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SHAPLEY VALUE-BASED ALLOCATION OF SYSTEM-LEVEL RISK IN THE HUNGARIAN BANKING SYSTEM

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Shapley value-based allocation of system-level risk in the Hungarian banking system

(A rendszerszintű kockázat Shapley-érték alapú allokációja a magyar bankrendszer esetében)

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Contents

Abstract

The aim of the paper is to estimate the allocation of system-level risk and capital buffers due to interconnectedness (due to the correlation structure) of Hungarian Other Systemically Important Institutions using a market information-based method. The approach of Tarashev et al. (2015) is followed, with the Shapley value used to allocate the risk measure (VaR and ES) calculated from the aggregate loss distribution of the Hungarian banking system. The methodology applied is employed to investigate the role of size, probability of default and correlation structure on systemic risk.

Our results confirm the important role of size in systemic risk. At the 99.9% confidence level, the Shapley value-based share of system-level risk for large banks exceeds the weights calculated based on asset value, while for smaller banks it is below the weights calculated based on asset value. As the confidence level is reduced, a significant shift in the allocation is observed due to the increasing importance of probabilities of default. The allocated share of banks with low probability of default diminishes, while the share of institutions with a higher probability of default increases.

Furthermore, it can be observed that for institutions with lower asset values and high comovement with the common factor, systemic risk due to interconnectedness explains a significant share of the allocated system-level risk. Conversely, for banks with higher asset values only approximately 10% of the estimated capital buffer is allocated due to systemic risk from interconnectedness, suggesting an important role for other factors (e.g. size, probability of default and foreign ownership).

Journal of Economic Literature (JEL) codes: G18, G21.
Keywords: Systemic risk, system-level risk, other systemically important institutions (O-SII), capital allocation, Shapley value.

Kivonat

A tanulmány célja piaci információalapú módszerrel becslést adni a rendszerszinten jelentős intézmények rendszerszintű kockázatára és összekapcsoltságból fakadó (korrelációs struktúra miatti) tőkepuffereire a magyar bankrendszer esetében. Tarashev et al. (2015) megközelítését követve a magyar bankrendszer aggregált veszteségeloszlásából számított kockázati mérték (VaR és ES) felosztásakor a kooperatív játékelméleten alapuló Shapley értéket használjuk. Az alkalmazott módszertan segítségével vizsgáljuk a méret, a csődvalószínűség és a korrelációs struktúra rendszerkockázati szerepét.

Eredményeink megerősítik a méret fontos rendszerkockázati szerepét, 99,9%-os konfidencia szinten a nagy bankok Shapley érték alapú részesedése a rendszerszintű kockázatból meghaladja, míg a kisebb bankok esetében nem éri el az eszközérték alapján számolt súlyokat. A konfidencia szint csökkenésével az allokáció a csődvalószínűségek növekvő jelentősége miatt jelentős mértékben átrendeződik, az alacsony csődvalószínűségű bankok esetében csökken, míg a magasabb csődvalószínűségű intézmények esetében nő az allokált kockázati mértékből a részesedés.

Továbbá megállapítható, hogy alacsonyabb eszközértékű és a közös faktorral erősen együttmozgó intézmények esetében az összekapcsoltságból fakadó rendszerkockázat magyarázza a puffer jelentős hányadát. A nagyobb eszközértékű bankok becsült tőkepufferéből viszont csak 10% körüli részt allokálunk az összekapcsoltságból fakadó rendszerkockázat következtében, ami más tényezők (pl. méret, csődvalószínűség és külföldi érdekeltségek) fontos szerepét sugallja.

Journal of Economic Literature (JEL) kódok: G18, G21.
Kulcsszavak: Rendszerkockázat, Rendszerszintű kockázat, Hazai rendszerszinten jelentős intézmények (O-SII), Tőkeallokáció, Shapley érték.

1 Introduction

The significance of systemically important financial institutions (SIFIs) was highlighted by the 2008 financial crisis, but the emergence of the problem is due to banking system developments in the decades preceding the crisis. During this period, many banks in developed countries[1] embarked on significant expansion and became increasingly active internationally as financial liberalisation spread. However, the revision of the former traditional banking model and the increase in cross-border activities resulted in structures that became less transparent to both external and internal stakeholders. Complex, opaque networks of cross-border financial intermediation developed, alongside the trading of innovative, complex products. The role of off-balance-sheet items (e.g. derivatives, guarantees) increased, and leverage rose dramatically. Newly created risk was not detected for a long time, and thus no adequate control mechanisms were put in place, neither by owners nor by regulators (*BCBS, 2012; BCBS, 2013; EBA, 2020*). This phenomenon was not only observed at the global level: similar processes have shaped the development of regional and national banking systems on a smaller scale.

The outbreak of the financial crisis highlighted the systemic importance of this situation, as well as the social and economic impacts (*BCBS, 2012; BCBS, 2013*). The financial distress and possible failure of a large financial institution can place a heavy burden on the financial system as a whole and threaten real economic activity through the temporary or permanent disruption of funding channels, with social consequences (e.g. significant unemployment, eviction of debtors unable to pay their loans). In addition, the recapitalisation of large financial institutions to ensure the continued functioning of critical financial functions can place a heavy burden on public finances and hence taxpayers.

In the interpretation adopted by the G20 countries,[2] a systemically important institution is defined as one whose failure or temporary difficulty[3] would trigger a widespread stress event by causing contagion directly (some contractual relationship between institutions, such as interbank claims) or indirectly (for example, positions built from highly correlated exposures) (*BCBS, 2012; BCBS, 2013).* However, this stress situation can be interpreted in multiple dimensions: some studies focus more on the financial system, while others consider the influence on real economic conditions and processes to be more important. The relevance of the two approaches is confirmed not only by theoretical and empirical research, but also by recent practice: in developing countries with less deep financial systems, authorities typically focus on potential macroeconomic effects, while regulators in more complex, interconnected and developed banking systems focus primarily on the financial system.

To address the negative externalities caused by SIFIs, a set of rules should be designed to both mitigate the likelihood of stress and default of such institutions and, in the event of insolvency, to reduce the negative impacts and the resulting taxpayer burden caused by disruptions to their systemically critical functions (see, for example, *BCBS, 2012; BCBS, 2013; Bongini et al., 2015; Weistoffer, 2011).* The probability of default of SIFIs can be reduced by improving their loss-absorbing capacity, i.e. by imposing an additional capital requirement. And in the event of a potential failure, an effective resolution regime needs to be developed to mitigate systemic risks and ensure the continuity of critical functions. Accordingly, the Basel Committee on Banking Supervision's framework for addressing the problem at both the global and local level focuses on three main priority areas (*BCBS, 2012; BCBS, 2013; Bongini et al., 2015; Weistoffer, 2011*): (1) it imposes additional capital requirements to ensure higher loss-absorbing capacity (mitigating the probability of default); (2) it requires the development of an effective resolution framework; and (3) it requires special supervisory attention (both at the microprudential and macroprudential levels).

[1] Over time, banks from developing countries have also emerged that fall into this category, with several large Chinese banks, for example, now being considered globally systemically important banks.

[2] IMF, BIS, FSB (2009): Report to G20 Finance Ministers and Governors, Guidance to Assess the Systemic Importance of Financial Institutions, Markets and Instruments: Initial Considerations.

[3] The concept can also be interpreted in a different way, whereby the relevance of a financial institution is measured by the extent to which its existence is essential for the proper functioning of the financial system. Current international economic policy regulation adopts the approach described above and seeks to create incentives to reduce the systemic importance of an institution.

In this study, the Shapley value-based approach of *Tarashev and Zhu (2008)* and *Tarashev et al. (2015)* is employed to allocate system-level risk to other systemically important institutions (O-SII) identified in the Hungarian banking system. With certain simplifications, this quasi-market information-based approach may be applicable to banking systems in countries where high frequency stock market data for the institutions concerned are not available. This is a particularly relevant issue for many banking systems in EU Member States. Furthermore, this approach can also be used in the case of relatively low direct interconnectedness, low interbank lending or swap market exposures and the high liquidity currently prevailing in the banking system. In such cases, the primary challenge in defining systemic risk is that the low direct network exposures and high liquidity buffers result in a relatively low direct "contagion" effect, thereby limiting the ability to define the interconnectedness dimension using these metrics. However, this approach has the advantage of allowing for the partial consideration of more indirect interconnectedness effects, such as network importance due to indirect asset price correlations. It is therefore suggested that this methodology could be a valuable addition to the indicator-based EBA methodology currently required by regulation, particularly regarding capital buffers for individual institutions.

The paper is structured as follows: Section 2 summarises the relevant literature related to systemic risk, the identification of systemically important financial institutions and the measurement of their risks, while Section 3 provides a brief overview of the regulatory framework employed by the European Union and Hungary regarding systemically significant institutions. In Section 4, the theoretical and practical approaches to the allocation of systemic risk, and the methodology that has been applied, are presented. Section 5 provides a description of the data set used, the simulation steps that were performed and a comprehensive summary of the results obtained.

2 Measuring the Risk of Systemically Important Financial Institutions

In recent years, there has been dynamic growth in the number of studies related to the identification and measurement of the risks of systemically important financial institutions. Three main directions are beginning to emerge in the literature: (i) market information-based methods, (ii) indicator-based methods, and (iii) network-based methods.

2.1 MARKET INFORMATION-BASED METHODS

Market information-based methods typically translate the risk measurement concept of credit institutions and investment activities into a systemic risk framework. These methods generally use high frequency data (e.g. CDS spreads, risk premiums on uninsured liabilities, return on equity, etc.). The advantage of this approach is that, on the one hand, their data requirements and data collection costs are not significant, as they work with commonly available market data, and on the other hand, due to the high frequency data, they can capture, at least in theory, the evolution of changes in systemic importance even on a daily basis (see, for example, *Brownless and Engle, 2012; Huang et al., 2010*). One drawback of these methods is that for many institutions these high-frequency data are not available or the trading depth of the relevant products (e.g. CDS markets) is not sufficiently deep. As a consequence, the regional and domestic applicability of this approach to SIFIs is severely limited. Moreover, market-based methods typically produce highly volatile results at different stages of the financial cycle, making objective risk assessment difficult (*Weistoffer, 2011).* Finally, it is important to underline that market assessment is highly endogenous with expectations of how the SIFI problem will be addressed, making it difficult to separate market assessment judgments of systemic importance from expectations of policy interventions (e.g. the systemic importance of an institution is not reduced based on market methods because market participants believe its importance will decline, but is indicated by the methodology because market participants expect that the risks of that institution will be appropriately addressed by the policymaker).

Adrian and Brunnermeier (2016) define the CoVaR[4] indicator as the Value-at-Risk (VaR) calculated for the aggregated banking system conditional on a certain asset return of a given financial institution. In their approach, the marginal contribution of an individual financial institution to systemic risk is defined by the ΔCoVaR (see also *Castro and Ferrari, 2014*), which is the difference between the CoVaR calculated at the p-ed order lower quantile and median asset return of the given institution.

