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A simple framework for analyzing the macroeconomic effects of inside money*

(A kereskedelmi banki pénzteremtés makrogazdasági hatásainak egyszerű elemzési kerete)

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Contents

Abstract

The majority of the New Keynesian DSGE literature assumes that the macroeconomic effects of monetary policy can be satisfactorily described by an interest rate rule without addressing the details of the money supply. We investigate whether this approach remains valid in the presence of inside money created by the banking system. To analyze this issue we present a framework based on the generalization of the *IS* and *LM* curves to dynamic general equilibrium models. We find that it is possible to implement a policy based on an interest rate rule even in the presence of inside money, although it requires a more complex toolkit of monetary policy implementation than it is assumed in models with only outside money. We also show that despite some current views, the existence of inside money does not invalidate the common macroeconomic wisdom that investments are linked to savings: both savings and financing matter in determining investments.

JEL: E51, E52, G21.

Keywords: monetary policy, interest rate rule, inside money, liquidity, money multiplier.

Összefoglaló

Az újkyenesi DSGE irodalomban többnyire azt feltételezik, hogy a monetáris politika makrogazdasági hatását kielégítően le lehet írni egy kamatszabállyal anélkül, hogy a pénzkínálatra ki kellene térni. Tanulmányunkban azt vizsgáljuk, hogy vajon ez a megközelítés érvényes marad-e a kereskedelmi banki pénzteremtés jelenlétében is. Bemutatunk egy, a dinamikus általános egyensúlyi modellekre alkalmazható, az *IS* és *LM* görbéken alapuló elemzési keretet. Azt találjuk, hogy kereskedelmi banki pénzteremtés esetén is megvalósítható egy kamatszabályra alapuló monetáris politika, azonban ez egy szélesebb eszköztárat igényel annál, mint amit azok a modellek feltételeznek, ahol a jegybank teljes kontroll alatt tartja a pénzkínálatot. Azt is megmutatjuk, hogy – ellentétben bizonyos véleményekkel – kereskedelmi banki pénzteremtés esetén is érvényes marad az uralkodó makroökonómiai konszenzus, miszerint a beruházások függnek a megtakarításoktól: mind a megtakarítás, mind a finanszírozás számít.

1 Introduction

Since the financial crisis of 2007-2008, a lot of effort has been made in macroeconomics to incorporate the specifics of the financial system into the models,[1] however, most of these models typically focus on the role of the banking system in financial intermediation and neglect the role it plays in the mechanism of money supply, assuming that the total amount of money is equal to the outside money issued by the central bank.

However, as is well known, the money stock is not only determined by the central bank, a significant part of the money supply is made up of inside money created by the banking system.[2] The aim of our study is to examine whether the prevailing academic view that the consequences of banks' money creation are macroeconomically negligible can be justified or, on the contrary, all macroeconomic models without inside money lead to fundamentally different results.

The mainstream academic approach is motivated by Poole (1970) who showed that if the money supply is stochastic and highly volatile then interest rate control is optimal as opposed to money supply control. In this case the stochastic fluctuation of the money supply money can be insulated from the real economy by interest rate control. This result confirmed the view of macroeconomists that the details of the money-creating process are secondary when aggregate demand is affected by monetary policy through the interest rate. However, Poole's formal analysis was based on outside money, and it was only a conjecture that his result can be generalized without any problem to any environment with inside money created by the banking system.[3]

In fact, the current practice of central banks is in line with Poole's finding and monetary policy is conducted by controlling interest rates, still central bankers have never been totally convinced by the approach of academic researchers focusing only on interest rate rules. Although not formally proven, they believe that ignoring the mechanism of money creation and focusing simply on interest rate rules leads to non-negligible errors in macroeconomic analyzes. According to central bankers, it is not only important that the central bank does not control the money supply perfectly, but also that the stochastic fluctuations in the money supply are a consequence of the functioning of the banking system and financial markets. This doubt has intensified since 2008 and central bank studies on the role of the banking system in money creation have proliferated, see for example Maclay, Radia and Thomas (2014), Deutsche Bundesbank (2017) and Jordan (2018). Furthermore, there is also a more radical view, e.g., represented by Werner (2016), which totally disagrees with the mainstream academic approach and claims that the process of money creation is not a negligible detail, and any analysis that omits it is fundamentally flawed. Both the central bank studies and Werner's paper focus on the description of the banking system and the macroeconomic context is only superficially considered, furthermore, they lack formal models.

Since the above debate is mainly based on conjectures and non-formal partial analyses, we create a simple formal macroeconomic model to study the effects of inside money and to examine whether the presence of inside money is negligible or significant from a macroeconomic point of view. Our formal and transparent framework makes it easy to understand the channels through which the effects of inside money creation take place. We show that the consequences of inside money can be easily captured by the *generalization* of the traditional *IS* ("investment-saving") and *LM* ("liquidity preference-money") curves, which can be interpreted in almost any *dynamic general equilibrium* macroeconomic model.

The starting point of our analysis is the observation that there are two main functions in the modern banking system: financial intermediation and the provision of transaction instruments for economic agents. When a bank provides investment loans from

[1] See, for example, Gertler and Kiyotaki (2015), Clerk et al. (2015) and Boissay, Collard and Smets (2016).

[2] Although there are macroeconomic models in which the money creation of the banking system appears explicitly, such as in the studies of Goodfriend (2004), Godley and Lavoie (2007) *chapter 10*, Jakab and Kumhof (2019), Piazzesi and Schneider (2018), Piazzesi, Rogers and Schneider (2021), these are rather exceptions.

[3] Although there is an interpretation that inside money is neglected by theoretical researchers simply because of their lack of knowledge, the apparent neglect is rather a consequence of the general modeling strategy, namely, that a model should not deal with every detail, only with those that are essential to the problem under study. In fact, macroeconomics has long recognized that the mechanism of money creation is much more complex than discussed in introductory textbooks, as, for example, Tobin (1963) attests.

long-term liabilities, it conducts financial intermediation. When it accepts liquid deposits, which are covered by central bank reserve on the asset side, the bank provides the depositor a transaction instrument. However, it is well known that the liquid liabilities of the modern banking system are not fully covered by central bank reserves: when long-term loans are financed by liquid liabilities, the two functions are mixed.

Examining the above issue in a macroeconomic context, the financial intermediation of the banking system is part of the relationship between savings and investments, or, translated into the language of modeling, it is part of the *IS* block. The provision of transaction instruments is part of the money supply process, i.e. liquid deposits are part of the *LM* block. From a macroeconomic point of view, the importance of inside money can be captured by the fact that due to the mixing of the two functions of the banking system, a new relationship is created between the *IS* and *LM* blocks.

In macroeconomic models with only outside money, the above two functions are clearly separated. The financial intermediation is independent of the money creation process and is therefore clearly only part of the IS block (see Woodford, 2010). The independent LM block is determined by the central bank's money supply and the money demand of economic agents (typically households). Consequently, monetary policy affects the IS curve only through the interest rate. Since the change of the money stock has no effect on the IS curve, it exerts its effect by shifting the LM curve along the unchanged IS curve. This mechanism becomes quite different if we add inside money to the model. Changing money stock shifts both the IS and LM curves.

But all this is true not only for monetary policy, but also for all exogenous shocks. In models with outside money one can consider exogenous shocks shifting either the IS curve only or the LM curve only. However, adding inside money to the model creates a new link between the IS and LM curves, and it is no longer possible to affect the two curves separately.

We examine the mechanism outlined above in a simple *general equilibrium model*. In the model, we deliberately used simplifications that allow our results to be expressed in simple analytical form, but at the same time the importance of the elements relevant to our analysis is preserved.

The central element of our model is the banking block based on Piazessi and Schneider (2018). The banking system provides investment loans, and its liability side consists of long-term deposits of households and liquid deposits of households and corporations. Liquid deposits fulfill the function of a transaction instrument, i.e., money. An individual bank has an incentive to use as many liquid deposits as possible to fund investment loans, as they are cheaper. At the same time, there is a risk associated with holding liquid deposits, since when a buyer withdraws his deposit from his bank during a transaction and it is transferred to the seller's bank at the end of the transaction, the movement of deposits must be accompanied by the movement of central bank reserves. Due to the resulting liquidity risk, banks are forced to cover part of their liquid deposits with central bank reserves. In the model, the money multiplier, i.e., the ratio of total money stock and central bank reserves, is the result of optimal liquidity management of banks. As liquidity and lending decisions, i.e., the provision of transaction instruments and financial intermediation are interrelated in the banking system, this creates a new, additional relationship between the IS and LM blocks of the model.

The way we model the banking system is in line with the view of central bankers. For example, Maclay, Radia, and Thomas (2014) from the Bank of England claim that a large part of commercial bank deposits, which represents money in the modern banking system, is created during the lending process, when banks create a deposit equal to the amount of credit granted on the liability side in an autonomous way. But since these deposits are liquid, they can be withdrawn shortly after their creation, thus the related liquidity risk should be managed by holding an adequate amount of central bank reserves. As a result, the total amount of the money stock is constrained by the availability of central bank's reserves, and the size of the money multiplier is determined by the liquidity risk management of banks. This approach is fully in line with our approach. Although we consider the specific form of deposit creation to be of secondary importance, the impact of liquidity risk management on money creation is a key factor in our analysis, and our model of the banking system is basically designed to reflect this.

First, we use the framework described above to study the impact of exogenous macroeconomic shocks in the case of passive monetary policy, that is, when the supply of central bank reserves is kept fixed. We examine how the responses of the IS and LM curves to different shocks change in the presence of inside money compared to the benchmark case with only outside money fully controlled by the central bank. As discussed, since in the former case an additional relationship is established between the IS and LM curves, there is always a numerical difference between the results.

Because our model is intentionally simple, it has only a limited ability to judge whether the differences between the numerical results of the two versions are significant. Therefore, we focus on qualitative differences. Specifically, we examine when a shock has a positive (negative) effect on output in the outside money version, whether this effect remains positive (negative) in the inside money version as well. We find that despite the quantitative differences, the results do not change qualitatively if we add inside money to the model.

Although the above analysis helps to understand the role of banks' money creation, it can be rightly criticized because monetary policy, if it can, typically does not behave passively. Hence, we examine whether the approach of the New Keynesian literature is valid, that is whether the macroeconomic effects of monetary policy can be satisfactorily described by an interest rate rule and the *IS* block of the model without addressing the details of the money supply. Of course, the interest rate rule implicitly assumes the existence of the LM curve. In a model with only outside money it is always possible to implement such a policy: if a shock shifts the IS curve, the latter will intersect the interest rate curve in a different point, but by changing the money supply, the LM curve can always be shifted to be consistent with the new intersection of the IS curve and the interest rate rule.

However, it is not obvious whether this implementation is possible in the presence of inside money. As discussed, in this case changing money supply shifts both the *IS* and the *LM* curves, but the implementation requires a policy which shifts the LM curve but holds the IS curve fixed. Of course, if such a policy exists, it must control other variables in addition to the money stock, such as interest paid on reserves. We show that such a policy mix exists, that is, despite the complexity of the creation of inside money, it is possible to implement a monetary policy perfectly based on the IS curve and an interest rate rule, although it requires a more complex toolkit of monetary policy implementation than it is assumed in models with only outside money.

However, the validity of the above equivalence of the inside and outside money models is limited to a certain range of the shocks. Moreover, the policy toolkit required for the appropriate policy is based on a very detailed knowledge of the economy and it is so complicated that legitimate doubts arise that it cannot be applied in practice. Because of that we take a less complicated approximation of the perfect policy rule and analyse its errors. We find that the error of the approximation is rather small for most shocks.

As mentioned, Werner (2016) claims that any analysis that omits inside money is fundamentally flawed.The essence of the argument is that the modern banking system generates its own resources during lending by *out-of-thin-air* creation of deposits and is able to create money essentially independently of the central bank's money supply, so the banking system does not provide financial intermediation, invalidating the traditional macroeconomic relationship between investments and savings. According to this view, investments are not related to savings, what matters is financing not saving.

Although his view is significantly different from ours, we consider the macroeconomic consequences of his approach. *Out-of-thin-air* money creation can be represented as a special case of our model, if it is assumed that the central bank reserve inflows and outflows at banks during economic transactions just offset each other at the end of the day, so the net change in reserves is zero for each bank. This assumption implies that banks have no liquidity risk. According to our results, financial intermediation does not disappear even in this case. This is because the creation of liquid deposits still has a constraint, namely, the demand for money by economic agents, as in Jakab and Kumhof (2019). As the banking system continues to provide financial intermediation, investments remain linked to savings, in other words, both savings and financing matter.

The paper is structured as follows. In *Section 2* the model is presented. *Section 3* analyzes the adjustment of the IS and LM curves in response to exogenous shocks if monetary policy is passive. In *Section 4* we investigate whether it is possible to implement a monetary policy determined by the IS curve and the interest rate rule in the presence of inside money. *Section 5* discusses the case when instead of implementing perfectly the above policy, it is only approximated. In *Section 6* we consider the implications of the absence of liquidity shocks in the banking system. Finally, *Section 7* concludes.

2 The model

In the model, households and firms are represented in a standard way. Investments are financed by the banking sector, the liabilities of which are stable long-term and liquid deposits, with the latter playing the role of money in the model. Households' savings portfolios include both stable and liquid deposits. Firms also hold liquid deposits for transaction purposes. The banking system has two types of assets: corporate loans and central bank reserves. Banks are actively involved in the money-creation process, as they hold more liquid deposits than central bank reserves.

Banks are subject to idiosyncratic liquidity shocks. If the outflow of liquid deposits from a given bank exceeds the amount of its central bank reserves, it has to borrow on the interbank market, which is relatively expensive. As a result, banks need to actively manage their liquidity risk, which is explicitly reflected in the model. The ratio of liquid deposits to the central bank reserve, i.e. the money multiplier, is determined in the model by liquidity management.

In addition to idiosyncratic liquidity shocks, the model also includes aggregate macroeconomic shocks. The model assumes sticky prices: only a subset of firms is able to respond to macroeconomic shocks with their prices in a given period. As a consequence of sticky prices, monetary policy has real effect in the model.

The timing of the shocks and economic decisions within a given time period is the following: First, firms set prices and some quantities on the basis of the expected values of macroeconomic shocks. Then the macroeconomic shocks are realized, some (but not all) firms can readjust their prices, the product, labor, loan and deposit markets open and monetary policy sets the relevant interest rate and the macroeconomic allocation decisions are made. Then the idiosyncratic liquidity shocks are realized and the interbank market opens, where monetary policy is also active.

2.1 PRODUCTION

The production of the final good takes place in three stages. First, an intermediate good is produced using physical capital. Firms in the input good producing sector use this intermediate good and labor to produce input goods for the final good producing sector. Finally, firms in the final good producing sector aggregate input goods and sell them for consumption and investment purposes.

INPUT AND FINAL GOOD PRODUCERS

The input goods ($y(j)$) are not perfect substitutes and are produced by infinitely many firms indexed by $j \in [0,1]$ acting on a market described by the concept of the Dixit-Stiglitz type monopolistic competition. Input goods are produced using intermediate goods (z) and labor (n) with a quasi linear technology

$$y_t(j) = a^n \left(n_t(j)\right)^{1-\alpha} + a^z z_t(j),$$

where a^n, $a^z > 0$ and $0 < \alpha < 1$.

The final good y is produced on a competitive market by a representative firm using infinitely many input goods and a CES production technology:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$.

Because of perfect competition, the price of the final good is the CES average of the input prices:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} \mathrm{d}j\right]^{\frac{1}{1-\theta}}.$$

Due to the assumption of perfect competition and the constant-return-to-scale technology the final goods producer earns zero profit.

It can be shown easily that demand for the *j*th input good is a function of its relative price and total output:

$$y_t(j) = \left(\frac{P_t}{P_t(j)}\right)^{\theta} y_t,$$

which implies that input goods producers operate on a *Dixit-Stiglitz* type *monopolistically competitive* market.

Some parameters of the model are driven by macroeconomics shocks, the vector of these shocks is denoted by ξ_t. Economic agents have uniform expectation for the shocks, at the beginning of date t the expected value of the shocks is ξ_t^0. At the beginning of period t the central bank announces its monetary policy and input good producer firms set their prices, $P_t^0(j)$. The aggregate price index becomes P_t^0, and wages and the price of the intermediate input good are also chosen. The prices and wages set at the beginning of the period are market clearing conditional on ξ_t^0 and the announced monetary policy. The allocation of goods consistent with the market clearing prices is the *flexible price* allocation described in details in *Appendix A.5*.

As shown in *Appendix A.1* the demand for labor is

$$n_t = \left(\frac{P_t^z}{W_t}\frac{a^n(1-\alpha)}{a^z}\right)^{\frac{1}{\alpha}},$$

and the demand for intermediate goods is

$$z_t = \frac{y_t - a^n n_t^{1-\alpha}}{a^z}, \tag{1}$$

where P^z is the price of the intermediate good, W is nominal wage.

Hence the cost function becomes

$$\mathcal{C}(W_t, P_t^z, y_t) = W_t n_t + \frac{P_t^z}{a^z}\left(y_t - a^n n_t^{1-\alpha}\right).$$

Since labor demand does not depend on the output, the marginal cost function is simply

$$\mathcal{MC}_t = \frac{P_t^z}{a^z}.$$

Observe that all firms in the input good producing sector face the same marginal cost. Profit maximization in the Dixit-Stiglitz type monopolistic competition model implies the following price formula:

$$P_t^0 = \vartheta \mathcal{MC}_t = \vartheta \frac{P_t^z}{a^z},$$

where

$$\vartheta = \frac{\theta}{\theta - 1} > 0$$

is the markup. Since the marginal cost is the same for all firms, prices and production quantities will also be uniform.

The above formula implies that the relative price of the intermediate goods is constant, that is

$$p_t^z = \frac{P_t^z}{P_t^0} = \frac{a^z}{\vartheta},$$

As a consequence, the flexible price labor demand is a function of the real wage,

$$n_t^0 = \left(\frac{a^n(1-\alpha)}{\vartheta w_t^0}\right)^{\frac{1}{\alpha}}, \tag{2}$$

where $w_t^0 = W_t^0/P_t^0$ and W_t^0 is the nominal wage set at the beginning of the period.

After setting the prices and wages the shocks, denoted by ξ_t^*, are realized. Of course, ξ_t^* is not necessarily equal to ξ_t^0. Since the the market of the input good producers can be described by the concept of *monopolistic competition*, the producers do not have to set the same price which allows *sticky prices* to be assumed. Specifically, we assume that only fraction γ of input good producers can adjust their price optimally after the shocks, while the other part of the firms (fraction $1-\gamma$) cannot. Wages are not sticky, they can be adjusted after the realization of the shocks.

We also assume that firms cannot adjust the quantity of z_t set at the beginning of the period, they can adjust only labor. Hence labor demand and the real marginal cost after the realization of ξ_t^* become

$$n_t = \left(\frac{y_t - a^z z_t}{a^n}\right)^{\frac{1}{1-\alpha}}, \tag{3}$$

$$mc(w_t, y_t) = w_t \frac{\partial n_t}{\partial y_t} = w_t \frac{(a^n)^{\frac{1}{\alpha-1}} y_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}. \tag{4}$$

Denote the prices set by firms that can adjust them after the occurrence of the shocks by $P_t^\star$. Then the aggregate price index is given by

$$P_t = \left[(1-\gamma)\left(P_t^0\right)^{1-\theta} + \gamma(P_t^\star)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Rearranging the previous equation yields

$$P_t^\star = \frac{\left[P_t^{1-\theta} - (1-\gamma)\left(P_t^0\right)_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\gamma^{\frac{1}{1-\theta}}}.$$

The optimal price adjustment after the realization of the shocks:

$$P_t^\star = \vartheta P_t mc(w_t, y_t).$$

The above equation can be expressed as

$$\frac{\left[P_t^{1-\theta} - (1-\gamma)\left(P_t^0\right)_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\gamma^{\frac{1}{1-\theta}} P_t} = \vartheta w_t \frac{(a^n)^{\frac{1}{\alpha-1}} y_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}. \tag{5}$$

The representative final good producer firm earns zero profit due to the constant-return-to-scale technology:

$$P_t y_t = \int_0^1 P_t(j) y_t(j)\,\mathrm{d}j$$

As a consequence, the aggregate profit of the final and input goods sectors can be expressed as

$$\Pi_t^y = P_t y_t - W_t n_t - P_t^z z_t,$$

since the profit of final goods production is zero.

INTERMEDIATE GOOD PRODUCERS

Intermediate good producer firms use only physical capital (k) for production with the following technology on a perfectly competitive market:

$$z_{t+1} = A_{t+1}\left(k_t - \omega k_t^2\right). \tag{6}$$

Their initial wealth is zero, hence they need bank loan to buy the necessary capital for production. The capital fully depreciates after production. In order to implement their investment projects, firms need transaction instruments, that is, money. We do not derive firms' money demand from an optimization problem, we simply assume that their demand for money, i.e. the demand for liquid deposits, is proportional to the size of their investment projects.

$$D_t^z = \eta_t^z P_t k_t. \tag{7}$$

As a consequence, the intermediate good producer firms' demand for bank loan will be:

$$L_t = \left(1 + \eta_t^z\right) P_t k_t.$$

Intermediate good producer solve the following profit maximization problem,

$$\max_{k_t} P_{t+1}^z z_{t+1} - \left(1 + i_t^L\right)\left(1 + \eta_t^z\right) P_t k_t,$$

where we assumed that companies do not expect to receive interest income on their deposits because they expect to have to spend their deposits at the beginning of the time period in order to carry out investment projects.

Since $z_{t+1} = A_{t+1}(k_t - \omega k_t^2)$, the first order condition is

$$P_{t+1}^z A_{t+1} - P_{t+1}^z A_{t+1} 2\omega k_t = \left(1 + i_t^L\right)\left(1 + \eta_t^z\right) P_t,$$

from which the demand for physical capital is

$$k_t = \frac{p_{t+1}^z A_{t+1} - \left(1 + r_t^L\right)\left(1 + \eta_t^z\right)}{2 p_{t+1}^z A_{t+1} \omega} \tag{8}$$

where $p_{t+1}^z \equiv P_{t+1}^z / P_{t+1}^0$ is the expected relative price of the intermediate good and r_t^L is the real loan rate. Substituting this expression into the production function yields the supply of z_{t+1} as a function of its (relative) price and the real loan rate.

