Reinhart, Rogoff, and Savastano, 2003; Besley and Persson, 2011); it tends to increase in σ (Catão and Kapur, 2004).

There appears to be little variation of debt intolerance in μ.

The result in σ is perhaps not surprising.

That in the ratio of government income to GDP reflects the dependence of government creditworthiness on government income.

μ, σ, Gov.Inc./GDP, Max.Debt/GDP, Max.PD, Selected Countries

Country	μ, %	σ, %	Gvt.Inc., %	d_M, %	PD_M, %
Argentina	2.5	6.0	28	61	3.01
Australia	3.1	1.6	33	166	0.62
Austria	2.1	1.5	49	224	0.58
Ecuador	3.1	3.4	24	79	1.5
France	1.8	1.4	49	222	0.53
Germany	1.7	2.0	45	172	0.82
Greece	1.4	2.9	34	107	1.28
Hungary	1.1	3.6	45	122	1.65
Italy	1.3	1.8	44	171	0.73
Japan	2.1	2.5	30	106	1.06
Spain	2.5	2.1	37	150	0.84
Russia	1.7	6.9	37	72	3.69
Sweden	2.3	2.2	56	216	0.91
Switzerland	1.8	1.7	34	140	0.66
Turkey	4.1	4.4	32	97	2.04
Ukraine	-0.8	10.8	39	53	6.81
U. Kingdom	2.3	2.2	38	148	0.89
U. States	2.6	2.0	32	135	0.81
Uruguay	2.5	4.6	30	80	2.18

We examine those four defaults for which we have detailed case studies (Sturzenegger and Zettelmeyer, 2006): Argentina (December 2001), Ecuador (January 1999), Russia (August 1998 and January 1999), Ukraine (September 1998), and Uruguay (May 2003).

All these countries' currencies were pegged, to a greater or lesser extent.

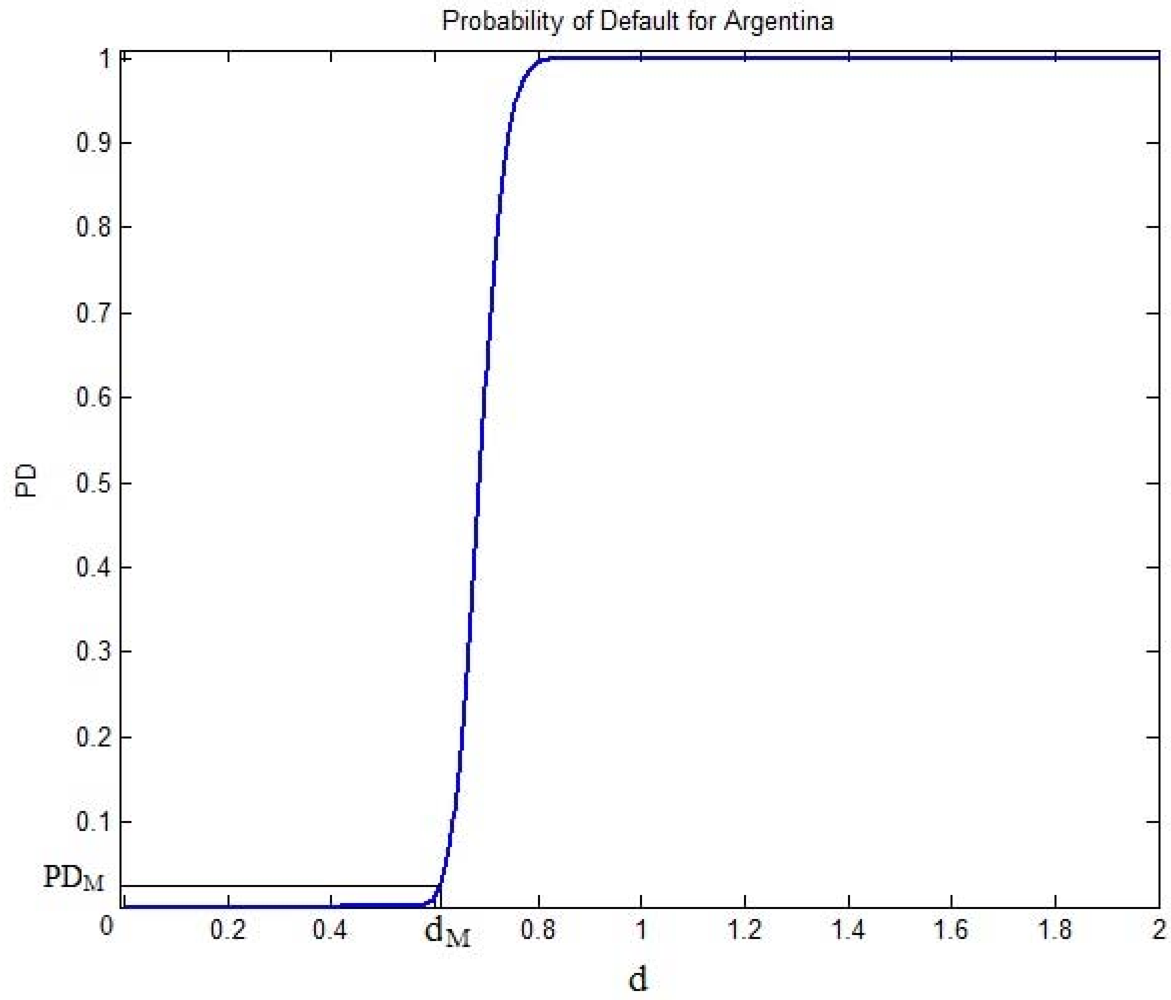
In all cases but Ukraine, currency devaluation transformed what had been a reasonable debt ratio (below d_M) into one that no longer was (above d_M).

