

Monetary Union with Voluntary Participation*

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Abstract

A Monetary Union is modeled as a technology that makes a *surprise* policy deviation impossible but requires voluntarily participating countries to follow the same monetary policy. Within a fully dynamic context, we identify conditions under which such arrangement may dominate a coordinated system with independent national currencies. Two new results are delivered by the voluntary participation assumption. First, optimal policy is shown to respond to the agents' incentives to leave the union by tilting both current and future policy in their favor. This contrasts with the static nature of optimal policy when participation is exogenously assumed and implies that policy in the union is not exclusively guided by area-wide developments but does occasionally take account of member countries' national developments. Second we show that there might exist states of the world in which the union breaks apart, as occurred in some historical episodes. The paper thus provides a first formal analysis of the incentives behind the formation, sustainability and disruption of a Monetary Union.

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1. Introduction

International macroeconomic interdependence raises the possibility, first formalized in the seminal work of Hamada (1974, 1976), that non-cooperative decisions by the policy makers of different countries produce inefficient outcomes. A large body of literature has used this insight to analyze international institutions and policy cooperation.¹

In the field of monetary economics the idea has provided a rationale for monetary unions (MU), an institutional arrangement in which countries relinquish autonomous control over national currencies to adopt a common one. Economic history offers several instances of countries that have deliberately given up monetary independence, jointly or unilaterally, to follow a common policy (Cohen, 1993). The European monetary union is the best known recent example, but the establishment of an MU is also being examined by the six states of the Gulf Cooperation Council, nine nations in South East Asia and a large group of African countries.² As argued by Persson and Tabellini (2000, Chapter 18), this phenomenon can be rationalized as a second-best institution-design problem when the cooperative first-best policy is not feasible. In this context, the MU may allow policy makers to alleviate the coordination problem at the expense of a reduced ability to stabilize idiosyncratic shocks.

The trade-off between coordination versus flexibility that emerges in the choice of the monetary regime has proved fruitful for the analysis of fixed exchange rate arrangements and monetary unions, e.g. Alesina and Grilli (1992), Canzoneri and Henderson (1991) and Persson and Tabellini (1995). These papers provide a useful foundation to understand the incentives to form a monetary union, but they suffer from two limitations that this paper tries to overcome.

First, the benefits of the MU are usually discussed in comparison to the welfare achievable under the repetition of the static Nash equilibrium, given the premise

¹For an encompassing survey of applications in the field of fiscal and monetary policy during the last two decades see Persson and Tabellini (1995). Canzoneri and Henderson (1991) use similar ideas to study international monetary arrangements.

²See IMF (2003) and Currency News (2003).

that the first-best coordination of policy is “not feasible”. This is not fully satisfactory. The restrictive context of one-shot games should be abandoned, to account for the fact that the underlying strategic environment is a repeated game. Dynamic provision of incentives should be properly analyzed to see what outcomes are sustainable by means of reputation. In practice, some degree of coordination is usually observed outside monetary unions, as one would expect if policy makers do not fully discount the future.³ Ideally, one would like to understand why a second-best arrangement, in which countries deliberately give up policy independence, may dominate some other form of coordination which does not involve the loss of flexibility.

A second shortcoming of previous contributions concerns how the MU can be sustained. The traditional approach is to assume that countries entering the MU are not allowed to quit it, what we label “enforced participation”. In other words, countries contemplating the formation of a union face a take-it-or-leave-it offer at time zero and are given no further choices afterwards. This is unsatisfactory on both theoretical and empirical grounds.⁴

We abandon the assumption of enforced participation to shed light into how joint policy-making may make the union sustainable even in the absence of an exogenous enforcement technology. The extensions we explore deliver new insights into the sources of the welfare benefits of a monetary union and the way optimal policy should respond to shocks given the countries’ option to leave the union.

By modeling the union as a technology that makes a *surprise* policy deviation impossible (e.g. an unexpected exchange rate devaluation), we show that an MU may be superior to policy coordination, despite the fact that it gives rise to a loss of flexibility. This occurs since the payoff of a deviation from the “coordinated policy” delivers a smaller payoff when it is anticipated than when it comes as a surprise to rival agents. As deviations become less tempting under the MU,

³In Europe, for instance, full monetary integration between the members of the Euro area was preceded by various cooperation arrangements (e.g. the European Monetary System).

⁴Persson and Tabellini (2000, page 467) recognize the necessity to complete this analysis: “It is not enough to demonstrate that the policy outcome under cooperative policy making is superior, though, as individual countries generally have incentives to deviate from cooperative policy. The argument is therefore incomplete unless coupled with an argument as to how the suggested solution might be enforced.”

better outcomes can be sustained along the equilibrium path on average. When there are no flexibility costs associated with the MU policy (as is the case with symmetric shocks), it immediately follows that reducing the payoff associated with a deviation allows a superior equilibrium to be sustained. With asymmetric shocks, there is a tradeoff between this benefit and the flexibility foregone by the common policy.

The optimal MU arrangement that emerges with voluntary participation differs markedly from the one under enforced participation. In the latter case, once the union is formed, policy is decided according to time-invariant “Pareto weights” and there are no changes in the way the benefits of belonging to the union are allocated to its members over time. In our case, instead, policy responds to the agents’ incentives to leave the union by tilting both current and future policy in their favor. This finding implies that the monetary policy rule in the MU without enforcement is not guided solely by a MU-wide “averages” but, in some instances, does take account of the member countries’ local conditions. This point is of interest for the ongoing debate on the role that national developments play in the conduct of monetary policy in the euro area (e.g. Heinemann and Hüfner, 2002; Aksoy et Al. 2002).

Finally, depending on the distribution of the shocks and discount factors, our model shows that the MU might be permanent or temporary. For the latter, there are some “fatal” states of the world in which the MU breaks apart along the equilibrium path and countries revert to national monetary policy. Intuitively, this occurs because a large asymmetric shock makes it very costly to follow a common policy in those states, even though this implies giving up the future benefits of the MU. The possibility that a break-up occurs along the equilibrium path highlights the importance of not assuming an “enforcement technology”.

This result is empirically relevant. Economic historians and political scientists have given serious consideration to the “sustainability” of currency unions.⁵ Bordo and Jonung (1997) and Cohen (1993) examine the historical record of several monetary regimes, including various forms of currency unions, some of which

⁵A related view was recently offered by Milton Friedman: “[...] I think that within the next 10 to 15 years the eurozone will split apart” (*Financial Times*, June 7 / June 8, 2003).

successfully lasted for as long as they could (the Belgium-Luxembourg monetary union, founded in 1922, was absorbed into EMU) and others which collapsed fairly quickly (the East African Community collapsed in 1977 after about a decade from its foundation). It emerges that major fiscal shocks, often linked to wars, seem to be fatal for monetary unions. The causes of a MU breakup, which remain largely untouched by formal economic analysis, are analyzed in this paper.

Recent contributions have revived interest on monetary unions. Alesina and Barro (2002) and Cooley and Quadrini (2003) present general equilibrium models of a currency union which allow welfare analysis to be based on the representative agent utility function. The analysis of our paper complements these studies by providing insights on the interplay of dynamic incentives that make a monetary union sustainable in the absence of an enforcement technology. In doing this, however, we abstracted from explicit microfoundations, as the basic ideas transcend a specific setting. The integration of the two approaches is a natural next step.

From a methodological point of view, our analysis relies on results from the literature on “limited commitment”, pioneered by Thomas and Worrall (1988) and Kocherlakota (1996) and originally applied to a risk-sharing environment.⁶ One important technical difference in comparison to those studies is that ours has an additional constraint requiring both agents to follow the same action as long as they remain in the MU. The loss of a policy instrument introduces a tradeoff that in certain circumstances may lead to a break-up of the MU contract along the equilibrium path. This increases the complexity of the problem significantly. Fortunately, we are able to prove that under the optimal policy, the set of states in which the union breaks apart is independent of the history and of the countries’ bargaining power at the union formation stage. This allows the problem to be analyzed in a relatively simple way. Other potential applications of this result are discussed in the concluding section.

The paper is organized as follows. The economic environment and the two monetary regimes considered are described in the next section. Section 3 demon-

⁶This literature has recently found fruitful applications in the international trade literature, e.g. Bond and Park (2002).

strates that a monetary union may be superior to coordinated independent monetary policy. After presenting the definition of a sustainable equilibrium, Section 4 shows that the monetary union problem can be given a recursive formulation. This result, which is mainly technical, is used in Section 5 to derive a convenient characterization of optimal policy in the voluntary MU. Section 6 illustrates the key features of our model using an example economy. The main findings and conclusions are summarized in Section 7.