Acharya et al. (2017) define a so-called marginal expected shortfall (MES) indicator, which estimates the expected loss of a given institution if the loss of the banking system is equal to or larger than the lower quantile of the system's return distribution (weighted average return distribution) of order p. Thus, in the case of MES, the expected loss of an individual bank is examined over the period in which the system performs with extreme losses, whereas in the case of CoVaR, the expected loss of the system is estimated for extreme losses of an individual institution.

The systemic expected shortfall (SES) indicator measures the expected contribution of an individual bank to the external economic costs of being undercapitalised, assuming that the system is below the target capital level. As a proxy for the expected social cost expressed by SES, SRISK (*Brownless and Engle, 2012*) measures the expected shortfall of the capital adequacy of a given bank below a bank-specific target level, assuming that the system's capital adequacy is below a systemic target level.

[4] In the CoVaR indicator, which is conceptually closer to a conditional VaR estimate, the prefix "Co" is intended by the authors to be an abbreviation of the systemic approach, which refers to the "conditional, contagion, comovement" characteristics.

One important advantage of the ΔCoVaR, MES, SES and SRISK methods is that the values of the indicators can be regressed on proportionate/share-based indicators that are commonly used in regulatory practice to measure relevance (e.g. size, leverage, maturity mismatch, return volatility, etc.), thus providing useful additional information on the relevance of the two approaches.

Huang et al. (2010) propose the so-called distress insurance premium (DIP) as a similar indicator to the ES measure. One specific feature of DIP compared to other statistical or structural risk measurement methods is that it uses the risk-neutral probability of default of institutions for systemic risk and individual marginal DIP estimates by using CDS spreads based on the default event.

Finally, according to *Drehmann and Tarashev (2013)*, previous ES measures defined under the assumption of systemic losses are limited in their ability to capture the systemic importance of an institution. Indeed, participation approach (PA) measures do not consider that an institution may be highly significant due to risks transmitted through its network of banking and interbank relationships, even if its individual losses during a crisis are less significant at the systemic level. By contrast, the contribution approach (CA) proposed by the authors identifies the risk of an institution not only in terms of the losses incurred by real economy agents. Instead, the systemic risk of an institution engaged in interbank intermediation is increased if it on-lends interbank funds from other credit institutions to credit institutions (or real economy agents) with high conditional expected losses (see also *Weistoffer, 2011*). The authors propose to use the Shapley value concept of cooperative game theory to measure risk allocation based on the contribution approach. According to this approach, the banking system can be decomposed into different combinations of possible subsystems. The risk measurement for a given institution is obtained by determining the average system-wide ES increment after adding the institution to different subsystems. One advantage of the Shapley measure is that it is efficient, i.e. the sum of the risks of each institution is equal to the risk measured at the system level. For a discussion of the application of this approach, see *Tarashev et al. (2015),* which is presented in more detail in Section 4.

2.2 INDICATOR-BASED METHODS

An important advantage of indicator-based methods based on supervisory reporting data is that they are easy to communicate, transparent, easy to interpret and simple to use for rule-making. Accordingly, the identification methods of international and EU recommendations are typically based on such methods. Indicator-based metrics using supervisory reporting data can cover a wide range of critical functions and negative externalities. In most cases, this is done by weighting institutions' size, critical economic functions (lending to the real economy, deposit taking, inter-bank intermediation, operation of financial infrastructure, etc.) and contagion risk activities (high activity in complex financial product markets, stock of cross-border transactions) by market share in one indicator. Hence, this type of measurement is often referred to as the share-based method. This methodology can also cope well with the need to define market shares in different units (e.g. loan stock in monetary terms, number of transactions in payment systems, number of customers in the case of deposits or number of branches) or to aggregate variables for which the concept of market shares is less straightforward (e.g. centrality indicators for networks, ratios describing the funding structure). It has the additional advantage of being generally applicable to non-listed institutions and of providing less volatile and more robust results than the market-based methodology. The disadvantages of this approach are that without additional estimations, the specific application can be arbitrary (e.g. weights of indicators, scope of indicators, setting of critical values, etc.) and it is often difficult to separate the risk contribution of the institution from the participation effects. Moreover, the problem of "gaming the rules" or "window dressing" also arises. Pre-fixed weights lead the institutions concerned to adapt in such a way as to appear as little systemically important as possible (see, for example, *Garcia et al., 2021*). This is partly why supervisory discretion in the identification of SIFIs and the use of relative participation is important.

Several studies raise the question of whether the use of market information-based indicators is worthwhile and provides substantial additional information over and above simpler, more commonly understood share-based methods. The studies outline that scores measuring systemic importance produced by market information-based models can be measured with reasonable accuracy using a suitably selected set of share-based variables (*Idier et al., 2013*), while market information-based scores can also help to quantify the extent to which the impact of a share-based indicator has contributed to systemic risk (*Drehmann and Tarashev, 2013;* for more on this topic, see also *Busch et al., 2021*).

2.3 NETWORK METHODS

Network analysis methods can be used to effectively investigate the contagion mechanisms of financial networks. In the event of a financial disruption, interconnectedness increases the likelihood of contagion, the channels of which can be very extensive. In the literature, the most common approach is to examine interbank credit market networks. The basic tool of the network approach is a relationship matrix describing the banking system, which contains the exposures of individual banks to each other. By representing the network as a graph, the implementation of different centrality indicators (proximity, intermediacy, degree, weighted degree, eigenvalue centrality, etc.) provides a measure of the systemic risk importance of each vertex (*Müller, 2006*). However, these approaches do not take into account the risk of contagion, but only measure the importance of the institution in a static network (*Upper, 2011; Allen and Babus, 2009*).

The consideration of contagion mechanisms is a promising area for further development. *Eisenberg and Noe (2001)* elaborated fictitious contagion algorithms in which contagion spreads through the network nodes simultaneously, while *Furfine (2003)* used a so-called sequential algorithm, in which the spread of contagion can be broken down into phases. The main drawback of this methodology is that it is very data-intensive and dealing with incomplete data can be a significant challenge. The measurement of the network effects of interbank markets and the potential channels of contagion have also been addressed by domestic researchers. *Lublóy (2005)* and *Berlinger et al. (2011)* investigated the network effects of the unhedged interbank market, while *Banai et al. (2013)* analysed the network effects of the foreign exchange swap market. One promising avenue of research is also the empirical investigation of the so-called multi-layer network approach (at the international level, see, for example, *Aldasoro and Alves, 2017* and *Montagna and Kok, 2016*). In his domestically focused research, *Szini (2021)* investigated two interbank markets (Hungarian unsecured interbank forint market and FX swap market) over the same period using network methods.

3 Regulation of Systemically Important Financial Institutions in the European Union

The leading international forum for international financial regulation, the Basel-based Financial Stability Board (FSB) and the Basel Committee on Banking Supervision (BCBS) of the Bank for International Settlements (BIS), has developed regulatory principles and recommendations on how to manage risks and identify systemically important institutions.[5] The international regulatory standards for systemically important financial institutions have been implemented by EU financial regulation since 2014 and are therefore applicable in the countries of the Central and Eastern European region and Hungary.

The international recommendation and the local regulations implementing it essentially require the introduction of an additional capital buffer to mitigate the risks of systemically important financial institutions. A capital buffer for systemically important institutions should be constituted by the institution concerned as an additional capital requirement on its total risk-weighted assets (RWA). The size of the buffer depends on the degree to which the institution is considered to be systemically important. The new macro-prudential instrument can exert an effect through four main channels (the main elements of these mechanisms and the overall impact of the increased cost of capital on lending are illustrated in Figure 3.1):

I. It can offset or dampen externalities (i.e. socially harmful excessive risk-taking) that are not reflected in the cost of funding for systemically important institutions;

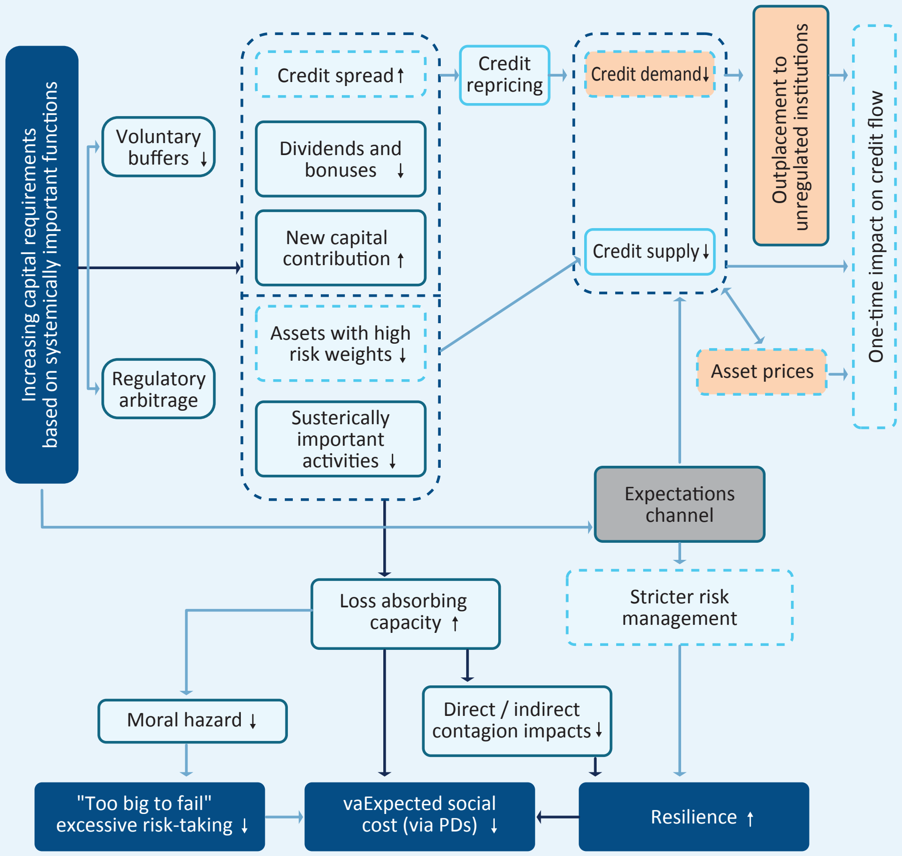
II. It can level the playing field between large banks with implicit state guarantees and smaller banks without such guarantees;

III. As an additional capital buffer, it reduces the likelihood and extent to which public money is used in the event of a potential insolvency;

IV. The increasing size of the buffer can act as a disincentive to further over-expansion.

[5] The identification methodology is detailed in Section 8.1 of the Appendix.

