ROMAN HORVATH | LORANT KASZAB | ALES MARSAL

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Fiscal Policy and the Nominal Term Premium

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Contents

Abstract

We estimate a New Keynesian model on post-war US data with generalised method of moments using either constant or time-varying debt and labor income taxes. We show that accounting for government debt and distortionary taxes help the New Keynesian model match the level of the nominal term premium with a lower relative risk-aversion than typically found in the literature.

JEL: E13, E31, E43, E44, E62.

Keywords: zero-coupon bond, nominal term premium, balanced budget rule, government debt, income taxation.

Összefoglaló

Egy új keynesi típusú makropénzügyi modellt becslünk USA adatokon (1960-2007) általánosított momentumok módszerével kiegyensúlyozott vagy deficites kormányzati költségvetést feltételezve. Azt találjuk, hogy a modell képes magyarázni a hosszú lejáratú (pl. 10 éves) államkötvények hozamában lévő kockázati prémiumot, amikor a költségvetés bevételei elsősorban munkához kapcsolódó adókból származnak. A korábbi irodalomhoz képest a mi modellünk a kockázat-elutasítás alacsonyabb mértéke mellett is sikeres.

1 Introduction

This paper explores how fiscal policy affects the term premium in the yields of long-term bonds from a macro-finance perspective. The yield on long-term nominal bonds (such as US Treasury securities or UK gilts) includes a term premium that investors require as compensation for nominal and real risks over the lifetime of the bond. Macro-finance models aim to jointly match a set of macroeconomic and finance moments, such as the variability of consumption growth and the sizable nominal term premium (NTP), respectively.

Our model is an extension of the New Keynesian macro-finance model developed by Rudebusch and Swanson (2012) (henceforth, RS) on the fiscal side. We have three departures from RS, who assume that the government budget is balanced through lump-sum taxes in each period. First, we consider constant government debt (balanced budget) and the more realistic case of time-varying debt. The latter implies that government budget deficits can occur, which is true for most countries and various time periods. Second, the government budget in our paper is consolidated through distortionary income taxes[1] in both scenarios. Third, we estimate the model instead of calibrating it as in RS. In particular, we take a third-order Taylor series approximation of the model and estimate it on US data for 1961-2007 using the Generalized Method of Moments (GMM).

Macro-finance models have long struggled to match finance moments, such as the NTP on long-run bonds (see RS and the papers cited therein). The RS model, as well as our model, features Epstein-Zin recursive preferences, which are used to disentangle risk aversion from the intertemporal elasticity of substitution. With EZ preferences, one can raise the risk aversion of the representative household to help produce high term premiums without reducing the intertemporal elasticity of substitution to counterfactually low levels. An important contribution of this paper is that our model, which is estimated on US data with detailed fiscal sector information, can match the NTP with lower risk aversion than can previous papers that operate with simpler fiscal setups (see, e.g., RS and Andreasen (2012)).

We contribute to the literature on the fiscal side as follows. In RS, the sources of the risks are temporary technology shocks that engineer negative covariance between consumption and inflation. For instance, a negative disturbance to technology raises production costs and, thus, the marginal cost of production but decreases consumption. With negative supply shocks, nominal bonds are risky in the sense that they deliver low real returns at a time of low consumption and output.

In our setting, fiscal policy has similar effects to those of productivity shocks in RS. On the one hand, additional government purchases induce higher taxes, marginal costs and inflation through the New Keynesian Phillips curve. On the other hand, higher taxes urge households to substitute labor for leisure, resulting in lower production and consumption. Uncertainty about the evolution of government spending therefore leads to inflation risks and higher bond yields. With a steady-state debt-to-GDP ratio of approximately 80 percent (not uncommon in the US before the outbreak of the financial crises in 2008) our models easily capture the observed term-premium of 106 basis points (see Kim and Wright (2005)) using the same calibration as RS. The RS model delivers a term-premium of 38 basis points.

[1] Income taxes distort the consumption-leisure trade-off in the model. In particular, the real wage, which governs this trade-off and is the opportunity cost of leisure, is smaller with positive taxes (also called the after-tax real wage).

2 The model

2.1 HOUSEHOLDS

Our model is based on the New Keynesian DSGE model of RS. The description of the households and firms' problems below closely follows RS.

The household maximizes the continuation value of its utility (V), which is of Epstein-Zin form and follows the specification of RS:

$$V_t = \begin{cases} U(C_t, L_t) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\ U(C_t, L_t) - \beta \left[E_t (-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0. \end{cases} \tag{1}$$

The households' problem is subject to its flow budget constraint:

$$B_t + P_t C_t = (1-\tau_t) W_t L_t + D_t + R_{t-1} B_{t-1}. \tag{2}$$

In equation (1), β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure ($1 - L_t$). E_t denotes expectations conditional on information available at time t. As the time frame is normalized to one, leisure time ($1 - L_t$) is what remains after spending some time working (L_t). $W_t L_t$ is labor income, R_t is the return on the one-period nominal bond, B_t, D_t is dividend income, and τ_t is taxes on labor income.

To be consistent with balanced growth, RS imposes the following functional form on U:

$$U(C_t, L_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-L_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0, \tag{3}$$

where Z_t is an aggregate productivity trend, and $\varphi, \chi, \chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1-\bar{L})/\chi\bar{L}$, where $\bar{L}$ is the steady state of hours worked.

It is of interest to report the relationship between the coefficient of relative risk-aversion RRA^c and the Epstein-Zin curvature parameter (α):

$$RRA^c = \frac{\varphi}{1 + \frac{\varphi}{\chi}\frac{(1-L)}{L}\left(\frac{1}{(1-\tau)}\right)} + \alpha \frac{1-\varphi}{\left(1 + (1-\tau)\frac{1-\varphi}{1-\chi}\frac{(1-L)}{L}\right)} \tag{4}$$

Equation 4 shows that the risk aversion measure is affected by the income tax, τ.

2.2 FIRMS

Intermediary firms maximize their profits and face Calvo style price-setting frictions. Accordingly, a $1 - \xi$ fraction of firms can set their price optimally in each period. There is a perfectly competitive sector that purchases a continuum of intermediary goods and turns them into a single final good using a CES aggregator.

Intermediary firm i produces output ($Y_t(i)$) using the following technology:

$$Y_t(i) = A_t [K_t(i)]^{1-\eta} [Z_t L_t(i)]^{\eta}, \tag{5}$$

which substituting for $Y_t(i)$ the demand for product i ($Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} Y_t$) and aggregating across firms, yields:

$$Y_t = \Delta_t^{-1} A_t [K_t]^{1-\eta} [Z_t L_t]^{\eta}, \; 0 < \eta < 1, \tag{6}$$

where $K_t = Z_t \bar{K}$ is the aggregate capital stock ($\bar{K}$ is fixed), η is the share of labor in production, $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} di$ is price dispersion. A_t is a stationary aggregate productivity shock:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

where ε_t^A is an independently and identically distributed (iid) stochastic technology shock drawn from Gaussian Normal distribution with mean zero and variance σ_A^2.
The market clearing equation reads as:

$$Y_t = C_t + \delta K_t + G_t,$$

where δ is the depreciation rate of capital and G_t is the government spending shock which is specified in the fiscal policy section below.

Based on a first-order Taylor series approximation of the firm's optimality condition, one can derive the New Keynesian Phillips curve that establishes a log-linear connection between the inflation rate ($\widehat{\pi}_t$) and the real marginal cost ($\widehat{mc}_t$)[2]:

$$\pi_t = \widetilde{\beta} E_t \widehat{\pi_{t+1}} + \kappa \widehat{mc}_t, \tag{7}$$

where $\pi_t \equiv \log(\Pi_t/\Pi^*)$, and $\widetilde{\beta}$ stands for the discount factor that is corrected by the growth rate (γ) of the productivity trend (Z_t), i.e., $\beta\gamma^{-\varphi}$.