2. The economic environment

We consider a symmetric setup with two infinitely lived *ex-ante* identical countries, named Home and Foreign, each controlling a policy instrument $\pi, \pi^* \in \left[\underline{\pi}, \bar{\pi} \right]$ (asterisks denote foreign variables).⁷

The state of the world s in period t is determined by the realization of a discrete and i.i.d. random variable with support $S = \{s_1, s_2, \dots, s_S\}$ with corresponding probabilities denoted by p_s . The state s affects the utility functions for each country in potentially different ways.⁸ We assume that the distribution of these effects over individual countries is symmetric.⁹

Let $U(\pi, \pi^*, s)$ and $U^*(\pi^*, \pi, s)$ be the per-period utility of, respectively, Home and Foreign in state s when the policy pair (π, π^*) is chosen. The functions $U(\pi, \pi^*, s)$ and $U^*(\pi^*, \pi, s)$ are assumed to be bounded, jointly differentiable with respect to π and π^* and to have a negative semi-definite Hessian. For there to be a coordination issue we also require some spillover between the agents' actions i.e. $U_2^*, U_2 \neq 0$. Each country maximizes the expected value of the intertemporal utility function $E_o \sum_{t=0}^{\infty} \delta^t U()$, where $\delta \in (0, 1)$ is the discount factor.

Given this general environment different games can be played depending on the monetary regime chosen. Two regimes are considered: Independent National

⁷This assumption is for technical purposes. We will consider bounds that are so large that this constraint will not affect policy.

⁸We can think of each state s as defined by a pair of country-specific variables, as in the example of Section 6.

⁹This assumption can easily be relaxed. Its purpose is simply to reduce notation by keeping the environment symmetric.

Monetary Policy (INMP) or formation of a Monetary Union (MU). Under the former each country has its own money printing machine and decides monetary policy unilaterally. Under the MU the individual country money prints are replaced by a commonly managed print, that is used to produce the MU single currency. The loss of a policy instrument (money print) inherent to the MU generates costs and benefits. The cost is that countries in the MU are forced to use the same policy, which may be inefficient when countries are hit by asymmetric shocks. On the other hand, the benefit arises from the fact that the single money-print makes unilateral “surprise” devaluations (deviations from an agreed policy) impossible. We assume that a country’s decision to abandon the union (re-installing its own money print and currency) does not come as a surprise to the other country. This is a realistic assumption, justified by noting that the decision to leave the MU takes more time and is more easily observed by the other parties than the decision to devalue under INMP. Since deviations no longer come as a surprise in the MU, they become less attractive. This facilitates cooperation. In the next subsections we will describe in greater detail the implications associated with these two monetary arrangements.

Finally, it should be stressed that the qualitative nature of the results presented below would not change if the model was modified to account for other potential benefits of forming an MU, such as a reduction in transaction costs (this can be done by adding an indicator variable to the agents’ utility functions). We decided to overlook such effects for clarity of presentation.

2.1. Independent National Monetary Policy

When countries retain control over their monetary instrument we have the following timing of events. At the beginning of each period s is observed, then Home and Foreign simultaneously choose the monetary instrument $\pi(h_t)$ and $\pi^*(h_t)$, respectively, where $h_t = (s_1, \dots, s_t; \pi_1, \dots, \pi_{t-1}; \pi_1^*, \dots, \pi_{t-1}^*)$ denotes the history at time t .

A policy plan Π is a stochastic vector process which determines π for each

history. $\Pi \in P$, where:

$$P \equiv \left[\begin{array}{c} \pi, \bar{\pi} \\ \underline{\quad} \end{array} \right]^S \times \left[\begin{array}{c} \pi, \bar{\pi} \\ \underline{\quad} \end{array} \right]^{S^2} \times \left[\begin{array}{c} \pi, \bar{\pi} \\ \underline{\quad} \end{array} \right]^{S^3} \times \dots$$

Definition 1. A subgame perfect policy pair $\tilde{\gamma} = (\Pi, \Pi^*) \in P \times P$ is a policy plan (strategy) for each country such that at every history h_t each country chooses a best response to the other player's strategy.

Proposition 1. A policy pair (Π, Π^*) is subgame perfect under INMP if and only if the following holds (for all $s \in S$ and $\tau = 0, 1, 2, \dots$):

$$U^*(\pi_\tau^*, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^*(\pi_{\tau+i}^*, \pi_{\tau+i}, s_{\tau+i}) \right] \geq U^*(\pi_\tau^{*d}, \pi_\tau, s_\tau) + \delta \underline{w} \quad (2.1)$$

$$U(\pi_\tau, \pi_\tau^*, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U(\pi_{\tau+i}, \pi_{\tau+i}^*, s_{\tau+i}) \right] \geq U(\pi_\tau^d, \pi_\tau^*, s_\tau) + \delta \underline{w} \quad (2.2)$$

Where π_τ^d and π_τ^{*d} stand for the optimal deviations and \underline{w} is the lowest value attainable with a subgame perfect policy pair.

Proof: Appendix A.

We will denote the set of subgame perfect policy pairs $\tilde{\gamma}$ with $\tilde{\Gamma}$.

Lemma 1. The set of subgame perfect policy pairs, $\tilde{\Gamma}$, is compact and convex.

Proof: Appendix A.

Let $w(\tilde{\gamma}), w^*(\tilde{\gamma})$ be the discounted expected utility from a pair of subgame perfect policy sequences for Home and Foreign, respectively, and denote by \tilde{W} the set of all such pairs. We will refer to \tilde{W} as the set of subgame perfect payoffs.

Lemma 2. *The set of subgame perfect payoffs, \tilde{W} , is compact.*

Proof: Appendix A.

Given a specific utility function and parameter values, we can use the methods developed by Abreu, Pearce and Stacchetti (1990) to find the set \tilde{W} . We will do this for the example economy analyzed in Section 6. We can show that in this setup Folk Theorem type results will hold. That is, as $\delta \rightarrow 1$ the policies corresponding to what a benevolent central planner could achieve would be sustainable. Therefore, the interesting cases for our analysis are those in which δ is sufficiently small, but greater than zero so that better-than-Nash outcomes can be sustained.

2.2. Monetary Union

As an alternative to independent monetary policies, countries can choose to form a Monetary Union. When forming a MU, local currencies are replaced by a common currency and monetary policy is jointly determined. To describe the monetary policy decision making process in the MU we assume that the decision making body (e.g. a governing council) is composed of national representatives who make policy announcements (during a council meeting). Implementation of any given announcement requires unanimity. Failure to find a unanimous agreement over an announcement (different announcements) leads to a breakup of the union. Each country would then print its own currency and set its own policy.

Therefore, forming a union changes the game in the following two important aspects. The first is the condition that a *common* policy $\pi = \pi^*$ must be chosen if the union is to be maintained. Second, the timing of the game is changed in the following way: as before countries first observe the state s but then, instead of each setting policy independently they make simultaneous announcements, $\hat{\pi}_s(h_t), \hat{\pi}_s^*(h_t) \in \left[\pi, \bar{\pi} \right]$, about the inflation level they wish to implement. If the announcements coincide, the unanimously proposed policy is implemented and the union is continued into the future. Otherwise, the union is dissolved. In autarky, each country is assumed to follow the Nash equilibria of the INMP stage game. The key aspect of the new timing is that there is no way a country can

surprise another on its policy choice since, in order to have an independent policy, first it needs to break out of the union and print its own currency.

Policies corresponding to the Nash equilibrium of the stage game will be denoted by $\pi_N(s)$ and $\pi_N^*(s)$. $U_N(s)$ and $U_N^*(s)$ will be used for the payoffs associated with these policies conditional on a given state s . The expected value of welfare under this Nash equilibrium is: $V_N \equiv \frac{E_s(U_N(s))}{1-\delta}$ (identical for both countries).

In general the value of belonging to the union could be higher if we allowed for a reunification after a breakup or if, instead of assuming that governments revert into autarky, we assumed reversion to some other point in the set of sustainable INMP payoffs. The assumptions we make actually decrease the potential value of the union. Since one of our goals is to provide a rationale for the existence of a union, this strengthens our argument.¹⁰

3. Rationalizing the formation of a Monetary Union.

As mentioned earlier, changing the timing of the game eliminates the ability of the countries to cheat on the agreed upon path of play. This is an advantage of the Union over the independent national monetary policy arrangement. However, this advantage must be compared with the costs incurred from losing a policy instrument and the cost of going to autarky if the union collapses. We now provide two propositions that show that there exist parameter settings for which forming a Union is preferable.

Proposition 2. *If the shocks affect countries identically, the symmetric first best policies are sustainable under the union for all $\delta \in (0, 1)$.*

Proof: *If countries face the same shocks the first best policies, in which each country is equally weighted, require both countries to choose the same inflation*

¹⁰Since the union is assumed to be optimal in expectations countries would have incentives to setup a new union immediately after the breakup. A delay before re-union occurs could be explained by high fixed costs of forming a union after it has broken. Future extensions of the model might allow for reunification after a given number of periods or after incurring a fixed cost.

rate. The key is to note that these policies are sustainable under the union since there are no profitable deviations for any of the countries. Formally the necessary conditions for the first best to be sustainable under the MU are (for all $s \in S$ and $\tau = 0, 1, 2, \dots$):

$$U(\pi_\tau^{fb}, \pi_\tau^{fb}, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U \left(\pi_{\tau+i}^{fb}, \pi_{\tau+i}^{fb}, s_{\tau+i} \right) \right] \geq U_N(s_\tau) + \delta V_N \quad (3.1)$$

$$U^*(\pi_\tau^{fb}, \pi_\tau^{fb}, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^* \left(\pi_{\tau+i}^{fb}, \pi_{\tau+i}^{fb}, s_{\tau+i} \right) \right] \geq U_N^*(s_\tau) + \delta V_N \quad (3.2)$$

where π^{fb} stands for the symmetric first best inflation rates. Note that with perfectly correlated shocks, the first term on the left hand side of (3.1) and (3.2) is always greater than the first term on the right hand side. Therefore, the left hand side is greater than the right hand side for all δ . Hence, since the Union achieves first best it must weakly dominate the best symmetric equilibrium under the INMP arrangement.

