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The effect of regulatory requirements and ESG promotion on market liquidity *

(A szabályozási előírások és az ESG szabályozás hatása a piaci likviditásra)

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Contents

Abstract

Liquidity and market risk are key considerations in financial markets, especially in times of financial crises. For this reason, regulatory attention to and measures in these fields have been on the rise for the past years. Based on practical experience, regulations aiming at ensuring funding liquidity or, in general, reducing certain risky positions have the side effect of reducing market liquidity. To understand this effect, we extend a standard general equilibrium model with transaction costs of trading, endogenous market liquidity, and the modeling of regulation. We prove that higher regulatory requirements or divesting bad ESG assets reduces market liquidity.

JEL: G11.

Keywords: Market liquidity, Market risk, Liquidity risk, General equilibrium model, Regulatory requirement, ESG related assets.

Összefoglaló

A pénzügyi válságok következményeként az elmúlt évtizedben középpontba került a szabályozás kérdése a likviditás és a piaci kockázat területén. Gyakorlati tapasztalat alapján, a finanszírozási likviditás biztosítása és a túlzott kockázatvállalás csökkentése érdekében bevezetett szabályozói lépések visszahatnak a piaci likviditásra. E hatás megértéséhez egy sztenderd általános egyensúlyelméleti modellt bővítünk a kereskedés tranzakciós költségeivel, endogén piaci likviditással és szabályozói előírással. Bizonyítjuk, hogy a szabályozás szigorítása, illetve a nem ESG-kompatibilis eszközök tartásának visszafogása csökkenti a piaci likviditást.

1 Introduction

The financial crises of the past decades such as Black Monday in 1987, the one related to the Iraq War in 1990, the collapse of LTCM in 1998 and the subprime mortgage crisis in 2007 (Brunnermeier and Pedersen, 2008) evidence the paramount importance of liquidity and capitalization of market risk in financial markets. The liquidity of assets and markets may fluctuate over time due to the varying level of transparency of information on asset values, the number and capital of intermediaries providing liquidity, and uncertainty. Therefore, it is important to capture liquidity risk in models (Amihud, Mendelson and Pedersen, 2013). Based on Acerbi and Scandolo (2008), the modelling of liquidity risk covers (1) the cash-flow risk of portfolios or companies, also called *funding liquidity*, (2) the risk of trading in illiquid markets, i.e. the risk of price impact related to *market liquidity* (Almgren and Chriss, 2001; Amihud, 2002; Acharya and Pedersen, 2005), and (3) the risk of drying up of the liquidity circulating in the financial system (see papers starting from Amihud, Mendelson and Wood (1990), Brunnermeier and Pedersen (2008), and Mitchell, Pedersen and Pulvino (2007)).

Many regulations are aiming at improving funding liquidity in particular and reducing certain risky positions in general. In January 2013, the Basel Committee (*Basel Committee on Banking Supervision, BCBS*) introduced two new measures, Liquidity Coverage Ratio and Net Stable Funding Ratio, as part of the Basel III international regulatory framework for banks (BCBS, 2013). For the appropriate capitalization of market risk BCBS published and revised the Fundamental Review of the Trading Book to replace the minimum capital requirements (BCBS, 2014, 2019). As an important improvement, the applied risk measure for market risk is the Expected Shortfall instead of the Value-at-Risk. Besides regulations for the banking system, the recommendations and good practices of IOSCO (*International Organisation of Securities Commissions*[1]) on the management of investment funds are also being updated. The objective of the recommendation IOSCO (2018) is to improve the management of liquidity risk of open-end investment funds with a view to protecting investors, increasing the efficiency of financial markets, and reducing systemic risk. In 2016, the SEC (*Securities and Exchange Commission*) adopted New Rule 22e-4 to regulate the liquidity risk of registered open-end funds[2]. The mission statement of the SEC[3] includes the aim of protecting households who borrow funds or invest in financial markets. Besides the regulation of financial institutions, the development of financial literacy of households and avoidance of excessive risk-taking and over-indebtedness are certainly also key to achieving this objective. Moreover, an interesting regulatory direction is to encourage a move towards sustainable finance, as outlined in IOSCO's and European Commissions' reports. Based on IOSCO (2020), global coordination and transparency are needed to deal with the most important tasks and challenges, which are multiple and diverse sustainability frameworks and standards, a lack of common definitions of sustainable activities, and greenwashing[4] and investor protection. IOSCO (2019b) approaches the issue from an emerging markets perspective and makes 11 recommendations for regulators to consider when regulating sustainable assets and ESG (*Environmental, Social, and Governance*) specific risks.

Anecdotal evidence suggests that regulations aiming at ensuring funding liquidity or, in general, reducing certain risky positions have the side effect of reducing market liquidity. To better understand this effect, we extend a standard two-period general equilibrium model with transaction costs (see, for instance, Le Roy and Werner (2001)). In the model, agents trade with financial assets to increase the utility of their initial endowments representing their stochastic income and initial investments. In order to model market liquidity and take transaction cost into account, we use the marginal supply-demand curve (MSDC) (Cetin, Jarrow, and Protter, 2004; Jarrow and Protter, 2005; Acerbi and Scandolo, 2008). For a given period, as a generalized order book capturing market liquidity, the MSDC of a risky asset expresses the marginal bid and ask prices at which a particular asset

[1] *"The International Organization of Securities Commissions (IOSCO) is the international body that brings together the world's securities regulators and is recognized as the global standard setter for the securities sector. IOSCO develops, implements, and promotes adherence to internationally recognized standards for securities regulation. It works intensively with the G20 and the Financial Stability Board (FSB) on the global regulatory reform agenda."* https://www.iosco.org/

[2] Securities and Exchange Commission's Investment Company Liquidity Risk Management Programs, 17 CFR Parts 210, 270, 274, pp. 90 and 195. https://www.sec.gov/rules/final/2016/33-10233.pdf

[3] *"The SEC enforces the securities laws to protect the more than 66 million American households that have turned to the securities markets to invest in their futures – whether it's starting a family, sending kids to college, saving for retirement or attaining other financial goals."* https://www.sec.gov/

[4] "Greenwashing usually refers to practices aimed to mislead investors or to give them a false impression about how well an investment is aligned with its sustainability goals." IOSCO (2020) p. 3.

can be traded. We assume that agents cannot trade directly with each other; there is a market maker who matches opposite orders for an asset. The market maker sets the marginal supply-demand curve as a transaction monopolist for each asset, thereby influencing market liquidity endogenously. We proxy regulation such that agents have to meet extra cash regulatory requirement given as a function of the expected shortfall (ES) (Acerbi and Tasche, 2002; Csóka, Herings and Kóczy, 2009), calculated as the average loss in a number of worst states. To ensure funding liquidity and reduce excessive risk-taking in general, ES is calculated for the whole portfolio of assets, called portfolio regulatory requirement. To reduce certain risky positions, ES is calculated for those assets, called asset regulatory requirement. The introduction of both regulatory requirements represents an additional constraint to the optimization problem of agents; thus, their previous optimal portfolio may not be attainable any longer. We show for both regulatory requirements that market liquidity will decrease. The main channel is that if the agent is constrained in its optimal decision by regulatory requirements, it makes sense for the market maker to increase transaction costs as long as the optimal portfolio of the agent under the given regulatory requirement does not change.

Our results can be interpreted for sustainable finance as follows. For fund managers and pension funds, it is common that they are not allowed to buy shares in certain companies that are considered "harmful"[5]. The logic can be reversed and ESG considerations can be introduced as an incentive in capital allocation decisions. Suppose that the trading-related (internal or regulation-based) rules of the investing institutions influence investors' decisions as a constraint. Using asset regulatory requirement and the regulators setting different regulatory parameters based on ESG risk, the regulatory move to discourage the holding of unsustainable assets (divestment) can be modelled. Our model predicts that those assets with bad ESG scores promoted for divestment will not only have a lower price, but also a lower market liquidity.

