Real effects of nominal shocks: a 2-sector dynamic model with slow capital adjustment and money-in-the-utility*

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Abstract

This paper develops a two-sector model to study the effect and incidence of nominal shocks (fiscal or exchange rate policies) on sectors and factors of production. I adopt a classical two-sector model of a small open economy and enrich its structure with gradual investment and a preference for real money holdings. An expansive nominal shock (fiscal expansion or a nominal appreciation) leads to an increase in nontraded prices, which translates into changes in factor rewards, capital labor ratios and sector-level employment of capital and labor. Higher nontraded prices lead to extra domestic income, validating some of the initial excess spending. This propagation mechanism leads to a persistent real effect (on relative prices, factor rewards, capital accumulation) of nominal shocks, which disappears gradually through money outflow (trade deficit). I also draw parallels with the NATREX approach of equilibrium real exchange rates and the literature on exchange rate based disinflations.
Keywords: two-sector growth model, money-in-the-utility, q-theory, real effects of nominal shocks, endogenous pass-through.

1 Introduction

This paper has a dual objective. One is to develop a two-sector model without price or wage rigidities in which various nominal shocks (nominal appreciation, fiscal expansion, the choice of the euro conversion rate) still have a medium-term impact on relative prices, factor rewards, investment and sectoral reallocation. For example, there is an endogenous gradual passthrough

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of a nominal appreciation into wages and nontradable prices, even with a full and immediate passthrough into tradable prices. The model also has a "real equilibrium path", which is essentially a two-sector neoclassical open economy growth model with asymmetric exogenous productivity growth in the two sectors.

Besides its theoretical aspects, the model seems to be capable of capturing actual price and wage dynamics after nominal appreciations and fiscal expansions, particularly the recent development of the Hungarian economy. The situation can be characterized by (1) a massive increase in wages (without a matching rise in TFP); (2) a halt in investment with a marked sectorial asymmetry: increase in service sector investments, fall in manufacturing; (3) slow or even reversed FDI flows; (4) export sector production costs (wages) not adjusting to the fall in revenues; (5) an increase in the nontraded-traded relative price; (6) an overall consumption boom, accompanied with a deteriorating trade balance. The policy environment can be summarized as (1) an increase in minimum wage legislation, (2) followed by a large nominal appreciation (monetary restriction), (3) followed by a massive fiscal expansion, partly in the form of public sector wage increases. The exact timing of the fiscal expansion is somewhat unclear: the rise in public sector wages unambiguously came after the monetary contraction, but the fiscal stance before and after the monetary developments is subject to heated political debates in Hungary.

The picture strongly suggests that $\frac{r}{w}$ has fallen. If we do not attribute this entirely to changes in minimum wages and public sector wages, then the monetary restriction ("revaluation") and the overall fiscal expansion should also play a role. The model successfully produces the same economic developments with the latter two policies, pointing to their potential role in the process. A similar moral applies to any exchange-rate based disinflation attempt, and its reverse conclusions are relevant to price and wage developments after large devaluations.

The main mechanism of the model is the following. Consider an appreciation of the nominal exchange rate. It changes the spending behavior of consumers, through influencing their intertemporal (savings) decisions. In particular, domestic (nominal) assets are revalued in terms of tradable goods. This is one "stickiness" in the model. Increased spending must lead to increased production of nontradables, while excess demand in tradables can be satisfied through imports as well. This shift in production leads to an increase in the relative price of nontradables as long as the short-term transformation curve is nonlinear. This is the second and last friction of the model, which can be attributed to gradual capital adjustment (q-theory), for example. The virtue of having only these two dynamic frictions is that one can clearly see the intuitive developments behind all results.

As the economy moves along its transformation curve, factor rewards must also change: if the nontraded sector is more labor-intensive, than r falls and w increases (Stolper-Samuelson theorem). There is a marked reallocation between the two sectors: both labor and capital migrate from tradables into nontradables. A lower $\frac{r}{w}$ increases capital intensity in both sectors. The decline in r initiates a fall in aggregate capital (slump in investment and FDI). Notice that this is compatible with an increase in sectorial capital intensities, since the expanding nontraded sector is less capital intensive than the contracting traded sector. Rising wages create extra income for consumers ("Dutch disease"), which makes the real effect persistent in the medium-term: excess spending slowly returns to equilibrium, through a gradual outflow of domestic money (assets).

The paper is organized as follows. The next section explains the basic building blocks of the model. Section 3 develops the full details of the two-sector growth model with money-in-the-utility, which is then adopted for numerical solution in Section 4. Section 5 describe the main results (nominal and real growth paths), which are discussed in Section 6. The final section concludes with some empirical considerations.

2 The basics of a gradual income and capital adjustment model

2.1 General considerations

This paper considers a dynamic adjustment of a two-sector small open economy model (the "dependent economy" model[1]). One of the sectors is traded, the other is nontraded. The two sectors differ in their pricing: traded prices are set by the law of one price (fixed international prices times the nominal exchange rate), while nontraded prices are determined through domestic market clearing. In traded goods, domestic supply and demand can temporarily deviate from each other, leading to a trade deficit or surplus. One could further distinguish between exportable and importable goods. This would serve as a base for a gradual entry model I will sketch later on.

There are two dynamic factors in the model. The first one is a gradual adjustment of expenditures to income – some sort of a nominal rigidity (illusion), which ensures that nominal shocks (nominal exchange rate movements, fiscal policy) will have a temporary effect on spending. Such a behavior is not necessarily inconsistent with consumer optimization: as we shall see, this can be rationalized by an explicit intertemporal maximization of a utility function containing real money balances as well (in this case, *the nominal money stock becomes a state variable*, which can be influenced by nominal policy choices). We would see a similar effect when consumer behavior reflects precautionary motivations as well: consumers would then try to build up an equilibrium stock (and portfolio) of wealth, and this accumulation process would be influenced by nominal shocks.

[1] See, for example, Dornbusch: Open Economy Macroeconomics, chapter 6.

The nominal effect does not come from the rigidity or stickiness of prices or wages, but from the gradual response of consumption expenditures. This does not imply that real-world prices or wages were flexible, or there were no inflation persistence – all is meant to show that there are systematic effects of nominal shocks on relative prices even under price flexibility. If the adjustment of expenditures (the resizing of real money holdings) is slower than price adjustments, one can interpret one period of such a flexible price model as a time interval during which prices have already been adjusted. Moreover, in a sticky price model, prices should also be adjusted to the equilibrium levels described by my model, giving a gradual passthrough of nominal exchange rate movements into the full CPI (services, wages, rental rates).

The other dynamic effect is the accumulation of capital, which is implied partly by a potential permanent exogenous technology improvement, and partly by a low initial capital stock. Initially, there is an excess return on capital relative to the rest of the world, which calls for a capital inflow. Due to adjustment costs (one could also interpret them as informational problems, lack of infrastructure, etc.), this inflow is gradual, like in a regular Tobin's q model. For simplicity, I assume that capital is owned 100% by foreigners (in other words: capital owners consume only tradables, their opportunity cost of funds is the fixed world interest rate, which then makes the nationality of capital owners irrelevant). It implies that changes in capital income will not affect domestic nontraded demand.

This is already sufficient to produce real effects of a nominal shock: under a nominal appreciation, for example, the value of domestic money holdings (wealth) in terms of tradable goods increases. This leads to more consumption of tradables and nontradables. Since country-level capital is fixed in the short-run, and nontraded consumption must equal production, this implies a change in relative prices between the two sectors, and also influences wages and the rental rate. This latter implies a change in the capital accumulation process, while the former has an impact on consumer income, which may reinforce or counteract the initial excess consumption. If wages increase (which happens if the nontraded sector is more labor intensive than the traded sector), then consumer income increases, creating some of the fundamental of the initial consumption boom, thus making the real effect of the nominal shock persistent. My objective is to quantify these dynamic mechanisms.

From the viewpoint of dynamic systems, we have two state variables in the model: the stock of money and of capital; and two jump variables: Tobin's q and consumption (or consumption expenditure, with more than one good). Fortunately, it turns out to be simple to eliminate consumption from the model, but we still need to solve an explicit saddle path system numerically. This necessitates the full specification of the production and consumption side. I will work with a Cobb-Douglas assumption on both sides, and I will also adopt certain simplifications on the dynamic equations of the model (neglect some second order effects) and linearization around the steady state (balanced growth path). These simplifications do not alter the behavior of the model: Benczúr and Kónya (2003) consider a continuous time, full optimization version of the model, with qualitatively similar (preliminary) results.

The formulation and numerical solution of the model offers many interesting and important applications. One is a quantification of the price level impact of fiscal policy: we shall see that a fiscal expansion generates extra spending, and prices do not adjust immediately. This is not a price rigidity, however, but the consequence of the nonlinearity of the short-term transformation curve: excess spending implies excess nontraded production, which leads to an increase in the cost of nontraded production. This modifies all equilibrium prices (traded-nontraded relative prices, wages, rental rates), and then gradually disappears through income dynamics (and money outflow). Due to forward-looking investment behavior, this process counteracts with capital accumulation in a complex way, leading to rich dynamic consequences of a fiscal expansion.

A second application concerns the quantitative consequences of a *monetary restriction (nominal appreciation)*, which in fact will have similar effects than a *fiscal expansion*. Interpreting the monetary restriction as a revaluation of a fixed exchange rate, traded prices will fall (assuming immediate, and potentially full passthrough of the nominal exchange rate to tradable prices). This increases the value of domestic money holdings in terms of tradables, leading to a similar consumption boom and dynamic implications as a fiscal expansion.[2] In particular, wages and nontraded prices will show an endogenous and gradual adjustment to the decreased tradable price level, which frequently puzzles central bankers.

A related but inherently fixed exchange rate situation is the choice of the EMU conversion rate. The model issues the warning that an overvaluation may imply a significant reduction in capital inflows, it may be persistent even with flexible prices and wages, and it has largely asymmetric effects on different sectors and different factors of production. Welfare implications are not clear-cut, since GDP growth may slow down, but consumers experience higher wages and consumption (financed by debt).