Proposition 2 states that if the shocks faced by the countries are identical, the best sustainable symmetric equilibrium under the union weakly dominates the best reputational symmetric equilibrium obtained under INMP for all δ . The incentive constraints (3.1) and (3.2) indicate the origin of the welfare gain of the MU. As discussed in Section 2, a deviation from the MU common policy does not come as a surprise to the other country, but instead involves reversion to the Nash equilibrium. This is captured by the value of the deviation equal to $U_N(s_\tau)$, which cannot be greater than $U(\pi_\tau^{fb}, \pi_\tau^{fb}, s_\tau)$, the first best period payoff delivered by the union.

Furthermore, the next proposition shows that the welfare gain delivered by the MU is weakly increasing in δ when the shocks are symmetric.

Proposition 3. *If the shocks affect countries identically, there exists a $\bar{\delta} > 0$, such that for all $\delta < \bar{\delta}$ the symmetric first best policies are not sustainable with INMP. Further more, as $\delta \rightarrow 0$ the only sustainable equilibrium becomes the repeated static Nash.*

Proof: Appendix A.

It is intuitive that when it is costless to loose a policy instrument (because shocks are symmetric) the MU is superior. Further more, the lower δ the greater the benefits from forming a MU. In general we care for cases in which the shocks are not perfectly correlated. The previous propositions required that the shocks were identical but we can depart from this assumption in a continuous way. Hence, in general there will exist parameter combinations with imperfectly correlated shocks under which the best symmetric equilibrium under the union will dominate that achievable with Independent National Monetary Policies.

We will revisit these issues with greater detail in our analysis of the example economy of Section 6. Before formally addressing them, we must first characterize the equilibrium in the Monetary Union game.

4. Sustainable policies and the efficient frontier

This section defines the equilibrium notion that is used to analyze the MU game and establishes a recursive representation of the problem that is useful to characterize its properties. To simplify the notation, we introduce the indicator variable I_t , which equals 1 if the union is active at the beginning of period t .

4.1. Sustainable policies

Let us adopt the following:

Definition 2. *A sustainable equilibrium is a strategy for each country such that:*

(i) *At every history h_t with $I_t = 1$, each country chooses an action which is a best response to the other country's strategy.*

(ii) *At every history h_t with $I_t = 0$, each country chooses a history independent inflation policy which is a best response to the other country's strategy.*

This definition is very close to the subgame perfect definition but we are constraining the set of equilibria by requiring strategies to be history independent outside of the union.¹¹

¹¹This is consistent with our assumption that countries revert to the repeated static Nash when they abandon the MU.

Sustainable policies are those consistent with the implementation of the strategies described above. Let us denote a sustainable policy sequence pair by $\gamma \equiv (\Pi, \Pi^*)$ and the set of all sustainable policies by Γ .

Proposition 4. *A policy sequence pair is sustainable if and only if it satisfies the following conditions:*

C1 : For all h_τ with $I_\tau = 1$ and $\hat{\pi}_\tau = \hat{\pi}_\tau^$:*

$$U^*(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^*(\pi_{\tau+i}^*, \pi_{\tau+i}, s_{\tau+i}) \right] \geq U_N^*(s_\tau) + \delta V_N$$

$$U(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U(\pi_{\tau+i}, \pi_{\tau+i}^*, s_{\tau+i}) \right] \geq U_N(s_\tau) + \delta V_N$$

C2 : For all h_τ with $I_\tau = 1$ and $\hat{\pi}_\tau \neq \hat{\pi}_\tau^$:*

$$\pi_\tau = \pi_N(s) \quad , \quad \pi_\tau^* = \pi_N^*(s)$$

C3 : For all h_τ , $\tau \geq t$ with $I_t = 0$:

$$\pi_\tau = \pi_N(s) \quad , \quad \pi_\tau^* = \pi_N^*(s)$$

Proof: Appendix A.

Lemma 3. *The set of sustainable policies, Γ , is compact and convex.*

Proof: Appendix A.

Let $w(\Pi, \Pi^*), w^*(\Pi, \Pi^*)$ be the *expected* utility from a pair of policy sequences for Home and Foreign, respectively, and let W be the set of all pairs $(w(\Pi, \Pi^*), w^*(\Pi, \Pi^*))$ such that $(\Pi, \Pi^*) \in \Gamma$. We will refer to W as the set of sustainable payoffs.

Lemma 4. *The set of sustainable payoffs, W , is compact.*

Proof: Appendix A.

Corollary 1. *The value associated to the static Nash equilibrium, V_N , is the lower bound of the set W .*

Proof: follows directly from Proposition 4.

4.2. Efficient frontier

To characterize the set of efficient policies we need the following:

Definition 3. *A policy pair $(\Pi, \Pi^*) \in \Gamma$ is efficient if there exists no other element in Γ that Pareto dominates it.*

We define V_{\max} to be the maximal level of utility available to one of the countries from a policy sequence in Γ . We define V_{\min} as follows:¹²

$$\begin{aligned} V_{\min} &= \max_{\tilde{w}} \tilde{w} \\ \text{subject to} & : \\ (\tilde{w}, \tilde{w}^*) &\in W \\ \tilde{w}^* &= V_{\max} \end{aligned}$$

Proposition 5. *For all pairs $(w, w^*) \in W$ with $w^* \geq V_{\min}$ there exists an efficient allocation in Γ which delivers the payoff vector (\bar{w}, w^*) , where \bar{w} is defined as follows:*

$$\begin{aligned} \bar{w} &= \max_{\tilde{w}, \tilde{w}^*} \tilde{w} \\ \text{subject to} & : \\ (\tilde{w}, \tilde{w}^*) &\in W \\ \tilde{w}^* &\geq w^* \end{aligned}$$

¹²By the symmetry of the setup these values are identical for Home and Foreign. The asterisk is thus suppressed.

Proof: Appendix A.

The key of this proposition is not the existence of a solution to the maximization problem¹³ but rather that in the solution the second constraint must be binding ($\tilde{w}^* = w^*$). That implies that the efficient frontier of the set W is decreasing in the range $[V_{\min}, V_{\max}]$.

We can characterize the Pareto frontier as follows. Let $V(w_0)$ denote the expected utility delivered by a social planner to Home conditional on having promised an expected utility level w_0 to Foreign, $V: [V_{\min}, V_{\max}] \rightarrow [V_{\min}, V_{\max}]$. Then:

$$V(w_0) = \max_{(\Pi, \Pi^*)} E_0 \left[\sum_{t=1}^{\infty} \delta^{t-1} U(\pi_t, \pi_t^*, s_t) \right] \quad (4.1)$$

subject to:

$$(\Pi, \Pi^*) \in \Gamma \quad (4.2)$$

$$E_0 \left[\sum_{t=1}^{\infty} \delta^{t-1} U^*(\pi_t^*, \pi_t, s_t) \right] \geq w_0 \quad (4.3)$$

Constraint (4.2) imposes that policy pairs are sustainable, (4.3) is the “promise keeping” constraint i.e. it requires the plan to deliver an expected utility level of at least w_0 to Foreign.

The function V is decreasing, strictly concave and continuous.¹⁴ Furthermore, monotonicity implies it is differentiable almost everywhere. Unfortunately the previous definition of V is not very useful to figure out the properties of the optimal policy. The next proposition establishes a recursive formulation of the sequential problem that is helpful to characterize the equilibrium.

¹³This follows from the compactness of W .

¹⁴Decreasing follows from Proposition 5. Concavity follows since we assumed the period utility function is strictly concave in π_s and the constraint set is convex. Continuity is implied by the Theorem of the Maximum.

Proposition 6. *The function V satisfies the functional equation:*

$$V(w_0) = \max_{(\pi_s, w_s, H)} \sum_{s \in H} p_s [U(\pi_s, \pi_s, s) + \delta V(w_s)] + \sum_{s \in H^C} p_s [U_N(s) + \delta V_N] \quad (4.4)$$

subject to:

$$w_0 \leq \sum_{s \in H} p_s [U^*(\pi_s, \pi_s, s) + \delta w_s] + \sum_{s \in H^C} p_s [U_N^*(s) + \delta V_N] \quad (4.5)$$

$$U_N^*(s) + \delta V_N \leq U^*(\pi_s, \pi_s, s) + \delta w_s \quad \forall s \in H \quad (4.6)$$

$$U_N(s) + \delta V_N \leq U(\pi_s, \pi_s, s) + \delta V(w_s) \quad \forall s \in H \quad (4.7)$$

$$w_s \in [V_{\min}, V_{\max}] \quad (4.8)$$

Where H denotes the set of states where the union is sustained (H^C is its complement).