Several other theoretical studies assess the effectiveness and potential costs of regulatory requirements. De Nicolò, Gamba and Lucchetta (2014) demonstrate in a partial market equilibrium model that the application of the Basel III Liquidity Coverage Ratio restricts lending and reduces the levels of efficiency and welfare. Begenau (2019) uses a dynamic general equilibrium model to determine the optimal level of capital requirement. Increasing capital requirement reduces the leverage, and thus the amount of coveted deposit funding of banks, which, through a reduction of deposit rates, reduces the cost of capital, increases profitability and, ultimately, lending. On the other hand, IOSCO (2019a) stresses that the regulation of the secondary market of corporate bonds has limited financial intermediaries in the provision of liquidity since the crisis. Stress test results show that market pressure may lead to more severe shifts in yields than before. Based on Petrella and Resti (2017), the adverse market conditions of the Basel III rules on liquidity strongly depend on individual bond's characteristics. Lara et al. (2021) find that regulatory reforms mainly imposed additional constraints on government debt holdings have negative impact on market liquidity, while rules designed to enhance transparency have positive effects. According to Sommer and Sullivan (2018), the abolition of tax credits for mortgage loans would result in a drop of real estate prices and the stock of mortgage loans, and an increase in welfare.

The structure of the paper is as follows. In Section 2 the augmented general equilibrium model is introduced. In Section 3, we show examples and derive more general results. Finally, in Section 4, we conclude and outline avenues for future research.

[5] A case in point: *"We exclude companies that produce or distribute tobacco, controversial weapons and recreational cannabis. We also exclude companies with significant revenue from coal and oil sands, and unsustainable palm oil production. The Storebrand Group has also chosen to exclude investments in companies within certain single product categories or industries that are unsustainable. These products or industries are associated with significant risks and liabilities from societal, environmental or health related harm. In these product categories there is also limited scope to influence companies to operate in a more sustainable way."* https://www.storebrand.no/en/asset-management/sustainable-investments/exclusions/product-based-exclusions

2 The general equilibrium model

As far as the structure of the general equilibrium model is concerned, the paper relies heavily on Le Roy and Werner (2001). New features introduced herewith include regulatory requirements as a function of expected shortfall, the use of endogenic MSDC and bid-ask spread of the market maker, and the distinction of cash from other assets, which makes saving in risk-free assets possible for all agents simultaneously.

2.1 NOTATION

In this section, we combine and adjust the notation of Csóka and Herings (2014) and Le Roy and Werner (2001). There are two periods in our model. An investor can hold cash, its amount hold denoted by θ_0, as well as risky assets belonging to a set $\mathcal{J} = \{1, \dots, J\}$. Assets are traded in period 0, while payoffs occur in period 1. The payoff of an asset is subject to uncertainty. One out of S possible states of nature materializes in the future, where state of nature $s \in \{1, \dots, S\}$ occurs with probability $\pi_s > 0$, such that $\sum_{s=1}^{S} \pi_s = 1$. The payoff of asset $j \in \mathcal{J}$ in state of nature $s \in \{1, \dots, S\}$ is denoted by $x_{js} \in \mathbb{R}$. Let us denote the payoff of asset $j \in \mathcal{J}$ by the vector $x_j = \left[x_{j1}, \cdots, x_{jS}\right] \in \mathbb{R}^{\mathbb{S}}$ and the payoff-matrix by the matrix $X \in \mathbb{R}^J \times \mathbb{R}^S$. The market is set to be complete if the rank of X is S. We do not assume complete markets.

A portfolio comprises J risky assets. Denote the space of risky portfolios by $\Theta = \mathbb{R}^J$ and a portfolio or position by $\theta \in \Theta$. Short selling is allowed in the model, so agents can construct portfolios with short positions as well. The value of a portfolio depends on the order books for the various assets to be specified as follows. We follow Cetin, Jarrow, and Protter (2004), Jarrow and Protter (2005) and Acerbi and Scandolo (2008) in modeling the order books for every asset $j \in \mathcal{J}$ by a marginal supply-demand curve (MSDC) m_j.

Definition 2.1. The *marginal supply-demand curve* (MSDC) for asset $j \in \mathcal{J}$ is given by the map $m_j : \mathbb{R} \setminus \{0\} \mapsto \mathbb{R}$ satisfying

1. $m_j(h) \geq m_j(h')$ if $h < h'$;
2. m_j is càdlàg (right continuous with left limits) at $h < 0$ and làdcàg (left continuous with right limits) at $h > 0$.

The MSDC can be used to calculate the liquidation value of a $\theta \in \Theta$ portfolio of risky assets. The amount $m_j(h)$ for $h > 0$ expresses the marginal bids at which asset $j \in \mathcal{J}$ can be sold. Similarly, $m_j(h)$ for $h < 0$ represents the marginal asks at which asset j can be bought. We denote by $m_j(0^-)$ the best (marginal) ask and by $m_j(0^+)$ the best (marginal) bid.

Definition 2.2. The *liquidation mark-to-market value* of a risky portfolio $\theta \in \Theta$ is defined by

$$\ell(\theta) = \sum_{j \in \mathcal{J}} \int_0^{\theta_j} m_j(h) dh. \tag{1}$$

We have agents/investors belonging to set $\mathcal{I} = \{1, \dots, I\}$. The *portfolio* $\theta^i \in \mathbb{R}^J$ *of investor* $i \in \mathcal{I}$ shows the amounts of assets held by investor i. Investor i consumes c_0^i in period 0 and $c_1^i = [c_{11}^i, \cdots, c_{1S}^i]$ in period 1, where c_{1s}^i represents consumption in state $s \in \{1, \cdots, S\}$. Investor i's endowment is given by ω_0^i capturing the cash in period 0 and $\omega_1^i = [\omega_{11}^i, \cdots, \omega_{1S}^i]$ representing the stochastic income and value of investments not captured by the assets traded in the model. We assume a continuous, strictly monotonic utility function $u^i : \mathbb{R}^{S+1} \to \mathbb{R}$ to indicate investor i's preferences.

Investor i's baseline consumption-portfolio choice problem without regulatory requirements is

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i) \tag{2}$$

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i \\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S, \end{aligned}$$

where 1^S is the row vector of ones. The agent determines optimal consumption level c_0^i and c_1^i, optimal portfolio θ^i and the amount of the risk free asset θ_0^i. Its utility maximization is subject to

1. its period 0 consumption being no more than initial endowments minus the amount of money needed to open position θ^i and keep risk-free asset (cash or bank deposit) θ_0^i,
2. its period 1 stochastic consumption being no more than its stochastic endowment plus the payoff of position θ^i plus θ_0^i.

2.2 THE ROLE OF THE MARKET MAKER

We assume that agents cannot trade directly with each other, there is a market maker who matches opposite orders for asset. The market maker sets the marginal supply-demand curve as a transaction monopolist for each asset, thereby influencing the liquidity of the markets for those assets.

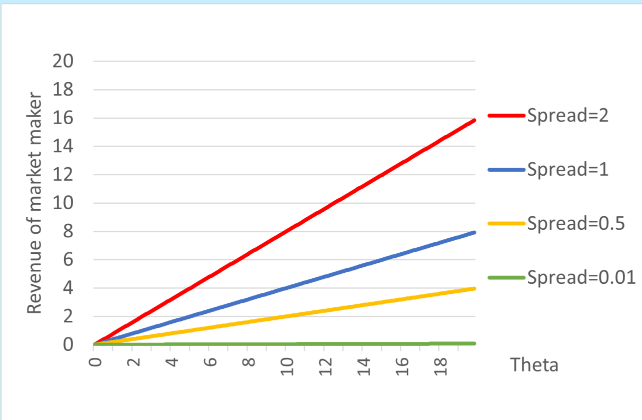
By placing limit orders, the market maker determines MSDCs based on which agents trade by submitting market orders and realizes revenue in the form of transaction fees when matching offers. To simplify, we approximate MSDCs with different functional forms and do not derive them directly from limit orders. The amount of the revenue depends on the functional form of the MSDC (the amount of bid-ask spread, and the distance between transaction price level and best price). Based on the definition of MSDC, the transaction cost function can be defined as follows.

Definition 2.3. For asset $j \in \mathcal{J}$, the *transaction cost function* $T_j : \mathbb{R}^I \to \mathbb{R}$ is defined as

$$T_j(\theta_j^1, \cdots, \theta_j^I) = -\sum_{i \in \mathcal{I}} \int_0^{-\theta_j^i} m_j(h) \mathrm{d}h. \tag{3}$$

Example 2.4. Suppose that asset $j \in \mathcal{J}$ is sold by a single agent and purchased by a single agent in the amount of $\theta_j > 0$.

Figure 1
Transaction cost function with bid-ask spread.



