László Békési–Csaba Köber–Henrik Kucsera–
Tímea Várnai–Balázs Világi

The macroeconomic forecasting model of the MNB

MNB Working Papers 4

2016

László Békési–Csaba Köber–Henrik Kucsera–
Tímea Várnai–Balázs Világi

The macroeconomic forecasting model of the MNB

MNB Working Papers 4

2016

MNB Working Papers 2016/4

The macroeconomic forecasting model of the MNB*

(Az MNB makrogazdasági előrejelző modellje)

Written by László Békési, Csaba Köber, Henrik Kucsera, Tímea Várnai, Balázs Világi

Budapest, December 2016

Published by the Magyar Nemzeti Bank

Publisher in charge: Eszter Hergár

H-1054 Budapest, Szabadság tér 9.

www.mnb.hu

ISSN 1585-5600 (online)

* László Békési (bekesil@mnb.hu) and Tímea Várnai (varnait@mnb.hu) are analysts, Világi Balázs (vilagib@mnb.hu) is the head of department, MNB Economic Modelling Department. Csaba Köber and Henrik Kucsera participated in the development of the model until September, 2015 and December, 2015. The authors would like to express their gratitude to György Molnár, Gábor Dániel Soós and Szabolcs Szentmihályi who supported the whole process of model development with their ideas and suggestions, to the members of the MNB's Monetary Council, Péter Bauer, Péter Gábriel, Gábor Pellényi and Barnabás Virág who discussed with the authors the results of the various versions of the model. Furthermore, to Ágnes Horváth who contributed to the creation of the paper's concept, to Tamás Briglevics the reviewer of the paper and to András Balatoni, Milán Horváth and Ferenc Tóth who contributed to the writing of the paper with their comments.

Contents

Abstract

The lessons of the financial and macroeconomic crisis of 2007-2008 made the development of a new macroeconomic forecasting model necessary in the MNB. The model represents a small open economy. It is based on the DSGE philosophy but it deviates from it at several points.

The new features of the model, compared to previous forecasting models of the MNB, are that the debt constraint and the heterogeneity of households and financial accelerator mechanism through the financing constraints of the firms appear. From methodological point of view, it is important that the model deviates from rational expectation hypothesis at several points and treats expectations pragmatically and flexibly.

The model parameters are calibrated according to experts' experience and SVAR estimations. The properties of the calibrated model are studied by impulse responses analysis, and the model fits into the MNB's forecasting framework successfully.

JEL: E21, E27, E31, E37, E44, E52.

Keywords: DSGE models, forecasting, precautionary motive, buffer stock model, heterogeneous households, financial accelerator, non-rational expectations.

Összefoglaló

A 2007-2008-as pénzügyi/makrogazdasági válság tanulságai szükségessé tették egy új makroökonómiai előrejelző modell fejlesztését az MNB-ben. A modell egy kis nyitott gazdaságot reprezentál, DSGE filozófián alapul, de több ponton eltér attól.

Az MNB eddigi előrejelzői modelljeihez képest újdonság, hogy a modellben megjelenik egyrészt a háztartások adósságkorlátja és heterogenitása, másrészt a vállalatok finanszírozási korlátainak hatásain keresztül a pénzügyi akcelerátor mechanizmus. Módszertani szempontból lényeges, hogy a racionális várakozások hipotézistől több ponton eltér a modell, a várakozásokat pragmatikusan és rugalmasan kezeli.

A modell paramétereit szakértői tapasztalatok és SVAR becslések alapján kalibráltuk. A kalibrált modell tulajdonságait impulzus válaszok elemzése segítségével vizsgáltuk, a modell sikeresen illeszkedik az MNB előrejelzői rendszerébe.

1 Introduction

The Magyar Nemzeti Bank (the central bank of Hungary, MNB) publishes macroeconomic forecasts since 2001, the adoption of inflation targeting, and uses formal macroeconomic models for these forecasts from the beginning. Initially, the forecast procedure was supported by traditional econometric models such as the NEM model (Benk et al. 2006) or the Delphi model (Horváth et al. 2009). From the beginning of the 2000s, *dynamic stochastic general equilibrium* (DSGE) models gained ground in the central banking practice too, in addition to the academic life.[1] As a result of these effects, a DSGE model estimated on the Hungarian economy's data was created in the MNB as well (Jakab and Világi, 2008), but this model did not become a part of the MNB's forecasting system. The NPM model, which was intoduced in 2011, was the first DSGE type model used for official forecasts (Szilágyi et al. 2016).

The financial crisis of 2007-08 and the following macroeconomic crisis made it inevitable for economists to reconsider some of their assumptions and certain implications which where derived from these conjectures. This process also affected macroeconomic modeling. As a result, the development of a new DSGE type model, which incorporates the lessons of the crisis, took place in the MNB.

The financial crisis of 2007-08 was unexpected by most economists because the main field of macroeconomics focused on the analysis of the 2-8 years long business cycles which were explained by technology and preference shocks. This narrow focus prevented the majority of macroeconomists from detecting a relatively new phenomenon, the financial cycle (Borio 2012, and Drehmann et al. 2012).

Financial cycles are longer and have larger amplitude than the usual business cycles. They tend to have larger credit growth in case of booms, during which risks build up endogenously, such as stock market or real estate price bubbles, which can cause systemic financial crises, and are followed by long recessions. Financial cycles strengthened significantly after the deregulations of the 1980s.

Currently, the main challenge of macroeconomic modeling is the incorporation of financial cycles in addition to usual business cycles into its models. For this, models have to display the characteristics of the financial intermediary system. It has become evident that leverage decisions and liquidity have crucial effects on the evolvement of financial cycles (see Geanakoplos 2009 and Gorton 2014).[2]

It should be emphasized that none of the models can fully integrate financial and usual business cycles or is able to represent all aspects of the financial cycle. Instead, the development of such models take place which can explain only some selected features of the financial cycle such as the build-up of bubbles, the outbreak of the crisis or the recession after the crisis. In the following, the recent results of the modeling of financial cycles will be illustrated with some examples.

A noteworthy example of the evolvement of macroeconomic bubbles can be found in De Grauwe (2012). The author's approach goes beyond the framework of mainstream macroeconomics, expectations are modeled through the results of behavioral economics. The advantage of this approach is that transitory shocks are also able to generate long macroeconomic fluctuations in the model economy. Furthermore, the approach has interesting economic policy implications. The model can demonstrate that the stabilization of inflation can help to stabilize the output gap, however, if the central bank tries to stabilize inflation only and does not focus on the output gap then in the long-run neither inflation, nor the output gap will be stabilized.

[1] See Beneš et al. (2005), Adolfson et al. (2008) and Christoffel et al. (2008).

[2] Although, before the crisis such models were created which analyzed the macroeconomic effects of leverage constraints, like Kiyotaki and Moore (1997), Bernanke et al. (1999), Iacoviello (2005), but these are not models of the financial cycle, they demonstrate how financial factors amplify the effects of usual business cycles.

Boissay et al. (2016) investigates how a long economic boom can lead to a financial crisis and why this type of crises has large macroeconomic costs.

The model of Eggertsson and Krugman (2012) explains how a large-scale financial crisis can create a macroeconomic recession. In addition to this, Eggertsson and Mehrotra (2014) demonstrate that the recession can be permanent. According to the models above, the leverage constraints of the indebted agents tighten due to the crisis and as a result of this their demand decreases. Normally, this lost demand can be compensated by the demand of non-indebted agents induced by decreasing interest rates due to monetary easing. However, if the crisis is severe enough then the required monetary easing is so large that it would violate the zero lower bound of the nominal interest rate. As a consequence, in such cases monetary policy is not able to stimulate the economy sufficiently. After Keynes this situation is called *liquidity trap*. The authors also show that in the case of a liquidity trap fiscal policy can boost the economy efficiently because the consumption of the indebted households co-moves with their income.

The economic modelers of the MNB had to react to these challenges, too. The development of several models have started such as the early warning system and other macroprudential models. In addition to that, the MNB's main macroeconomic forecasting model has also been substantially revised. In the following, the latter development will be described.

Naturally, the horizon of an inflation forecasting model adapts to the horizon of inflation targeting which is much shorter than the length of financial cycles. Due to this, an inflation forecasting model cannot aim to explain the evolvement of financial cycles. Nevertheless, this does not mean that the research results described above can not be used in the course of the development of the model. In fact, it should be emphasized once again that the main motivation of the development of the new forecasting model was to model the economy more plausibly with the help of the new lessons derived from the recent crisis.

In the course of the model development two areas of the recent research results were used. First, after the crisis it became evident that economic agents' debt and leverage constraints have significant macroeconomic effects, thus the explicit incorporation of these constraints' effects cannot be neglected. Furthermore, behavioral economic considerations have become fruitful in understanding financial cycles. Due to this, the model differs in some points from the mainstream approach.

Concretely, this means the following. First, in the model the leverage constraints of households and the heterogeneity of households' indebtedness appear. Furthermore, financing constraints of firms are also considered. From a methodological point of view, the deviation from the rational expectations hypothesis is essential. Adaptive expectations are assumed in several parts of the model.[3]

The remaining part of the paper is organized as follows. *Section 2* presents the model and the method of its solution. In *section 3* the model's impulse responses are analyzed. Finally, *section 4* concludes.

[3] The rational and the adaptive expectations are the two extreme cases of the modeling of expectations. While the former attributes unrealistic cognitive abilities to economic agents, the latter attributes too little rationality to them. Therefore neither of them are good models of expectations formation. Due to the fact that there isn't a consensus concerning the modeling of expectations formation, a hybrid approach is used in the model where rational and adaptive elements are both present. In this way we expect the disadvantageous features of the two extreme cases to be eliminated.

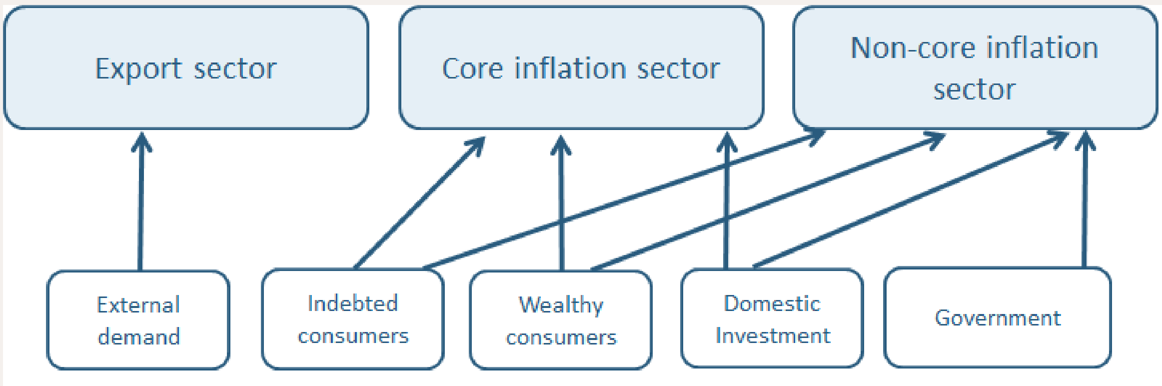
2 The model

The new model resembles for the most part to the principles of a medium-sized DSGE models but throughout the process of model development deviations from these principles took place if the forecasting performance of the model was found to be improved by this. Being a model of a small open economy, most foreign factors are treated exogenously in a small-scale block representing the rest of the world. The production side of the model is multi-sectoral. The sectors are determined by the need of inflation forecasting. The sector of core inflation products and the sector of non-core inflation products are separated. The effect of the households' net wealth on their consumption-savings decisions is also considered. Furthermore, the effect of the firms' leverage constraints on their financing possibilities are also incorporated.

2.1 AGGREGATE DEMAND

In the model the following agents' demand for products and services defines aggregate demand: households (indebted and with positive net wealth), firms, government, and the rest of the world. Aggregate demand is directed to three sectors. The sectors are defined according to the needs of inflation forecasting: the sector producing core inflation products, the sector producing non-core inflation products, and the sector producing export products are separated.

Figure 1
The structure of aggregate demand



The demand of the two types of households is directed to a final consumption good which uses the products of core inflation and the non-core inflation sectors as inputs. The investment demand of domestic firms is directed to a different good, the investment good which also uses core inflation and non-core inflation products as inputs, but the ratio of the two inputs is different from the consumption good's ratio. The government's demand has the same structure. The demand for the export sector's products are generated by the rest of the world.

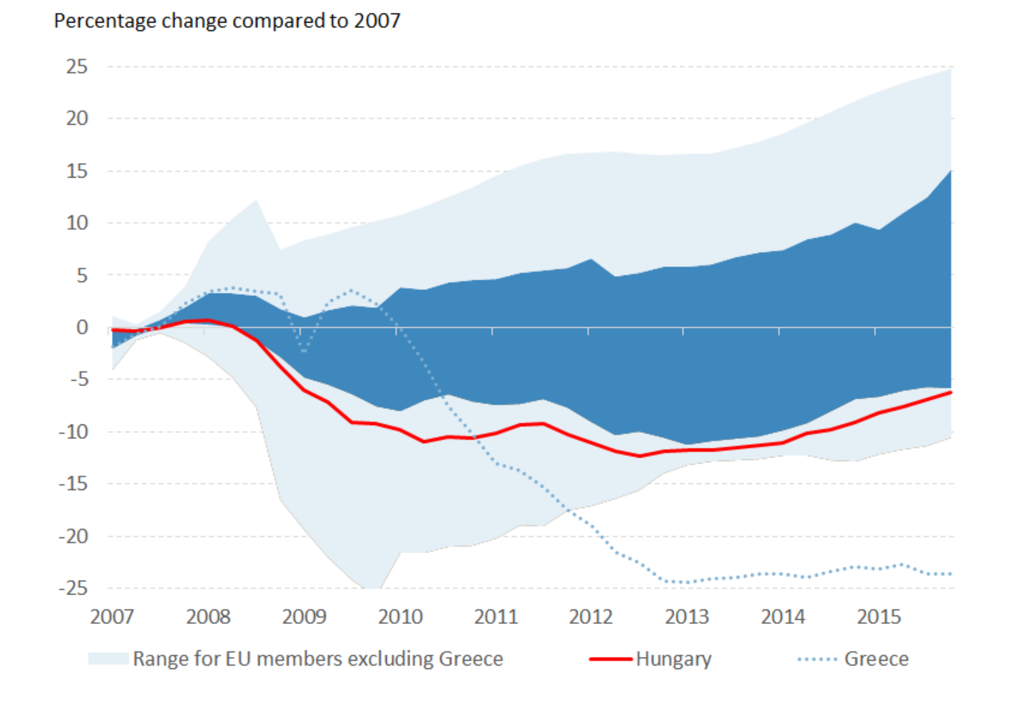
2.1.1 HOUSEHOLDS

In most DSGE models consumption demand is derived from the behavior of a representative household. One of the main drawbacks of this approach is that it cannot take into consideration the effect of households' wealth heterogeneity on consumption. As Eggertsson and Krugman (2012) and Eggertsson and Mehrotra (2014) showed heterogeneity of wealth and indebtedness of households have significant macroeconomic effects, especially after a financial crisis. This is especially important in case of the

Hungarian economy. As it is known, the Hungarian households' indebtedness was extremely high at the outbreak of the crisis. Due to this consumption also dropped as *Figure 2* demonstrates.

Figure 2
Household consumption compared to its pre-crisis level

(Source: MNB.
The inner darker area on the figure represents the inner 60% of distribution of consumption development in the EU countries.)



The other problem with the modeling of consumption comes from the application of linear approximation to DSGE models in the forecasting practice. However, in a linearized model consumers become risk-neutral (even if they were risk-averse in the original model) and due to this the precautionary motive, which affects consumer behaviour significantly, can't be taken into consideration.

The version of the standard consumption models which assumes deterministic or risk-averse consumers has two main problems. On the one hand, the *marginal propensity to consume* (the amount of money spent on current consumption from a unit income increase) is very low. On the other hand, the marginal propensity to consume is independent of the level of wealth or indebtedness. If the precautionary motive is incorporated into the model then the above features of the model, which are not inline with empirical results, can be eliminated. The average marginal propensity to consume will increase and will be in line with the empirical results. Furthermore, the marginal propensity to consume for the poorer (indebted) households will be higher while for richer households it will be lower. These will be discussed in more detail in the next section and in *Appendix A.2*.

To solve the above problem, the household block of the model is able to handle the precautionary motive and takes into consideration the households' wealth heterogeneity and its implications. The household block is based on the models of Carroll (2001, 2009, 2012). The model's assumptions are for the most part standard, consumer demand can be derived from an infinite time horizon utility maximization problem. Significant degree of precautionary motive comes from the assumption of significant negative idiosyncratic income shocks (e.g. unemployment) in addition to macroeconomic shocks. As discussed in more detail in the following, this assumption ensures that the precautionary motive significantly influences the consumption-savings decision of households.

THE PRECAUTIONARY MOTIVE – AN ILLUSTRATIVE MODEL

In this section, the effect of the precautionary motive on consumer behavior will be illustrated non-technically. As a starting point, let's analyze the two period deterministic consumer problem without precautionary motive.

$$\begin{aligned}
&\max_{c_1,c_2,s_1} \log(c_1) + \beta \log(c_2),\\
&c_1 + s_1 = n,\\
&c_2 = y_2 + (1+r)s_1.
\end{aligned}$$

where $0 < \beta < 1$ is the discount factor of the household. $n = y_1 + (1+r)s_0$ is the disposable resources of the consumer in the first period which is composed of two elements, $(1+r)s_0$ is past savings and the income on it, where r is the interest rate, s_0 is past savings, and y_1 is the consumers current income in the first period. Furthermore, y_2 is the income in the second period, c_1, c_2 are consumption in the first and the second period, and s_1 is savings in the first period.

The first order condition of the solution is the well-known Euler equation,

$$MU_1 = (1+r)MU_2,$$

where MU_1 and MU_2 are the marginal utility of consumption in the first and the second period, respectively. Because of the logarithmic utility function, the above equation implies the following formula:

$$\frac{1}{c_1} = (1+r)\frac{\beta}{c_2}.$$

Substituting the budget constraints:

$$\frac{1}{n-s_1} = \frac{(1+r)\beta}{y_2 + (1+r)s_1}. \tag{1}$$

Using this equation, the solution for s_1 can be calculated. Knowing s_1, one can calculate c_1 and c_2 with the help of the budget constraints. Rearranging equation (1):

$$y_2 + s_1(1+r) = \beta(1+r)(n-s_1)$$

This implies that

$$s_1 = \frac{\beta(1+r)n - y_2}{(1+\beta)(1+r)}. \tag{2}$$

The functions describing optimal consumptions can be derived using equation (2),

$$\begin{aligned}
c_1 &= n - s_1 = \frac{n + \frac{y_2}{1+r}}{1+\beta}, \\
c_2 &= (1+r)s_1 + y_2 = \frac{\beta\left[(1+r)n + y_2\right]}{1+\beta}.
\end{aligned} \tag{3}$$

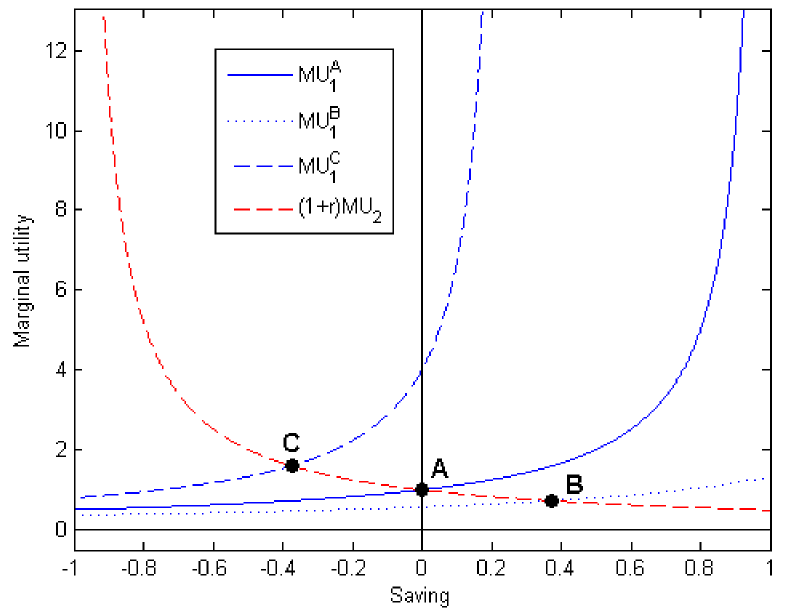
Figure 3 illustrates graphic solution for s_1. In the figure MU_1 is represented with three possible values for n, furthermore it is assumed that $(1+r)\beta = 1$. If $n = y_2$ then the solution is point A, and the consumer's savings are zero, $c_1 = n$ and $c_2 = y_2$. If $n > y_2$, then $s_1 > 0$, so savings are positive. If $n < y_2$, then $s_1 < 0$, so the consumer takes on debt.