Proof: Appendix A.

Constraint (4.5) is the promise keeping constraint, constraints (4.6) and (4.7) are the sustainability (participation) constraints for Foreign and Home, respectively, so that they do not leave the union. Condition (4.8) imposes that promised continuation values have to be in W .

5. A characterization of the equilibrium in the Monetary Union

This section establishes some results to characterize the MU equilibrium. First we derive an important result regarding the sustainability of the MU and secondly we study policy dynamics inside the union.

5.1. Sustainability of the Monetary Union

The following is one of our main results and is key in simplifying the problem.

Proposition 7. *There exists an optimal set of states (\bar{H}) where the union is sustained which is independent of the promised value w_0 for $w_0 \in [V_{\min}, V_{\max}]$.*

Proof: Suppose that for two different promised values $w, \tilde{w} \in [V_{\min}, V_{\max}]$, the optimal solution has two different sets $H \neq \tilde{H}$ on which the union holds. Consider any state $s \in H, s \notin \tilde{H}$. Since $s \in H, \exists (\pi_s, w_s)$ such that the participation constraints hold. Hence if we included s in \tilde{H} , the participation constraint for Foreign would imply that its promise keeping constraint must be relaxed. Moreover, Home's participation constraint being satisfied would imply that Home must be weakly better off. The same argument holds for any $\tilde{s} \in \tilde{H}, \tilde{s} \notin H$. Therefore $\tilde{H} \cup H$ is optimal for both initial promised values.

This Proposition implies that, regardless of the initial bargaining power of the two countries in the initial institution design phase of the union, they would both agree on the states of the world in which to sustain the union and on which not. This property is quite appealing, the players will remain in the union as long as they find it mutually profitable in expectation. Though, as we will show later, their individual values of being part of the union will be changing as time goes by. From a technical standpoint the proposition facilitates the analysis of the Pareto frontier, since we need only find one optimal set of states on which the union holds.

Proposition 7 allows us to divide the problem into two sub-problems. The first one consists in finding the optimal set \bar{H} over which the union can be sustained. The second is to determine the optimal policy and continuation values (π_s, w_s) given this set.

5.2. Optimal policy and dynamics in the MU

Let us take \bar{H} as given and solve for the optimal policy inside the union. Consider the problem:

$$V(w_0) = \max_{(\pi_s, w_s)} \sum_{s \in \bar{H}} p_s [U(\pi_s, \pi_s, s) + \delta V(w_s)] + \sum_{s \in \bar{H}^c} p_s [U_N(s) + \delta V_N] \quad (5.1)$$

subject to:

$$w_0 \leq \sum_{s \in \bar{H}} p_s [U^*(\pi_s, \pi_s, s) + \delta w_s] + \sum_{s \in \bar{H}^C} p_s [U_N^*(s) + \delta V_N] \quad (5.2)$$

$$U_N^*(s) + \delta V_N \leq U^*(\pi_s, \pi_s, s) + \delta w_s \quad \forall s \in \bar{H} \quad (5.3)$$

$$U_N(s) + \delta V_N \leq U(\pi_s, \pi_s, s) + \delta V(w_s) \quad \forall s \in \bar{H} \quad (5.4)$$

$$w_s \in [V_{\min}, V_{\max}] \quad (5.5)$$

For any feasible allocation that promises a value of w_0 to Foreign, we can divide the state space in the following partition:

S_1 = states in which neither (5.3) nor (5.4) is binding

S_2 = states in which (5.3) is binding but not (5.4)

S_3 = states in which (5.4) is binding but not (5.3)

S_4 = states in which the union cannot be sustained.

The states in S_4 are such that either both countries mutually prefer to break the union or the country that prefers to remain in the union is unable (or unwilling) to provide the necessary incentives to prevent the other country from abandoning the union.¹⁵ Those states correspond exactly to the ones that belong to \bar{H}^C . As we have shown in Proposition 7, this set is independent of w_0 . Instead the sets S_1, S_2, S_3 are indexed by the initial value w_0 .

A useful characterization of the equilibrium properties of this problem is obtained from the Lagrangian representation of the functional equation that appeared above. Before doing so we must first address one last technical point. So far, we have shown that V is differentiable almost everywhere but, for the analysis that follows we actually need it to be differentiable everywhere. Koepl (2003) shows how things can go wrong in the environment of Kocherlakota (1996) if V is not differentiable everywhere. He also provides sufficient conditions to guarantee differentiability of V . We will consider parameter settings such that these

¹⁵By construction in Kocherlakota (1996) model it is never the case that both participation constraints bind at the same time. Hence $S_4 \equiv \emptyset$ in his setup. Instead, we impose the additional constraint that countries must choose the *same* policy while in the MU. This creates the possibility that some INMP outcomes cannot be replicated by the MU.

conditions are met. Let us write the Lagrangian:

$$\mathcal{L} \equiv \max_{\pi_s, w_s} \sum_{s \in \bar{H}} p_s [U(\pi_s, \pi_s, s) + \delta V(w_s)] + \sum_{s \in \bar{H}^C} p_s [U_N(s) + \delta V_N] \quad (5.6)$$

$$+ \lambda \left[\sum_{s \in \bar{H}} p_s (U^*(\pi_s, \pi_s, s) + \delta w_s) - w_0 \right] \quad (5.7)$$

$$+ \sum_{s \in \bar{H}} \mu_s [U^*(\pi_s, \pi_s, s) + \delta w_s - U_N^*(s) - \delta V_N] \quad (5.8)$$

$$+ \sum_{s \in \bar{H}} \nu_s [U(\pi_s, \pi_s, s) + \delta V(w_s) - U_N(s) - \delta V_N] \quad (5.9)$$

The first order conditions with respect to w_s give:

$$\begin{aligned} (p_s + \nu_s) V'(w_s) + \lambda p_s + \mu_s &= 0 \text{ if } w_s \in (V_{\min}, V_{\max}) \\ &\geq 0 \text{ if } w_s = V_{\max} \\ &\leq 0 \text{ if } w_s = V_{\min} \end{aligned} \quad (5.10)$$

The one with respect to π_s yields:

$$(p_s + \nu_s) U_\pi + (\lambda p_s + \mu_s) U_\pi^* = 0 \quad (5.11)$$

Note that at an internal solution (5.10) and (5.11) imply:

$$V'(w_s) = \frac{U_\pi}{U_\pi^*} \quad (5.12)$$

an efficiency condition equating the agents' marginal rate of substitution to the technical rate of transformation (the slope of the efficient frontier, V'). Let us study the implications of the first order conditions in the different regions of the state space:¹⁶

Region S_1 : Neither participation constraint binds, hence $\mu_s = \nu_s = 0$ which implies $V'(w_s) = -\lambda < 0$. Note, moreover, that the envelope condition

¹⁶The analytical derivation of the equilibrium properties in regions S_1, S_2 and S_3 is analogous to the analysis developed by Kocherlakota (1996) for a risk-sharing problem.

(Benveniste-Scheinkman) yields $V'(w_0) = -\lambda$, which gives:

$$V'(w_0) = V'(w_s). \quad (5.13)$$

It follows from the strict concavity of V that $w_0 = w_s$. Hence, when neither participation constraint binds, the expected utility promised to each country in the union is the same one with which the country entered the period, i.e. the promised value is kept constant at w_0 for Foreign and at $V(w_0)$ for Home. Moreover, equations (5.12) and (5.13) show that current policy (π) in the states of this region is such that a constant ratio between the marginal utilities of Home and Foreign is maintained. Note how this last result is isomorphic to the one that emerges as the internal optimum of a planner's problem in which each country's utility function is given a time-invariant Pareto weight.

Region S_2 : The participation constraint of Foreign binds, i.e. $\mu_s > 0, \nu_s = 0$. This yields:

$$V'(w_s) = V'(w_0) - \frac{\mu_s}{p_s} \quad (5.14)$$

which implies that $w_s > w_0$ (by the concavity of V). Hence in states of the world belonging to S_2 the promised utility to Foreign increases (the expected utility of Home decreases). It follows from equation (5.12) that the current policy choice is also closer to Foreign's preferred policy. This contrasts with the constant weighting observed in the presence of an enforcement technology (i.e. problem without participation constraints).