1. If we do not examine the depth of the market and we simply model market liquidity through the bid-ask spread, then the transaction cost of trading asset $j \in \mathcal{J}$ is linear function of the traded quantity θ_j

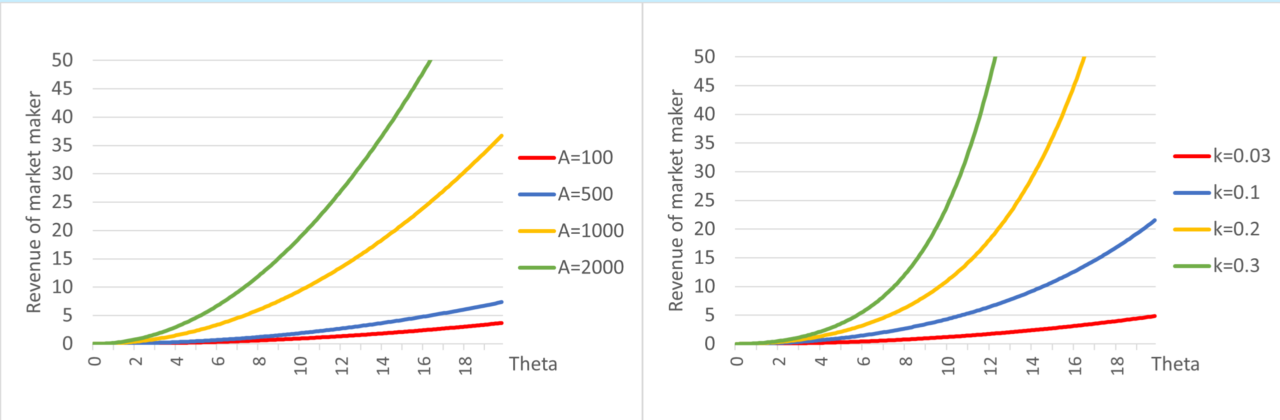
$$T_j(\theta_j, -\theta_j) = \theta_j m_j(0^-) - \theta_j m_j(0^+) = \theta_j(m_j(0^-) - m_j(0^+)),$$

where $(m_j(0^-) - m_j(0^+))$ is the bid-ask spread.

2. Suppose an exponential marginal supply-demand curve, i.e. $m_j(\theta_j^i) = A_j e^{-k_j\theta_j^i}$. The transaction cost function can be given as

$$
\begin{aligned}
T_j(\theta_j, -\theta_j) &= \int_{-\theta_j}^{0} A_j e^{-k_j x}\mathrm{d}x - \int_{0}^{\theta_j} A_j e^{-k_j x}\mathrm{d}x = \\
&= -A_j\left[\frac{1}{k_j}e^{-k_j x}\right]_{-\theta_j}^{0} + A_j\left[\frac{1}{k_j}e^{-k_j x}\right]_{0}^{\theta_j} = \\
&= \frac{A_j}{k_j}\left(e^{-k_j\theta_j} + e^{-k_j(-\theta_j)} - 2\right).
\end{aligned}
$$

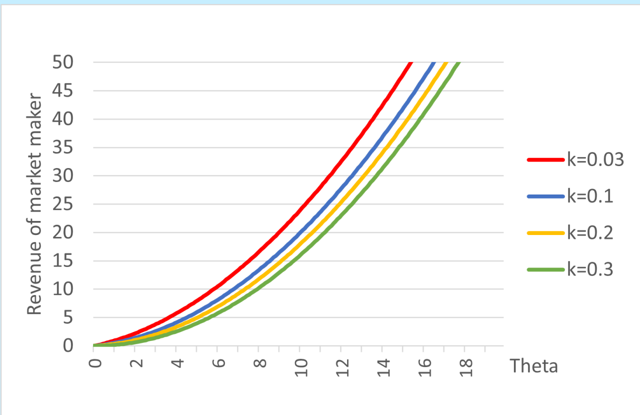
Figure 2
Transaction cost function with exponential MSDC and varying parameter *A* (k=0.003) (left panel) and varying parameter *k* (A=0.4) (right panel)



3. Let the marginal supply-demand curve be a linear function with slope (-1) and discontinuity at 0

$$
m_j(\theta_j^i) = \begin{cases} m_j(0^-) - \theta_j^i, & \text{if } \theta_j^i < 0 \\ m_j(0^+) - \theta_j^i, & \text{if } \theta_j^i > 0. \end{cases}
$$

Figure 3
Transaction cost function with linear MSDC.



If the market maker simultaneously sells and buys $\theta_j > 0$ of the asset $j \in \mathcal{J}$, then the transaction cost function is

$$
T_j(\theta_j, -\theta_j) \quad = \quad (m_j(0^-) - m_j(0^+))\theta_j + \theta_j^2.
$$

We define the revenue of the market maker, i. e. the total transaction cost collected, as follows.

Definition 2.5. *The revenue of the market maker* is given by the function $T : \mathbb{R}^I \times \mathbb{R}^J \to \mathbb{R}$ as

$$T(\theta^1, \cdots, \theta^I) = \sum_{j \in \mathcal{J}} T_j(\theta_j^1, \cdots, \theta_j^I) = -\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}} \int_0^{-\theta_j^i} m_j(h) \mathrm{d}h. \tag{4}$$

Next, we show that the revenue of the market maker can be calculated as the opposite of the sum of the liquidation mark-to-market values of the portfolios of agents.

Proposition 2.6.

$$T(\theta^1, \cdots, \theta^I) = \sum_{j \in \mathcal{J}} T_j(\theta_j^1, \cdots, \theta_j^I) = \sum_{i \in \mathcal{I}} -\ell(-\theta^i). \tag{5}$$

Proof. By swapping the sums and using Definition 2.2, the proposition follows. ■

In the model, the market maker sets the MSDC by maximizing its profit, thus its optimization problem is

$$\max_{m_j() \, \forall j \in \mathcal{J}} \sum_{j \in \mathcal{J}} T_j(\theta_j^1, \cdots, \theta_j^I), \tag{6}$$

subject to each agent maximizing its utility when determining its portfolio θ^i. For a given traded volume, increasing the bid-ask spread or the parameters of the exponential MSDCs (A_j and k_j) leads to an increase in transaction costs and thus in the revenue of the market maker. However, the trade-off is that in their portfolio optimization, agents also consider the transaction cost.

In general, we cannot capture that the order book is changing after each maching of orders if more than one agent buys from an asset using the same offered MSDC by the market maker. To avoid this problem, we assume that there are two agents, $I = 2$. Since the market maker does not hold inventory, it follows that each asset is sold and purchased by one agent only. [6]

2.3 REGULATORY REQUIREMENTS

Both investors in our model try to smooth their consumption by trading assets. We model a regulator discouraging risk taking in specific assets or the portfolio of the assets by requiring the holding of extra cash as follows.

First, consider the case when investors are required to meet a cash liquidity requirement specified as a function of assets' payoffs. Define δ_j as the regulatory parameter for asset j, so regulation determines different δ_js for different markets.

Definition 2.7. Denote the *function of regulatory requirement for asset* $j \in \mathcal{J}$ by $r_j : \mathbb{R} \times \mathbb{R}^S \to \mathbb{R}$. The required amount of the risk free asset is given by inequality

$$\theta_0^i \geq \sum_{j \in \mathcal{J}} r_j \left[\delta_j, \theta_j^i x_j\right],$$

where the *regulatory requirement function* $r_j \left[\delta_j, \theta_j^i x_j\right]$ is a function of the payoff $\theta_j^i x_j$ of asset $j \in \mathcal{J}$ and the regulatory parameter δ_j.

Investor i's consumption and portfolio choice problem with *asset regulatory requirement* is defined as

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i) \tag{7}$$

[6] Hevér (2020) introduces a model variant using the bid-ask spread only, where we do not have this problem. An exogenous constraint on the decisions of agents pre-defining which agent can trade which asset, as in Faias and Luque (2016), would be an alternative solution.