Figure 3.1
Simplified transmission mechanism of the capital buffer for systemically important institutions



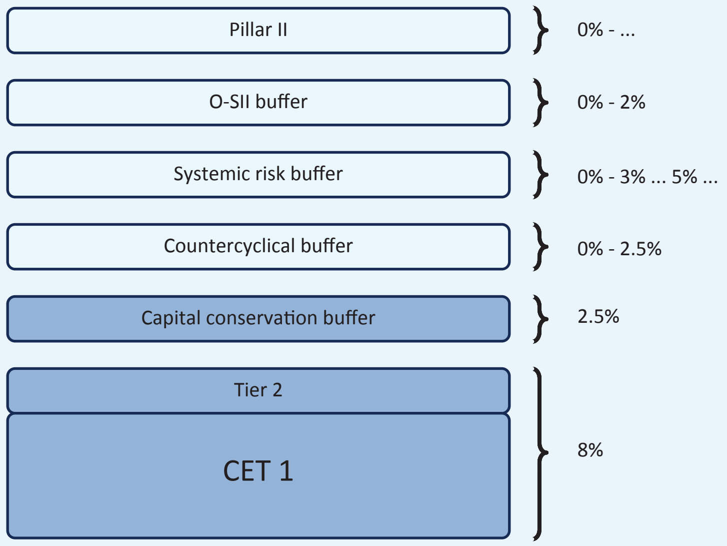
Source: ESRB, MNB

Of course, the identification of systemically important institutions and the application of the capital buffer may also have unintended negative effects. On the one hand, the cost of holding an additional capital buffer may encourage market participants to shift activities towards financial institutions with less stringent rules than banks ("shadow banking"). On the other hand, the explicit declaration of SIFI "status" by the authorities may indirectly increase the moral hazard (SIFI "recognition") associated with the institutions concerned. Finally, it may also create substantive entry barriers into the market for globally systemically important financial institutions, as it may discourage institutions that do not yet have SIFI status from establishing international networks (stronger international activity would impose SIFI burdens that the institution would not want to bear). Increasing entry barriers may thus have a detrimental effect on global competition, which is undesirable from a financial stability perspective (*Weistoffer, 2011*).

In the Central and Eastern European region, mitigating the risks of Other Systemically Important Institutions (O-SIIs) is of key relevance. Regulation of the capital buffers that may be required for these institutions was introduced in the CRD IV/CRR regulatory package,[6] the relevant parts of which have been implemented in regional and domestic legislation.

[6] Directive 2013/36/EU of the European Parliament and of the Council of 26 June 2013 on access to the activity of credit institutions and the prudential supervision of credit institutions and investment firms, amending Directive 2002/87/EC and repealing Directives 2006/48/EC and 2006/49/EC and Regulation (EU) No 575/2013/EU of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012.

Figure 3.2
Capital adequacy requirements under Basel III and CRD IV/CRR



Note: Under CRD V, a 3% requirement may already be imposed for O-SII capital buffers with supervisory discretion.
Source: Own compilation based on CRD IV/CRR

The regulation allows for the imposition of different types of capital buffers, depending on the macro-prudential risk(s) that the authorities wish to address (Figure 3.2). Under the current CRD V rule, the systemic risk capital buffer cannot be used to address O-SII risks, and thus the systemic risk capital buffer and the O-SII/G-SII buffer are always additive. If the institution concerned fails to comply with these additional capital requirements, restrictions on dividend and performance-related payouts/bonuses are triggered.

In addition to the O-SII buffer (i), the risks of systemically important financial institutions can be mitigated by an appropriately designed and effective resolution framework (ii), and by a focused, specific micro-prudential and macro-prudential supervisory attention (iii). Significant progress has already been made in the EU towards an effective resolution framework with the adoption of the BRRD[7] and Member State implementations, but it will take longer for a full resolution regime to be in place and to function effectively and in a tested manner. Additionally, detailed, fully harmonised implementation of the application of specific supervisory attention at the EU level is still in progress.

[7] Directive 2014/59/EU of the European Parliament and of the Council of 15 May 2014 establishing a framework for the recovery and resolution of credit institutions and investment firms and amending Council Directive 82/891/EEC, Directives 2001/24/EC, 2002/47/EC, 2004/25/EC, 2005/56/EC, 2007/36/EC, 2011/35/EU, 2012/30/EU and 2013/36/EU and Regulations (EU) No 1093/2010 and (EU) No 648/2012 of the European Parliament and of the Council.

4 Preliminaries and Methodology

In this paper, the approach of *Tarashev et al. (2015)* is followed, with the Shapley value based on cooperative game theory used to allocate the risk determined from the aggregate loss distribution of the Hungarian banking system. In addition to the allocation of systemic risk, the determination of systemic risk capital buffer levels due to interconnectedness (due to correlation structure) is performed. The methodology is then applied to examine the role of size, probability of default and correlation structure on systemic risk.

One significant challenge in applying market information approaches is the limited availability of high-frequency market data (e.g. stock prices, CDS spreads) in the EU, even for systemically important institutions. The situation is more challenging for smaller banking systems, where there are hardly any O-SIIs for which high-frequency data are available. This Section presents the empirical application of a methodology that is likely to require input data that are available to the responsible national authorities.

In the current analysis, an alternative, market information-based methodology is applied, which diverges from the prevailing EBA guidelines. This methodology is employed for the estimation of the systemic risk share and capital buffer to be determined among the identified O-SIIs. The EBA methodology is retained for the identification of systemically important institutions, as specified in the current regulation. The rationale behind this approach, which can be regarded as a hybrid strategy, is multifaceted. On the one hand, the current regulatory framework already offers a high degree of flexibility for identifying O-SIIs, empowering responsible authorities to designate institutions as O-SIIs at their discretion (they are even able to identify institutions as O-SIIs on the basis of a qualitative approach with suitable justification). Consequently, the alternative identification methodology's added value is more limited. On the other hand, technical implementation of the simulation used to estimate the capital buffers may be limited for the inclusion of 30 to 40 institutions or even more.

In accordance with the methodology employed by *Tarashev et al. (2015)*, a simulation was performed to determine the distribution of losses incurred at the time of default for each bank under review, as well as for the broader banking system. To achieve these objectives, we simulate the change in the asset value of each bank by employing estimated common and individual factors, and examining whether the value of the assets of the bank falls below the default threshold. The magnitude of the loss realised in the event of default is determined by the composition of the banks' debt. The loss distribution for the banking system in its entirety, as well as for each subunit, is obtained by aggregating the loss distributions of the individual institutions. As a risk measure, VaR and ES are calculated based on the aggregated loss distributions for the different subunits. Subsequently, risk is allocated to individual banks using the Shapley value.

In this Section, we present a brief overview of the game-theoretic risk allocation methods that can be applied for the allocation of systemic risk, as outlined in the extant literature. We also introduce the notation that will be used throughout this study, provide a concise overview of the Shapley value and its properties, and outline the methodology for modelling loss distributions.

4.1 RISK CAPITAL ALLOCATION METHODS AND SYSTEMIC RISK

The issue of risk allocation has been the subject of extensive research in the academic literature, presumably due to its numerous potential applications (see, among others, *Denault, 2001; Valdez and Chernih, 2003; Kalkbrener, 2005; Homburg and Scherpereel, 2008; Buch and Dorfleitner, 2008; Kim and Hardy, 2009; Boonen et al., 2012; Van Gulick et al., 2012* or *Hougaard and Smilgins, 2016*). *Csóka et al. (2009)* investigated the applicability of game-theoretic methods to risk allocation. They noted that there is always a stable risk allocation method and that, even in the presence of a changing risk environment, risk can be allocated in such a way that no coalition of actors opposes it (see also *Csóka and Pintér, 2016*). The various risk allocation approaches are presented in a comprehensive manner by *Balog et al. (2017)* and *Balog (2018)*. In their analysis, they assess the most important features of the Activity-based method, the Beta method, the

Incremental method, the Cost gap method, the Euler method, the Shapley method and the Nucleolus method. In a similar vein, *Dehez (2017)* conducted an examination of the Harsányi dividend method.

The potential applications of risk allocation are manifold and include such areas as capital allocation issues for individual investment portfolios, performance measurement, allocation of capital across business lines, product pricing and even the allocation of systemic risk (see, for example, *Balog et al., 2011; Huang et al., 2010; Acharya et al., 2017; Tarashev et al., 2009*). When applying allocation methods to allocate systemic risk, the financial system is considered as a portfolio of individual institutions (see, for example, *Goodhart and Segoviano, 2008* or *Webber and Willison, 2011*). *Brunnermeier and Cheridito (2019)* define the allocation of systemic risk across institutions based on the Euler method. In the context of systemic risk allocation, *Kadan et al. (2016)* demonstrate that the Aumann-Shapley value can be applied to ascertain the systemic risk component of the return on a security (for further information on this subject, see also *Koyluoglu and Stoker, 2002*; for a comparison of the Shapley value and the Aumann-Shapley value from a risk allocation perspective, see *Denault, 2001*). *Tarashev et al. (2009)* and *Drehmann and Tarashev (2013)* argue that previous ES measures defined under the assumption of systemic losses are limited in their ability to capture the systemic importance of an institution. They therefore propose the use of the Shapley value concept of cooperative game theory for contribution-based measures of risk allocation. Use of the Shapley value in the context of systemic risk allocation is also observed in *Aldasoro et al. (2017),* in which aggregate default risk is allocated through this approach. By contrast, *Borsos and Mérő (2020)* employ the Shapley value approach to define the set of systemically important institutions. In terms of practical application, it is important to mention the research of *Gauthier et al. (2012),* in which a Shapley value was determined for several large Canadian banks.

4.2 NOTATION

The following notation is introduced based on *Balog et al. (2017)*, *Balog (2018)* and *Tarashev et al. (2015).* In this study, the banking system is defined as consisting of a finite number of subunits (banks). The set of banks is denoted by $N = \{1, 2, 3 \dots n\}$. The loss/gain for bank $i \in N$ is denoted by $X_i \in \boldsymbol{X}$, where $\boldsymbol{X}$ is the set of probability variables interpreted on the finite probability space $(\Omega, \mathcal{M}, P)$.[8] Subsequently, the loss/gain of a coalition $S \subseteq N$ can be given by the probability variables $X_S = \Sigma_{i \in S} X_i \in \boldsymbol{X}$.

The measure of risk for each bank is represented by the function $\rho: \boldsymbol{X} \rightarrow \mathbb{R}$. The measure of risk is defined as a function that assigns a real number to a probability variable representing the gains/losses of a given coalition (*Csóka, 2003; Csóka et al., 2007; Balog, 2018*). The amount of capital captured by the measure of risk represents the sum needed to mitigate risk to an acceptable level, based on specific criteria. We use Value at Risk[9] (VaR) and Expected Shortfall[10] (ES) as measures of risk, following the study by *Tarashev et al. (2015)*.

The risk capital allocation situation can be given as $X_N^\rho = \{N, \{X_i\}_{i \in N}, \rho\}$, which is a set of subunits/banks N, probability variables describing the profit/loss of banks $\{X_i\}_{i \in N}$ and the measure of risk ρ. While the capital allocation method is given by the function $\varphi: D\varphi \rightarrow \mathbb{R}^N$ defined on the domain $D\varphi$, where $D\varphi$ is a subset of risk capital allocation situation with the set of subunits N (*Balog et al., 2017; Balog, 2018*).

In allocating the measure of risk (VaR and ES) determined from the aggregate loss distribution, we use the Shapley value as the capital allocation method. For a risk capital allocation situation X_N^ρ and for the subunit/bank $i \in N$, the Shapley value can be given as

$$\varphi_i^{Shapley}(\mathrm{X}_N^\rho) = \frac{1}{n} \sum_{n_S=1}^{n} \left[\frac{(n - n_s - 1)!\, n_s!}{n!} \sum_{S \subseteq N \setminus \{i\}} \left[\rho(X_{S \cup \{i\}}) - \rho(X_S) \right] \right]$$

[8] Due to the finite probability space assumption $\mathbf{X}$ can be represented by a realisation vector that gives the return of the subunit in each possible state of nature.