Region S_3 : This yields symmetric opposite results to those in Region S_2 .¹⁷

These results illustrate the nature of optimal policy in a monetary union with voluntary participation. Policy obeys a state contingent rule which only gets revised when one of the countries has the incentive to leave the union (i.e. the

¹⁷Participation constraint of Home binds, i.e. $\mu_s = 0, \nu_s > 0$.

$$V'(w_s) = \frac{p_s}{p_s + \nu_s} V'(w_0)$$

which implies $w_s < w_0$ (by the concavity of V and recalling $V' < 0$). Therefore, in states of the world belonging to S_3 both the current and promised utility delivered to Foreign decrease.

participation constraint binds). When no such incentives arise, the rule is analogous to the efficient one produced by a planner who maximizes the utility of the two countries assigning each of them a Pareto weight. If one country has the incentive to leave the union, then the new policy rule for the current and future periods is closer to that country's unilateral optimal choice. The new rule increases the country's weighting in the current policy decision and the expected continuation value from remaining in the union, making the country indifferent between remaining or leaving. This rule remains in place until the next "renegotiation", i.e. until a state is again reached where one participant has an incentive to leave.

Depending on the primitive features of the problem, these dynamics may continue forever, may eventually reach a state where the union collapses, or may converge to a region where participation constraints never bind and "renegotiations" cease to occur. This last case is explored in the next subsection.

5.3. When is participation not a problem?

Given the previous characterization of optimal policy we can explore the consequences for the dynamics of a country's (ex-ante) time- t value of being in the union, conditional on the MU not breaking up.

Let \underline{w} be the lowest value $w \in W$ such that for all $s \in S$ the participation constraint for Foreign does not bind when $w_s = \underline{w}$ (therefore $\frac{U_{\pi_s}}{U_{\pi_s^*}}$ is constant). Now, if Home's participation constraint does not bind for $V(\underline{w})$, it means that once Foreign is assigned a promised value in the range $[\underline{w}, V(\underline{w})]$, then the participation constraint will never bind again. This leads us to:

Proposition 8. *Suppose that the interval $[\underline{w}, V(\underline{w})]$ is non-empty then:*

- i) If $w_0 \in [\underline{w}, V(\underline{w})]$, $w_t = w_0$ for all t .*
- ii) If $w_0 < \underline{w}$ then w_t converges monotonically to \underline{w} . If instead, $w_0 > V(\underline{w})$ then w_t converges monotonically to $V(\underline{w})$.*

Proof: Appendix A.

Intuitively what is going on is that the agent with $w_0 > V(\underline{w})$ is so well off that his constraint does not bind regardless of the state s . On the other hand, the

other agent's constraint for sure binds in at least one state of the world. Hence, given the previous characterization of the optimal policy and conditional on not hitting any state in S_4 , we know that the continuation value must increase for the agent that was not very well off to start with and vice-versa for the other agent.

If the premise of this proposition holds true, then eventually policy in the MU would just become a constant weighting between the countries' preferred policies. This result identifies the conditions under which the results by Canzoneri and Henderson (2000, chapter 2), in which monetary policy in the union obeys a constant Pareto weighting of the players' preferred policy, are justified in the absence of an enforcement technology.

6. An example economy

This section utilizes a stylized two-country economy to illustrate, by means of simple algebra and numerical computations, some of the results that were discussed above in a more general context.

Let Home's objectives be described by the intertemporal objective function $V = \sum_{t=0}^{\infty} \delta^t U_t$. The period utility function U_t is given by:¹⁸

$$U(\pi_t, \pi_t^*, s_t) = \left[-\frac{(\pi_t - \varepsilon_t)^2}{2} + \alpha(\pi_t - \pi_t^*) \right] (1 - \delta) \quad (6.1)$$

where π_t and π_t^* denote the policy instruments set, respectively, by Home and Foreign, and ε_t is a desired target for Home's instrument in period t (an analogous utility expression holds for Foreign). The linear term $\pi_t - \pi_t^*$ posits that, irrespective of the desired target ε_t , Home benefits from setting the instrument "above" the level chosen by Foreign. For concreteness we can think of π_t as denoting Home's inflation, over which policy makers have perfect control. This abstraction provides a stylized way to describe a country's motive to surprise its neighbor by means of an unanticipated monetary expansion. This simple mechanism gives rise to a coordination problem.

¹⁸Since the objective of this section is mainly to illustrate the previous theory, we chose not to do a formal derivation of this particular objective function.

The random variable ε_t in (6.1) captures, in a convenient way, the time-varying priorities of the monetary policy authority with regard to inflation. There are S states of the world, each characterized by the pair $s \equiv (\varepsilon_s, \varepsilon_s^*)$. It is assumed that the random variables ε and ε^* have the following properties:

$$\begin{aligned} E(\varepsilon) &= E(\varepsilon^*) = \bar{\varepsilon} \\ \text{var}(\varepsilon) &= \text{var}(\varepsilon^*) = \sigma^2 \end{aligned}$$

with covariance $\text{cov}(\varepsilon_s, \varepsilon_s^*)$. We will focus on an ex-ante symmetric case, so that even though the realizations of ε_s and ε_s^* may differ, their joint distribution is symmetrical.

We will next consider the equilibria which emerge from this setup under alternative equilibrium notions and assumptions about the enforcement technology.

6.1. Symmetric first-best (Ramsey)

It is useful as a benchmark to note that the symmetric first best strategies that maximize the welfare of Home and Foreign prescribe that, in each period, countries set their policy according to: $(\pi_t = \varepsilon_t, \pi_t^* = \varepsilon_t^*)$. The expected value delivered by adherence to this strategy (identical for both Home and Foreign) is: $V_{best} = \sum_s \alpha (\varepsilon_t - \varepsilon_t^*) p_s = 0$. Without a commitment technology, however, countries may have an incentive to deviate from the proposed strategy, as shown next.

6.2. Equilibrium of the one-shot game (Nash)

In the Nash equilibrium each country sets its policy instrument (π_t, π_t^*) after the shock $(\varepsilon_s, \varepsilon_s^*)$ is realized, taking the other country's instrument as given. This yields the following strategies for the players:

$$\begin{aligned} \pi_t &= \varepsilon_t + \alpha \\ \pi_t^* &= \varepsilon_t^* + \alpha \end{aligned}$$

which imply the period payoff:

$$\begin{aligned} U_N(s) &= \left[-\frac{\alpha^2}{2} + \alpha (\varepsilon_t - \varepsilon_t^*) \right] (1 - \delta) \\ U_N^*(s) &= \left[-\frac{\alpha^2}{2} - \alpha (\varepsilon_t - \varepsilon_t^*) \right] (1 - \delta) \end{aligned}$$

Expected utility under Nash is $U_N^e = -\frac{\alpha^2}{2} (1 - \delta)$, hence the expected utility enjoyed by each country under the Nash equilibrium is:

$$V_N \equiv \frac{U_N^e}{1 - \delta} = -\frac{\alpha^2}{2}.$$

It is immediate to note that the presence of the spillover effect ($\alpha \neq 0$) causes welfare under the Nash equilibrium to be lower than is achievable with the first best.

6.3. Subgame perfect equilibria in the repeated game (INMP)

The repeated nature of the game allows countries to sustain reputational equilibria that dominate the Nash equilibrium in terms of welfare. We seek to characterize these equilibria to describe the instance in which countries coordinate their independent national monetary policy (INMP) and improve upon the Nash outcomes.

Equations (2.1) and (2.2) in Proposition 1 characterize sustainable strategies in this repeated game. They state that it must be in each country's interest to stick to the proposed policy in all period and for all states of the world. The right side of these equations states that a deviation from the optimal plan is punished in the future with the reversion to a "bad equilibrium", which has an expected value of \underline{w} .¹⁹

The credibility of this threat requires that the pair of strategies that yields \underline{w} is itself a subgame perfect equilibrium satisfying equations (2.1) and (2.2). Computing the value of the "bad equilibrium" \underline{w} is thus key to characterize sustainable equilibria. Focusing on the symmetric equilibria of our example economy, the worst (symmetric) subgame perfect equilibrium that can be used as a threat to

¹⁹The root of this idea is in the "stick and carrot" strategy first proposed by Abreu (1988).

sustain efficient outcomes satisfies the following conditions:

$$\begin{aligned}
\underline{w} &\equiv \min_{\pi, \pi^*, w_s} \Sigma_s [U^*(\pi^*, \pi, s) + \delta w_s] p_s & (6.2) \\
&\text{subject to :} \\
U^*(\pi^*, \pi, s) + \delta w_s &\geq U^*(\pi^{*d}, \pi, s) + \delta \underline{w} & \forall s \\
U(\pi, \pi^*, s) + \delta V(w_s) &\geq U(\pi^d, \pi^*, s) + \delta \underline{w} & \forall s \\
w_s &\in \tilde{W}
\end{aligned}$$

where \tilde{W} is the set of sustainable payoffs, $V(w_s)$ is the maximum value attainable by Home conditional on the promised value w_s to Foreign and π^d (π^{*d}) denotes the optimal deviation from the policy plan for Home (Foreign).²⁰ The two incentive constraints impose the SPE requirement that both countries have an incentive to stick with the optimal plan. The recursive formulation is achieved expressing the continuation strategy by means of its value, following Abreu, Pearce and Stacchetti (1990).