In the deterministic case the *marginal propensity to consume* (*mpc*) is constant in the first period. Independent of the level of initial wealth and y_1, one unit income increase increases c_1 in the same magnitude. Differentiating the consumption function (3) with respect to n:

$$mpc = \frac{\partial c_1}{\partial n} = \frac{1}{1+\beta} < 1.$$

In the following the precautionary motive is discussed. Suppose that in the second period there are two states of the world. In the first one, income is $y_2(1) = y_2 > 0$, while in the second one it is $y_2(2) = 0$. The second state can be interpreted as if the

Figure 3
The effect of income and/or wealth (n) on savings



household has lost its job. In this case, the optimization problem is:

$$\max_{c_1,c_2(1),c_2(2),s_1} \log(c_1) + \beta\left[(1-\pi)\log(c_2(1)) + \pi\log(c_2(2))\right],$$
$$c_1 + s_1 = n,$$
$$c_2(1) = y_2 + (1+r)s_1,$$
$$c_2(2) = (1+r)s_1,$$

where the probability of the first state of the world is $1-\pi$, while the second's is π. Then the following expression will be the Euler equation:

$$MU_1 = (1+r)\mathrm{E}_t\left[MU_2\right] = (1+r)\left[(1-\pi)MU_2(1) + \pi MU_2(2)\right],$$

so

$$\frac{1}{c_1} = (1-\pi)\frac{(1+r)\beta}{c_2(1)} + \pi\frac{(1+r)\beta}{c_2(2)}.$$

After substituting the budget constraints:

$$\frac{1}{n-s_1} = (1-\pi)\frac{(1+r)\beta}{y_2+(1+r)s_1} + \pi\frac{(1+r)\beta}{(1+r)s_1}.$$

The Euler equation above can be expressed in the following way:

$$\frac{1}{n-s_1} = \frac{(1+r)\beta}{y_2+(1+r)s_1} + \pi\frac{(1+r)\beta y_2}{(1+r)s_1\left(y_2+(1+r)s_1\right)}.$$

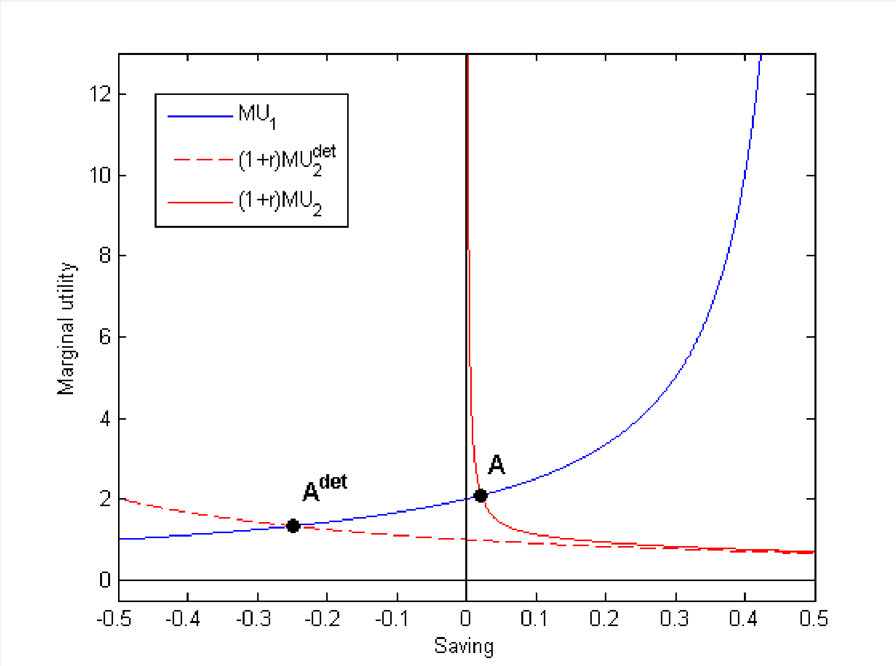
The left hand side and the first term of the right hand side of the equation are identical to the deterministic problem's solution. The difference comes from the second term of the right hand side. This term expresses the precautionary motive. See *Figure 4* where the red dashed line represents the marginal utility of saving in the deterministic case. The red solid line represents the marginal utility of saving in case if income uncertainty. The difference between the two lines can be expressed with the following formula:

$$\pi\frac{\beta(1+r)y_2}{(1+r)s_1\left(y_2+(1+r)s_1\right)}.$$

If $s_1 \to 0$ then the expression above converges to infinity. Hence, the line representing the marginal utility of saving approaches the vertical axis of the coordinate system.

Figure 4 shows that the optimal choice of the household without income uncertainty would be indebtedness in the first period (A^{det}). However, she saves due to income uncertainty to be able to consume in case of unemployment as well (A).

Figure 4
The precautionary motive



To derive the numeric solution of the problem we use the Euler equation,

$$\frac{1}{n - s_1} = \frac{\beta(1+r)}{y_2 + (1+r)s_1} + \pi \frac{\beta(1+r)y_2}{(1+r)s_1\left(y_2 + (1+r)s_1\right)}.$$

Rearranging this gives a quadratic equation, and the positive root of this provides the solution:

$$\mathrm{A}s_1^2 + \mathrm{B}s_1 + \mathrm{C} = 0, \tag{4}$$

where

$$\begin{aligned} \mathrm{A} &= (1+\beta)(1+r), \\ \mathrm{B} &= y_2 - \beta(1+r)n + \pi\beta y_2, \\ \mathrm{C} &= -\pi\beta n y_2. \end{aligned}$$

Figure 5 illustrates, how the optimal saving decision changes due to the changes of n (what can be caused by changes in y_1 income or s_0 savings). Point A shows the starting point, point B illustrates the effect of a relatively small increase of n, and point C represents the effect of the further increase of n. The initial growth of n increases savings minimally. This income increase almost fully increases consumption, that is, the marginal propensity to consume is close to one. However, if the income increase is big then the marginal utility of c_1 decreases, and the household significantly increases its savings, hence, the marginal propensity to consume will be significantly lower than one.

Due to this, if one illustrates c_1 as a function of n, then in the beginning its slope will be close to one, because the increase of income does not increase savings significantly, most of it increases c_1. In case of higher n values the slope of the function will be much lower, because an income increase raises savings significantly and c_1 increases only slightly. Using the solution of (4) the

Figure 5
The effect of changes in income and/or wealth (*n*)

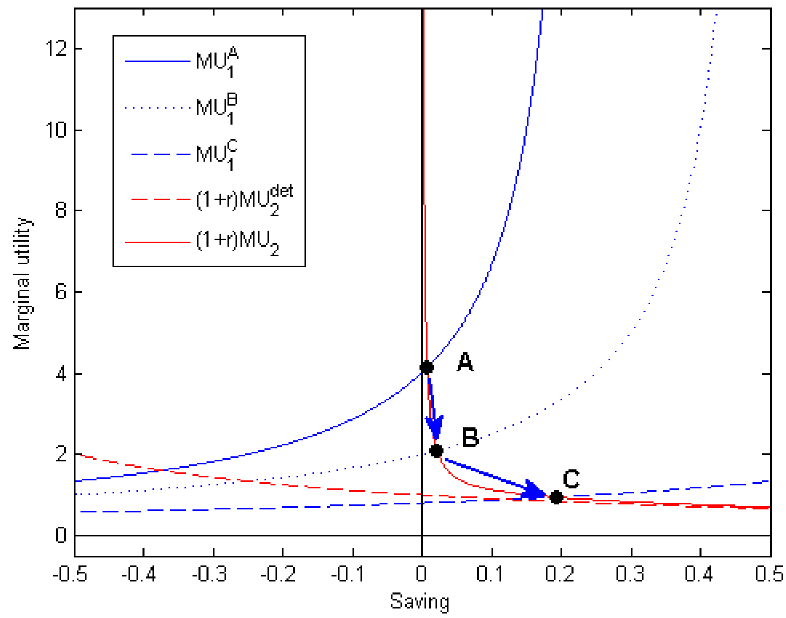
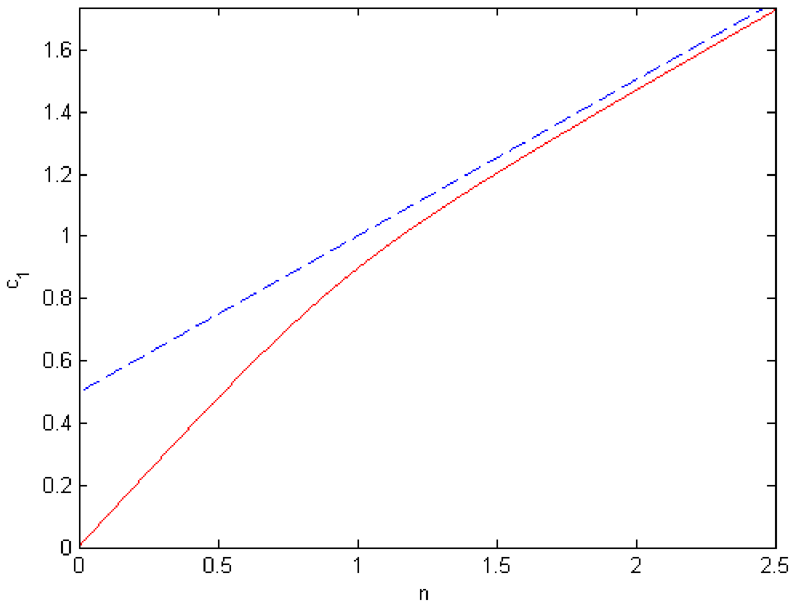


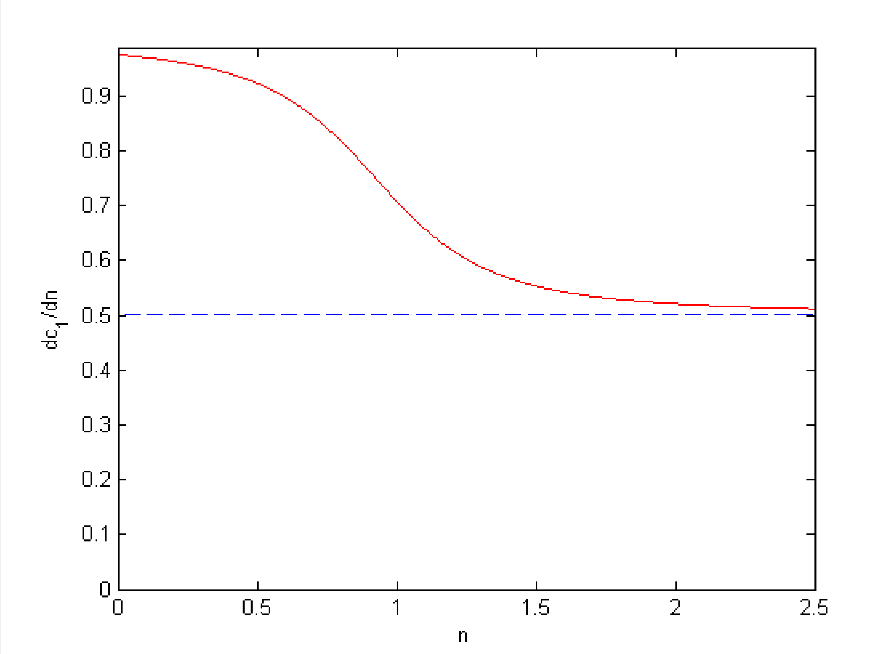
Figure 6
Consumption function



function can be calculated numerically as well. *Figure 6* shows the concave consumption function in case of the precautionary motive. *Figure 7* illustrates the changes of the marginal propensity to consume.[4].

[4] It should be emphasized that the consumption function presented here is derived from an illustrative model. It illustrates the effects of the precautionary motive qualitatively but the presented values of the marginal propensity to consume are not plausible empirically.

Figure 7
Marginal propensity to consume



In *Appendix A.2.* it is shown that the optimal decision can be indebtedness even in the presence of the precautionary motive if in the second state of the world the consumer's income is positive. The level of debt even in this case is smaller than in the deterministic case. It is also shown that if income in the second state of the world is close to y_2 then the effect of the precautionary motive becomes negligible. Furthermore, it is also demonstrated that the implications of the precautionary motive are similar to a model where an exogenous debt constraint prevents the consumers to take on as much debt as in the deterministic case.

PRECAUTIONARY MOTIVE – DYNAMIC MODEL

After illustrating the effect of the precautionary motive on the consumer decision in the previous section, we now turn to the discussion of the actual household block of the model. The optimization problem of a household is formally the following:

$$\max \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right],$$

$$c_t + a_t = \left(1 + r_{t-1}^h\right) a_{t-1} + d_t,$$

where $\sigma > 0$ is the parameter of the utility function, $0 < \beta < 1$ is the discount factor of the household, c_t is real consumption, t is the time index, a_t is the household's net wealth, d_t is the real disposable income of the household and r_t^h is the relevant real interest rate from household's perspective.[5]

From the household's perspective r_t^h and d_t are exogenous random variables[6], both affected by macroeconomic shocks. As mentioned in the previous section, only macroeconomic shocks are not enough for the precautionary motive to influence households' behavior significantly. If d_t's value fluctuates only a few percentages then the households will not increase their savings significantly for precautionary reasons. Therefore it is assumed that d_t's value can be influenced by household specific shocks as well, not just by macroeconomic shocks. It is assumed that in each period d_t's value can decrease significantly with non-zero probability. This state of the world can be interpreted as unemployment, what has low but non-negligible probability, and the unemployment benefit is significantly lower than the household's normal income.

[5] It is assumed that interest rates on the household's deposit and debt are different.

[6] The labor supply decisions are not analyzed in the household block, so incomes are considered exogenous.

For technical reasons, it is practical to reformulate the problem to solve for the solution in the following way:

$$\max \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left[\frac{c_t^{1-\sigma}}{1-\sigma} \right],$$
$$a_t = n_t - c_t,$$
$$n_{t+1} = \left(1 + r_t^h\right) a_t + d_{t+1},$$

where n_t is defined similarly to that as in the previous section.

It is assumed that households are rational to the extent that if they know the initial value of n_t and the expected path of d_t and r_t^h then they will be able to find the optimal solution of the above problem. However, it is not assumed that households can forecast the exact path of d_t and r_t^h according to the rational expectation hypothesis. In case of rational, i.e., model consistent expectations, the households' expectations regarding to d_t and r_t^h are totally in line with the endogenous path of d_t and r_t^h generated by the full macroeconomic model. The calculation of such model consistent paths is pushing at the current frontiers of economics. Therefore, it is assumed that households' income and interest rate expectations are not model consistent. This assumption, besides that it makes it easier to solve the model, is closer to the empirical evidence on households' behavior than rational expectations according to the authors' view. For the expectations of r_t^h and d_t the followings are assumed. The household observes the r_t^h real interest rate and the d_t real income, and it assumes that the two variables return to their steady state values in given T^r and T^d periods. The decay of the two variables is assumed to be linear. Furthermore, it is also assumed that the household's considerations about the probability of unemployment are also independent from the full macroeconomic model.

The solution of the optimum problem is non-trivial even with the assumptions above. It can be found with numerical simulations as described in Carroll (2001, 2009, 2012). The simulation is carried out independently from other parts of the model. For the numerical simulations it is worth formulating the model in the following simplified way. Let's define the normalized variables $\widehat{o}_t = o_t/d$.[7] With the help of these, the household's optimum problem is the following:

$$\max \sum_{t=0}^{\infty} \beta^t d^{1-\sigma} \mathrm{E}_0 \left[\frac{\widehat{c_t^{1-\sigma}}}{1-\sigma} \right],$$
$$\widehat{a}_t = \widehat{n}_t - \widehat{c}_t$$
$$\widehat{n_{t+1}} = \left(1 + r_t^h\right) \widehat{a}_t + \widehat{d_{t+1}}.$$

One can get the following consumption function with numerical simulations

$$\widehat{c}_t = f\left(\widehat{n}_t, r_t^h, \widehat{d}_t\right). \tag{5}$$

The consumption function is increasing and concave in n_t. In the formula the n_t variable expresses the effect of current income and the initial wealth or debt on consumption. If n is small (the net debt is big) then the function is steep and the marginal propensity to consume is big. If n is big (big positive net wealth), then the function is flat and the marginal propensity to consume is small.

The consumption function (5) summarizes the households' behavior in case of the precautionary motive. This can be described intuitively in the following way. The households aim to hold a certain size of wealth, i.e., *buffer stock*. The size of this buffer stock depends on several parameters, but income uncertainty is a key factor. The higher is this uncertainty the larger is the aimed buffer stock. Therefore, if the household's wealth falls below the aimed buffer stock then it decreases its consumption relative to its income, and at the same time, it increases its savings to reach the target value of its wealth. Conversely, if its wealth increases above the target value, then the household decreases its savings. If households' income increases then their consumption also increases but this does not mean that they consume the increase completely. Households would like to smooth their consumption so they try to distribute the increase over time, and have its benefit in the future as well.

Finally, we would like to draw attention to the fact that with this model we are able to simulate the effect of such factors which cannot be analyzed in most DSGE models. For example, the consumption effect of variations in economic uncertainty can be investigated. This uncertainty can be modeled by the changes of the subjective probability of unemployment, which influences the size of buffer stock savings and consumption.

[7] The steady state value of the model variables corresponds to the trend growth path of the same variables' empirical counterpart.

THE INCORPORATION OF THE CONSUMER MODEL INTO THE MACROECONOMIC ENVIRONMENT

The consumption function (5) can easily be incorporated into the full macroeconomic model. If n_t, r_t^h and d_t are determined by other parts of the model then one can calculate the level of consumption with the help of the consumption function.

At the same time, it is a well-known empirical fact that aggregate consumption reacts relatively slowly to the changes of income and interest rates. To solve this problem, the hypothesis of *habit formation* was built into DSGE models, see Smets and Wouters (2003, 2007). Formally, this means that in addition to current consumption, past consumption is also present in the utility function of household. Therefore, not just current and future, but past consumption is also in the *Euler equation*. As a result of this, the accommodation of consumption to any kind of shock is smooth without jumps.

In this model, the gradual accommodation of consumption is reached by another approach. At this point, it was assumed again that consumers' rationality is bounded. It is assumed that they know perfectly the level of their nominal income and wealth. However, they know only imperfectly the real value of their income and wealth, since they do not know the exact price level. As a result of this, perceived real income and perceived real wealth converge to their true value gradually. Formally, equations (52)–(53) and (104)–(105) describe this process in *Appendix A.1.*

As discussed previously, one of the key features of the Hungarian economy was the high indebtedness of consumers at the outbreak of the crisis. Therefore, two types of households are distinguished in the macroeconomic model. A group of indebted households and a group of households with positive net wealth. The preferences of the two groups are identical, thus, consumption function (5) represents the behavior of both groups. The difference is in their initial wealth a_{-1}^1 and a_{-1}^2 (as a result of this n_0^1 and n_0^2 are also different) and the path of their income d_t^1 and d_t^2 as well.

2.1.2 INVESTMENT

It is assumed that an independent competitive sector is responsible for investments. This sector buys investment goods, and at the end of each period, it buys the used capital from the entrepreneurs described in *section 2.3*. From the combination of investment goods and capital, they produce a new good and sell it to the entrepreneurs. Following the models of Smets and Wouters (2003, 2007), the equation below describes the production of new capital

$$k_t = (1-\tau)k_{t-1} + \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{6}$$

where k_t is physical capital, I_t is investment, the Φ function represents investment adjustment cost such that $\Phi(1) = \Phi'(1) = 0$, $\Phi'' > 0$ and τ is the depreciation rate of capital.

Smets and Wouters showed that profit maximization in the sector implies an investment function where the investment decision depends on past and current investment and on the real price of physical capital. At this point, we deviate from the strict rules of DSGE models and do not derive the investment equation of the model from a formal profit maximization problem, we only gain inspiration from that. The forward-looking term is left out and the coefficients are not derived from 'deep parameters',

$$I_t = I_{t-1}^{\omega^I} Q_t^{\omega^Q}, \tag{7}$$

where Q_t is the real value of capital, ω^I and ω^Q are positive parameters.

In the next part, the determination of Q_t is described. The definition of the return on physical capital is,

$$\mathrm{E}_t\left[R_{t+1}^k\right] = \frac{(1-\tau)\mathrm{E}_t\left[Q_{t+1}\right] + \mathrm{E}_t\left[z_{t+1}\right]}{Q_t}, \tag{8}$$

where R_{t+1}^k is the return on capital and z_{t+1} is the rental rate of capital. Equation (36) in *section 2.3* shows how R_{t+1}^k depends on the real interest rate.

In standard DSGE models, the rental rate of capital is determined by the marginal product of capital (based on *section 2.2.3*),

$$z_t = p_t^{ci} \alpha \left(\frac{l_t}{k_{t-1}} \right)^{1-\alpha}, \tag{9}$$

where p_t^{ci} is the real price of the composite good produced by labor and capital, and l_t is labor input. Equations (6), (13), (10) and (9) describe the process of investment and capital accumulation.

However, the investment block above is not built into the model in this form because it is not in line with the views of the MNB's experts. Explicitly, in case of certain shocks, real GDP increases permanently, while investments decrease. Therefore, the investment model was modified, we deviated from the strict DSGE philosophy.