Default probabilities literally exploded.

Default generally preceded rather than followed devaluation. The exceptions were Uruguay and, to a lesser extent, Russia.

Debt and Default Probabilities around Actual Default, Default Countries

Country	d_{-1}, %	PD_{-1}, %	d_0, %	PD_0, %	d_1, %	PD_1, %	d_M, %	PD_M, %
Argentina	45	0	53	0	150	100	61	3.01
Ecuador	67	0	101	100	91	98	79	1.5
Russia	54	0	68	0.33	90	92	72	3.69
Ukraine	33	0	37	0	53	6.81	53	6.81
Uruguay	94	94	109	100	100	100	80	2.18



Probability of Default for Argentina
PD
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
PD_M
0
0.2
0.4
d_M
0.8
1
1.2
1.4
1.6
1.8
2
d

General Model

We now assume the government is reelected to another term in office with probability θ.

The government's value function $V(D_t, y_t)$ is such that

$$V(D_t, y_t) = u(y_t + B_t - D_t) + \frac{\theta}{1+r} E[V(D_{t+1}, y_{t+1})]$$

The government recognizes that borrowing B_t is determined by debt D_{t+1}.

We shall set $u(c) = \frac{c^{1-a}}{1-a}$. This makes $V(D, y)$ homogeneous

$$V(D, y) = V\left(\frac{D}{y}, 1\right) y^{1-a} \equiv v(d) y^{1-a}$$

The value function is obtained by iterating the contraction mapping

$$Tv(d') = \max_{d} u(1 + B(d) - d') + \frac{\theta}{1+r} E\left[g^{1-a} v\left(\frac{d}{g} \right) \right]$$

where

$$B(d) \equiv \frac{1}{1+r}\left[\left[1 - F\left(\frac{d}{1+b_M} \right) \right] d + \int_0^{d/(1+b_M)} g dF(g) \right]$$

is government borrowing and

- d' is the current debt ratio,
- d is the target debt ratio, and
- d/g will be the realized debt ratio.

All are expressed per unit of current disposable income.

Note that $\theta = 0$ (the government will not be reelected) corresponds to the case of the myopic government: $d = d_M$ and $B(d) = B(d_M) = b_M$.