The marginal cost is defined—in log-linear terms—as the difference between the real wage and the marginal product of labor:

$$\begin{aligned} \widehat{mc}_t &\equiv \log(mc_t/\overline{mc}) = \widehat{w}_t - \widehat{mpl}_t \\ &= \widehat{w}_t - (\widehat{a}_t + (\eta - 1)\widehat{l}_t). \end{aligned} \tag{8}$$

In equation (8), $\widehat{a}_t \equiv \log(A_t/\bar{A})$, $\widehat{w}_t = \log(W_t^r/\bar{W}^r)$, $W_t^r \equiv W_t/P_t$ and $\widehat{mpl}_t \equiv \log(MPL_t/\overline{MPL})$ denote the log-deviations of the technology shock, the real wage and the marginal product of labor from the corresponding steady-states values (captured by an upper bar), respectively. The first row contains the definition of the real marginal cost in log-linear form. The second row contains the marginal product of labor based on the Cobb-Douglas functional form. For the real wage the intratemporal condition is substituted in:

$$\widehat{mc}_t = \varphi \widehat{c}_t + \frac{\bar{L}}{(1-\bar{L})} \chi \widehat{l}_t + d\tau_t^i - \widehat{mpl}_t \tag{9}$$

where $\widehat{c}_t \equiv \log(C_t/\bar{C})$, $\widehat{l}_t \equiv \log(L_t/\bar{L})$, $d\tau_t^i \equiv \tau_t^i - \tau^i$ and $\widehat{mpl}_t \equiv \log(MPL_t/\overline{MPL})$. Variables with an upper bar mean steady-state. Equation (34) shows that higher taxes imply higher marginal costs.

Monetary and fiscal policy are described in the section below.

2.3 MONETARY AND FISCAL POLICY

Monetary Policy. The New Keynesian model is closed by a monetary policy rule (a so-called Taylor rule):

$$dR_t = \rho_i dR_{t-1} + (1-\rho_i)[\bar{R} + \log \widetilde{\Pi}_t + g_\pi(\log(\widetilde{\Pi}_t) - log(\Pi^*)) + g_y(Y_t - Y_t^*)/Y_t^*] + \varepsilon_t^i, \tag{10}$$

where dR_t is the deviation of the policy rate from its steady-state, $\widetilde{\Pi}_t$ is a four-quarter moving average of inflation (defined below), and Y_t^* is the trend level of output $\bar{Y}Z_t$ (where $\bar{Y}$ denotes the steady-state level of Y_t/Z_t). Here, $\bar{R}$ is the steady-state gross interest rate, which equals $\log(\bar{\Pi}^*/\widetilde{\beta})$, Π^* is the target rate of inflation, and ε_t^i is an iid shock with mean zero and variance σ_i^2. ρ_i captures the motive for interest rate smoothing.

The four-quarter moving average of inflation ($\widetilde{\Pi}_t$) can be approximated by a geometric moving average of inflation:

$$\log \widetilde{\Pi}_t = \theta_\pi \log \widetilde{\Pi}_{t-1} + (1-\theta_\pi) \log \Pi_t, \tag{11}$$

[2] Here, we use the log-linear version of the Phillips curve for illustration purposes. The model is solved using the Phillips curve in its non-linear form.

where the calibration of $\theta_\pi = 0.7$ in RS ensures that the geometric average in equation (11) has an effective duration of approximately four quarters. Below we also estimate θ_π by GMM.

Fiscal Policy. Government spending follows the process:

$$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G, \quad 0 < \rho_G < 1, \tag{12}$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an iid shock with mean zero and variance σ_G^2.

In one of our fiscal scenarios, the government can issue debt in each time t. The evolution of debt from time $t-1$ to time t is described by the government's budget constraint:

$$b_t + \tau_t w_t L_t = \frac{\gamma^{-1} R_{t-1} b_{t-1}}{\Pi_t} + g_t, \tag{13}$$

where b_t and w_t represent de-trended real government debt and real wages, respectively. All quantities are expressed in real terms, except for the nominal interest rate (R_t). Here, $R_{t-1}b_{t-1}$ denotes interest payments on the previous period's debt.

Our second fiscal scenario is the case of balanced budget (with either positive or zero steady-state government debt). If one imposes a restriction $b_t = b_{t-1} = 0$ for all t, then expression (13) boils down to the balanced budget case ($g_t = \tau_t w_t L_t$ for all t in the absence of steady-state debt $b = 0$). In both fiscal scenarios, the government budget is consolidated through distortionary labor income tax revenue.

To observe the role of steady-state debt, we linearize equation (13) to the first order:

$$\widehat{b}_t + \eta d\tau_t + \bar{\tau}\eta\widehat{w}_t + \bar{\tau}\eta\widehat{l}_t = \gamma^{-1}\gamma_b(dR_{t-1} - \bar{R}\pi_t) + \gamma^{-1}\bar{R}\widehat{b}_{t-1} + \widehat{g}_t, \tag{14}$$

where $\eta \equiv \bar{w}\bar{L}/\bar{y}$, $\widehat{b}_t \equiv (b_t - \bar{b})/\bar{y}$, and $\gamma_b \equiv \bar{b}/\bar{y}$ is the government debt-to-GDP ratio. The rest of the variables are as defined above. Note that the deviations of debt and government spending from their respective steady states are defined relative to the steady-state output. When steady-state debt is zero, i.e., $\gamma_b = 0$, the real interest rate ($dR_{t-1} - \bar{R}\pi_t$) does not have a direct effect on taxes ($d\tau_t$). Positive and increasing γ_b is shown to raise the nominal term premium (see the Results section below).

Our fiscal rule is motivated by the evidence in Romer and Romer (2010), which estimates the effects of exogenous tax changes on output and emphasizes that ignoring the influences of economic activity on tax policy leads to biased estimates of the macroeconomic effects of tax changes. To address these concerns, we allow the tax rate at time t to respond to previous period output, allowing for long delays in legislation and reactions to previous period debt to prevent the build-up of large debt-to-GDP ratios:

$$d\tau_t = \rho_\tau d\tau_{t-1} + \rho_{\tau b}\widehat{b}_{t-1} + \rho_{\tau y}\widehat{y}_{t-1} + \varepsilon_t^\tau. \tag{15}$$

Our specification of the fiscal policy rule captures the four main features suggested by Leeper et al. (2010) and Zubairy (2014). First, the response of taxes to the deviations of output from its steady-state captures some 'automatic stabilizer' component of fiscal policy (see parameter $\rho_{\tau y}$). Second, we allow the income tax rate to respond to the state of government debt (see parameter $\rho_{\tau b}$). Third, government spending and tax rates evolve persistently, which is allowed for in the form of autoregressive terms, ρ_g and ρ_τ, in equations (12) and (15), respectively. Fourth, unexpected movement in the tax rate is reflected by the tax shock ε_t^τ, which has a mean of zero and variance σ_τ^2.

Finally, we note that goods and labor markets clear in equilibrium and that the transversality condition regarding bond holdings is satisfied.