A deviation from the strategy prescribed by the “worst equilibrium” is punished with the future reversion to the same equilibrium (Abreu, 1988). As is known from the work of Abreu, Pearce and Stacchetti, such punishments can be harsher than the reversion to the static Nash equilibrium and thus allow a “good” equilibrium to be sustained. The best (symmetric) sustainable equilibrium satis-

²⁰The computation of the worst value \underline{w} thus utilizes the value function $V(w_s)$, which traces the frontier of the maximal utility attainable by Home provided the utility delivered to Foreign is w_s . Formally, the value function $V(w_s)$ is defined as follows:

$$\begin{aligned}
V(w_o) &\equiv \max_{\pi, \pi^*, w_s} \Sigma_s [U(\pi, \pi^*, s) + \delta V(w_s)] p_s \\
&\text{subject to :} \\
w_o &= \Sigma_s [U^*(\pi, \pi^*, s) + \delta w_s] p_s \\
U^*(\pi^*, \pi, s) + \delta w_s &\geq U^*(\pi^{*d}, \pi, s) + \delta \underline{w} & \forall s \\
U(\pi, \pi^*, s) + \delta V(w_s) &\geq U(\pi^d, \pi^*, s) + \delta \underline{w} & \forall s \\
w_s, \underline{w}, w_o &\in \tilde{W}
\end{aligned}$$

fies:

$$\begin{aligned} \bar{w} &\equiv \max_{\pi, \pi^*} \Sigma_s [U(\pi, \pi^*, s) + \delta \bar{w}] p_s & (6.3) \\ \text{subject to } & : \\ U^*(\pi^*, \pi, s) + \delta \bar{w} &\geq U^*(\pi^{*d}, \pi, s) + \delta \underline{w} & \forall s \\ U(\pi, \pi^*, s) + \delta \bar{w} &\geq U(\pi^d, \pi^*, s) + \delta \underline{w} & \forall s \\ \bar{w} &= \Sigma_s [U^*(\pi, \pi^*, s) + \delta \bar{w}] p_s \end{aligned}$$

where the last constraint imposes the symmetry requirement. The “best” equilibrium is “self rewarding”, i.e. adherence to the prescribed strategy is rewarded with the continuation of the same strategy tomorrow.

With reputation, the first best can be sustained provided the discount factor is sufficiently large. In the example economy, it is easy to show that for a given “punishment value” \underline{w} , the first best is sustainable if $\delta \geq \frac{\alpha^2}{\alpha^2 - 2\underline{w}}$. For instance, if the Nash equilibrium was chosen as a punishment for deviations ($V_N = -\frac{\alpha^2}{2}$), the first best can be sustained with reputation provided $\delta \geq \frac{1}{2}$. Even if the discount is smaller than this value, however, the first best might still be supported if a credible (i.e. SPE) punishment more severe than reversion to Nash exists. In general, finding the “best” (possibly smaller than the first-best) and the “worst” sustainable values from the solution of problems (6.2) and (6.3) can be done numerically for a given model parametrization. A few examples are discussed in Section (6.5).

6.4. Monetary Union with an enforcement technology

Let us next define the Monetary Union as an arrangement in which both countries abandon sovereignty over their own instrument and adopt a common instrument so that $\pi_t = \pi_t^*$ forever (i.e. no possibility of reverting to autarky is admitted). In this setting the period utility each country derives from the union is given by:

$$U(\pi_t, \pi_t, s) = -\frac{(\pi_t - \varepsilon_t)^2}{2} (1 - \delta) \quad (6.4)$$

$$U^*(\pi_t, \pi_t, s) = -\frac{(\pi_t - \varepsilon_t^*)^2}{2} (1 - \delta) \quad (6.5)$$

Simple algebra shows that if membership in the union is externally enforced

there may exist ex-ante welfare gains from participating to it. This amounts to solving the following Pareto problem (with enforcement the dynamic problem breaks down into a sequence of static problems):

$$\max_{\pi_s} E_s [\kappa U(\pi_s, \pi_s, s) + (1 - \kappa)U^*(\pi_s, \pi_s, s)]$$

subject to $\pi_s = \pi_s^*$ where κ is the Pareto weight. Straightforward algebra reveals that the optimal policy takes the form:

$$\pi_s = \kappa \varepsilon_s + (1 - \kappa) \varepsilon_s^* \tag{6.6}$$

Note that the Pareto weight κ determines the degree to which the rule is tilted towards the welfare of Home versus Foreign. It is simple to compute expected welfare from joining the MU, naturally a function of κ :

$$\begin{aligned} V_{MU}(\kappa) &= -(1 - \kappa)^2 [\sigma^2 - cov(\varepsilon_s, \varepsilon_s^*)] \\ V_{MU}^*(\kappa) &= -(\kappa)^2 [\sigma^2 - cov(\varepsilon_s, \varepsilon_s^*)] \end{aligned} \tag{6.7}$$

Note how the expected welfare in the union is increasing in $\frac{cov(\varepsilon_s, \varepsilon_s^*)}{\sigma^2}$, the linear correlation coefficient between the shocks hitting the two countries.

A comparison of the expected welfare under the Nash equilibrium with expected welfare in the “union with-enforcement” reveals that the union dominates autarky in welfare terms provided α is sufficiently high (i.e. the coordination problem is relevant) or $\frac{cov(\varepsilon_s, \varepsilon_s^*)}{\sigma^2}$ is sufficiently large (i.e. shocks are similar across country and hence the flexibility costs of the union are low). This comparison provides a rationale for a monetary union. But it may be criticized for being biased because the “alternative” option considered (Nash) can be improved upon if countries can sustain a reputational equilibrium.

Interestingly, as showed in the numeric examples of Section 6.5, a MU may turn out to be welfare improving even in comparison to the best sustainable reputational equilibrium. This point, which was illustrated analytically for the case of symmetric shocks in Section 3, provides a more robust rationale for a monetary union than the one obtained under the restriction that Nash is the only alternative to the MU.

α	\underline{w}	V_{nash}	V_{MU}	\bar{w}
0.3	-0.12	-0.05	-0.06	-0.001
1	-1.4	-0.50	-0.06	-0.014
3	-12.7	-4.5	-0.06	-0.12
5	-35.4	-12.5	-0.06	-0.33

6.5. Numeric examples

Assume the state $s \equiv (\varepsilon, \varepsilon^*)$ is i.i.d. and that there are three possible states of the world: $s \equiv (\varepsilon, \varepsilon^*) \in \{(0, 0), (0, 1), (1, 0)\}$. Let the probability mass of each state be respectively $p_s \equiv \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$ and the intertemporal discount be $\delta = 0.2$. The rows of Table 1 report the welfare values of alternative subgame perfect symmetric equilibria. Each row is computed for a different value of the externality α (first column). Greater values of this parameter imply that the externality problem is more relevant, as is reflected in the worsening of the Nash equilibrium value (third column). Note that the discount factor was chosen to be sufficiently low so that the first best could not be sustained by reputation. However, the first two rows in the Table show that when the externality is sufficiently small the value of the best reputational equilibrium (last column) is very close to the value of the first best (zero) and, more importantly, that it is greater than the value delivered by a symmetric monetary union ($V_{MU}(\kappa)$ with $\kappa = \frac{1}{2}$). Note however that as the externality gets sufficiently large (third row), welfare under the MU dominates the value of the best (symmetric) reputational equilibrium.

Figure 1 depicts the efficient welfare frontier under the reputational equilibria (INMP) and under the MU (dotted line) for the case in which $\alpha = 3$. The Nash value is depicted in the bottom-left corner of the figure. It appears that welfare for Home and Foreign improves substantially under both the INMP and the MU regime in comparison to the Nash equilibrium. Moreover, note that the set of values which is sustainable under the monetary union Pareto dominates the corresponding values attained with the INMP. This point, as we mentioned, provides a rationale for a monetary union even when “reputation” is feasible.

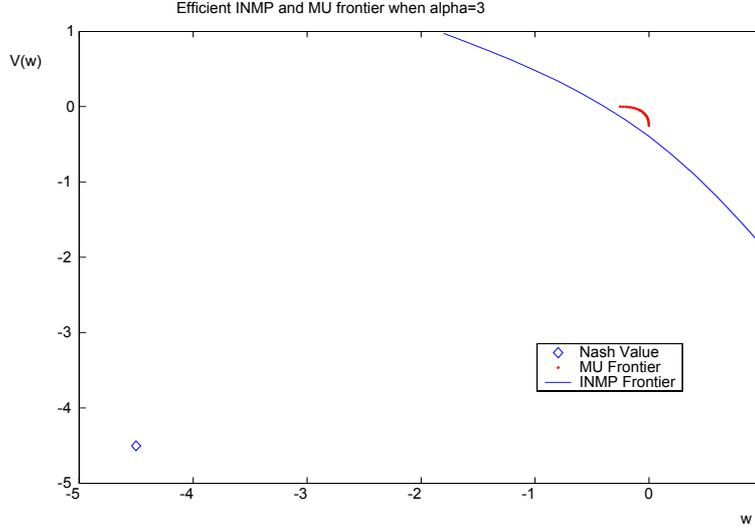


Figure 1

6.6. Monetary Union without enforcement technology

The results of Section 6.4 were derived under the assumption that countries did not have an option to leave the MU. Relaxing that assumption is important to gain further insights into the mechanism that allows the MU to be sustained.