A fourth application comes from a surprising similarity between the dynamic equations of the model and different equilibrium concepts of the NATREX approach.[3] In this sense, the model can be viewed as an (almost) explicit optimization-based version of a NATREX model. The modifier "almost" applies only because I will have to adopt certain simplifications (approximations) of the full optimization model to ensure tractability (these are eliminated

[2] The behavior of the CPI will differ: a fiscal expansion leaves traded prices unchanged, so the CPI increases by the increase in relative prices times their weight. A nominal appreciation leads to a decline in traded prices and an increase of relative prices. Consequently, the CPI is likely to fall, but by less than the drop in traded prices.

[3] This framework was developed by Jeremy Stein, in Stein (1994) and various additional papers.

in Benczúr and Kónya (2003)). More precisely, the long-term equilibrium NATREX concept matches the steady state (balanced growth path) of my model (when both capital and money holdings are at their balanced growth path, which corresponds to the "traditional flexible" Balassa-Samuelson framework); while the medium-term equilibrium concept corresponds to the nonmonetary version of my model (when the adjustment of money holdings is much faster than that of capital, thus the income and expenditure of consumers are always equal to each other, and money does not influence any real variables).

More generally, the long-term NATREX concept is the balanced growth path of a model with many state variables. The medium-term NATREX corresponds to such a transition path where some of the state variables adjust immediately, and only a subset of the laws of motion drives the dynamics. Disequilibrium (realized behavior of the economy) is then described by the full model, where all state variables adjust slowly, though at a different speed.

This is illustrated on Figure 1, for two state variables (H and K – money and capital). The left panel plots the phase diagram in the $H - K$ space. The long-term equilibrium point is the intersection of $\frac{d}{dt}K = 0$ and $\frac{d}{dt}H = 0$. The medium-term NATREX path moves towards the long-term point along the $\frac{d}{dt}H = 0$ curve: for any given level of K, there is a corresponding $H(K)$, and $\frac{d}{dt}K$ describes the dynamics of the system. The observed path starts from any initial K *and* H, and moves towards the long-term point as a two-dimensional stable dynamic system. Underneath the state variables, there is a corresponding value of the real exchange rate (the relative price), $p(H, K)$. The medium-term NATREX path implies $p_t = p(H(K_t), K_t) = p(K_t)$, while the observed path comes with $p_t = p(H_t', K_t')$. The evolution of capital is different in the two scenarios, so the right measure of the misalignment of the real exchange rate in the nominal (observed) economy is $p(H(K_t'), K_t') - p((H_t', K_t')$.

2.2 Behavioral equations

Production

- Traded sector: $Y_T = (A_T L_T)^\beta K_T^{1-\beta}$; $A_T(t) = A_T(0)(1+g)^t$.
- Nontraded sector: $Y_{NT} = (A_{NT} L_{NT})^\alpha K_{NT}^{1-\alpha}$. Let us keep A_{NT} constant for simplicity ($A_{NT} = 1$).

Demand

- For a given money stock $H(t)$ (money, or wealth), consumption expenditure is proportional to money holdings: $E(t) = VH(t)$, where V is the (fixed) velocity of money.

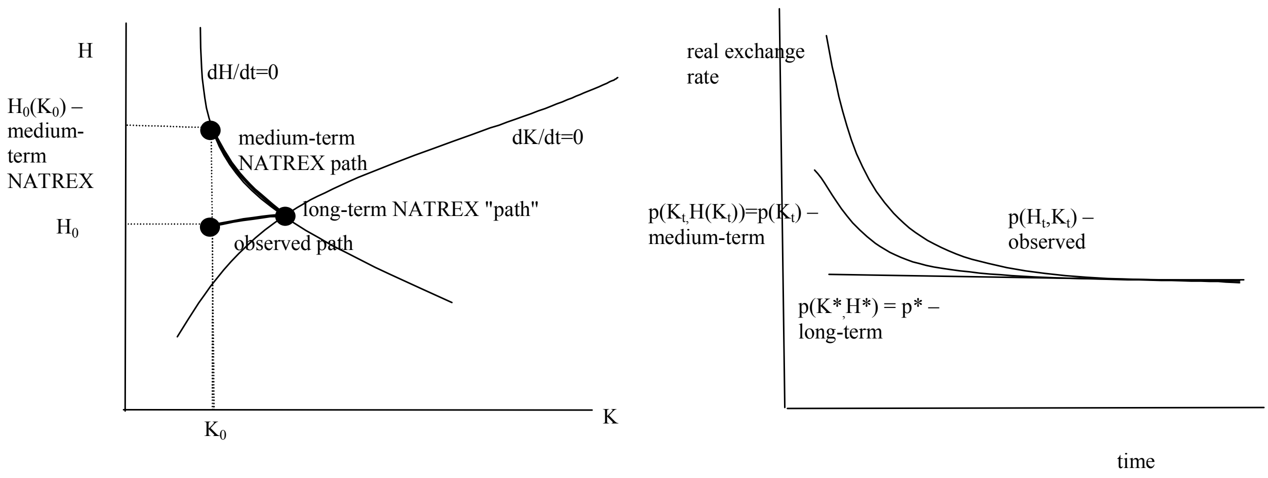

Figure 1: Long-term equilibrium, medium-term equilibrium and observed behavior

- For a given E, traded and nontraded consumption is driven by a Cobb-Douglas utility function: $p_T C_T = (1-\lambda) VH$, $p_{NT} C_{NT} = \lambda VH$. I could also assume a different degree of substitutability between the two goods ($C_T/C_{NT} = (p_T/p_{NT})^{\mu}$, where μ is not necessarily -1, but still a constant) – this would not make the model conceptually different, only the equations would become more complicated, maybe beyond numerical tractability. For this reason, I work with the $\mu = -1$ choice. Section 6 discusses how my results would change with different substitutability in production or preferences.

Such a consumption-money behavior can follow from a precise intertemporal maximization framework: if $u(t) = \int v(\tau) e^{-\delta(\tau - t)}$, $v(t) = E(t)^{\gamma} H(t)^{1-\gamma}$ (money-in-the-utility specification of Sidrauski), then it is true for this special (Cobb-Douglas) case that E/H is constant along the saddle path (Dornbusch-Mussa (1975)). One needs to adjust the utility specification for the two good case:

$$v(t) = \left(C_T(t)^{1-\lambda} C_{NT}(t)^{\lambda}\right)^{\gamma} \left(H(t)/P(t)\right)^{1-\gamma}.$$

The price level variable P corresponds to the domestic price index ($P = P_T^{1-\lambda} P_{NT}^{\lambda}$). With some extra work, one can reestablish the property that $E = VH$ along the saddle path (the key observation is that for a given E, the per period problem implies fixed expenditure shares, so one ends up with an intertemporal objective function expressed in terms of H and E again).

If there is inflation (P changes), then this constant velocity in fact depends on inflation: $V = V(\pi)$. In particular, $E = VH = (\delta + \pi) \frac{\gamma}{1-\gamma} H$, where π is the CPI-inflation (the change of P), and δ is the discount factor. My full model is one extra step more complicated, since inflation is not constant in the short-run (approaches the balanced growth path value from above or below). Then it is no longer true that E/H is constant along the saddle path, because the term $\dot{V}$ shows up in optimality conditions and destroys linearity. Approximately it remains true that $E = (\delta + \pi) \frac{\gamma}{1-\gamma} H$, which means that velocity increases with inflation.

Under the assumption of a fixed V, we get that a nominal expansion implies excess spending (a consumption boom), which changes prices and inflation. A fiscal expansion increases the CPI, while a nominal appreciation is likely to decrease it (the fall in tradable prices dominates the increase in relative prices). With its extra feedback to V, a fiscal expansion would lead to an even stronger consumption boom. *Ceteris paribus,* this would generate a larger impact effect on nontraded prices, but also smaller persistence (for the same H, there is more excess spending, thus the trade deficit is larger, so money stocks adjust faster). Similarly, a nominal appreciation would have a smaller impact effect but more persistence. Since capital accumulation is also forward-looking, the dynamic effect of a larger but less persistent shock on capital accumulation is unclear. The size of the capital stock then also influences all other equilibrium variables – but this cross-effect is likely to be of second order, so it remains true that the effect on nontraded prices and wages is larger on impact but less persistent, or exactly the opposite.

For simplicity, I will neglect the change in V, although I will explore the sensitivity of the results to the choice of V. The full optimization of consumers (where $\dot{V}$ also plays a role) will be neglected in any case, since it would seriously complicate all calculations. In fact, I will adopt a similar simplification of the Tobin's q approach, but that choice will leave the forward-lookingness clearly visible (with consumption, it is reflected by the temporary deviation of income and expenditure). Benczúr and Kónya (2003) considers the continuous time version of the same model, with full optimization both on consumer and investor side. There is no qualitative difference in the results, and the main intuitions also carry through: nominal shocks influence intertemporal consumption decisions, which moves the economy along a nonlinear short-term transformation curve.

Prices

In the traded sector $p_T = ep_T^* = e$, while p_{NT} comes from goods market clearing. In other words, there is an immediate and full passthrough of the nominal exchange rate into tradable prices, but not necessarily into nontradables. Wages and the rental rate are also determined through factor market clearing.

This completes the description of the per period equilibrium: for a fixed $K(t)$ and $H(t)$, the above considerations determine the per period values of r, w, p_{NT}/p_T, K_T, L_T, K_{NT}, L_{NT}, C_T and C_{NT}. The original dependent economy model solves such a per period model (though with fixed sectorial capital stocks, giving two different quasi-rental rates). To complete the model, I need to write down the laws of motion.

Money (H) dynamics

$$H(t+1) = H(t) + eY_T + p_{NT}Y_{NT} - r(t)K(t) - eC_T - p_{NT}C_{NT} + DH(t) \quad (1)$$
$$= H(t) + e(Y_T - C_T) - r(t)K(t) + DH(t).$$

This is purely an accumulation equation (identity): money stock in the next period is equal to initial money holding, plus GNP minus expenditure, plus a potential exogenous term. GNP is the sum of traded and nontraded production (GDP) minus capital rents (that belongs to foreigners). If the nontraded sector is in equilibrium, then the value of nontraded production must equal the value of nontraded consumption. Change in money holding thus equals the excess production of tradables, minus capital rents, plus the exogenous term DH.