3 Bond pricing

The price of a default-free n-period zero-coupon bond that pays \$1 at maturity can be described with a recursive formula:

$$p_t^{(n)} = E_t\{m_{t+1} p_{t+1}^{(n-1)}\},$$

where $m_{t+1} \equiv m_{t,t+1}$ is the stochastic discount factor, $p_t^{(n)}$ denotes the price of the bond at time t with maturity n, and $p_t^{(0)} \equiv 1$, i.e., the time t price of \$1 delivered at time t is \$1. To calculate the term premium, we need the bond price expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years (in case a bond with 10-year maturity). The risk -neutral bond price can be expressed through the expectations hypothesis of the term structure:

$$\widehat{p_t}^{(n)} = e^{-i_t} E_t \widehat{p}_{t+1}^{(n-1)}, \tag{16}$$

where $\widehat{p_t}^{(0)} \equiv 1$. Equation (16) is also recursive and states that the current period price of the bond is the present discounted value of the next period bond price, where the discount factor is the risk-free rate rather than the stochastic discount factor.

The continuously compounded yield to maturity of the n-period zero-coupon bond is defined as:

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}.$$

The implied term premium is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield expected by the risk-neutral investor ($\widehat{\iota}_t^{(n)}$):

$$TP_t^{(n)} = i_t^{(n)} - \widehat{\iota}_t^{(n)}.$$

We also report two imperfect but frequently used measures of the risk of nominal bonds. The first one is the slope of the term structure, which is defined as the difference between the yield with maturity n and the short yield (3-month yield). The second alternative riskiness indicator is the excess holding period return, which can be written as:

$$x_t^{(n)} = \frac{p_t^{(n-1)}}{p_{t-1}^{(n)}} - i_{t-1}. \tag{17}$$

In equation (17), $p_t^{(n-1)}$ is the period t price of a bond that matures in $n-1$ quarters, i_{t-1} is the 3-month yield in period $t-1$, and $p_{t-1}^{(n)}$ is the period $t-1$ price of a bond that matures in n quarters.

4 Data and GMM Estimation

To discipline the choice of model parameters, we estimate our models (with either constant of time-varying debt) with the GMM toolbox of Andreasen et al. (2018) using the following quarterly US time series for 1961-2007 (the sample period follows RS to facilitate comparison): i) the per capita consumption growth dC_t (d denotes the temporal difference operator), ii) the one-quarter nominal interest rate i_t, iii) the per capita hours growth dL_t, iv) the growth rate of real wage $d(W_t/P_t)$, v) inflation Π_t, vi) the slope of the term structure proxied by the difference between the 10-year nominal interest rate $i_t^{(40)}$ and the one-quarter nominal interest rate i_t, vii) the 10-year nominal term premium from Adrian et al. (2013), and viii) the growth rate of labor tax revenue divided by GDP ($d(\tau_t W_t L_t/Y_t)$). The Appendix provides more information about the data used in the estimation.

Similarly to Andreasen et al. (2018) and Bretscher et al. (2016), we consider three types of unconditional moments for the GMM estimation: i) sample means $m_1(y_t) = y_t$, contemporaneous covariances $m_2(y_t) = vech(y_t y_t')$, and own autocovariances $m_3(y_t) = \{y_{i,t} y_{i,t-k}\}_{i=1}^{n_y}$ for $k = 1$ and $k = 5$. The total set of moments used in the estimation are, therefore, given by $m(y_t) = [m_1(y_t)\ m_2(y_t)\ m_3(y_t)]'$.

Letting θ denote the structural parameters, the GMM estimator is given by:

$$\arg\min_{\theta\in\Theta} \left(\frac{1}{T}\sum_{t=1}^{T} q_t - E(q_t(\theta))\right)' W \left(\frac{1}{T}\sum_{t=1}^{T} q_t - E(q_t(\theta))\right). \tag{18}$$

In equation (18), W is a positive definite weighting matrix, $\frac{1}{T}\sum_{t=1}^{T} q_t$ are data moments and $E(q_t(\theta))$ are moments computed from the model. We follow a conventional two-step procedure to implement GMM. In the first step, we set $W_T = diag(\widehat{S}^{-1})$ to obtain $\widehat{\theta}^{(1)}$, where $\hat{S}$ denotes the long-run variance-covariance matrix of $\frac{1}{T}\sum_{t=1}^{T} q_t$ when centered around its sample mean. In the second (final) step, we obtain $\widehat{\theta}^{(2)}$ using the optimal weighting matrix $W_T = \widehat{S}^{-1}_{\widehat{\theta}^{(1)}}$, where $\widehat{S}^{-1}_{\widehat{\theta}^{(1)}}$ denotes the long-run variance of our moments re-centered around $E\left(q_t(\widehat{\theta}^{(1)})\right)$. The long-run variances in both steps are produced with the Newey-West estimator using five lags, and our results are robust to the inclusion of, e.g., ten lags.

We estimate three models and present the results in Table 1. The first two columns contain the models with time-varying and constant debt (both with distortionary labor taxes), respectively. The third column entails the RS model with balanced budget and lump-sum taxes. For the time-varying debt model 20 parameters and one steady-state quantity (hours worked) are estimated using 49 moments[3] by GMM. In the case of the constant debt model with distortionary taxes and the RS model (constant debt and lump-sum taxes) 16 parameters and two steady-state quantities are estimated using 39 moments[4] by GMM. The rest of parameters and steady-state quantities are not estimated but calibrated as follows: $\gamma_b = 2.4$ is consistent with a yearly debt-to-GDP ratio of 60 percent. The steady-state inflation rate is zero ($\Pi^* = 1$). The steady-state capital-to-GDP ratio is calibrated to ten as in RS and capital stock has a depreciation rate of ten per cent per annum. The government spending-to-GDP ratio is calibrated 27 per cent.

The estimated coefficients in the tax rule and the government purchases are close to those of Leeper et al. (2010) and Zubairy (2014).[5] Importantly, both models estimate lower relative risk-aversion coefficients (see the implied CRRA of 45 and 31 for the time-varying and constant debt models respectively) than earlier papers (see RS for a value of 110 and Andreasen (2012) for a value of 168). Regarding our other estimates of the parameters, they are largely in line with those in Andreasen et al. (2018) and Bretscher et al. (2016). Similarly to the findings of Andreasen et al. (2018) and Bretscher et al. (2016), the curvature parameter of recursive preferences is not precisely estimated.

[3] For seven observables the 49 moments are composed of the seven means, standard deviations, first- and fifth-order autocorrelations plus 21 covariances in the symmetric variance-covariance matrix.

[4] For six observables the 39 moments are the six means, standard deviations, first- and fifth-order autocorrelations plus 15 covariances in the symmetric variance-covariance matrix.

[5] The only exception is the coefficient on the lagged tax variable which is estimated to be magnitudes lower than in Zubairy (2014).