Without the “enforcement technology”, the following participation constraints need to be satisfied for countries to remain in the Union (in each period and for each state):

$$U^*(\pi, \pi, s) + \delta w_s \geq U_N^*(s) + \delta V_N \quad (6.8)$$

$$U(\pi, \pi, s) + \delta V(w_s) \geq U_N(s) + \delta V_N \quad (6.9)$$

where w_s and $V(w_s)$ are, respectively, the promised values for Foreign and Home.

Proposition 9. *Policy in the example economy is a convex combination of the policies preferred by Home (ε_s) and Foreign (ε_s^*):*

$$\pi_s = \kappa_s \varepsilon_s + (1 - \kappa_s) \varepsilon_s^* \quad (6.10)$$

where the weight κ_s is given by:

- (i) $\kappa \equiv \frac{1}{1+\lambda}$ when neither participation constraint binds (Region S_1)

- (ii) $\kappa_s^F \equiv \frac{p_s}{p_s(1+\lambda)+\mu_s}$ when Foreign's participation constraint binds (Region S_2)
 (iii) $\kappa_s^H \equiv \frac{p_s+\nu_s}{p_s(1+\lambda)+\nu_s}$ when Home's participation constraint binds (Region S_3).

Proof: Follows from the first order condition (5.11) and equation (6.4) by noting that the Lagrange multiplier μ_s and ν_s are zero when their respective constraint does not bind.

When no participation constraint binds policy obeys a time-invariant weighting of the policies preferred by Home (ε_s) and Foreign (ε_s^*), with weights κ and $(1-\kappa)$, respectively. This obviously resembles the outcomes obtained when participation is not an issue (Section 6.4). More interestingly, the proposition indicates that if a state is reached where the participation constraints of a country binds, then the optimal policy rule (6.10) prescribes that this country is given a greater weight in decision process (note that $\kappa_s^H > \kappa$ and that $\kappa_s^F < \kappa$). As was discussed for the general case in Section 5.2, when a country's participation constraint binds the optimal rule provides incentives to remain in the union by increasing *both* the future value of belonging to the union (the country is promised a greater "expected utility") and the current return. In the example, the latter mechanism takes a simple linear form. Optimal policy without enforcement thus resembles the solution of a planning problem with time-varying Pareto weights. After hitting a state where its participation constraint binds, Home is assigned a greater importance in today's decision and is promised a correspondingly greater weight in future. From this period onwards, until another state is reached in which the participation constraint of Home or Foreign binds, policy in the union is conducted according to these new "weights".

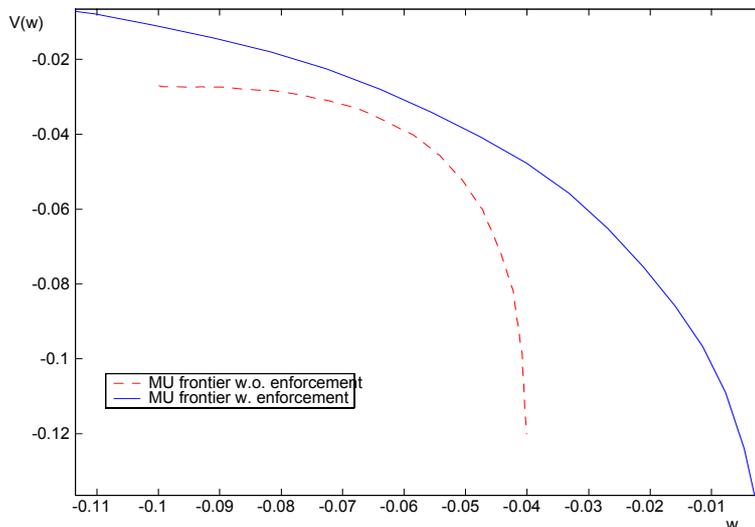


Figure 2: Utility frontier

The workings of optimal policy can be illustrated by means of a numerical example.²¹ The efficient utility frontier under the MU with and without enforcement for this example are shown in Figure 2. Under the chosen parameterization, no portion of the efficient frontier is sustainable, as indicated by the fact that the latter frontier lies below the efficient one. This indicates that participation constraints bind, at least in some states. Note that while the countries agree on the policy to be followed in state s_1 (in which they share the same inflation objective) they have different views on policy in s_2 and s_3 .²² The optimal incentive scheme reported in Table 1 shows how such diverging views are balanced in a voluntary MU. When a country's participation constraint binds the incentive to remain in the union is provided by increasing *both* the current return and the future value of belonging to the union, i.e. the country is promised a greater continuation value (expected utility). For example, suppose Foreign entered the MU with a relatively low expected utility level (w_o), equal to -0.08 (the first line of Table 1). Foreign is stuck with this value as long as the economy remains in s_1 . If state s_2 is reached, the scheme prescribes that Foreign expected utility from participating

²¹As in the examples considered above, we assume that there are three possible states of the world $s \equiv (\varepsilon, \varepsilon^*) \in \{s_1 = (0, 0), s_2 = (0, 1), s_3 = (1, 0)\}$. The results reported in Table 1 are obtained under the assumption that the probability mass of each state is, respectively, $p_s \equiv \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$, the intertemporal discount $\delta = 0.8$ and the externality $\alpha = 0.6$.

²²The preferred policy profile $\{\pi_1, \pi_2, \pi_3\}$ for Home is $\{0, 0, 1\}$, for Foreign $\{0, 1, 0\}$.

Table 1. Policy in a voluntary MU

<i>Initial promise</i>	<i>F's promised values</i>			<i>H's promised values</i>			<i>Current Policy</i>		
	w_1	w_2	w_3	$V(w_1)$	$V(w_2)$	$V(w_3)$	π_1	π_2	π_3
w_o	w_1	w_2	w_3	$V(w_1)$	$V(w_2)$	$V(w_3)$	π_1	π_2	π_3
-0.08	-0.08	-0.06	-0.08	-0.02	-0.04	-0.02	0.0	0.6	0.8
-0.07	-0.07	-0.06	-0.07	-0.03	-0.04	-0.02	0.0	0.6	0.7
-0.06	-0.06	-0.06	-0.07	-0.03	-0.04	-0.03	0.0	0.6	0.6
-0.05	-0.05	-0.05	-0.06	-0.06	-0.05	-0.04	0.0	0.7	0.4
-0.04	-0.04	-0.04	-0.06	-0.08	-0.08	-0.04	0.0	0.9	0.4

in the union is raised to -0.06 (in expected terms). A comparison of the first and third line of Table 1 shows that this corresponds to assigning Foreign a greater weight on current policy decisions in s_3 , as inflation in that state gets closer to Foreign desired value (i.e. π_3 is reduced from 0.8 to 0.6). This policy remains in place until the economy eventually reaches s_3 , the state where Home participation constraint binds. At this point current policy is shifted towards Home preferred policy (Foreign weight on current policy in s_3 decreases from 0.4 to 0.3) and Home continuation utility is raised (Foreign expected utility is reduced from -0.06 to -0.07). In the parametrization of this example such swings continue forever. Other examples may be constructed in which the MU eventually collapses or, alternatively, reaches a point on the MU efficient frontier (and remains there forever).

The results highlight an important feature of optimal policy in a voluntary MU, namely that MU members may occasionally be given “special consideration” to preserve the value of the union to all participants.

7. Concluding remarks

History offers several examples of countries participating in international agreements that constrain unilateral policy actions, such as exchange rate interventions, therefore removing one adjustment mechanism otherwise available to policy makers. This paper explored the motives behind a country’s choice to voluntarily adopt such a constraint, as it occurs in a monetary union.

We model the MU as a technology which precludes policy surprises (e.g. an

unexpected exchange rate realignment) at the cost of foregoing a policy instrument. It is shown that this technology may dominate a coordinated system with independent national currencies, hence providing a rationale for the formation of an MU.

Departing from the previous literature on international monetary arrangements we abandon the assumption that countries are exogenously bound to the monetary union and explicitly model their incentives to remain within the union or to leave it. This leads to two novel results.

First, while optimal policy when participation is exogenously assumed obeys a time-invariant weighted average of both countries' preferred policies, optimal policy in a "voluntary" MU responds to a country's incentive to abandon the union by tilting current and future policy in its favor.²³ This enriches policy dynamics significantly and may provide insights into the workings of decision making within supra-national institutions, such as the European central bank, where "national interests" are compounded in the choice of the common policy. Our result suggests that policy, besides depending on MU "average" economic conditions, should occasionally respond to the conditions of the member country for whom adherence to the common policy is costly. This is consistent with the findings of Heinemann and Hüfner (2002) who report descriptive and econometric evidence that national divergence from euro area averages matters for the decisions of the ECB Governing Council.