The exogenous term will play a dual role: one is to allow for a fiscal expansion. The other is related to growth: if there is a permanent productivity growth $g > 0$, consumption must be growing and hence H grows as well. If we do not want this increase to come only through a permanent money inflow, then the government must generate a fixed growth rate of domestic money. In principle, one should also worry about the distribution method of this money, but that would definitely complicate things (introducing new players like banks), and might even influence the outcome (if there are any distortions). To cut it short, I revert to the classical "helicopter drop" method, which gives the fresh money lumpsum to consumers. This does not at all indicate that the distribution of money is this simple, and its method and efficiency are irrelevant – on the contrary, it is so complex that it is better to isolate money distribution from the issues I want to address.

Capital (K) accumulation

One of the cornerstones of the "standard", "long-run" Balassa-Samuelson model (the one advocated by chapter 4 of the Obstfeld-Rogoff textbook) is the full mobility of capital. It implies that the rental rate at home equals the international rental rate. However, this implies a very fast and also mechanical capital accumulation and adjustment process. If we add the standard labor flexibility assumption ($w_T = w_{NT}$), the real exchange rate (traded-nontraded relative price) is fully supply-determined. The transformation curve is linear, and nominal variables (or preferences) have no effect on relative prices.

Let us slow down this capital adjustment process: it means allowing a temporary deviation of domestic rental rates from world rental rates. This could come from some risk premium – then the convergence process would imply a gradual decline of this risk factor, otherwise there would be no long-run equalization of rental rates. One could explicitly model such a process, and use this to determine capital accumulation: as the premium declines, there is a corresponding capital inflow. This approach would require an endogenous risk premium, because that term would be responsible for the forward looking behavior of investment.

I will adopt an other alternative – though qualitatively it describes a similar, slow and gradual adjustment.[4] This is the framework of Tobin's q. Capital inflow does not immediately eliminate excess returns because that would imply too large adjustment costs. Gradual investment behavior reflects the balance between excess returns and adjustment costs, current and future. This generates the desired forward looking behavior of capital accumulation.

The q-theory approach assumes that the cost of investment is not just the price of capital, but there is an installation cost as well. Investors maximize the present discounted value of their profit stream, including adjustment costs. This intertemporal maximization leads to a standard saddle path solution: the state variable is the capital stock, and the jump variable is q, which measures the difference between the internal and external value of a unit of capital. If the internal value is higher, then the firm invests, if the external, then disinvests (I or I/K equals $f(q)$, where f is increasing, and $f(1) = 0$). The interpretation of q is the extra profit implied by a marginal unit of extra capital, evaluated along the future optimal path. This is determined by two factors: one is the future marginal product, and the other is future saving on adjustment costs. Around steady-state (near constant K), the latter is negligible (second order), so q is approximately the present value of future per period returns, discounted by the world interest rate (r^*). The larger its value, the more investment firms do.

I will employ this latter approximation to my open economy model: investment depends on q, where q is the present value of future equilibrium rental rates ($r(\tau)$, $\tau \in [\tau, \infty]$), discounted by r^*. I shift the no investment point from $q = 1$ to $q = 0$, which means that my q is the present value of the excess yield, and not the yield itself.

This is in fact quite similar to the specification with capital responding to per period excess returns – but it is more forward-looking (there is investment today even if excess yields become positive only in the future), moreover, it produces a larger shock response, since it is not just the current yield that matters, but also the future. Measuring r^* in traded goods (foreign currency)

[4] A lumpy adjustment model is a third possibility, moreover, the current literature on investment strongly supports such a specification. That approach would also imply interesting (and complicated) investment responses to nominal shocks: if there is a small investment bust in the convex adjustment world, then a lumpy world might imply an even larger reaction if many firms are moved to their adjustment margins. This approach, however, would be even more complicated, and the answer would significantly depend on the initial position of firms relative to their desired capital holdings. For this reason, I will not incorporate this into my model. It is important to still remember that a lumpy adjustment world could give even larger, or delayed shock responses relative to the convex adjustment world.

and $r(t)$ in local currency, the equations become

$$K_{t+1} = K_t + f(q_t)$$
$$q_t = \frac{q_{t+1}}{1+r^*} + \frac{r(t)/e - r^*}{1+r^*} = \frac{r(t)/e - r^*}{1+r^*} + \frac{r(t+1)/e - r^*}{(1+r^*)^2} + \dots$$

In principle, f should be determined by the functional form of adjustment costs, but since it does not show up anywhere else, I can choose f directly (obeying $f(0) = 0$, $f' > 0$). This formulation corresponds to the adjustment cost being a function of I itself. An alternative is that it depends on relative investment (I/K), when the investment equation becomes

$$K_{t+1} = K_t (1 + f(q_t)).$$

3 Model details

To pin down the per period equilibrium, we need to determine all prices and quantities given a fixed level of $K(t)$ and $H(t)$ (the two state variables). Let us go through all the optimization and equilibrium conditions, with comments whenever necessary. Throughout these calculations, I will drop all time indices, and reintroduce them only at the summary of the per period solution.

Profit maximization in the two sectors (X=T, NT; $\delta_T = \beta$, $\delta_{NT} = \alpha$):

$$\max\ p_X K_X^{1-\delta_x} (A_X L_X)^{\delta_x} - wL_X - rK_X.$$

It is immediately visible that I have assumed the indifference of both factors *between the two sectors*, so $w_T = w_{NT} = w$, $r_T = r_{NT}$. The first implies that labor is mobile within the country. I would not argue that this is necessarily true in reality, or that the adjustment of labor was fast enough (compared to the adjustment of capital and nominal spending) to validate such an approximation. One could also set up a model with slow labor adjustment. I still refrain from this, for multiple reasons. One is that having three sources of slow adjustments would be clearly the most realistic treatment, but also the most complicated. For a real effect of nominal shocks, we need to have slow adjustment of nominal spending. The behavior of capital flows is also a specific object of interest of the paper, so it is necessary to include its gradual adjustment.

Gradual labor flows are also more difficult to handle technically. One potential way would be to introduce search and matching into the model, which looks quite complicated. Another, more compelling and intuitive way is to assume that labor flows between the sectors in response to wage differentials. This could be handled by the specification that (past) wage differentials determine the degree to which labor flows close the current wage gap between sectors.

This looks similar to my q theory approximation – the full analogue would be to condition labor flows on discounted future earnings. The problem with such a q theory is that while investment decisions have no upper bounds, labor flows most obey an upper bounds (L_T,$L_{NT} \leq \bar{L}$). Krugman (1991) contains such an approach, without taking this effect into account, and it was shows later on, that his results are seriously affected by this problem (Benabou-Fukao (1993)). As far as I know, the approach can be rescued, but only in a very complicated way. One could also introduce job search, but also on the expense of large extra difficulties.

My other crucial assumption is that capital is indifferent between the *two domestic sectors*, but not necessarily between home and foreign. This is again not an obvious assumption, and one may have reasons to doubt its validity. For its support, I would argue the following way. Provided that the initial difference in sectorial returns of capital is not "too large", their equalization is feasible entirely through new investment. So it is not necessary to move installed capital between sectors, all we need is that new investment flows to the more productive sector, and it is abundant enough to equalize returns of the last marginal unit of capital across sectors. This looks a plausible assumption. If per period investment is sufficiently large, then the equality of sectorial returns to capital can be sustained even after large shocks. It is possible that a too large shock necessitates disinvestment in one of the sectors, thus making the indifference assumption problematic. Then one needs to assume that capital is mobile between sectors up to this degree.

A further alternative would be to consider two separate q-theories in the two sectors, and then make marginal capital movements indifferent between the two sectors (the two q-values should be equal). This does not necessarily mean the equalization of per period returns to capital in the two sectors, only their present discount value. For simplicity, I will work with the per period equalization assumption.

From profit maximization (using $A_{NT} = 1$, $p_T = ep_T^* = e$):

$$w = p_{NT}\alpha K_{NT}^{1-\alpha} L_{NT}^{\alpha-1} = \alpha p_{NT} k_{NT}^{1-\alpha} \tag{2}$$

$$w = p_T A_T^{\beta} \beta K_T^{1-\beta} L_T^{\beta-1} = \beta e A_T^{\beta} k_T^{1-\beta} \tag{3}$$

$$r = (1-\alpha)\, p_{NT} k_{NT}^{-\alpha} \tag{4}$$

$$r = (1-\beta)\, e A_T^{\beta} k_T^{-\beta}. \tag{5}$$

This specification assumes that all prices are expressed in home currency. For this reason, the domestic rental rate (r) must be divided by e, in order to be comparable to the world interest rate. This is why the q-theory expressions from earlier had $\frac{r}{e}$ in their arguments.

We could neglect changes in A_T (and choose $A_T = const = 1$), or assume that they are only transitory. In the latter case, the long run value of A_T would be 1, and it would be possible to have $A_T(t) \neq 1$ temporarily.

The more interesting case is to allow for a permanent trend in A_T (even if not permanent, at least long enough to be treated constant at the model's horizon). This makes it necessary to interpret the long-term equilibrium situation appropriately: instead of a steady state, we will have a balanced growth path. Just like in a one-sector Ramsey model with growth, we need to introduce effective variables, i.e., divide all variables by an appropriate power of productivity growth. In a one sector model, it means the same first power for all variables. In our asymmetrical two sector model (A_{NT} is constant), this will mean a different power for some variables (p_{NT}).[5]

The model can easily incorporate temporary deviations from the fixed growth trend of A_T. All this means is that some deviations may still be present in the effective versions of the profit maximization conditions. I will not consider such a scenario, so my specification satisfies $A_T(t) = A_T(t-1)(1+g)$. Let us look at the specifics of deriving the balanced growth path formulation. For this, we need to transform all variables into effective variables.

Starting with (5), it is immediate that the transformation to effective labor gives the steady state:

$$r = (1-\beta)\, e A_T^{\beta} \left(\frac{K_T}{L_T}\right)^{-\beta} = (1-\beta)\, e \left(\frac{K_T}{A_T L_T}\right)^{-\beta} = (1-\beta)\, e \hat{k}_T^{-\beta}.$$

The variable $\hat{k}_T$ is the amount of capital per effective worker in the traded sector.