Table 1
GMM estimates of the models parameters and steady-states

Parameters and steady-states	Time-varying debt	Constant debt	RS model Zero debt
Household			
$\tilde{\beta}$	0.9903 (0.0024)	0.9851 (0.0005)	0.9911 (0.0043)
φ	1.9923 (0.48)	2.0083 (0.1)	1.9841 (0.2)
χ	2.8499 (0.99)	2.8493 (0.13)	2.9649 (0.99)
α	−61.2441 (31.53)	−39.1276 (33.34)	−149.3743 (33.62)
$\bar{L}$	0.3667 (0.0048)	0.3666 (0.00062)	0.3391 (0.00032)
CRRA (implied)	43.14	30.63	75.18
Firm			
η	0.6513 (0.0013)	0.6004 (0.00162)	0.7031 (0.00038)
ξ	0.7505 (0.0027)	0.7915 (0.00024)	0.7807 (0.00031)
ε	4.07 (0.065)	4.12 (0.021)	4.98 (0.0172)
Monetary Policy			
ρ_i	0.5502 (0.36)	0.5314 (0.0015)	0.6134 (1.06)
g_π	0.5021 (2.13)	0.5132 (0.0014)	0.5257 (19.66)
g_y	0.9299 (3.5)	0.9224 (0.0009)	0.9315 (33.80)
θ_π	0.22 (0.13)	0.28 (0.32)	0.32 (0.03)
Fiscal Policy			
ρ_τ	0.02 (0.0055)	– (–)	– (–)
$\rho_{\tau b}$	0.011 (0.0021)	– (–)	– (–)
$\rho_{\tau y}$	0.009 (0.03)	– (–)	– (–)
Shock processes			
ρ_a	0.9758 (0.0024)	0.9519 (0.00121)	0.9516 (0.00137)
ρ_g	0.8101 (1.5)	0.9629 (0.11)	0.9845 (0.16)
σ_a	0.0058 (0.0076)	0.0053 (0.00083)	0.0054 (0.00065)
σ_g	0.011 (0.0031)	0.0094 (0.0029)	0.0093 (0.0038)
σ_τ	0.0033 (0.0064)	– (–)	– (–)
σ_i	0.0231 (0.0202)	0.0001 (0.34)	0.0003 (0.37)

The numbers below the parameter estimates denote the standard deviation of the estimate as a percent. — indicates parameters that do not appear in the constant debt model or in the RS model. The implied CRRA parameter can be calculated from equation 4.

The estimates of the technology shock in the case of the time-varying debt model are in accordance with those of King and Rebelo (1999), as well as Basu, Fernald and Kimball (2006), while the estimates in the case of the constant debt model closer to the GMM estimates of Andreasen (2012). The estimates of the AR(1) term and the size of the shock for the government spending process are reasonably close to the single equation estimates reported in the online Appendix.

The estimates of the parameters in the Taylor rules, as well as the monetary policy shock, are in line with those of Rudebusch (2002) and Andreasen (2012) in the case of the time-varying debt model while they are somewhat lower for the constant debt model. The steady state hours work and government spending-to-GDP ratios are estimated to be somewhat higher than the ones in RS.

It is important to note that the time-varying debt model is successful in matching the NTP with lower CRRA not only because of the introduction of a richer fiscal setup but also because the GMM estimates the technology and government spending shocks with higher autocorrelation and shock size parameters. In particular, we find that the reduction in the risk-aversion parameter is made possible by the introduction of a more detailed fiscal structure for 38 percent of the total effect in case of the time-varying debt model. The higher estimated AR(1) parameter in the technology shock process is responsible for 62 percent of the total decrease[6] in the case of the time-varying debt model.

In the case of the constant debt model, however, seventy percent of the reduction in the CRRA is made possible by the lower estimate of the inflation persistence ($\theta_{\bar{\pi}}$) as well as the interest-rate smoothing (ρ_i) parameters. The lower estimate of interest rate smoothing imply that the central bank reacts to shocks by greater changes in the nominal interest rate (in a given quarter) meaning that monetary policy leans more against changes in inflation and output gap and, thus, equilibrium interest rates, inflation and output gap will be more volatile implying a rise in risk-premiums. Lower inflation smoothing parameter also implies larger reaction of inflation as well as the nominal interest rate and, therefore, its workings are similar to the effects of lower interest rate smoothing. The last column indicates that the RS model is estimated with a risk-aversion coefficient at least two times higher in magnitude than those obtained by the model versions with richer fiscal sector.

Table 2 presents selected macro and finance moments calculated from US data 1961-2007[7] and three model variants (the model with time-varying debt, constant debt, and the RS model which assumes zero steady-state government debt and lump-sum taxes). The reported unconditional moments are based on simulated time series of 10 000 periods utilising a third-order approximation of the model (pruning was also used to avoid explosive paths). Beyond macro and finance variables, model fit is assessed on the basis of fiscal moments such as the unconditional correlation of the labor tax revenue and the debt-to-output ratio with output, as well as the standard deviation, first-order autocorrelation of labor tax revenue and the debt-to-output ratio. In general, we find that the richer model structure with time-varying debt better matches the data. In particular, constant debt approximates the empirical correlation between tax revenue and GDP but fails to capture the standard deviation and the autocorrelation of the tax revenue. A shortcoming of the time-varying debt and constant debt models is that they generate high inflation uncertainty and, thus, inflation and the short-term policy rate display more variability than what we see in the data.

In the last two rows of Table 2 we measure of the overall fit of the models. In particular, we follow Rudebusch and Swanson (2012) and calculate the sum of squared differences between the model moments and the data as:

$$\nu^2 = \sum_{i=1}^{n} \omega_i (x_i^M - x_i^D)^2 \tag{19}$$

where $n = 15$ stands for the number of moments considered, x^M is the first or second moment from the model and x^D is the data based counterpart. The $\omega_i = 1/n$ is the weighting parameter and we assume it to be common for all moments.

We calculate two measures of fit: i) the average distance to the data moments (how well our estimated models perform with respect to the data), ii) average distance to the estimated RS model (how well our estimated models perform compared to the estimated RS model).

We calculate the measure of fit only for the moments which have counterpart in the estimated RS model to ensure fair comparison (meaning that the comparison exercise is done for the macroeconomic variables without the fiscal ones which are not

[6] This result is not reported in table 2.

[7] We focus on data before the great recession to avoid complications posed by the fact that the US policy rate reached its zero lower bound at the end of 2008.

Table 2
Unconditional moments from the simulated models

Unconditional Moments	US data, 1961-2007	Time-varying debt	Constant debt	RS model
SD(dC)	2.78	2.38	3.60	1.86
SD(L)	0.80	0.50	0.62	0.67
SD(dW^r)	0.97	3.30	2.53	1.26
SD(π)	2.52	4.05	3.96	1.89
SD(R)	2.71	4.23	4.66	1.73
SD(R^{real})	2.30	0.75	1.16	1.14
SD(R^{40})	2.41	3.44	2.96	1.03
Mean($NTP^{(40)}$)	1.06	1.14	1.05	0.73
SD($NTP^{(40)}$)	0.54	0.33	0.17	0.07
Mean($R^{(40)} - R$)	1.43	1.00	0.73	0.37
SD($R^{(40)} - R$)	1.33	0.98	1.97	0.86
Mean($X^{(40)}$)	1.76	1.08	0.87	0.38
SD($X^{(40)}$)	23.43	11.73	13.17	5.24
Mean($IRP^{(40)}$)	0.80	1.01	0.95	0.44
Corr(dC, π)	-0.34	-0.13	-0.11	-0.40
SD($d(\tau WN)$)	2.84	1.25	1.61	–
SD((τWN))	0.99	3.22	5.79	–
Corr($d(\tau WN, dY)$)	-0.07	-0.14	0.53	–
Corr($d(\tau WN, dC)$)	-0.11	-0.16	0.37	–
AutoCorr($d\tau WN$)	-0.17	-0.44	0.03	–
SD(D/Y)	2.13	16.56	–	–
SD(d(D/Y))	0.69	0.30	–	–
Corr(d(D/Y),dY)	-0.46	-0.63	–	–
Corr(d(D/Y),dC)	-0.16	-0.57	–	–
AutoCorr(D/Y)	0.99	1.00	–	–
Fit to Data	0.00	12.32	10.85	18.45
Fit to RS	0.00	8.06	9.28	–

Mean, SD, Corr and Autocorr denote the unconditional mean, standard deviation, correlation and first-order autocorrelations, respectively. $NTP^{(40)}$=nominal term premium on a 40-quarter bond, $R^{(40)} - R$ is the slope, $IRP^{(40)}$ is the inflation risk premium, and $X^{(40)}$ is the excess holding period return for a 10-year bond. The model parameters are based on the GMM estimates in table 1. All variables are expressed in per cent except for consumption growth, inflation and interest rates of various maturities which are expressed in annualised percentage. — denotes statistics that are not available for the constant debt model.

present in the RS model and also for the finance variables). In this manner, the measure of fit indicator does not capture that our estimated fiscal models are matching fiscal moments too.