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i \\ \theta_0^i &\geq \sum_{j\in\mathcal{J}} r_j\left[\delta_j, \theta_j^i x_j\right] \\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S. \end{aligned}$$

To further specify the regulatory requirement, we will follow Acerbi and Tasche (2002) and Csóka, Herings and Kóczy (2009) to define the expected shortfall (*ES*) of an asset. For asset $j \in \mathcal{J}$, denote the ordered values of payoffs $x_{j1}, \cdots, x_{jS}$ by $x_{j,s:S}$, that is, $\{x_{j,1:S}, \cdots, x_{j,S:S}\} = \{x_{j1}, \cdots, x_{jS}\}$ and $x_{j,1:S} \leq x_{j,2:S} \cdots \leq x_{j,S:S}$. Denote $\pi_{j,s:S}$ the probability of the state where the expected payoff of asset j is $x_{j,s:S}$.

Definition 2.8. The *n-expected shortfall* ($n \in \{1, \dots, S\}$) of a realization vector x_j for asset $j \in \mathcal{J}$ is defined by

$$ES_n(x_j) = -\sum_{s=1}^{n} \frac{\pi_{j,s:S}}{\sum_{l=1}^{n} \pi_{l,s:S}} x_{j,s:S}.$$

The *n*-expected shortfall is the average loss in the worst *n* states. The regulatory requirement function is given by

$$r_j\left[\delta_j, \theta_j^i x_j\right] = \delta_j ES_n(\theta_j^i x_j).$$

According to the requirement, agents are required to hold risk-free assets corresponding to the amount of the capital requirement aggregated for all assets $j \in \mathcal{J}$

$$\theta_0^i \geq \sum_{j\in\mathcal{J}} \delta_j ES_n(\theta_j^i x_j).$$

In this case, the regulator aims to discourage holding risky assets with significant negative payoff in adverse states of nature without taking into account the risk-mitigating effects of portfolio diversification, and the stochastic endowment. The use of different regulatory parameters for each asset allows the promotion of ESG aspects, as the regulator can stimulate the demand for preferred assets at the expense of that for others.

The regulator can also determine the regulatory requirement as a function of the realization vector of portfolio $\theta^i X$.

Definition 2.9. Denote the *function of regulatory requirement* by $r : \mathbb{R} \times \mathbb{R}^S \to \mathbb{R}$. The required amount of the risk free asset is given by inequality

$$\theta_0^i \geq r\left[\delta, \theta^i X\right],$$

where the *regulatory requirement function* $r\left[\delta, \theta^i X\right]$ is a function of payoff of the risky portfolio $\theta^i X$ and the regulatory parameter δ.

Investor i's consumption and portfolio choice problem with *portfolio regulatory requirement* is defined as

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i) \tag{8}$$

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i \\ \theta_0^i &\geq r\left[\delta, \theta^i X\right] \\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S. \end{aligned}$$

It is straightforward to define the ES at portfolio level, too. Let us denote by $(\sum_{j\in\mathcal{J}} \theta_j^i x_{js})_{1:S} \cdots, (\sum_{j\in\mathcal{J}} \theta_j^i x_{js})_{S:S}$ the ordered values of payoffs $\sum_{j\in\mathcal{J}} \theta_j^i x_{j1}, \cdots, \sum_{j\in\mathcal{J}} \theta_j^i x_{jS}$ and by $\pi_{\theta^i X, z:S}$ the probability of the state where the expected payoff of portfolio θ^i is $(\sum_{j\in\mathcal{J}} \theta_j^i x_{js})_{z:S}$.

Definition 2.10. Given a portfolio $\theta^i \in \Theta$, the *n-expected shortfall* ($n \in \{1, \dots, S\}$) of a realization vector $\theta^i X$ is defined by

$$ES_n(\theta^i X) = -\sum_{z=1}^{n} \frac{\pi_{\theta^i X, z:S}}{\sum_{l=1}^{n} \pi_{\theta^i X, l:S}} \left(\sum_{j \in \mathcal{J}} \theta_j^i x_{js} \right)_{z:S} .$$

If the regulator aims to contain the risk of the total asset portfolio of the agents, it determines the regulatory requirement as a function of the ES quantified based on realization vector of the portfolio $\theta^i X$ as

$$r\left[\delta, \theta^i X\right] = \delta ES_n(\theta^i X).$$

The requirement becomes

$$\theta_0^i \geq \delta ES_n(\theta^i X),$$

where the regulatory parameter δ determines what proportion of the expected shortfall of the portfolio should be kept in the risk-free asset. If the average expected payoff of the portfolio calculated for the worst n states is positive, i.e. it provides a profit, the ES will be negative. Borrowing is possible, but its extent is limited by the expected profit. The regulator takes into account the diversification resulting from the holding of the portfolio, and formulates a lower (or equal) capital requirement compared to the regulatory requirement formulated at the asset level.

2.4 EQUILIBRIUM

Definition 2.11. Let $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ denote the equilibrium where portfolio allocation $\{\theta^{*i}, \theta_0^{*i}\}$ and consumption plan $\{c_0^{*i}, c_1^{*i}\}$ are a solution to optimization problem of agent i and MSDCs $\{m_1^*(), \dots, m_J^*()\}$ are a solution to optimization problem of the market maker. In equilibrium

$$\sum_{i \in \mathcal{I}} \theta^i = 0, \tag{9}$$

$$\sum_{i \in \mathcal{I}} c_0^i = \sum_{i \in \mathcal{I}} \omega_0^i - T(\theta^1, \cdots, \theta^I) - \sum_{i \in \mathcal{I}} \theta_0^i, \tag{10}$$

$$\sum_{i \in \mathcal{I}} c_1^i = \sum_{i \in \mathcal{I}} \omega_1^i + \sum_{i \in \mathcal{I}} \theta_0^i 1^S, \tag{11}$$

so the portfolio market clearing[7] and the consumption market clearing conditions hold.

Proposition 2.12. *In equilibrium, when portfolio market clears*

$$\sum_{i \in \mathcal{I}} \theta^i = 0,$$

the consumption market-clearing conditions hold as well.

Proof. Summarize the budget constraints of the agents for periods 0 and 1

$$\begin{aligned} \sum_{i \in \mathcal{I}} c_0^i &\leq \sum_{i \in \mathcal{I}} \omega_0^i + \sum_{i \in \mathcal{I}} \ell(-\theta^i) - \sum_{i \in \mathcal{I}} \theta_0^i \\ \sum_{i \in \mathcal{I}} c_1^i &\leq \sum_{i \in \mathcal{I}} \omega_1^i + \sum_{i \in \mathcal{I}} \theta^i X + \sum_{i \in \mathcal{I}} \theta_0^i 1^S. \end{aligned}$$

The term $\sum_{i \in \mathcal{I}} \theta^i X$ represents the aggregated payoff of the agents' risky portfolios. By using portfolio market-clearing condition $\sum_{i \in \mathcal{I}} \theta^i = 0$, proposition 2.6 and the assumption of the strictly monotonic utility function, the consumption market-clearing conditions follow. ■

[7] In period 0, the sum of the cash/risk free deposits of the agents is not equal 0. We assume that there is an outside agent (e.g. bank) who ensures the required amount.

To understand what happens to market liquidity upon the introduction of regulatory requirement, we have to compare two equilibria: the one determined by the decision of agents in optimization problem (2) without regulatory requirement, and the ones resulting from agent decisions in optimization problem (7) with asset regulatory requirement and optimization problem (8) with portfolio regulatory requirement. In the remainder of this subsection, we analyze adding the asset regulatory requirement and consider two cases.

First, the optimum determined without regulatory requirements meets the introduced constraint, i.e. the asset regulatory requirement is redundant.

Remark 2.13. Let $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ denote the equilibrium where θ^{*i}, θ_0^{*i}, c_0^{*i}, and c_1^{*i} are solutions to optimization problem (2) of the agents. If for all $i \in \mathcal{I}$ we have that

$$\theta_0^i \geq \sum_{j \in \mathcal{J}} r_j \left[\delta_j, \theta_j^i x_j\right],$$

then $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ remains the equilibrium, if agents make decisions according to optimization problem (7).

Second, if there is an agent in the equilibrium determined without regulatory requirement who breaches the constraint introduced as an asset regulatory requirement, the portfolio chosen earlier will not be attainable to it after the introduction of the regulatory requirement.