Continuing with this transformation:

$$\begin{aligned} \hat{w} &= \frac{w}{A_T} = \beta e \hat{k}_T^{1-\beta} \\ r &= (1-\alpha)\, \hat{k}_{NT}^{-\alpha} \frac{p_{NT}}{A_T^{\alpha}} \\ \hat{w} &= \alpha \hat{k}_{NT}^{1-\alpha} \frac{p_{NT}}{A_T^{\alpha}}. \end{aligned}$$

We can see that the nontraded relative price should be divided by A_T^{α} instead of A_T itself. This is in line with the "canonical flexible Balassa-Samuelson" result of chapter 4 of Obstfeld and Rogoff: in their formulation, TFP in the traded sector was growing at a rate γ, which implied a rate of $\frac{\alpha}{\beta}\gamma$ for the change in the relative price of nontradables. A rate of γ for TFP growth corresponds to a rate of $\gamma\beta$ in labor productivity increase, so the relative price should grow at

[5] Under $\alpha \neq \beta$, even $A_T = A_{NT}$ implies such an asymmetry, since productivity growth is labor augmenting, and the two sectors use labor with different intensity. If we assume a common growth rate of the *TFP* of the two sectors, we get back to full symmetry.

α times the rate of growth in A_T. Let us introduce $\hat{p}_{NT} = \frac{p_{NT}}{A_T^{\alpha}}$, and get a fully homogenous system for the effective variables::

$$\hat{w} = \alpha \hat{p}_{NT} \hat{k}_{NT}^{1-\alpha} \tag{6}$$

$$\hat{w} = \beta e \hat{k}_T^{1-\beta} \tag{7}$$

$$r = (1-\alpha)\, \hat{p}_{NT} \hat{k}_{NT}^{-\alpha} \tag{8}$$

$$r = (1-\beta)\, e \hat{k}_T^{-\beta}. \tag{9}$$

This is a system of four equations with five unknowns. If we fix r, for example, then we get back the "flexible Balassa-Samuelson" result, where supply completely determines the relative price of nontradables, wages and capital-labor ratios. The role of demand is the reduced to the determination of the size of the sectors. We can repeat the same procedure with any of the variables, say, $\hat{k}_T$, since $r = r\left(\hat{k}_T\right)$ and $\hat{k}_T = \hat{k}_T(r)$ is a bijection under (6)-(9).

In the model, however, r is endogenous, and it is the total capital stock, $K_T + K_{NT}$, that is fixed (within the period). We also know the behavior of demand, since nominal spending is proportional to the other state variable, H. This enables a straightforward determination of r: for a given r, (6)-(9) defines $p_{NT}(r)$. Using H, we get demand (expenditure) for traded and nontraded goods. Nontraded demand must equal supply, so we have the value of nontraded production. With $\hat{k}_{NT}(r)$, we then also obtain $L_{NT}(r)$ and $K_{NT}(r)$. Labor market clearing defines $L_T(r) = L - L_{NT}(r)$. Combining this result with $\hat{k}_T(r)$ gives $K_T(r)$ as well. The last step is to write the market clearing condition for capital: $K = K_T(r) + K_{NT}(r)$, which defines the per period equilibrium value of r.

Let us look into the details of this procedure – for the sake of easier calculations (and simulations later on), I will work with $\hat{k}_T$ instead of r. From profit maximization in T:

$$r = (1-\beta)\, e \hat{k}_T^{-\beta}$$

$$\hat{w} = \beta e \hat{k}_T^{1-\beta}.$$

Using the labor optimality condition in NT:

$$\alpha \hat{p}_{NT} \hat{k}_{NT}^{1-\alpha} = \beta e \hat{k}_T^{1-\beta},$$

so

$$\hat{p}_{NT} = \frac{\beta e \hat{k}_T^{1-\beta}}{\alpha \hat{k}_{NT}^{1-\alpha}}.$$

Plugging into the capital optimality conditions:

$$(1-\beta)\, e\hat{k}_T^{-\beta} = r = (1-\alpha)\, \hat{p}_{NT}\hat{k}_{NT}^{-\alpha} = (1-\alpha)\, \beta e \hat{k}_T^{1-\beta} \frac{\hat{k}_{NT}^{-\alpha}}{\alpha \hat{k}_{NT}^{1-\alpha}} = \frac{1-\alpha}{\alpha} \beta e \hat{k}_T^{1-\beta} \hat{k}_{NT}^{-1},$$

so

$$\hat{k}_{NT} = \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta} \hat{k}_T$$

and

$$\hat{p}_{NT} = \frac{\beta e \hat{k}_T^{1-\beta}}{\alpha \hat{k}_{NT}^{1-\alpha}} = e\frac{\beta}{\alpha} \left(\frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta} \right)^{\alpha-1} \hat{k}_T^{\alpha-\beta}.$$

Let us write down the demand for nontraded goods:

$$\lambda V H = p_{NT} C_{NT}. \tag{10}$$

In the nontraded sector, demand must equal supply (one cannot import or export nontradables by definition), so

$$C_{NT} = Y_{NT} = L_{NT}^{\alpha} K_{NT}^{1-\alpha} = K_{NT} k_{NT}^{-\alpha}.$$

Substituting this into (10):

$$K_{NT} = C_{NT} k_{NT}^{\alpha} = \frac{\lambda V H}{p_{NT}} k_{NT}^{\alpha} = \frac{\lambda V H}{\left(p_{NT}/A_T^{\alpha}\right)} \left(k_{NT}/A_T\right)^{\alpha}$$

$$\frac{K_{NT}}{A_T} = \lambda V \hat{H} \frac{\hat{k}_{NT}^{\alpha}}{\hat{p}_{NT}} = \frac{\lambda V \hat{H}}{e} \frac{\left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\hat{k}_T\right)^{\alpha}}{\frac{\beta}{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)^{\alpha-1} \hat{k}_T^{\alpha-\beta}} = \frac{\lambda V \hat{H}}{e} \frac{1-\alpha}{1-\beta} \hat{k}_T^{\beta}.$$

We already have the effective capital-labor ratio in the nontraded sector, and the effective capital stock as well, so we can obtain an expression for labor:

$$L_{NT} = \frac{K_{NT}}{k_{NT}} = \frac{K_{NT}/A_T}{k_{NT}/A_T} = \frac{\frac{\lambda V \hat{H}}{e}\frac{1-\alpha}{1-\beta}\hat{k}_T^{\beta}}{\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\hat{k}_T} = \frac{\lambda V \hat{H}}{e} \frac{\alpha}{\beta} \hat{k}_T^{\beta-1},$$

so labor market clearing implies

$$L_T = 1 - L_{NT} = 1 - \lambda V \frac{\hat{H}}{e} \frac{\alpha}{\beta} \hat{k}_T^{\beta - 1},$$

and

$$K_T = L_T k_T = L_T A_T \hat{k}_T.$$

Write down the capital market clearing condition:

$$\begin{aligned} K &= K_T + K_{NT} = L_T A_T \hat{k}_T + \lambda V \frac{\hat{H}}{e} \frac{1-\alpha}{1-\beta} \hat{k}_T^{\beta} A_T = A_T \hat{k}_T - \lambda V \frac{\hat{H}}{e} \frac{\alpha}{\beta} \hat{k}_T^{\beta} A_T + \lambda V \frac{\hat{H}}{e} \frac{1-\alpha}{1-\beta} \hat{k}_T^{\beta} A_T \\ &= A_T \hat{k}_T + \lambda V A_T \frac{\hat{H}}{e} \hat{k}_T^{\beta} \left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta} \right). \end{aligned}$$

For a given K and H, this expression pins down the per period equilibrium value of $\hat{k}_T$. It is thus clear that K and H are indeed the state variables of the model. For consistency, we should also transform these variables into effective variables (which will then have to be taken into account in the dynamic equations):

$$\hat{K} = \hat{k}_T + \lambda V \frac{\hat{H}}{e} \hat{k}_T^{\beta} \left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta} \right). \tag{11}$$

Let us turn now to the dynamic laws of motion. Money accumulation follows from the individual per period budget constraint (equation (1)):

$$H_{t+1} = H_t + eY_T(t) + p_{NT}(t) Y_{NT}(t) - r_t K_t - eC_T(t) - p_{NT}(t) C_{NT}(t) + DH(t).$$

Starting from past money holdings, we add total national income (GNP) – which equals the value of traded and nontraded production minus capital income (by assumption, all capital is held by foreigners) –, then subtract consumption expenditures (traded and nontraded), and add a potential exogenous money growth term. Noting that consumption equals production in the nontraded sector, the money accumulation equation reduces to

$$H_{t+1} = H_t + e(Y_T(t) - C_T(t)) - r_t K_t + DH(t). \tag{12}$$

Apart from the exogenous money growth term, this expression is closely related to the balance of payments: $e(Y_T - C_T)$ is the trade balance, while $-rK$ is investment income paid to foreigners. This determines the dynamics of financial wealth ("net foreign financial assets"). It is important to make the qualification "financial", because K will have its own accumulation equation, and a full balance of payments would reflect capital flows as well.

Equation (12) also offers a direct reinterpretation of the dual concept of equilibrium exchange rates (traded-nontraded relative price) in the NATREX approach. The system has two state variables, H and K. In the long-term equilibrium situation, both variables have reached their steady state levels (with growth, it applies to their effective versions – this necessitates the exogenous money growth term, which will be discussed later on). There is a corresponding relative price $\hat{p}_{NT}$, which implies a long-term path (a growth trend and a fixed "level") for the relative price. The medium-term equilibrium allows for $K \neq K^*$, but requires effective money holdings to be constant at every moment (the economy is always on the $\frac{d}{dt}\hat{H} = 0$ locus). In other words, consumers satisfy their budget constraint in every period, so traded consumption also equals traded production less capital income. According to (12), this is equivalent to the balance of payments "without capital flows", which is the condition defining the medium-term NATREX value:

$$0 = e\left(Y_T - C_T\right) - rK.$$

Consequently, the medium-term NATREX equilibrium restricts the use of money, wealth or any other means of intertemporal consumption reallocation. Putting differently, money adjustment is so fast that the economy is practically always along the $\frac{d}{dt}\hat{H} = 0$ curve, and it converges towards $\frac{d}{dt}\hat{K} = 0$ along this curve. In "reality", money adjustment is slower, so the economy may be out of its medium-term equilibrium. In every moment (i.e., for every K_t), one can still define the corresponding medium-term equilibrium value of the real exchange rate $p_{NT}\left(H\left(K_t\right), K_t\right)$.