A lower measure of fit indicator means that our estimated fiscal models fits the data better than the estimated RS model (i.e. smaller squared differences between our model and the RS model).

The moments reported from the estimated version of the RS model (the last column of Table 2) are reasonably close to the numbers presented originally in RS It needs to be noted that the estimated version of the RS model produces lower nominal term premium (with an estimated risk-aversion of about 75) than its fiscal extensions which are estimated with risk-aversion in the range of 30 to 43.[8] Still, it needs to be stressed that our estimated RS model performs better in terms of matching the mean of the nominal term premium than the calibrated RS model since our interest rate and inflation smoothing coefficient estimates are lower and, thus, inflation risks are higher. Section 7.1 of the Appendix contains all data and model moments used for the GMM estimations so that the overall performance of the models can be fully assessed.

[8] Note that the nominal term premium is higher in our paper than in the baseline calibrated version of RS because our GMM procedure estimates slightly higher standard deviation for the innovation of the technology shock but similar levels of risk-aversion.

5 The distortionary income tax channel

In this section, we explain how the richer fiscal structure (distortionary labor income taxation with either constant or time-varying debt) helps generate a higher mean nominal term premium. To better understand the distortionary tax channel, we first point to the fact that fiscal policy has workings similar to those of temporary technology shocks.

In the RS model, the main source of nominal and real risks are temporary and persistent technology shocks, which facilitate a negative correlation between consumption and inflation. In bad times (low realizations of technology), consumption is low and inflation is high; thus, real returns on bonds are also low. In other words, nominal bonds provide a poor hedge against technology shocks. However, we show below that government spending shocks also give rise to inflation risks with income taxation under either constant or time-varying government debt.

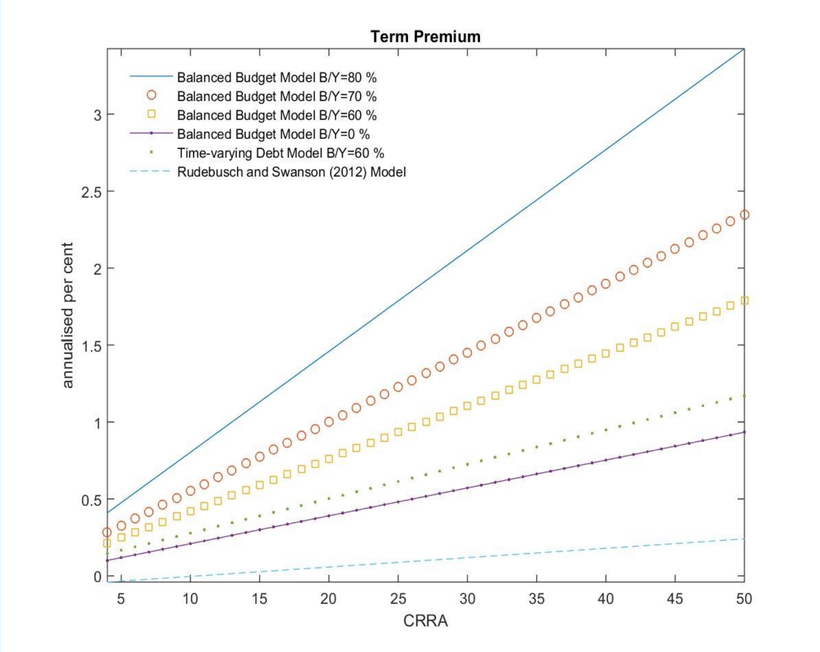
To elaborate on the distortionary income tax channel, let us study what happens after a positive innovation in government spending that needs to be financed by income taxes either on a balanced-budget basis or allowing for a budget deficit (time-varying debt). Higher taxes on income imply fewer hours worked and lower output because households substitute away from labor to leisure. Additionally, higher income taxes imply higher real marginal costs and higher inflation through the New Keynesian Phillips curve (see equations (7) and (8)). Hence, government spending shocks with either constant or time-varying debt and income taxation reproduce the negative pattern between consumption and inflation that underlies the riskiness of nominal bonds.

The effect of the distortionary income tax channel is magnified by positive steady-state debt (see $\gamma_b > 0$ in equation (14)), establishing a direct connection between taxes and real interest rates, which rise after a surge in government purchases, based on the logic of the Taylor rule to curb inflation expectations. In particular, the larger the debt-to-GDP ratio, the higher the cost of servicing debt, the larger the increases in taxes and inflation and the lower the output. Hence, the negative covariance between inflation and output is magnified by the fact that positive steady-state debt increases inflation risks.[9]

Figure 1 illustrates the positive connection between the CRRA (horizontal axis) and the mean of the NTP for various debt-to-yearly-output ratios using the estimated models. The straight line reproduces the results of RS, where lump-sum taxes cover spending. The remaining cases depicted in the figure are based on our setup with income taxation. A debt-to-yearly-output ratio of approximately 70 percent (which was not uncommon in countries such as the US or UK over the last decade) matches the NTP of 106 basis points for the US (see RS) and 92 basis points for the UK (see Andreasen (2012)) with a risk aversion coefficient of thirty.

[9] Linnemann (2005) also argues that the distortionary tax channel and steady-state government debt creates substantial inflation risks, and the Taylor rule may not be sufficient to avoid multiple equilibria. Our results are also in line with those of Dai and Philippon (2006), who used an arbitrage-free affine term-structure model to trace the effects of fiscal shocks on the prices of bonds.

Figure 1
The link between the coefficient of relative risk-aversion (CRRA) and the nominal term premium using the estimated models.



6 Concluding Remarks

When government debt—either constant or time-varying—is retired by income taxes, inflation risks are substantial and increasing with the long-run debt-to-output ratio. Our fiscal extension of the New Keynesian model with Epstein-Zin preferences helps to reduce the high risk-aversion coefficient used in the literature to match the high mean nominal term premium. If the debt-to-GDP ratio is sufficiently high, our macro-finance model largely matches the empirical value of the NTP on long-term bonds.

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7 Appendix

7.1 ALL MOMENTS

7.1.1 MEANS

In the vectors and matrices below $\Delta C, \Delta N, Slope, \pi, NTP, \Delta W$ and $\Delta \frac{\tau^W WN}{Y}$ denote consumption growth, hours growth, slope of the term structure, inflation, nominal term premium, real wage growth and growth rate of the labour tax revenue per GDP, respectively.

Means D, Means M tv, Mean M bb and Means M RS denote the means from the data, the model with time-varying debt (M tv), the model with distortionary taxation and balanced budget (M bb), and the RS model (M RS) respectively. 'na' means that statistic is not available from and, thus, reported in the RS model.

Table 3
Means

	Means D	Means M tv	Means M bb	Means M RS
ΔC	2.41	-0.0027	0.0023	-0.009
ΔN	0.0183	0.0003	0.0007	0.0005
$Slope$	1.43	1.11	0.78	1.03
π	3.70	-0.45	0.18	0.32
NTP	1.06	1.05	1.04	0.64
ΔW	0.48	0.478	0.72	0.7902
$\Delta \frac{\tau^W WN}{Y}$	0.0008	0.0034	0.0097	na

One can see that the models have a poor match in terms of matching the means of the listed macro variables. However, the time-varying debt and the balanced budget models do improve over the RS model regarding the mean of inflation and the nominal term premium.