Let's rearrange equation (8):

$$Q_t = \mathrm{E}_t \left[\frac{(1-\tau)Q_{t+1} + z_{t+1}}{R_{t+1}^k} \right]. \tag{10}$$

As a first step, using equation (10), one can get the following equations with recursive substitution,

$$\begin{aligned}
Q_t &= \mathrm{E}_t \left[\frac{(1-\tau)^2 Q_{t+2}}{R_{t+2}^k R_{t+1}^k} + \frac{(1-\tau) z_{t+2}}{R_{t+2}^k R_{t+1}^k} + \frac{z_{t+1}}{R_{t+1}^k} \right], \\
Q_t &= \mathrm{E}_t \left[\frac{(1-\tau)^3 Q_{t+3}}{R_{t+3}^k R_{t+2}^k R_{t+1}^k} + \frac{(1-\tau)^2 z_{t+3}}{R_{t+3}^k R_{t+2}^k R_{t+1}^k} + \frac{(1-\tau) z_{t+2}}{R_{t+2}^k R_{t+1}^k} + \frac{z_{t+1}}{R_{t+1}^k} \right], \\
&\vdots \\
Q_t &= \mathrm{E}_t \left[\frac{(1-\tau)^T Q_{t+T}}{R^k(t+1, t+T)} \right] + \sum_{i=1}^{T} \mathrm{E}_t \left[\frac{(1-\tau)^{i-1} z_{t+i}}{R^k(t+1, t+i)} \right],
\end{aligned}$$

where $R^k(t, t') = R_t^k R_{t+1}^k \ldots R_{t'}^k$. If $T \to \infty$ then the formula will be the following,

$$Q_t = \sum_{i=1}^{\infty} \mathrm{E}_t \left[\frac{(1-\tau)^{i-1} z_{t+i}}{R^k(t+1, t+i)} \right].$$

At this point, the authors deviate from the rational expectations hypothesis. It is assumed that the agents of the investment sector cannot see forward over an infinite horizon, only over a T^Q finite horizon. Furthermore, it is also assumed that the forward-looking variables of the model are not determined by model consistent rational expectations, but with adaptive expectations which is indicated with operator $\bar{\mathrm{E}}$. With these modifications, the formula above takes the following form

$$Q_t^\star = \sum_{i=1}^{T^Q} \frac{\bar{\mathrm{E}}_t \left[(1-\tau)^{i-1} z_{t+i}^\star \right]}{\bar{\mathrm{E}}_t \left[R(t+1, t+i) \right]}. \tag{11}$$

In the equation above, not just the time horizon and expectations but the formula of the rental price of capital is also modified to ensure that the expansion of output is indeed increasing the rental price. $z_t^\star$ is defined in the following way,

$$z_t^\star = a^z z_t + (1 - a^z)\, z^\star \frac{p_t^{ci} ci_t}{p^{ci} ci}, \tag{12}$$

where $0 < a^z < 1$, ci_t is the composite good defined in *section 2.2.3*, which contains physical capital. The higher the revenue from selling composite good produced with the help of capital, the higher the rental price is. The variables without time index $z^\star$, p^{ci} and ci stand for the steady state value of the appropriate variable. Observe that in the steady state $z^\star = z$.

The investment equation is modified in the following way:

$$I_t = I_{t-1}^{\omega^I} Q_t^{\omega^Q} \left(Q_t^\star \right)^{\omega^{Q^\star}} gdp_t^{\omega^{gdp}}, \tag{13}$$

where gdp_t is real GDP.

2.1.3 EXPORT DEMAND AND GOVERNMENT DEMAND

The export demand of foreigners are not modeled in detail. The demand for the products of the export sector depends on global growth and the prices of the export products relative to foreign ones,

$$x_t = x_{t-1}^{\theta^1} \left(y_t^*\right)^{\theta^2} \left(q_t^x\right)^{\theta^3}, \tag{14}$$

where x_t is the export, y_t^* is the foreign real GDP, $q_t^x = e_t P_t^* / P_t^x$ is the relevant relative price (i.e., real exchange rate) from the export's perspective, θ^1, θ^2 and θ^3 are positive parameters.

In the model, government expenditure is totally exogenous, its value is g_t. Government expenditure is financed by *lump-sum taxes*.

2.1.4 DEMAND FOR THE PRODUCTS OF THE DIFFERENT SECTORS

In the model, three sectors of production are distinguished. The sector producing core inflation products (sector 1), the sector producing non-core inflation products (sector 2) and the sector producing export goods.

The final consumption good is assumed to be a combination of the products of sector 1 and sector 2. Formally, this is represented in the model by assuming that a competitive sector exists which produces the final consumption good and uses the two sectors' products as input in a CES production function,

$$c_t = \left[\left(\chi^c\right)^{\frac{1}{\varrho^c}} \left(y_t^{1c}\right)^{\frac{\varrho^c - 1}{\varrho^c}} + \left(1 - \chi^c\right)^{\frac{1}{\varrho^c}} \left(y_t^{2c}\right)^{\frac{\varrho^c - 1}{\varrho^c}} \right]^{\frac{\varrho^c}{\varrho^c - 1}},$$

where y_t^{1c} and y_t^{2c} are inputs from sector 1 and sector 2, χ^c and ϱ^c are positive parameters, and the latter measures the elasticity of substitution between the two inputs.[8]

The demand for y_t^{1c} and y_t^{2c} and the price index determining the price of the final consumption good can be derived from the profit maximization problem of the sector producing the final consumption good:

$$y_t^{1c} = \chi^c \left(\frac{P_t^c}{P_t^1}\right)^{\varrho^c} c_t, \tag{15}$$

$$y_t^{2c} = \left(1 - \chi^c\right) \left(\frac{P_t^c}{P_t^2}\right)^{\varrho^c} c_t, \tag{16}$$

$$P_t^c = \left[\chi^c \left(P_t^1\right)^{1-\varrho^c} + \left(1 - \chi^c\right) \left(P_t^2\right)^{1-\varrho^c} \right]^{\frac{1}{1-\varrho^c}}, \tag{17}$$

where P_t^1 and P_t^2 are the prices of sector 1's and sector 2's products, respectively.

Similarly, it is assumed that the investment good is also produced from the products of sector 1 and sector 2 by a competitive sector with CES technology:

$$I_t = \left[\left(\chi^I\right)^{\frac{1}{\varrho^I}} \left(y_t^{1I}\right)^{\frac{\varrho^I - 1}{\varrho^I}} + \left(1 - \chi^I\right)^{\frac{1}{\varrho^I}} \left(y_t^{2I}\right)^{\frac{\varrho^I - 1}{\varrho^I}} \right]^{\frac{\varrho^I}{\varrho^I - 1}},$$

where y_t^{1I} and y_t^{2I} are the inputs from sector 1 and sector 2, respectively, χ^I and ϱ^I are positive parameters, and the latter measures the elasticity of substitution between the two inputs.

[8] The sector aggregating intermediate products can be interpreted as the retail sector, but the authors prefer to interpret this sector as a technical assumption. The retail sector as an interpretation is problematic from two aspects. On the one hand, labour is not an input of this sector. On the other hand, it is a competitive sector with flexible pricing. In contrast to this, the retail sector's pricing is sticky. In the model sector 1's and the export sector's pricing is sticky.

The foregoing implies that the demand for y_t^{1I} and y_t^{2I}, and the investment price index are defined by the following equations,

$$y_t^{1I} = \chi^I \left(\frac{P_t^I}{P_t^1}\right)^{\varrho^I} I_t, \tag{18}$$

$$y_t^{2I} = (1-\chi^I)\left(\frac{P_t^I}{P_t^2}\right)^{\varrho^I} I_t, \tag{19}$$

$$P_t^I = \left[\chi^I \left(P_t^1\right)^{1-\varrho^I} + (1-\chi^I)\left(P_t^2\right)^{1-\varrho^I}\right]^{\frac{1}{1-\varrho^I}}. \tag{20}$$

Furthermore, the government consumption good is also assumed to be produced from the products of sector 1 and sector 2 by a competitive sector with CES technology:

$$g_t = \left[(\chi^g)^{\frac{1}{\varrho^g}} \left(y_t^{1g}\right)^{\frac{\varrho^g-1}{\varrho^g}} + (1-\chi^g)^{\frac{1}{\varrho^g}} \left(y_t^{2g}\right)^{\frac{\varrho^g-1}{\varrho^g}}\right]^{\frac{\varrho^g}{\varrho^g-1}}.$$

Similarly to the previous steps:

$$y_t^{1g} = \chi^g \left(\frac{P_t^g}{P_t^1}\right)^{\varrho^g} g_t, \tag{21}$$

$$y_t^{2g} = (1-\chi^g)\left(\frac{P_t^g}{P_t^2}\right)^{\varrho^g} g_t, \tag{22}$$

$$P_t^g = \left[\chi^g \left(P_t^1\right)^{1-\varrho^g} + (1-\chi^g)\left(P_t^2\right)^{1-\varrho^g}\right]^{\frac{1}{1-\varrho^g}}. \tag{23}$$

Based on the above, the demand for the products of sector 1 and sector 2 is defined by the following equations:

$$y_t^1 = y_t^{1c} + y_t^{1I} + y_t^{1g}, \tag{24}$$

$$y_t^2 = y_t^{2c} + y_t^{2I} + y_t^{2g}. \tag{25}$$

2.2 AGGREGATE SUPPLY

As discussed in the section describing *aggregate demand*, the model distinguishes three sectors of production. The separation is based on the need of inflation forecasting. The sectors producing core inflation products, non-core inflation products and export products are distinguished. Domestic demand is directed to core inflation products and non-core inflation products. The household and government consumption goods, as well as investment goods are the aggregation of these sectors' products. Evidently, the export sector produces the products exported to the rest of the world.

The core inflation and the export sector uses three types of inputs for production: domestic composite goods produced from capital and labor, imported goods and imported energy (oil). The difference between the two sectors is that the export sector has higher import utilization.

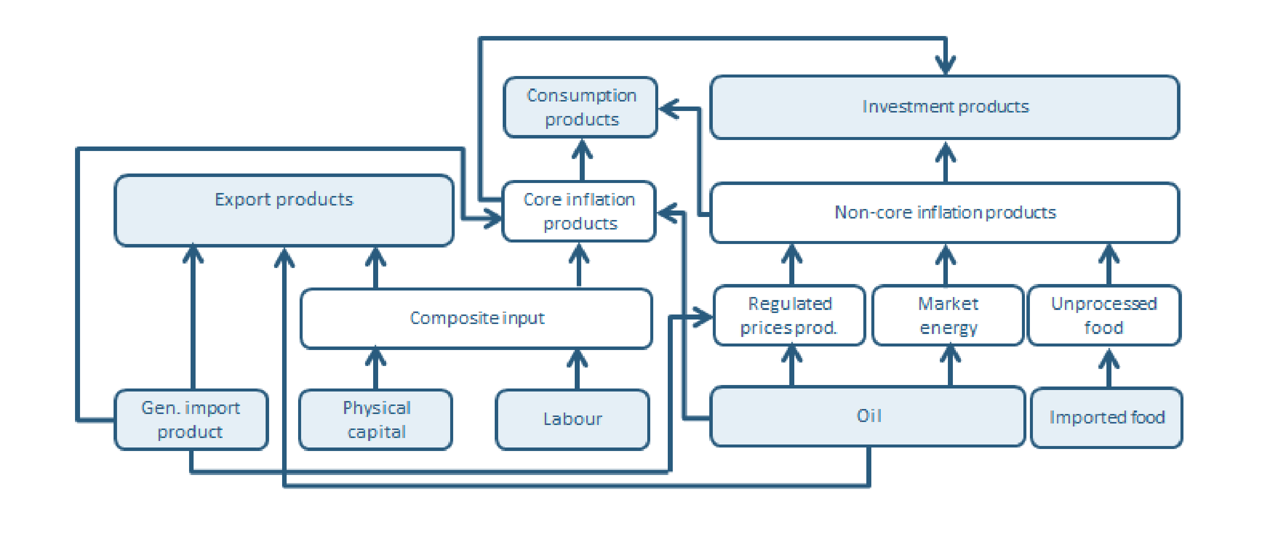
The sector of non-core inflation products can be divided into three subsectors: the sectors of products with regulated prices, sector of market energy, as well as sector of non-processed food. For simplicity, it is assumed that these sectors use only one input, energy or imported food.

2.2.1 CORE INFLATION SECTOR

The sector producing core inflation products (henceforth: sector 1) uses two inputs for producing its final product, domestic composite goods (ci_t what is defined exactly in *section 2.2.3*) and imported goods (m_t). From the two inputs sector 1 produces its final product with CES technology:

$$y_t^1 + \mathrm{F}^1 = \left[\left(\mathrm{a}^1\right)^{\frac{1}{\varrho}} \left(m_t^1\right)^{\frac{\varrho-1}{\varrho}} + \left(1-\mathrm{a}^1\right)^{\frac{1}{\varrho}} \left(ci_t^1\right)^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}},$$

Figure 8
The structure of aggregate supply



where y_t^1 is the output of sector 1, m_t^1 and ci_t^1 are the sector's input utilization from imported and domestic composite goods. Furthermore, $0 < a^1 < 1$ and $\varrho > 0$ are the parameters of the production function, and the latter measures the elasticity of substitution. F^1 is the fixed costs of production. Assuming profit maximization the demand for m_t^1 and ci_t^1 is defined by the following equations,

$$m_t^1 = \mathrm{a}^1 \left(\frac{MC_t^1}{P_t^m}\right)^{\varrho} y_t^1, \tag{26}$$

$$ci_t^1 = \left(1-\mathrm{a}^1\right)\left(\frac{MC_t^1}{P_t^{ci}}\right)^{\varrho} y_t^1, \tag{27}$$

$$MC_t^1 = \left[\mathrm{a}^1 \left(P_t^m\right)^{1-\varrho} + \left(1-\mathrm{a}^1\right)\left(P_t^{ci}\right)^{1-\varrho}\right]^{\frac{1}{1-\varrho}},$$

where MC_t^1 is the marginal cost of the CES technology, P_t^{ci} is the price of the domestic composite goods, and P_t^m is the price of imported goods in domestic currency.

The pricing of sector 1 follows the standard Calvo framework supplemented with indexation, see Smets and Wouters (2003, 2007). Therefore, core inflation (π_t^1) can be expressed with the following log-linearized equation,

$$\left(1+\beta\gamma^{p1}\right)\pi_t^1 = \beta \mathrm{E}_t\left[\pi_{t+1}^1\right] + \gamma^{p1}\pi_{t-1}^1 + \bar{\xi}^{p1}\left(\widetilde{MC}_t^1 - \widetilde{P}_t^1\right) + \left(1+\beta\gamma^{p1}\right)\varepsilon_t^{p1}, \tag{28}$$

where

$$\bar{\xi}^{p1} = \frac{\left(1-\xi^{p1}\right)\left(1-\beta\xi^{p1}\right)}{\xi^{p1}},$$

and $0 < \xi^{p1} < 1$ is the Calvo parameter, $0 < \gamma^{p1} \leq 1$ is the indexation parameter, the tilde shows the percentage deviation from the steady state for the specific variables and ε_t^{p1} shows the magnitude of the exogenous effects which are not incorporated into the model.

2.2.2 EXPORT SECTOR

The sector producing the export products also uses two inputs, ci_t and m_t. From the two inputs it produces the final product with CES technology:

$$x_t + \mathrm{F}^x = \left[\left(\mathrm{a}^x\right)^{\frac{1}{\varrho}}\left(m_t^x\right)^{\frac{\varrho-1}{\varrho}} + \left(1-\mathrm{a}^x\right)^{\frac{1}{\varrho}}\left(ci_t^x\right)^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}},$$

where m_t^x and ci_t^x are the sector's input utilization from import and domestic composite goods, respectively. Furthermore, $0 < a^x < 1$ and $\varrho > 0$. F^x is the fixed costs of production. Assuming profit maximization, the demand for m_t^x and ci_t^x is defined by the following equations,

$$m_t^x = \mathrm{a}^x \left(\frac{MC_t^x}{P_t^m}\right)^{\varrho} x_t, \tag{29}$$

$$ci_t^x = (1-\mathrm{a}^x)\left(\frac{MC_t^x}{P_t^{ci}}\right)^{\varrho} x_t, \tag{30}$$

$$MC_t^x = \left[\mathrm{a}^x \left(P_t^m\right)^{1-\varrho} + (1-\mathrm{a}^x)\left(P_t^{ci}\right)^{1-\varrho}\right]^{\frac{1}{1-\varrho}},$$

where MC_t^x is the marginal cost of the CES technology.

The price setting of the export sector follows the standard Calvo framework supplemented with indexation as well. π_t^x can be expressed with the following log-linearized equation,

$$(1+\beta\gamma^x)\,\pi_t^x = \beta \mathrm{E}_t\left[\pi_{t+1}^x\right] + \gamma^x \pi_{t-1}^x + \bar{\xi}^x\left(\widetilde{MC}_t^x - \widetilde{P}_t^x\right), \tag{31}$$

where

$$\bar{\xi}^x = \frac{\left(1-\xi^x\right)\left(1-\beta\xi^x\right)}{\xi^x},$$

$0 < \xi^x < 1$ is the Calvo parameter, and $0 < \gamma^x \leq 1$ is the indexation parameter.

2.2.3 PRODUCTION OF COMPOSITE INPUTS

The composite aggregate of imported goods used by sector 1 and the export sector are produced by CES technology in a competitive market from general imported goods (u_t) and imported energy (o_t):

$$m_t = \left[\left(\mathrm{b}^1\right)^{\frac{1}{\rho}} u_t^{\frac{\rho-1}{\rho}} + \left(1-\mathrm{b}^1\right)^{\frac{1}{\rho}} o_t^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$

where $m_t = m_t^1 + m_t^x$ is the total output of the sector, $0 < \mathrm{b}^1 < 1$ and $\rho > 0$.

From profit maximization, the equations defining the demand for inputs and P_t^m can be derived:

$$u_t = \mathrm{b}^1\left(\frac{P_t^m}{e_t P_t^{u*}}\right)^{\rho} m_t,$$

$$o_t = \left(1-\mathrm{b}^1\right)\left(\frac{P_t^m}{e_t P_t^{o*}}\right)^{\rho} m_t,$$

$$P_t^m = \left[\mathrm{b}^1\left(e_t P_t^{u*}\right)^{1-\rho} + \left(1-\mathrm{b}^1\right)\left(e_t P_t^{o*}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{32}$$

where e_t is the nominal exchange rate, P_t^{u*} and P_t^{o*} are the inputs' prices in foreign currency.

The domestic composite input is produced in a competitive market with Cobb-Douglas technology from the combination of labour and capital:

$$ci_t = k_{t-1}^{\alpha} l_t^{1-\alpha},$$

where $ci_t = ci_t^1 + ci_t^x$ is the sector's total output, k_{t-1} and l_t are capital and labour.

The price of the composite good ci_t is determined by the marginal cost of the companies. It is assumed that the amount of capital used in period t must be determined in period $t-1$ by the companies. Therefore, companies can react to unexpected changes of demand in period t only by adjusting their labour input. Due to this, the technology becomes decreasing return to scale in the short-run, that is, the marginal cost depends on the sector's output:

$$P_t^{ci} = MC_t^{ci} = \frac{W_t}{1-\alpha} ci_t^{\frac{\alpha}{1-\alpha}} k_{t-1}^{\frac{-\alpha}{1-\alpha}}, \tag{33}$$

where W_t is the nominal wage. The demand for labour is determined by the following equation,

$$l_t = ci_t^{\frac{1}{1-\alpha}} k_{t-1}^{\frac{-\alpha}{1-\alpha}}. \tag{34}$$

2.2.4 NON-CORE INFLATION SECTOR

The sector of non-core products is the fix-weighted (Leontyev) aggregate of three subsectors, thus, the relative prices of the subsectors do not influence their demand. The three subsectors are the market energy, the non-processed food and the products with regulated prices.

The only input of the energy sector is imported energy (oil) which has an exogenous price. The pricing of the sector is flexible, taxes are the only factor which makes the pass-through of the exchange rate and the energy price imperfect. The only input of the food sector is imported food. The pricing of the sector is flexible similar to the energy sector. Equations (72) and (73) represents the pricing of the two sectors in *section A.1*.

The sector of products with regulated prices can be divided into two parts, regulated energy and regulated non-energy. Their input is the imported energy and the general imported goods. Their pricing is exogenous from the model's point of view, see equations (74) and (75).

2.2.5 LABOR SUPPLY

The labor supply equations of the model are not derived explicitly from the optimisation problem because labor supply does not appear in the households' utility function. The labor supply block of the model consists of two equations. Equation (78) is a Phillips-curve type wage equation. Equation (80) describes the relationship of working hours and employment. Both of the equations are inspired by the model of Smets and Wouters (2003).

2.3 FINANCIAL ACCELERATOR

As discussed in the introduction, before the crisis of 2007-2008 financial frictions were missing from most macroeconomic models but the crisis made it evident that without them economic fluctuations cannot be understood completely. Concretely, Christiano et al. (2014), Christiano et al. (2015) and Lindé et al. (2016) show that adding financial shocks to DSGE models improves their forecasting performance and they can explain normal business cycles and the last crisis better. Due to this, the financial accelerator mechanism of Bernanke et al. (1999) is incorporated into the model.

To illustrate the effects of the financial accelerator, a new type of economic agent is introduced, the entrepreneurs. Entrepreneurs are independent from households. It is assumed that the physical capital produced by the investment sector cannot be used directly to produce composite input. To create a factor of production from physical capital, the contribution of the entrepreneurs' sector is necessary. It is assumed that infinitely many entrepreneurs exists. A given entrepreneur is indicated with index l, where $l \in [0, 1]$.

The activity of entrepreneurs can be described in the following way. At the end of period t, entrepreneur l buys $k_t(l)$ amount of physical capital from the investment sector and transforms it to capital which is suitable for production. The available technology is stochastic, the amount of capital produced is influenced by an idiosyncratic shock. Therefore, the amount of capital produced is $\omega(l)k_t(l)$ where $\omega(l)$ is the idiosyncratic shock.[9] After the transformation, the entrepreneur lends the capital in period $t+1$ to produce ci_{t+1}, and then he sells the used capital to the investment sector.

It follows that the returns of the entrepreneurs activity is

$$R^k_{t+1}(l) = \omega(l)\frac{z_{t+1} + Q_{t+1}(1-\tau)}{Q_t},$$

where z_{t+1} is the rental rate of capital (see equation 9), Q_{t+1}, Q_t are the real price of capital, and τ is the depreciation rate of capital.