Remark 2.14. Let $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ denote the equilibrium where θ^{*i}, θ_0^{*i}, c_0^{*i}, and c_1^{*i} are solutions to optimization problem (2) of the agents, and suppose that there exists an $\bar{i} \in \mathcal{I}$, for which

$$\theta_0^{*\bar{i}} < \sum_{j \in \mathcal{J}} r_j \left[\delta_j, \theta_j^{*\bar{i}} x_j\right].$$

- In this case, equilibrium $\{\theta^{**i}, \theta_0^{**i}, c_0^{**i}, c_1^{**i}, m_j^{**}()\}$ where the agents decide according to optimization problem (7), is not identical to equilibrium $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$.
- There exists an agent $\hat{i} \in \mathcal{I}$, for which the constraint will be satisfied with equality

$$\theta_0^{**\hat{i}} = \sum_{j \in \mathcal{J}} r_j \left[\delta_j, \theta_j^{**\hat{i}} x_j\right].$$

In the second case, the equilibrium changes due to the introduction of the regulatory requirement; the question is what happens to it.

3 The effects on market liquidity

3.1 AN EXAMPLE WITH PORTFOLIO REGULATORY REQUIREMENT

On top of two agents, $I = 2$, assume $J = 2$ and $S = 2$, so there are two assets, and two states of nature. The specific model in which the endowments of agents are inverse in period 1 ($\omega^2_{12} = \omega^1_{11} = \omega_1$) and identical in period 0 ($\omega^2_0 = \omega^1_0 = \omega_0$) simplifies calculations and is suitable for the analysis of the relationship between regulatory constraints and market liquidity. In this case, the problem is symmetric, and for consumption, $c^1_0 = c^2_0 = c_0$, $c^1_{11} = c^2_{12}$ and $c^2_{11} = c^1_{12}$ hold true, while $\theta^1_0 = \theta^2_0 = \theta_0$ and $\theta_1 = \theta_2 = \theta$ hold for the assets in the optimal portfolios. Suppose that $x_{12} = x_{21} = 0$ and $x_{11} = x_{22} = x$ for the payoffs of the two assets. The market maker prices two assets with inverse payoffs, and the target portfolios of the agents are inverted. The market maker buys and sells quantities θ of both assets; therefore, setting the same exponential MSDC ($A_1 = A_2 = A$ and $k_1 = k_2 = k$) for both assets is a precondition to the existence of an equilibrium.[8]

Let us consider the portfolio regulatory requirement with $n = 1$, that is expected shortfall is calculated in the worst state. The portfolio of risky assets of agent 1 is $\theta^1 = (-\theta, \theta)$, and that of agent 2 is $\theta^2 = (\theta, -\theta)$, thus the liquidation values of the portfolios are identical in equilibrium.

$$
\begin{aligned}
\ell(-\theta^1) = \ell(-\theta^2) &= \int_0^{\theta} Ae^{-kx}\mathrm{d}x + \int_0^{-\theta} Ae^{-kx}\mathrm{d}x = \\
-A\left[\frac{1}{k}e^{-kx}\right]_0^{\theta} + A\left[\frac{1}{k}e^{-kx}\right]_{-\theta}^{0} &= -\frac{A}{k}\left(e^{-k\theta} - 1\right) + \frac{A}{k}\left(1 - e^{k\theta}\right) \\
&= \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right).
\end{aligned}
$$

We can suppose that, when optimizing, agents know that they can only choose portfolios $\theta^1 = (-\theta, \theta)$ and $\theta^2 = (\theta, -\theta)$. In this case, the conditional optimization problems of the agents are identical and can be defined as

$$
\max_{\theta,\theta_0} \ln\left(\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0\right) + \frac{1}{2}\ln(\omega_1 - \theta x + \theta_0) + \frac{1}{2}\ln(\theta x + \theta_0)
$$

subject to

$$
\theta_0 \geq \delta\theta x.
$$

The first order conditions are given by

$$
\begin{aligned}
\frac{Ae^{-k\theta} - Ae^{k\theta}}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} - \frac{\frac{1}{2}x}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}x}{\theta x + \theta_0} + \lambda\delta x &= 0, & (12)\\
\frac{-1}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} + \frac{\frac{1}{2}}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}}{\theta x + \theta_0} - \lambda &= 0, & (13)\\
\theta_0 - \delta\theta x &\geq 0, & (14)\\
\lambda[\theta_0 - \delta\theta x] &= 0, & (15)\\
\lambda &\geq 0. & (16)
\end{aligned}
$$

Example 3.1. Similarly to the case of the specific model, assume two agents, two assets and $S = 2$ states of nature in period 1. Agents have endowments of $\omega_0 = 10$ in period 0 and $\omega^1_1 = (20, 0)$ and $\omega^2_1 = (0, 20)$ in period 1. The payoffs of the risky assets are $x_1 = (2, 0)$ and $x_2 = (0, 2)$, respectively. The regulatory parameter is $\delta = 0.3$. First, suppose that the market maker sets an exogenous exponential MSDC in the functional form of $m(\theta) = Ae^{-k\theta}$.

Table 1
Equilibrium for $k = 0.003$ and exponential MSDCs at various values of parameter A.

A	θ	θ_0	c_0	$c^1_{12} = c^2_{11}$	$c^1_{11} = c^2_{12}$	$u^1 = u^2$	$\theta_0 - \delta\theta x$
2000	0.12	3.49	6.4	3.7	23.2	4.09	3.41
1000	0.24	3.32	6.5	3.8	22.8	4.10	3.18
500	0.47	3.00	6.7	3.9	22.1	4.13	2.72
100	1.86	1.23	7.7	5.0	17.5	4.28	0.12
50	2.37	1.42	7.7	6.2	16.7	4.36	0.00

The equilibrium portfolios of agents using exponential MSDCs with various values of parameters A and k can be calculated. For a given parameter k, increasing the value of parameter A results in an increase in transaction cost (Table 1). With transaction cost increasing, the smoothing of the stochastic endowment of period 1 is less and less feasible. The difference between the consumptions of the two future states of nature increases and the attainable level of utility decreases. No transaction is carried out when the level of utility drops to the level attainable without trading. Trading and, consequently, risk sharing are constrained by the introduction of regulatory requirement. In a favourable transaction environment ($k = 0.003$ and $A \leq 50$), traded amount θ in risky assets is constrained by regulatory requirements in reaching the optimum from risk sharing point of view. The impact is similar in case of a fixed parameter A and an increasing parameter k (Table 2).

Table 2
Equilibrium in case of exponential MSDCs with $A = 500$ and various values of parameter k.

k	θ	θ_0	c_0	$c^1_{12} = c^2_{11}$	$c^1_{11} = c^2_{12}$	$u^1 = u^2$	$\theta_0 - \delta\theta x$
0.1	0.01	3.64	6.35	23.6	3.7	4.08	3.63
0.01	0.14	3.45	6.4	23.2	3.7	4.09	3.37
0.005	0.28	3.25	6.5	22.7	3.8	4.11	3.08
0.003	0.47	3.00	6.7	22.1	3.9	4.13	2.72
0.001	1.25	1.96	7.3	19.4	4.5	4.21	1.20
0.0001	3.07	1.84	7.7	15.7	8.0	4.45	0.00

When setting the optimal MSDC, the market maker considers the optimal decisions of agents with the given MSDC. Therefore, for the maximization of transaction revenue, we can assume that the first order conditions to the consumption-portfolio choice problem of agents are met. Without regulatory constraints, the optimization problem of the market maker is given by

$$\max_{A,k} -\ell(-\theta^1) - \ell(-\theta^2) = \frac{2A}{k}\left(e^{-k\theta} + e^{k\theta} - 2\right),$$

subject to

$$\frac{Ae^{-k\theta} - Ae^{k\theta}}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} - \frac{\frac{1}{2}x}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}x}{\theta x + \theta_0} = 0, \text{ and}$$

$$\frac{-1}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} + \frac{\frac{1}{2}}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}}{\theta x + \theta_0} = 0.$$

With regulatory constraints, the market maker optimizes the transaction cost function under first order conditions (12)-(16). Let us see whether the optimal MSDC of the market maker changes upon introduction of regulatory constraints. If, in equilibrium, $\lambda = 0$, then $\theta_0 - \delta\theta x \geq 0$ is met, and the optimization problem of the market maker does not change. However, $\theta_0 - \delta\theta x = 0$

[8] The market maker prices assets with inverse payoffs; therefore, it could seem intuitive to substitute the two assets for a single one with payoff x in state of nature 1 and payoff $-x$ in state of nature 2. The market maker would price an asset with payoff $[x, -x]$ by setting parameters A and k of the exponential MSDC. However, the liquidation values of the portfolios of the two agents with symmetric positions trading through the same market maker would be different, thus this would not be an equilibrium. In the case of bid-ask spread, the assumption of a single asset will be possible.

and $\lambda > 0$ can also apply in equilibrium. In this case, the inequality $\theta_0 - \delta\theta x < 0$ applies in the equilibrium determined without regulatory constraints. The constraint can be binding in the new equilibrium, provided that

- θ_0 increases, or
- θ_0 is unchanged and θ decreases, or
- θ_0 decreases and θ decreases.