7.1.2 CORRELATIONS AND STANDARD DEVIATIONS FROM THE DATA AND THE MODELS

Note that the main diagonal of the matrix contains the standard deviations and the off-diagonal elements are the correlations. The standard deviations are in per cent (annualised per cent for inflation, interest rates, slope, nominal term premium and excess holding period return). Results are displayed in Tables (4), (5), (6) and (7).

7.1.3 FIRST- AND FIFTH-ORDER AUTOCORRELATION IN THE DATA AND THE MODELS

Ac1D, Ac5D, Ac1M and Ac5M denote the first (Ac1) and fifth (Ac5) order autocorrelations of the data (D) and the models (M). The statistics are listed for the time-varying debt (M tv), the model with distortionary taxation and balanced budget (M bb), and the RS model (M RS), respectively. 'na' means that statistic is not available from and, thus, reported in the RS model. Results are displayed in Tables (8) and (9).

Table 4
Correlations and Standard Deviations from the Data

	ΔC	ΔN	*Slope*	π	*NTP*	ΔW	$\Delta\frac{\tau^W WN}{Y}$
ΔC	2.69	0.1037	0.2328	-0.3422	-0.0769	0.1149	0.5369
ΔN		1.14	0.1186	-0.0424	0.0377	0.0038	0.2546
Slope			1.33	-0.3965	0.5406	-0.0502	0.0468
π				2.52	0.2119	-0.0414	-0.2827
NTP					0.54	-0.0224	-0.1847
ΔW						0.82	0.1238
$\Delta\frac{\tau^W WN}{Y}$							3.06

Table 5
Correlation Matrix from the Estimated Time-varying Debt Model

	ΔC	ΔN	*Slope*	π	*NTP*	ΔW	$\Delta\frac{\tau^W WN}{Y}$
ΔC	2.3822	-0.9172	-0.3674	-0.1255	-0.1047	0.0714	-0.1963
ΔN		0.5014	0.3766	0.0904	0.0643	-0.0127	0.3121
Slope			0.9813	-0.7607	-0.7578	0.7755	0.0565
π				4.0576	0.9886	-0.9469	0.0378
NTP					0.3382	-0.9485	0.0216
ΔW						3.3071	0.0776
$\Delta\frac{\tau^W WN}{Y}$							1.2564

Table 6
Correlation Matrix from the Estimated Balanced Budget Model

	ΔC	ΔN	*Slope*	π	*NTP*	ΔW
ΔC	3.6047	-0.1984	-0.4025	-0.0778	-0.0835	0.4005
ΔN		0.6203	0.1440	-0.0085	-0.0820	-0.1861
Slope			0.7323	-0.7407	-0.5064	-0.5191
π				3.9668	0.7272	0.2017
NTP					0.1714	0.6449
ΔW						2.5330

Table 7
Correlation Matrix from the Estimated Rudebusch-Swanson Model

	ΔC	ΔN	*Slope*	π	*NTP*	ΔW
ΔC	1.1777	-0.6095	-0.2530	-0.4320	0.0332	-0.4939
ΔN		0.7126	0.2750	0.2438	-0.0138	0.4814
Slope			0.9798	-0.5192	0.0825	0.9244
π				2.0711	-0.1153	-0.2661
NTP					0.10	0.0602
ΔW						1.4425

Table 8
First-order Autocorrelations from the Estimated Models

	Ac1D	Ac1M tv	Ac1M bb	Ac1M RS
ΔC	0.9853	0.0970	0.0875	0.3035
ΔN	0.9459	-0.0625	0.0182	-0.0914
Slope	0.9483	0.8867	0.9456	0.9109
π	0.9424	0.9718	0.9753	0.8878
NTP	0.9920	0.9847	0.9675	0.9582
ΔW	0.8251	0.9643	0.3445	0.8490
$\Delta \frac{\tau^W WN}{Y}$	0.8463	-0.4451	-0.0460	na

Table 9
Fifth-order Autocorrelations from the Estimated Models

	Ac5D	Ac5M tv	Ac5M bb	Ac5M RS
ΔC	0.7612	-0.0031	-0.0093	-0.0034
ΔN	0.5548	-0.0020	-0.0315	-0.0357
Slope	0.3080	0.7012	0.7602	0.6428
π	0.4232	0.9101	0.8467	0.6257
NTP	0.9645	0.9203	0.8352	0.7874
ΔW	-0.1924	0.8692	0.3473	0.5141
$\Delta \frac{\tau^W WN}{Y}$	-0.1947	-0.031	-0.0468	na

7.2 CONSTRUCTION OF TIME SERIES

To construct the following time series, we follow the procedures in Christoffel et al. (2013) and Leeper et al. (2010):

PY: Gross Domestic Product. Bureau of Economic Analysis (BEA). Nipa Table 1.1.5, line 1.

P: GDP deflator personal consumption expenditures. Source: BEA, Nipa Table 1.1.4, line 2.

C: Private Consumption. Source: BEA, Nipa Table 1.1.6, line 2.

L: hours, measure of the labour input. This is computed as $L = H \times (1 - U/100)$, where H and U are the average over monthly series of hours and unemployment, respectively. Source: BLS, series LNU02033120 for hours and LNS14000000 for unemployment.

INT: Net Interest Payments of Federal Government Debt. Source: BEA, Nipa Table 3.2 (line 29-line13).

G: Government consumption is computed as current consumption expenditures (line 21)+gross government investment (line 42)+net purchases of non-produced assets (line 44)-consumption of fixed capital (line 45). Source: BEA, Nipa Table 3.2

W: Wage and Salary Disbursement. BEA. Series ID A576RC1.

WL: labour income tax base. Source: Nipa Table 1.12 (line 3).

τ: average effective labour income tax rate as in Jones (2002) and Leeper et al. (2010). We follow the procedure in the appendix of Leeper et al. (2010) to construct τ_t.

B/Y: government-debt-to-GDP ratio. St. Louis Fed Database.

7.3 DERIVATION OF THE NEW KEYNESIAN PHILLIPS CURVE, THE AGGREGATE PRODUCTION FUNCTION AND THE PRICE DISPERSION

Here we follow the steps from chapter two of the PhD dissertation by Kaszab (2014). Intermediary firms which maximise their profits face price-setting frictions of Calvo style. With Calvo frictions a $1 - \xi$ fraction of firms can set its price optimally in each period. Intermediary firm i chooses a state-contingent plan for prices that maximises its current and future profits:

$$E_t \left\{ \sum_{T=0}^{\infty} (\xi \beta)^T \mathcal{K}_{t,t+T} \left[P_t(i) Y_{t+T}(i) - W_{t+T} L_{t+T}(i) \right] \right\} \tag{20}$$

where $\mathcal{K}_{t,t+T}$ is the representative household's (nominal) stochastic discount factor (or pricing kernel defined below in equation (27)). The first term in the squared bracket in equation (20) is the revenue of the firm while the latter one is the cost of labour.