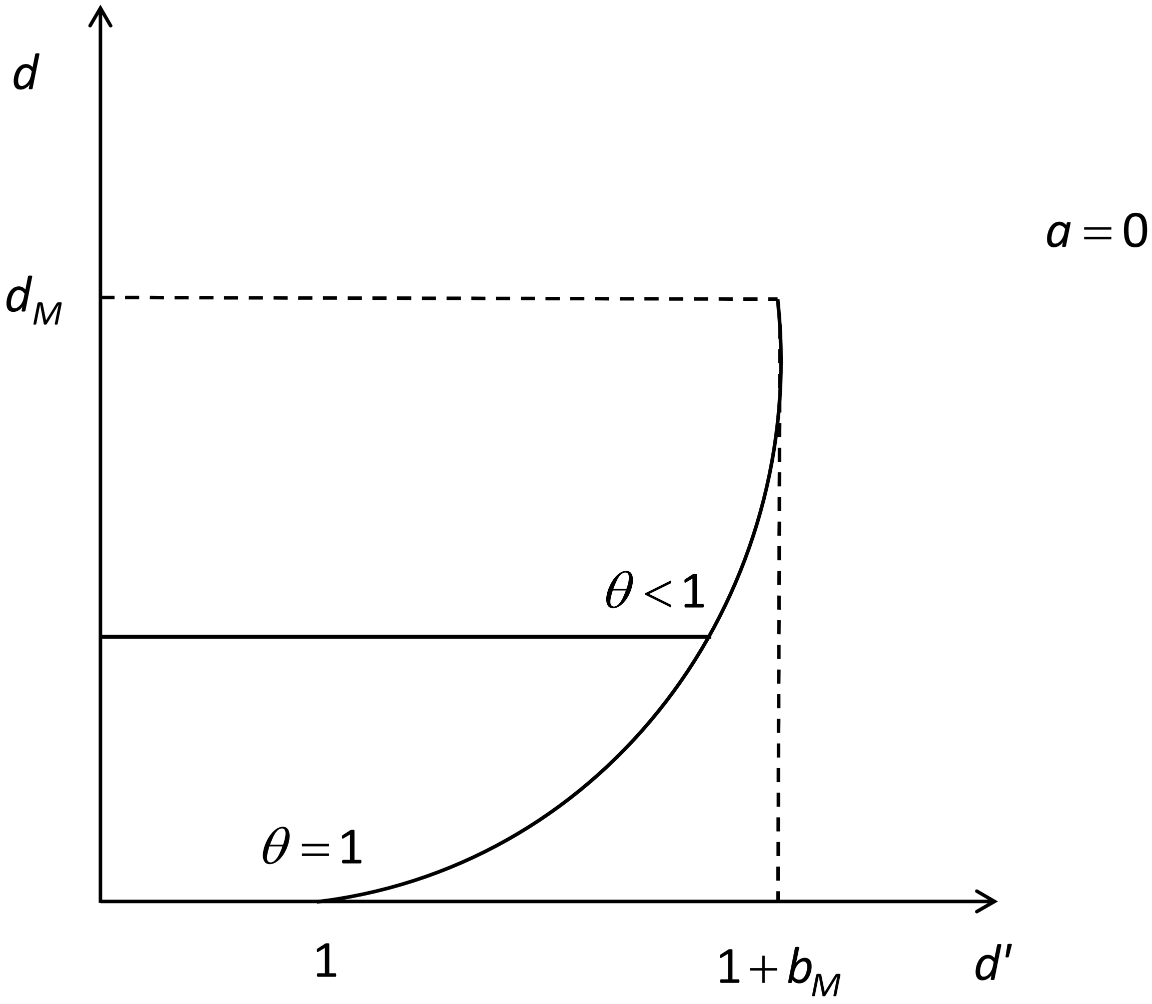
Proposition 5: Government borrowing $b^* \equiv B(d^*)$, government debt d^*, and the probability of default $PD^* \equiv F(d^*/(1+b_M))$ are decreasing in θ.

Corollary 1: $b^* < b_M$, $d^* < d_M$, and $PD^* < PD_M$ for $\theta > 0$.