As the entrepreneurs are independent from households they can use their own net worth and loans to buy capital. If $a^e_t(l)$ is the net wealth of a given entrepreneur in period t then the loan needed for buying capital is

$$B^e_t(l) = Q_t k_t(l) - a^e_t(l).$$

[9] $\log \omega(l)$'s expected value is zero, its variance is σ^2.

At the same time, the relationship of a given entrepreneur and its creditor is complicated by asymmetric information. While entrepreneur l is evidently able to observe the value of $\omega(l)$, the creditor can do this with costs only (costly state verification). As Bernanke et al. (1999) shows, with these assumptions the optimal credit contracts are formed as follows. The entrepreneur and the creditor agrees in the $\overline{R}$ interest rate. At the end of period $t+1$, the entrepreneur realizes his yield, $\omega(l)R^k_{t+1}$. If it covers the credit costs then he pays back $\overline{R}B^e_t$ and keeps the rest what increases his net wealth. If $\omega(l)$ is smaller than a given $\bar{\omega}$ then the yield does not cover the loan repayments. In this case, the entrepreneur declares bankruptcy. In the case of bankruptcy, the creditor pays a given value of monitoring cost for being able to monitor the entrepreneurs yield and takes the full yield, $\omega(l)R^k_{t+1}$. It is assumed that the full monitoring cost is μ share of the total gross yield, where $0 < \mu < 1$.

The financial contract above has two implications. Firstly, the entrepreneur has to pay a premium in addition to the risk-free rate due to the asymmetric information and the monitoring cost. Secondly, at the macroeconomic level the premium can be expressed in the following way,

$$s_t = s\left(\frac{a_t}{Q_t K_t}\right), \qquad s'(\cdot) < 0, \tag{35}$$

where the premium s_t is defined in the following way,

$$s_t \equiv \mathrm{E}_t\left[\frac{R^k_{t+1}}{1+r_t}\right], \tag{36}$$

and r_t is the real interest rate derived from the short-term risk-free rate.

Formula (35) summarizes the most important features of the financial accelerator mechanism: in equilibrium the premium depends negatively on the net wealth of the entrepreneurs' sector. Intuitively, this can be explained in the following way: if an entrepreneur has higher leverage (higher loan relatively to its net wealth) then she will go bankrupt with a higher probability, which implies that the expected monitoring cost of the creditor is higher so she expects a higher premium from the entrepreneur. This mechanism amplifies business cycles. For example, in case of recession, the entrepreneurs' net wealth is lower so they pay a higher premium, which lowers their wealth further. Furthermore, in case of higher premiums investments also decrease, further deepening the recession.

It is assumed that entrepreneurs stay on the market for a finite time horizon, and they are risk-averse. The (exogenous) probability that the entrepreneur remains on the market in the next period is θ^e. This guarantees that the entrepreneurs net worth can never be as high that he is able to finance his capital purchases from his own wealth without loans. In each period, $1-\theta^e$ entrepreneurs leave the market and consumes their net worth. On the other hand, in each period $1-\theta^e$ new entrepreneurs enter the market so the number of entrepreneurs is constant. For technical reasons, it is assumed that a new entrepreneur enters the market with wealth a^e.

The aggregate wealth of entrepreneurs in period t is,

$$a^e_t = \theta^e v_t + \bar{a}^e, \tag{37}$$

where $\bar{a}^e = \left(1-\theta^e\right) a^e$, and v_t is the aggregate wealth of existing entrepreneurs,

$$v_t = R^k_t Q_{t-1} k_{t-1} - \left(1 + r_{t-1} + \mathcal{M}_t\right)\left(Q_{t-1}k_{t-1} - a^e_{t-1}\right), \tag{38}$$

where $\mathcal{M}_t$ is the expected monitoring cost. Equation (38) expresses that wealth is the difference of the yield of capital and the financing cost where $\mathcal{M}_t(Q_{t-1}k_{t-1} - a^e_{t-1})$ represents the external finance premium. By combining formulas (37) and (38), one can get the equation describing the evolution of aggregate wealth,

$$a^e_t = \theta^e\left[R^k_t Q_{t-1}k_{t-1} - \left(1 + r_{t-1} + \mathcal{M}_t\right)\left(Q_{t-1}k_{t-1} - a^e_{t-1}\right)\right] + \bar{a}^e. \tag{39}$$

2.4 MONETARY POLICY RULE, NOMINAL EXCHANGE RATE

In the model, monetary policy is represented by an interest rate rule. According to the rule, the decision makers determine the domestic short-term interest rate based on the past interest rate, inflation expectations and the output gap. Formally,

$$i_t = (1-\mathrm{r})\, i + \mathrm{r} i_{t-1} + \frac{1-\mathrm{r}}{4}\left(\mathrm{r}^{\pi}\mathrm{E}_t\left[\pi^4_{t+4}\right] + \mathrm{r}^{gdp}\widetilde{gdp}_t\right) + \varepsilon^i_t, \tag{40}$$

where $0 < \mathrm{r} < 1$, $0 < \mathrm{r}^{\pi}$ and $0 < \mathrm{r}^{gdp}$ are parameters, i_t is the domestic nominal short-term interest rate, i is the target level of the nominal interest rate (neutral interest rate), π_t^4 is the deviation of the annual inflation from its target level, $\widetilde{gdp}_t$ is the output gap, ε_t^i is the exogenous variable representing monetary policy's deviation from its systematic behavior.

The nominal exchange rate is derived from the *modified uncovered interest rate parity* (MUIP). The basic principle of the MUIP is the following equation:

$$e_t = \eta\left(\mathrm{E}_t\left[e_{t+1}\right] - di_t\right) + (1-\eta)e_{t-1},$$

where $e_t = \log(E_t)$ is the logarithm of the nominal exchange rate, $di_t = i_t - i_t^* - pr_t$ is the difference of the domestic short-term interest rate and the foreign interest rate adjusted with the premium, furthermore, $0 < \eta < 1$. For the MUIP relationship, see the paper of Adolfson et al. (2008).[10] *Section A.3* of the *Appendix* shows that in case of rational expectations the equation above implies that

$$de_t = -\sum_{i=0}^{\infty} \bar{\eta}^{i+1}\mathrm{E}_t\left[di_{t+i}\right].$$

In contrast to the expression above, in the model it is assumed that the agents in foreign exchange rate markets have bounded rationality. They can't see forward over an infinite horizon, so the expression above is modified to a finite sum,

$$de_t = -\sum_{i=0}^{T^e} \bar{\eta}^{i+1}\mathrm{E}_t\left[di_{t+i}\right], \tag{41}$$

where $0 < T^e < \infty$ is an exogenous parameter.

2.5 THE REST OF THE WORLD

As usual in models of small open economies, the behavior of the rest of the world is assumed to be exogenous, i.e., the domestic variables are assumed not to affect the foreign economy. The behavior of the foreign economy affects the domestic one in the model in the following way. Equation (14) in *section 2.1.3* shows which factors affect the export demand of the rest of the world. *Section 2.2* discusses which kind of goods are imported by domestic companies. They take the prices of these products as exogenously given. *Section 2.4* discusses how foreign investors influence the evolution of the nominal exchange rate. In addition to this, the behavior of foreigners is represented with four other equations, see (108)–(111) in *section A.1*. These equations describe the evolution of foreign CPI inflation, foreign core inflation, foreign output gap and foreign short-term interest rate.

2.6 THE SOLUTION OF THE MODEL

The easiest way to solve a DSGE model is to solve the log-linearized version of it. The advantage of it is its quickness arising from the simplicity of the method while its disadvantage is that certain economic issues are related to the non-linearity of the original model. This is the case in our model as well. As described in *section 2.1.1* the effects of the precautionary motive cannot be analyzed in a linear model. The concavity of the consumption function (5), which summarizes the behavior of households, is an important feature. If the model is linearized then this feature is lost.

The disadvantage of the algorithms which solve the problem non-linearly is that they are complicated and time-consuming. This increases the operational risks in case of a model which is used for forecasting and economic policy analysis, when results often have to produced with tight deadlines.

Therefore, a compromise was applied which is fast and reliable just like the algorithms which solve the problem linearly but does not loose the information in the consumption function. The cost of this solution is that expectations are not perfectly model consistent. Due to the fact that the authors have deviated from model consistent expectations at other points as well, this is not considered as a significant problem.

[10] If $\eta = 1$ then one would get the usual uncovered interest rate parity.

The process of the solution is the following. As a starting point, the model described in *section A.1* is solved. Formally, the model described with equations (45)–(111) can be expressed as

$$\mathbf{A}_{+1}\widetilde{\mathcal{X}_{t+1}} + \mathbf{A}_0\left(\widehat{n}^1, \widehat{n}^2\right)\widetilde{\mathcal{X}_t} + \mathbf{A}_{-1}\widetilde{\mathcal{X}_{t-1}} + \mathbf{B}\widetilde{\mathcal{Z}_t} = 0, \tag{42}$$

where $\mathbf{A}_{+1}$, $\mathbf{A}_0$, $\mathbf{A}_{-1}$ and $\mathbf{B}$ are coefficient matrices, $\widetilde{\mathcal{X}_t}$ is the vector of endogenous variables in the linear model and $\widetilde{\mathcal{Z}_t}$ is the vector of exogenous shocks. The fact that the coefficient matrix $\mathbf{A}_0\left(\widehat{n}^1, \widehat{n}^2\right)$ is a function of the variables $\widehat{n}^1$ and $\widehat{n}^2$ expresses that in the linearized equations (45)–(46) the values of the partial derivatives of function *f* depend on $\widehat{n}^1$ and $\widehat{n}^2$. Equation (42) can be solved with any linear algorithm such as the method of undetermined coefficients in Uhlig (1999). The solution provides the following equation:

$$\widetilde{\mathcal{X}_t} = \mathbf{P}\left(\widehat{n}^1, \widehat{n}^2\right)\widetilde{\mathcal{X}_{t-1}} + \mathbf{Q}\widetilde{\mathcal{Z}_t}. \tag{43}$$

The model is solved via iteration based on the linear algorithm above. The process of iteration is the following. As a first step, calculate the values of $\widehat{n}_0^1$ and $\widehat{n}_0^2$ which are consistent with the two households' initial wealth. Then solve equation (42) by substituting the $\mathbf{A}_0\left(\widehat{n}_0^1, \widehat{n}_0^2\right)$ coefficient matrix. After this, the value of $\widetilde{\mathcal{X}_1}$ can be calculated by using $\mathbf{P}\left(\widehat{n}_0^1, \widehat{n}_0^2\right)$ and $\mathbf{B}$ coefficient matrices from the solution as well as the value of $\widetilde{\mathcal{X}_0}$ with the help of equation (43). From the vector generated this way, one can select $\widetilde{n}_1^1$ and $\widetilde{n}_1^2$, and from these the value of $\widehat{n}_1^1$ and $\widehat{n}_1^2$ can be calculated.[11]

Then after substituting the $\mathbf{A}_0\left(\widehat{n}_1^1, \widehat{n}_1^2\right)$ coefficient matrix into equation (42), the model is solved again linearly. In the next step, the values of $\widehat{n}_2^1$ and $\widehat{n}_2^2$ are calculated with a similar process to the above one. Then the model is solved again with coefficient matrix $\mathbf{A}_0\left(\widehat{n}_2^1, \widehat{n}_2^2\right)$. This iteration can be repeated arbitrary times.

The algorithm above assumes agents with bounded rationality because the agents of the model calculate the future path of the economy at each period of time in such a way that they assume that household's marginal propensity to consume does not change even if their wealth changes in the future. Then in the next period, when they are faced with the unforeseen change, they recalculate their expectation but they still assume unchanged behavior in the future.

The algorithm described above gives an inprecise approximation if the household's wealth is far from the aimed buffer stock, or in other words, from its steady state value. For example, the case of indebted households are just like this. In case of indebted households (second type of households) the log-linearized consumption equation (46) from *section A.1* of the *Appendix* can be formulated in the following way (for simplicity the effect of r_t^h and d is neglected),

$$c_t^2 - c^2 = f_n\left(\widehat{n}_{t-1}^2, \cdot\right)\left(n_t^2 - n\right),$$

where the variables without time index indicate steady state values and $n_t^2 << n$. Since function *f* is concave, its slope is big in case of negative n_t^2 and significantly higher than in the neighbourhood of n. As a result of this, if the derivative of *f* is calculated in n_t^2's neighbourhood then the above formula significantly overestimates the change of consumption (while if it is calculated in n's neighbourhood then it significantly underestimates the slope).

This problem is solved by dividing the $n_t^2 - n$ distance into two parts, $n_t^2 - \bar{n}^2$ and $\bar{n}^2 - n$ where $\bar{n}^2 > n_t^2$ and $\bar{n}^2$ is close to n_t^2. The effect of the move from n_t^2 to $\bar{n}^2$ on consumption is

$$f_n\left(\widehat{n}_{t-1}^2, \cdot\right)\left(n_t^2 - \bar{n}^2\right),$$

while the effect of the other move is

$$\Delta_t\left(\bar{n}^2 - n\right)$$

where Δ_t is the approximation of

$$\frac{f(\bar{n}^2, \cdot) - f(n, \cdot)}{\bar{n}^2 - n}.$$

The problem is how to choose $\bar{n}^2$. According to our point of view, the $n_t^2 - \bar{n}^2$ move should reflect a move typical to dates close to t. Because n_t^2 moves slowly the $n_{t-2}^2 - n_{t-1}^2, n_{t-1}^2 - n_t^2, n_t^2 - n_{t+1}^2$ values are close to each other. Using this, one can represent the $n_t^2 - \bar{n}^2$ move by $(1 - \Psi_t)\left(n_t^2 - n\right)$ and $\bar{n}^2 - n$ by $\Psi_t\left(n_t^2 - n\right)$ where

$$1 - \Psi_t = \frac{\left|n_{t-2}^2 - n_{t-1}^2\right|}{\left|n_{t-1}^2 - n\right|}.$$

[11] Recall that $\widetilde{n}_t^i + 1 = \widehat{n}_t^i d/n$, $i = 1, 2$.

Using this the approximation of the change in c_t^2 provides the following formula,

$$c_t^2 - c^2 = f_n\left(\widehat{n}_{t-1}^2, \cdot\right)(1-\Psi_t)\left(n_t^2 - n\right) + \Delta_t \Psi_t \left(n_t^2 - n\right), \tag{44}$$

and

$$\Delta_t = \frac{f\left(\Psi_t\left(n_{t-1}^2 - n\right) + n, r^h, d\right) - f\left(n, r^h, d\right)}{\Psi_t\left(n_{t-1}^2 - n\right)}.$$

If equation (46) in the log-linearized model is substituted by the formula (44) above then the iteration algorithm described in the beginning of the section can be applied to this modified system, as well.[12]

2.7 THE CALIBRATION OF THE MODEL

The parameters of the model are calibrated. The value of those parameters which affect the steady state of the model are determined by the components' share within GDP. The parameters influencing the dynamic features of the model are partly calculated from the SVAR estimations of the Hungarian economy[13] and partly from the MNB experts' forecasting experience.

For the calibration of the household model, both aggregate and individual data on household wealth and debt were used. The process of calibration was aimed to be consistent with the newest estimations of experts which claim that the pass-through of the nominal exchange rate become slower since the crisis. Furthermore, the wide-spread international evidence was taken into consideration which argues that the Phillips-curve has become flatter so real variables have smaller effect on inflation[14]. During the calibration of the monetary policy rule it was taken into account that the current monetary policy of the MNB supports economic growth more than in the past.

[12] It should be noted additionally that if n_t^2 is negative then equations (49) and (51) are also inaccurate approximations so in this case the coefficient of $\widetilde{r}_t^{hlag}$ is substituted by $\left(1+r^h\right) a_t^{h2}$, and this parameter is also refreshed as a part of the discussed algorithm.

[13] See for example Vonnák (2010).

[14] See Szentmihályi and Világi (2015).

3 Impulse response analysis

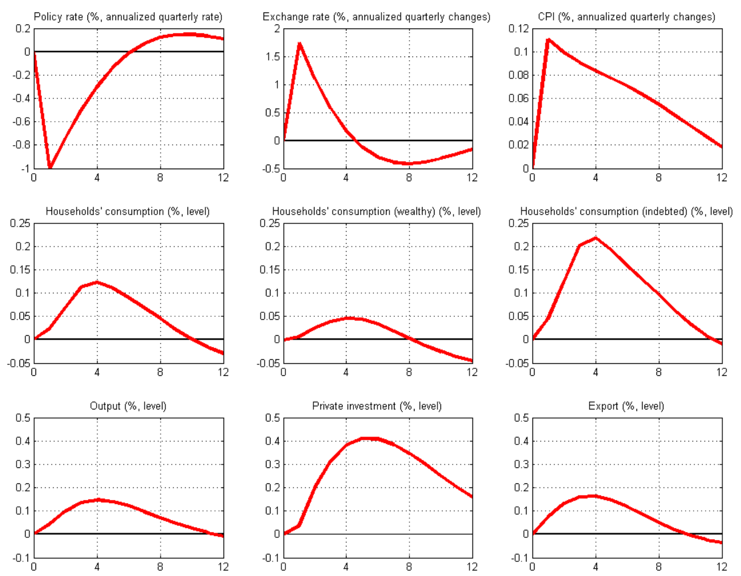
In this section the economic features of the model are analyzed with the help of its impulse responses.

In case of linear models it is well-known that the transition dynamics and the impulse responses can be calculated independently.[15] In other words, if arbitrary shocks are added to the baseline path of a linear model then the difference between the baseline path and the path with shocks will be always the same. As a result of this, in case of linear models it is worth calculating the impulse responses compared to the model's steady state.

However, this model – even if only to a low degree – is non-linear, therefore the transition dynamics affects the impulse responses. The simulations are based on the following transition paths: the initial wealth of the two household groups differs from the stable value of wealth, which is defined by the buffer stock. The initial values of indebted and wealthy households' net worth are calibrated using Hungarian household data. The initial values of the other state variables do not differ from their steady state values.

MONETARY POLICY SHOCK

Figure 9
Monetary policy shock



Due to a monetary policy shock[16] (shock ε_t^i in equation (90)), the exchange rate depreciates and domestic demand increases. As a result of these effects, inflation rises and because of improved competitiveness export also increases. Higher output leads to

[15] Transition dynamics are those paths of the endogenous variables which can be created without shocks but the initial values of some variables are not equal to their steady state values. In contrast to this, impulse responses describe how the paths of endogenous variables change due to a shock compared to the baseline without shocks.

[16] 25 basis point shock of the quarterly interest rate which corresponds to a 100 basis point shock in the annualized rate.

an increase of investments as well. The consumption decision of the two types of households is determined by the substitution and the wealth effect of the interest rate decrease. In case of the indebted households both of the effects lead to an incentive to increase current consumption. While for households with positive net wealth the two channels has opposite effects, and initially they increase their consumption but from the second year the wealth effect dominates and consumption decreases.

Figure 10
The effect of financial accelerator mechanism in case of a monetary policy shock

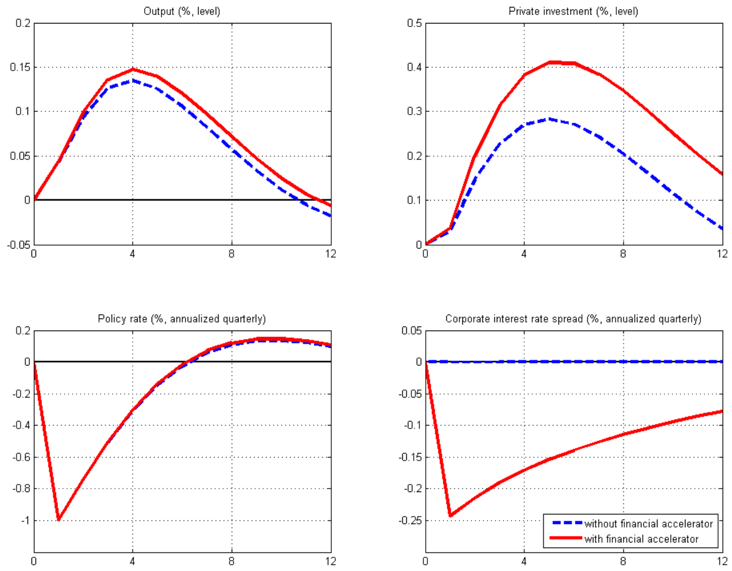


Figure 11
The effect of wealth heterogeneity in case of a monetary policy shock

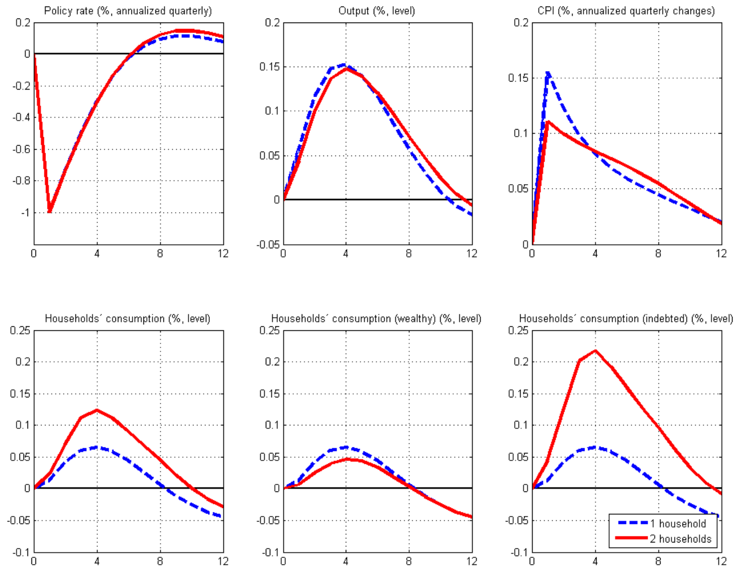


Figure 10 illustrates the financial accelerator mechanism. In the figure, the blue dashed line represents the effect of monetary policy shock without the financial accelerator mechanism ($\varkappa = 0$ in equation (88)). The red solid line shows the case with the financial accelerator mechanism. As the figure reveals the financial accelerator mechanism strengthens the effect of the

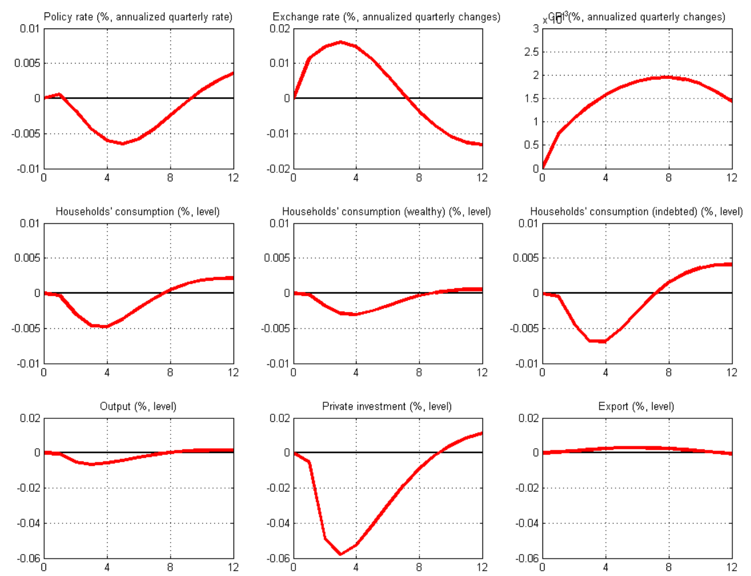
monetary policy shock because the external financing premium decreases. As a result of this, the reaction of investments is higher, which leads to larger increase in output.