Another reinterpretation is to assume that the nominal exchange rate is pinned down by the balance of payment condition $0 = e\left(Y_T - C_T\right) - rK$. This is clearly an exaggeration, but we do expect the nominal exchange rate to "react" to the balance of payments, so under "neutral interest rate policy", it should quickly adjust to a level compatible with the balance of payments. Let us assume a neutral interest rate policy, and also that the nominal exchange rate immediately adjusts to external imbalances. The corresponding value of e is then such that $\hat{H}/e$ remains unchanged – the medium-term NATREX equilibrium then also corresponds to a monetary model where the nominal exchange rate immediately adjusts to ensure the balance of payments (no change in $\hat{H}/e$).

Let us now return to the transformation of (12) into effective variables:

$$\frac{H_{t+1}}{A_T(t+1)} = \frac{H_t}{A_T(t+1)} + e\left(\frac{Y_T(t)}{A_T(t+1)} - \frac{C_T(t)}{A_T(t+1)}\right) - r_t\frac{K_t}{A_T(t+1)} + \frac{DH(t)}{A_T(t+1)}$$

$$\hat{H}_{t+1} = \frac{\hat{H}_t}{(1+g)} + \frac{e}{1+g}\left(\hat{Y}_T(t) - \hat{C}_T(t)\right) - \frac{r_t\hat{K}_t}{1+g} + \frac{DH(t)}{A_T(t+1)}$$

$$\hat{H}_{t+1} = \frac{1}{1+g}\left(\hat{H}_t + e\left(L_T^{\beta}\hat{K}_T^{1-\beta} - (1-\lambda)V\frac{\hat{H}_t}{e}\right) - r_t\hat{K}_t + \widehat{DH}(t)\right).$$

Along the balanced growth path, effective traded and nontraded production is constant, so effective expenditure ($e\hat{H}$) is also constant. The per period budget constraint still implies that $e\left(L_T^{\beta}\hat{K}_T^{1-\beta} - (1-\lambda)V\hat{H}_t\right) - r_t\hat{K}_t$ is zero, so

$$\hat{H} = \frac{\hat{H}}{1+g} + \frac{\widehat{DH}(t)}{(1+g)}$$

along the balanced growth path. This implies $\widehat{DH}(t) = g\hat{H}$. In order for the monetary model to reproduce the balanced growth path of the corresponding real model, there must be an exogenous growth of money at the rate of $(1+g)$. Productivity growth implies increasing consumption and money holdings. Unless the consumer gets the extra money exogenously, she would have a trade surplus every period to ensure growing money balances. This cannot coincide with the real equilibrium, because that has balanced trade every period. To reproduce this situation, one must assume that the domestic government prints gH extra money every period. This is the level of money growth that is in line with productivity growth – so foreign investors will have full confidence in the convertibility of domestic money at a fixed nominal exchange rate e.

In summary, we must have $\widehat{DH}(t) = g\hat{H}_t$, which then yields

$$\hat{H}_{t+1} = \hat{H}_t + \frac{e}{1+g}\left(L_T^{\beta}\hat{K}_T^{1-\beta} - (1-\lambda)V\frac{\hat{H}_t}{e}\right) - \frac{r_t\hat{K}_t}{1+g}.$$

Using our previous results:

$$L_T^{\beta}\hat{K}_T^{1-\beta} = L_T\hat{k}_T^{1-\beta} = \left(1 - \lambda V\frac{\hat{H}}{e}\frac{\alpha}{\beta}\hat{k}_T^{\beta-1}\right)\hat{k}_T^{1-\beta} = \hat{k}_T^{1-\beta} - \lambda V\frac{\hat{H}}{e}\frac{\alpha}{\beta}$$

$$r_t\hat{K}_t = (1-\beta)e\hat{k}_T^{-\beta}\left(\hat{k}_T + \lambda V\frac{\hat{H}}{e}\hat{k}_T^{\beta}\left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right)\right) = (1-\beta)e\hat{k}_T^{1-\beta} + \lambda V\hat{H}\left(1 - \alpha - \frac{\alpha-\alpha\beta}{\beta}\right).$$

Plugging these back to the law of motion for $\hat{H}_t$:

$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g}\left(\hat{k}_T^{1-\beta} - \lambda V \frac{\hat{H}}{e}\frac{\alpha}{\beta} - (1-\lambda) V \frac{\hat{H}}{e} - (1-\beta)\hat{k}_T^{1-\beta} - \lambda V \frac{\hat{H}}{e}\left(1 - \alpha - \frac{\alpha}{\beta} + \alpha\right)\right)$$
$$= \frac{e}{1+g}\beta \hat{k}_T^{1-\beta} - \frac{1}{1+g} V \hat{H}_t \left(\lambda \frac{\alpha}{\beta} + 1 - \lambda + \lambda - \lambda \frac{\alpha}{\beta}\right) = \frac{e}{1+g}\beta \hat{k}_T^{1-\beta} - \frac{V\hat{H}_t}{1+g}.$$

In the background we still have (11), so $\hat{k}_T(t) = \hat{k}_T\left(\hat{K}_t\right)$ and

$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g}\beta \hat{k}_T\left(\hat{K}_t\right)^{1-\beta} - \frac{V\hat{H}_t}{1+g}.$$

This is the final form of the law of motion for H, which only has state variables on its right hand side (K and H). The medium-term value of $\hat{H}\left(\hat{K}_T\right)$ is thus:

$$\frac{\hat{H}}{e} = \frac{\beta}{V}\hat{k}_T\left(\hat{K}_t\right)^{1-\beta}.$$

Let us plug this into the expression for $\hat{k}_T$:

$$\hat{K}_t = \hat{k}_T + \frac{\lambda V \hat{H}}{e}\hat{k}_T^{\beta}\left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right) = \hat{k}_T + \lambda\beta\hat{k}_T\left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right)$$
$$\hat{K}_t = \hat{k}_T\left(1 + \lambda\beta\left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right)\right).$$

This reassures the interpretation that the medium-term equilibrium (where H was eliminated) is independent from nominal variables (e). Moreover, we can see that $\hat{k}_T$ deviates from its steady state value in proportion to the deviation of $\hat{K}_t$.

Now we can turn to capital accumulation:

$$K_{t+1} = K_t + f(q_t)$$
$$q_t = \frac{q_{t+1}}{1+r^*} + \frac{r_t/e - r^*}{1+r^*}.$$

In principle, any f that satisfies certain criteria (its value is zero in zero, it is increasing) can be derived from some appropriate adjustment cost function. Simple power functions, however, imply such an f that is not differentiable in zero ($f(q) = q^{-c}$ for some positive c). For tractability, I will try to eliminate q from the system, by using $q_t = f^{(-1)}(K_{t+1} - K_t)$ and $q_{t+1} = f^{(-1)}(K_{t+2} - K_{t+1})$. For this reason, it is important to work with the simplest possible specification of f. At the end, I will also linearize the equations, where the nondifferentiability of f is also problematic. Hence it is convenient to work with a linear specification ($f(q) = cq$).

The next necessary step is to rewrite capital accumulation in effective terms. The current formulation of q-theory is not compatible with balanced growth: we would like to see q as zero and $\hat{K}$ as constant. This latter would imply perpetual investment: constant effective capital means exponentially growing normal capital. If this investment is also subject to adjustment costs, then q should never reach zero, but rather, settle at $gK = f(q)$. As K still grows here, this is also incompatible with a constant q – but it can be easily rescued. One way is to assume that the installation of new capital is also subject to the same productivity growth as the traded sector. Alternatively, if the adjustment cost depends on I/K, then q converges to $g = f(q)$.

A similar alternative is to assume that the 'natural growth" of capital (at a rate of $1 + g$) is costless. In other words, one can write the same q-theory formalism in terms of effective capital:

$$\begin{aligned} \hat{K}_{t+1} &= \hat{K}_t + f(q_t) \\ q_t &= \frac{q_{t+1}}{1+r^*} + \frac{r_t/e - r^*}{1+r^*}. \end{aligned}$$

Here $q = 0$ is indeed compatible with a fixed $\hat{K}$. Adopting this assumption, the full dynamic system becomes

$$\begin{aligned} \hat{H}_{t+1} - \hat{H}_t &= \frac{e}{1+g}\beta\hat{k}_T\left(\hat{K}_t\right)^{1-\beta} - \frac{V\hat{H}_t}{1+g} \\ \hat{K}_{t+1} &= \hat{K}_t + cq_t \\ q_t &= \frac{q_{t+1}}{1+r^*} + \frac{r_t/e - r^*}{1+r^*} \\ \text{transversality condition} &: \; q_\infty = 0. \end{aligned}$$

The system consists of two state variables ($\hat{K}$ and $\hat{H}$) and one jumping variable (q). For any $\hat{K}_0$ and $\hat{H}_0$, q_0 is such that the terminal condition ($q_\infty = 0$) is met – the regular saddle path solution. Such a system is often hard to solve numerically, since the software must end up with the single value of q_0 that leads to a nonexplosive solution. A useful trick is to eliminate q from the system:

$$\begin{aligned} q_t &= \frac{\hat{K}_{t+1} - \hat{K}_t}{c} \\ q_{t+1} &= \frac{\hat{K}_{t+2} - \hat{K}_{t+1}}{c} \\ \frac{\hat{K}_{t+1} - \hat{K}_t}{c} &= \frac{\hat{K}_{t+2} - \hat{K}_{t+1}}{c(1+r^*)} + \frac{r_t/e - r^*}{1+r^*}. \end{aligned}$$

The remaining system has only two state variables ($\hat{K}$ and $\hat{H}$), but the dimension of $\hat{K}$ has increased: there is now a forward-looking term $\hat{K}_{t+2}$. The saddle property here implies that two initial and one terminal condition ($\hat{K}_\infty$ is constant, i.e., $q_\infty = 0$) should pin down the solution. Formally, the system should have two convergent and one divergent eigenvalue. Rearranging the equation for capital accumulation yields

$$\begin{aligned} \hat{K}_{t+1} &= \frac{\hat{K}_{t+2}}{2+r^*} + \frac{\hat{K}_t\left(1+r^*\right)}{2+r^*} + \frac{c}{2+r^*}\left(r_t\left(\hat{K}_t\right)/e - r^*\right) \\ \hat{H}_{t+1} - \hat{H}_t &= \frac{e}{1+g}\beta\hat{k}_T\left(\hat{K}_t\right)^{1-\beta} - \frac{V\hat{H}_t}{1+g}. \end{aligned} \tag{13}$$

This is already a dynamic system without an explicit jumping variable. It is three dimensional, but two initial conditions and asymptotic boundedness is sufficient for a unique solution (one of the eigenvalues is divergent, so a general stable solution is the linear combination of two eigenvectors).