[9] Value at Risk (VaR) is the maximum potential loss over a given time horizon for a given significance level $\alpha \in [0,1)$, i.e. for $X \in \boldsymbol{X}$, $VaR_\alpha(X) = -inf\{x \in \mathbb{R}: P(X \leq x) \geq 1-\alpha\}$. As an interpretation, based on the probability distribution of future losses, the probability that losses will not exceed the amount of VaR is α (see *Csóka, 2003*).

[10] Expected Shortfall (ES) also takes the edges of the distribution into account (see, for example, *Acerbi, 2002; Acerbi and Tasche, 2002; Tasche, 2002* or, in the context of back-measuring and elicitability, *Tasche, 2014* and *Acerbi and Székely, 2014*). ES gives the average losses for the worst outcomes at a given level of significance, i.e. for $X \in \boldsymbol{X}$ and $\alpha \in [0,1)$, $ES_\alpha(X) = \frac{1}{1-\alpha} \int_0^{1-\alpha} VaR_\beta(X) d\beta$.

where $|S|=n_s$ denotes the number of banks in coalition S. The method allocates to each subunit its average marginal contribution to the risk of the coalitions of the preceding subunits along all permutations (see, for example, *Shapley, 1953; Mas-Colell et al., 1995; Tarashev et al., 2015*).

Balog et al. (2017) show that when using a coherent measure of risk, the Shapley value satisfies the properties of *Full Domain, Diversification*, *Efficiency*, *Equal Treatment*, *Strong Monotonicity*, *Incentive Compatibility*, *Covariance* and *Riskless Portfolio*, but fails the properties of *Core Compatibility* and *Decomposition Invariance*. In the context of a banking system, the calculation of Shapley value is possible for each individual bank, due to the full domain property. It is evident that, due to its efficiency, the total systemic risk can be allocated to individual banks. Furthermore, the sum of these Shapley values corresponds exactly to the risk of the whole system. The *Diversification* property ensures that the method allocates to each institution at most the amount of their stand-alone risk. However, *Core Compatibility* is not satisfied (in the case of *Deault (2001)* "no undercut"). This means if the total risk is allocated, there may be a coalition of banks that has an interest in not accepting the allocation (*Balog et al., 2017; Balog, 2018*). The Shapley method satisfies the *Equal Treatment (or Symmetry) property*,[11] i.e. if two subunits make the same contribution to all coalitions that do not contain them, then the risk allocated to them is the same. This method also satisfies the *Strong Monotonicity Requirement*. If two coalitions have the same risk value and the contribution of one subunit to the risk of the others does not decrease (including the risk of the coalitions themselves), then the risk allocated to that subunit cannot be smaller. Strong monotonicity implies fulfilment of the *Incentive Compatibility* requirement (see *Csóka and Pintér, 2016*), so the Shapley method also fulfils it. This property implies that if the allocation situation changes such that the risk of only one subunit decreases, the risk allocated to it cannot increase either (see *Csóka and Pintér, 2016; Balog, 2018*). The *Covariance* property implies that, given the allocation ratios, multiplying portfolios by a non-negative scalar has no effect (scale invariance) and that, if a risk-free portfolio providing the same payoff in all states of nature is added to a given subunit, the risk allocated to that subunit varies with its risk (translation invariance) (*see Balog et al., 2017; Tarashev et al., 2015*). Furthermore, the Shapley method also satisfies the requirement of *Riskless Portfolio* (see *Balog et al., 2017; Balog, 2018*). The Shapley value does not satisfy the *Decomposition Invariance* property, i.e. the risk allocated to each institution depends not only on that institution's and the entire system's loss distribution, but also on the risk allocated to other institutions (see also *Balog et al., 2017; Balog, 2018*). In our case, the failure to fulfil this property is beneficial, as the risk allocated to each institution in the financial system depends on the other institutions.

Tarashev et al. (2015) define the risk to be allocated using VaR and ES measures of risk of the aggregate loss distribution

$$VaR_\alpha(X_N) = -inf\{x \in \mathbb{R}: P(X_N \leq x) \geq 1-\alpha\}$$

$$ES_\alpha(X_N) = \frac{1}{1-\alpha}\int_0^{1-\alpha} VaR_\beta(X_N)d\beta$$

while using two different capital allocation methods to determine the risk of coalitions and allocate the measure of risk. In its approach, the two characteristic functions are related to variable and fixed tail approaches.[12]

The variable tail approach uses the Shapley value as the capital allocation method, while the risk of the $S \subseteq N$ coalition is measured using the following risk measures

$$VaR_\alpha(X_S) = -inf\{x \in \mathbb{R}: P(X_S \leq x) \geq 1-\alpha\}$$

$$ES_\alpha(X_s) = \frac{1}{1-\alpha}\int_0^{1-\alpha} VaR_\beta(X_S)d\beta$$

Since ES is a coherent measure of risk, the allocation $\varphi_i^{Shapley}(X_N^{ES})$ using ES satisfies the previously described properties of the Shapley value.

[11] For details, see for example *Denault (2001)* or *Tarashev et al. (2015)*.

[12] According to *Tarashev et al. (2015)*, under the fixed tail approach, tail events are defined at the level of the entire system as follows

$$\varphi_i^{Ft}(X_N^{VaR}) = E(X_i|X_N < -VaR_\alpha(X_N))$$
$$\varphi_i^{Ft}(X_N^{ES}) = E(X_i|X_N < -ES_\alpha(X_N))$$

4.3 MODELLING THE LOSS DISTRIBUTION OF BANKS AND THE ENTIRE BANKING SYSTEM

In order to apply the methodology, the distribution of individual losses of domestic banks and the aggregate loss distribution of the banking system as a whole are simulated.

As a first step, a factor model is employed, based on the work of *Tarashev and Zhu (2008)* and *Morokoff, Yang and Islam (2012)*, in order to model the change in asset values. In the model, the normalised asset value change (asset return) v_i of bank $i \in N$ can be expressed as a function of a common and an idiosyncratic factor

$$v_i = r_i M + \sqrt{1 - r_i^2}\, Z_i$$

where M is the common factor, Z_i is the idiosyncratic factor for $\forall\, i \in N$, r_i is the common factor loading, and Z_i, M, and $Z_{i \neq j}$ are mutually independent standard normal variables. The correlation between asset value changes of bank i and bank j can be given as $r_{ij} = r_i r_j$, while r_i^2 is the percentage of the asset returns variance due to systemic risk. The factor model can be specified by providing r_i.

The effect of a shock (e.g. macroeconomic) is reflected in the model through both common and individual factors. The assumption of orthogonality between the factors implies that all effects affecting the asset value changes of all banks are included in the common factor. The common factor is a probability variable with an expected value of 0 and a variance of 1. Therefore, its value in an average year would be around 0. The effect of an extremely low common factor in the model depends on the magnitude of the common factor M itself, as well as on the loading values of the factor r_i. For a highly correlated banking system (high r_i values in the model), the spillover effect through the common factor is larger. Whereas, if we assume no correlation between the asset value changes of the banks (factor loading values $r_i = 0$), the shock does not transmit through the common factor. Shock can also have an impact through the idiosyncratic factors. However, this effect is orthogonal between banks.

4.3.1 The ASRF model

Tarashev and Zhu (2008) apply and calibrate the ASRF (asymptotic-single-risk-factor) model of *Gordy (2003)*. We derive the value of the capital buffers defined in the ASRF framework based on *Tarashev and Zhu (2008)*, in order to understand the differences between the methodology used in this paper and the ASRF model.

Let λ_i denote the characteristic function, which equals 1 if the bank $i \in N$ is in default and 0 otherwise. In the model, default occurs when a bank's asset value reduction exceeds the default threshold. If $N \to \infty$, then the expected value of λ_i conditional on the value of the common factor M can be calculated as

$$E(\lambda_i\,|M) = P\,(v_i < \mathcal{F}^{-1}(PD_i)|M) =$$

$$= P\,(r_i M + \sqrt{1 - r_i^2}\, Z_i < \mathcal{F}^{-1}(PD_i)|M) =$$

$$= \mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - r_i M}{\sqrt{1 - r_i^2}}\right)$$

where PD_i is the probability of default of bank i, $\mathcal{H}$ is the distribution function of Z_i, $\mathcal{F}$ is the distribution function of v_i, and $\mathcal{F}^{-1}(PD_i)$ is the default threshold of bank i. The last equation follows from the independence of Z_i and M. $E(\lambda_i\,|M)$ expresses the probability of bank i's default for a given value of the common factor M.

Due to the Law of Large Numbers, the conditional expected value of the total loss (TL) of the banking system is deterministic for a given value of M:

$$TL|M = \sum_i \omega_i \, E(LGD_i) \, E(\lambda_i|M) = \sum_i \omega_i \, E(LGD_i)\mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - r_i \, M}{\sqrt{1-r_i^2}}\right)$$

where $\omega_i E(LGD_i)$ is the expected loss given default of bank i.

Since the conditional aggregate loss is a decreasing function of the common factor M, the $(1\text{-}\alpha)$-th percentile of the unconditional aggregate loss distribution can be expressed as

$$TL_{1-\alpha} = \sum_i \omega_i \, E(LGD_i)\mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - r_i \, \mathcal{G}^{-1}(\alpha)}{\sqrt{1-r_i^2}}\right) = TL|M_\alpha$$

where $\mathcal{G}$ is the distribution function of the common factor M, and M_α is $\mathcal{G}^{-1}(\alpha)$ is the α-th percentile of the distribution of the common factor M. α is the probability that M is less than M_α; hence, $TL_{1-\alpha}$ represents the VaR at the $(1 - \alpha)$ confidence level.

A capital buffer, which theoretically covers the unexpected loss of the entire system with probability $(1 - \alpha)$, can be calculated as

$$\mathcal{K} = TL_{1-\alpha} - \sum_i \omega_i \, E(LGD_i) \, PD_i =$$

$$= \sum_i \omega_i \, E(LGD_i) \left[\mathcal{H}\left(\frac{\mathcal{F}^{-1}(PD_i) - r_i \, \mathcal{G}^{-1}(\alpha)}{\sqrt{1-r_i^2}}\right) - PD_i\right] = \sum_i \omega_i \; \kappa_i.$$

The capital allocation $\{\kappa_1 \dots \kappa_n\}$ satisfies the *Decomposition Invariance* property, meaning the amount of capital allocated to each bank is independent of the amount allocated to the others. Assuming that the distribution functions $\mathcal{H}$, $\mathcal{F}$, and $\mathcal{G}$ follow standard normal distribution, we can compute the capital buffers based on the Basel II IRB approach ($\alpha = 0{,}1$ and $\omega_i E(LGD_i)$ 45% of asset value).

4.3.2 The simulated model version

Given that the condition $N \to \infty$ of the ASRF model is not fulfilled for the Hungarian O-SII institutions, the model version of *Tarashev et al. (2015)* is employed, and loss distributions are simulated in the study. Over the period, the asset value v_i of bank $i \in N$ changes as a function of a common and an idiosyncratic factor:

$$\frac{V_i - V_0}{V_0} = \sigma_i \left(r_i \, M + \sqrt{1 - r_i^2} \; Z_i\right)$$

where V_0 is the initial asset value and σ_i is the volatility of the asset value change for bank $i \in N$.