Although individual firms expect early withdrawal of their deposits, we assume that the aggregate deposit holding of the sector remains constant, hence contrary to the expectation the realized revenue of the sector contains the interest income $(1 + i_t^D) D_t^z$. Hence the realized aggregate profit of the sector:

$$\begin{aligned} \Pi_{t+1}^z &= \left(1 + i_t^D\right) D_t^z + P_{t+1}^z z_{t+1} - \left(1 + i_t^L\right)\left(P_t k_t + D_t^z\right) \\ &= \left(1 + i_t^D\right) D_t^z + P_{t+1}^z z_{t+1} - \left(1 + i_t^L\right) L_t. \end{aligned}$$

2.2 HOUSEHOLDS

Households' instantaneous utility function is given by

$$\mathcal{U}\left(c_t, n_t, D_t^h, \zeta_t\right) = \frac{c_t^{1-\nu}}{1-\nu} + \frac{\zeta_t \left(D_t^h / P_t\right)^{1-\nu}}{1-\nu} - \varphi n_t.$$

where c is consumption, D^h/P is real money holding, n_t is labor and ζ_t is a time varying preference parameter.

Households maximize the following objective function,

$$\max_{\left\{c_t, n_t, F_t^h, D_t^h\right\}} \sum_{t=1}^{T} \mathrm{E}_t \left[\Gamma_{t-1} \mathcal{U}\left(c_t, n_t, D_t^h, \zeta_t\right)\right],$$

subject to the budget constraints:

$$P_t c_t + F_t^h + D_t^h = W_t n_t + \Pi_t + \mathcal{D}_t + T_t + (1 + i_{t-1}) F_{t-1}^h + \left(1 + i_{t-1}^D\right) D_{t-1}^h,$$

where

- We assume that the economy only exists for a finite time, and T is the final date of the economy.
- $\Gamma_t = \beta_t \Gamma_{t-1}$, $\Gamma_0 = 1$, and $0 < \beta_t < 1$ is the time varying discount factor of households,
- F^h and D^h denotes time and demand deposits of households, i_t and i_t^D are the nominal interest rates paid on them,
- $W_t n_t + \Pi_t + \mathcal{D}_t + T_t$ is the income of households which consists of labor income ($W_t n_t$), the profit of the production sector ($\Pi_t = \Pi_t^y + \Pi_t^z$), dividend from banks ($\mathcal{D}_t$) and transfer from the government (T_t).

The solution of the above optimization problem is derived in *Appendix A.2*.

2.3 THE BANKING SECTOR

The banking block of the model is inspired by Piazzesi and Schneider (2018). The main feature of their model is that the banks make decisions on their balance sheet facing liquidity risk and subject to a cost function which we will specify later. Banks can create liquid deposits, that is money, but this money creation is constrained by the costs of expanding their balance sheet as well as by the need to maintain a liquidity buffer for future liquidity shocks.

The banking system is formed by a continuum of homogenous banks owned by the households. Banks are operated by independent managers, whose decisions are not influenced by the owners. The task of the managers is to maximize the discounted net real cash flow (dividends) of households. Households take the cash flow stream as given by passively collecting positive cash flow from the banks and providing the necessary resources if the cash flow is negative.

ASSETS AND LIABILITIES OF BANKS

The liability side of the banks' balance sheet consists of time deposits (F_t) of households, demand deposits of households and firms ($D_t = D_t^h + D_t^z$), and loans from the interbank market (B_t^b). On the asset side they hold reserves (M_t), loans to the intermediate goods producing corporate sector (L_t) and loans to other banks (B_t^l). The net interbank position is denoted by B_t with positive value indicating a net lender position. Consequently, lending and borrowing can be written as $B_t^l = \max[0, B_t]$ and $B_t^b = \max[0, -B_t]$, respectively.

The interest rate paid on reserves, interbank loans, corporate loans, demand and time deposits are denoted by i_t^R, i_t^B, i_t^L, i_t^D and i_t, respectively.

At the beginning of date t the bank collects deposits F_t and D_t, provides loans to firms producing intermediate goods L_t and borrows reserves M_t from the central bank for expected future transactions with other banks. At this stage the interbank market is not open yet, and thus the balance sheet constraint is

$$M_t + L_t = D_t + F_t. \tag{9}$$

IDIOSYNCRATIC LIQUIDITY SHOCKS

After having decided on their balance sheets, banks are hit by an idiosyncratic liquidity shock $\widehat{\lambda}_t$. This idiosyncratic shock is different from the macroeconomic shocks discussed in *section 2.1*. The idiosyncratic liquidity shock aims to capture the real life fact that banks' customers often initiate payments to counterparties having account at another bank, so the payer's bank has to transfer the corresponding amount in reserves to the payee's bank account at the central bank. We assume that $\widehat{\lambda}_t D_t$ has to be paid by the bank. If $\widehat{\lambda}_t > 0$, the depositors withdraw part of their deposits. If $\widehat{\lambda}_t < 0$, new deposits arrive to the bank. We assume that $\widehat{\lambda}_t$ has a continuous distribution over the $[-\bar{\lambda}_t, \bar{\lambda}_t]$ interval described by the cumulative distribution function G and the corresponding probability density function g.

Holding reserves is costly because the interest paid on it is less than the interest on corporate loans. Therefore, banks may hold less reserves than what would cover outflows. Those banks that do not have enough reserves to make interbank payments have to borrow on the interbank market. If we define the reserve ratio as $\lambda_t = M_t/D_t$, a bank must borrow on the interbank market if $M_t < \widehat{\lambda}_t D_t$ or $\lambda_t < \widehat{\lambda}_t$. That is, a bank with liquidity shock $\widehat{\lambda}_t$ borrows at least $B^b\left(\widehat{\lambda}_t\right)$ amount,[4]

$$B^b\left(\widehat{\lambda}_t\right) \geq \widehat{\lambda}_t D_t - M_t. \tag{10}$$

It is assumed that a bank with net deposit outflow does not pay interest on $\widehat{\lambda}_t D_t$. On the other hand, it receives the interest paid by the central bank on $B^b\left(\widehat{\lambda}_t\right)$.

A bank does not have extra liquidity need if $M_t \geq \widehat{\lambda}_t D_t$, that is, if $\lambda_t \geq \widehat{\lambda}_t$. In this case the bank can lend part of its excess liquidity on the interbank market

$$B^l\left(\widehat{\lambda}_t\right) \leq M_t - \widehat{\lambda}_t D_t. \tag{11}$$

A bank with net deposit inflow has to pay interest on $-\widehat{\lambda}_t D_t$ and it will lose the interest paid by the central bank on $B^l\left(\widehat{\lambda}_t\right)$.

[4] The variables $B_t(\widehat{\lambda}_t)$, $B_t^l(\widehat{\lambda}_t)$, $B_t^b(\widehat{\lambda}_t)$ are functions of $\widehat{\lambda}_t$. Whenever it is necessary to avoid confusion, we will explicitly indicate this in the notation, and the letters without the $(\widehat{\lambda}_t)$ extension will represent the related aggregate variables. However, to simplify notation, when it does not result in confusion we will omit the term $(\widehat{\lambda}_t)$ even in the case of individual, non-aggregate variables.

OPERATION COST OF BANKING

The main focus of the banking block in our model is representing liquidity risk management and considering its consequences. On the other hand, we do not want to provide a deeper understanding of other aspects of banking behavior. Therefore, following Cúrdia and Woodford (2016) and Piazzesi and Schneider (2018), we simply posit a reduced-form intermediation technology represented by a cost function to capture the operation of banks.

Specifically, we assume the banks' have the following real cost function:

$$\begin{aligned} \kappa_t &= \bar{\kappa} + \tau^L \frac{L_t}{P_t} + \chi_t \phi^L \left(\frac{L_t}{P_t}\right)^2 + \tau^D \frac{D_t}{P_t} + \phi^D \left(\frac{D_t}{P_t}\right)^2 \\ &+ \tau^F \frac{F_t}{P_t} - \phi^{DL} \frac{D_t L_t}{P_t^2} + \phi^B \frac{\left(B^I/P_t\right)^2}{\left(M_t - \widehat{\lambda}_t D_t\right)/P_t}. \end{aligned} \tag{12}$$

The term $\bar{\kappa}$ represents the fixed cost of banking. The second to sixth terms on the right hand side represent the operation cost of collecting deposits and providing loans, including the marketing cost.

The time varying coefficient χ_t is a shortcut for capturing the shock to default risk on outstanding corporate loans. In *Section 2.1* we do not explicitly model the uncertainty of corporate investment. The representative intermediate goods producer describes the aggregate behavior of the sector, but we assume that individual firms are subject to idiosyncratic profit shocks that are not explicitly reflected in the model. The distribution of shocks is such that they do not affect the size of the expected profit, only the variance of individual profits (mean preserving spread). We assume that the worst performing companies go bankrupt, although this is not reflected explicitly in the model, but the effect of this is captured by the variable χ_t. As the standard deviation of shocks increases, so does the number of companies that go bankrupt, and this effect is represented by the increase in χ_t.

The term $-\phi^{DL} D_t L_t / P_t^2$, ($\phi^{DL} > 0$) can have two interpretations:

- First, as in Piazzesi and Schneider (2018), it takes resources to convince the owners of demand deposits that their claims are satisfied on demand at any time. Moreover, we assume that convincing depositors is cheaper if the bank owns more assets to back the commitments, especially if those assets are relatively safe. As discussed above, corporate loans are not immune to uncertainty and are therefore not considered to be safe assets, but we assume that bankruptcy losses do not jeopardize the ability of banks to repay their deposits. According to this interpretation having more assets, that is, more L_t, reduces the cost of deposit creation:

$$\left(\tau^D - \phi^{DL} \frac{L_t}{P_t}\right) \frac{D_t}{P_t} + \phi^D \left(\frac{D_t}{P_t}\right)^2.$$

- Just the other way around, according to the second interpretation more demand deposits reduce the cost of lending:

$$\left(\tau^L - \phi^{DL} \frac{D_t}{P_t}\right) \frac{L_t}{P_t} + \phi^L \left(\frac{L_t}{P_t}\right)^2.$$

  This approach can be justified by the following line of thought. Beyond liquidity risk, banks have to manage their solvency risk as well. This can be captured by the *value-at-risk* approach of banks to keep the probability of default within reasonable limits, as in the models in *chapters 2* and *3* of Shin (2010). Taking *value-at-risk* decisions into account implies that not only the marginal cost of funding, but also the average cost of liabilities determine lending since *ceteris paribus* smaller repayment reduces the probability of default. Therefore, more cheap funding by demand deposits facilitates corporate lending since the *value-at-risk* constraint becomes looser and the higher leverage is allowed.
  However, this mechanism does not appear explicitly in our model. Instead of representing the above mechanism in detail we capture this feature by a shortcut, that is, by assuming that D_t reduces the cost of L_t.

The final term represents the cost of interbank lending. Since the interbank market is a standardized and organized market, this cost is not proportional to the magnitude of lending. Rather, this term wants to capture the phenomenon that it is easier to lend overnight if the bank has abundant liquidity, and it is more difficult if the bank's liquidity is scarce. This term is positive if the bank is a net lender on the interbank market and zero if she is a net borrower. We assume that borrowing has no operating costs because interbank borrowing is a coercive decision, if the outflow of deposits is large enough and a bank wants to avoid bankruptcy, it must do so, in which case no sophisticated liquidity management considerations are required. However, it is exactly the sophisticated liquidity management that we assume to be costly. Although the denominator can take negative values for banks that do not have enough reserves to make the necessary transfer payments, the whole term cannot go below zero, because if a bank is net borrower on the market, the numerator will be zero. As a consequence of this type of cost, banks with excess liquidity will hold reserves even if the interest on reserves is lower than the interbank lending rate.

OPTIMIZATION PROBLEM OF INDIVIDUAL BANKS

At date t banks collect the principal and interest on their assets and pay the principal and interest on their liabilities. The banks' net income is the dividend which is transferred to the household sector:

$$\begin{aligned}\mathcal{D}_t &= \left(1+i^L_{t-1}\right)L_{t-1} + \left(1+i^R_{t-1}\right)\left(M_{t-1} - \widehat{\lambda}_{t-1}D_{t-1} - B_{t-1}\right) \\ &+ \left(1+i^B_{t-1}\right)B_{t-1} - (1+i_{t-1})F_{t-1} - (1+i^D_{t-1})\left(1-\widehat{\lambda}_{t-1}\right)D_{t-1} - P_t\kappa_t.\end{aligned} \tag{13}$$

Individual banks take as given the interest rates i^R_t, i^L_t, i_t, i^D_t, i^B_t and the price level P_t. The problem of a bank is to maximize the discounted value of the real dividends paid to the households.

This is basically a static decision. First, banks have only one-period assets and liabilities, therefore the balance sheet and liquidity constraints contain only date t variables. Second, physical capital formed in the previous period does not constraint the operation of banks, hence the operation cost also contains only date t variables. Finally, there is no accumulation of equity (since all dividends are paid automatically to the household), as a consequence, banks do not have to take into account expectations for future variables.

Therefore, at each date t banks solve the following optimization problem:

$$\max_{x_t, B_t} \mathrm{E}_t\left[\bar{\beta}_t\frac{\mathcal{D}_{t+1}}{P_{t+1}} + \frac{\mathcal{D}_t}{P_t}\right]$$

subject to the constraints:

$$M_t + L_t = D_t + F_t,$$
$$M_t - \widehat{\lambda}_t D_t \geq B_t,$$

and

$$x_t \geq 0,$$

where $x_t = L_t, M_t, D_t, F_t$ and

$$\bar{\beta}_t = \beta_t\frac{c_{t+1}^{-\nu}}{c_t^{-\nu}}$$

is the discount factor of the household (recall, that banks are owned by households). The first constraint represents the balance sheet of the bank, the second liquidity constraint is derived from equations (10) and (11).

Here we characterize the most important properties of the solution of an individual banks' optimization problem. For more details see *Appendix A.3*.

First of all, the optimal solution does not depend on the absolute level of the interest rate, only relative interest rate matters, that is, the spreads between the different interest rates and the interest rate on time deposits,

$$\Delta^R_t \equiv \frac{1+i^R_t}{1+i_t}, \qquad \Delta^L_t \equiv \frac{1+i^L_t}{1+i_t}, \qquad \Delta^B_t \equiv \frac{1+i^B_t}{1+i_t}, \qquad \Delta^D_t \equiv \frac{1+i^D_t}{1+i_t}.$$

Consider the conditions characterizing interbank lending and borrowing (B_t):

$$\Delta^B_t = \Delta^R_t + 2\phi^B\rho_t \qquad \text{if} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t], \tag{14}$$
$$\Delta^B_t = \Delta^R_t + \mu_t \qquad \text{if} \qquad \widehat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t], \tag{15}$$

where μ_t is the shadow price of liquidity, that is, the Lagrange multiplier of the liquidity constraint ($M_t - \widehat{\lambda}_t D_t \geq B_t$), and $2\phi^B \rho_t$ is the marginal cost of interbank lending,

$$\rho_t = \frac{B_t^l\left(\widehat{\lambda}_t\right)}{M_t - \widehat{\lambda}_t D_t} \qquad \text{for all} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t], \tag{16}$$

The above conditions describe a symmetric solution, all banks' lending on the interbank market have the same ρ_t. Later it will be shown that this symmetric solution is consistent with an equilibrium on the interbank market.[5]

In equilibrium $0 < \rho_t < 1$, therefore, the interbank interest rate contains a premium over the reserve rate. Furthermore, conditions (14) and (15) implies that $\mu_t > 0$, that is, if a bank borrows on the interbank market its liquidity constraint is binding,

$$B_t^b\left(\widehat{\lambda}_t\right) = \widehat{\lambda}_t D_t - M_t. \tag{17}$$

Now consider the optimal allocation of assets (M_t, L_t). The first order condition determining reserve holding is the following:

$$\Delta_t^R + \mu_t\left[1 - G(\lambda_t)\right] + \phi^B \rho_t^2 G(\lambda_t) = 1 + \tau^F, \tag{18}$$

recall that $G(\lambda_t)$ is the probability that $\widehat{\lambda}_t \leq \lambda_t$. The left-hand and the right-hand sides of the above equation represent the benefit and cost of reserve holding. The first term of the left-hand side is the interest paid on reserves, the second captures the benefit of not borrowing on the interbank market, the third represents that more reserves will reduce the cost of interbank lending. The right-hand side reveals that keeping everything else fixed one extra unit of reserves requires one extra unit of funding.

To get closed form solutions we assume that $\widehat{\lambda}_t$ is drawn from a uniform distribution over the interval $[-\bar{\lambda}_t, \bar{\lambda}_t]$. As shown in the *Appendix A.3*, equation (18) can then be expressed in the following way,

$$\Delta_t^R + \frac{\delta_t}{2\bar{\lambda}_t}\mu_t + \frac{\varsigma_t}{2\bar{\lambda}_t}\phi^B \rho_t^2 = 1 + \tau^F, \tag{19}$$

where $\delta_t \equiv \max\left[0, \bar{\lambda}_t - \lambda_t\right]$ and $\varsigma_t \equiv \max\left[2\bar{\lambda}_t, \bar{\lambda}_t + \lambda_t\right]$.

The first order condition with respect to corporate loans is the following:

$$\Delta_t^L = 1 + \bar{\tau}^L + 2\chi_t \phi^L \frac{L_t}{P_t} - \phi^{DL}\frac{D_t}{P_t}, \tag{20}$$

where $\bar{\tau}^L \equiv \tau^F + \tau^L$. The interpretation of this condition is that the spread of lending rate should reflect the cost of lending.

Considering the liability side, the first order condition determining demand deposits (D_t):

$$\begin{aligned}
&\Delta_t^D + \tau^D + 2\phi^D \frac{D_t}{P_t} - \phi^{DL}\frac{L_t}{P_t} + \mu_t \int_{\lambda_t}^{\bar{\lambda}_t} \widehat{\lambda} g\left(\widehat{\lambda}\right) \mathrm{d}\widehat{\lambda} \\
&= 1 + \tau^F - \phi^B \rho_t^2 \int_{-\bar{\lambda}_t}^{\lambda_t} \widehat{\lambda}_t g\left(\widehat{\lambda}\right) \mathrm{d}\widehat{\lambda}.
\end{aligned} \tag{21}$$

The first line of the above equation describes the cost of demand deposits, the interest payment, the operation cost associated with deposit collection, and the cost of illiquidity, namely that the more demand deposits the bank has the more likely that it has to borrow on the interbank market. The second line represent the benefits. The first term indicates that demand deposits are substitutes of other types of liabilities, the second term represents that the more demand deposits the bank has the more the cost of interbank lending is reduced.

As shown in the *Appendix A.3*, assuming uniform distribution implies that equation (21) becomes

$$\Delta_t^D = 1 - \bar{\tau}^D - 2\phi^D \frac{D_t}{P_t} + \phi^{DL}\frac{L_t}{P_t} - \left(\mu_t - \phi^B \rho_t^2\right)\frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}, \tag{22}$$

where $\bar{\tau}^D \equiv \tau^D - \tau^F$.

[5] Although the symmetric solution is not the only equilibrium solution.

EQUILIBRIUM ON THE INTERBANK MARKET

Since the banking sector is homogeneous and banks are similar before the liquidity shock, in equilibrium all banks choose the same M_t, L_t, D_t, F_t and, as a consequence, the same reserve ratio λ_t. Since there is a continuum of banks in the model, the cross sectional distribution of $\widehat{\lambda}_t$ can be described by the probability distribution of $\widehat{\lambda}_t$.

The interbank equilibrium condition is

$$B_t^l + B_t^{CB} = B_t^b, \tag{23}$$

where $0 \leq B_t^{CB} \leq B_t^b$ is central bank lending on the interbank market and

$$\begin{aligned} B_t^l &= \int_{-\bar{\lambda}_t}^{\lambda_t} B_t^l\left(\widehat{\lambda}\right) \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\widehat{\lambda}, \\ B_t^b &= \int_{\lambda_t}^{\bar{\lambda}_t} B_t^b\left(\widehat{\lambda}\right) \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\widehat{\lambda}, \end{aligned}$$

where $1/\left(2\bar{\lambda}_t\right)$ is the uniform probability density function. Equation (17) implies that

$$B_t^b = \int_{\lambda_t}^{\bar{\lambda}_t} \left(\widehat{\lambda} - \lambda_t\right) D_t \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\widehat{\lambda}.$$

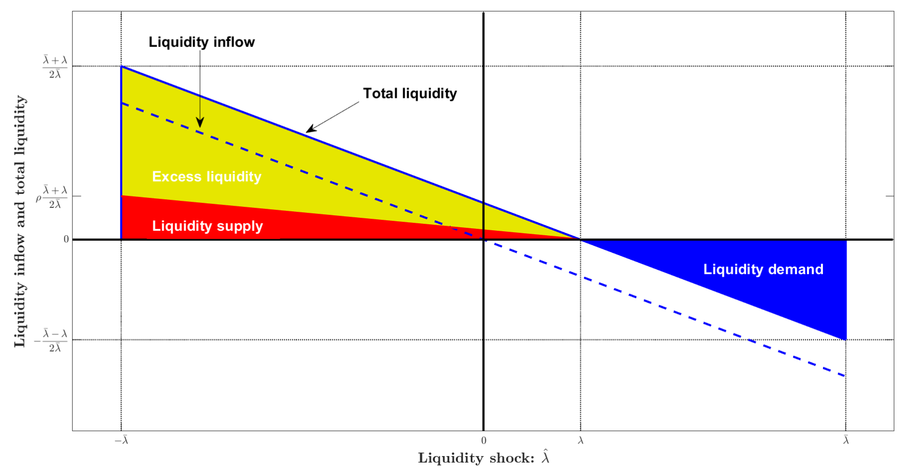

Figure 1
Liquidity demand and supply

In *Figure 1* the blue triangle represents B_t^b/D_t, the total demand for liquidity per unit of demand deposit on the interbank market. Since the area of the triangle is equal to $\delta_t^2/\left(4\bar{\lambda}_t\right)$,

$$B_t^b = \frac{\delta_t^2}{4\bar{\lambda}_t} D_t. \tag{24}$$

In *Figure 1* the inflow per unit of demand deposit as a function of $\widehat{\lambda}_t$ is represented by the dashed line. However, the total liquidity of potential lenders ($\widehat{\lambda}_t < \lambda_t$) is greater than the aggregate liquidity inflow, since they can lend their reserves plus the $\widehat{\lambda}_t D_t$. The total liquidity per unit of demand deposit is represented by sum of the yellow and red triangle, its area is equal to $\varsigma_t^2/\left(4\bar{\lambda}_t\right)$. Hence the total excess liquidity (TL_t) is clearly greater than the market demand for liquidity,

$$TL_t = \frac{\varsigma_t^2}{4\bar{\lambda}_t} D_t > \frac{\delta_t^2}{4\bar{\lambda}_t} D_t = B_t^b.$$

We assumed that each lender supplies the same ρ_t fraction of its available liquidity as equation (16) indicates. As a consequence, $\rho_t < 1$. The total liquidity supply is equal to ρ_t times the total liquidity, that is,

$$B_t^l = \rho_t \int_{\lambda_t}^{\bar{\lambda}_t} \left(\widehat{\lambda} - \lambda_t\right) D_t \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\widehat{\lambda} = \rho_t TL_t. \tag{25}$$

Now we can express equilibrium condition (23) as

$$\rho_t TL_t + B_t^{CB} = B_t^b.$$

Rearranging it yields

$$\rho_t = \frac{B_t^b - B_t^{CB}}{TL_t} < 1,$$

since $TL_t > B_t^b$ and $0 \leq B_t^{CB} \leq B_t^b$.