A final step to reach numerical tractability is to linearize the laws of motion around the steady state. It means that one approximates r_t and $\hat{k}_T\left(t\right)$ around steady state with the linear combination of the state variables, by adopting a first order Taylor approximation.

4 Details of the numerical solution

4.1 The minimized dynamic system

After simplifying the per period equilibrium conditions, we are left with the following system to be solved by winsolve (all hat variables correspond to effective variables, adjusted for TFP growth):

$$\begin{aligned} \hat{K}_t &= \hat{k}_T\left(t\right) + \lambda V\frac{\hat{H}}{e}\hat{k}_T\left(t\right)^\beta\left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right) \\ r_t &= \left(1-\beta\right)e\hat{k}_T\left(t\right)^{-\beta} \\ \hat{K}_{t+1} &= \frac{\hat{K}_{t+2}}{2+r^*} + \frac{\hat{K}_t\left(1+r^*\right)}{2+r^*} + \frac{c}{2+r^*}\left(r_t\left(\hat{K}_t\right)/e - r^*\right) \\ \hat{H}_{t+1} - \hat{H}_t &= \frac{e}{1+g}\beta\hat{k}_T\left(\hat{K}_t\right)^{1-\beta} - \frac{V\hat{H}_t}{1+g}. \end{aligned} \tag{14}$$

The first two equations simultaneously determine r_t and $\hat{k}_T\left(t\right)$ for a given H and K. The second two govern dynamics.

For given initial values of H and K, we look at the implied paths of p_{NT}, K etc. The model converges to the stationary point (it is a point in terms of effective variables, and a path in normal variables), which also corresponds to the long-term NATREX relative price of tradables and nontradables. Along the transition path, however, it does not strictly follow the medium-term NATREX path. This medium-term NATREX path can be obtained by replacing the law of motion of H with

$$\frac{\hat{H}_t}{e} = \frac{\beta}{V} \hat{k}_T \left(\hat{K}_t\right)^{1-\beta}. \tag{15}$$

It means endogenizing H in such a way that we move along the medium-term equilibrium (along the $\frac{d}{dt}\hat{H} = 0$ curve, and it is only capital that adjusts slowly). The complete system becomes

$$\begin{aligned}
\hat{k}_T(t) &= \frac{\hat{K}_t}{\left(1 + \lambda\beta \left(\frac{1-\alpha}{1-\beta} - \frac{\alpha}{\beta}\right)\right)} \\
r_t &= (1-\beta)\, \hat{k}_T(t)^{-\beta} \\
\hat{K}_{t+1} &= \frac{\hat{K}_{t+2}}{2+r^*} + \frac{\hat{K}_t (1+r^*)}{2+r^*} + \frac{c}{2+r^*} \left(r_t\left(\hat{K}_t\right) - r^*\right).
\end{aligned}$$

One can then check the persistence of deviations from the medium-term equilibrium path.

Notice that the medium-term equilibrium path (the real economy) does not depend on g, the rate of technology growth. It means that *the equilibrium real exchange rate can be decomposed into a TFP term ("standard Balassa-Samuelson" effect) and a capital accumulation term, and there is no interaction between the two.* This does not apply to the non-NATREX path.

I will consider the two basic shocks: a nominal appreciation and an exogenous increase in money accumulation (fiscal expansion, financed by foreign debt, to be repaid in the future, or maybe not). One can see that the medium-term NATREX path is unaffected, but the actual realization changes. We can check the shock responses of inflation (prices), wages and capital accumulation – for example, a nominal appreciation may slow down the convergence to the steady state $\hat{K}$. It might also have an inflationary effect: traded prices are down, but the resulting consumption boom pushes the nontraded-traded relative price up. This dies out only gradually, as a trade deficit restores the equilibrium level of $\hat{H}$ (measured in traded goods).

4.2 Linearization

In order to solve the model numerically, I had to linearize the laws of motion around the steady state. Though winsolve is capable of solving forward-looking models, its algorithm might not converge fast (or at all). More precisely: if a given initial condition is consistent with an asymptotically stable solution, winsolve tends to find it. This applies 100% to linear models, but less for nonlinear specifications.

4.3 Calibration

So far, this is just a first guess calibration, which attempts to show that under reasonable parameter constellations, the model produces quantitatively relevant results. For actual policy simulations or implications, one would have to back up some of the parameters from actual data (elasticities, expenditure shares, capital and labor shares, etc.), or estimate approximate static or dynamic equations of the model. One likely pitfall of such an approach is the apparent nonhomothetic behavior of traded-nontraded relative prices and consumption, often found in many economies, including Hungary. One cannot simply incorporate nonhomothetic preferences into growth models, since they do not lead to well-defined steady states or balanced growth paths. Nonetheless, such a consumer behavior may have important implications for the evolution of relative prices, which is worth exploring in the future.

Parameters – $\alpha, \beta, \lambda, r^*, g, c, V$; initial conditions – H_0, K_0. One period is chosen to be approximately one *tenth* of a calendar year. It is not necessarily true that such a short period would be enough for all price adjustments to be over. One can recalibrate the model in a way that one period corresponds to one quarter. I have also run some simulations for this latter scenario, without any significant changes in the results.

- $\alpha = 0.8$ – labor intensity of the nontraded sector
- $\beta = 0.5$ – labor intensity of the traded sector – one could look at more detailed data on these. All this starting assumption does is to assume that $\alpha > \beta$, which is a standard choice, though there are signs that it might not fully hold in certain countries (including Hungary). Another hint is the share of capital from GDP. With the current choices, it is 37.5%.
- $\lambda = 2/3$ NT expenditure share; this is no an unreasonable assumption, particularly if we take into account that traded prices also have large service components
- $r^* = 0.005$ – required real rate of return on capital (assuming that one year is ten periods, then it means 5% annually). With one period being one quarter, this value is 0.0125.
- g – I choose $g = 0.001$ (0.0025), which implies a growth of percentage point per year.
- c, V – one can choose one of them "freely" (say, c), by matching an a priori speed of adjustment. Based on this, the choice of $c = 3000$ is consistent with one year being 10 periods (the half-life of an innovation to the capital stock is 2 years). Then I select V in such a way that the speed of nominal adjustment be sufficiently faster than that of real adjustment. This led to $V = 0.1$ (a 10% shock to H has a half life of around one year).

- $H_0/H^*, K_0/K^*$. The latter measures the ratio of current to steady state per capita capital stock. I chose its value to be 100 and 90. In various runs, I have chosen H_0 to be 100, 90 and 110 percent of H^*. Another possibility is to reproduce the initial trade deficit.

5 Results

5.1 The behavior of the real exchange rate during convergence

5.1.1 The medium-term equilibrium path

As discussed earlier, we get profiles for effective variables, which are to be added one in one to the growth part. For the non-monetary ("flexible exchange rate") version, this is independent from the speed of growth; but the same does not apply to the monetary version. This also implies that there is no choice of the fixed exchange rate that is compatible with converging along the medium-term NATREX path. We will see small deviations, which are partly related to the approximation that V is constant (and the negligence of the $\dot{V}$ term), and partly inherent to the nominal economy.[6]

Convergence implies an appreciating real exchange rate, if the nontraded sector is more labor-intensive. If convergence involves both TFP growth and capital accumulation, then what the model produces is the excess real appreciation relative to the standard Balassa-Samuelson situation. If the labor intensities are equal across sectors, then capital accumulation has no impact on the equilibrium real appreciation, while if the nontraded sectors is less labor-intensive, real appreciation should be smaller than the standard Balassa-Samuelson.

All these are fully consistent with international trade theory: as long as capital is scarce, it has a high factor price. In the flexible Balassa-Samuelson model, an increase in world interest rates increases the relative price of that sector which uses capital more intensively (inverse Stolper-Samuelson theorem). For high rental rates, if the nontraded sector is more labor-intensive, then the NT relative price starts from a low relative price, thus must increase. It means a positive but vanishing excess inflation (real appreciation) relative to the standard Balassa-Samuelson case.

The two figures (2 and 3) show the evolution of the nontraded relative price and price change (per period, so annual measures are tenfold larger). The steeper path (Run3) corresponds to $K_0 = 0.8K^*$, while the choice for the other is $K_0 = 0.9K^*$. We can see that there is a large

[6] The real economy moves along the $\frac{d}{dt}\hat{H}\left(\hat{K}\right) = 0$ curve. As $\hat{K}$ grows, this leads to an increase in $\hat{H}$ as well. The nominal economy cannot satisfy $\frac{d}{dt}\hat{H}\left(\hat{K}\right) = 0$ and still produce an increase in effective money holdings, unless there is an extra exogenous increase in $\hat{H}$.