The asset value of bank $i \in N$ can be expressed as

$$V_i = V_0 \, \sigma_i \left(r_i \, M + \sqrt{1 - r_i^2} \; Z_i\right) + V_0$$

The probability of default PD_i of bank $i \in N$ as a function of the default threshold of the asset value V_i can thus be given as follows.

$$PD_i = P\left(V_i < \overline{V_i}\right) = P\left(V_0\,\sigma_i\left(r_i\,M + \sqrt{1 - r_i^2}\;Z_i\right) + V_0 < \overline{V_i}\right) = P\left(r_i\,M + \sqrt{1 - r_i^2}\;Z_i < \frac{\overline{V_i} - V_0}{V_0\,\sigma_i}\right).$$

The value of the characteristic function λ_i is 1 if the asset value change of bank $i \in N$ falls below the default threshold, and 0 otherwise.

$$\lambda_i = \begin{cases} 1, & if\;\; r_i\,M + \sqrt{1 - r_i^2}\;Z_i < \phi^{-1}\left(PD_i\right) \\ 0, & \text{otherwise.} \end{cases}$$

For bank $i \in N$, the loss is $X_i = LGD_i\,\lambda_i$, where LGD_i is the amount of losses[13] incurred by a bank in the event of default. The aggregate loss distribution of the subsystem of banks $S \subseteq N$ is $X_S = \sum_{i \in S} X_i = \sum_{i \in S} LGD_i \lambda_i$, and the loss distribution of the entire banking system is $X_N = \sum_{i \in N} X_i = \sum_{i \in N} LGD_i \lambda_i$.

[13] In the ASRF model, this value is denoted by $\omega_i E(LGD_i)$.

5 Simulation and Results

In the present analysis, data from the seven other systemically important institutions (O-SIIs) in Hungary are collected, namely OTP, MBH, UniCredit, K&H, Raiffeisen, Erste and CIB. The data are then used to conduct a Monte Carlo simulation of the loss distribution of the banks X_i. The loss distributions of the entire banking system and of each subsystem are obtained by aggregating the loss distributions of each institution. VaR and ES are calculated as measures of risk from the loss distributions. Subsequently, risk was allocated to individual banks using the Shapley value method. The results and robustness check are presented in this section, along with the data used and the simulation steps.

5.1 DATA

Following the approach of *Tarashev et al. (2015),* the value of bank assets (V_i) is simulated using factor loading values r_i determined from the covariance matrix among banks' asset value changes. The simulation incorporated both a common factor (M) and idiosyncratic factors (Z_i) with the distribution of all factors assumed to be normal. Given that Moody's KMV data are only available for a few domestic systemically important institutions, the MNB Bank Balance Sheet (Table 01 of the F01 Supervisory Balance Sheet) databases are used to estimate bank asset values, bank losses and common factor loadings r_i.

The initial asset values of the institutions (V_0) are based on the balance sheet data as of December 2024 (F01/01 Total assets). The mean asset value for the institutions under review is HUF 8,456 billion, with a standard deviation of HUF 6,394 billion. The institution-level standard deviations (σ_i) and the covariance matrix of institutions are estimated from annual asset value changes between 2004 and 2024. Despite the correlations being calculated using book value-based, non-market asset value data, the seven largest institutions in the Hungarian banking system are still highly correlated: the average correlation coefficient per pair is 0.58, the minimum is 0.23 and the maximum is 0.9. As an alternative approach, the annual return on assets (ROA) data of banks is used to estimate the covariance matrix. In this case, the average correlation coefficient is of a similar magnitude of 0.52.

The factor loading values (r_i) are determined through calculation using the covariance matrix derived from the annual asset value changes of banks. This calculation is conducted using the built-in factor analysis of Matlab's computing environment.[14] The mean r_i value is 0.76, with a minimum value of 0.52 and a maximum of 0.97. For the covariance matrix calculated from annual bank ROA data, the average factor loading value is 0.73.

The value of the characteristic function λ_i can be derived from the change in asset value and the default threshold, which is calculated using the default probabilities. The annual probability of default (PD_i) for each systemically important institution is determined based on the risk ratings[15] of the institution by using the FitchRatings transition matrix[16] for the period between 1990 and 2023. The probability of default ranges from 0.1% for A3 Moody's and A- Fitch ratings to 0.29% for Baa3 Moody's and BBB- Fitch ratings.

In order to simulate the distribution of losses X_i realised in the event of default, the loss given default LGD_i is multiplied by the characteristic function λ_i. The estimation of LGD_i is based on the liability side of each bank, taking into account not only the external debt, but also the level of insured deposits[17] as one of the most important liability-side elements. This is a novel element of our research, because similar estimates (e.g. *Tarashev et al., 2015*) calculate the LGD_i considering only the external debt stock of the institution. This approach provides a more precise estimate of realised losses, as

[14] See factoran command description in Matlab.

[15] For the risk ratings, we used data available from Bloomberg. Primarily using ratings from Fitch and, if missing, Moody's.

[16] Based on the table in Appendix D - Global Corporate Finance Transition Matrices (Continued) Financial Institutions Average Annual Transition Matrix: 1990-2023 of FitchRatings (2024).

[17] For further insight into the matter of insured deposits and bank defaults, see for example *Wheelock and Wilson (1995)* or Correia et al. (2024).

these are mitigated by insured deposits from households and non-financial companies. In the event of losses, the deposit insurance scheme ensures that affected clients (households and non-financial companies) receive compensation. Despite the potential impact of these payouts on the banking system, such as higher deposit insurance contributions due to the need to replenish the Deposit Guarantee Fund, this does not result in realised losses for real economic agents in the short term.[18]

The same insured deposit rates are assumed for each institution with separate rates applied to household and non-financial companies (corporate) deposits. However, heterogeneity between institutions still appeared, due to the different composition of deposits. For household deposits, a 90% insured deposit rate is assumed, while a 30% insured deposit rate is assumed for corporate deposits. In the robustness check, the extent to which the simulation results vary according to different household insured deposit rates is examined.

5.2 STEPS OF THE SIMULATION

The collected data are used to conduct a Monte Carlo simulation, which is employed to generate the distribution of banks' losses $\{X_i\}_{i \in N}$.

1. The first step in the simulation is to employ Matlab's built-in factor analysis tool[19] to determine the factor loading values based on the covariance matrix derived from the annual asset value changes of banks.

2. In order to estimate the loss distribution functions of the banks, 2 million states of nature are simulated. For each state of nature, common and individual components are generated with standard normal distribution, and then asset value changes are calculated for each bank using the generated components and factor loading values. We examine which bank defaults in the given state of nature and then determine the individual loss for each institution in that state of nature.

3. Based on all runs, the loss distribution function for each institution and the aggregate loss distribution functions for all possible coalitions of institutions are derived.

4. As *Tarashev et al. (2015)* demonstrate, VaR and ES are calculated at 99.9%, 99.5% and 99% confidence levels from the individual loss distribution functions, distribution functions for each coalition and the aggregate loss distribution function for all banks under study.

5. Finally, the VaR and ES values determined for the entire banking system are allocated to each systemically important institution using the Shapley value.

5.3 RESULTS AND ROBUSTNESS CHECK

The simulation results are tested under different assumptions. For each scenario, the system-level risk measures (ES and VaR) and their Shapley value-based allocations at different significance levels are demonstrated. Following a comparison of the fixed tail and variable tail approaches, it was decided that a detailed analysis of the results obtained under the ES measure of risk and variable tail approach would be the most beneficial research method. In order to test the robustness of our results, we examine how the Shapley value-based shares of the system-level risk measure change when we assume identical bank default probabilities, calculate different insured deposit ratios for household deposits and use a covariance matrix calculated from ROA instead of one calculated from annual asset value changes. Finally, we present a solution for the allocation of the systemic risk capital buffer due to interconnectedness using the methodology previously outlined.

[18] The implicit assumption is that OBA (National Deposit Insurance Fund of Hungary) will always have enough funds for compensations, because if it does not, it would borrow and be repaid later by the banks via higher deposit insurance premiums. In this case, although the state guarantees the OBA bonds, it does not have to finance them.

[19] See factoran command description in Matlab.

5.3.1 Scenarios and the system-level risk measures

The simulation is executed under six alternative sets of assumptions, which are outlined in Table 5.1. In order to estimate the loss given default, different household insured deposit ratios (70%, 80% or 90%) are assumed for the different scenarios. The annual probabilities of default are based on the Fitch transition matrix. However, to perform a robustness test, the results are also examined with the same probability of default of 0.11%. Alternatively, the estimation of the covariance matrix between banks' assets is performed based on the annual change in asset value and ROA. In the final scenario, it is assumed that the asset value changes are uncorrelated.

Table 5.1
Calculated system-level risk measures at different significance levels for the six scenarios (HUF billion)

Scenarios	Households' insured deposits ratios	PD	Correlations	ES			VaR		
				0.1%	0.5%	1.0%	0.1%	0.5%	1.0%
1.	90%	Fitch/Moody's ratings	Derived from asset value changes	15,611	10,102	5,824	12,437	3,998	0
2.	80%	Fitch/Moody's ratings	Derived from asset value changes	16,124	10,298	5,972	12,940	4,138	0
3.	70%	Fitch/Moody's ratings	Derived from asset value changes	16,552	10,349	6,123	13,444	4,277	0
4.	90%	0.11%	Derived from asset value changes	14,937	7,703	4,253	12,437	3,395	0
5.	90%	Fitch/Moody's ratings	Derived from banks' annual ROA	15,675	10,145	5,803	12,437	3,998	0
6.	90%	Fitch/Moody's ratings	0	12,497	8,847	5,770	12,437	3,998	0

In order to determine the total system-level risk of domestic O-SIIs, ES and VaR risk measures[20] are calculated at confidence levels of 99.9% (α = 0,001), 99.5% (α = 0,005) and 99% (α = 0,01) based on the simulations which were run. In the first five scenarios, ES ranges from HUF 14,900 to 16,600 billion at the 99.9% confidence level, from HUF 7,700 to HUF 10,350 billion at the 99.5% confidence level, and from HUF 4,200 to 6,150 billion at the 99% confidence level. In other words, the losses of the seven O-SIIs identified in Hungary average between HUF 14,900 billion and HUF 16,600 billion over one year at the worst 0.1% of the outcomes. If we define the probabilities of default based on the Fitch transition matrix, the VaR is estimated to be between HUF 12,400 billion and HUF 13,500 billion at the 99.9% confidence level, and between HUF 4,000 billion and HUF 4,300 billion at the 99.5% confidence level. Given that the maximum probability of default for the banks under review is 0.29%, the estimated systemic VaR is already 0 at the 99% confidence level. In other words, for the seven O-SIIs identified in Hungary, the probability that the aggregate loss is at least HUF 12,400-13,500 billion over one year is 0.1%, and the probability that the aggregate loss exceeds HUF 4000-4300 billion is 0.5%.