Using equation (13) it is easy to show that the aggregate dividend of the banking sector is given by

$$\begin{aligned} \mathcal{D}_t \quad = \quad & \left(1 + i_{t-1}^L\right) L_{t-1} + \left(1 + i_{t-1}^R\right) M_{t-1} - (1 + i_{t-1}) F_{t-1} - (1 + i_{t-1}^D) D_{t-1} \\ + \quad & \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB} - P_t \kappa_t, \end{aligned}$$

since

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \widehat{\lambda} D_t \,\mathrm{d}\widehat{\lambda} = 0, \qquad \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} B_t\left(\widehat{\lambda}\right) \,\mathrm{d}\widehat{\lambda} = B_t^l - B_t^b = -B_t^{CB}.$$

2.4 THE GOVERNMENT AND THE CENTRAL BANK

MONETARY POLICY

At the beginning of each period monetary policy sets its instruments in such a way that $P_t^0 = P_{t-1}$. After the realization of ξ_t^* monetary policy adjusts its instruments governed by a standard interest rate rule:

$$\frac{i_t}{i_t^0} = \left(\frac{y_t}{y_t^0}\right)^{\psi_y} \left(\frac{P_t}{P_{t-1}}\right)^{\psi_\pi}, \tag{26}$$

where y_t^0, i_t^0 are the real output and the nominal interest rate consistent with the flexible price allocation.

To implement the required policy the central bank has three instruments. First, it determines the total quantity of reserves (M_t) available for banks at the beginning of date t. In practice, the aggregate quantity of reserves is often controlled by open market operations. In our model this option is not available since there is no government debt. Instead, we assume that the central bank lends to commercial banks before the realization of the liquidity shock. From the point of view of an individual bank households ex ante lending F_t^h and central bank ex ante lending $F_t^{CB} = M_t$ are perfect substitutes, thus total time deposits (F_t) are the sum of the two.[6]

Second, the central bank sets the interest rate paid on reserves (i^R) which determines Δ_t^R.

Finally, the central bank lends on the interbank market after the realization of the liquidity shock (B_t^{CB}) in order to control the interest rate on the market (i^B) which determines Δ_t^B.

M_t, Δ_t^R and Δ_t^B determine ρ_t, λ_t and D_t. To see this, rearrange equation (14) to get

$$\rho_t = \frac{\Delta_t^B - \Delta_t^R}{2\phi^B}. \tag{27}$$

[6] Perfect substitution implies that lending from households and from the central bank have the same operation cost, see *Section 2.3*. This is just a simplifying assumption.

Combining equations (14) and (15) provides

$$\mu_t = 2\phi^B \rho_t. \tag{28}$$

Then substitute the above expression into equation (19)

$$\frac{\bar{\lambda}_t - \lambda_t}{2\bar{\lambda}_t} 2\phi^B \rho_t + \frac{\bar{\lambda}_t + \lambda_t}{2\bar{\lambda}_t} \phi^B \rho_t^2 = 1 + \tau^F - \Delta_t^R.$$

Rearranging it yields a solution for λ_t,

$$\lambda_t = \bar{m}_t \bar{\lambda}_t, \tag{29}$$

where

$$\bar{m}_t \equiv \frac{\phi^B \rho_t (2 + \rho_t) - 2\left(1 + \tau^F - \Delta_t^R\right)}{\phi^B \rho_t (2 - \rho_t)} > 0.$$

Furthermore, by using the definition of λ_t,

$$D_t = \frac{M_t}{\lambda_t}.$$

As shown, M_t, Δ_t^R and Δ_t^B clearly defines ρ_t, λ_t and D_t However, this is also true the other way round:

$$\begin{aligned}
\Delta_t^R &= 1 + \tau^F - \frac{\bar{\lambda}_t - \lambda_t}{2\bar{\lambda}_t} 2\phi^B \rho_t - \frac{\bar{\lambda}_t + \lambda_t}{2\bar{\lambda}_t} \phi^B \rho_t^2, \\
\Delta_t^B &= 2\phi^B \rho_t + \Delta_t^R, \\
M_t &= \lambda_t D_t.
\end{aligned}$$

That is, $(M_t, \Delta_t^R, \Delta_t^B)$ and (ρ_t, λ_t, D_t) mutually unambiguously determine each other. As a consequence, it is possible to represent monetary policy by ρ_t, λ_t and D_t as well.

CONSOLIDATED BUDGET CONSTRAINT

The central bank's profit at date t has two components. First, the difference between the revenue on lending at the beginning of a time period and the interest paid on reserves: $(1+i_{t-1})F_{t-1}^{CB} - \left(1 + i_{t-1}^R\right) M_{t-1}$. Second, the difference between the revenue and expenditure related to B_{t-1}^{CB}. Recall that after the settlement of interbank payments B_{t-1}^{CB} is held as a reserve, hence the central bank has to pay the reserve rate on it, hence this component of the profit is given by $\left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}$. Since $M_{t-1} = F_{t-1}^{CB}$, the central banks' profit:

$$\left(i_{t-1} - i_{t-1}^R\right) M_{t-1} + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}.$$

The central bank pays the profit to the central government. We assume that there is no government consumption and the government's budget is always balanced, the central bank's profit is transferred to the household sector:

$$T_t = \left(i_{t-1} - i_{t-1}^R\right) M_{t-1} + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}.$$

2.5 EXOGENOUS SHOCKS

As discussed, there are two types of shocks in the model: macroeconomic shocks, ξ_t, and an idiosyncratic liquidity shock $\widehat{\lambda}_t$.

The vector of macroeconomic shocks consists of the following variables:

$$\xi_t = \left[\bar{\lambda}_t, \eta_t, \beta_t, A_{t+1}, \chi_t, \eta_t^z\right],$$

where $\bar{\lambda}_t$ is the upper limit of the liquidity shock's absolute value, $\eta_t \equiv \zeta_t^{\frac{1}{\nu}}$ is a parameter of households' money demand, β_t is the discount factor of households, A_{t+1} is the productivity factor in the intermediate goods producing sector, χ_t measures the risk of corporate loans and η_t^z is the parameter of firms' money demand.

We will also apply the following notations:

$$\begin{aligned}\xi_t^{LM} &= \left[\bar{\lambda}_t, \eta_t\right],\\ \xi_t^{IS} &= \left[\beta_t, A_{t+1}, \chi_t, \eta_t^z\right],\end{aligned}$$

We will define the IS and LM curves later. The shocks in the ξ_t^{LM} vector have an effect primarily on the LM curve, while those within the ξ_t^{IS} have an effect primarily on the IS curve.[7]

The timing of the shocks and economic decisions is the following:

- First, firms set prices and the quantity of the intermediate good on the basis of ξ_t^0, M_t^0, Δ_t^{R0} and Δ_t^{B0}.
- Then the macroeconomics shocks are realized (ξ_t^*) and the product, labor, loan and deposit markets open and the macroeconomic allocation decisions are made. Monetary policy sets i_t by choosing M_t, Δ_t^R and announcing Δ_t^B.
- Then the liquidity shocks are realized and the interbank market opens.

[7] It may be somewhat surprising that households' money demand is primarily affects the LM curve, while corporate money demand primarily affects the IS curve. We will explain this later.

Table 1
Transactions flow matrix of the model

	Households	Final and input goods producers	Intermediate goods producers	Banking system	The government and the central bank	Σ
Production	$-P_t c_t$	$P_t y_t$	$-P_t k_t$	$-P_t \kappa_t$	0	0
Factor income	$W_t n_t$	$-W_t n_t - P^z_t z_t$	$P^z_t z_t$	0	0	0
Profit	$\Pi_t + \mathcal{D}_t$	$-\Pi^y_t$	$-\Pi^z_t$	$-\mathcal{D}_t$	0	0
Transfers	T_t	0	0	0	$-T_t$	0
Interest rate on						
demand deposits	$i^D_{t-1} D^h_{t-1}$	0	$i^D_{t-1} D^z_{t-1}$	$-i^D_{t-1} D_{t-1}$	0	0
time deposits	$i_{t-1} F^h_{t-1}$	0	0	$-i_{t-1} F_{t-1}$	$i_{t-1} F^{CB}_{t-1}$	0
corporate loans	0	0	$-i^L_{t-1} L_{t-1}$	$i^L_{t-1} L_{t-1}$	0	0
interbank CB loans	0	0	0	$-i^B_{t-1} B^{CB}_{t-1}$	$i^B_{t-1} B^{CB}_{t-1}$	0
reserves	0	0	0	$i^R_{t-1}\left(M_{t-1} + B^{CB}_{t-1}\right)$	$-i^R_{t-1}\left(M_{t-1} + B^{CB}_{t-1}\right)$	0
Change of						
demand deposits	$D^h_{t-1} - D^h_t$	0	$D^z_{t-1} - D^z_t$	$D_t - D_{t-1}$	0	0
time deposits	$F^h_{t-1} - F^h_t$	0	0	$F_t - F_{t-1}$	$F^{CB}_{t-1} - F^{CB}_t$	0
corporate loans	0	0	$L_t - L_{t-1}$	$L_{t-1} - L_t$	0	0
interbank CB loans	0	0	0	$B^{CB}_t - B^{CB}_{t-1}$	$B^{CB}_{t-1} - B^{CB}_t$	0
reserves	0	0	0	$M_{t-1} - M_t + B^{CB}_{t-1} - B^{CB}_t$	$M_t - M_{t-1} + B^{CB}_t - B^{CB}_{t-1}$	0
Σ	0	0	0	0	0	0

2.6 SOLUTION OF THE MODEL

IS, LM, aggregate demand and aggregate supply curves are common graphical tools for solving textbook macroeconomic models and illustrating their operation. However, with the spread of modern dynamic macroeconomics models, they have been pushed out of the analytical tools of academic research, as they can basically only be applied to static models.

Although they are no longer used as a graphical tool, they continue to help structure macroeconomic thinking, and it is no coincidence that Galí (2015) calls one of the key equations of his basic model a dynamic IS curve, or that Bernanke, Gertler and Gilchrist (1999) group the equations of their model into aggregate supply and demand blocks (see pages 1361–1362).

In addition, these tools can be used even in dynamic models: if a certain date is chosen and the values of the predetermined state variables and expectations are given at this date, then the equilibrium allocations of the model can be calculated and illustrated with them. For example, Eggertsson and Krugman (2012) solve their model at a given date by constructing aggregate demand and supply curves. Woodford (2010) also illustrates the implications of financial intermediation in dynamic models by IS and LM curves.

Woodford's paper demonstrated that this analytical tool helps understand the macroeconomic aspects of financial intermediation, although he neglected the issue of the money creation of the banking system. In this paper we generalize this approach to models with inside money. We think this a fruitful approach in this context, since the financial intermediation function of banks is part of the IS relationship, while the provision of transaction instrument function is part of the LM relationship. As discussed, in the presence of inside money, these two functions of the banking system cannot be insulated, which creates relationships between the IS and LM curves that do not exist when there is only outside money in the model. Thus, we can understand the macroeconomic significance of inside money by analyzing how the aforementioned relationships change the shape of the IS and LM curves and their sensitivity to monetary policy and other exogenous shocks.

In this paper we present an intentionally simple model to be able to express the IS and LM curves as explicit functions. Our approach can be generalized to more complex models, but in those cases the IS and LM curves can be defined only by implicit functions.

IS AND LM CURVES

In this model if the value of the state variable k_{t-1} (which determines z_t) and the expected value of households' income y^h_{t+1} are known, one can construct the IS and LM curves to calculate the date t equilibrium allocation.

We will assume that $t+1 = T$, that is the subsequent period is the final date. This simplifies the derivation of y^h_{t+1} as a function of date t variables. (See *Appendix A.5*.)

Consider equation (8) and recall that $p^z = \alpha^z/\vartheta$. Since expected inflation for date $t+1$ is zero by assumption, $r^L_t = i^L_t$ and thus

$$k_t = \frac{\frac{\alpha^z}{\vartheta}A_{t+1} - \left(1+\eta^z_t\right)\left(1+i^L_t\right)}{2\frac{\alpha^z}{\vartheta}A_{t+1}\omega},$$

Recall that $L_t/P_t = (1+\eta^z_t)k_t$. Inserting it into the corporate loan rate spread equation (20) yields

$$1+i^L_t = (1+i_t)\Delta^L_t = (1+i_t)\left[1+\bar{\tau}^L + 2\chi_t\phi^L\left(1+\eta^z_t\right)k_t - \phi^{DL}\frac{D_t}{P_t}\right]. \tag{30}$$

Combining the above equations provides an expression for k_t:

$$k_t = \frac{\frac{\alpha^z}{\vartheta}A_{t+1} - (1+i_t)\left(1+\eta^z_t\right)\left(1+\bar{\tau}^L - \phi^{DL}\frac{D_t}{P_t}\right)}{2\frac{\alpha^z}{\vartheta}A_{t+1}\omega + 2(1+i_t)\chi_t\phi^L\left(1+\eta^z_t\right)^2}. \tag{31}$$

As the above equation reveals, the nominal interest rate i_t and monetary policy, represented by D_t determines corporate loans and the new capital stock. This expression describes the financial intermediation function of banks, however, as discussed in the introduction, in the modern banking system financial intermediation cannot be insulated from provision of transaction instruments, as the presence of D_t reveals.

Substituting the balance sheet constraint (9) and terms $L_t/P_t = (1+\eta_t^z)k_t$ and $M_t = \lambda_t D_t$ into equation (12) yields the following expression for the operation cost of banking:

$$\begin{aligned}\kappa_t &= \kappa^0 + \bar{\tau}^D \frac{D_t}{P_t} + \phi^D \left(\frac{D_t}{P_t}\right)^2 + \bar{\tau}^L \left(1+\eta_t^z\right) k_t + \chi_t \phi^L \left[\left(1+\eta_t^z\right) k_t\right]^2 \\ &- \phi^{DL} \frac{D_t}{P_t} \left(1+\eta_t^z\right) k_t + \kappa_t^{\lambda\rho},\end{aligned} \tag{32}$$

where

$$\kappa_t^{\lambda\rho} \equiv \tau^F \lambda_t \frac{D_t}{P_t} + \phi^B \rho_t \frac{B_t^I}{P_t} = \left(\tau^F \lambda_t + \phi_t^B \rho_t^2 \frac{\varsigma_t^2}{4\bar{\lambda}_t}\right) \frac{D_t}{P_t},$$

and the second equation of the above formula is a consequence of equation (25). Observe that κ_t is completely determined by k_t and monetary policy which controls ρ_t, λ_t and D_t.

Combining equations (22) and (28) yields

$$\Delta_t^D = 1 - \bar{\tau}^D - 2\phi^D \frac{D_t}{P_t} + \phi^{DL} \left(1+\eta_t^z\right) k_t - \Psi_t, \tag{33}$$

where

$$\Psi_t \equiv \phi^B \rho_t (2-\rho_t) \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}.$$

In *Appendix A.2* it is shown that real savings is given by the following function,

$$s_t = \frac{\mathcal{B}_t}{1+\mathcal{B}_t}\left(y_t - \kappa_t\right) - \frac{1}{1+\mathcal{B}_t} \frac{y_{t+1}^h}{1+i_t},$$

where y_{t+1}^h, the future income of households is a function of i_t, Δ_t^D, k_t, D_t/P_t, and

$$\mathcal{B}_t \equiv (1+\eta_t)\beta_t^\sigma \left(1+r_t\right)^{\sigma-1} + \eta_t \left(1-\Delta_t^D\right)^{1-\sigma}, \qquad \sigma \equiv \frac{1}{\nu}, \qquad \eta_t \equiv \zeta_t^\sigma,$$

and where we used that the real and the nominal interest rates are the same (as discussed, the expected inflation rate is zero, $P_t = P_{t+1}^0$). Observe that $\mathcal{B}_t$ is also determined by i_t and the monetary policy.

It is also shown in *Appendix A.2* that

$$k_t = s_t,$$

that is, the real savings of households is equal to the capital stock (which is quite clear intuitively since k_t is equal to investments in this model). Combining the above formulas yields the IS curve:

$$y_t = \frac{1}{\mathcal{B}_t}\left(\left(1+\mathcal{B}_t\right) k_t + \frac{y_{t+1}^h}{1+i}\right) + \kappa_t, \tag{34}$$

Households' demand for real money is derived in *Appendix A.2*:

$$\frac{D_t^h}{P_t} = \frac{\eta_t}{\left(1-\Delta_t^D\right)^\sigma} \frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+r_t}}{1+\mathcal{B}_t}.$$

Combine the equilibrium condition $D_t = D_t^h + D_t^z$ with the above formula and equation (7) to obtain the LM curve:

$$y_t = \frac{\left(1-\Delta_t^D\right)^\sigma \left(1+\mathcal{B}_t\right)}{\eta_t}\left(\frac{D_t}{P_t} - \eta_t^z k_t\right) - \frac{y_{t+1}^h}{1+i_t} + \kappa_t. \tag{35}$$

We will also apply the following notations for the equations (34) and (35):

$$y_t = y^{IS}\left(i_t, \frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right), \tag{36}$$

$$y_t = y^{LM}\left(i_t, \frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right). \tag{37}$$

The above formulas emphasize that the IS and LM curves are functions mapping i_t to the real output, y_t. Furthermore, they also depend on monetary policy represented by ρ_t, λ_t, D_t, as well as on the price level (P_t) and the exogenous shocks.

If the price level is given, the IS and LM curves define two equations for y_t and i_t. We denote the solution, that is, the intersection of the two curves by

$$y_t^{islm} = y^{islm}\left(\frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right), \tag{38}$$

$$i_t^{islm} = i^{islm}\left(\frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right). \tag{39}$$

The above formulas reveal that the equilibrium values of y_t and i_t are functions of monetary policy and the exogenous shocks. Then using the equilibrium value of i_t^{islm}, one can calculate the equilibrium values i_t^L, k_t, κ_t and Δ_t^D by equations (30)–(33).

Observe that

$$y_t^0 = y^{islm}\left(\frac{D_t^0}{P_t^0}, \lambda_t^0, \rho_t^0, \xi_t^{LM0}, \xi_t^{IS0}\right),$$

$$i_t^0 = i^{islm}\left(\frac{D_t^0}{P_t^0}, \lambda_t^0, \rho_t^0, \xi_t^{LM0}, \xi_t^{IS0}\right),$$

that is, the initial flexible-price GDP and nominal interest rate can be calculated by the IS and LM curves when they are evaluated at ξ_t^0 and the initial price level and monetary policy.

AGGREGATE SUPPLY AND DEMAND CURVES

The IS and the LM curves provide the equilibrium allocation if the price level is given. To find the date t price level one has to construct the *aggregate demand* and *aggregate supply curves*.

In fact, the aggregate demand curve is already given by equation (38). The aggregate supply curve is derived in *Appendix A.5*:

$$\frac{\left[P_t^{1-\theta} - (1-\gamma)\left(P_t^0\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\gamma^{\frac{1}{1-\theta}} P_t} = \vartheta\varphi\left(\frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+i_t}}{1+\mathcal{B}_t}\right)^{\nu} \frac{(a^n)^{\frac{1}{\alpha-1}} y_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}. \tag{40}$$

The above formula defines an implicit function which maps P_t to y_t. Hence, both the aggregate demand and supply curves are functions in the (y_t, P_t) space, and their intersection provides the equilibrium solutions for y_t and P_t.

2.7 THE OUTSIDE-MONEY VERSION OF THE MODEL

In the New Keynesian literature, the money supply typically does not appear, only the interest rate rule, or when it appears explicitly, it is in the form of outside money. Specifically, this means in the models that the money supply is controlled exclusively by the central bank, the development of the money supply is independent of the banking system, and it has no direct effect on the behavior of the banking system. That is, the provision of transaction instruments is independent from the financial intermediation.

In models where money appears in an explicit form, the demand for money is typically derived from the money-in-the-utility function approach, and it is most often assumed that in the utility function the term for money is additively separable. This assumption implies that the behavior of households is influenced by the money supply only through the interest rate, not directly.

It follows from the above assumptions that the IS curve is independent of the money stock.

The above description of outside money can be represented as a special case of our model. If $\Delta^R = 1 + \tau^F$ then equations (19), (23), (24) and (25) imply that $\lambda_t = \bar{\lambda}_t$, $\mu_t = \rho_t = 0$ and $B_t^b = B_t^l = B_t^{CB} = 0$. In this case banks' reserves holding always cover liquidity outflow therefore the interbank market does not open. Furthermore, if $\bar{\lambda} = 1$ then $M_t = D_t$, that is, each unit of demand deposits are backed by central bank reserves. As a consequence, the central bank controls exclusively the money supply.

We also assume that $\tau^D = \phi^D = \phi^{DL} = 0$, which assures that behavior of the money supply does not affect the financial intermediation function and the operation cost of the banking system.