If θ decreases, the market maker sets a less liquid MSDC to maximize transaction revenue. We have to investigate whether θ can remain unchanged or increase in the new equilibrium. By rearranging the first order condition (13), we get that

$$c_0 = \frac{2(\lambda + 1)}{\frac{1}{c_{11}^1} + \frac{1}{c_{12}^1}} \tag{17}$$

for the consumptions of the periods. As $\lambda > 0$, the smoothing of consumption across periods is less feasible; the relative consumption of period 0 will be higher. For increasing θ_0 and unchanged θ, condition (17) is breached. As the liquidation value $(2 - e^{-k\theta} - e^{k\theta})$ decreases upon an increase in θ, the condition cannot be met if θ_0 and θ increase, either. Quantity θ necessarily decreases in the new equilibrium, i.e. the introduction of a regulatory constraint results in a decrease in market liquidity.

Example 3.2. Continue example 3.1. Suppose that the market maker sets an exponential MSDC in the functional form of $m(\theta) = Ae^{-k\theta}$ endogenously. Supposing a fixed parameter $k = 0.003$, the market maker determines the optimal parameter value A for various regulatory parameters δ, while if parameter $A = 100$ is fixed rather than k, the market maker will determine the optimal value of k for each δ (Table 3). For a given parameter δ, the same level of transaction cost will be attained by setting optimal MSDCs of various shapes.

Table 3
Optimal decision of the market maker for exponential MSDCs with parameters A and k and various values of parameter δ.

δ	A ($k = 0.003$)	k ($A = 100$)	θ	θ_0	c_0	$c_{12}^1 = c_{11}^2$	$c_{11}^1 = c_{12}^2$	$u^1 = u^2$	$T()$
0.6	140.7	0.0042	1.4	1.7	10.0	18.8	4.6	4.54	1.75
0.5	126.0	0.0038	1.6	1.6	10.3	18.4	4.7	4.56	1.86
0.3	95.0	0.0029	1.9	1.2	11.1	17.3	5.0	4.63	2.12
0.2	78.2	0.0023	2.2	0.9	11.4	16.5	5.3	4.66	2.26
0.1	59.3	0.0018	2.6	0.5	11.9	15.3	5.7	4.71	2.37
0	53.1	0.0016	2.7	0.4	12.0	14.9	5.9	4.72	2.38

In the example, the optimal parameters k and A of the market maker are lower without regulatory constraints, the MSDC set is flatter and the risky asset is more liquid. In the optimum on a more liquid market, agents trade a higher quantity θ of risky assets, and their utility increases due to the partial realization of risk sharing. The market maker is compensated through higher traded volume on the more liquid market, the total transaction fee collected is higher than in a less liquid market.

Using the MSDC optimal for the market maker, the regulatory constraint introduced will be binding for all values of regulatory parameters $\delta \geq 0.1$ in Table 3 ($\theta_0 - \delta\theta = 0$). The equilibrium changes due to the regulatory constraint. When determining their optimal portfolios, agents hit the constraint, and the prior optimal portfolio is not attainable any more. Due to regulation, agents trade a lower quantity θ of risky assets, thus the optimal MSDC of the market maker is less liquid.

If the value of the regulatory parameter is lower than the 0.1 used in the table, for example $\delta = 0.05$, the regulatory constraint is also met in the optimum determined without constraint ($\theta_0 - \delta\theta = 0, 12$). Equilibrium and market liquidity do not change.

3.2 MORE GENERAL RESULTS WITHOUT REGULATION

On top of $I = 2$, suppose that $J = 2$, so two investors trade two risky assets. The consumption-portfolio choice problem of agent $i \in \mathcal{I}$ is

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i), \tag{18}$$

subject to

$$\begin{aligned} c_0^i &= \omega_0^i + \ell(-\theta^i) - \theta_0^i \\ c_1^i &= \omega_1^i + \theta^i X + \theta_0^i 1^S. \end{aligned}$$

When the portfolio market clears, the sum of cash is not necessarily 0. The market-clearing condition can be given by the equation

$$\theta^1 + \theta^2 = 0,$$

which implies that $-\theta_j^1 = \theta_j^2$ for all risky asset $j \in \mathcal{J}$.[9] Using $\theta_1 = -\theta_1^1 = \theta_1^2$ and $\theta_2 = \theta_2^1 = -\theta_2^2$ we get the transaction cost function as

$$\begin{aligned} T_1(\theta_1^1, \theta_1^2) + T_2(\theta_2^1, \theta_2^2) &= -\ell(-\theta^1) - \ell(-\theta^2) = \\ = -\int_0^{-\theta_1^1} m_1(h)\mathrm{d}h - \int_0^{-\theta_1^2} m_1(h)\mathrm{d}h - \int_0^{-\theta_2^1} m_2(h)\mathrm{d}h - \int_0^{-\theta_2^2} m_2(h)\mathrm{d}h &= \\ = -\int_0^{\theta_1} m_1(h)\mathrm{d}h + \int_{-\theta_1}^{0} m_1(h)\mathrm{d}h - \int_0^{-\theta_2} m_2(h)\mathrm{d}h + \int_{\theta_2}^{0} m_2(h)\mathrm{d}h &= \\ \int_0^{\theta_1} (m_1(-h) - m_1(h))\,\mathrm{d}h + \int_0^{\theta_2} (m_2(-h) - m_2(h))\,\mathrm{d}h&. \end{aligned}$$

The market maker maximizes the transaction cost function

$$\max_{m_1(), m_2()} T_1(\theta_1^1, \theta_1^2) + T_2(\theta_2^1, \theta_2^2),$$

subject to

$$\begin{aligned} \frac{\partial u^1(\omega_0^1 + \ell(-\theta^1) - \theta_0^1, \omega_1^1 + \theta^1 X + \theta_0^1 1^S)}{\partial \theta_1} &= 0, \\ \frac{\partial u^1(\omega_0^1 + \ell(-\theta^1) - \theta_0^1, \omega_1^1 + \theta^1 X + \theta_0^1 1^S)}{\partial \theta_2} &= 0, \\ \frac{\partial u^1(\omega_0^1 + \ell(-\theta^1) - \theta_0^1, \omega_1^1 + \theta^1 X + \theta_0^1 1^S)}{\partial \theta_0^1} &= 0, \\ \frac{\partial u^2(\omega_0^2 + \ell(-\theta^2) - \theta_0^2, \omega_1^2 + \theta^2 X + \theta_0^2 1^S)}{\partial \theta_1} &= 0, \\ \frac{\partial u^2(\omega_0^2 + \ell(-\theta^2) - \theta_0^2, \omega_1^2 + \theta^2 X + \theta_0^2 1^S)}{\partial \theta_2} &= 0, \text{ and} \\ \frac{\partial u^2(\omega_0^2 + \ell(-\theta^2) - \theta_0^2, \omega_1^2 + \theta^2 X + \theta_0^2 1^S)}{\partial \theta_0^2} &= 0. \end{aligned}$$

If ceteris paribus $|\theta_1|$ or $|\theta_2|$ increases, then the transaction cost of the market maker increases. Of course, ceteris paribus, increasing the MSDC $m_1()$ also leads to an increase in the transaction cost. But at the same time, the increase in MSDC will

[9] Generally, we have consumption market clearing conditions (one in period 0 and S in the S states of the period 1)

$$\begin{aligned} c_0^1 + c_0^2 &= \omega_0^1 + \omega_0^2 - T(\theta^1, \theta^2) - \theta_0^1 - \theta_0^2 \\ c_1^1 + c_1^2 &= \omega_1^1 + \omega_1^2 + \theta_0^1 1^S + \theta_0^2 1^S \end{aligned}$$

as well. By summing up budget constraints, they trivially hold in this specific case.

reduce the volume traded, $|\theta_1|$. In equilibrium, the marginal benefit of increasing MSDCs equals the marginal cost of reducing traded volume. Marginal profit and marginal cost can be determined based on the optimization problem of the market maker, while the extent of the decrease in traded volume can be determined based on the optimization problem of agents.