For comparability, the aggregate asset value of the banks under review as of 31 December 2024 is HUF ~59,200 billion, the aggregate regulatory capital is ~6,500 billion and the minimum capital requirement is ~2,800 billion.

5.3.2 Allocations of system-level ES and VaR under fixed and variable tail approaches

In the process of allocating system-level ES and VaR via application of the Shapley value, the fixed tail and variable tail approaches of *Tarashev et al. (2015)* are employed. The fixed tail approach defines tail events at the level of the entire system and keeps these events fixed at each subsystem breakdown. By contrast, the variable tail approach defines tail events at the level of the subsystem, i.e. it may vary for each subsystem.[21] The variable tail assumption is consistent with the use of the Shapley value as a capital allocation method, as introduced in the methodological framework. Furthermore, ES is a coherent measure of risk under the variable tail assumption only. Consequently, this allocation possesses the positive properties of the Shapley value outlined above.

The simulation results also confirm the advantages of the ES risk measure and the variable tail approach. The fixed tail approach is shown to result in more concentrated risk allocation, whilst the variable tail approach is found to be more balanced, i.e. not only the most significant actor is subject to a meaningful risk allocation (see Figure 8.1 for ES and Figure

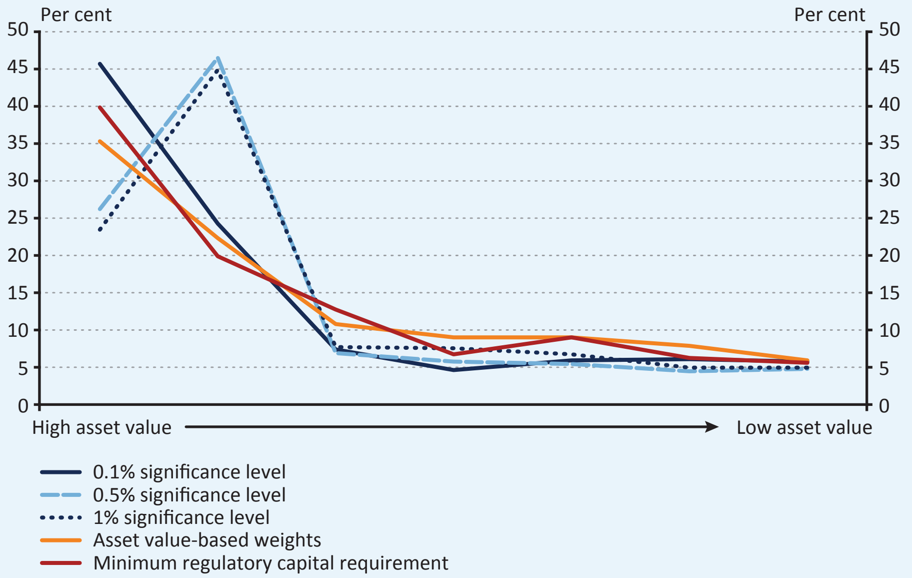
[20] By definition, the calculated ES is always at least as large as the VaR risk measure.
[21] For details of the two approaches with respect to systemic importance, see *Tarashev et al. (2015)*.

8.2 for VaR). Furthermore, it can be observed that the ES risk measure provides more stable allocations. This is because VaR does not take the full tail of the distribution into account, with the result that discrete jumps in the VaR measure can occur as the confidence level is varied. For these reasons, the detailed analysis essentially presents the results using the ES risk measure and the variable tail approach.

5.3.3 Allocation of system-level ES at different significance levels

As the next step, the Shapley value-based allocation of system-level risk determined by the ES risk measure is examined at 99.9%, 99.5% and 99% confidence levels under the variable tail approach (see Figure 5.1).

Figure 5.1
Asset value and minimum capital requirement-based shares, and the allocation of system-level ES among the O-SIIs

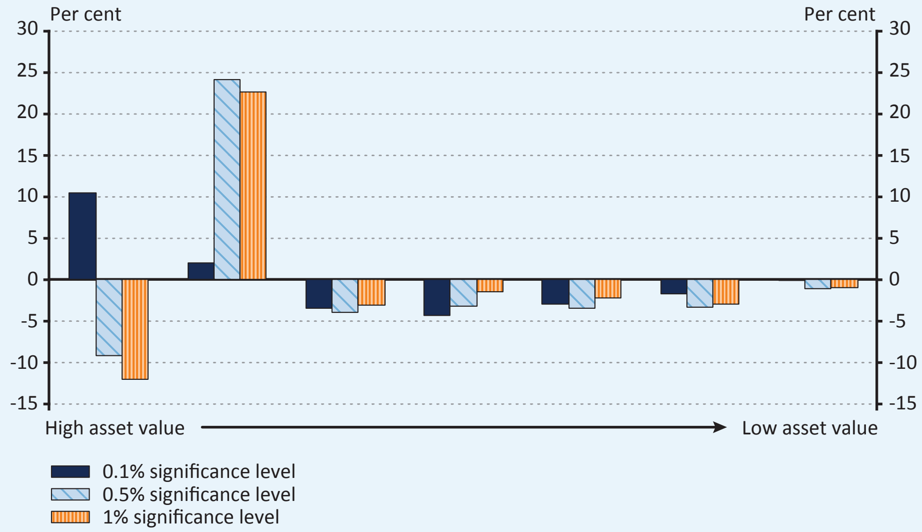


Note: Using ES and variable tail approach. In the figure, each O-SII is plotted in descending order of asset value.

Large banks also have a high share of regulatory capital requirements, asset values and, at the 99.9% confidence level, system-level risk measure. Similarly, relatively small banks have low shares of minimum capital requirements, asset values and system-level risk due to their size. At the 99.9% confidence level, the distribution of the allocation calculated using the Shapley value is comparable to the share calculated based on asset value or the regulatory capital requirement. However, it slightly exceeds the shares calculated based on asset value for large banks and is below the shares calculated based on the asset value for smaller banks.

At the 99.5% and 99% confidence levels, the allocation shows a substantially different distribution, propelled by the increasing significance of the probability of default, i.e. the role of riskiness. In order to facilitate a more comprehensive understanding of the role of size and probability of default, a plot is presented of the difference between the Shapley value-based shares and asset value-based shares for the O-SII institutions under review (Figure 5.2). In addition, the system-level risk allocated based on Shapley value as a proportion of asset value is shown, sorted by probability of default (Figure 5.3).

Figure 5.2
Difference between Shapley value-based shares and asset value-based shares sorted by asset value in percentage points

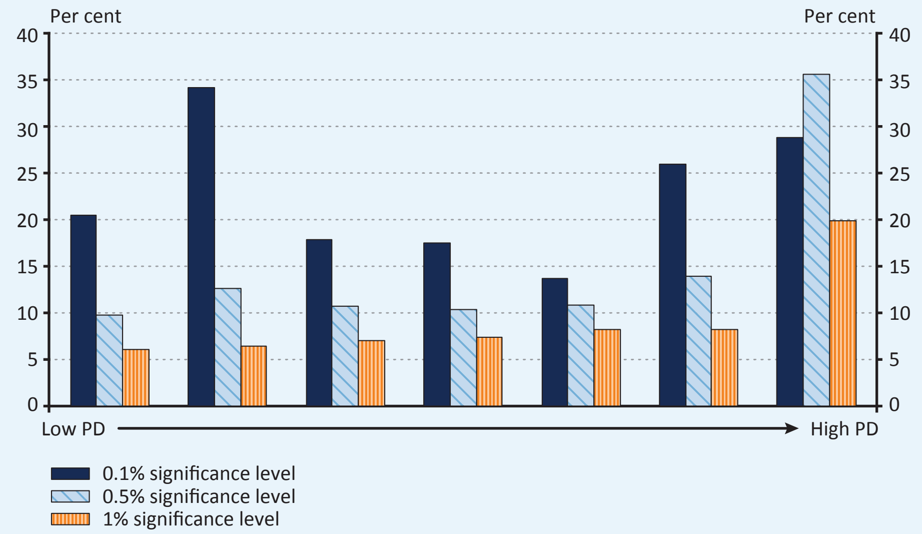


Note: Using ES and variable tail approach. In the figure, each O-SII is plotted in descending order of asset value.

At the 99.9% confidence level, the tail of the distribution is considered, meaning that even a large bank with a low probability of default can realise losses that are systemically large due to the size of the market participants. A Shapley value-based share of system-level risk above the asset value share of large banks confirms the important systemic risk role of size (Figure 5.2), which is also reflected in the O-SII capital buffer share.

As the confidence level decreases, the Shapley value-based allocation of system-level risk shifts significantly, with a decrease in the shares for banks with a low probability of default and an increase in the share of allocated risk for banks with a higher probability of default (Figure 5.3). The decision to employ a confidence level of 99.9% is supported by the significance of the tails of the distribution when capturing systemic risk. Regarding the allocation method that is currently under consideration, it is possible to observe the effect of both size and probability of default at the 99.9% confidence level.

Figure 5.3
Shapley value as a percentage of asset value (sorted by PD)



Note: Using ES and variable tail approach. In the figure, each O-SII is plotted in increasing order of PD.

5.3.4 Robustness check

In order to further investigate the effect of bank default probabilities, simulations are run assuming both individual bank default probabilities (Scenario 1) and identical bank default probabilities (Scenario 4). Then, the percentage point difference in the Shapley value allocations is determined for the two scenarios. In accordance with prior expectations, in Scenario 4 a higher Shapley value is obtained for institutions with a lower probability of default, using a PD of 0.11%, ceteris paribus. At the 99.9% confidence level, the effect is strongly attenuated, with only one institution showing a significant effect (Figure 8.3). Conversely, for institutions with a high probability of default, the effect is reversed, resulting in a lower Shapley value in Scenario 4, ceteris paribus. In both cases, the difference increases as confidence levels are reduced.

In the next step, the extent to which the Shapley value-based allocation of system-level risk varies under different assumptions regarding the proportion of insured household deposits is examined. As demonstrated in Figure 8.4, while there is some heterogeneity in the change in allocation across institutions, even the largest deviation is negligible, falling well below one percentage point, and both the mean and median are approximately zero. In summary, whilst the reduction in the insured deposit ratio increases the loss of institutions in the event of default (and consequently aggregate system-level risk), the effect on the share of each O-SII is not significant, even with the heterogeneity in the funding structure across banks.

Finally, the extent to which the Shapley value-based allocation of system-level risk is altered when, in the simulation, the covariance matrix calculated from the annual change in asset value is replaced by a covariance matrix calculated from ROA is investigated. The estimated risk of the entire banking system is estimated at HUF 15,611 billion under the assumptions of Scenario 1 and HUF 15,675 billion under the assumptions of Scenario 5, with a difference of less than 0.5% between the two estimates. As demonstrated in Figure 8.5, at confidence levels of 99% and 99.5%, the difference is negligible when examining the allocation determined by using the two different covariance matrices. At a confidence level of 99.9%, in Scenario 5, one bank has an 8-percentage point higher share of the allocated system-level risk, while another bank has a share that is 6 percentage points lower. The difference is already substantial in the estimated factor loading values: the values of 0.84 and 0.57, respectively, were obtained instead of the values of 0.60 and 0.74. Consequently, it can be concluded that, at the level of the banking system, similar results are obtained for two alternative asset return approaches in our simulation for the Shapley value-based allocation of system-level risk of domestic systemically important institutions.