In this outside-money version of model the IS and LM curves will take the following forms:

$$y_t = \frac{1}{\mathcal{B}_t^{om}}\left(\left(1+\mathcal{B}_t^{om}\right)k_t^{om} + \frac{y_{t+1}^h}{1+i_t}\right) + \kappa_t^{om}, \tag{41}$$

$$y_t = \frac{\left(1-\Delta_t^{D,om}\right)^{\sigma}\left(1+\mathcal{B}_t^{om}\right)}{\eta_t}\left(\frac{M_t}{P_t} - \eta_t^z k_t^{om}\right) - \frac{y_{t+1}^h}{1+i_t} + \kappa_t^{om}, \tag{42}$$

where

$$\begin{aligned}
\Delta^{D,om} &= 1+\tau^F, \\
\mathcal{B}_t^{om} &= (1+\eta_t)\beta_t^{\sigma}(1+i_t)^{\sigma-1} + \eta_t\left(1-\Delta^{D,om}\right)^{1-\sigma}, \\
k_t^{om} &= \frac{\frac{\alpha^z}{\vartheta}A_{t+1} - (1+i_t)\left(1+\eta_t^z\right)\left(1+\bar{\tau}^L\right)}{2\frac{\alpha^z}{\vartheta}A_{t+1}\omega + 2(1+i_t)\chi_t\phi^L\left(1+\eta_t^z\right)^2}, \\
\kappa_t^{om} &= \kappa^0 + \bar{\tau}^L\left(1+\eta_t^z\right)k_t + \chi_t\phi^L\left[\left(1+\eta_t^z\right)k_t\right]^2.
\end{aligned}$$

In the outside-money version the term $\phi^{DL}D_t/P_t$ is missing from the capital stock equation since financial intermediation is independent from the supply of demand deposits. The assumption $\tau^D = \phi^D = \phi^{DL} = 0$ assures the independence of financial intermediation and provision of liquidity, thus the money stock is missing from the deposit rate spread and the cost of intermediation equations.

As a consequence, monetary policy only affects the IS curve and aggregate demand through interest rates, the money stock does not have a direct impact on it. An essential difference between the inside-money and the outside money versions is that the IS curve reacts differently to changes in monetary policy: changing D_t shifts the IS curve in the general case, but not in the outside-money version.

If monetary policy is kept fixed, that is, M_t is fixed, then in the outside money version Δ^D remains fixed, and $\mathcal{B}_t$ depends on i_t only through the term $(1+\eta_t)\beta_t^{\sigma}(1+i_t)^{\sigma-1}$. In the general case when commercial banks can create money, $\mathcal{B}_t$ depends on i_t via the deposit rate spread, too. However, with our parameter choice, this difference between the two cases is negligible, hence the IS curves in the outside money version and the general case have very similar shape, the difference is visually undetectable in *Figure 2*.

In the general case Δ_t^D is decreasing in i_t, and, consequently, the term $\left(1-\Delta_t^D\right)^{\sigma}(1+\mathcal{B}_t)\eta_t$ in the LM curve is increasing in i_t. On the other hand, its impact on $\mathcal{B}_t$ is negligible. Hence, the LM curve, as a function of i_t, is flatter in the general case than in the outside money case, see *Figure 2*.

We will also apply the following notations for equations (41) and (42):

$$y_t = \widetilde{y}^{IS}\left(i_t, \eta_t, \xi_t^{IS}\right), \tag{43}$$

$$y_t = \widetilde{y}^{LM}\left(i_t, \frac{M_t}{P_t}, \eta_t, \xi_t^{IS}\right). \tag{44}$$

The above formulas emphasize that the outside-money IS and LM curves are independent of $\bar{\lambda}_t$, λ_t and ρ_t, because $M_t = D_t$ and the interbank market is closed. Furthermore, equation (43) reveals that the outside-money IS curve does not depend on M_t either.

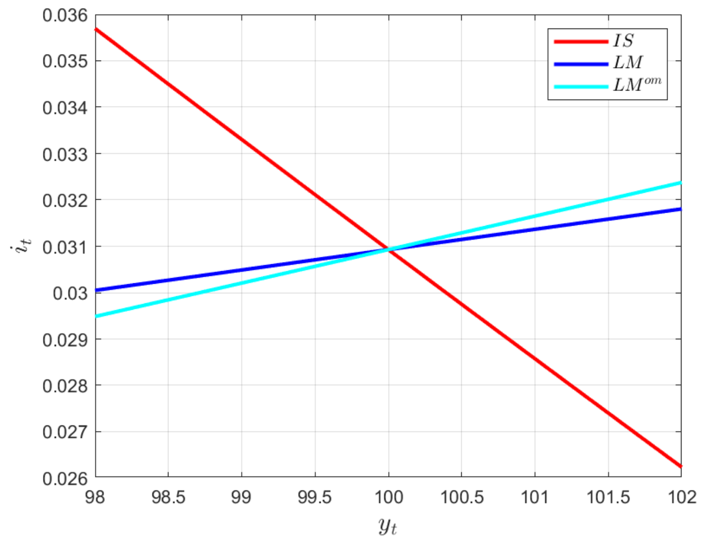

Figure 2
The IS and LM curves in the outside money and the general versions of the model. Note that the IS curves almost perfectly coincide in the two versions if monetary policy is fixed.

It is important to note that our approach is different from the way Lagos (2006) defines outside money. According to his classification, the case where banks are involved in money creation is not considered outside money, even if demand deposits are fully backed by central bank reserves. However, with the above parameter restrictions our approach is isomorphic to Lagos' outside money concept, at least within our framework.[8]

2.8 THE "OUT-OF-THIN-AIR" VERSION OF THE MODEL

Some authors (e.g. Werner, 2016) assume that loans can always be financed by demand deposits since the volume of demand deposits is not constrained by reserves at all. Although we do not agree with this view, it can also be represented as a special case of our model: if there are *no liquidity shocks* in the model ($\bar{\lambda}_t = 0$), there will be no need for the interbank market, and thus, $\rho_t = 0$, $B^I_t = 0$ and $\Psi_t = 0$.

In this case there is no connection between M_t and D_t since banks do not need reserves to diminish liquidity risk. Therefore, we assume $M_t = 0$.

In the general version of the model the volume of demand deposits is controlled by monetary policy. In the *no-liquidity-shock* version deposit holdings (D^{nls}_t) becomes endogenous. Since we have the same number of equations, to remain consistent we need another variable controlled by policy. Hence we assume that the deposit rate is controlled by regulation: $i^{D,reg}_t$.

As a consequence the deposit rate spread is given by

$$\Delta^{D,nls}_t = \frac{1 + i^{D,reg}_t}{1 + i_t},$$

[8] According to Lagos' classification, in the case of outside money, the central bank holds government securities on its balance sheet. Of course, this feature is missing from our model, as we have no government consumption and government debt. However, this is an irrelevant issue to the problem we are examining. In principle, the creation of fiat money does not require government securities to be on the balance sheets of central banks, even if this is often the case in practice. The asset side of the central bank's balance sheet may consist of loans to any other economic agents or any other asset, such as gold or foreign reserves. For example, in the new Keynesian models, money is most often created by direct central bank transfers to households.

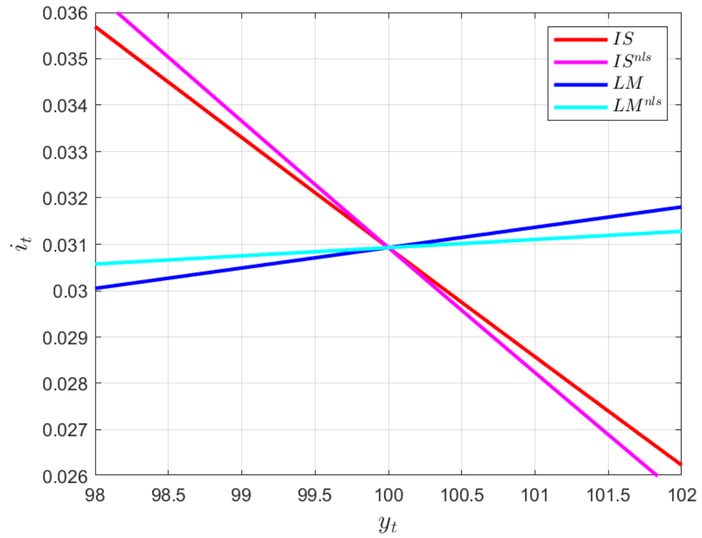

Figure 3
The IS and LM curves in the no-liquidity-shock and the general versions of the model

and it is not endogenous anymore.

Variables k_t^{nls} and D_t^{nls} are jointly determined by the following equations,

$$\begin{aligned}\Delta_t^{D,nls} &= 1-\bar{\tau}^D - 2\phi^D \frac{D_t^{nls}}{P_t} + \phi^{DL}\left(1+\eta_t^z\right)k_t^{nls},\\ k_t^{nls} &= \frac{\frac{\alpha^z}{\vartheta}A_{t+1} - \left(1+i_t\right)\left(1+\eta_t^z\right)\left(1+\bar{\tau}^L - \phi^{DL}\frac{D_t^{nls}}{P_t}\right)}{2\frac{\alpha^z}{\vartheta}A_{t+1}\omega + 2\left(1+i_t\right)\chi_t\phi^L\left(1+\eta_t^z\right)^2}.\end{aligned}$$

Furthermore,

$$\begin{aligned}\mathcal{B}_t^{nls} &= \left(1+\eta_t\right)\beta_t^\sigma\left(1+i_t\right)^{\sigma-1} + \eta_t\left(1-\Delta_t^{D,nls}\right)^{1-\sigma},\\ \kappa_t^{nls} &= \kappa^0 + \bar{\tau}^D\frac{D_t^{nls}}{P_t} + \phi^D\left(\frac{D_t^{nls}}{P_t}\right)^2\\ &+ \bar{\tau}^L\left(1+\eta_t^z\right)k_t^{nls} + \chi_t\phi^L\left[\left(1+\eta_t^z\right)k_t^{nls}\right]^2\\ &- \phi^{DL}\frac{D_t^{nls}}{P_t}\left(1+\eta_t^z\right)k_t^{nls}.\end{aligned}$$

Then the no-liquidity-shock IS and LM curves become:

$$\begin{aligned}y_t &= \frac{1}{\mathcal{B}_t^{nls}}\left(\left(1+\mathcal{B}_t^{nls}\right)k_t^{nls} + \frac{y_{t+1}^h}{1+i_t}\right) + \kappa_t^{nls},\\ y_t &= \frac{\left(1-\Delta_t^{D,nls}\right)^\sigma\left(1+\mathcal{B}_t^{nls}\right)}{\eta_t}\left(\frac{D_t}{P_t} - \eta_t^z k_t^{nls}\right) - \frac{y_{t+1}^h}{1+i_t^h} + \kappa_t^{nls}.\end{aligned}$$

Figure 3 compares the IS and LM curves in the no-liquidity-shock and the general versions of the model.

2.9 PARAMETRIZATION

The parameter values of the model are chosen in such a way to match the most important stylized facts of the banking system and the aggregate economy.

Table 2 displays the baseline (flexible price allocation) numerical values of balance sheet items of the banking system. The items represented in the table imply that

- The value of the money multiplier $D_t^0/M_t^0 = 5$, see the online dataset of Maclay, Radia and Thomas (2016).
- The ratio of stable and liquid liabilities $F_t^{h0}/D_t^0 = 1$ which is in line with Bigio and Weil (2016, page 5) who claim that demand deposit correspond to 50-60% of banks' liabilities. The empirical share of time deposits is significantly smaller, only 10-20%. However, in our model F_t^{h0} represents all other types of stable liabilities, hence it is not the exact theoretical counterpart of time deposits in reality. That is why we choose higher share for F_t^{h0}.
- The ratio of households' and firms' demand deposits $D_t^{h0}/D_t^{z0} = 1$, which is confirmed by the calculations of the MNB based on Hungarian data.[9]

Table 2
Balance sheet of the banking system – baseline values

Assets	Liabilities
$M_t^0 = 2.67$	$F_t^{CB0} = 2.67$
$L_t^0 = 26.67$	$F_t^{h0} = 13.33$
	$D_t^{h0} = 6.67$
	$D_t^{z0} = 6.67$

In our model there is no explicit lower bound for nominal interest rate (there is no cash in the model), however, we still want to avoid zero nominal interest rates in the simulations. Therefore we choose a relatively high baseline value for the nominal interest rate: $i_t^0 = 0.0309$. We choose β^0 to be consistent with the baseline interest rates, that is, $\beta^0 = 1/(1 + i_t^0)$, since the expected inflation rate is zero. Since this is a stylized model, there is no clear interpretation of the length of time periods, therefore, it should not be inferred from the magnitude of households' discount factor either.

$\Delta_t^{R0} = \Delta_t^{D0} = 0.97$ implying $i_t^{R0} = i_t^{D0} = 0$. $\Delta_t^{L0} = 1.08$, that is, we assume 8% premium on risky corporate loans which is roughly consistent with the equity premium literature. (In our model there are no equities, corporate loans represent all types of risky assets.)

We assume that the volume of loans and deposits have very moderate direct impact on the above spreads: one percent increase of loans/deposits induce 10 basis points increase in the loan rate/deposit rate spread, that is $\phi^L = 0.0019$ and $\phi^D = 0.0039$. We assume that the cost function parameter capturing the cross effect of D_t and L_t is weaker than the parameters of the direct effects, that is, $\phi^{DL} = 0.0015$. If we also assume that one percent increase in the loan stock implies on average 0.43 percent increase in demand deposits, these parameters are consistent with Calice and Zhou (2018), who estimated the effect of gross loans on the net interest margin from bank-level panel data on more than 14,000 commercial banks in 160 countries for the period 2005-2014.

The baseline consumption is 78.75%, the baseline investment is 20%, the baseline cost of intermediation is 1.25% of the real GDP.

The particular parameters values which are consistent with the above baseline (flexible price) allocation can be found in *Appendix A.4*.

[9] We would like to thank Bálint Dancsik for the data and the calculations.

3 The effect of shocks with unchanged monetary policy

In this section we show how shocks to some model parameters shift the IS and LM curves when the monetary policy does not react to these shocks. Recall that this means unchanged M_t^0, Δ_t^{R0} and Δ_t^{B0}. Although the main objective of our paper is to understand how interest rate rule based monetary policy works in the presence of inside money, it is worth breaking it down into two steps. First, we look at what happens as a result of macroeconomic shocks if monetary policy does not respond. In the second step, we examine the impact of the monetary policy response.

As discussed, an analysis based on the IS and LM curves takes the price level P_t given, but it can be generalized for endogenous price level by combining them with the aggregate supply curve (40). However, the main issues regarding inside money are related to the IS and LM curves, hence we restrict our attention to them. In the forthcoming analysis we assume $P_t = P_t^0$, which is an extreme form of sticky prices when $\gamma = 0$, that is, firms are unable to adjust the prices they set at the beginning of date t after the realization of the macroeconomic shocks.

We restrict our analysis to cases where moderate shocks hit the economy. In the case of shocks that cause an excessive recession, the economy reaches the zero lower bound of the nominal interest rate, and this problem is not addressed in our model. In the case of shocks that cause excessively large expansion, the economy runs into capacity constraints, which has a strong inflationary effect and the assumption of rigid prices is not plausible.

Our starting point is the flexible price allocation which is determined by the IS and LM curves at ξ_t^0. We then examine how changes in exogenous variables (ξ_t^*) shift the IS and LM curves and thus change economic allocation.

We compare the effects between the outside money and the inside money versions of the model. The first two shocks are traditionally associated with the LM curve, nevertheless, as we will show, they affect the IS curve, too, even if just slightly. Then we discuss parameter changes that affect directly the IS curve, but also the LM curve indirectly.

3.1 LM SHOCKS

First, we consider the effect of the unexpected change of the size of liquidity shocks, captured by the $\bar{\lambda}$ parameter. Note that $\bar{\lambda}$ does not appear in the formulas of the IS and LM curves of the outside money version. As a consequence, the $\bar{\lambda}$ shock does not have any impact on these curves.

In the general case $\lambda_t^* = \bar{m}_t \bar{\lambda}_t^*$ and $\bar{m}_t > 0$ (see equation (29)) and $D_t^* = M_t/\lambda_t^*$. As a consequence, a decrease of $\bar{\lambda}_t$ results in an increase of the money supply, therefore the LM curve shifts downward. Since $\phi^{DL} > 0$, the capital stock k_t increases when the liquidity shocks become smaller. The intuition behind this is the following: with lower expected deposit outflow, banks are more willing to create deposits via lending, because the probability of running short of reserves is less. This implies that the IS curve shifts upward, see *Figure 4*.

The next experiment is a decrease in money demand, that is, in η_t. Although the variable η_t appears in $\mathcal{B}_t$, and shifts the IS curves in both cases, its impact is negligible with our parameter choice.

Obviously, the main effect of a negative money demand shock is shifting the LM curve. In both cases decreasing money demand shifts the LM curve downward. The effect will be expansionary in both cases, but in the outside money world the expansion is larger, see *Figure 5*.

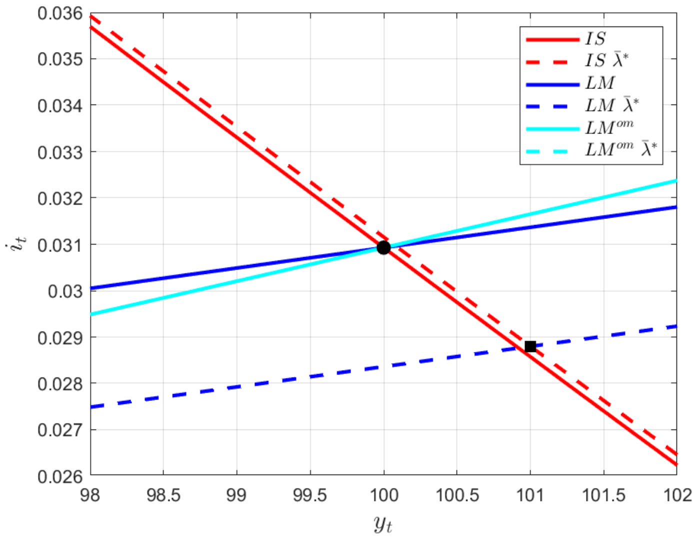

Figure 4
The effect of the decrease in the liquidity shocks on the IS and LM curves

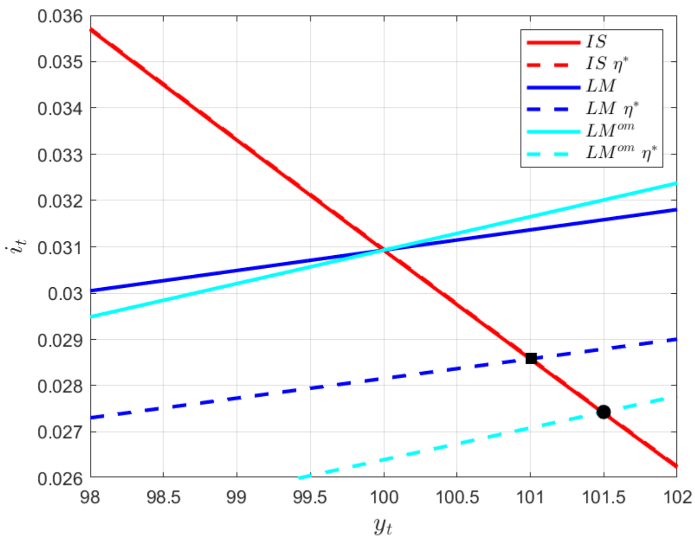

Figure 5
The effect of the decrease in money demand on the IS and LM curves

3.2 IS SHOCKS

Our next shock is an unexpected change in the discount factor (β_t), which affects the propensity to save, and thus, has direct impact on the IS curve. Smaller β_t implies less saving in period t, as the relative marginal utility of future consumption decreases. As a consequence, consumption and money demand increases.

Formally, if β_t decreases, $\mathcal{B}_t$ decreases as well, which shifts the IS curve upward. Because of the increase in money demand, with unchanged money supply the LM curve will shift upward, too. The overall effect is expansionary, however, in the inside-money case it is larger, because of its smaller slope.

A_{t+1} is the productivity in the intermediary good sector. If A_{t+1} increases, firms invest more and k_t increases which shifts the IS curve to the right.

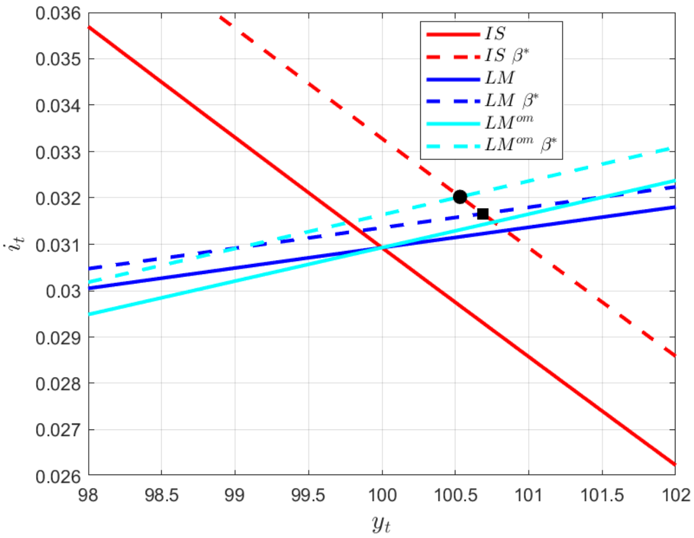

Figure 6
The effect of the decrease of the discount factor on the IS and LM curves

If k_t increases, firms' money demand will increase. Furthermore, the production will increase at date $t+1$, hence the expected income y^h_{t+1} of households will increase as well. As a consequence, both sectors' money demand increases which shifts the LM curve upward. The overall effect will be slightly expansionary in both cases, see *Figure 7*.

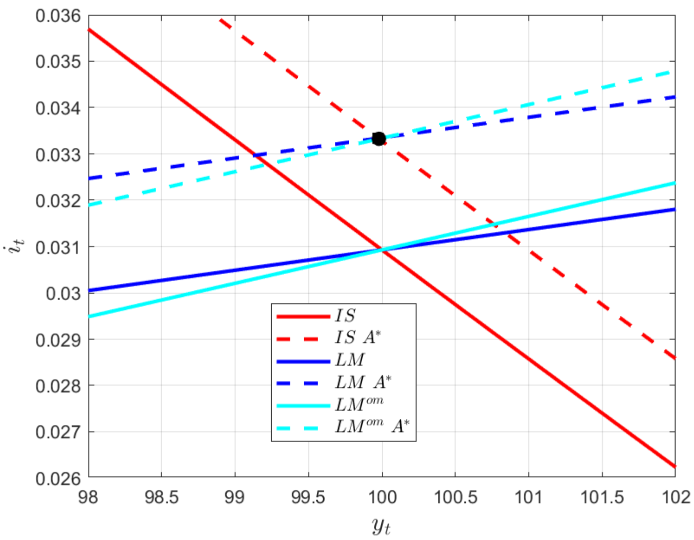

Figure 7
The effect of an increase in productivity on the IS and LM curves

Shocks to the credit risk of corporate loans (χ_t) and firms' money demand (η^z_t) have similar affect to that of productivity discussed above. The details can be found in *Appendix A.7*.