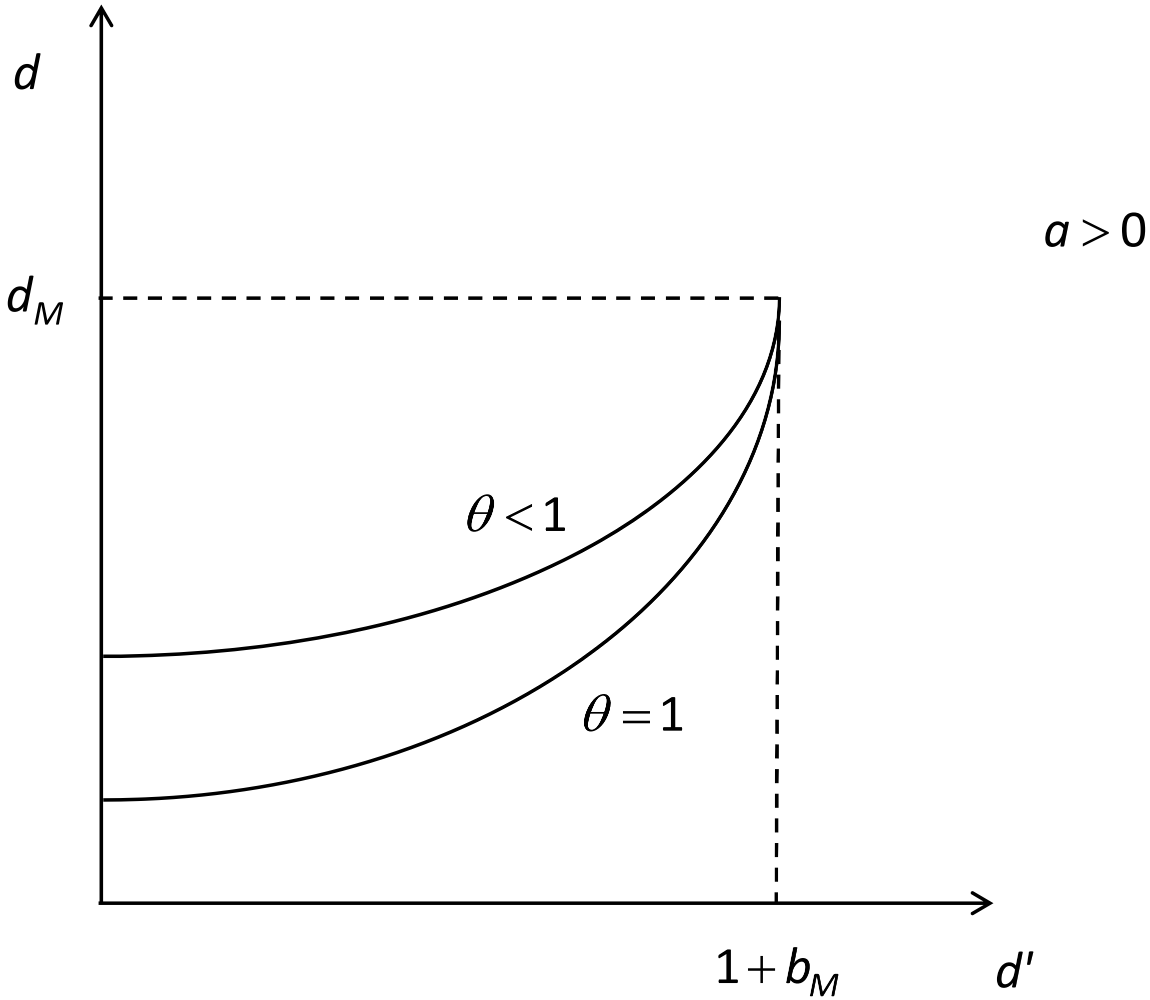
The prospect of reelection decreases government borrowing and government debt: governments realize that default jeopardizes their prospect of future terms in office; they decrease the probability of default by decreasing borrowing.

We refer to θ as 'the patience of politicians.'

It reflects not only the length of politicians' expected term in office, but also the 'quality' of the political system, as reflected in politicians' greater or lesser concern with developments beyond their term.