5.3.4. Allocation of systemic risk capital buffer due to interconnectedness

In the simulated model, the asset value varies as a function of both the common factor and the individual factor. The common factor, which is a probability variable with an expected value of 0 and a variance of 1, affects the asset value change of all banks. By contrast, individual factors are defined as independent probability variables, meaning that the asset value change of each bank is dependent on a different individual factor.

The impact of the common factor on banks in the model depends both on the magnitude of the common factor M itself and on the loading values of the factor r_i. In the case of a highly correlated banking system in the model (as indicated by high r_i values), the spillover effect through the common factor is larger. Conversely, in the absence of a correlation between changes in the asset values of banks (i.e. if the factor loadings r_i are 0), the common factor does not impact asset values.

In order to capture the systemic risk due to interconnectedness, the magnitude of the additive risk associated with the effect of the correlation structure between banks' assets is investigated. Accordingly, for the Shapley value-based allocation of the systemic risk capital buffer for O-SII institutions due to interconnectedness, the results of Scenarios 1 and 6 are compared.

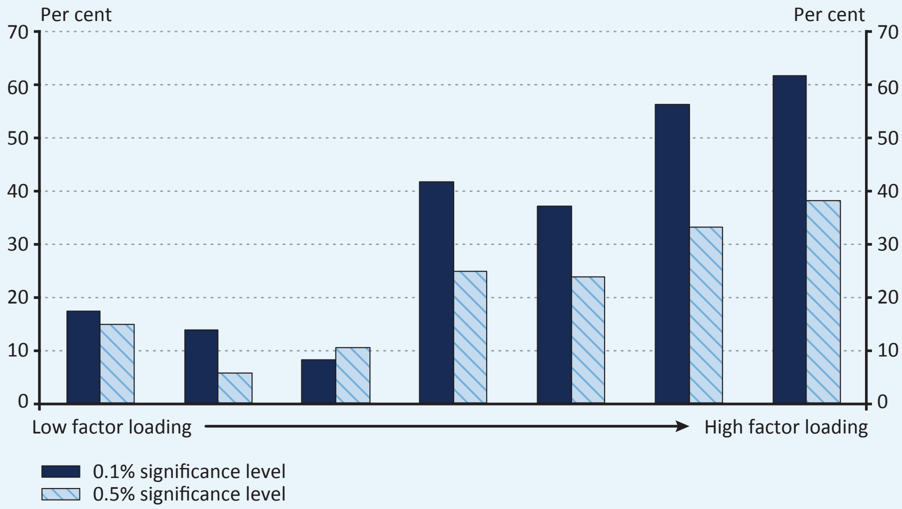
Based on the simulation, the aggregate systemic risk capital buffer resulting from interconnectedness at the banking system-level can be estimated as the difference between the ES risk measures defined for the two scenarios at the 99.9% confidence level. The ES calculated from the determined correlation matrix in Scenario 1 is HUF 15,611 billion. This loss takes into account the correlations between the asset returns of individual institutions and the resulting additive risks in the model. By contrast, the hypothetical ES based on Scenario 6 is HUF 12,497 billion. This systemic loss does not take into account the dimension of systemic risk arising from correlation between institutions. The systemic risk capital buffer due to interconnectedness aggregated at the banking system-level obtained from the methodology is ~ HUF 3,100 billion. At the 99.5% confidence level, the estimated aggregate capital buffer amount is ~ HUF 1,300 billion, while it is HUF 54 billion at the 99% confidence level.

In order to allocate the systemic risk arising from interconnectedness, the amount allocated to individual institutions in Scenarios 1 and 6 are deducted from each other. The ratio of the difference and the system-level risk allocated to each institution in Scenario 1 is then calculated.

As illustrated in Figures 5.4 and 5.5, for institutions with lower asset values and high factor loadings, the correlation structure explains 50-60% of the institutional capital buffer allocated at the 99.9% confidence level. This indicates that, in these cases, more than half of the systemic risk buffer is allocated to these institutions due to systemic risk arising from interconnectedness. At the 99.5% confidence level, this proportion drops to 30-40%. This finding may be explained by the increasing role of the probability of default, based on the results presented earlier.

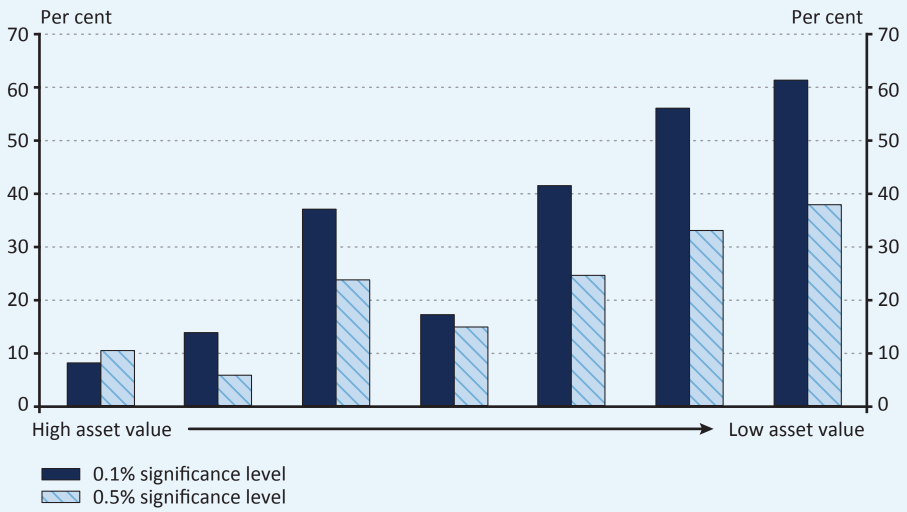
For banks with higher asset values, lower factor loading values are estimated, thereby reducing the impact of the common factor, since a number of specific factors affect these institutions. Consequently, systemic risk due to interconnectedness accounts for a mere 10% of the allocated capital from the system-level risk, suggesting the involvement of other factors (e.g. size).

Figure 5.4
Systemic risk capital buffer due to interconnectedness as a proportion of the capital buffer allocated from system-level risk to the institutions (sorted by factor loading values)



Note: Using ES risk measure and variable tail approach.

Figure 5.5
Systemic risk capital buffer due to interconnectedness as a proportion of the capital buffer allocated from system-level risk to the institution (sorted by size)



Note: Using ES risk measure and variable tail approach.

6 Summary

In this study, the Shapley value, a cooperative game theory method, was applied to allocate the system-level risk (measured as VaR and ES of the aggregate loss distribution) to domestic O-SIIs in the Hungarian banking system. In addition to the allocation of system-level risk, systemic risk capital buffers due to correlation structure are calculated. The results obtained enable the examination of the role of size, the probability of default and the correlation structure on systemic risk.

Following the methodology of *Tarashev et al. (2015)*, the distribution of losses realised in the event of default is simulated for each bank under review to determine the individual and aggregate loss distributions of domestic banks and the banking system. To do so, we simulate the change in asset value of each bank using the estimated common and individual factors and examine whether the value of the assets of the bank falls below the default threshold. The magnitude of the loss realised in the event of default is determined based on the composition of the banks' debt. The loss distribution for the banking system in its totality, as well as for each specific subsystem, is derived through the aggregation of the loss distributions of the individual institutions involved. As risk measures, VaR and ES are calculated based on the aggregated loss distributions for the different subsystems. Finally, the allocation of risk to individual banking institutions is determined by the Shapley value methodology.

Due to the theoretical properties (i. e. consideration of the full distribution tail and coherence) and the robustness of the simulation, we present the results using the variable tail approach and the coherent expected shortfall (ES) risk measure. For Scenarios 1 to 5, the losses of the seven O-SIIs identified in Hungary averaged between HUF 14,900 billion and 16,500 billion over one year in the worst 0.1% of outcomes. At the 99.5% confidence level, the ES ranges from HUF 7,700 to 10,300 billion, while at the 99% confidence level, it ranges from HUF 4,300 to 6,000 billion.

At the 99.9% confidence level, the distribution of the allocation calculated using the Shapley value is comparable to the share calculated based on asset value or the regulatory capital requirement. However, it slightly exceeds the shares calculated based on asset value for large banks and is below the shares calculated on the basis of the asset value for smaller banks. The result confirms the important role of size in systemic risk. At 99.5% and 99% confidence levels, the increasing importance of probabilities of default leads to a significant reallocation of the Shapley value-based allocation of system-level risk, with a decrease in the share of allocated risk for banks with low probabilities of default and an increase for institutions with higher probabilities of default.

In determining the systemic risk capital buffers arising from the interconnectedness of the O-SIIs identified in Hungary, the aggregate capital buffer amount at the banking system-level was approximated by the difference between the total system-level risk measure estimated from the correlation matrix calculated from annual asset value changes and a hypothetical system-level risk measure assuming a zero correlation between asset value changes. The aggregate systemic risk capital buffer due to interconnectedness at the banking system level is estimated to be ~ HUF 3,100 billion at 99.9% confidence level and ~ HUF 1,300 billion at 99.5% confidence level, based on the methodology employed.

An examination of the allocation suggests that for institutions with lower asset values and high factor loadings, at the 99.9% confidence level, more than half of the buffer is allocated due to systemic risk from interconnectedness. At the 99.5% confidence level, this ratio diminishes to 30-40% due to the increasing role of the probability of default. For banks with higher asset values, lower factor loading values are estimated. This is due to the lower comovement with the common factor, which is to be expected given the impact of numerous specific factors on such institutions. Consequently, systemic risk due to interconnectedness explains only around 10% of the system-level capital buffer, suggesting an important role of other factors (e.g. size, foreign interests).

In summary, the key advantage of the approach lies in its ability to use quasi-market information methodology in banking systems within countries where high-frequency stock market data is not available for the institutions under review. This is a particularly relevant issue for many banking systems in the European Union. Secondly, it is important to note that this approach can also be used with relatively low levels of direct interconnectedness, low interbank lending or swap exposures and the high liquidity currently prevailing in the banking system. Finally, this methodology helps to understand the role of different factors (probability of default, size, correlation structure) in the allocation of system-level risk. It could therefore be a good complement to the indicator-based EBA methodology currently required by regulation, in particular regarding capital buffers for individual institutions.

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8 Appendix

8.1 MAIN ELEMENTS OF THE STANDARDISED IDENTIFICATION METHODOLOGY DEVELOPED BY THE EUROPEAN BANKING AUTHORITY

In order to ensure that the treatment of O-SIIs is as harmonised as possible across the European Union and consistent with the requirements for G-SIIs, the EBA has issued guidelines on the methodology for the identification and assessment of O-SIIs.[22] According to the guidelines, the identification will be based on a set of four sets of indicators, summarized into ten share-based indicators (indicator-based approach). For each of the four indicator groups, a score between 0 and 10,000 will be assigned, on the basis of which each institution concerned will also receive an aggregate score (the four indicator groups will receive the same weighting of 25% for the aggregate score, so that scores between 0 and 10,000 are possible for the aggregate score). The four groups include indicators representing size, importance (substitutability) of functions in financial intermediation, complexity (derivatives, cross-border exposures) and interconnectedness with financial institutions. In any case, institutions scoring above a fixed threshold (350 points) will be considered as O-SIIs and, in addition, Member State authorities may adjust the 350-point standardised threshold within a range of -/+75 points, depending on the specificities of the domestic banking system and the evolution of the scores of individual banks. In addition, institutions with a qualitative assessment and additional indicators that better capture domestic specificities may be assigned an O-SII status with a limited deviation from the threshold, provided that the authority of the Member State provides adequate justification.