3.3 SUMMARY

Using the IS and LM curves, we compared how the economy responds to macroeconomic shocks in the presence of outside and inside money. Since the curves in the two cases differ already in the baseline scenario and also react differently to the shocks, it is not surprising that we obtain numerically different results in the two cases. On the other hand, if we focus on the direction of change in real output, the results will not differ qualitatively: if the effect of a shock is expansive for inside money, it will remain expansive for outside money.

4 Implementation of an interest rate rule

In the previous section, we have shown how the presence of inside money modifies the macroeconomic effects of exogenous shocks if monetary policy does not respond to them. At the same time, it may rightly be argued that all of this is not really important, as if monetary policy responds to shocks, these differences may disappear or become insignificant. The objective of monetary policy is to facilitate the achievement of different inflation-output combinations in response to exogenous shocks, and the central bank controls its monetary policy instruments to achieve this macroeconomic objective. In the case of inside money, monetary policy must be implemented differently in order to achieve the same macroeconomic goal compared to the case of outside money, but the exact mechanism of implementation is not necessarily interesting from a macroeconomic point of view.

Of course, this line of reasoning is based on the implicit assumption that the same macroeconomic objective can always be achieved with monetary policy, regardless of the role of the banking system in the process of money creation. In this section, we will investigate the validity of this assumption.

We assume that the monetary policy behavior required to achieve the above objectives can be described by the interest rate rule (26), which becomes simpler in the context of IS and LM curves, where prices are completely rigid, that is, $P_t = P_t^0 = P_{t-1}$,

$$\frac{i_t}{i_t^0} = \left(\frac{y_t}{y_t^0}\right)^{\psi_y},$$

Therefore the rule is a relationship between the output and an interest rate, and it can be represented by an increasing curve in the output-interest rate space. The actual output-interest rate combination desirable to monetary policy is given by the intersection of the IS and the interest-rate-rule curves.

We introduce the following notation for the output and interest rate determined by the interest rate rule and the IS curve:

$$y_t^{ir} = y^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right), \tag{45}$$

$$i_t^{ir} = i^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right). \tag{46}$$

Since the IS curve depends on the shocks and D_t, λ_t, ρ_t (recall equation (36)), y_t^{ir} and i_t^{ir} also depend on these variables. Without loss of generality, we assumed that $P_t = P_t^0 = 1$.

4.1 LM SHOCKS

As pointed out by Poole (1970), an important advantage of conducting monetary policy on the basis of an interest rate rule is that it stabilizes the real output in the presence of fluctuations in the supply and demand of money. This is illustrated by *Figure 8* in the case of outside money.

In the figure, the LM curve shifts because money supply changes from M_t^0 to M_t^* due to an exogenous shock. If monetary policy did not react, the equilibrium output would be the intersection of the IS and LM curves, so the output would increase. If, on the other hand, monetary policy reacts in line with the interest rate rule, then it must implement the allocation (y_t^0, i_t^0) given by the intersection of the unchanged IS and the interest-rate-rule curves.

If we denote the reaction of monetary policy by M_t^+, the *post-reaction* money supply becomes $M_t^* + M_t^+$. Monetary policy must choose M_t^+ so that the LM curve returns exactly to its original position.[10] Using the notation introduced in equation (44), we

[10] Suppose monetary policy controls the amount of money, but it can only do so with some stochastic error (ϵ_t^M) After the error is realized ($M_t^* = M_t^0 + \epsilon_t^M$), it tries to correct it ($M_t^* + M_t^+$).

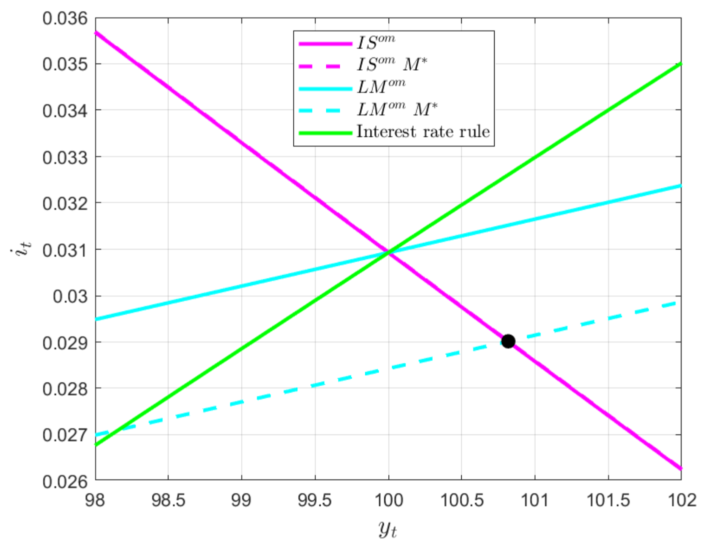

Figure 8
Money supply shock in the outside-money version of the model

can express the formal condition for implementing the interest rate rule:

$$y_t^0 = \overline{y}^{LM}\left(i_t^0, M_t^* + M_t^+, \eta_t^0, \xi_t^{IS0}\right).$$

The above condition is obviously satisfied if

$$M_t^+ = -\left(M_t^* - M_t^0\right),$$

that is, if monetary policy reduces the money supply by exactly as much as it increased as a result of the exogenous shock.

The example above illustrates why it is advisable to follow an interest rate rule in the presence of inside money in the case of an LM curve shocks (i.e. money market shocks). This is because by doing so the turbulence of the money market does not cause unnecessary fluctuations in the real economy, and the effects of the shocks can be completely eliminated.

In what follows, we examine whether the above implementation is feasible in the presence of inside money in the case of shocks to money supply or demand. However, the problem is now more complicated, since in contrast to the case of outside money, the IS curve also reacts to changes in the amount of money. That is, when monetary policy pushes the LM curve back to its original position, the IS curve is also shifted, and it is not certain that it will eventually return to its original position.

Although the task of monetary policy is more complicated in the case of inside money, it has more instruments at its disposal: beyond the money stock, it also controls λ and ρ. Therefore, as equations (32), (33), and (34) reveal, monetary policy is able to determine the values of $\mathcal{B}_t$ and κ_t in the formula of the IS curve. As a consequence, the change of $\mathcal{B}_t$ and κ_t can offset the effect of changing money supply.

Let D_t^+ denote how much monetary policy changes the money supply in response to shocks, and λ_t^+ and ρ_t^+ the values of the variables in question as determined by the monetary policy response. Then, in order to stabilize output, the monetary policy response must meet the following conditions:

$$\begin{aligned} y_t^0 &= y^{IS}\left(i_t^0, D_t^* + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM*}, \xi_t^{IS0}\right), \\ y_t^0 &= y^{LM}\left(i_t^0, D_t^* + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM*}, \xi_t^{IS0}\right) \end{aligned}$$

where we applied the notations introduced in equations (36) and (37). As shown in *Appendix A.6*, there exist D_t^+, λ_t^+, ρ_t^+ which satisfy the above conditions. Generally, the solution only makes economic sense if $0 < \lambda_t^+ \leq \bar{\lambda}_t$ and $0 < \rho_t < 1$. However, there is no guarantee that an economically meaningful solution will be found for shocks of any magnitude. If not, the monetary policy that would stabilize the output cannot be implemented.[11]

So it is not generally the case that all monetary policies that can be implemented in the case of outside money can also be implemented in the case of inside money. In the following we examine in what range of shocks $\bar{\lambda}_t^*$ and η_t^* the interest rate rule can be implemented, or, in other words, the output can be fully stabilized. Recall that y_t^{islm} is the output determined by the intersection of the IS and LM curves, see equation (38), that is, the output level that the shocks would cause without a monetary policy response. As discussed in the previous section, we limit our attention to moderate shocks, that is, to the range of shocks where $|y_t^{islm}/y_t^0 - 1|$ is not larger than 5 percent (0.05). We also exclude policies which require unrealistically low values of Δ^R for their implementation: the smallest possible value of Δ^R we consider is 0.95.[12]

First, consider the exogenous change of $\bar{\lambda}_t$, which can be interpreted as a money supply shock in the inside money case. As discussed in *section 2.4*, a decrease of $\bar{\lambda}_t$ results in a decrease of λ_t and an increase in the money supply, D_t. It is easy to show that in this case, if λ_t^+ and ρ_t^+ are chosen in such a way that they restore the pre-shocks value of κ_t and Ψ_t in equations (32) and (33), and

$$D_t^+ = -\left(D_t^* - D_t^0\right),$$

then the money supply shock can be eliminated and the output remains equal to y_t^0.

Figure 9 displays the range [0.7059, 0.7929] around the baseline value ($\bar{\lambda}^0 = 0.7464$) where the deviation of y_t^{islm} from y_t^0 is no more than 5 percent (see the left panel). The right panel reveals that over the whole range $0 < \lambda_t^+ < \bar{\lambda}_t$ and $0 < \rho_t < 1$, that is, the interest rate can be implemented, and the output can be stabilized at y_t^0.

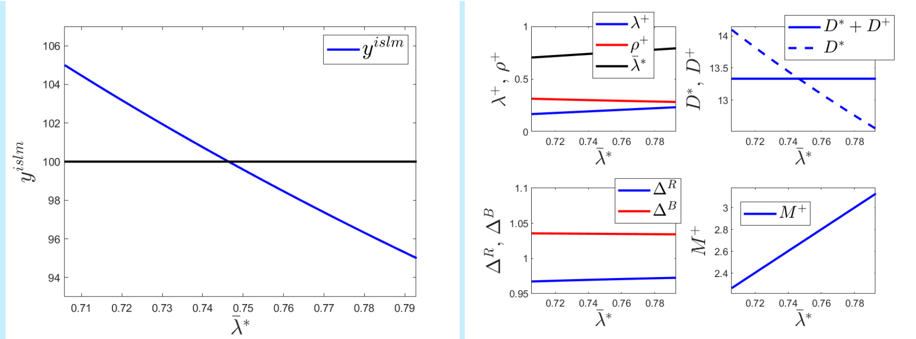

Figure 9
Implementation of the interest rate rule – money supply shock

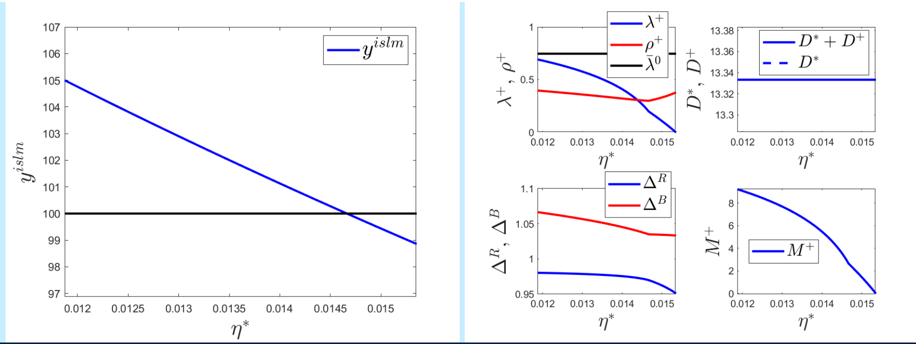

Figure 10
Implementation of the interest rate rule – households' money demand shock

[11] If we take the interest rate rule strictly, in the case of inside money, monetary policy should not push back the curves to the starting point. This is because in the presence of inside money, the shocks of the money market shifts IS curve as well, and instead of the starting point, the LM curve should be pushed to the intersection of the IS and the interest-rate-rule curve. At the same time, this point is very close to the starting point, and if the starting point is targeted, we retain the useful feature seen in the case of outside money that turbulences in the money market do not cause real economic fluctuations at all.

[12] As discussed, there is no cash in our model, so the zero lower bound on nominal interest rates does not appear explicitly. However, we want to avoid examining cases that are irrelevant in practice. Therefore, we exclude from our analysis the cases where the interest paid on the central bank reserve is unrealistically low.

The next shock we investigate is a shock to the households' money demand. Contrary to the $\bar{\lambda}$ case, it is not always possible to neutralize the effect of the shock within the range that would result in less than 5 percent change in the output without the response of monetary policy. The baseline value of η_t is 0.0147. As it turns out, monetary policy can fully offset shocks that are within the range of $[0.0119, 0.0154]$, which would correspond to a change in output between -1.14 and 5 percent without monetary policy reaction (see *Figure 10*).

The figure also reveals that the appropriate policy in this case leaves the money supply unchanged, that is, $D_t^+ = 0$, and the positions of the IS and the LM curves are adjusted only by λ_t^+ and ρ_t^+.

4.2 IS SHOCKS

As an illustrative example, let us first examine how the monetary policy based on the interest rate rule can be implemented in the outside money version of the model in the presence of households' discount factor shock. In this case, monetary policy should implement the output determined by the intersection of the moving IS curve and the interest rate rule (see *Figure 11*). This is possible by shifting the LM curve to this point by changing the money supply.

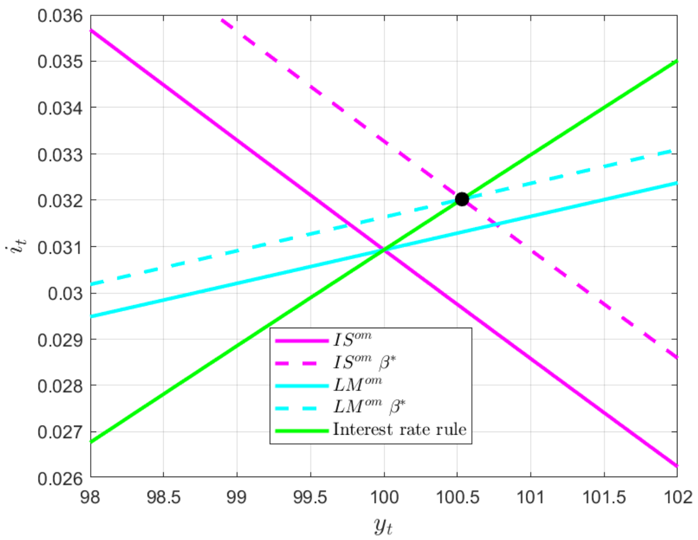

Figure 11
Households' discount factor shock in the outside-money version of the model

Formally, M_t^+ must be chosen so that the following condition is met,

$$\widetilde{y}_t^{ir*} = y^{LM,om}\left(\widetilde{\iota}_t^{ir*}, M_t + M_t^+, \eta_t, \xi_t^{IS*}\right),$$

where $\widetilde{y}_t^{ir*}$ and $\widetilde{\iota}_t^{ir*}$ are determined by the IS and the interest-rate-rule curves.

In the case of inside money, the problem is similar to that of the previous section: if the LM curve is shifted to the desired point by changing the money supply, then the IS curve will move away from the intersection point. This can still be handled by changing monetary policy to affect λ_t and ρ_t. Formally, the following conditions must be met:

$$\begin{aligned} y_t^{ir*} &= y^{IS}\left(i_t^{ir*}, D_t + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM0}, \xi_t^{IS*}\right), \\ y_t^{ir*} &= y^{LM}\left(i_t^{ir*}, D_t + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM0}, \xi_t^{IS*}\right), \end{aligned}$$

where, using the notations introduced in equations (45) (46), y_t^{ir*} and i_t^{ir*} are defined as

$$\begin{aligned} y_t^{ir*} &= y^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS*}\right), \\ i_t^{ir*} &= i^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS*}\right), \end{aligned}$$

that is they are determined by the intersection of the post shock IS curve and interest rate rule.

In the following, we examine the size of the shocks for which the monetary policy defined by the interest rate rule can be implemented in the manner defined by the above equations, and we measure the output effect of the shocks by the change of y_t^{ir*} relative to y_t^0. Again, we focus on the range of shocks when $|y_t^{ir}/y_t^0 - 1|$ is no more than 5 percent (0.05), and exclude policies which require too low values of Δ^R to implement.

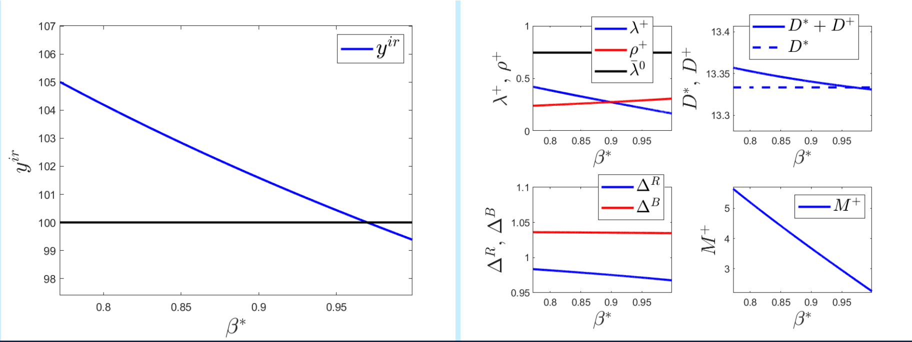

Figure 12
Implementation of the interest rate rule – households' discount factor shock

First, we investigate shocks to the discount factor (β_t). Its baseline value equals to 0.97. Although, in finite time horizon it is not necessary to assume that $\beta_t < 1$, we use this widespread assumption, hence the highest value of β_t we consider is 0.999. In a wide range of β_t the interest rate rule can be implemented. *Figure 12* displays the range of [0.7722, 0.999] where at the lower limit the output is higher by 5 percent than its baseline value, as the left panel reveals.

In the case of the productivity shock it is not always possible to implement the monetary policy rule within the range that would result in less than 5 percent change in the output without the response of monetary policy. The central bank can implement the interest rate rule over the interval [1.7767, 1.826] (baseline value is 1.8178).

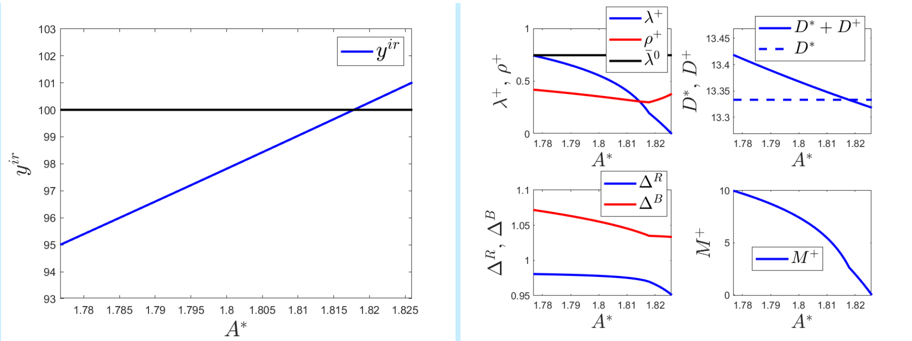

Figure 13
Implementation of the interest rate rule – productivity shock

The implementation of the interest rate rule in the presence of credit risk and corporate money demand shocks is discussed in *Appendix A.7*.

4.3 SUMMARY

In this section, we have demonstrated that the interest rate rule can be implemented for inside money as well, but requires a more sophisticated monetary policy than for outside money. It needs all the three instruments of monetary policy used in a coordinated way.

In the case of money supply and discount factor shocks, the above implementation is possible for a fairly wide range of shocks.

In the other cases, the interest rate rule can only be implemented in a narrower range of possible values of shocks. In these cases, the problem is that for large enough shocks, unrealistically low values of i_t^R (interest paid on reserves) would be required for implementation.

5 Approximation of the interest rate rule

In the previous section, we saw that it is possible to implement a monetary policy based on an interest rate rule even in the case of inside money, but we have also shown that this is only true for a limited range of shocks.

But there is another problem with the implementation. In order to be perfectly able to implement the interest rate rule, the central bank must know the exact structure of the economy and the numerical values of the parameters, and on this basis it must coordinate the control of its three instruments with extreme precision. In reality, central banks do not have such an accurate knowledge of the economy and cannot conduct such a sophisticated monetary policy.

Therefore, in this section, we examine the consequences of the limited ability to implement the interest rate rule. Specifically, we assume that the central bank responds to shocks only with the money supply, and as a result, it can only approximately stabilize the output in the event of LM shocks and reach the (y_t^{ir*}, i_t^{ir*}) allocation in the presence of IS shocks.

Let us denote the central bank's post shock reaction in money supply by D_t^a. Assume that the central bank chooses this so that the IS, LM, and interest-rate-rule curves intersect each other at the same point, but this point does not necessarily match the (y_t^0, i_t^0) or (y_t^{ir*}, i_t^{ir*}) allocations. That is, D_t^a is chosen in such a way that the resulting (y_t^a, i_t^a) allocation satisfies the following conditions:

$$
\begin{aligned}
y_t^a &= y^{ir}\left(D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right), \\
i_t^a &= i^{ir}\left(D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right), \\
y_t^a &= y^{LM}\left(i_t^a, D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right).
\end{aligned}
$$

The first two conditions guarantee that (y_t^a, i_t^a) is at the intersection of the IS and the interest-rate-rule curves. The third assures that the LM curve is also on this point.

5.1 LM SHOCKS

Figure 14 displays the approximate implementation of the interest rate rule for the money supply shock. The left panel shows the shift of the IS and LM curves as a result of the shock without monetary policy response. The symbol '⍰' represents the (y_t^0, i_t^0) allocation that monetary policy would achieve if the interest rate rule were perfectly implemented.

The right panel shows the shift of the IS and LM curves as a result of the monetary policy response. The symbol '♦' represents the (y_t^a, i_t^a) allocation. The figure reveals that the two allocations are very close to each other, so in the case of a money supply shock, the simpler monetary policy closely approximates the results of the sophisticated one.

This is confirmed by *Table 3*. The table shows the fluctuation of output for different sizes of the shock if there is no monetary policy reaction, and the extent to which the approximate implementation of the interest rate rule will stabilize output. It is clear that the stabilization is quite successful: even in the case of shocks capable of causing 5 percent change in output, the approximation deviates from y_t^0 at most by 0.01 percent.

Figure 15 displays the approximate implementation of the interest rate rule in the case of households' money demand shock. It can be seen visually that the error of the approximation is now larger than in the previous case.