There is a perfectly competitive sector that purchases the continuum of intermediary goods and turns them into a single final good using a CES aggregator:

$$Y_t = \left[\int_0^1 [Y_t(i)]^{\frac{1}{1+\theta}} di \right]^{1+\theta}.$$

Each intermediary firm i faces a downward-sloping demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t \tag{21}$$

where the economy-wide price index P_t is a CES aggregator of the individual prices:

$$P_t \equiv \left[\int_0^1 [P_t(i)]^{-\frac{1}{\theta}} di \right]^{-\theta}.$$

After taking the derivative of the profit equation (20) with respect to the optimal price $P_t(i)$ we obtain the standard optimality condition in case of sticky prices:

$$P_t(i) = \frac{(1+\theta)\sum_{T=0}^{\infty}(\xi\beta)^T \mathcal{K}_{t,t+T} MC_{t,t+T}(i) Y_{t,t+T}(i)}{\sum_{T=0}^{\infty}(\xi\beta)^T \mathcal{K}_{t,t+T} Y_{t,t+T}(i)} \tag{22}$$

where $1+\theta$ is the gross markup and the (nominal) marginal cost of firm i can be written as:

$$MC_t(i) = \frac{W_t L_t(i)}{\eta Y_t(i)} \tag{23}$$

Also the optimality condition in equation (22) can alternatively be expressed in recursive (and aggregate) form as the equations in the next proposition show.

Lemma 1. *It can be shown that the optimality condition in equation (22) can be rewritten as*

$$\begin{aligned} \widetilde{P}_t^{1+\frac{(1+\theta)(1-\eta)}{\theta\eta}} &\equiv \left(\frac{P_t(i)}{P_t}\right)^{1+\frac{(1+\theta)(1-\eta)}{\theta\eta}} \\ &= \frac{(1+\theta)\sum_{T=0}^{\infty}(\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} MC^{\text{real}}_{t,t+T} \pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}} Y_{t+T}}{\sum_{T=0}^{\infty}(\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \pi_{t,t+T}^{\frac{1}{\theta}} Y_{t+T}} \end{aligned} \tag{24}$$

where the (real) stochastic discount factor is

$$\mathcal{K}^{\text{real}}_{t,t+T} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha}$$

and the (real) marginal cost is

$$MC^{\text{real}}_{t,t+T} \equiv \frac{W_{t+T}/P_{t+T}}{\eta}\left(\frac{Y_{t+T}}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_{t+T}^{-\frac{1}{\eta}}.$$

Proof. One can decompose the nominal marginal cost of an individual firm in the following way:

$$\begin{aligned} MC_t(i) &= \frac{W_t/P_t}{\eta} P_t \left(\frac{Y_t(i)}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_t^{-\frac{1}{\eta}} \\ &= \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \frac{W_t/P_t}{\eta} P_t \left(\frac{Y_t}{\bar{K}}\right)^{\frac{1-\eta}{\eta}} A_t^{-\frac{1}{\eta}} \\ &= \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} P_t MC_t^{\text{real}} \end{aligned} \tag{25}$$

where the second line made use of the demand function in equation (21).
The previous equation can be substituted for $MC_t(i)$ in equation (22) to obtain:

$$\begin{aligned} & E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T} P_t(i) Y_{t+T}(i) \\ & = (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}_{t,t+T} \left(\frac{P_t(i)}{P_t}\frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times \\ & MC^{\text{real}}_{t,t+T} P_{t+T} Y_{t+T}(i) \end{aligned} \tag{26}$$

where

$$\mathcal{K}_t \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha} \frac{1}{\pi_{t,t+T}} = \mathcal{K}^{\text{real}}_{t,t+T} \frac{1}{\pi_{t,t+T}}. \tag{27}$$

Then using the demand for an individual product (equation (21)) we can express equation (26) as:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \frac{1}{\pi_{t,t+T}} P_t(i) \left(\frac{P_t(i)}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}$$

$$= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \frac{1}{\pi_{t,t+T}} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times$$

$$MC^{\text{real}}_{t,t+T} P_{t+T} \left(\frac{P_t(i)}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.$$

Next we multiply both sides of the previous equation by $P_t(i)^{\frac{1+\theta}{\theta}}$ and $P_t^{-\frac{1+\theta}{\theta}}$ and derive:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \frac{1}{\pi_{t,t+T}} P_t(i) \left(\frac{P_t}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}$$

$$= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times$$

$$MC^{\text{real}}_{t,t+T} [\pi^{-1}_{t,t+T} P_{t+T}] \left(\frac{P_t}{P_{t,t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.$$

We can make use of the identity

$$[\pi^{-1}_{t,t+T} P_{t+T}] = [P_t]$$

to rewrite the previous expression as:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \pi^{-1}_{t,t+T} P_t(i) \left(\pi_{t,t+T}\right)^{\frac{1+\theta}{\theta}} Y_{t+T}$$

$$= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \left(\frac{P_t(i)}{P_t} \frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \times$$

$$MC^{\text{real}}_{t,t+T} [P_t] \left(\frac{P_t}{P_{t+T}}\right)^{-\frac{1+\theta}{\theta}} Y_{t+T}.$$

Regarding $\frac{P_t}{P_{t+T}}$ terms in the previous equation the following algebraic manipulation can be carried out on the RHS:

$$\left(\frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}}\right)^{-\frac{1+\theta}{\theta}}$$

$$= \left(\frac{P_t}{P_{t+T}}\right)^{\frac{(1+\theta)(\eta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)}{\theta\eta}} \left(\frac{P_t}{P_{t+T}}\right)^{-\frac{1+\theta}{\theta}}$$

$$= \left(\frac{P_t}{P_{t+T}}\right)^{-\frac{(1+\theta)}{\theta\eta}} = \pi_{t,t+T}^{\frac{(1+\theta)}{\theta\eta}}$$

and the inflation term on the LHS can be written as:

$$\pi^{-1}_{t,t+T} \left(\pi_{t,t+T}\right)^{\frac{1+\theta}{\theta}} = \pi_{t,t+T}^{-\frac{\theta}{\theta}} \left(\pi_{t,t+T}\right)^{\frac{1+\theta}{\theta}} = \pi_{t,t+T}^{\frac{1}{\theta}}.$$

Using the previous result the optimality condition can be transformed as:

$$E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \pi_{t,t+T}^{\frac{1}{\theta}} Y_{t+T} P_t(i)$$

$$= (1+\theta) E_t \sum_{T=0}^{\infty} (\xi\beta)^T \mathcal{K}^{\text{real}}_{t,t+T} \left(\frac{P_t(i)}{P_t}\right)^{-\frac{(1+\theta)(1-\eta)}{\theta\eta}} MC^{\text{real}}_{t,t+T} [P_t] \pi_{t,t+T}^{\frac{(1+\theta)}{\theta\eta}} Y_{t+T}$$

which is the same as the expression in the Lemma. □

The preceding lemma has shown that the optimality condition of the firm in equation (22) can be rewritten as equation (24).