Table 8.1
Main sets of indicators at the standardised identification methodology of the European Banking Authority (EBA)

	Type of risk	Basis indicators
Complexity	Extent of cross-border and cross-market activities	1. Cross-border exposures/claims 2. Cross-border liabilities 3. OTC derivatives
Size	Relative size of potential losses and contagion impacts	4. Balance sheet total-based share
Interconnectedness	Leverage risks arising from the financial network structure	5. Exposures between financial institutions 6. Liabilities between financial institutions 7. Debt securities portfolio
Importance (Substitutability)	Magnitude of the risk arising from the failure of the real economy's intermediary role	8. Participation in payments 9. Private sector deposits (EU level) 10. Private sector loans (EU level)

Source: EBA

The methodology in the guidelines can be supplemented with country-specific additional indicators (this is certainly justified for the countries of the euro area and Hungary; see network analysis), on the other hand, it only applies to the identification methodology, the imposition, determination and differentiation of the specific capital buffer is basically a matter for national authorities to decide in the case of non-euro area Member States. Within the euro area, due to the framework of the Banking Union, this discretion is somewhat shared between the ECB and the national authorities: the ECB expects national authorities to comply with a minimum threshold approach when setting O-SII capital buffer rates.

The elements of the current indicator list may be revised in the light of experience in the coming years. As a general principle, the regulator's objective is to compile a "set" of indicators that, while being consistent, simple and transparent, captures as accurately as possible the relative magnitude of the loss that a crisis in a systemically important institution

[22] https://www.eba.europa.eu/-/eba-publishes-criteria-to-assess-other-systemically-important-institutions-o-siis-

causes to other actors in the financial system and to real economy participants through impairment to financial intermediation and financial infrastructure.

8.2 COMPARISON OF ALLOCATION UNDER FIXED AND VARIABLE TAIL APPROACHES

Figure 8.1
System-level risk allocation based on Shapley value (using Expected Shortfall risk measure, Scenario 1)

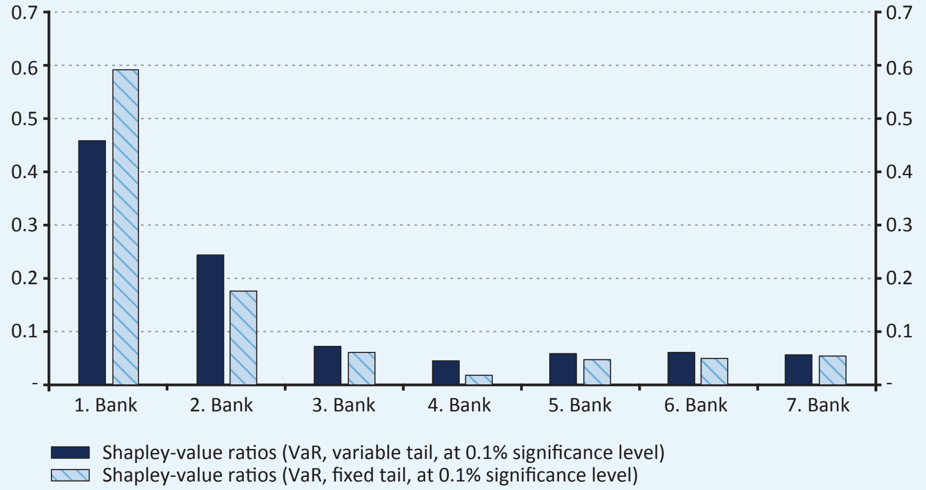
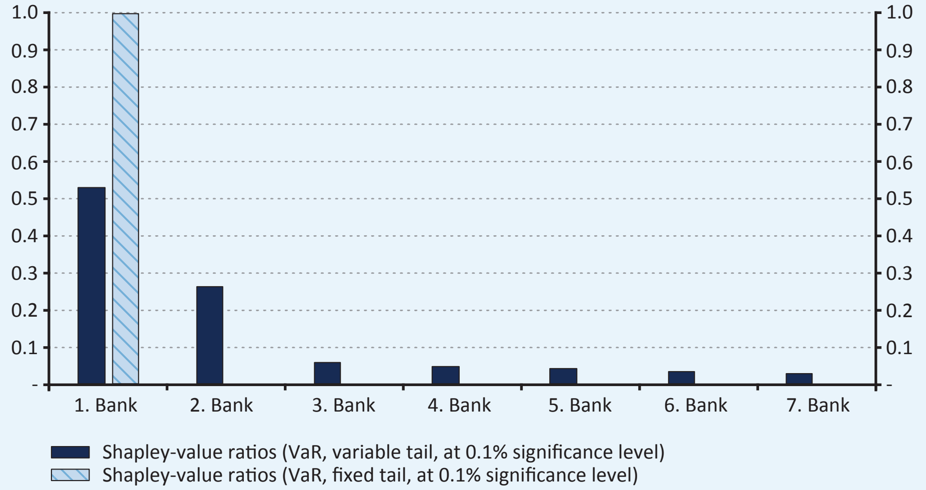
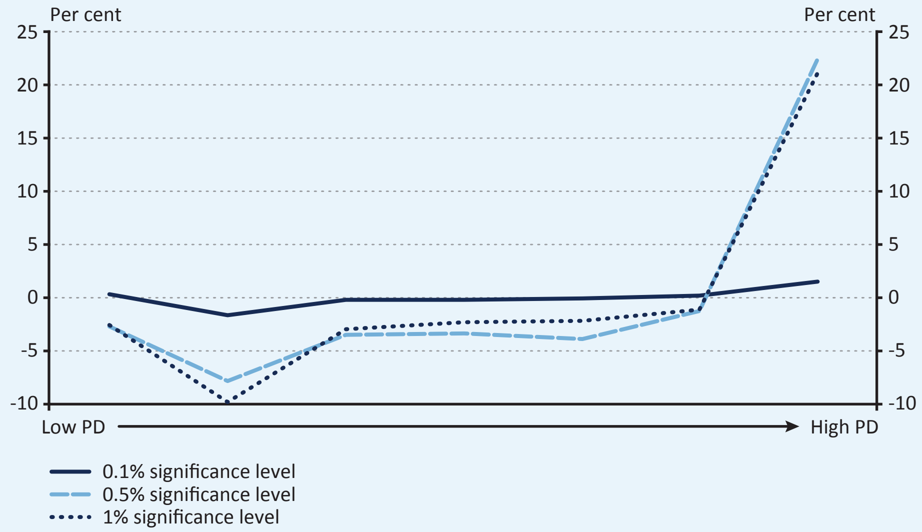


Figure 8.2
System-level risk allocation based on Shapley value (using Value at Risk measure, Scenario 1)



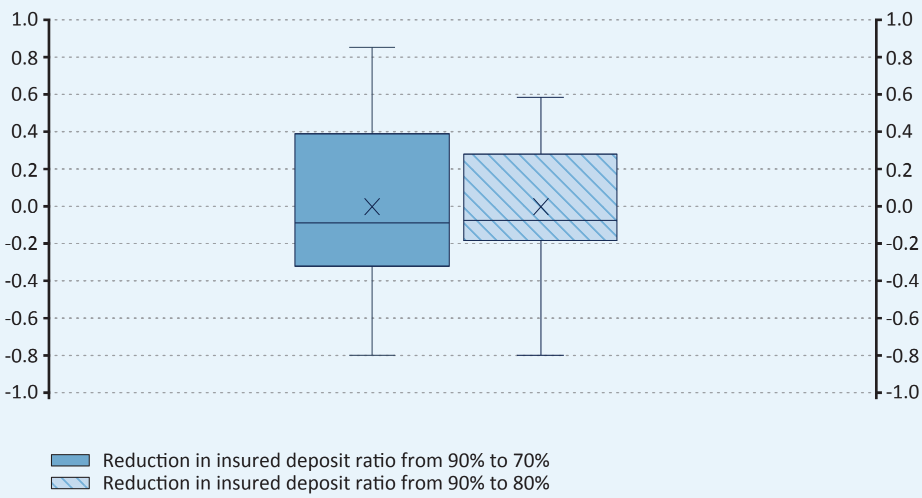
8.3 FIGURES FOR ROBUSTNESS ANALYSIS

Figure 8.3
Shapley value-based system-level risk allocation of domestic O-SIIs under different bank PDs



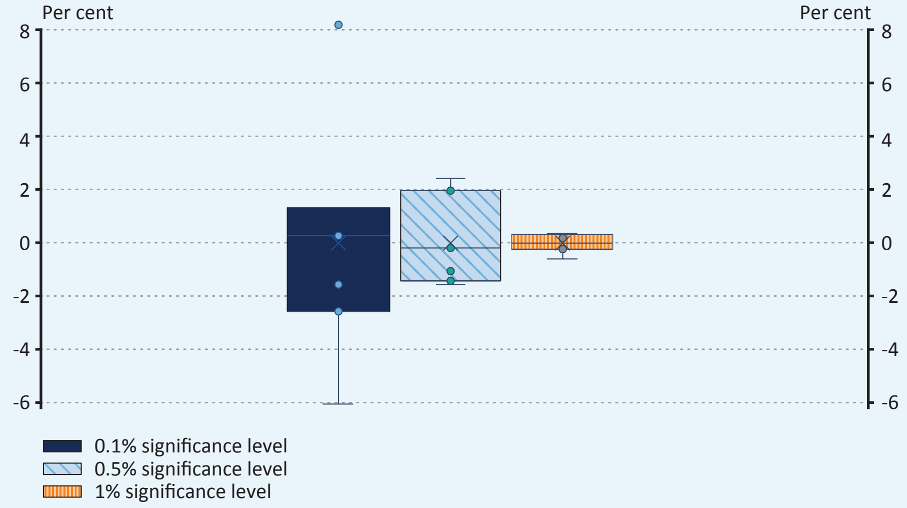
Note: Using ES as risk measure and variable tail approach. Percentage point change is the difference between the Shapley value-based allocation estimated using individual bank PDs and the Shapley value-based allocation estimated using the same PD for all institutions. In the figure, the Shapley value change data for each O-SII are plotted in ascending order by PD.

Figure 8.4
Change in the Shapley value-based system-level risk allocation under different household insured deposit ratios



Note: Using ES as risk measure and variable tail assumption at 99.9% confidence level. The figure plots the mean, median, lower and upper quartile, minimum and maximum of the Shapley value difference for O-SII institutions. The vertical axis shows the data in percentage points.

Figure 8.5
Difference in the Shapley value-based allocation using a correlation matrix calculated from annual asset value changes and from ROAs



Note: Using ES as risk measure and variable tail assumption. The figure plots the mean, median, lower quartile, upper quartile, minimum and maximum of the Shapley value difference for O-SII institutions. The vertical axis shows the data in percentage points.

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