This is confirmed in *Table 4*. Even in the case of shocks that could potentially cause 5 percent output fluctuations, monetary policy allows only around 0.25 percent fluctuations. So it can neutralize around 95 percent of the output impact of the shock. Although this is an order of magnitude larger fluctuation than in the previous case, it is still a fairly successful stabilization of the economy.

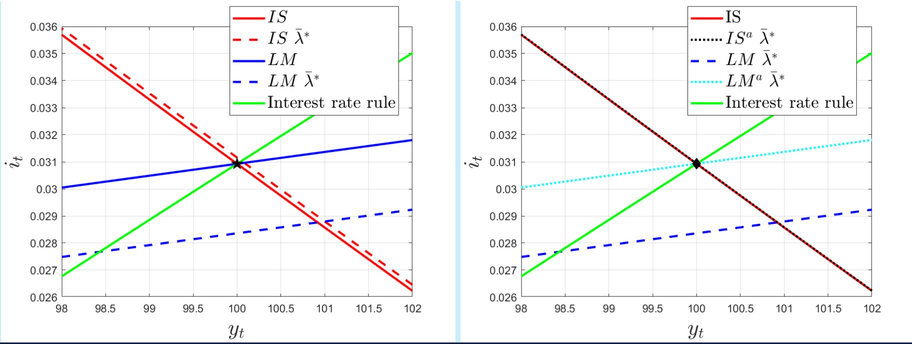

Figure 14
Approximate implementation of the interest rate rule – money supply shock

Table 3

$\bar{\lambda}_t^*$	0.7929	0.7735	0.7552	0.7379	0.7215	0.7059
y_t^{islm}	95	97	99	101	103	105
y_t^0	100	100	100	100	100	100
y_t^a	99.99	99.99	100	100	100.01	100.01

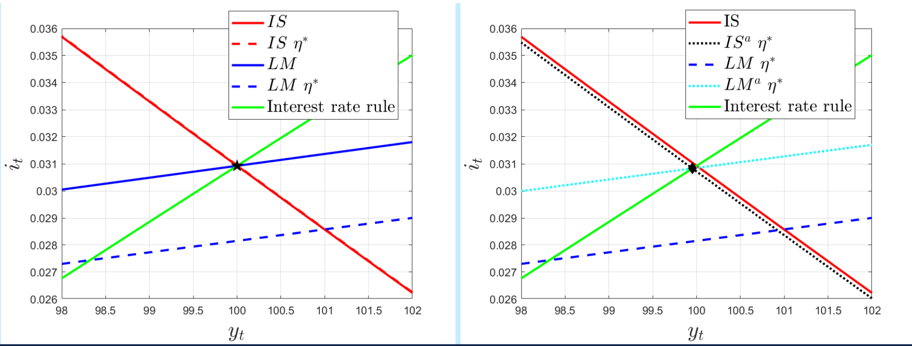

Figure 15
Approximate implementation of the interest rate rule – households' money demand shock

Table 4

η_t^*	0.0178	0.0165	0.0153	0.0141	0.0129	0.0119
y_t^{islm}	95	97	99	101	103	105
y_t^0	100	100	100	100	100	100
y_t^a	100.25	100.15	100.05	99.95	99.86	99.77

5.2 IS SHOCKS

Figure 16 displays the approximate implementation of the interest rate rule for the discount factor shock. As can be seen, similarly to the money supply shock, the approximation is almost perfect in this case as well, which is also confirmed by *Table 5*

In contrast to the previous case, as *Figure 17* reveals, in the case of a productivity shock, the error of approximation is no longer negligible, although it is still not very large. The figure also shows that in the case of an approximate implementation, the shock causes more fluctuation in output than in the case of a perfect implementation.

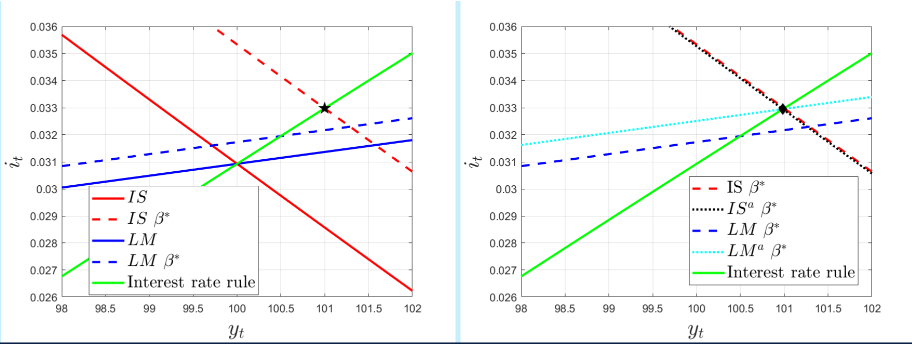

Figure 16
Approximate implementation of the interest rate rule – households' discount factor shock

Table 5

β_t^*	0.999	0.9253	0.844	0.7722
y_t^{ir}	99.39	101	103	105
y_t^a	99.39	100.98	102.95	104.92

Table 6 also demonstrates that the approximate implementation amplifies the output effect of shocks: it increases the output effect of the perfect implementation by about 1.06 times.

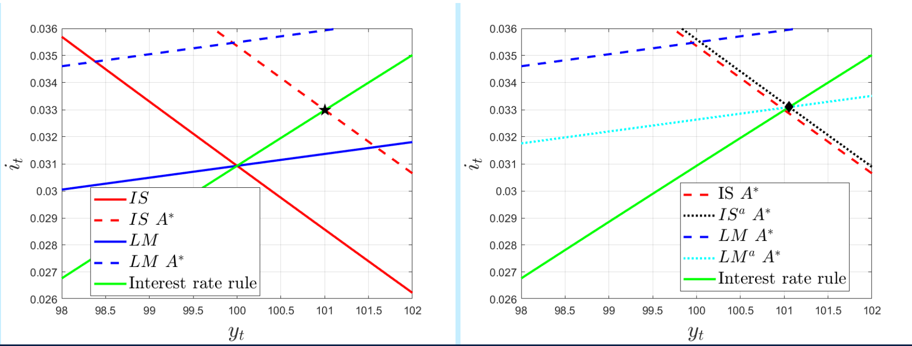

Figure 17
Approximate implementation of the interest rate rule – productivity shock

Table 6

A_t^*	1.7767	1.7933	1.8097	1.8259	1.8419	1.8578
y_t^{ir}	95	97	99	101	103	105
y_t^a	94.73	96.84	98.95	101.06	103.16	105.28

The case of credit risk and corporate money demand shocks can be found in *Appendix A.7*.

5.3 SUMMARY

In this section we considered what happens when the central bank has limited ability to pursue sophisticated monetary policy and controls only the supply of reserves.

We found that, of course, it is not possible to perfectly implement the monetary policy rule in this case, only to approximate it, but the error of the approximation does not seem significant from a practical point of view.

6 “Out-of-thin-air” money creation

As discussed in *section 2.8*, if banks are not hit by liquidity shocks, monetary policy will not constrain banks’ creation of demand deposits, meaning that banks will be able to generate unlimited amount of money “out of thin air”. According to some authors, such as Werner (2016), this is an empirically plausible description of the operation of commercial banks and the consequences of this invalidates some of the standard claims of macroeconomics. As a result, macroeconomic models that ignore the money creation of commercial banks lead to completely false conclusions.

The argument in support of this is as follows: As the creation of banks’ demand deposits is not restricted by monetary policy, banks are able to create their own resources. When granting a loan, they create a demand deposit of the same size as the loan. So banks do not need external liabilities, therefore the function of financial intermediation ceases. As a result, investments are not affected by savings, they are only determined by bank financing.

We do not agree that liquidity shocks are empirically negligible and that “out-of-thin-air” money creation is an empirically relevant description of banks’ behavior. For example, the yearly turnover on the overnight interbank market in the euro area exceeds the stock of households’ and non-financial corporates’ demand deposits, see Arciero et al. (2016).

But as discussed in *section 2.8*, the lack of liquidity shocks can be represented as a special case of our model ($\bar{\lambda}_t = 0$), hence we are able to examine the validity of the statement that standard macroeconomics inferences lose their validity in this case. The operation of the model in the case of *no-liquidity-shocks* is illustrated by the response of the *IS* and *LM* curves to the shock of households’ demand for money (η_t^*) and willingness to save (β_t^*). *Figures 18* and *19* compares the *no-liquidity-shocks* and the general case under passive monetary policy.

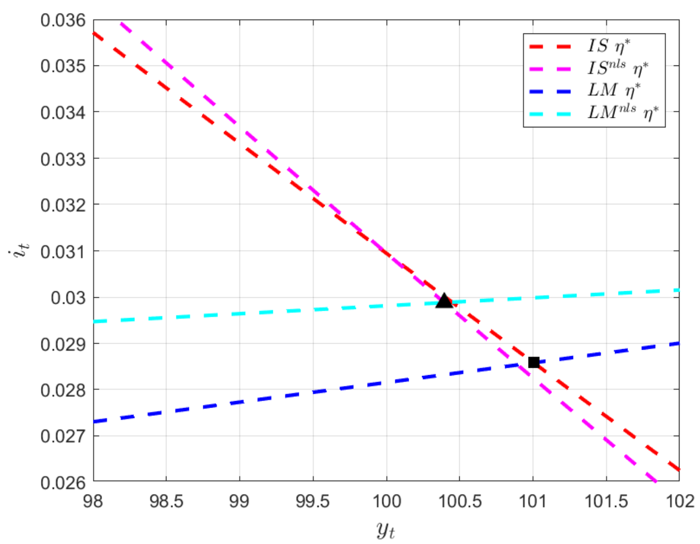

Figure 18
Comparison of the no-liquidity-shock and the general cases: reaction to money demand shock

The figures obviously reveal that “out-of-thin-air” money creation quantitatively modifies the results but leads to qualitatively similar results to the general case. So it is not true that the assumption of “out-of-thin-air” money creation would fundamentally change macroeconomic reasoning.

This is especially interesting in the case of a shock to the willingness to save (discount factor shock). *Figure 19* shows that an exogenous change in the saving behavior of households affects the economy in a very similar way in both cases. This finding refutes the claim that savings have no effect on investment in the “out-of-thin-air” case.

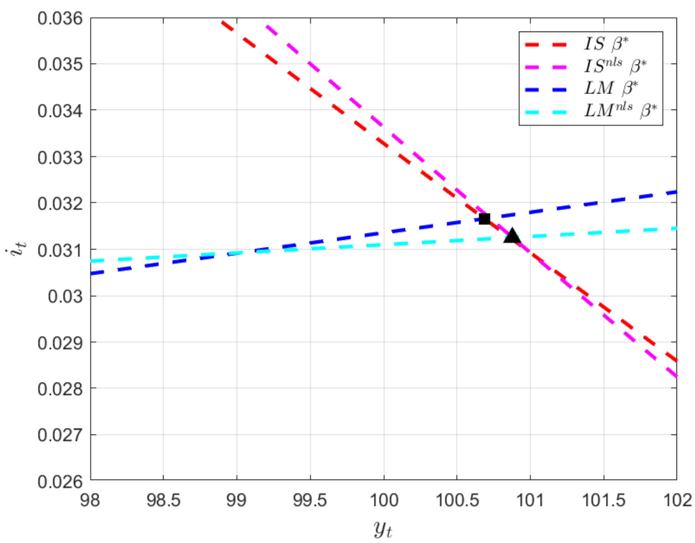

Figure 19
Comparison of the no-liquidity-shock and the general cases: discount factor shock

The explanation for this is simple. On the one hand, as Jakab and Kumhof (2019) emphasize, although without liquidity shocks the amount of central bank reserves does not limit deposit creation, the demand for money by households and corporations does. As a result, households' and firms' demand deposits do not fully finance corporate loans, hence banks still need external long-run liabilities (F_t^h), so financial intermediation will not cease. On the other hand, as shown in *Appendix A.2*, $k_t = D_t^h/P_t + F_t^h/P_t = s_t$, so demand deposits are also part of household savings, i.e. they are also part of financial intermediation.

So in the "out-of-thin-air" case, just as in the general case, investments are simultaneously determined by firms' demand for capital, the behavior of banks and the willingness of households to save.

7 Conclusions

We generalized the traditional IS and LM curves to dynamic general equilibrium models to examine the macroeconomic consequences of banks' creation of inside money. We used a simple two-period model to study the problem, however, our framework based on the generalized IS and LM curves can be applied in more complex general equilibrium models, too.

The starting point of our analysis was the observation that financial intermediation and the provision of transaction instruments cannot be separated in the modern banking system, they are inherently mixed. The close connection of the two function creates a link between the IS and LM curves since the financial intermediation function is part of the relationship between savings and investment, or, translated into the language of modeling, of the IS block of macroeconomics models, while the provision of transaction instruments is part of the LM block. Hence, unlike in models only with outside money, changing the money supply affects both the IS and LM curves. Moreover, this is true not only for monetary policy, but also for all exogenous shocks. In models with only outside money, one can imagine exogenous shocks which shift either the IS curve only or the LM curve only. However, adding inside money to the model creates a new link between the IS and LM curves, and it is no longer possible to affect the two curves separately.

First, we studied the impact of exogenous macroeconomic shocks in the case of passive monetary policy. Due to the above additional relationship between the two curves, there is always quantitative difference between the impact of shocks in a model version with only outside money and the version with inside money. However, despite the quantitative differences, the results are qualitatively similar in the two model versions.

Then we examined whether the approach of the New Keynesian literature is valid, namely, whether the macroeconomic effects of monetary policy can be satisfactorily described by an interest rate rule and the IS block of the model without addressing the details of the money supply. We have shown that despite the complexity of the creation of inside money, it is possible to implement perfectly a monetary policy based on the IS curve and an interest rate rule, although it requires a more complex toolkit of monetary policy implementation than assumed in models with only outside money.

However, the above equivalence result is valid only in certain limited ranges of the shocks. That is why, in addition to the perfect implementation of a policy based on the interest rate rule, we also examined its approximation and we have found that the error of the approximation is rather small for most shocks.

We have also shown that despite some current views the existence of inside money does not invalidate the common macroeconomic wisdom that investment are linked to savings: both savings and financing matter in determining investments.

This paper has demonstrated that a framework based on the generalized IS and LM curves is suitable for investigating problems where the details of the money creation process of the banking system matter. We have shown that the approach of the New Keynesian macroeconomics to examine the effects of monetary policy using the IS block and the interest rate rule of the model, abstracted from money creation, is justified.

In our paper, we examined the role of inside money under normal circumstances when the economy is not hit by extreme shocks and the nominal interest rate does not reach its zero lower bound. A natural extension of this research could be to use the framework of generalized IS and LM curves to examine situations where the nominal interest rate has reached its lower bound, the economy is in liquidity trap, and the abundance of liquidity makes monetary policy ineffective. The applied framework is also suitable for analyzing issues related to the money creation process such as unconventional monetary policies or central bank digital currency.

References

[1] Arciero, L., R. Heijmans, R. Heuver, M. Massarenti, C. Picillo and F. Vacirca, (2016). How to Measure the Unsecured Money Market: The Eurosystem's Implementation and Validation Using TARGET2 Data, *International Journal of Central Banking*, 12(1), 247-280.

[2] Bernanke, B., M. Gertler and S. Gilchrist (1999). The Financial Accelerator in a Quantitative Business Cycle Framework, in J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, Amsterdam: North-Holland.

[3] Bigio, S. and P.O. Weill (2016). A Theory of Bank Balance Sheets, UCLA.

[4] Boissay, F., F. Collard and F. Smets (2016). Booms and Banking Crises. *Journal of Political Economy*, 124(2), 489-538.

[5] Calice,P. and N. Zhou (2018). Benchmarking costs of financial intermediation around the world, Policy Research Working Paper Series 8478, The World Bank.

[6] Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolove, L. Stracca, J. Suarez and A.P. Vardoulakis (2015). Capital Regulation in a Macroeconomic Model with Three Layers of Default, *International Journal of Central Banking*, 11(3), 9-63.

[7] Cúrdia, V. and M. Woodford (2016). Credit Frictions and Optimal Monetary Policy, *Journal of Monetary Policy*, 84, 30-65.

[8] Deutsche Bundesbank (2017). The role of banks, non-banks and the central bank in the money creation process, Monthly Report, April 2017.

[9] Eggertsson, G.B and P.R. Krugman (2012). Debt, Deleveraging and the Liquidity Trap: a Fisher-Minsky-Koo Approach, *Quarterly Journal of Economics*, 127(3), 1469-1513.

[10] Galí, J. (2015). *Monetary Policy, Inflation and the Business Cycle – An Introduction to the New Keynesian Framework and its Applications*, Princeton University Press.

[11] Gertler, M. and N. Kiyotaki (2015). Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, *American Economic Review*, 105(7), 2011–2043.

[12] Godley, W. and M. Lavoie (2007).*Monetary Economics – An Integrated Approach to Credit, Money, Income, Production and Wealth*, Palgrave Macmillan.

[13] Goodfriend, M. (2004). Narrow Money, Broad Money, and the Transmission of Monetary Policy, Federal Reserve Bank of Richmond.

[14] Jakab, Z. and M. Kumhof (2019). Banks Are Not Intermediaries of Loanable Funds — Facts, Theory and Evidence. Bank of England Staff Working Paper No. 761.

[15] Jordan, T.J. (2018). How money is created by the central bank and the banking system, Swiss National Bank.

[16] Lagos, R. (2006). Inside and outside money, Federal Reserve Bank of Minneapolis Research Department Staff Report 374.

[17] Maclay, M., A. Radia and R. Thomas (2014). Money creation in the modern economy, Bank of England Quarterly Bulletin, 2014 Q1.

[18] Piazzesi, M. and M. Schneider (2018). Payments, Credit and Asset Prices, BIS WP 734.

[19] Piazzesi, M., C. Rogers and M. Schneider (2021). Money and banking in a New Keynesian model, manuscript, Stanford University.

[20] Poole, W. (1970). Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model, *Quarterly Journal of Economics*, 84(2), May, 197-216.

[21] Shin, H.Y. (2010). *Risk and Liquidity*, Oxford University Press.

[22] Tobin, J. (1963). Commercial banks as creators of 'money', Cowles Foundation Discussion Papers No. 159.

[23] Werner, R. (2016). A lost century in economics: Three theories of banking and the conclusive evidence, *International Review of Financial Analysis* 46, 361–379.

[24] Woodford, M. (2010). Financial Intermediation and Macroeconomic Analysis, *Journal of Economic Perspectives*, 24(4), Fall, 21–44.

Appendix A

A.1 THE COST MINIMIZATION PROBLEM OF INPUT GOOD PRODUCERS

The cost minimization problem of an input producer is the following:

$$\min_{n_t, z_t} W_t n_t + P^z_t z_t,$$

subject to

$$a^n n_t^{1-\alpha} + a^z z_t \geq y_t,$$

where W_t is the nominal wage. Here we dropped the (j) index to simplify the notation.

The Lagrangian of the cost minimization:

$$\mathcal{L} = W_t n_t + P^z_t z_t + \upsilon \left(y_t - a^n n_t^{1-\alpha} - a^z z_t \right),$$

where υ is the multiplier.

The first order conditions with respect to labor and intermediate goods are

$$W_t = \upsilon a^n (1-\alpha) n_t^{-\alpha}$$

$$P^z_t = \upsilon a^z.$$

Eliminating υ yields the demand for labor,

$$n_t = \left(\frac{P^z_t}{W_t} \frac{a^n (1-\alpha)}{a^z} \right)^{\frac{1}{\alpha}},$$

and by substituting it into the production function we get the demand for intermediate goods:

$$z_t = \frac{y_t - a^n n_t^{1-\alpha}}{a^z}.$$

Hence the cost function:

$$\mathcal{C}(W_t, P^z_t, y_t) = W_t n_t + \frac{P^z_t}{a^z} \left(y_t - a^n n_t^{1-\alpha} \right).$$

Since labor demand does not depend on the output, the marginal cost function is simply

$$\mathcal{MC}_t = \frac{P^z_t}{a^z}.$$

A.2 THE SOLUTION TO THE HOUSEHOLDS' PROBLEM

The instantaneous utility function of households' is given by

$$\mathcal{U}\left(c_t, n_t, D^h_t, \zeta_t \right) = \frac{c_t^{1-\nu}}{1-\nu} + \frac{\zeta_t \left(D^h_t / P_t \right)^{1-\nu}}{1-\nu} - \varphi n_t.$$

Since we assumed that $t+1 = T$ is the final date, households have to solve the following two-period problem:

$$\max \mathcal{U}\left(c_t, n_t, D^h_t, \zeta_t \right) + \beta_t \mathcal{U}\left(c_{t+1}, n_{t+1}, D^h_{t+1}, \zeta_t \right),$$

subject to the budget constraints:

$$\begin{aligned} P_t c_t + F_t^h + D_t^h &= W_t n_t + \Pi_t + \mathcal{D}_t + T_t + (1+i_{t-1})F_{t-1}^h + \left(1+i_{t-1}^D\right) D_{t-1}^h, \\ P_t c_{t+1} + F_t^h + D_{t+1}^h &= W_t n_{t+1} + \Pi_{t+1} + \mathcal{D}_{t+1} + T_{t+1} + (1+i_t)F_t^h + \left(1+i_t^D\right) D_t^h, \end{aligned}$$

where we assumed $\zeta_{t+1} = \zeta_t$. Observe that $F_{t+1}^h = 0$ since in the final period households do not want to save. On the other hand, D_{t+1} is positive because they need demand deposit as a transaction instrument.