3.3 MORE GENERAL RESULTS WITH REGULATION

The introduction of regulatory requirements adds constraints defined through inequalities to the optimization problem of agents[10]. The regulator can define the requirement as a function of the expected shortfall of assets or portfolios. In case of asset regulatory requirement, the consumption-portfolio choice problem of agent 1 is

$$\max_{c_0^1, c_1^1, \theta^1, \theta_0^1} u^1(c_0^1, c_1^1), \tag{19}$$

subject to

$$\begin{aligned} c_0^1 &= \omega_0^1 + \ell(-\theta^1) - \theta_0^1 \\ \theta_0^1 &\geq \delta_1 ES_n(\theta_1^1 x_1) + \delta_2 ES_n(\theta_2^1 x_2) \\ c_1^1 &= \omega_1^1 + \theta^1 X + \theta_0^1 1^S, \end{aligned}$$

and for agent 2 is

$$\max_{c_0^2, c_1^2, \theta^2, \theta_0^2} u^2(c_0^2, c_1^2), \tag{20}$$

subject to

$$\begin{aligned} c_0^2 &= \omega_0^2 + \ell(-\theta^2) - \theta_0^2 \\ \theta_0^2 &\geq \delta_1 ES_n(\theta_1^2 x_1) + \delta_2 ES_n(\theta_2^2 x_2) \\ c_1^2 &= \omega_1^2 + \theta^2 X + \theta_0^2 1^S. \end{aligned}$$

Assume $S = 2$ with two equiprobable states of nature and $n = 1$ that is expected shortfall is calculated in the worst state. Applying that $\theta_1 = -\theta_1^1 = \theta_1^2$ and $\theta_2 = \theta_2^1 = -\theta_2^2$,

$$\begin{aligned} \theta_0^1 &\geq -\delta_1 \min\{-\theta_1 x_{11}; -\theta_1 x_{12}\} - \delta_2 \min\{\theta_2 x_{21}; \theta_2 x_{22}\} \\ \theta_0^2 &\geq -\delta_1 \min\{\theta_1 x_{11}; \theta_1 x_{12}\} - \delta_2 \min\{-\theta_2 x_{21}; -\theta_2 x_{22}\}. \end{aligned}$$

We have the following results.

Proposition 3.3. *By introducing an asset regulatory requirement, the regulated assets will be less liquid compared to no regulation.*

Proof. The Karush-Kuhn-Tucker conditions can be used to find the solution to the optimization problems (19) and (20) (Boyd and Vandenberghe, 2004). The Lagrangian functions of the agents are

$$\begin{aligned} G_1(\theta_1, \theta_2, \theta_0^1, \lambda_1) = u^1\left(\omega_0^1 + \ell(-\theta^1) - \theta_0^1, \omega_1^1 + \theta^1 X + \theta_0^1 1^S\right) - \\ -\lambda_1[\theta_0^1 - \delta_1 ES_n(-\theta_1 x_1) - \delta_2 ES_n(\theta_2 x_2)] \end{aligned}$$

and

$$\begin{aligned} G_2(\theta_1, \theta_2, \theta_0^2, \lambda_2) = u^2\left(\omega_0^2 + \ell(-\theta^2) - \theta_0^2, \omega_1^2 + \theta^2 X + \theta_0^2 1^S\right) - \\ -\lambda_2[\theta_0^2 - \delta_1 ES_n(\theta_1 x_1) - \delta_2 ES_n(-\theta_2 x_2)], \end{aligned}$$

[10] No capital requirement applies to the market maker because it matches opposite orders without taking risk on its own balance sheet once the market has cleared.

where λ_1 and λ_2 are the Lagrange multipliers. In addition to the first order conditions, due to complementary slackness and dual feasibility the following conditions also hold

$$\begin{aligned}\lambda_1[\theta_0^1 - \delta_1 ES_n(-\theta_1 x_1) - \delta_2 ES_n(\theta_2 x_2)] &= 0,\\ \lambda_2[\theta_0^2 - \delta_1 ES_n(\theta_1 x_1) - \delta_2 ES_n(-\theta_2 x_2)] &= 0,\\ \lambda_1 &\geq 0, \text{and}\\ \lambda_2 &\geq 0.\end{aligned}$$

When comparing the two equilibria, we use the notation of Remarks 2.13 and 2.14. If the regulatory requirement is redundant, i.e. the optimum determined without the regulatory requirement complies with the new constraint, then for the Lagrange multipliers of the Karush-Kuhn-Tucker optimization, we have that $\lambda_1 = \lambda_2 = 0$. We get back the first order conditions of the model without regulatory requirements. The optimization problem of the market maker is unchanged, and the MSDCs describing market liquidity do not change upon the introduction of the regulatory requirement *(Remark 2.13)*.

If $\lambda_1 \neq 0$ and/or $\lambda_2 \neq 0$, then

$$\theta_0^{**1} - \delta_1 ES_n(-\theta_1^{**} x_1) - \delta_2 ES_n(\theta_2^{**} x_2) = 0$$

and/or

$$\theta_0^{**2} - \delta_1 ES_n(\theta_1^{**} x_1) - \delta_2 ES_n(-\theta_2^{**} x_2) = 0.$$

due to complementarity. When optimizing, agents hit the new constraint, which will be binding. The optimal decisions of the agents change, thus the constraints limiting the decision of the market maker are also modified *(Remark 2.14)*.

Suppose $\delta_1 > 0$ and $\delta_2 = 0$ so the regulator defines the requirement as a function of the expected shortfall of asset 1 and compare the new equilibrium under regulatory constraints to the one without regulation. For optimal portfolios determined without regulatory constraints, inequalities

$$\theta_0^{*1} - \delta_1 ES_n(-\theta_1^* x_1) < 0$$

or/and

$$\theta_0^{*2} - \delta_1 ES_n(\theta_1^* x_1) < 0$$

hold true, when the regulatory requirement is not redundant. In the new equilibrium, at least one of the constraints on asset 1 is binding.

If $ES_n(-\theta_1^* x_1) > 0$, to satisfy the inequalities

$$\theta_0^{*1} - \delta_1 ES_n(-\theta_1^* x_1) < 0$$

and

$$\theta_0^{**1} - \delta_1 ES_n(-\theta_1^{**} x_1) \geq 0,$$

three cases need to be examined.

- *Case 1.* $\theta_0^{**1} > \theta_0^{*1}$.
- *Case 2.* $\theta_0^{**1} = \theta_0^{*1}$ and $\theta_1^{**} < \theta_1^*$.
- *Case 3.* $\theta_0^{**1} < \theta_0^{*1}$ and $\theta_1^{**} < \theta_1^*$.

The question is whether $\theta_1^{**} = \theta_1^*$ or $\theta_1^{**} > \theta_1^*$ can hold true for equilibrial positions of asset 1 while $\theta_0^{**1} > \theta_0^{*1}$. If the utility function (e. g. CRRA[11] or as a special case, a logarithmic utility function) ensures consumption smoothing, and the payoff of

[11] For details on consumption smoothing in case of the CRRA utility function see Donaldson and Mehra (2008).

asset 1 and the payoff of asset 2 are linearly independent vectors, then $\theta_1^{**} < \theta_1^*$ and the monopolist market maker sets a less liquid MSDC when maximizing transaction cost. With the new MSDC, θ_1^{**} would be optimal even without regulatory constraints.

If $ES_n(-\theta_1^* x_1) < 0$, then $ES_n(\theta_1^* x_1) > 0$ and inequalities

$$\begin{array}{rcl} \theta_0^{*2} - \delta_1 ES_n(\theta_1^* x_1) & < & 0 \\ \theta_0^{**2} - \delta_1 ES_n(\theta_1^{**} x_1) & \geq & 0 \end{array}$$

should be analyzed analogously.