The effect of the introduction of the two household types can be illustrated with the help of the monetary policy shock. *Figure 11* shows how consumption and real GDP reacts to a monetary policy shock if the indebted households are substituted with households who has positive net worth, that is, wealth heterogeneity is eliminated. The figure reveals if heterogeneity is taken into consideration then the maximum reaction of consumption will be twice higher than in the case of homogeneous consumers. Furthermore, the change in the reaction of inflation is also significant, near 5 basis points.

EXTERNAL FINANCE PREMIUM SHOCK

If creditors perceive that entrepreneurs' projects will be riskier, i.e. the variance of shock $\omega(l)$ in *section 2.3* increases, then entrepreneurs' external finance premium increases. This process is represented by the increase of ε_t^s in equation (88). If corporate credit spreads increase by 100 basis points[17] due to the reasons above then investment costs increase, which decreases the investment demand of firms. The decrease of production slightly lowers households' consumption through the decrease of labor demand. Monetary policy reacts to this by lowering the interest rate to increase demand. This leads to an exchange rate depreciation. Prices increase slightly because the disinflationary effect of the decreasing demand cannot offset the price increase due to the exchange rate pass-through because the coefficient of the real marginal cost in equation (69) is low, i.e. the Phillips curve is flat.

Figure 12
External finance premium shock



FOREIGN DEMAND SHOCK

Due to a 1% decrease of foreign demand (shock $\varepsilon_t^{y^*}$ in equation (110)) the demand of export products declines, which lowers the output of the domestic economy. The decreasing investment demand of the export sector further lowers economic activity. Labor demand of the export sector also declines, which decreases real wages and the wage bill. Households perceive the decrease of disposable income and lower their consumption. Indebted households, who have higher marginal propensity to consume, lower their consumption more. Monetary policy, which aims to support economic growth, tries to compensate

[17] 25 basis point increase of the quarterly premium what is 100 basis points in the annualized premium.

decreasing demand (and its disinflationary effect) so starts to ease. As a result of this, the financial market reacts with an exchange rate depreciation. Due to the flat Phillips curve the disinflationary effect of decreasing demand is small, hence the disinflationary effect of decreasing demand and imported inflation is compensated by the immediate inflationary effect of the exchange rate decrease.

Figure 13
Foreign demand shock

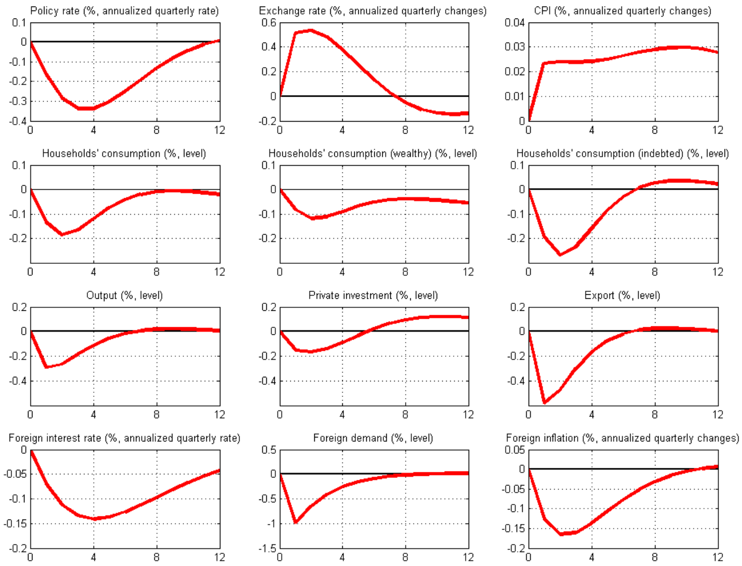
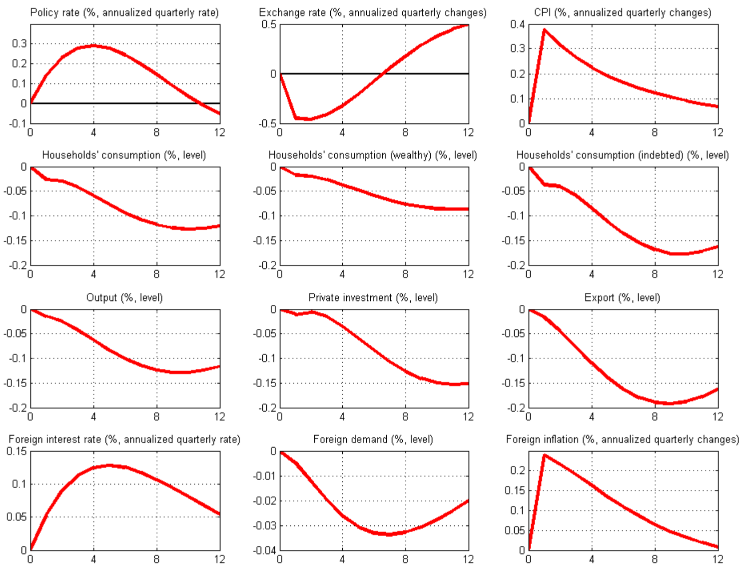


Figure 14
Oil price shock



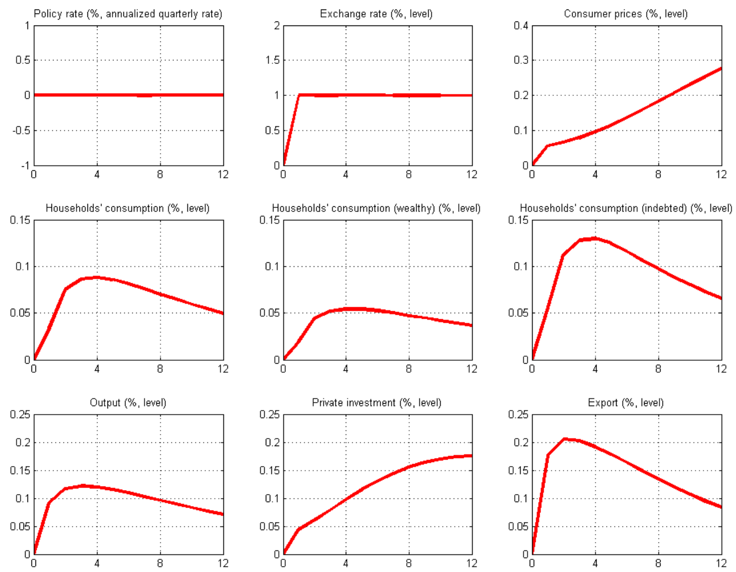
OIL PRICE SHOCK

A 10% increase of the oil price (shock π_t^{o*}), which decays gradually, increases production costs and this raises imported inflation. With regard to price setting, it is important that the oil price increases domestic energy prices immediately. This appears in the pricing of core inflation products as well, through second-round effects with delays. Beside this, firm's profit decreases because of increasing costs. They adjust to this by increasing their prices, decreasing their labour demand and decreasing their production. The reaction of monetary policy is higher to inflation than to the change in the output gap. As a result of this, it reacts to the oil price shock with an interest rate hike. This decreases domestic inflation through the exchange rate channel. Due to the increase of consumer prices the real income of households decreases. Households, who have adaptive expectations, perceive this only gradually, so the reaction of consumption is slower.

EXCHANGE RATE SHOCK

The effect of a permanent 1% depreciation of the exchange rate (shock ε_t^{de} in equation (91)) is analyzed by keeping the nominal interest rate fixed. The degree of price rigidity is calibrated with the slow exchange rate pass-through observed recently. As a result of this, the price level increases with less than 0.2% over two years time. The real economy picks up due to the depreciated exchange rate. The export products become more competitive and the increasing profit increases the export sector's production. This leads to the increase of investment and labour demand of exporting companies. Households perceive the increase of their income and increase their consumption permanently. Indebted consumers react more than the consumers with positive net wealth due to their higher propensity to consume.

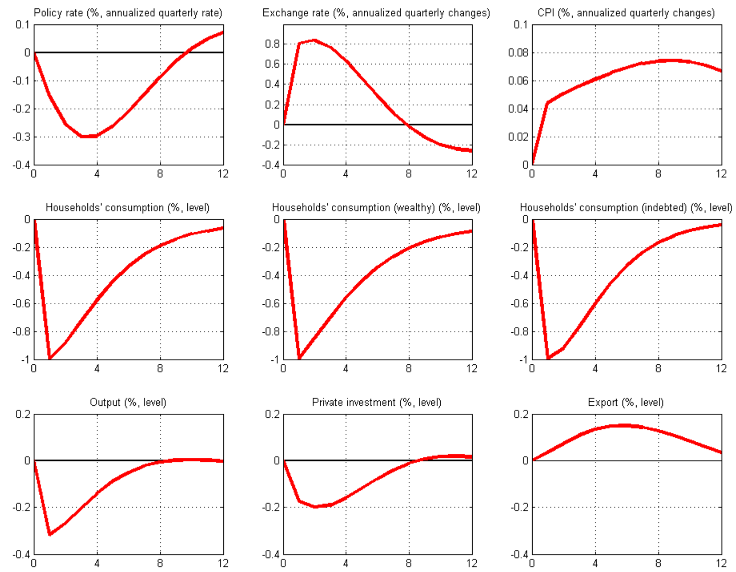
Figure 15
Exchange rate shock (with fixed interest rate)



CONSUMPTION SHOCK

If households' consumption decreases by 1% (equally for both types, shocks ε_t^{h1} and ε_t^{h2} in equations (45) and (46)) then this leads to the decrease of demand, production and firms' investments. As a result of the decrease in demand, monetary policy starts to ease, which leads to the depreciation of the exchange rate. Due to the flat Phillips-curve the immediate inflationary effect of the exchange rate depreciation is higher than the disinflationary effect of the demand decrease.

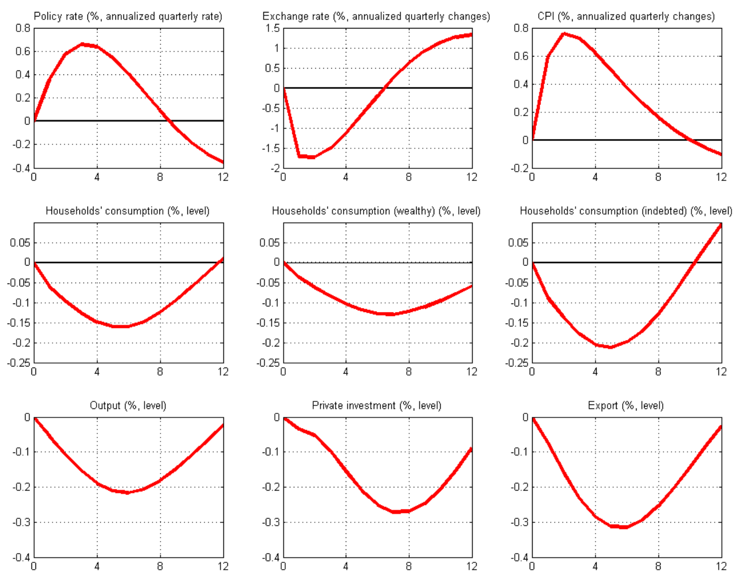
Figure 16
Consumption shock



CORE INFLATION SHOCK

Suppose that the mark-up above the marginal cost increases (shock ε_t^{p1} in equation (69)), and as a result inflation also rises. As a result monetary policy tightens, which pushes inflation back towards its equilibrium value. Due to increasing interest rates and the appreciation of the exchange rate domestic demand and export decrease, resulting in a negative output gap.

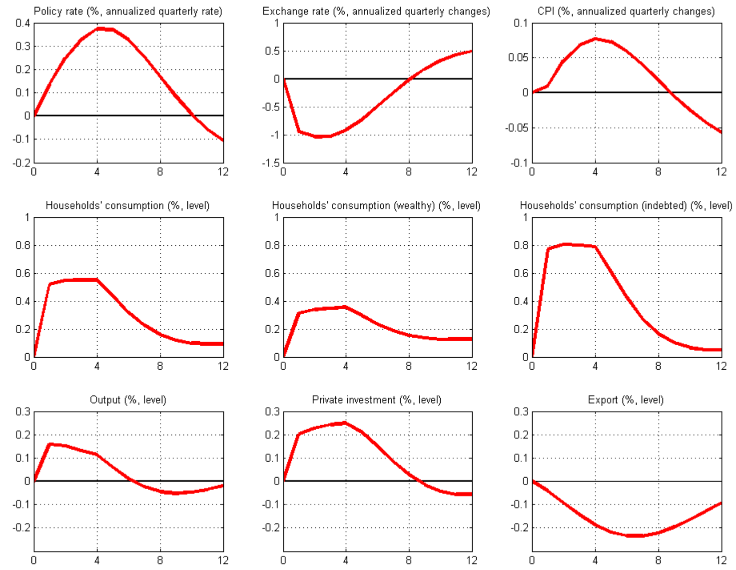
Figure 17
Core inflation shock



REAL WAGE SHOCK

If real wages are higher by 1% for a year (shock ε_t^w in equation (78)), then the marginal cost of firms increase, which raises consumer prices through increasing core inflation. Increasing real wages raise households' income and leads to higher consumption which enhances demand. This strengthens the former process. The reaction of indebted households is higher due to their higher marginal propensity to consume. However, monetary policy tightens due to the increasing demand and inflation which leads to the appreciation of the currency. This decreases inflation directly and in the medium term indirectly through second round effects.

Figure 18
Real wage shock



4 Conclusion

The lessons of the financial and macroeconomic crisis of 2007-2008 made necessary the development of a new macroeconomic forecasting model in the MNB. The model represents a small open economy. It is based on DSGE philosophy but deviates from that at several points.

The main innovations of the model compared to the MNB's previous forecasting models is that it takes into consideration the effects of households' indebtedness and heterogeneity on consumption, it incorporates the financial accelerator mechanism and represents expectations more realistically.

The calibration of model parameters is based on experts' experience and SVAR estimations. The features of the calibrated model are analyzed with the help of impulse response analysis. The results are in line with MNB's experts' views based on empirical analysis. The model fits successfully into the MNB's forecasting system.

Further research will be directed towards the more detailed modeling of fiscal policy and its effects and the incorporation of the banking system such as in Bokan et al. (2016).

References

[1] Adolfson, M., S. Laséen, J. Lindé and M. Villani, 2008. Evaluating an Estimated New Keynesian Small Open Economy Model, *Journal of Economic Dynamics and Control*, 32(8), 2690-2721.

[2] Beneš, J., T. Hlédik, M. Kumhof and D. Vávra, 2005. An Economy in Transition and DSGE: What the Czech National Bank's New Projection Model Needs, Czech National Bank Working Paper 12.

[3] Benk, Sz., M.Z. Jakab, M.A. Kovács, B. Párkányi, Z. Reppa and G. Vadas G., 2006. The Hungarian Quarterly Projection Model (NEM), MNB Occasional Paper 60.

[4] Bernanke, B., M. Gertler and S. Gilchrist, 1999. The Financial Accelerator in a Quantitative Business Cycle Framework, in J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics* Vol. 1, Amsterdam: North-Holland.

[5] Boissay, F., F. Collard and F. Smets, 2016. Booms and Banking Crises, *Journal of Political Economy*, 124(2), 489-538.

[6] Bokan, N., A. Gerali, S. Gomes, P. Jacquinot and M. Pisani, 2016. EAGLE-FLI – A macroeconomic model of banking and financial interdependence in the euro area, ECB Working Paper 1923.

[7] Borio, C., 2012. The Financial Cycle and Macroeconomics: What Have We Learnt?, BIS Working Paper 395.

[8] Brunnermeier, M.K., 2009. Deciphering the Liquidity and Credit Crunch 2007-2008, *Journal of Economic Perspectives*, 23(1), 77-100.

[9] Carroll, C., 2001. A Theory of the Consumption Function, with and without Liquidity Constraints, *Journal of Economic Perspectives*, 15(3) (Summer), 23-45.

[10] Carroll, C., 2009. Precautionary Saving and the Marginal Propensity to Consume Out of Permanent Income, *Journal of Monetary Economics*, 56(6), 780-790

[11] Carroll, C., 2012. Theoretical Foundations of Buffer Stock Saving, manuscript, John Hopkins University.

[12] Christiano, L.J., R. Motto and M. Rostagno, 2014. Risk Shocks, *American Economic Review*, 104(1), 27-65.

[13] Christiano, L.J., M.S. Eichenbaum and M. Trabandt, 2015. Understanding the Great Recession, *American Economic Journal: Macroeconomics*, 7(1), 110–167.

[14] Christoffel, K., G. Coenen and A. Warne, 2008. The New Area-Wide Model of the Euro Area – A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis, ECB Working Paper 944.

[15] De Grauwe, P., 2012. *Lectures on Behavioral Macroeconomics*, Princeton University Press.

[16] Drehmann, M., C. Borio and K. Tsatsaronis, 2012. Characterising the Financial Cycle: Don't Lose Sight of the Medium Term!, BIS Working Paper 380.

[17] Eggertsson, G.B. and P.R. Krugman, 2012. Debt, Deleveraging and the Liquidity Trap: a Fisher-Minsky-Koo Approach, *Quarterly Journal of Economics*, 127(3), 1469-1513.

[18] Eggertsson, G.B. and N.R. Mehrotra, 2014. A Model of Secular Stagnation, NBER Working Paper 20574.

[19] Geanakoplos, J., 2009. The leverage cycle, in D. Acemoglu, K. Rogoff, M. Woodford (eds.), *NBER Macroeconomics Annual*, Daron Acemoglu, Kenneth Rogoff and Michael Woodford, University of Chicago Press.

[20] Gorton, G.B., 2010. *Slapped by the Invisible Hand: The Panic of 2007*, Oxford University Press.

[21] Gorton, G.B., 2012. *Misunderstanding Financial Crises – Why We Don't See Them Coming*, Oxford University Press.

[22] Gorton, G.B. and G. Ordoñez, 2014. Collateral crises, *American Economic Review*, 104(2), 343–378.

[23] Gorton, G.B. and G. Ordoñez, 2016. Good booms, bad booms, NBER Working Paper 22008.

[24] Horváth, Á., Á. Horváth, B. Krusper, T. Várnai and V. Várpalotai, 2009. A Delphi modell (The Delphi model), MNB manuscript.

[25] Iacoviello, M., 2005. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle, *American Economic Review* 95(3), 739-764.

[26] Jakab, M.Z. and B. Világi, 2008. An estimated DSGE model of the Hungarian economy, MNB Working Paper 2008/9.

[27] Kiyotaki, N. and J. Moore, 1997. Credit cycles, *Jornal of Political Economy*, 105(2), 211-248.

[28] Lindé, J., F. Smets and R. Wouters, 2016. Challenges for Central Banks' Macro Models, in J.B. Taylor and H. Uhlig (eds.) *Handbook of Macroeconomics*, forthcoming.

[29] Smets, F. and R. Wouters, 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association*, 1(5), 1123-1175.

[30] Smets, F. and R. Wouters, 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review*, 97(3), 586-606.

[31] Szentmihályi, Sz. and B. Világi, 2015. The Phillips curve –- history of thought and empirical correlations, *Financial and Economic Review*, 14(4), december.

[32] Szilágyi, K., D. Baksa, J. Beneš, Á. Horváth, Cs. Köber and G.D. Soós, 2013. The Hungarian Monetary Policy Model, MNB Working Paper 2013/1.

[33] Uhlig, H., 1999. A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily, in R. Marimon and A. Scott (eds.) *Computational Methods of Dynamic Economies*, Oxford University Press.