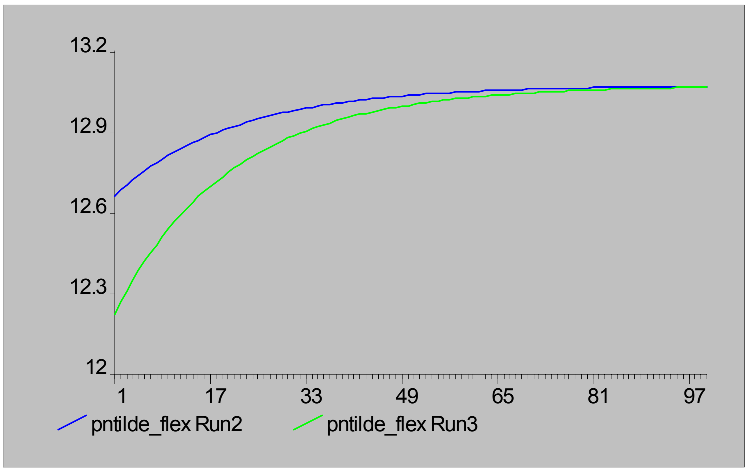

Figure 2: Convergence paths – the NT-T relative price

difference between the two, but it disappears as capital approaches K^*. Under slower capital accumulation, the same cumulative difference (the relative price is determined by K/K^*, so both the initial and the terminal price level is independent from the speed of adjustment) is distributed along a longer time period, so it is more persistent, but also smaller.

5.1.2 Disequilibrium (nominal) paths

I show the results of two different scenarios, and compare them to the medium-term equilibrium convergence path. In both cases, I choose $K_0 = 0.9K^*$. The initial value of H is set such that $e = 1$ yields $p_{NT}(1) = p_{NT}^{eq}(1)$ In other words, if $e = 1$ (the first nominal scenario), then the nominal economy starts along the (real) equilibrium convergence path. Numerically, it means that $H_0 = 0.949H^{st}$. In scenario 2, we have the same H_0, but the nominal exchange rate is 10% stronger ($e = 0.9$). The results are displayed on figures (4-8).

We can see that the nominal path starting from the real equilibrium departs from the medium-term equilibrium path, but the deviation is minor. The largest difference seems to correspond to capital, which depends on the discounted sum of future returns, so all deviations are added up in some sense. Overall, the exchange rate (relative price of nontradables) is undervalued during convergence. This is also shown by the difference between actual money stocks and the equilibrium values calculated according to equation (15): the disequilibrium path involves smaller money stocks, thus lower consumption. That benefits the price and accumulation of capital, while keeps wages low. Undervaluation has no systematic cause: the interaction between the two dynamic effects and the precision of my approximations jointly determine the deviation between the real and nominal paths.

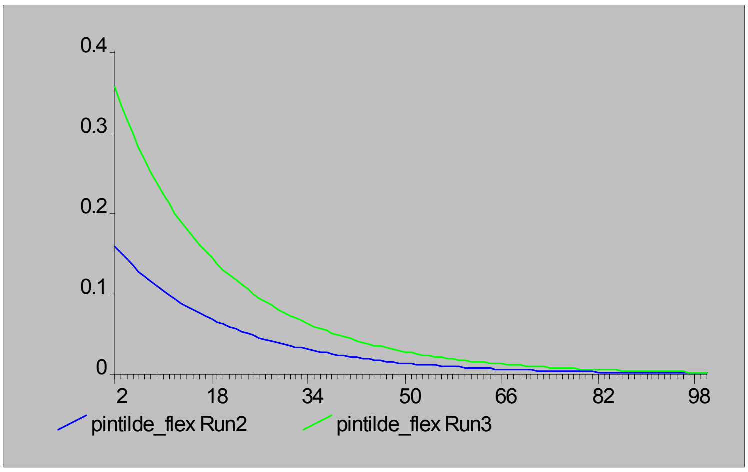

Figure 3: Convergence paths – excess nontraded inflation

A stronger initial exchange rate (by 10%) leads to overvaluation in the first 10 quarters of convergence (its impact is 170 basis point on the relative price), which then turns into an undervaluation. This undervaluation implies a boost to capital accumulation in two years. We can see a similar dual effect on most variables. In the next subsection, I will compare two such nominal paths in more detail (though with $H_0 = 0.9H^{st}$). Section 6 offers some interpretations, and relates all the results to international trade theory.

5.2 A shock to the nominal exchange rate

5.2.1 Base values

We start at the steady state capital and money stock, but there is a 10% revaluation at the beginning. This leads to a 170 basis points increase of nontraded-traded relative prices, and the effect disappears gradually in near 3 years. The stock of capital is reduced by 35 basis points for this time period (in fact, even longer), and there is an entire year with a loss of 45 basis points. A back of the envelope calculation:

$$\begin{aligned} rK &= 0.375 * GDP \\ K &= \frac{0.375}{0.05} * GDP = 7.5 * GDP \\ 0.0035K &= 0.0265 * GDP, \end{aligned}$$

so the loss of capital is equivalent to 265 basis points of GDP. In the worst year, it is 337 basis points. Actual estimates of Hungary's net capital stock to GDP (not necessarily consistent with

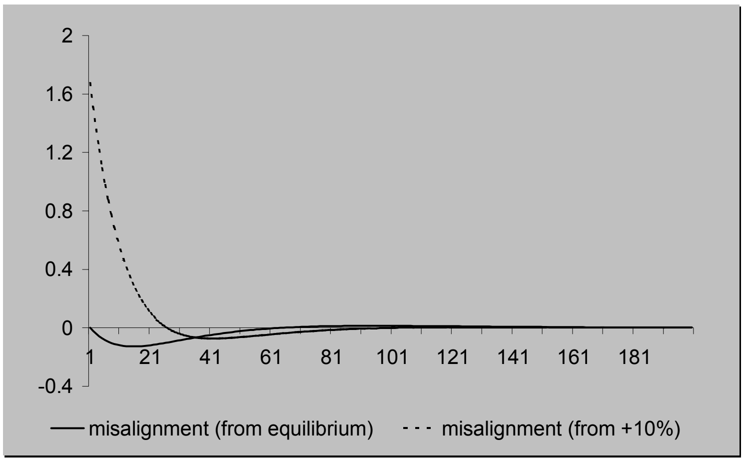

Figure 4: Percent deviation of the relative price from equilibrium

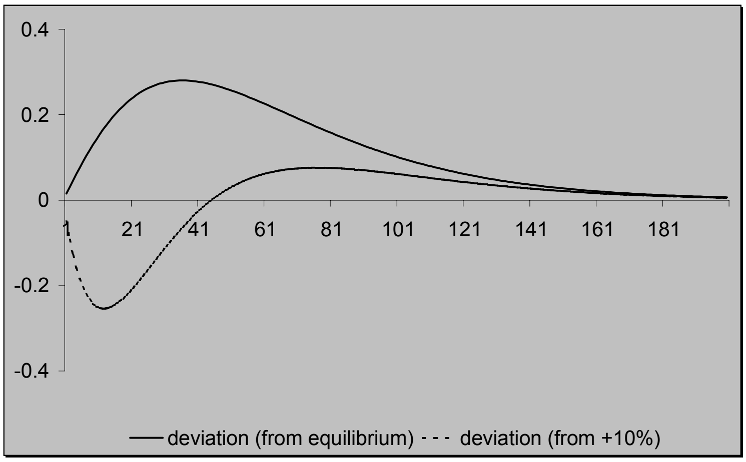

Figure 5: Percent deviation of the capital stock from equilibrium

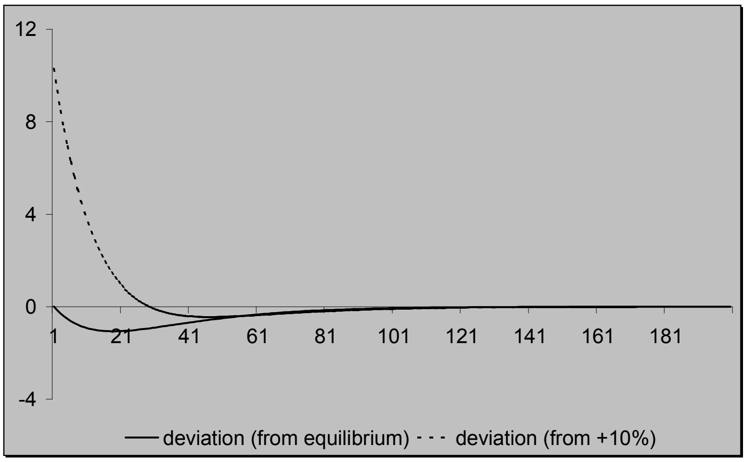

Figure 6: Percent deviation of the money stock from equilibrium

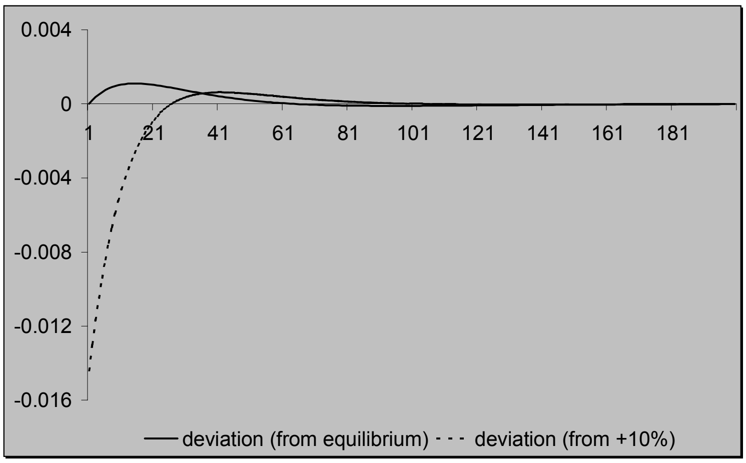

Figure 7: Percent deviation of the rental rate from equilibrium

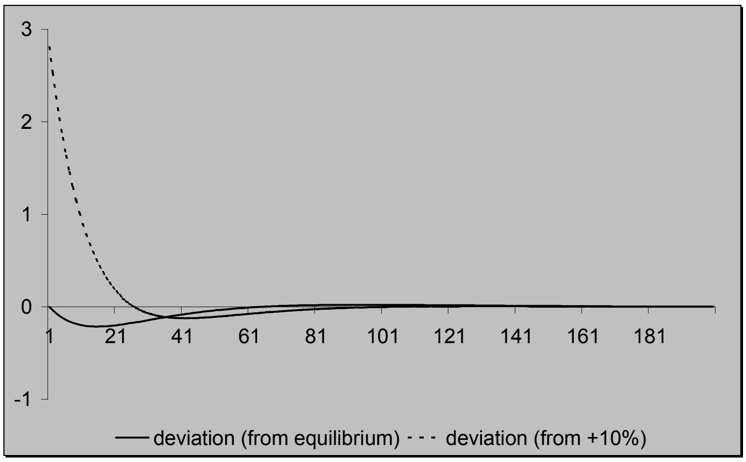

Figure 8: Percent deviation of wages from equilibrium

a fixed share of capital income and a rental rate of 5%) are around 1.2-2.9, so the 35 basis points reduction in capital stocks is equivalent to 40-100 basis points of GDP.