■

In case of portfolio regulatory requirement, the following conditions will limit agent 1 and agent 2

$$\begin{array}{rcl} \theta_0^1 & \geq & \delta ES_n(\theta^1 X), \\ \theta_0^2 & \geq & \delta ES_n(\theta^2 X). \end{array}$$

The regulatory requirement is δ times the portfolio loss realised in the state of nature resulting in the lower payoff, i.e.

$$\begin{array}{rcl} \theta_0^1 & \geq & -\delta \min\{-\theta_1 x_{11} + \theta_2 x_{21}; -\theta_1 x_{12} + \theta_2 x_{22}\}, \\ \theta_0^2 & \geq & -\delta \min\{+\theta_1 x_{11} - \theta_2 x_{21}; +\theta_1 x_{12} - \theta_2 x_{22}\}. \end{array}$$

We have the following results.

Proposition 3.4. *By introducing a portfolio regulatory requirement, assets will be less liquid compared to no regulation.*

Proof. Consider the case, when at least one optimizing agents hits the new constraint, so the equilibrium changes upon the introduction of regulatory requirement. For optimal portfolios determined without regulatory constraints, inequalities

$$\theta_0^{*1} - \delta ES_n(\theta^{*1} X) < 0$$

and/or

$$\theta_0^{*2} - \delta ES_n(\theta^{*2} X) < 0$$

hold true, when the regulatory requirement is not redundant. In the new equilibrium,

$$\theta_0^{**1} - \delta ES_n(\theta^{**1} X) \geq 0$$

and

$$\theta_0^{**2} - \delta ES_n(\theta^{**2} X) \geq 0.$$

At least one of the constraints is binding.

Suppose that agent 1 has positive endowment in state of nature 1, while its endowment is 0 in state of nature 2. Conversely, agent 2 has no endowment in state of nature 1 and positive endowment in state of nature 2. Our assumption models the very case where the natural exposures of agents are inverted, thus they can reciprocally reduce the uncertainty of future payoffs through trading. We can assume that $x_{11} > 0$ and $x_{22} > 0$, while $x_{12} = 0$ and $x_{21} = 0$. Agent 1 sells asset 1 ($-\theta_1 < 0$) and buys asset 2 ($\theta_2 > 0$), whereas agent 2 buys asset 1 ($\theta_1 > 0$) and sells asset 2 ($-\theta_2 < 0$). For this specific problem, the regulatory requirement defined at the level of portfoliosare

$$\begin{array}{rcl} \theta_0^1 & \geq & \delta \theta_1 x_{11}, \\ \theta_0^2 & \geq & \delta \theta_2 x_{22}. \end{array}$$

For optimal portfolios determined without regulatory constraints, inequalities

$$\theta_0^{*1} - \delta\theta_1^{*}x_{11} < 0$$

and/or

$$\theta_0^{*2} - \delta\theta_2^{*}x_{22} < 0$$

hold true, when the regulatory requirement is not redundant. In the new equilibrium, we have that

$$\theta_0^{**1} - \delta\theta_1^{**}x_{11} \geq 0$$

and

$$\theta_0^{**2} - \delta\theta_2^{**}x_{22} \geq 0.$$

If the utility function ensures consumption smoothing, then $\theta_1^{**} < \theta_1^{*}$ in all three cases, so the market maker sets a less liquid MSDC when maximizing transaction cost. With the new MSDC, θ_1^{**} would be optimal even without regulatory constraints. Analogously, for asset 2, $\theta_2^{**} < \theta_2^{*}$ holds true. ■

4 Conclusions

We extend a standard two-period general equilibrium model with transaction costs of trading, endogenous MSDCs, and the modeling of asset or portfolio regulation. Our model has the empirically testable prediction that assets related to regulation ensuring funding liquidity and assets with bad ESG scores promoted for divestment will have a lower market liquidity. In real life, regulation is much more complex, and intervention is justified by market imperfections. Nevertheless, our paper confirms that intervention has its costs and market liquidity is impacted by regulatory requirements, which should be considered during the impact assessment of regulatory proposals.

References

Acerbi, C., and G. Scandolo (2008), Liquidity risk theory and coherent measures of risk, *Quantitative Finance*, 8 (7), 681–692.

Acerbi, C., and D. Tasche (2002), On the coherence of expected shortfall, *Journal of Banking & Finance*, 26(7), 1487-1503.

Acharya, V. V. and L. H. Pedersen (2005), Asset pricing with liquidity risk, *Journal of Financial Economics*, 77 (2), 375–410.

Almgren, R. and N. Chriss (2001), Optimal execution of portfolio transactions, *Journal of Risk*, 3 (2), 5–39.

Amihud, Y. (2002), Illiquidity and stock returns cross-section and time-series effects, *Journal of Financial Markets*, 5 (1), 31–56.

Amihud, Y., H. Mendelson and L. H. Pedersen (2013), Market Liquidity: Asset Pricing, Risk and Crises, *Cambridge University Press*.

Amihud, Y., H. Mendelson and R. Wood (1990), Liquidity and the 1987 Stock Market Crash, *Journal of Portfolio Management*, 16 (3), 65–69.

Basel Committee on Banking Supervision (2013), Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools, *Bank for International Settlements*, January 2013.

Basel Committee on Banking Supervision (2014), Consultative Document: Fundamental review of the trading book: A revised market risk framework, *Bank for International Settlements*, January 2014.

Basel Committee on Banking Supervision (2019), Minimum capital requirements for market risk, *Bank for International Settlements*, February 2019.

Begenau, J. (2019), Capital Requirements, Risk Choice, and Liquidity Provision in a Business-Cycle Model, *Journal of Financial Economics*, 136 (2), 355–378.

Boyd, S. and L. Vandenberghe (2004), Convex Optimization, *Cambridge University Press*.

Brunnermeier, M. K. and L. H. Pedersen (2008), Market Liquidity and Funding Liquidity, *Review of Financial Studies*, 22 (6), 2201–2238.

Çetin, U., R.A. Jarrow and P. Protter (2004), Liquidity risk and arbitrage pricing theory, *Finance and Stochastics*, 8, 311–341.

Csóka P. and P.J.J. Herings (2014), Risk Allocation under Liquidity Constraints, *Journal of Banking and Finance*, 49, 1–9.

Csóka P., P. J. J. Herings and L. Á. Kóczy (2009), Stable allocations of risk, *Games and Economic Behavior*, 67 (1), 266–276.

De Nicolò, G., A. Gamba and M. Lucchetta (2014), Microprudential Regulation in a Dynamic Model of Banking, *The Review of Financial Studies*, 27 (7), 2097–2138.

Donaldson J. and R. Mehra (2008), CHAPTER 2 - Risk-Based Explanations of the Equity Premium, *In Handbooks in Finance, Handbook of the Equity Risk Premium, Elsevier*, 37–99.

Faias, M. and J. Luque (2016), Endogenous formation of security exchanges, *Economic Theory* 64 (2), 331–355.

Hevér J. (2020), The effect of regulation on market liquidity: a general equilibrium approach, *Economic Review (Közgazdasági Szemle)*, LXVII (7-8), 708–733.

International Organization of Securities Commissions (2018), Recommendations for Liquidity Risk Management for Collective Investment Schemes, FR01/2018 February.

International Organization of Securities Commissions (2019), Liquidity in Corporate Bond Markets Under Stressed Condition, FR10/2019 June.

International Organization of Securities Commissions (2019), Sustainable finance in emerging markets and the role of securities regulators, CR01/2019 February.

International Organization of Securities Commissions (2020), Sustainable Finance and the Role of Securities Regulators and IOSCO, FR04/2020 April.

Jarrow, R. A. and P. Protter (2005), Liquidity risk and risk measure computation, *Review of Futures Markets*, 11, 27–39.

Lara, J. L., F. López-Gallo, S. Lord and A. Romero (2021), Effects of the international regulatory reforms over market liquidity of Mexican sovereign debt, *Journal of Financial Stability*, 52, 100807.

Le Roy, S. F. and J. Werner (2001), Principles of Financial Economics, *Cambridge University Press*.

Mitchell, M., L. H. Pedersen and T. Pulvino (2007), Slow Moving Capital, *American Economic Review*, 97 (2), 215–220.

Petrell, G. and A. Resti (2017), What drives the liquidity of sovereign bonds when markets are under stress? An assessment of the new Basel 3 rules on bank liquid assets, *Journal of Financial Stability*, 33, 297–310.

Sommer, K. and P. Sullivan (2018), Implications of US Tax Policy for House Prices, Rents, and Homeownership *American Economic Review*, 108 (2), 241–274.

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