[34] Vonnák, B., 2010. Risk premium shocks, monetary policy and exchange rate pass-through in the Czech Republic, Hungary and Poland, MNB Working Paper 2010/1.

Appendix A

A.1 LOG-LINEARIZED MODEL

This section discusses the log-linearized version of the model. As described in *section 2.6*, non-linearities are considered when solving the model. However, the following linearized version of the model is used in the solution process. In the linearized equations the variables with tilde indicate the percentage deviation of the variable from its steady state. The variables without time index denote the steady state.[18]

AGGREGATE DEMAND

Households' consumption demand can be derived from the consumption function (5) in *section 2.1.1*,

$$\widetilde{c}_t^1 = \frac{n^1}{c^1} f_n \widetilde{n}_t^1 + \frac{d^1}{c^1}(1+r^h) f_{r^h} \widetilde{r}_t^h + \frac{d^1}{c^1} f_d \widetilde{d}_t^1 + \varepsilon_t^{h1}, \tag{45}$$

$$\widetilde{c}_t^2 = \frac{n^2}{c^2} f_n \widetilde{n}_t^2 + \frac{d^2}{c^2}(1+r^h) f_{r^h} \widetilde{r}_t^h + \frac{d^2}{c^2} f_d \widetilde{d}_t^2 + \varepsilon_t^{h2}. \tag{46}$$

where f_n, f_{r^h}, and f_d are the partial derivatives of the consumption function with respect to $\widehat{n}$, r^h and $\widehat{d}$, respectively.[19] Aggregate consumption is the weighted sum of the two groups' consumptions:

$$\widetilde{c}_t = \frac{c^1}{c}\widetilde{c}_t^1 + \frac{c^2}{c}\widetilde{c}_t^2. \tag{47}$$

The key variable in consumption equation (5) is n_t which is the sum of households' current income, their past real net worth and the interest rate income earned on that. The evolution of households' net real wealth is described by the following equations:

$$a^1\widetilde{a}_t^1 = (1+r^h)a^1\left(\widetilde{a}_{t-1}^1 + \widetilde{r}_t^{hlag}\right) - c^1\widetilde{c}_t^1 + D^1\widetilde{D}_t^1, \tag{48}$$

$$a^2\widetilde{a}_t^2 = (1+r^h)a^2\left(\widetilde{a}_{t-1}^2 + \widetilde{r}_t^{hlag}\right) - c^2\widetilde{c}_t^2 + D^2\widetilde{D}_t^2, \tag{49}$$

where a_t^i, $(i = 1, 2)$ is the net real wealth, D_t^i is current disposable income, r_t^{hlag} is the realized ex-post real interest rate. Define variables N_t^i :

$$N^1\widetilde{N}_t^1 = (1+r^h)a^1\left(\widetilde{a}_{t-1}^1 + \widetilde{r}_t^{hlag}\right) + D^1\widetilde{D}_t^1, \tag{50}$$

$$N^2\widetilde{N}_t^2 = (1+r^h)a^2\left(\widetilde{a}_{t-1}^2 + \widetilde{r}_t^{hlag}\right) + D^2\widetilde{D}_t^2. \tag{51}$$

As described in *section 2.1.1* households have bounded rationality in the sense that they can identify the exact value of their real income and wealth only with a lag. Let n_t^i denote the perceived value of N_t^i. This variable is assumed to be determined by the following adaptive learning mechanism:

$$\widetilde{n}_t^1 = g^n\widetilde{N}_t^1 + (1-g^n)\widetilde{n}_{t-1}^1, \tag{52}$$

$$\widetilde{n}_t^2 = g^n\widetilde{N}_t^2 + (1-g^n)\widetilde{n}_{t-1}^2. \tag{53}$$

where $0 < g^n < 1$. The return on capital is defined by equation (8) which has the following log-linearized version,

$$\widetilde{r}_t^k = \frac{z\widetilde{z}_t + (1-\tau)\widetilde{Q}_t}{z + (1-\tau)} - \widetilde{Q}_{t-1}, \tag{54}$$

[18] If X_t is a general variable then X indicates its steady state, $\widetilde{X} = (X_t - X)/X$. If j_t stands for the interest rate then $\widetilde{j}_t = j_t - j$.

[19] Recall that $\widehat{X}_t = X_t/d$, i.e. the ratio of the variable of interest and d's steady state. As a result $\widetilde{X}_t = \widehat{X}_t d/X - 1$.

where $\widetilde{r}^k_t = (R^k_t - R^k)/R^k$. One can get the following formula by log-linearizing equation (11),

$$\widetilde{Q}^*_t = \sum_{i=1}^{T^Q} \frac{z(1-\tau)^{i-1}}{(r^k)^i} \left(\bar{\mathrm{E}}_t\left[\widetilde{z}^*_{t+i}\right] - \bar{\mathrm{E}}_t\left[r^k_{t+1}\right] - \cdots - \bar{\mathrm{E}}_t\left[r^k_{t+i}\right]\right). \tag{55}$$

The real rental rate of capital is defined by equation (9) which has the following log-linearized version,

$$\widetilde{z}_t = (1-\alpha)\left(\widetilde{l}_t - \widetilde{k}_{t-1}\right). \tag{56}$$

Log-linearizing and combining equation (9) and equation (12) leads to:

$$\widetilde{z}^*_t = \widetilde{mc}^{ci}_t + a^z (1-\alpha)\left(\widetilde{l}_t - \widetilde{k}_{t-1}\right) + (1-a^z)\,\widetilde{ci}_t, \tag{57}$$

where the equality of price and the marginal cost of the composite good is used. Log-linearizing the capital accumulation equation (6) yields

$$\widetilde{k}_t = (1-\tau)\widetilde{k}_{t-1} + \tau\widetilde{I}_t. \tag{58}$$

The log-linearization of equation (13) gives the following formula,

$$\widetilde{I}_t = \omega^I \widetilde{I}_{t-1} + \omega^Q \widetilde{Q}_t + \omega^{Q^*} \widetilde{Q}^*_t + \omega^{gdp} \widetilde{gdp}_t. \tag{59}$$

Export is determined by the log-linearization of equation (14):

$$\widetilde{x}_t = \theta^1 \widetilde{x}_{t-1} + \theta^2 \widetilde{y}^*_t + \theta^3 \widetilde{q}^x_t, \tag{60}$$

Log-linearizing equations (15)–(17) one can get the demand for $\widetilde{y}^{1c}_t$ and $\widetilde{y}^{2c}_t$:

$$\widetilde{y}^{1c}_t = \varrho^c (1-\chi^c)\widetilde{P}^{21}_t + \widetilde{c}_t, \tag{61}$$
$$\widetilde{y}^{2c}_t = -\varrho^c \chi^c \widetilde{P}^{21}_t + \widetilde{c}_t, \tag{62}$$

where $\widetilde{P}^{21}_t = \widetilde{P}^2_t - \widetilde{P}^1_t$. The log-linearization of equations (21)–(23) gives the following demand equations:

$$\widetilde{y}^{1g}_t = \varrho^g (1-\chi^g)\widetilde{P}^{21}_t + \widetilde{g}_t, \tag{63}$$
$$\widetilde{y}^{2g}_t = -\varrho^g \chi^g \widetilde{P}^{21}_t + \widetilde{g}_t. \tag{64}$$

Log-linearizing equations (18)–(20) yields the demand for $\widetilde{y}^{1I}_t$ and $\widetilde{y}^{2I}_t$,

$$\widetilde{y}^{1I}_t = \varrho^I (1-\chi^I)\widetilde{P}^{21}_t + \widetilde{I}_t, \tag{65}$$
$$\widetilde{y}^{2I}_t = -\varrho^I \chi^I \widetilde{P}^{21}_t + \widetilde{I}_t, \tag{66}$$

Log-linearizing equations (24) and (25) yields the domestic demand for the products of sector 1 and 2:

$$y^1 \widetilde{y}^1_t = y^{1c}\widetilde{y}^{1c}_t + y^{1I}\widetilde{y}^{1I}_t + y^{1g}\widetilde{y}^{1g}_t, \tag{67}$$
$$y^2 \widetilde{y}^2_t = y^{2c}\widetilde{y}^{2c}_t + y^{2I}\widetilde{y}^{2I}_t + y^{2g}\widetilde{y}^{2g}_t. \tag{68}$$

AGGREGATE SUPPLY

As discussed in *section 2.2.1*, the price setting behaviour of sector 1 is described by the Phillips curve equation (28), that is

$$\left(1+\beta\gamma^{p1}\right)\pi^1_t = \beta \mathrm{E}_t\left[\pi^1_{t+1}\right] + \gamma^{p1}\pi^1_{t-1} + \bar{\xi}^{p1}\widetilde{mc}^1_t + \left(1+\beta\gamma^{p1}\right)\varepsilon^{p1}_t, \tag{69}$$

where

$$\bar{\xi}^{p1} = \frac{\left(1-\xi^{p1}\right)\left(1-\beta\xi^{p1}\right)}{\xi^{p1}},$$

and $\widetilde{mc}^1_t = \widetilde{MC}^1_t - \widetilde{P}^1_t$ is the real marginal cost. As *section A.4* of the *Appendix* shows

$$\widetilde{mc}^1_t = \mathrm{a}^1 \widetilde{q}^1_t + \left(1-\mathrm{a}^1\right)\left[\widetilde{mc}^{ci}_t + (1-\chi^c)\widetilde{P}^{21}_t\right],$$

where the real marginal cost of composite input is

$$\widetilde{mc}_t^{ci} = \widetilde{w}_t + \frac{\alpha}{1-\alpha}\widetilde{ci}_t - \frac{\alpha}{1-\alpha}\widetilde{k}_{t-1}, \tag{70}$$

which can be calculated by log-linearizing equation (33). As shown in *section 2.2.2*, price setting in the export sector is represented by the Phillips curve (31), that is

$$(1+\beta\gamma^x)\,\pi_t^x = \beta \mathrm{E}_t\left[\pi_{t+1}^x\right] + \gamma^x\pi_{t-1}^x + \bar{\xi}^x\widetilde{mc}_t^x, \tag{71}$$

where

$$\bar{\xi}^x = \frac{\left(1-\xi^x\right)\left(1-\beta\xi^x\right)}{\xi^x},$$

and $\widetilde{mc}_t^x = \widetilde{MC}_t^x - \widetilde{P}_t^x$ is the real marginal cost. As *section A.4 of Appendix* shows

$$\widetilde{mc}_t^x = \mathrm{a}^x\left(\widetilde{q}_t^1 + \widetilde{P}_t^{1x}\right) + (1-\mathrm{a}^x)\left[\widetilde{mc}_t^{ci} + (1-\chi^c)\widetilde{P}_t^{21} + \widetilde{P}_t^{1x}\right].$$

The price setting equations of the market energy and non-processed food sectors are

$$\pi_t^{2e} = \mathrm{p}^e\left(d\widetilde{e}_t + \pi_t^{o*}\right), \tag{72}$$

$$\pi_t^{2a} = \mathrm{p}^a\left(d\widetilde{e}_t + \pi_t^{a*}\right), \tag{73}$$

where π_t^{o*} and π_t^{a*} are the inflation of imported energy and imported food denominated in foreign currency which are considered exogenous in the model. Furthermore, $0 < \mathrm{p}^e < 1$ and $0 < \mathrm{p}^a < 1$. The inflation of regulated energy and regulated non-energy sectors are treated as exogenous factors,

$$\pi_t^{2re} = \varepsilon_t^{p2re}, \tag{74}$$

$$\pi_t^{2r} = \varepsilon_t^{p2r}. \tag{75}$$

The inflation of non-core products is the weighted sum of the items above,

$$\pi_t^2 = \mathrm{c}^e\pi_t^{2e} + \mathrm{c}^a\pi_t^{2a} + \mathrm{c}^{re}\pi_t^{2re} + \mathrm{c}^r\pi_t^{2r}, \tag{76}$$

where c^e, c^a, c^{re} and c^r are the shares of the market energy's, non-processed food's, regulated energy's and regulated priced products' subsectors in sector 2.

CPI inflation is the weighted sum of sector 1's and sector 2's inflation,

$$\pi_t^c = \chi^c\pi_t^1 + (1-\chi^c)\pi_t^2. \tag{77}$$

The evolution of wages is described by the following Phillips curve type relationship.

$$\begin{aligned}(1+\beta)\widetilde{w}_t &= \beta\mathrm{E}_t\left[\widetilde{w}_{t+1}\right] + \widetilde{w}_{t-1} + \beta\pi_t^{ce} - (1+\beta\gamma^w)\,\pi_t^c + \gamma^w\pi_{t-1}^c \\ &+ \frac{\left(1-\beta\xi^w\right)\left(1-\xi^w\right)}{\left(1+\theta^w\sigma^l\right)\xi^w}\left(\sigma^l\widetilde{emp}_{t-1} - \widetilde{w}_t\right) + (1+\beta)\varepsilon_t^w.\end{aligned} \tag{78}$$

Labor demand can be calculated by log-linearizing equation (34),

$$\widetilde{l}_t = \frac{1}{1-\alpha}\widetilde{ci}_t - \frac{\alpha}{1-\alpha}\widetilde{k}_{t-1}. \tag{79}$$

In the production function of the model working hours are used as input instead of employment. The changes of employment are described by an additional equation as in Smets and Wouters (2003). In this equation the evolution of employment gradually follows the adjustment in hours:

$$\widetilde{emp}_t = \mathrm{g}^{emp}\widetilde{l}_t + (1-\mathrm{g}^{emp})\,\widetilde{emp}_{t-1}. \tag{80}$$

The demand for the composite input is given by log-linearizing and combining equations (27) and (30),

$$\widetilde{ci} = \varrho\left\{\frac{\mathrm{a}^1 ci^1 + \mathrm{a}^x ci^x}{ci}\left[q_t^1 + (1-\chi^c)\widetilde{P}_t^{21} - \widetilde{mc}_t^{ci}\right]\right\} + \frac{ci^1\widetilde{y}_t^1 + ci^x\widetilde{x}_t}{(1+\mathrm{f})ci} - \varepsilon_t^{\mathcal{A}}, \tag{81}$$

where the magnitude of the fix costs is assumed to be $\mathrm{F}^1 = \mathrm{f}y^1$ and $\mathrm{F}^x = \mathrm{f}x$. One can get the demand of sector 1 and the export sector for import goods ($\widetilde{m}_t$) by log-linearizing equations (26) and (29),

$$\begin{aligned} \widetilde{m}_t &= \varrho\left\{\left(1 - \frac{\mathrm{a}^1 m^1 + \mathrm{a}^x m^x}{m}\right)\left[\widetilde{mc}_t^{ci} - q_t^1 + (1-\chi^c)\widetilde{P}_t^{21}\right]\right\} \\ &+ \frac{m^1\widetilde{y}_t^1 + m^x\widetilde{x}_t}{(1+\mathrm{f})m} - \varepsilon_t^{\mathcal{A}}, \end{aligned} \tag{82}$$

Due to the assumptions on sector 2's technology, the import demand functions of the sector do not depend on relative prices just on the magnitude of production,

$$\widetilde{o}_t^2 = \frac{\widetilde{y}_t^2}{(1+\mathrm{f})}, \tag{83}$$

$$\widetilde{a}_t = \frac{\widetilde{y}_t^2}{(1+\mathrm{f})}, \tag{84}$$

$$\widetilde{m}_t^2 = \frac{\widetilde{y}_t^2}{(1+\mathrm{f})}, \tag{85}$$

where o_t^2, a_t and m_t^2 are the sector's demand for imported energy, agricultural product and general import goods.

REAL GDP

In *section A.4* it is shown that with the combination of y_t^1, y_t^2, export and import, the real GDP can be expressed by the following log-linearized equation,

$$P^{gdp} gdp\widetilde{gdp}_t = y^1\widetilde{y}_t^1 + y^2\widetilde{y}_t^2 + x\widetilde{x}_t - P^{m*}\left(m\widetilde{m}_t + m^2\widetilde{m}_t^2 + o^2\widetilde{o}_t^2 + a\widetilde{a}_t\right). \tag{86}$$

FINANCIAL ACCELERATOR

The financial accelerator mechanism described in *section 2.3* is represented by the equations describing the evolution of the external finance premium and the entrepreneurs' wealth. The difference between the return of capital and the real interest rate defines the external finance premium as the log-linearized version of equation (36) shows,

$$\mathrm{E}_t\left[\widetilde{r}_{t+1}^k\right] = \widetilde{s}_t + \widetilde{r}_t, \tag{87}$$

where $\widetilde{r}_t^k = (R_t^k - R^k)/R^k$. The evolution of the premium is described by the following equation which is the log-linearized version of equation (35),

$$\widetilde{s}_t = -\varkappa\left(\widetilde{a}_t - \widetilde{k}_t - \widetilde{Q}_t\right) + \varepsilon_t^s. \tag{88}$$

The evolution of entrepreneurs' net worth is described by equation (39). If it is assumed that $\mathcal{M}_t$ and $\bar{a}^e$ are relatively small then the log-linearization of the equation gives the following relationship,

$$\begin{aligned} \widetilde{a}_t^e &= \theta^e\frac{k}{a^e}(1+r)\left(s\widetilde{r}_t^k - \widetilde{r}_{t-1}\right) + \theta^e\frac{k}{a^e}(1+r)(s-1)\left(\widetilde{q}_{t-1} + \widetilde{k}_{t-1}\right) \\ &+ \theta^e(1+r)\left(\widetilde{r}_{t-1} + \widetilde{a}_{t-1}^e\right). \end{aligned} \tag{89}$$

MONETARY POLICY RULE

The monetary policy rule (40) represented in *section 2.4* can be expressed in the following way,

$$\widetilde{\imath}_t = \mathrm{r}\widetilde{\imath}_{t-1} + \frac{1-\mathrm{r}}{4}\left(\mathrm{r}^\pi\mathrm{E}_t\left[\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4}\right] + \mathrm{r}^{gdp}\widetilde{gdp}_t\right) + \varepsilon_t^i, \tag{90}$$

where $\widetilde{\imath}_t = i_t - i$.

NOMINAL EXCHANGE RATE

Equation (41) of *section 2.4* implies that

$$d\widetilde{e}_t = -\sum_{i=1}^{T^e} \bar{\eta}^i \mathrm{E}_t \left[di_{t+i-1}\right] + \varepsilon_t^{de}, \tag{91}$$

because $de_t = (e_t - e) - (e_{t-1} - e) = \widetilde{e}_t - \widetilde{e}_{t-1} = d\widetilde{e}_t$, furthermore $di_t = i_t - i_t^* - prem_t = \widetilde{\iota}_t - \widetilde{\iota}^* - \widetilde{prem}_t$, because $i = i^* + prem$.

RELATIVE PRICES, REAL EXCHANGE RATE INDICATORS

The log-linearized model equations utilize the following relative prices,

$$\widetilde{P}_t^{21} - \widetilde{P}_{t-1}^{21} = \pi_t^2 - \pi_t^1, \tag{92}$$
$$\widetilde{P}_t^{uo} - \widetilde{P}_{t-1}^{uo} = \pi_t^{u*} - \pi_t^{o*}. \tag{93}$$
$$\widetilde{P}_t^{1x} - \widetilde{P}_{t-1}^{1x} = \pi_t^1 - \pi_t^x. \tag{94}$$

The first relative price is the ratio of the prices of sector 2 and 1. The second relative price is the ratio of the prices of general import goods and imported energy both denominated in foreign currency. The third one is the ratio of the prices of sector 1 and the export sector.