Debt Dynamics under Risk Neutrality



Debt Dynamics under Risk Aversion

Calibration: General Model

Minimum debt, d_m, prevails where politicians are least risk averse ($a \approx 0$), most patient ($\theta = 1$), and unencumbered by inherited debt ($d' = 0$).

Low actual default probabilities for most countries may be explained by the ‘margin of safety’ due to the difference between actual and maximum sustainable debt.

Default probabilities mostly remain quite low as long as government debt remains below its maximum sustainable level. They increase very quickly afterwards.

Debt/GDP, Max.Debt/GDP, Min.Debt/GDP, Selected Countries

Country	D, 1980-2011, %	D, 2011, %	d_m, %	d_M, %
Argentina	73	45	7	61
Australia	20	24	21	166
Austria	64	72	43	224
Ecuador	34	18	12	79
France	50	86	61	222
Germany	61	80	24	172
Greece	85	165	15	107
Hungary	65	80	19	122
Italy	108	120	23	171
Japan	117	229	20	106
Russia	29	12	7	72
Spain	47	69	27	150
Sweden	55	38	52	216
Switzerland	50	47	26	140
Turkey	53	39	13	97
Ukraine	32	36	7	53
U. Kingdom	45	82	21	148
U. States	65	103	25	135
Uruguay	73	55	14	80

Debt/GDP, PD, Max.Debt/GDP, Max.PD, Selected Countries (%)

Country	d_m	PD_m	D, 80-11	PD, 80-11	D, 11	PD, 11	d_M	PD_M
Argentina	7	0	73	82	45	0	61	3.01
Australia	21	0	20	0	24	0	166	0.62
Austria	43	0	64	0	72	0	224	0.58
Ecuador	12	0	34	0	18	0	79	1.5
France	61	0	50	0	86	0	222	0.53
Germany	24	0	61	0	80	0	172	0.82
Greece	15	0	85	0	165	100	107	1.28
Hungary	19	0	65	0	80	0	122	1.65
Italy	23	0	108	0	120	0	171	0.73
Japan	20	0	117	96	229	100	106	1.06
Russia	7	0	29	0	12	0	72	3.69
Spain	27	0	47	0	69	0	150	0.84
Sweden	52	0	55	0	38	0	216	0.91
Switzerland	26	0	50	0	47	0	140	0.66
Turkey	13	0	53	0	39	0	97	2.04
Ukraine	7	0	32	0	36	0	53	6.81
U. Kingdom	21	0	45	0	82	0	148	0.89
U. States	25	0	65	0	103	0	135	0.81
Uruguay	14	0	73	0	55	0	80	2.18

The marked asymmetry between the probability of default's very slow increase below the maximum sustainable debt level and very fast increase above is a natural consequence of the trade-off involved in computing the maximum sustainable debt level under the assumption of lognormality.

We know the interest rate and the probability of default to be positively related.

As the ‘average’ interest rate reaches its minimum at maximum sustainable debt, the average interest rate should decrease for debt lower than maximum sustainable debt and increase for debt higher.

Although the interest rate itself always increases in debt, it should increase slowly for debt lower than maximum sustainable debt and rapidly for debt higher.

The same is true of the probability of default.

This is especially so where the probability of default is lognormally distributed with very low volatility.

Maximum sustainable debt is around the point at which the probability of default starts its dramatic increase.

Debt levels that exceed the maximum sustainable level are therefore on or beyond that part of distribution where the probability of default increases very quickly.

Conclusion

Different countries have different degrees of debt (in)tolerance, represented in our model by the maximum sustainable debt ratio d_M.

A country's debt tolerance depends on the mean and especially the volatility of the country's growth rate, its government income to GDP ratio, its government disposable income to government income ratio, and the world risk-free interest rate.

A country's actual debt ratio differs from the country's maximum sustainable ratio because

- politicians realize that default jeopardizes their prospect of future terms in office
- politicians may have concerns that extend beyond their expected term in office.

The difference between a country's actual and maximum sustainable debt ratios provides the country with a 'margin of safety:' a country that has not yet reached its maximum sustainable debt ratio can increase debt at relatively little cost.

Such is no longer the case once the margin of safety has been exhausted and debt is larger than its maximum sustainable level.