The second new result is that our model may deliver a break-up of the union along the equilibrium path. Given the second best nature of the policy choices available in the MU, this result stems from the fact that, even when the union is desirable ex-ante, there may be some states of the world in which a country's incentive to abandon the common policy and its future benefits are irresistible ex-post. The paper shows that the introduction of this new feature, which at first appears as a potentially serious complication of the problem considered, does not impair the tractability of the problem. This result is important because, as

²³Hence, optimal policy is history dependent in this setting and only in the long run, for some special cases, we can replicate the result, obtained when participation is exogenously assumed, that policy obeys a time-invariant weighted average of both countries' preferred policy (see Proposition 8).

mentioned in the introduction, history provides us with examples of supra-national monetary arrangements, including currency unions, that eventually broke apart (see Cohen, 1993). Our framework provides a first formal analysis of a country's incentives to voluntarily participate in a monetary union.

The distinguishing aspect of what we called a “union” is that, while the agents belong to it, they must choose the *same* action. Therefore, even though belonging to the union might be preferred in expectations, the lack of flexibility introduced by this constraint introduces ex-post incentives to leave the union. In some instances, a compromise regarding the common action to be taken will be reached but in others the union will be dissolved. While we focussed in this paper on monetary policy (and occasionally mentioned exchange rate policy), the key features of our analysis also appear in other settings where coordination on a single action matters, such as fiscal policies in a MU (consider e.g. the choice of the excessive deficit in the Stability and Growth Pact), political parties in a coalition or firms in a joint venture. Our results may find fruitful application in those fields. We leave this task for future research.

A. Appendix: Proofs

Proof of Proposition 1: Consider a policy pair that satisfies (2.1) and (2.2) for all histories; then since there are no profitable deviations at any history it implies that players are playing a best response to each other.

Conversely given that the players are playing a best response to each other, it must be the case that they cannot find any profitable deviation at any given history hence (2.1) and (2.2) must be satisfied.

Proof of Lemma 1:

$\tilde{\Gamma}$ is compact since it is a closed subset of $P \times P$ which is compact. Convexity follows from the concavity of $U(\cdot)$.

Proof of Lemma 2:

\tilde{W} is bounded since the per period utility is bounded and $\delta \in (0, 1)$ to prove compactness we therefore need only prove that it is closed. Consider a sequence of discounted utility vectors (w_n, w_n^*) that converges to (\tilde{w}, \tilde{w}^*) for each n , let (Π_n, Π_n^*) be the associated policies with these payoffs. Since $\tilde{\Gamma}$ is compact there is a convergent subsequence $(\Pi_{n_k}, \Pi_{n_k}^*)$, let $(\tilde{\Pi}, \tilde{\Pi}^*)$ denote its limit. The subsequence $(w_{n_k}, w_{n_k}^*)$ must also converge to (\tilde{w}, \tilde{w}^*) . By the continuity of U over policies the payoff from $(\tilde{\Pi}, \tilde{\Pi}^*)$ is given by (\tilde{w}, \tilde{w}^*) , hence by definition it is an element of \tilde{W} .

Proof of Proposition 3:

For the first best policies to be sustainable under INMP the following must hold for all $s \in S$ and $\tau = 0, 1, 2, \dots$:

$$U^*(\pi_\tau^{fb}, \pi_\tau^{fb}, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^*(\pi_{\tau+i}^{fb}, \pi_{\tau+i}^{fb}, s_{\tau+i}) \right] \geq U^*(\pi_\tau^{*d}, \pi_\tau^{fb}, s_\tau) + \delta \underline{w} \quad (\text{A.1})$$

$$U(\pi_\tau^{fb}, \pi_\tau^{fb}, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U(\pi_{\tau+i}^{fb}, \pi_{\tau+i}^{fb}, s_{\tau+i}) \right] \geq U(\pi_\tau^d, \pi_\tau^{fb}, s_\tau) + \delta \underline{w} \quad (\text{A.2})$$

where π^{fb} stands for the symmetric first best inflation level, π^d, π^{*d} stand for the optimal deviations and \underline{w} is the lowest value in \tilde{W} .

The first term on the right hand side of A.1 (A.2) is always greater than the corresponding first term on the left hand side (by the assumption that the first best is not the Nash equilibrium of the stage game). Furthermore, \underline{w} is a weakly

increasing function of δ . Hence, as $\delta \rightarrow 0$ the constraints become binding and will eventually be violated for all $\pi_\tau \neq \pi_N(s)$ and $\pi_\tau^* \neq \pi_N^*(s)$. Therefore, clearly the advantage of the Union over the INMP will increase as δ decreases. In the extreme case of $\delta = 0$ only V_N is subgame perfect under INMP but first best is attainable with the MU.

Proof of Proposition 4:

Consider a policy pair (Π, Π^*) that satisfies (C1), (C2) & (C3). We see immediately that part (ii) of Definition 2 is satisfied iff (C3) is satisfied. If (C1) & (C2) hold then it follows that players are playing a best response to each other. Given that Home proposes $\hat{\pi}_\tau$, Foreign would only propose $\hat{\pi}_\tau^* = \hat{\pi}_\tau$ if it is weakly better than autarky (where Nash equilibrium strategies π_N are played).

Conversely given that the players are playing a best response to each other, there are two cases. Either they announce the same π_τ and remain in the union, in which case the expected utility must be higher than autarky (as from C1). Or, announcements differ and Nash best responses $\pi_N(s), \pi_N^*(s)$ are played from then on.

Proof of Lemma 3:

Γ is compact since it is a closed subset of $P \times P$ which is compact. Convexity follows from the concavity of $U(\cdot)$.

Proof of Lemma 4:

W is bounded since the per period utility is bounded and $\delta \in (0, 1)$ to prove compactness we therefore need only prove that it is closed. Consider a sequence of discounted utility vectors (w_n, w_n^*) that converges to (\tilde{w}, \tilde{w}^*) for each n , let (Π_n, Π_n^*) be the policies associated with these payoffs. Since Γ is compact there is a convergent subsequence $(\Pi_{n_k}, \Pi_{n_k}^*)$, let $(\tilde{\Pi}, \tilde{\Pi}^*)$ denote its limit. The subsequence $(w_{n_k}, w_{n_k}^*)$ must also converge to (\tilde{w}, \tilde{w}^*) . By the continuity of U over policies, the payoff from $(\tilde{\Pi}, \tilde{\Pi}^*)$ is given by (\tilde{w}, \tilde{w}^*) , hence by definition it is an element of W .

Proof of Proposition 5:

Suppose that the constraint was not binding. This implies that there is at least one state where the participation constraint is slack:

$$U^*(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^*(\pi_{\tau+i}^*, \pi_{\tau+i}, s_{\tau+i}) \right] > U_N^*(s_\tau) + \delta V_N$$

Now let $\bar{\pi}_\tau$ denote the optimal level of inflation that Home would choose if it could unilaterally set a given π for both countries. First note that if $\pi_\tau \neq \bar{\pi}_\tau$, the

value to Home can be increased by bringing policy closer to $\bar{\pi}_\tau$, hence decreasing the value to Foreign until the constraint binds.

If $\pi_\tau = \bar{\pi}_\tau$ and $\tilde{w}^* > w^*$, future policy can be tilted towards Home's preferred policy, until the second term becomes δV_{\min} . The proof is completed by noting that it is not possible to have $\tilde{w}^* > w^* \geq V_{\min}$ and that for all s_τ for which $U(\bar{\pi}_\tau, \bar{\pi}_\tau, s_\tau) + \delta V_{\max} > U_N(s_\tau) + \delta V_N$ the following holds:

$$U^*(\bar{\pi}_\tau, \bar{\pi}_\tau, s_\tau) + \delta V_{\min} > U_N^*(s_\tau) + \delta V_N$$

By definition V_{\max} is the upper bound in W . Since the proposed policy and continuation values $(\bar{\pi}, V_{\max})$ cannot be improved upon, they must deliver V_{\max} . By the definition of V_{\min} , this implies that $\tilde{w}^* = V_{\min}$, which delivers the contradiction.

Proof of Proposition 6:

Given Propositions 4 and 5 and our sequential formulation of the problem this result follows directly.

Proof of Proposition 8:

i) Follows directly from the definition of \underline{w} and the policy characterization for states in S_1

ii) Consider any infinite sequence of shock realizations. With probability one any such sequence must include infinite realizations of every shock. We show that if $w_0 < \underline{w}$ then $w \rightarrow \underline{w}$. The other case follows by symmetry.

For $w_0 < \underline{w}$ Home participation constraint does not bind for any state s . But there is at least one state, say s' , in which the participation constraint binds for Foreign. In this state then $w_{s'} > w_0$ must hold. If $w_{s'} < \underline{w}$, we start over with our argument. Note, from Home's problem, that $w_t > \underline{w}$ cannot be a solution because of an efficiency argument: \underline{w} is all that Home needs to promise Foreign to keep it in the union; since Home continuation value is decreasing in this promise, there is never an incentive to assign Foreign a value greater than \underline{w} . Therefore, promised values for Foreign are a (stochastically) increasing and bounded sequence, converging to \underline{w} with probability 1.

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