The real exchange rate indicators represent different relative prices of the home and foreign economies. These are discussed in more detail in *section A.4* of the *Appendix*. The first real exchange rate indicator below is the relative price of the products of sector 1 and general import goods. The second one is the relative price of the export sector's product and the foreign price level.

$$\widetilde{q}_t^1 - \widetilde{q}_{t-1}^1 = d\widetilde{e}_t + \mathrm{b}^1 \pi_t^{u*} + \left(1 - \mathrm{b}^1\right) \pi_t^{o*} - \pi_t^1, \tag{95}$$
$$\widetilde{q}_t^x = \widetilde{q}_t^1 + \widetilde{P}_t^{1x} + \left(\mathrm{b}^* - \mathrm{b}^1\right) \widetilde{P}_t^{uo}. \tag{96}$$

INTEREST RATES, INTEREST RATE MARGINS, REAL INTEREST RATES

In the model the stickiness of banks' deposit rates is assumed so they do not follow immediately the adjustment of financial market rates. The evolution of domestic deposit rates is described by the following Calvo-type equation,

$$\widetilde{\iota}_t^h - \widetilde{\iota}_{t-1}^h = \beta \mathrm{E}_t \left[\widetilde{\iota}_{t+1}^h - \widetilde{\iota}_t^h\right] + \frac{\left(1 - \xi^i\right)\left(1 - \beta \xi^i\right)}{\xi^i} \left(\widetilde{\iota}_{t-2} - \widetilde{\iota}_{t-2}^h\right). \tag{97}$$

Households are assumed to have bounded rationality, and their adaptive inflation expectations are captured by the following equation,

$$\pi_t^{ce} = \mathrm{g}^\pi \pi_t^c + \left(1 - \mathrm{g}^\pi\right) \pi_{t-1}^{ce}, \tag{98}$$

where π_t^{ce} is the inflation expectation of households for the next period, $0 < \mathrm{g}^\pi < 1$. Based on this, the real interest rate perceived by households is

$$\widetilde{r}_t^h = \widetilde{\iota}_t^h - \pi_t^{ce}. \tag{99}$$

But the ex-ante real interest rate perceived by the households is not identical to the ex-post realized real interest rate,

$$\widetilde{r}_t^{hlag} = \widetilde{\iota}_{t-1}^h - \pi_t^c. \tag{100}$$

In case of firms the inflation and real interest rate expectations are assumed to be described by the model consistent Fisher equation,

$$\widetilde{r}_t = \widetilde{\iota}_t - \mathrm{E}_t \left[\pi_{t+1}\right]. \tag{101}$$

INCOMES, PROFITS

The components of the real income of households are the following items: labor income, dividends and taxes financing government expenditure. λ^{w1} and λ^{pro1} indicates the share of households with positive net worth in labor income and dividends, λ^{dom} denotes the share of the household sector in profits, the expression $\widetilde{g}_t - (\chi^g - \chi^c)\widetilde{P}_t^{21}$ represents the effect of taxes, see *section A.4* of the *Appendix* for more details.

$$\begin{aligned} D^1\widetilde{D}_t^1 &= \lambda^{dom}\lambda^{pro1} div\widetilde{div}_t \\ &+ \lambda^{w1}\left[wE\left(\widetilde{E}_t + \widetilde{w}_t\right) - \lambda^g g\left(\widetilde{g}_t - (\chi^g - \chi^c)\widetilde{P}_t^{21}\right)\right]. \quad (102)\\ D^2\widetilde{D}_t^2 &= \lambda^{dom}\left(1-\lambda^{pro1}\right) div\widetilde{div}_t \\ &+ \left(1-\lambda^{w1}\right)\left[wE\left(\widetilde{E}_t + \widetilde{w}_t\right) - \lambda^g g\left(\widetilde{g}_t - (\chi^g - \chi^c)\widetilde{P}_t^{21}\right)\right]. \quad (103)\end{aligned}$$

In the model households are assumed to have bounded rationality so the adjustment of their perceived real income follows with lags the changes of their true real income,

$$\begin{aligned} \widetilde{d}_t^1 &= \mathrm{g}^d\widetilde{D}_t^1 + \left(1-\mathrm{g}^d\right)\widetilde{d}_{t-1}^1, \quad (104)\\ \widetilde{d}_t^2 &= \mathrm{g}^d\widetilde{D}_t^2 + \left(1-\mathrm{g}^d\right)\widetilde{d}_{t-1}^2, \quad (105)\end{aligned}$$

where $0 < \mathrm{g}^d < 1$.

In the model it is assumed that households only receive a share in profit produced in sector 1 and the export sector. In *section A.4* of the *Appendix* the log-linearized equation describing the profit of sector 1 is derived in detail,

$$\begin{aligned} pro\widetilde{pro}_t &= y^{1c}\widetilde{y}_t^{1c} + y^{1I}\widetilde{y}_t^{1I} + y^{1g}\widetilde{y}_t^{1g} - wl\left(\widetilde{w}_t + e\widetilde{mp}_t\right) \\ &+ x\left(\widetilde{x}_t - \widetilde{P}_t^{1x}\right) - P^{m*}m\left(\widetilde{q}_t^1 + \widetilde{m}_t\right) \\ &- \left(y^{1c} + y^{1I} + y^{1g} + x - P^{m*}m\right)(1-\chi^c)\widetilde{P}_t^{21}. \quad (106)\end{aligned}$$

The dividend expression smoothes the fluctuation of profits:

$$\widetilde{div}_t = \mathrm{g}^{div}\widetilde{pro}_t + \left(1-\mathrm{g}^{div}\right)\widetilde{div}_{t-1}, \quad (107)$$

where $0 < \mathrm{g}^{div} < 1$.

THE REST OF THE WORLD

In addition to the export demand equation (60), the following four log-linearized equations describe the behavior of the rest of the world. The foreign CPI inflation rate is the weighted average of the inflation rates of the foreign general good and energy.

$$\pi_t^* = \mathrm{b}^*\pi_t^{u*} + \left(1-\mathrm{b}^*\right)\left(\widetilde{P}_t^{o*} - \widetilde{P}_{t-1}^{o*}\right). \quad (108)$$

Inflation of the foreign general good is described by a Phillips curve which captures the relationship between inflation and the output gap,

$$\pi_t^{u*} = \phi^1\mathrm{E}_t\left[\pi_{t+1}^{u*}\right] + \left(1-\phi^1\right)\pi_{t-1}^{u*} + \phi^2\widetilde{y}_t^*. \quad (109)$$

The foreign output gap is determined by an Euler equation type relationship:

$$\widetilde{y}_t^* = \phi^3\mathrm{E}_t\left[\widetilde{y}_{t+1}^*\right] + \left(1-\phi^3\right)\widetilde{y}_{t-1}^* + \phi^4\mathrm{E}_t\left[i_t^* - \pi_{t+1}^*\right] + \varepsilon_t^{y^*}. \quad (110)$$

The foreign nominal interest rate is also determined by a monetary policy rule similar to the domestic one,

$$\widetilde{\imath}_t^* = \phi^5\widetilde{\imath}_{t-1}^* + \frac{1-\phi^5}{4}\left(\phi^6\mathrm{E}_t\left[\pi_{t+1}^* + \pi_{t+2}^* + \pi_{t+3}^* + \pi_{t+4}^*\right] + \phi^7\widetilde{y}_t^*\right). \quad (111)$$

A.2 THE EFFECT OF THE PRECAUTIONARY MOTIVE ON CONSUMER BEHAVIOR

This section supplements the discussion of the precautionary motive in *section 2.1.1*. It shows that taking on debt is possible in the case of the precautionary motive, furthermore it compares the model based on the precautionary motive with the model using an exogenous debt constraint.

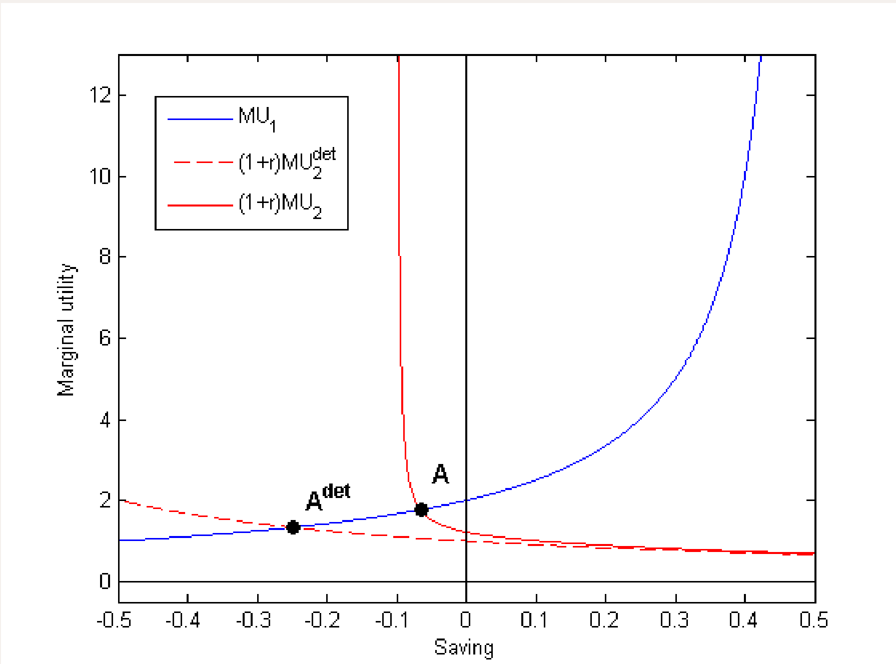
The drawback of the main text's precautionary motive model is that it does not allow households to take on debt which is in contradiction with the empirical results. However, a small modification of the model allows indebtedness in the optimal solution. Assume that in the second state of the world income is not zero but $0 < u_2 < y_2$ (unemployment benefit). In this case, the Euler equation takes the following form,

$$\frac{1}{n-s_1} = (1-\pi)\frac{(1+r)\beta}{y_2+(1+r)s_1} + \pi\frac{(1+r)\beta}{u_2+(1+r)s_1},$$

which can be expressed in the following way,

$$\frac{1}{n-s_1} = \frac{(1+r)\beta}{y_2+(1+r)s_1} + \pi\frac{(1+r)\beta(y_2-u_2)}{[u_2+(1+r)s_1]\,[y_2+(1+r)s_1]}. \tag{112}$$

Figure 19
Precautionary motive in case of positive unemployment benefit ($u_2 > 0$)



The second term of the right hand side of equation (112) expresses the precautionary motive, see *Figure 19*. The optimal solution (*A*) still deviates from the deterministic case (A^{det}) but if n is small enough then indebtedness is possible even in the presence of the precautionary motive. However, it is smaller than in the deterministic model.

The consumption function remains concave, but its starting point shifts to the negative region. See *Figures 20* and *21*.

As equation (112) reveals, if u_2 is really close to y_2 then the effect of the precautionary motive is negligible. This explains why macroeconomic shocks are not sufficient to imply significant precautionary motive. In the case of macroeconomic shocks the difference of incomes in the two states of the world are relatively small. However, specific shocks (such as unemployment) can induce large enough income fluctuations that the consumption-saving decision changes significantly due to the precautionary motive.

Figure 20
Consumption function in case of positive unemployment benefit ($u_2 > 0$)

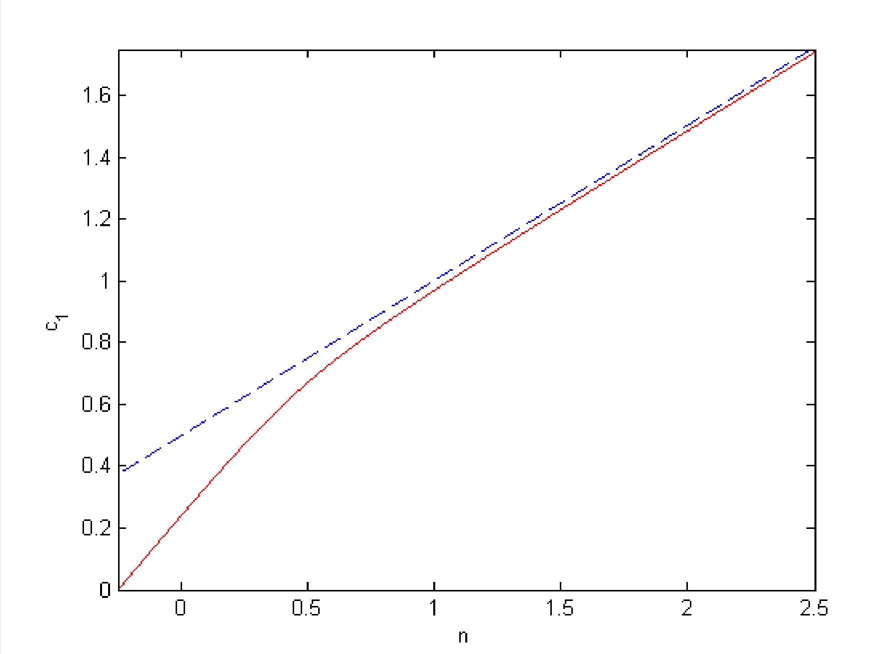
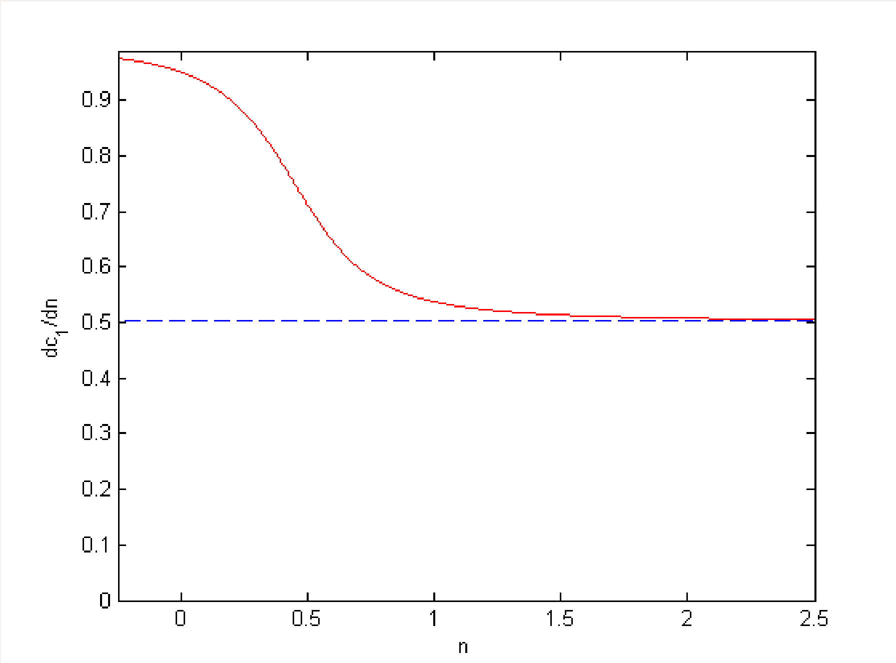


Figure 21
Marginal propensity to consume in case of positive unemployment benefit ($u_2 > 0$)



In the next step, compare the model based on the precautionary model with a model where there is no uncertainty but instead an exogenous debt constraint influences consumer behavior. The optimization problem is:

$$\max_{c_1,c_2,s_1} \log(c_1) + \beta \log(c_2),$$
$$c_1 + s_1 = n,$$
$$c_2 = y_2 + (1+r)s_1,$$
$$-s_1 \leq d,$$

where $d \geq 0$ is the exogenous debt constraint. The consumer cannot have higher debt than this in the first period. If $d = 0$ then consumers cannot have debt at all, they can only save. If $d > 0$ then some amount of debt is allowed. *Figure 22* shows a case when the debt constraint binds. In the figure $d = 0.1$ and the debt constraint is represented by a vertical line. The consumer would consume more in the first period and would take on more debt (point A') than what is allowed by the exogenous constraint (point A).

Figure 22
Positive debt constraint

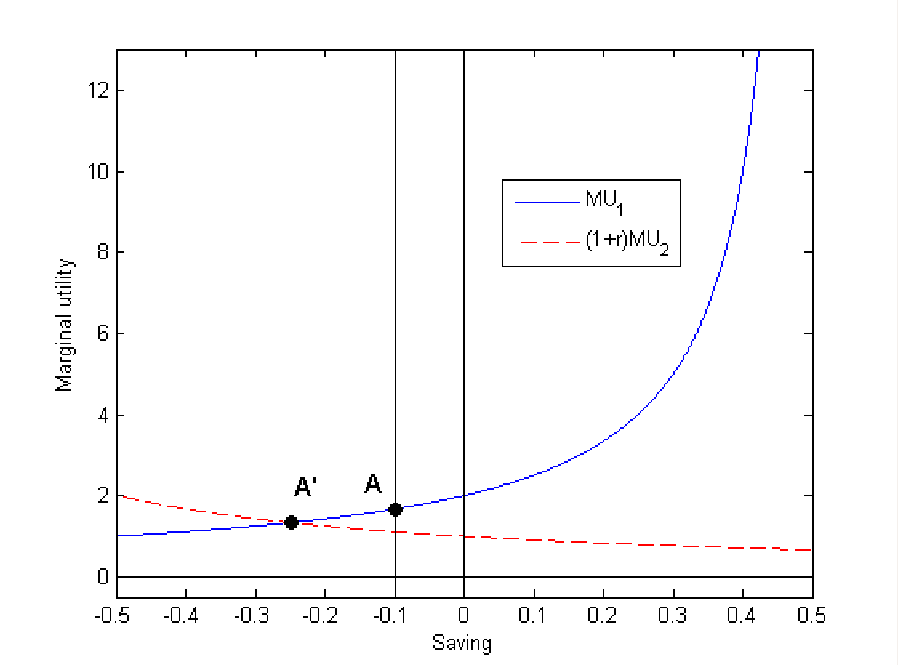


Figure 23
Zero debt constraint

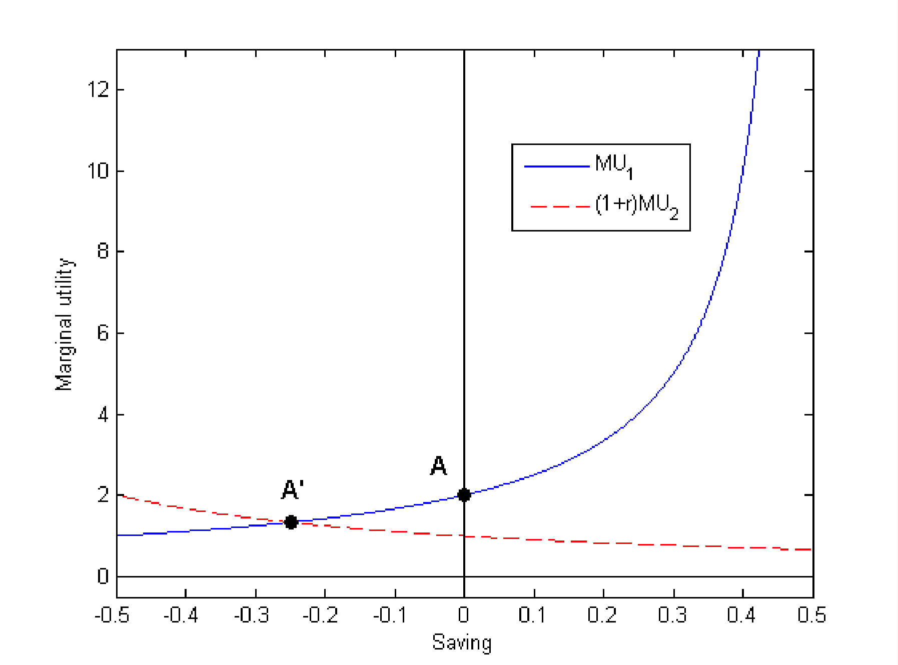


Figure 23 shows the effect of a more strict constraint where $d = 0$ so taking on debt is not possible at all. The solution of the model is similar to that when the consumer does not have income in the bad state of the world in the precautionary motive case. If the consumer has a debt constraint then her consumption is determined by her budget constraint not the Euler equation:

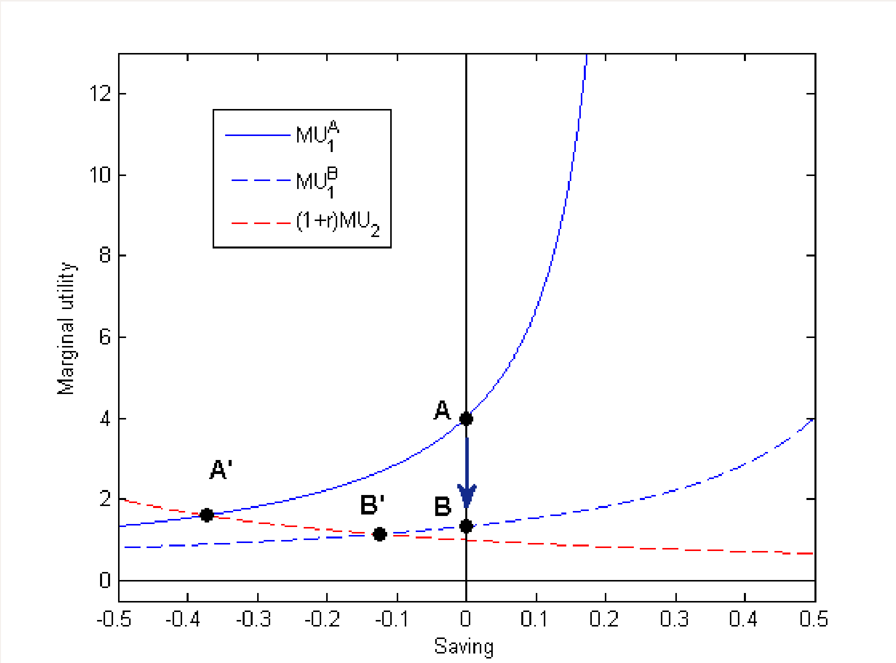
$$c_1 = n + d, \qquad c_2 = y_2 - (1 + r)d.$$

Without the debt constraint the marginal propensity to consumption does not depend on n. It is constant and smaller than 1. Recall that $mpc = 1/(1 + \beta) < 1$.