The Lagrangian of the household's problem:

$$\begin{aligned} \mathcal{L} = &\left(\frac{c_t^{1-\nu}}{1-\nu} + \frac{\zeta_t \left(D_t^h/P_t\right)^{1-\nu}}{1-\nu} - \varphi n_t \right) + \beta_t \left(\frac{c_{t+1}^{1-\nu}}{1-\nu} + \frac{\zeta_t \left(D_{t+1}^h/P_t\right)^{1-\nu}}{1-\nu} - \varphi n_{t+1} \right) \\ &+ \upsilon_t \left(W_t n_t + \Pi_t + \mathcal{D}_t + T_t + (1+i_{t-1})F_{t-1}^h + \left(1+i_{t-1}^D\right) D_{t-1}^h - P_t c_t - D_t^h - F_t^h \right) \\ &+ \upsilon_{t+1} \left(W_{t+1} n_{t+1} + \Pi_{t+1} + \mathcal{D}_{t+1} + T_{t+1} + (1+i_t)F_t^h + \left(1+i_t^D\right) D_t^h \right) \\ &- \upsilon_{t+1} \left(P_{t+1} c_{t+1} + D_{t+1}^h \right). \end{aligned}$$

The first order conditions with respect to c_t, c_{t+1}:

$$\begin{aligned} c_t^{-\nu} &= P_t \upsilon_t \\ \beta_t c_{t+1}^{-\nu} &= P_{t+1} \upsilon_{t+1}; \end{aligned}$$

with respect to n_t, n_{t+1}:

$$\begin{aligned} \varphi &= W_t \upsilon_t \\ \beta_t \varphi &= W_{t+1} \upsilon_{t+1}; \end{aligned}$$

with respect to F_t^h:

$$\upsilon_t = (1+i_t)\upsilon_{t+1};$$

and with respect to D_t^h, D_{t+1}^h:

$$\begin{aligned} \zeta_t \left(d_t^h\right)^{-\nu} &= P_t \left[\upsilon_t - \upsilon_{t+1}\left(1+i_t^D\right)\right] \\ \beta_t \zeta_t \left(d_{t+1}^h\right)^{-\nu} &= P_{t+1} \upsilon_{t+1}, \end{aligned}$$

where $d_t^h \equiv D_t^h/P_t$ is the real money holdings.

Combining the first order conditions one can easily derive the Euler equation

$$c_t^{-\nu} = \beta_t (1+r_t) c_{t+1}^{-\nu},$$

where

$$1 + r_t \equiv \frac{1+i_t}{P_t/P_{t+1}}$$

is the real interest rate. The Euler equation can also be expressed as

$$c_{t+1} = \beta_t^\sigma (1+r_t)^\sigma c_t, \tag{47}$$

where $\sigma \equiv \nu^{-1}$. From the first order conditions with respect to c_t and n_t one obtains the labor supply:

$$\varphi c_t^\nu = \frac{W_t}{P_t} \equiv w_t. \tag{48}$$

For date 2, it follows from the first order condition with respect to D_{t+1}^h that money demand is proportional to consumption:

$$d_{t+1}^h = \eta_t c_{t+1}. \tag{49}$$

where $\eta_t \equiv \zeta_t^{\sigma}$.

Consider the first order condition with respect to D_t and divide both side by υ_t:

$$\frac{\zeta_t \left(d_t^h\right)^{-\nu}}{\upsilon_t} = P_t \left[1 - \frac{\upsilon_{t+1}}{\upsilon_t}\left(1 + i_t^D\right)\right]$$

Using the first order conditions with respect to F_t^h and c_t, this can be rewritten as

$$\frac{\zeta_t \left(d_t^h\right)^{-\nu}}{c_t^{-\nu}} = 1 - \Delta_t^D,$$

where $\Delta_t^D \equiv (1 + i^D)/(1 + i_t)$. Rearranging it yields the money demand at date 1:

$$d_t^h = \frac{\eta_t}{\left(1 - \Delta_t^D\right)^{\sigma}} c_t. \tag{50}$$

Define

$$\begin{aligned} Y_t^h &\equiv W_t n_t + \Pi_t + \mathcal{D}_t + T_t, \\ y_t^h &\equiv \frac{Y_t^h}{P_t}. \end{aligned} \tag{51}$$

and

$$\begin{aligned} \bar{Y}_t^h &\equiv Y_t^h + (1 + i_{t-1})F_{t-1}^h + \left(1 + i_{t-1}^D\right) D_{t-1}^h, \\ \bar{y}_t^h &\equiv \frac{\bar{Y}_t^h}{P_t}. \end{aligned}$$

Recall that

$$\begin{aligned} \Pi_t^y &= P_t y_t - W_t n_t - P_t^z z_t, \\ \Pi_t^z &= \left(1 + i_{t-1}^D\right) D_{t-1}^z + P_t^z z_t - \left(1 + i_{t-1}^L\right) L_t. \\ \mathcal{D}_t &= \left(1 + i_{t-1}^L\right) L_{t-1} + \left(1 + i_{t-1}^R\right) M_{t-1} - (1 + i_{t-1})F_{t-1}^{CB} - (1 + i_{t-1})F_{t-1}^h \\ &- (1 + i_{t-1}^D)D_{t-1}^z - (1 + i_{t-1}^D)D_{t-1}^h + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB} - P_t \kappa_t, \\ T_t &= \left(i_{t-1} - i_{t-1}^R\right) M_{t-1} + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}, \end{aligned}$$

where $M_t = F_t^{CB}$. Then

$$\begin{aligned} Y_t^h &= W_t n_t + \Pi_t^y + \Pi_t^z + \mathcal{D}_t + T_t = P_t y_t - P_t \kappa_t - (1 + i_{t-1})F_{t1-}^h - (1 + i_{t-1}^D)D_{t-1}^h. \\ y_t^h &= y_t - \kappa_t - (1 + r_{t-1})f_{t-1}^h - (1 + r_{t-1}^D)d_{t-1}^h, \\ \bar{Y}_t^h &= P_t y_t - P_t \kappa_t, \\ \bar{y}_t^h &= y_t - \kappa_t, \end{aligned}$$

where

$$f_t^h = \frac{F_t^h}{P_t}, \qquad d_t^h = \frac{D_t^h}{P_t}, \qquad 1 + r_t^D \equiv \frac{1 + i_t^D}{P_t/P_{t+1}}.$$

The budget constraints can be written in real terms as follows:

$$\begin{aligned} c_t &= \bar{y}_t^h - f_t^h - d_t^h. \\ c_{t+1} &= y_{t+1}^h + (1 + i_t)\frac{P_t}{P_{t+1}} f_t^h + \left(1 + i_t^D\right)\frac{P_t}{P_{t+1}} d_t^h - d_{t+1}^h \\ &= y_{t+1}^h + (1 + r_t) f_t^h + \left(1 + r_t^D\right) d_t^h - d_{t+1}^h. \end{aligned}$$

Expressing f_t^h from the first budget constraint and substituting into the second one yields:

$$c_{t+1} = y_{t+1}^h + (1+r_t)\left(\bar{y}_t^h - c_t - d_t^h\right) + \left(1+r_t^D\right) d_t^h - d_{t+1}^h.$$

Using that $\Delta_t^D = \left(1+i^D\right)/(1+i) = \left(1+r^D\right)/(1+r)$ the previous equation can be written as the present value budget constraint:

$$c_t + \frac{c_{t+1}}{1+r_t} = \bar{y}_t^h + \frac{y_{t+1}^h}{1+r_t} - \left(1-\Delta_t^D\right) d_t^h - \frac{d_{t+1}^h}{1+r_t}.$$

Substituting the Euler equation (47) and the money demand equations (49), (50) into the above formula yields

$$\left[1 + (1+\eta_t)\beta_t^\sigma (1+r_t)^{\sigma-1} + \eta_t \left(1-\Delta_t^D\right)^{1-\sigma}\right] c_t = \bar{y}_t^h + \frac{y_{t+1}^h}{1+r_t}.$$

After rearranging it we obtain the following consumption function:

$$c_t = \frac{\bar{y}_t^h + \frac{y_{t+1}^h}{1+r_t}}{1+\mathcal{B}_t}, \tag{52}$$

where

$$\mathcal{B}_t \equiv (1+\eta_t)\beta_t^\sigma (1+r_t)^{\sigma-1} + \eta_t \left(1-\Delta_t^D\right)^{1-\sigma}.$$

The aggregate real saving is given by

$$s_t \equiv y_t - \kappa_t - c_t = \bar{y}_t^h - c_t.$$

Combining the above expression with equation (52) yields

$$s_t = \frac{\mathcal{B}_t}{1+\mathcal{B}_t}\bar{y}_t^h - \frac{1}{1+\mathcal{B}_t}\frac{y_{t+1}^h}{1+r_t}. \tag{53}$$

Substituting the definition of $\bar{y}_t^h$ into formula (52) yields the following consumption demand function:

$$c_t = \frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+r_t}}{1+\mathcal{B}_t}, \tag{54}$$

Substituting the definition of $\bar{y}_t^h$ into formula (53) yields the aggregate saving function:

$$s_t = \frac{\mathcal{B}_t}{1+\mathcal{B}_t}\left(y_t - \kappa_t\right) - \frac{1}{1+\mathcal{B}_t}\frac{y_{t+1}^h}{1+r_t}. \tag{55}$$

Finally combine equations (50) and (52) to get a formula for real money demand:

$$d_t^h = \frac{\eta_t}{\left(1-\Delta_t^D\right)^\sigma}\frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+r_i^h}}{1+\mathcal{B}_t}.$$

The budget constraint of households implies

$$f_t^h + d_t^h = y_t^h + (1+r_{t-1})f_{t-1}^h + (1+r_{t-1}^D)d_{t-1}^h - c_t = y_t - \kappa_t - c_t = s_t. \tag{56}$$

The aggregate balance sheet of the banking system:

$$M_t + L_t = F_t + D_t = F_t^h + F_t^{CB} + D_t^h + D_t^z.$$

Since

$$D_t = D_t^h + D_t^z, \qquad L_t = P_t k_t + D_t^z, \qquad M_t = F_t^{CB},$$

the balance sheet equation can be expressed as

$$P_t k_t = L_t - D_t^z = F_t^h + D_t^h = P_t\left(f_t^h + d_t^h\right).$$

Combining the above expression with formula (56) yields

$$k_t = s_t,$$

the real savings of households is equal to investments (recall physical capital is fully depreciates after production).

A.3 THE SOLUTION TO THE BANKS' PROBLEM

Recall that banks solve the following problem:

$$\max_{x_t, B_t} \mathrm{E}_t \left[\bar{\beta}_t \frac{\mathcal{D}_{t+1}}{P_{t+1}} + \frac{\mathcal{D}_t}{P_t} \right]$$

subject to

$$M_t - \widehat{\lambda}_t D_t \geq B_t,$$
$$M_t + L_t = D_t + F_t,$$

and

$$x_t \geq 0,$$

where $x_t = L_t, M_t, D_t, F_t$ and

$$\begin{aligned}
\mathcal{D}_t &= \left(1 + i_{t-1}^L\right) L_{t-1} + \left(1 + i_{t-1}^R\right)\left(M_{t-1} - \widehat{\lambda}_{t-1} D_{t-1} - B_{t-1}\right) + \left(1 + i_{t-1}^B\right) B_{t-1} \\
&- (1 + i_{t-1}) F_{t-1} - (1 + i_{t-1}^D)\left(1 - \widehat{\lambda}_{t-1}\right) D_{t-1} - P_t \kappa_t, \\
\mathcal{D}_{t+1} &= \left(1 + i_t^L\right) L_t + \left(1 + i_t^R\right)\left(M_t - \widehat{\lambda}_t D_t - B_t\right) + \left(1 + i_t^B\right) B_t \\
&- (1 + i_t) F_t - (1 + i^D)\left(1 - \widehat{\lambda}_t\right) D_t - P_{t+1} \kappa_{t+1},
\end{aligned}$$

and

$$\bar{\beta}_t = \beta_t \frac{c_{t+1}^{-\nu}}{{}^{-\nu}c_t} = \frac{1}{1 + r_t}.$$

Multiplying the objective function by a positive constant does not alter the results, therefor we can multiply it by P_t

$$\frac{1}{1 + r_t} \frac{P_t}{P_{t+1}} \mathcal{D}_{t+1} + \mathcal{D}_t = \frac{1}{1 + i_t} \mathcal{D}_{t+1} + \mathcal{D}_t.$$

As discussed in *section 2.3*, date $t - 1$ variables do not constraint the date t decisions. Hence they can be treated as constants from the point of view of optimization. Hence all date $t - 1$ terms can be omitted from the objective function. On the other, hand date t decisions do not have any impact on κ_{t+1}, therefore we can omit is as well.

Expressing F_t from the balance sheet constraint and substituting into the modified objective function yields

$$\begin{aligned}
&\frac{1 + i_t^L}{1 + i_t} L_t + \frac{1 + i_t^R}{1 + i_t}\left(M_t - \widehat{\lambda}_t D_t - B_t\left(\widehat{\lambda}_t\right)\right) + \frac{1 + i_t^B}{1 + i_t} B_t\left(\widehat{\lambda}_t\right) \\
&- \frac{1 + i_t^D}{1 + i_t}\left(1 - \widehat{\lambda}_t\right) D_t + D_t - M_t - L_t - P_t \kappa_t.
\end{aligned}$$

We can form the Lagrangian of the optimization problem by the above expression and the liquidity constraint:

$$\begin{aligned}
\mathcal{L}\left(\widehat{\lambda}_t\right) &= \Delta_t^L L_t + \Delta_t^R \left(M_t - \widehat{\lambda}_t D_t - B_t\left(\widehat{\lambda}_t\right)\right) + \Delta_t^B B_t\left(\widehat{\lambda}_t\right) \\
&- \Delta_t^D \left(1 - \widehat{\lambda}_t\right) D_t + D_t - M_t - L_t - P_t \kappa_t + \\
&+ \mu_t\left(\widehat{\lambda}_t\right)\left(M_t - \widehat{\lambda}_t D_t - B_t\left(\widehat{\lambda}_t\right)\right),
\end{aligned}$$

where

$$\Delta_t^L \equiv \frac{1 + i_t^L}{1 + i_t}, \qquad \Delta_t^R \equiv \frac{1 + i_t^R}{1 + i_t}, \qquad \Delta_t^B \equiv \frac{1 + i_t^B}{1 + i_t}, \qquad \Delta_t^D \equiv \frac{1 + i_t^D}{1 + i_t}.$$

The Lagrangian is a function of the liquidity shock $\widehat{\lambda}_t$ since the variable B_t and the multiplier μ_t are also functions of it. The expected Lagrangian can be calculated as

$$\mathrm{E}_1[\mathcal{L}] = \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \mathcal{L}\left(\widehat{\lambda}\right) g\left(\widehat{\lambda}\right) \, \mathrm{d}\widehat{\lambda}.$$

where $g(\widehat{\lambda}) = 1/(2\bar{\lambda})$ is the density function of the uniform distribution on the $[-\bar{\lambda}, \bar{\lambda}]$ interval.

FIRST ORDER CONDITIONS

The variables L_t, M_t, D_t, F_t are independent of $\widehat{\lambda}_t$, which is the result of the timing of decisions, since they are determined prior to the realization of $\widehat{\lambda}_t$. On the other hand, when B_t and μ_t are determined, $\widehat{\lambda}_t$ is already observed. Thus, while the first order conditions for interbank lending have to be met for all possible realizations of the liquidity shock, for the other choice variables only in expectation.

Formally, the first order condition with respect to B_t:

$$\frac{\partial \mathcal{L}\left(\widehat{\lambda}_t\right)}{\partial B_t\left(\widehat{\lambda}_t\right)} = 0, \text{ for all } \widehat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

The first order conditions with respect to L_t, M_t, D_t and F_t:

$$\frac{\partial \mathrm{E}_1\left[\mathcal{L}\right]}{\partial x_t} \leq 0,$$

where $x_t = L_t$, M_t, D_t, F_t. The inequalities in the above conditions are due to the non-negativity constraints. A *strict inequality* implies that $x_t = 0$.

To find the solution beyond the first order conditions one also needs the constraints and the complementary slackness condition:

$$\mu_t\left(\widehat{\lambda}_t\right)\left(M_t - \widehat{\lambda}_t D_t - B_t\left(\widehat{\lambda}_t\right)\right) = 0, \text{ for all } \widehat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

That is, a positive μ_t implies a binding constraint, $M_t = \widehat{\lambda}_t D_t + B_t$. On the other hand, if $M_t > \widehat{\lambda}_t D_t + B_t$ then $\mu_t = 0$.

To derive the first order condition with respect to B_t, first calculate the marginal cost of B_t. Using equation (12) one can obtain

$$\frac{\partial P_t \kappa_t}{\partial B_t\left(\widehat{\lambda}_t\right)} = 2\phi^B \rho_t\left(\widehat{\lambda}_t\right),$$

where

$$\rho_t\left(\widehat{\lambda}_t\right) = \frac{B_t^l\left(\widehat{\lambda}_t\right)}{M_t - \widehat{\lambda}_t D_t}.$$

Therefore the first order condition:

$$\Delta_t^B - \Delta_t^R - 2\phi^B \rho_t\left(\widehat{\lambda}_t\right) - \mu_t = 0 \qquad \text{for all} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

First, consider the case when the bank has enough reserves to meet the interbank payment obligations due to the liquidity shock, if any. This is the case when $\widehat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}]$. These banks are potential lenders on the interbank market. We assume symmetric solution, that is,

$$\rho_t\left(\widehat{\lambda}_t\right) = \rho_t < 1 \qquad \text{for all} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t].$$

In *section 2.3* it is shown that such a symmetric solution is consistent with an equilibrium on the interbank market. Since $\rho_t < 1$

$$B_t^l\left(\widehat{\lambda}_t\right) < M_t - \widehat{\lambda}_t D_t \qquad \text{for all} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t].$$

Then the complementary slackness condition implies that $\mu\left(\widehat{\lambda}_t\right) = 0$ for all $\widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$. Therefore the first order condition for the lenders becomes

$$\Delta_t^B = \Delta_t^R + 2\phi^B \rho_t \qquad \text{for all} \qquad \widehat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t].$$

Now, consider the case when the bank has to borrow on the interbank market, because its reserves are not sufficient to cover the deposit outflow, that is when $M_t < \widehat{\lambda}_t D_t$ or, equivalently $\widehat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$. For such a bank $B_t^l = 0$, thus $\rho_t\left(\widehat{\lambda}_t\right) = 0$ and, consequently, the first order condition becomes

$$\Delta_t^B = \Delta_t^R + \mu_t \qquad \text{for all} \qquad \widehat{\lambda}_t \in (-\lambda_t, \bar{\lambda}_t].$$

Since $\mu_t = \Delta^B - \Delta_t^R > 0$, the liquidity constraint will bind and $B_t^b = \hat{\lambda}_t D_t - M_t$.

The first order condition with respect to M_t is

$$\int_{-\bar{\lambda}}^{\bar{\lambda}} \left(\Delta_t^R - 1 - \tau^F + \mu_t + \phi^B \rho_t^2\right) g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} \leq 0.$$

Since we have just shown that if $\hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$ then $\mu_t = 0$, $\rho_t > 0$, and if $\hat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$ then $\mu_t > 0$, $\rho_t = 0$, and both are independent of $\hat{\lambda}$ inside these two intervals, the above integral can be decomposed as

$$\left(\Delta_t^R - 1 - \tau^F\right) \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} + \mu_t \int_{\lambda_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} + \phi^B \rho_t^2 \int_{-\bar{\lambda}_t}^{\lambda_t} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} \leq 0.$$

Since

$$\int_{\lambda_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} = 1 - G(\lambda_t), \qquad \int_{-\bar{\lambda}_t}^{\lambda_t} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} = G(\lambda_t),$$

and focusing only on solutions in which M_t is positive, the above condition simplifies to the following equation:

$$\Delta_t^R + \mu_t \left[1 - G(\lambda_t)\right] + \phi^B \rho_t^2 G(\lambda_t) = 1 + \tau^F.$$

The first order condition with respect to L_t is

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}} \left(\Delta_t^L - 1 - \tau^F - \tau^L - 2\chi_t \phi^L \frac{L_t}{P_t} + \phi^{DL} \frac{D_t}{P_t}\right) g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} \leq 0.$$

Since all terms inside the integral are independent of $\hat{\lambda}_t$, in equilibrium with non-zero lending the previous expression simplifies to

$$\Delta_t^L = 1 + \tau^F + \tau^L + 2\chi_t \phi^L \frac{L_t}{P_t} - \phi^{DL} \frac{D_t}{P_t}.$$

The first order condition with respect to D_t is

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \left(\Delta_t^D - 1 - \tau^F + \tau^D + 2\phi^D \frac{D_t}{P_t} - \phi^{DL} \frac{L_t}{P_t}\right) g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} +$$
$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \hat{\lambda} \left(\mu_t + \phi^B \rho_t^2 + \Delta_t^R - \Delta_t^D\right) g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda} \geq 0.$$

Taken into account that if $\hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$ then $\mu_t = 0$, $\rho_t > 0$, and if $\hat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$ then $\mu_t > 0$, $\rho_t = 0$, in equilibrium with positive demand deposit the above condition can be rewritten as

$$\Delta_t^D + \tau^D + 2\phi^D \frac{D_t}{P_t} - \phi^{DL} \frac{L_t}{P_t} + \mu_t \int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda} g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda}$$
$$= 1 + \tau^F - \phi^B \rho_t^2 \int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}_t g\left(\hat{\lambda}\right) \,\mathrm{d}\hat{\lambda}.$$

Finally, time deposits (F_t) are determined by the balance sheet constraint: $F_t = M_t + L_t - D_t$.

IMPLICATIONS OF THE UNIFORM DISTRIBUTION

In this section the closed form expressions are derived for the probabilistic terms in equations (19), (22) and (24) assuming that $\hat{\lambda}_t$ can be described by a uniform distribution. Its cumulative distribution function is:

$$\begin{aligned} G\left(\hat{\lambda}\right) &= \frac{\hat{\lambda} + \bar{\lambda}_t}{2\bar{\lambda}_t}, \text{ if } \hat{\lambda} \in [-\bar{\lambda}_t, \bar{\lambda}_t], \\ &= 0 \text{ if } \hat{\lambda} < -\bar{\lambda}_t, \\ &= 1 \text{ if } \bar{\lambda}_t < \hat{\lambda}. \end{aligned}$$

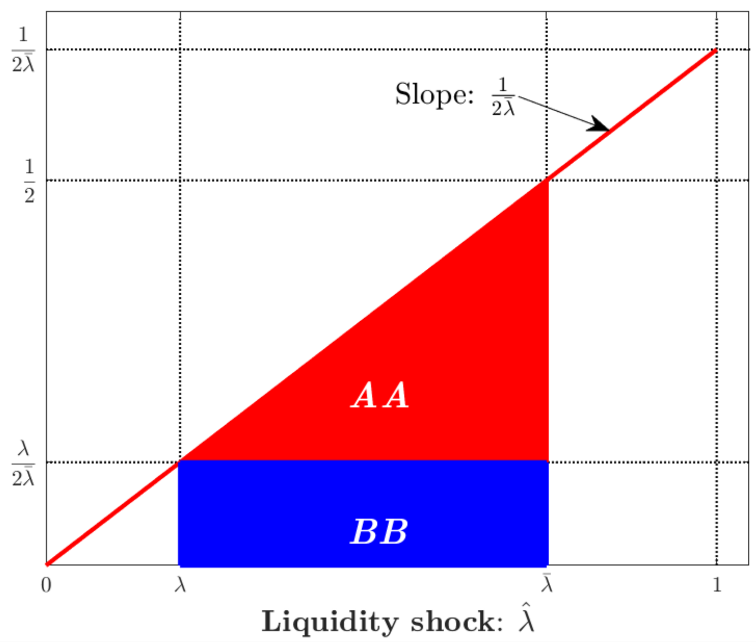

Figure 20
Uniform distribution

The probability density function:

$$\begin{aligned} g\left(\hat{\lambda}\right) &= \frac{1}{2\bar{\lambda}_t}, \text{ if } \hat{\lambda} \in [-\bar{\lambda}_t, \bar{\lambda}_t], \\ &= 0, \text{ otherwise.} \end{aligned}$$

Using the definition of G, the terms in equation (18) become

$$G\left(\lambda_t\right) = \frac{\varsigma_t}{2\bar{\lambda}_t}, \qquad 1 - G\left(\lambda_t\right) = \frac{\delta_t}{2\bar{\lambda}_t},$$

where $\varsigma_t \equiv \max\left[2\bar{\lambda}, \bar{\lambda}_t + \lambda_t\right]$ and $\delta_t \equiv \max\left[0, \bar{\lambda}_t - \lambda_t\right]$.