Proposition 2. *Here we demonstrate that equation (24) can be expressed recursively as follows (so that we can input them into Dynare):*

$$\widetilde{P}_t^{1+\frac{(1+\theta)(1+\eta)}{\theta\eta}} = \frac{Zn_t}{Zd_t}$$

where

$$Zn_t = (1+\theta)MC_t^{\mathrm{real}}Y_t + \xi\beta\mathcal{K}_t^{\mathrm{real}}\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}Zn_{t+1} \tag{28}$$

and

$$Zd_t = Y_t + \xi\beta\mathcal{K}_t^{\mathrm{real}}\pi_{t+1}^{\frac{1}{\theta}}Zd_{t+1}. \tag{29}$$

Proof. The nominator of the fraction in equation (24) can be tagged as *Zn*:

$$Zn_t \equiv (1+\theta)E_t\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t,t+T}^{\mathrm{real}}MC_{t,t+T}^{\mathrm{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T}$$

which can also be written as:

$$Zn_t = (1+\theta)MC_t^{\mathrm{real}}Y_t + (1+\theta)E_t\sum_{T=1}^{\infty}(\xi\beta)^T\mathcal{K}_{t+T}MC_{t+T}^{\mathrm{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T}. \tag{30}$$

After iterating the definition of *Zn* one period ahead we obtain:

$$\begin{aligned} Zn_{t+1} &= (1+\theta)E_{t+1}\sum_{T=0}^{\infty}(\xi\beta)^T\mathcal{K}_{t+1+T}^{\mathrm{real}}MC_{t+1+T}^{\mathrm{real}}\pi_{t+1,t+1+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T+1} \\ &= (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^{T-1}\mathcal{K}_{t+T}^{\mathrm{real}}MC_{t+T}^{\mathrm{real}}\pi_{t+1,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T} \\ &= (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^{T-1}\mathcal{K}_{t+T}^{\mathrm{real}}MC_{t+T}^{\mathrm{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}\pi_{t+1}^{-\frac{1+\theta}{\theta\eta}}Y_{t+T} \end{aligned} \tag{31}$$

where the last line made use of

$$\pi_{t+1,t+T} \equiv \frac{P_{t+T}}{P_t}\frac{P_t}{P_{t+1}} = \pi_{t,t+T}\pi_{t+1}^{-1}.$$

Finally let us multiply equation (31) by $\xi\beta\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}$:

$$\xi\beta\pi_{t+1}^{\frac{1+\theta}{\theta\eta}}Zn_{t+1} = (1+\theta)E_{t+1}\sum_{T=1}^{\infty}(\xi\beta)^T\mathcal{K}_{t+T}^{\mathrm{real}}MC_{t+T}^{\mathrm{real}}\pi_{t,t+T}^{\frac{1+\theta}{\theta\eta}}Y_{t+T} \tag{32}$$

and recognise that the resulting expression is the the second term on the RHS of equation (30).
Hence the combination of equations (30) and (32) give way to equation (28) in the proposition. Similar derivation can be used to obtain equation (29). □

The average real marginal cost is defined as follows:

$$MC_t^{\mathrm{real}} = \frac{W_t/P_t}{MPL_t} = \frac{\frac{C_t^{\varphi}(1-L_t)^{-\chi}}{1-\tau_t^i}}{MPL_t}. \tag{33}$$

where MPL_t denotes the marginal product of labour and can be obtained from equation (25). For the real wage the intratemporal condition is substituted in. Equation (33) can be loglinearised as in the main text of the paper:

$$\widehat{mc}_t = \varphi\hat{c}_t + \frac{\bar{L}}{(1-\bar{L})}\chi\hat{l}_t + d\tau_t^i - \widehat{mpl}_t \tag{34}$$

where $\widehat{c}_t \equiv \log(C_t/\bar{C})$, $\widehat{l}_t \equiv \log(L_t/\bar{L})$, $d\tau_t^i \equiv \tau_t^i - \tau^i$ and $\widehat{mpl}_t \equiv \log(MPL_t/\overline{MPL})$. Variables with an upper bar mean steady-state.

Aggregation

Recall that the production function is given by,

$$Y_t(j) = A_t \bar{K}^{1-\eta} N_t^{\eta}(j)$$

Using the aggregate production function we can integrate over j–goods to obtain:

$$\left(\frac{Y_t(j)}{A_t \bar{K}^{1-\eta}}\right)^{\frac{1}{\eta}} = N_t(j)$$

Since the workers are all the same the sum is simply, $N_t = \int_0^1 N_t(j)dj$. Note that the rest of derivation uses the notation $\varepsilon = \frac{1+\theta}{\theta}$ which defines the relationship between the elasticity of substitution between intermediary goods, ε and the net markup, θ. Plugging in the demand function

$$\left(\frac{\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t}{A_t \bar{K}^{1-\eta}}\right)^{\frac{1}{\eta}} = N_t(j)$$

Integrating over j–goods

$$N_t = \int_0^1 \left[\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t \frac{1}{A_t \bar{K}^{1-\eta}}\right]^{\frac{1}{\eta}} dj$$

Taking variables independent from j out of the integral,

$$N_t = \left(\frac{Y_t}{A_t \bar{K}^{1-\eta}}\right)^{\frac{1}{\eta}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} dj$$

Now expressing this equation for Y_t,

$$N_t A_t^{\frac{1}{\eta}} \bar{K}^{\frac{1-\eta}{\eta}} = Y_t^{\frac{1}{\eta}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} dj$$

$$N_t^{\eta} A_t \bar{K}^{1-\eta} = Y_t \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} dj\right]^{\eta}$$

where the term in the squared bracket is the price dispersion and it creates a wedge between inputs and output in the aggregate.

Price dispersion

Lets define price dispersion, S_t:

$$S_t^{\frac{1}{\eta}} = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} dj$$

where $1-\eta$ is the capital share of output and ε is the elasticity of substitution between differenciated good j. Next, using the 'Calvo result' (proportion of firms changing its price), we can write price dispersion recursively as:

$$\begin{aligned}
S_t^{\frac{1}{\eta}} &\equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} dj = (1-\xi)\left(\frac{P_t^*(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} + \xi(1-\xi)\left(\frac{P_{t-1}^*(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} + \xi^2(1-\xi)\left(\frac{P_{t-2}^*(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} + \ldots \\
&= (1-\xi)\left(\frac{P_t^*(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} + \xi\left(\frac{P_{t-1}}{P_t}\right)^{\frac{-\varepsilon}{\eta}}\left[(1-\xi)\left(\frac{P_{t-1}^*(j)}{P_{t-1}}\right)^{\frac{-\varepsilon}{\eta}} + \xi(1-\xi)\left(\frac{P_{t-2}^*(j)}{P_{t-1}}\right)^{\frac{-\varepsilon}{\eta}} + \ldots\right] \\
S_t^{\frac{1}{\eta}} &\equiv (1-\xi)\left(\frac{P_t^*(j)}{P_t}\right)^{\frac{-\varepsilon}{\eta}} + \xi\left(\frac{P_{t-1}}{P_t}\right)^{\frac{-\varepsilon}{\eta}} S_{t-1}^{\frac{1}{\eta}} \\
S_t^{\frac{1}{\eta}} &\equiv (1-\xi)\left(p_t^*\right)^{\frac{-\varepsilon}{\eta}} + \xi\left(\pi_t\right)^{\frac{\varepsilon}{\eta}} S_{t-1}^{\frac{1}{\eta}} \qquad (35)
\end{aligned}$$

where $(1-\xi)$ is the probability that the firm will be able to change price. Price dispersion can be written recursively as

$$S_t^{\frac{1}{\eta}} = (1-\xi)\left(\frac{P_t^*(j)}{P_t}\right)^{\frac{-\epsilon}{\eta}} + \xi(\pi_t)^{\frac{\varepsilon}{\eta}} S_{t-1}^{\frac{1}{\eta}}$$

Thus, we can write the aggregate production function as,

$$N_t^{\eta} A_t \bar{K}^{1-\eta} = Y_t S_t$$

Now we can use the domestic aggregate price index to substitute out the ratio of prices and write everything in terms of inflation. Using the definition of aggregate price index and $p_t^* = \left(\frac{P_t^*(j)}{P_t}\right)$.

Law of motion for prices

$$\begin{aligned}
P_t &= \int_0^1 \left[P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \\
P_t^{1-\varepsilon} &= (1-\theta)\left(P_t^*(j)\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \\
1 &= (1-\theta)\left(\frac{P_t^*(j)}{P_t}\right)^{1-\varepsilon} + \theta\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} \\
p_t^* &= \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \qquad (36)
\end{aligned}$$

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