In the next step, we analyze the magnitude of mpc if the consumer's debt constraint binds. *Figure 24* compares the optimal saving decision in case of two levels of n, $n^A < n^B$. The difference between n^A and n^B is assumed to be small. In this case the optimal solution moves from point A to B. Debt is d in both cases, due to this $c_1^A = n^A + d$ and $c_1^B = n^B + d$. So

$$mpc = \frac{c_1^B - c_1^A}{n^B - n^A} = 1.$$

Figure 24
High marginal propensity to consume ($mpc = 1$)



Nevertheless, a threshold value for n exists, such that for higher values than this the consumer's debt constraint does not bind. In *Figure 25* point C represents the n^C where debt is exactly d, but due to the consumer's own decision. In this case the marginal utilities are identical, the consumption-saving decision is determined by the Euler equation. This $\bar{n}$ threshold value can be calculated using equation (2):

$$-d = s_1 = \frac{\beta(1 + r)\bar{n} - y_2}{(1 + \beta)(1 + r)}.$$

So,

$$\bar{n} = \frac{y_2 - (1 + \beta)(1 + r)d}{\beta(1 + r)}.$$

$$mpc = \frac{\partial c_1}{\partial n} = \frac{1}{1 + \beta} < 1$$

If $n^D > n^C$ (see *Figure 26*), then the consumer's debt constraint does not bind any more. The consumption decision is determined by the unconstrained problem:

Figure 25
Threshold value of income and/or wealth (*n*)

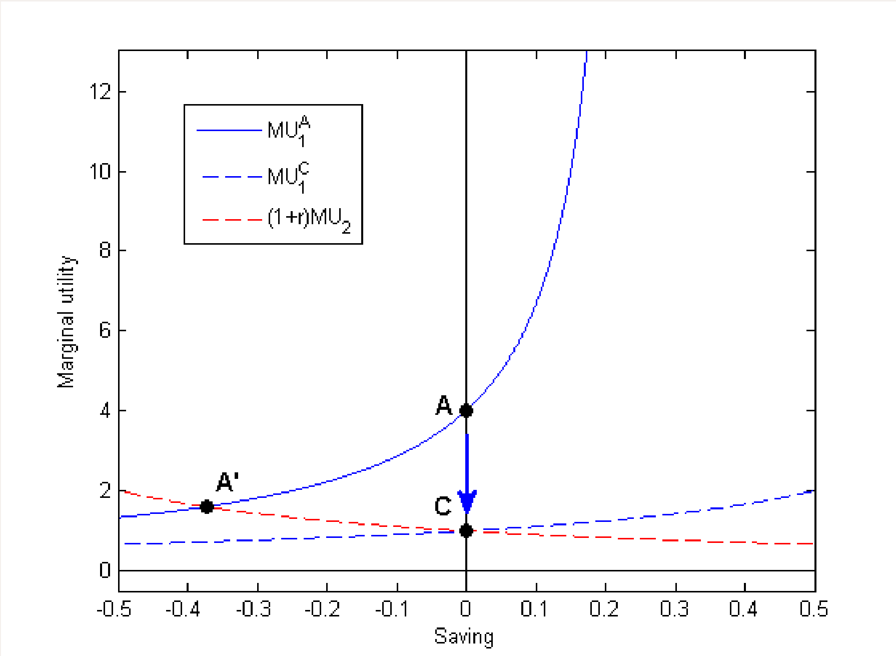
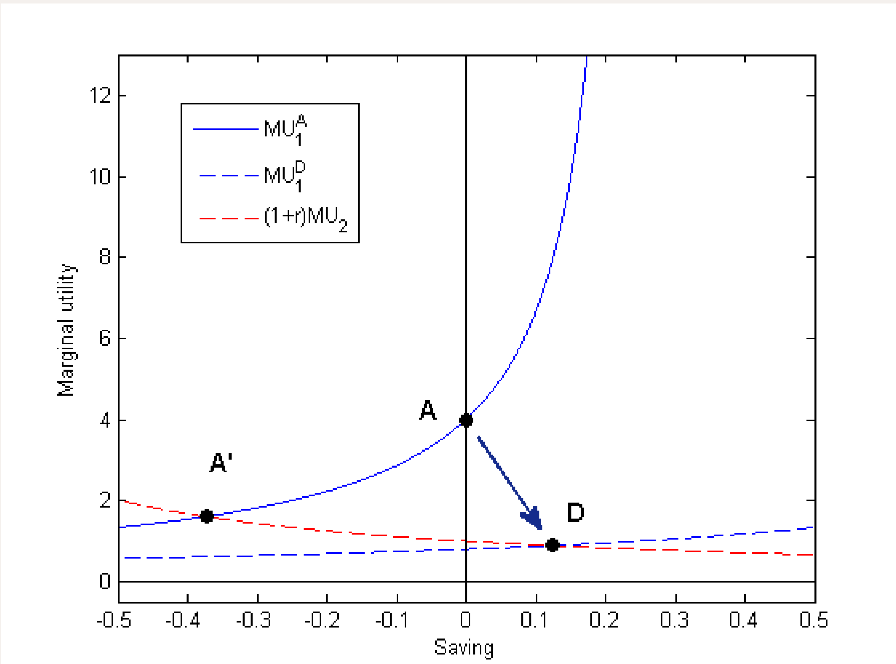


Figure 26
Low marginal propensity to consume ($mpc = 1/(1+\beta)$)

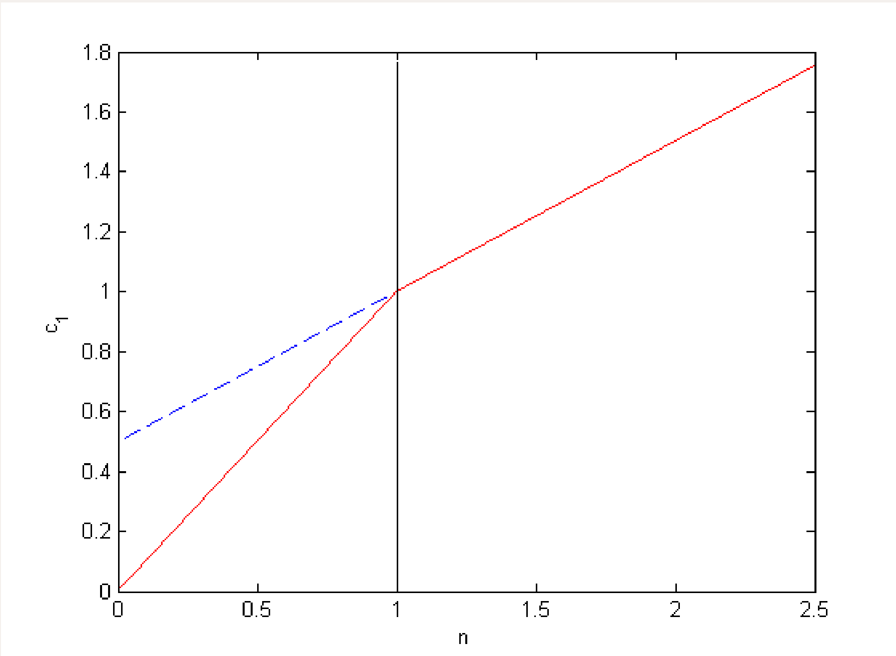


Due to this, consumption in the first period can be represented by the following function:

$$c_1 = \begin{cases} n+d, & \text{if } -d \le n < \bar{n}, \\ \frac{n+\frac{y_2}{1+r}}{1+\beta}, & \text{if } \bar{n} \le n. \end{cases}$$

Figure 27 shows that this function is concave similarly to the model based on the precautionary motive.

Figure 27
Consumption function



A.3 THE MODIFIED UNCOVERED INTEREST RATE PARITY

This section compares the implications of the covered and uncovered interest rate parity. The uncovered interest rate parity (UIP) is defined by the following equation,

$$e_t = \mathrm{E}_t\left[e_{t+1}\right] - di_t,$$

where $e_t = \log(E_t)$ and $di_t = i_t - i_t^* - pr_t$. Assuming rational expectations and using recursive substitution:

$$\begin{aligned}
e_t &= \mathrm{E}_t\left[e_{t+2}\right] - di_t - \mathrm{E}_t\left[di_{t+1}\right]. \\
e_t &= \mathrm{E}_t\left[e_{t+3}\right] - di_t - \mathrm{E}_t\left[di_{t+1}\right] - \mathrm{E}_t\left[di_{t+2}\right]. \\
e_t &= \mathrm{E}_t\left[e_{t+4}\right] - di_t - \mathrm{E}_t\left[di_{t+1}\right] - \mathrm{E}_t\left[di_{t+2}\right] - \mathrm{E}_t\left[di_{t+3}\right]. \\
&\vdots \\
e_t &= \mathrm{E}_t\left[e_{t+T+1}\right] - \sum_{i=0}^{T} \mathrm{E}_t\left[di_{t+i}\right].
\end{aligned}$$

Continue the iteration to $T = \infty$. Suppose that $\lim_{T\to\infty} \mathrm{E}_t\left[e_T\right] = e$. Then the UIP condition implies the following:

$$e_t - e = -\sum_{i=0}^{\infty} \mathrm{E}_t\left[di_{t+i}\right].$$

The modified uncovered interest rate parity (MUIP) is expressed by the following equation:

$$e_t = \eta\left(\mathrm{E}_t\left[e_{t+1}\right] - di_t\right) + (1-\eta)e_{t-1},$$

where $0 < \eta < 1$. Rearranging this:

$$de_t = \bar{\eta}\left(\mathrm{E}_t\left[de_{t+1}\right] - di_t\right),$$

where $\bar{\eta} = \eta/(1-\eta)$ and $de_t = e_t - e_{t-1}$. Assuming rational expectations and using recursive substitution:

$$\begin{aligned}
de_t &= \bar{\eta}^2 \mathrm{E}_t\left[de_{t+2}\right] - \bar{\eta} di_t - \bar{\eta}^2 \mathrm{E}_t\left[di_{t+1}\right]. \\
de_t &= \bar{\eta}^3 \mathrm{E}_t\left[de_{t+3}\right] - \bar{\eta} di_t - \bar{\eta}^2 \mathrm{E}_t\left[di_{t+1}\right] - \bar{\eta}^3 \mathrm{E}_t\left[di_{t+2}\right]. \\
de_t &= \bar{\eta}^4 \mathrm{E}_t\left[de_{t+4}\right] - \bar{\eta} di_t - \bar{\eta}^2 \mathrm{E}_t\left[di_{t+1}\right] - \bar{\eta}^3 \mathrm{E}_t\left[di_{t+2}\right] - \bar{\eta}^4 \mathrm{E}_t\left[di_{t+3}\right]. \\
&\vdots \\
de_t &= \bar{\eta}^{T+1} \mathrm{E}_t\left[de_{t+T+1}\right] - \sum_{i=0}^{T} \bar{\eta}^{i+1} \mathrm{E}_t\left[di_{t+i}\right].
\end{aligned}$$

Apply recursion to $T = \infty$. Define $\lim_{T\to\infty} \mathrm{E}_t\left[e_T\right] = e$. Due to this $\lim_{T\to\infty} \mathrm{E}_t\left[de_T\right] = 0$. So the MUIP implies that

$$de_t = -\sum_{i=0}^{\infty} \bar{\eta}^{i+1} \mathrm{E}_t\left[di_{t+i}\right].$$

A.4 RELATIVE PRICES, PRICE INDICES AND REAL EXCHANGE RATE INDICATORS

During the derivation of the log-linearized version of the model (see *section A.1* of the *Appendix*) several variables were expressed in terms of some relative price or real exchange rate variables. In this section these relationships are derived.

As a reminder, the following (log-linearized) relative prices are used, see equations (92), (93) and (94).

$$\begin{aligned}
\widetilde{P}_t^{21} &= \widetilde{P}_t^2 - \widetilde{P}_t^1, \\
\widetilde{P}_t^{uo} &= \widetilde{P}_t^{u*} - \widetilde{P}_t^{o*}, \\
\widetilde{P}_t^{1x} &= \widetilde{P}_t^1 - \widetilde{P}_t^x.
\end{aligned}$$

The $\widetilde{q}_t^1$ log-linearized real exchange rate indicator is the ratio of the import price index and the core inflation sector's price index:

$$\widetilde{q}_t^1 = \widetilde{e}_t + \widetilde{P}_t^{m*} - \widetilde{P}_t^1 = \widetilde{e}_t + \mathrm{b}^1 \widetilde{P}_t^{u*} + \left(1 - \mathrm{b}^1\right) \widetilde{P}_t^{o*} - \widetilde{P}_t^1.$$

The $\widetilde{q}_t^x$ log-linearized real exchange rate indicator is the ratio of the foreign price index and the domestic export price index:

$$\begin{aligned}
\widetilde{q}_t^x &= \widetilde{e}_t + \widetilde{P}_t^* - \widetilde{P}_t^x = \left(\widetilde{P}_t^* - \widetilde{P}_t^{m*}\right) + \left(\widetilde{e}_t + \widetilde{P}_t^{m*} - \widetilde{P}_t^1\right) + \left(\widetilde{P}_t^1 - \widetilde{P}_t^x\right) \\
&= \left(\mathrm{b}^* - \mathrm{b}^1\right) \widetilde{P}_t^{uo} + \widetilde{q}_t^1 + \widetilde{P}_t^{1x},
\end{aligned}$$

since using equations (32) and (108) implies that

$$\begin{aligned}
\widetilde{P}_t^* - \widetilde{P}_t^{m*} &= \mathrm{b}^* \widetilde{P}_t^{u*} + \left(1 - \mathrm{b}^*\right) \widetilde{P}_t^{o*} - \mathrm{b}^1 \widetilde{P}_t^{u*} - \left(1 - \mathrm{b}^1\right) \widetilde{P}_t^{o*} \\
&= \left(\mathrm{b}^* - \mathrm{b}^1\right) \left(\widetilde{P}_t^{u*} - \widetilde{P}_t^{o*}\right) = \left(\mathrm{b}^* - \mathrm{b}^1\right) \widetilde{P}_t^{uo}.
\end{aligned}$$

The ratio of the consumer price index and the price index of sector 1 can be expressed by the relative price $\widetilde{P}_t^{21}$:

$$\widetilde{P}_t^c - \widetilde{P}_t^1 = \chi^c \widetilde{P}_t^1 + (1 - \chi^c) \widetilde{P}_t^2 - \widetilde{P}_t^1 = (1 - \chi^c) \left(\widetilde{P}_t^2 - \widetilde{P}_t^1\right) = (1 - \chi^c) \widetilde{P}_t^{21}$$

The real marginal cost from sector 1's Phillips-curve equation (69) can be formulated in the following way using the above relationships,

$$\begin{aligned}
\widetilde{mc}_t^1 &= \widetilde{MC}_t^1 - \widetilde{P}_t^1 = \mathrm{a}^1 \left(\widetilde{e}_t + \widetilde{P}_t^{m*}\right) + \left(1 - \mathrm{a}^1\right) \widetilde{MC}_t^{ci} - \widetilde{P}_t^1 \\
&= \mathrm{a}^1 \left(\widetilde{e}_t + \widetilde{P}_t^{m*} - \widetilde{P}_t^1\right) + \left(1 - \mathrm{a}^1\right) \left(\widetilde{MC}_t^{ci} - \widetilde{P}_t^c\right) + \left(1 - \mathrm{a}^1\right) \left(\widetilde{P}_t^c - \widetilde{P}_t^1\right) \\
&= \mathrm{a}^1 \widetilde{q}_t^1 + \left(1 - \mathrm{a}^1\right) \widetilde{mc}_t^{ci} + \left(1 - \mathrm{a}^1\right) (1 - \chi^c) \widetilde{P}_t^{21}.
\end{aligned}$$

The marginal cost from equation (71) which describes the pricing of the export sector can be expressed in the following way using the above relationships,

$$\begin{aligned}
\widetilde{mc}_t^x &= \widetilde{MC}_t^x - \widetilde{P}_t^x = a^x\left(\widetilde{e}_t + \widetilde{P}_t^{m*}\right) + (1-a^x)\,\widetilde{MC}_t^{ci} - \widetilde{P}_t^x \\
&= a^x\left(\widetilde{e}_t + \widetilde{P}_t^{m*} - \widetilde{P}_t^1\right) + (1-a^x)\left(\widetilde{MC}_t^{ci} - \widetilde{P}_t^1\right) + \widetilde{P}_t^1 - \widetilde{P}_t^x \\
&= a^x\widetilde{q}_t^1 + (1-a^x)\left(\widetilde{MC}_t^{ci} - \widetilde{P}_t^1\right) + \widetilde{P}_t^{1x} \\
&= a^x\widetilde{q}_t^1 + (1-a^x)\left(\widetilde{mc}_t^{ci} + \widetilde{P}_t^c - \widetilde{P}_t^1\right) + \widetilde{P}_t^{1x} \\
&= a^x\widetilde{q}_t^1 + (1-a^x)\,\widetilde{mc}_t^{ci} + (1-a^x)\,(1-\chi^c)\widetilde{P}_t^{21} + \widetilde{P}_t^{1x}.
\end{aligned}$$

In equations (102) and (103), which describes households' disposable income, the formula $\widetilde{g}_t - (\chi^g - \chi^c)\widetilde{P}_t^{21}$ is present. This formula describes the lump-sum tax equal to government spending's real value adjusted with the consumer price index. This is explained in detail below. The log-linearized nominal government spending is $\widetilde{G}_t$. This can be expressed in the following way with the help of the real government consumption and the price index of government products: $\widetilde{G}_t = \widetilde{g}_t + \widetilde{P}_t^g$. Recall that $\widetilde{P}_t^g = \chi^g\widetilde{P}_t^1 + (1-\chi^g)\,\widetilde{P}_t^2$ and $\widetilde{P}_t^c = \chi^c\widetilde{P}_t^1 + (1-\chi^c)\,\widetilde{P}_t^2$. Using this,

$$\begin{aligned}
\widetilde{G}_t - \widetilde{P}_t^c &= \widetilde{g}_t + \widetilde{P}_t^g - \widetilde{P}_t^c = \widetilde{g}_t + \chi^g\widetilde{P}_t^1 + (1-\chi^g)\,\widetilde{P}_t^2 - \chi^c\widetilde{P}_t^1 - (1-\chi^c)\,\widetilde{P}_t^2 \\
&= \widetilde{g}_t + (\chi^g - \chi^c)\left(\widetilde{P}_t^1 - \widetilde{P}_t^2\right) = \widetilde{g}_t - (\chi^g - \chi^c)\widetilde{P}_t^{21}.
\end{aligned}$$

Equation (106), which describes the real profit of sector 1 and the export sector, also contains several relative prices and exchange rate indicators. The following part explains this. The nominal profit of sector 1 and the export sector is

$$PRO_t = P_t^1\left[y_t^{1c} + y_t^{1I} + y_t^{1g}\right] + P_t^x x_t - P_t^c w_t emp_t - e_t P_t^{m*} m_t.$$

From this the real profit is:

$$pro_t = \frac{P_t^1}{P_t^c}\left[y_t^{1c} + y_t^{1I} + y_t^{1g} + \frac{P_t^x}{P_t^1}x_t - \frac{e_t P_t^{m*}}{P_t^1}m_t\right] - w_t emp_t.$$

Rearranging this:

$$pro_t = \frac{P_t^1}{P_t^c}\left[y_t^{1c} + y_t^{1I} + y_t^{1g} + \frac{x_t}{P_t^{1x}} - q_t^1 m_t\right] - w_t emp_t.$$

Taking into consideration that $P^1 = P^2 = P^c = P^x = 1$, $P^{1x} = 1$, $e = 1$, $q^1 = P^{m*}$ the log-linearized version is the following:

$$\begin{aligned}
pro\widetilde{pro}_t &= y^{1c}\widetilde{y}_t^{1c} + y^{1I}\widetilde{y}_t^{1I} + y^{1g}\widetilde{y}_t^{1g} - wl\left(\widetilde{w}_t + \widetilde{emp}_t\right) \\
&+ x\left(\widetilde{x}_t - \widetilde{P}_t^{1x}\right) - P^{m*}m\left(\widetilde{q}_t^1 + \widetilde{m}_t\right) \\
&- \left(y^{1c} + y^{1I} + y^{1g} + x - P^{m*}m\right)(1-\chi^c)\widetilde{P}_t^{21}.
\end{aligned}$$

The nominal GDP in the model is defined in the following way:

$$GDP_t = P_t^1 y_t^1 + P_t^2 y_t^2 + P_t^x x_t - \left[P_t^m\left(m_t + m_t^2\right) + P_t^o o_t^2 + P_t^a a_t\right].$$

If a properly defined GDP deflator P_t^{gdp} exists then one can define real GDP. For this the following expression is true,

$$P_t^{gdp} gdp_t = P_t^1 y_t^1 + P_t^2 y_t^2 + P_t^x x_t - \left[P_t^m\left(m_t + m_t^2\right) + P_t^o o_t^2 + P_t^a a_t\right].$$

For a well-defined deflator it is true that the log-linearized deflator is the average of the sectors' log-linearized prices weighted by the shares of the related sectors to the aggregate GDP, that is,

$$\begin{aligned}
\widetilde{P}_t^{gdp} &= \frac{P^1 y^1}{P^{gdp}gdp}\widetilde{P}_t^1 + \frac{P^2 y^2}{P^{gdp}gdp}\widetilde{P}_t^2 + \frac{P^x x}{P^{gdp}gdp}\widetilde{P}_t^x \\
&- \frac{P^m(m+m^2)}{P^{gdp}gdp}\widetilde{P}_t^m - \frac{P^o o^2}{P^{gdp}gdp}\widetilde{P}_t^o - \frac{P^a a}{P^{gdp}gdp}\widetilde{P}_t^a.
\end{aligned} \tag{113}$$

The log-linearized real GDP equation has the following form:

$$\begin{aligned} P^{gdp} gdp \left(\widetilde{P}_t^{gdp} + \widetilde{gdp}_t \right) &= P^1 y^1 \left(\widetilde{P}_t^1 + \widetilde{y}_t^1 \right) + P^2 y^2 \left(\widetilde{P}_t^2 + \widetilde{y}_t^2 \right) + P^x x \left(\widetilde{P}_t^x + \widetilde{x}_t \right) \\ &- P^m (m + m^2) \widetilde{P}_t^m + P^m \left(m \widetilde{m}_t + m^2 \widetilde{m}_t^2 \right) \\ &- P^o o^2 \left(\widetilde{P}_t^o + \widetilde{o}_t^2 \right) - P^a a \left(\widetilde{P}_t^a + \widetilde{a}_t \right). \end{aligned} \tag{114}$$

If one combines equations (113) and (114) and uses that $P^1 = P^2 = P^x = 1$ and $P^m = P^{m*}$, $P^o = P^{o*}$, $P^a = P^{a*}$ (because $e = 1$) and $P^{m*} = P^{o*} = P^{a*}$, then one can get equation (86) from *section A.1*.

MNB WORKING PAPERS 2016/4

THE MACROECONOMIC FORECASTING MODEL OF THE MNB

Print: Prospektus–SPL consortium

H-8200 Veszprém, Tartu u. 6.