The term

$$\int_{\lambda_t}^{\bar{\lambda}_t} \left(\hat{\lambda} - \lambda_t\right) \frac{1}{2\bar{\lambda}_t} \, \mathrm{d}\hat{\lambda}.$$

in equation (25) is equal to the red shaded area AA in *Figure 26*, that is,

$$AA = \frac{1}{2}(\bar{\lambda}_t - \lambda_t)\left(\frac{1}{2} - \frac{\lambda_t}{2\bar{\lambda}_t}\right) = \frac{\delta_t}{4}\left(1 - \frac{\lambda_t}{\bar{\lambda}_t}\right) = \frac{\delta_t^2}{4\bar{\lambda}_t}.$$

Observe that the term

$$\int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda} \frac{1}{2\bar{\lambda}_t} \, \mathrm{d}\hat{\lambda}.$$

in equation (21) is equal to the sum of areas AA and BB in *Figure 26*,

$$BB = (\bar{\lambda}_t - \lambda_t)\frac{\lambda_t}{2\bar{\lambda}_t} = \frac{\delta_t \lambda_t}{2\bar{\lambda}_t}.$$

Hence,

$$AA + BB = \frac{\delta_t}{2\bar{\lambda}_t}\left[\frac{\delta_t}{2} + \lambda_t\right] = \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}.$$

Furthermore, consider the term

$$\int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}.$$

in equation (21). First observe that

$$\int_{-\lambda_t}^{0} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = -\int_{0}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}.$$

As a consequence,

$$\int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = \int_{-\bar{\lambda}_t}^{-\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}. = -\int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = -\frac{\delta_t}{2\bar{\lambda}_t}\left[\frac{\delta_t}{2}+\lambda_t\right] = -\frac{\delta_t\varsigma_t}{4\bar{\lambda}_t}.$$

A.4 PARAMETER VALUES

The following tables display the values of the parameters and the baseline values of the exogenous variable used in the model.

Table 7
Parameter values of the model

Name	a^n	a^z	α	θ	ϑ	ω	ν	σ	φ
Value	34.43	1	0.67	6	1.2	0.0005	2	0.5	0.001

Name	$\bar{\kappa}$	τ^F	$\bar{\tau}^L$	ϕ^L	$\bar{\tau}^D$	ϕ^D	ϕ^{DL}	ϕ^B
Value	0.272	0	0	0.0019	-0.0396	0.0037	0.0015	0.1084

Table 8
Baseline values of the exogenous shocks

Name	$\bar{\lambda}^0$	η^0	β^0	A^0	χ^0	η^{z0}
Value	0.7464	0.0147	0.97	1.8178	1	1/3

A.5 MODEL SOLUTION

FLEXIBLE PRICE ALLOCATION AND AGGREGATE SUPPLY CURVE

In this section we derive the flexible price allocation and the aggregate supply curve.

Combining the labor supply equation (48) and the consumption demand equation (54) yields the following expression:

$$\varphi c_t^{\nu} = \varphi\left(\frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+i_t}}{1+\mathcal{B}_t}\right)^{\nu} = w_t^0. \tag{57}$$

Taken as given the values of k_{t-1} and A_t they determine z_t by equation (6). If the variables controlled by monetary policy are also given then the six aggregate demand equations (30)–(35) and equations (1), (2) and (57) determine the flexible price values of y_t, n_t, w_t, k_t, i_t, i_t^L, Δ_t^D, κ_t, d_t ($d_t = D_t/P_t^0$) since y_{t+1}^h is a function of date t variables (see the previous section). As discussed, monetary policy chooses D_t in such a way that $P_t^0 = P_{t-1}$, that is,

$$D_t = P_{t-1}d_t.$$

Combining the optimal price adjustment equation (5) with formula (57) yields the aggregate supply curve:

$$\frac{\left[P_t^{1-\theta} - (1-\gamma)\left(P_t^0\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\gamma^{\frac{1}{1-\theta}}P_t} = \vartheta\varphi\left(\frac{y_t - \kappa_t + \frac{y_{t+1}^h}{1+i_t}}{1+\mathcal{B}_t}\right)^{\nu}\frac{(a^n)^{\frac{1}{\alpha-1}}y_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}.$$

DERIVATION OF Y^H_{T+1}

We assume that $t+1=T$ is the final date. In the last period there is no need for investment and capital accumulation, hence there is no financial intermediation. So we assume the banking system disappears at date T. The transaction instrument is provided by the central bank and transferred directly to households as helicopter money. That is, the final date is represented by an economy only with outside money, $D_t = M_t$. Government transfer to households becomes

$$T_{t+1} = \left(i^B_t - i^R_t\right) B^{CB}_t + \left(i_t - i^R_t\right) M_t + M_{t+1}.$$

We also assume that the final date can be described by the flexible price allocation, the expected values of the shocks are the same as their actual realization. In fact, at date T only one exogenous shock remains η_{t+1}, and we assume that $\eta_{t+1} = \eta_t$.

Since there is no capital accumulation, the product market equilibrium condition becomes

$$c_{t+1} = y_{t+1}.$$

To calculate the flexible price allocation, consider the labor supply equation (48) and take into account that $c_{t+1} = y_{t+1}$,

$$\varphi c^{\nu}_{t+1} = \varphi y^{\nu}_{t+1} = w_{t+1}. \qquad (58)$$

Recall that z_{t+1} is determined by k_t, see equation (6). Taking the predetermined value of z_{t+1}, equations (1), (2) and (58) provide a solution for y_{t+1}, n_{t+1} and w_{t+1}.

The money demand equation (49) implies that

$$\frac{M_{t+1}}{P^0_{t+1}} = \frac{D_{t+1}}{P^0_{t+1}} = \eta_t y_{t+1}.$$

We assume that monetary policy sets M_{t+1} in such a way that $P^0_{t+1} = P_t$. Therefore

$$M_{t+1} = P_t \eta_t y_{t+1}.$$

The real income of households is given by $y^h_{t+1} + (1+i_t)f^h_t + (1+i^d_t)d^h_t$, where y^h_t is defined by equation (51). Since households' income is spent for consumption and real money holdings,

$$c_{t+1} + d_{t+1} = y^h_{t+1} + (1+i_t)f^h_t + (1+i^d_t)d^h_t.$$

Substituting the money demand equation (49) and the product market equilibrium condition into the above formula yields

$$y^h_{t+1} = (1+\eta_t)y_{t+1} - (1+i_t)f^h_t - \left(1+i^D_t\right)d^h_t.$$

Equation (7) implies that $d^z_t = \eta^z_t k_t$, where $d^z_t = D^z_t/P_t$. Since $d_t = d^h_t + d^z_t$,

$$d^h_t = d_t - \eta^z_t k_t.$$

Equation (**??**) in *Appendix A.2* implies that

$$k_t = s_t = f^h_t + d^h_t,$$

that is,

$$f^h_t = k_t - d^h_t = \left(1+\eta^z_t\right)k_t - d_t.$$

Therefore

$$\begin{aligned} y^h_{t+1} &= (1+\eta_t)y_{t+1} - (1+i_t)\left[\left(1+\eta^z_t\right)k_t - d_t\right] - \left(1+i^D_t\right)\left(d_t - \eta^z_t k_t\right) \\ &= (1+\eta_t)y_{t+1} - (1+i_t)k_t + (1+i_t)(1-\Delta^D_t)(d_t - \eta^z_t k_t). \end{aligned}$$

A.6 IMPLEMENTATION OF AN INTEREST RATE RULE

As discussed in *section 4*, if monetary policy wants to implement an interest rate rule, its instruments have to satisfy the following conditions:

$$\begin{aligned} y_t^{\star} &= y^{IS}\left(i_t^{\star}, D_t^{*} + D_t^{+}, \lambda_t^{+}, \rho_t^{+}, \xi_t^{LM*}, \xi_t^{IS*}\right), \\ y_t^{\star} &= y^{LM}\left(i_t^{\star}, D_t^{*} + D_t^{+}, \lambda_t^{+}, \rho_t^{+}, \xi_t^{LM*}, \xi_t^{IS*}\right), \end{aligned}$$

where

$$y_t^{\star} = y^{ir}\left(D_t^0, \lambda_t^0, \rho_t^0, \xi_t^{LM0}, \xi_t^{IS*}\right),$$
$$i_t^{\star} = i^{ir}\left(D_t^0, \lambda_t^0, \rho_t^0, \xi_t^{LM0}, \xi_t^{IS*}\right),$$

if $\xi^{IS0} \neq \xi^{IS*}$, and

$$y_t^{\star} = y_t^0, \qquad i_t^{\star} = i_t^0,$$

if $\xi^{IS0} = \xi^{IS*}$.

Observe that λ_t^{+} and ρ_t^{+} influences the IS and LM curves only via $\kappa_t^{\lambda\rho}$ and Ψ_t in equations (32) and (33). Slightly changing our notation, express the above two conditions as functions of $\kappa_t^{\lambda\rho+}$ and Ψ_t^{+}:

$$\begin{aligned} y_t^{\star} &= y^{IS}\left(i_t^{\star}, D_t + D_t^{+}, \kappa_t^{\lambda\rho+}, \Psi_t^{+}, \xi_t^{LM*}, \xi_t^{IS*}\right), \\ y_t^{\star} &= y^{LM}\left(i_t^{\star}, D_t + D_t^{+}, \kappa_t^{\lambda\rho+}, \Psi_t^{+}, \xi_t^{LM*}, \xi_t^{IS*}\right), \end{aligned}$$

and add the following auxiliary condition:

$$\kappa_t^{\lambda\rho+} \geq 0.$$

If one chooses $\kappa_t^{\lambda\rho+}$ which satisfies the above inequality than the above two equations provide a solution for D_t^{+} and Ψ_t^{+}.

If $\tau^F = 0$ then one can calculate the values of λ_t^{+} and ρ_t^{+} in the following way: Define

$$\begin{aligned} \mathtt{A} &= \frac{4\kappa_t^{B+}\bar{\lambda}_t^{*}}{\phi^B D_t} \\ \mathtt{B} &= \frac{4\Psi_t^{+}\bar{\lambda}_t^{*}}{\phi^B}. \end{aligned}$$

Then equations (32) and (33) imply that

$$\begin{aligned} \mathtt{A} &= \left(\rho_t^{+}\right)^2\left(\varsigma_t^{+}\right)^2, \\ \mathtt{B} &= 2\rho_t^{+}\delta_t^{+}s_t^{\lambda+} - \left(\rho_t^{+}\right)^2\delta_t^{+}\varsigma_t^{+}. \end{aligned}$$

Combining them results in

$$\mathtt{B} = 2\sqrt{\mathtt{A}}\left(\bar{\lambda}_t^{*} - \lambda_t^{+}\right) - \mathtt{A}\frac{\bar{\lambda}_t^{*} - \lambda_t^{+}}{\bar{\lambda}_t^{*} + \lambda_t^{+}},$$

rearranging it yields

$$2\sqrt{\mathtt{A}}\left(\lambda_t^{+}\right)^2 + (\mathtt{B} - \mathtt{A})\lambda_t^{+} + (\mathtt{B} + \mathtt{A})\bar{\lambda}_t^{*} - 2\sqrt{(\mathtt{A})}\left(\bar{\lambda}_t^{*}\right)^2 = 0.$$

The above quadratic equation provides a solution for λ_t^{+} and then one can solve for ρ_t^{+} as well. If the solutions are real numbers and satisfy $0 < \lambda_t^{+} \leq \lambda_t^{*}$ and $0 < \rho_t^{+} < 1$, then the interest rate rule is implementable.

A.7 DISCUSSION OF THE EFFECTS OF CREDIT RISK AND CORPORATE MONEY DEMAND SHOCKS

UNCHANGED MONETARY POLICY

Shocks to the credit risk of corporate loans (χ_t) have similar effect to that of productivity discussed above. When there is an unexpected decrease in the credit risk, firms in the intermediate good producing sector can invest more, and the higher k_t will increase money demand. The difference between the two shocks is that the productivity shock has a direct impact on y^h_{t+1}.

As a consequence, both the IS and LM curve shifts upwards. The overall effect is slightly contractionary in both cases, see *Figure 21*.

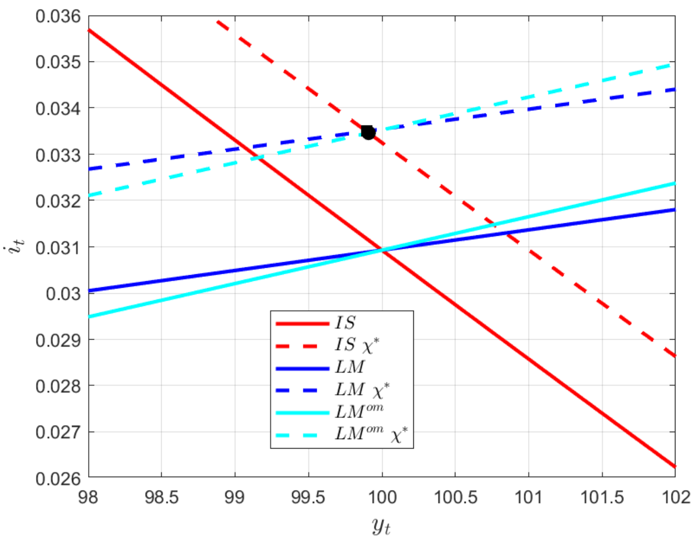

Figure 21
The effect of a decrease in credit risk on the IS and LM curves

Although η^z_t is the money demand of firms, its unexpected changes have very similar macroeconomic effects to other types of IS shocks. This is the reason why classified it as an IS shock,

When firms' money demand for transactional purposes decreases, they can borrow more for investment purposes without increasing the banks' cost associated with corporate lending. More investment implies more capital and a rightward shift of the IS curve.

However, more capital results higher money demand. As a result, the LM curve will shift upwards, but compared to the shift of the IS curve, not as much as in the case of a productivity shock. The difference can be explained by the fact that the increase in the households' money demand is partly offset by the decrease in the firms' money demand. The overall effect of first period's output will be expansionary with no significant difference between the outside money and the general case, see *Figure 22*.

IMPLEMENTATION OF AN INTEREST RATE RULE

The central bank can implement the interest rate rule in the presence of plausible credit risk and corporate money demand shocks. The corresponding intervals are [0.9546, 1.3082] for credit risk χ (baseline value is 1), and [0.3236, 0.3633] for firms' money demand η^z_t (baseline value is 1/3), see also *Figures 23–24*.

APPROXIMATION OF AN INTEREST RATE RULE

As *Figure 25* and *Table 9* show, the situation is very similar in the case of credit risk and productivity shocks, although the approximation error of the credit risk shock is somewhat larger.

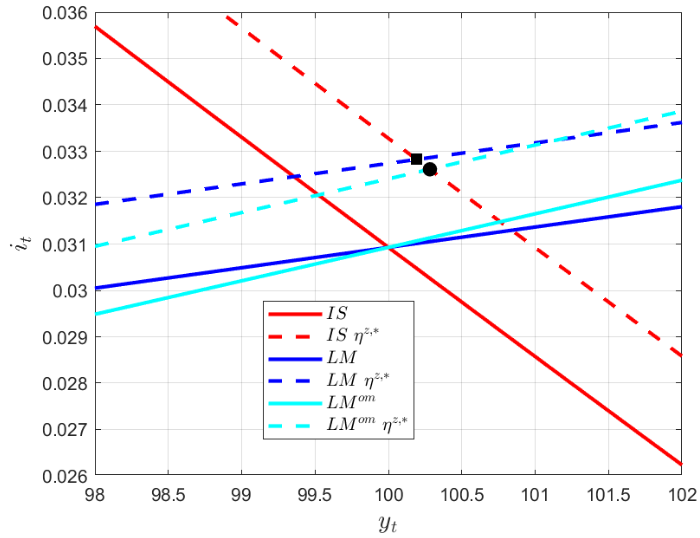

Figure 22
The effect of a decrease in firms' money demand on the IS and LM curves

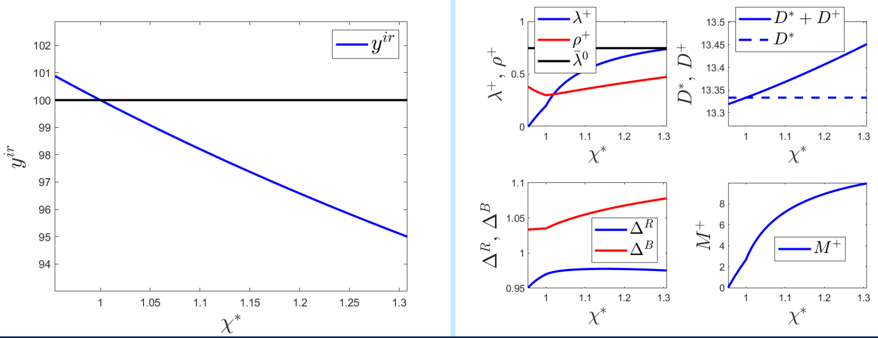

Figure 23
Implementation of the interest rate rule – credit risk shock

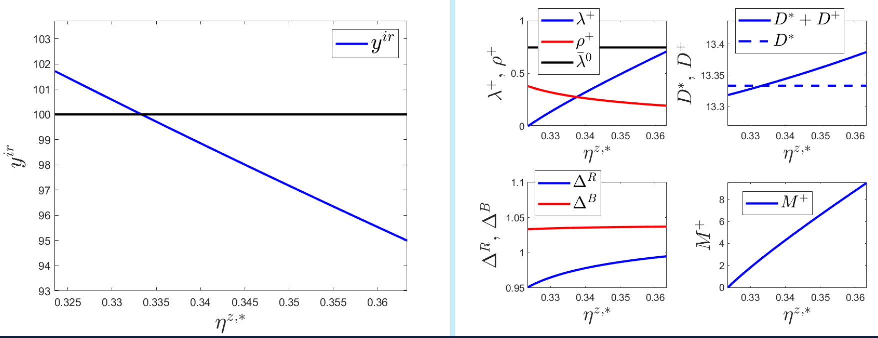

Figure 24
Implementation of the interest rate rule – corporate money demand shock

Results related to the corporate money demand shock are displayed in *Figure 26* and *Table 10*. We see qualitatively similar results as in the case of the productivity and credit risk shocks. However, the error of the approximate implementation is now the smallest.

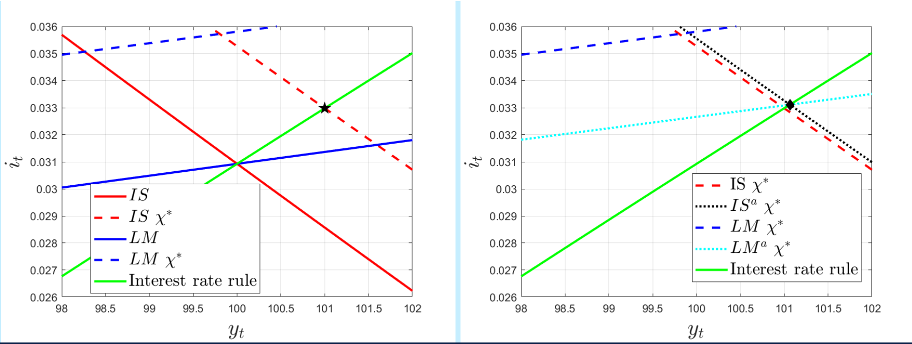

Figure 25
Approximate implementation of the interest rate rule – credit risk shock

Table 9

χ_t^*	1.3082	1.1734	1.0544	0.9486	0.8540	0.7690
y_t^{ir}	95	97	99	101	103	105
y_t^a	94.72	96.83	98.94	101.06	103.2	105.36

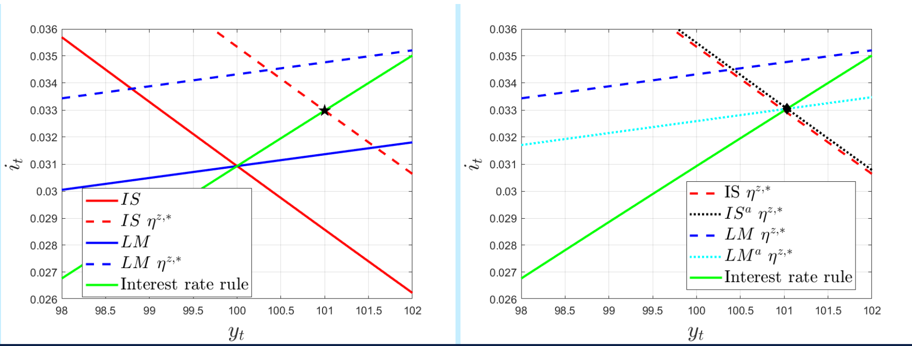

Figure 26
Approximate implementation of the interest rate rule – corporate money demand shock

Table 10

η_t^{z*}	0.3633	0.3511	0.3392	0.3276	0.3164	0.3055
y_t^{ir}	95	97	99	101	103	105
y_t^a	94.83	96.89	98.97	101.03	103.10	105.16

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