The impact on the two sectors is much stronger: the nontraded capital stock increases by 15%, while its traded counterpart falls by 5%. The corresponding numbers of employment is +7.5% and -10%, and 6% for capital-labor ratios in both sectors. The price of capital (measured in "euros") falls by 3%, while wages (also in euros) increase by 3%.

The volume of consumption increases in both sectors, which implies an equal increase in nontraded production. Traded production, on the other hand, falls, so part of the excess tradable consumption is imported, showing up in a trade deficit and a money outflow. This reflects partly increased spending, and partly decreased production of tradables.

One can also calculate the balance of GDP in fixed euro prices: we have the volumes of both sectors, and aggregate them using $p_T^* = 1$, $p_{NT}^* = p_{NT}^{st.st.}$ (which corresponds to steady state prices). This shows an initial loss of 5 basis points, which then accelerates, and the cumulative sum is 929 basis points. This is measured in per period GDP, so in terms of annual GDP, the loss is 93 basis points. One cannot really obtain a sacrifice ratio from this, since there was no disinflation, but just a correction of the price level.

Detailed results are displayed on Figures 9 – 17.

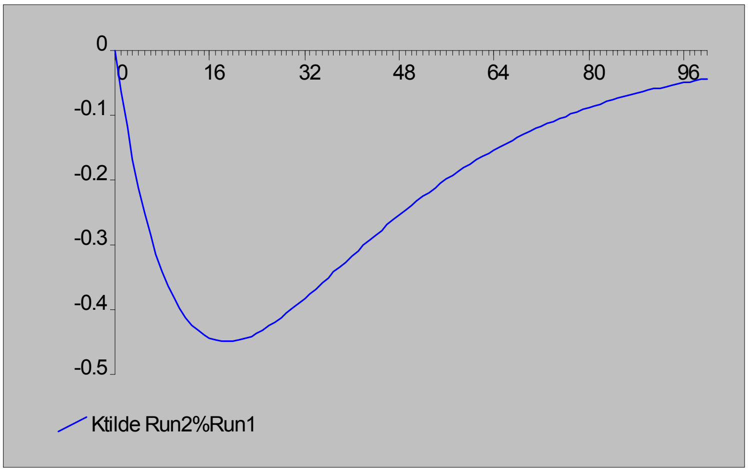

Figure 9: Shock response: capital

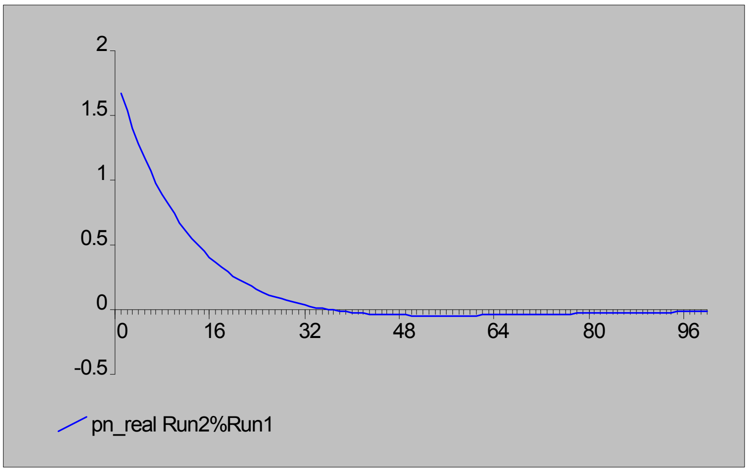

Figure 10: Shock response: NT-T relative prices

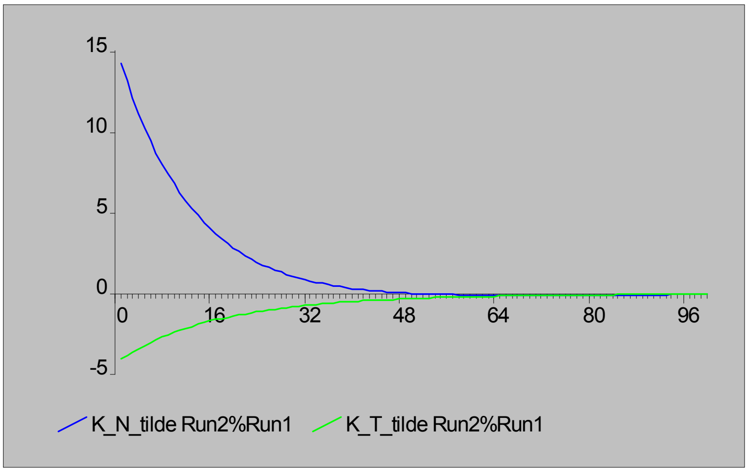

Figure 11: Shock response: sectorial capital employment

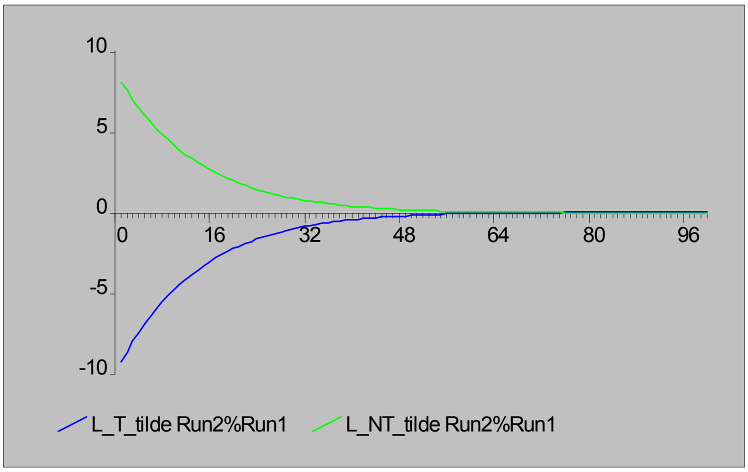

Figure 12: Shock response: sectorial labor employment

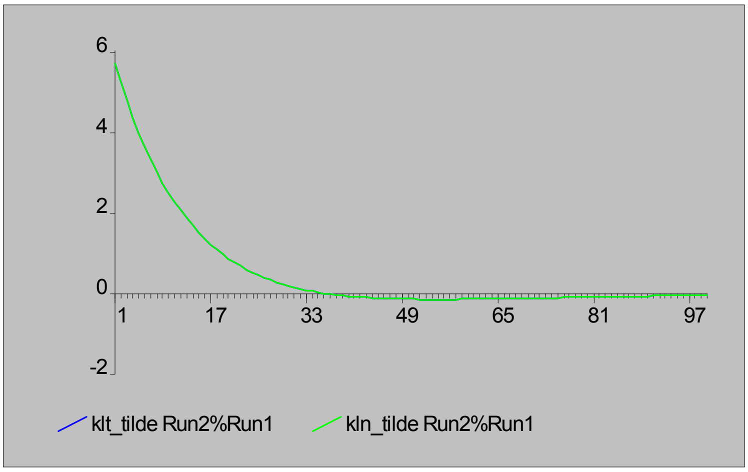

Figure 13: Shock response: sectorial capital-labor ratios (identical)

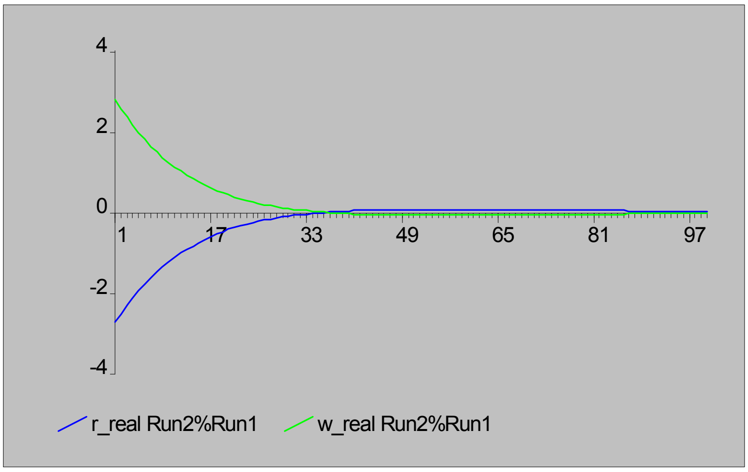

Figure 14: Shock response: factor prices in euros

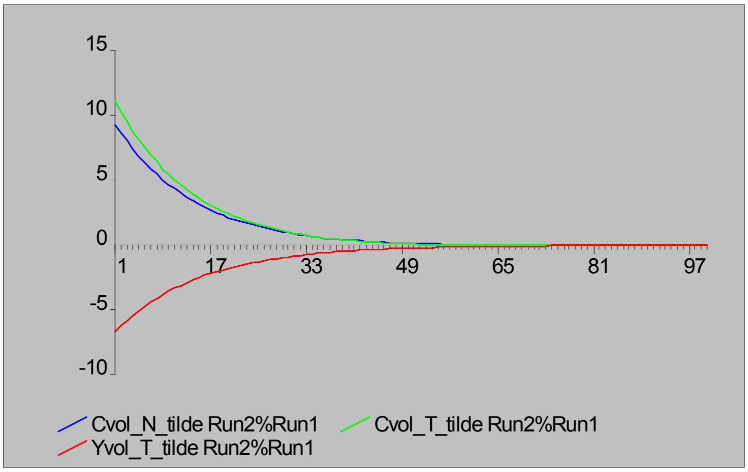

Figure 15: Shock response: volume of T and NT production and consumption

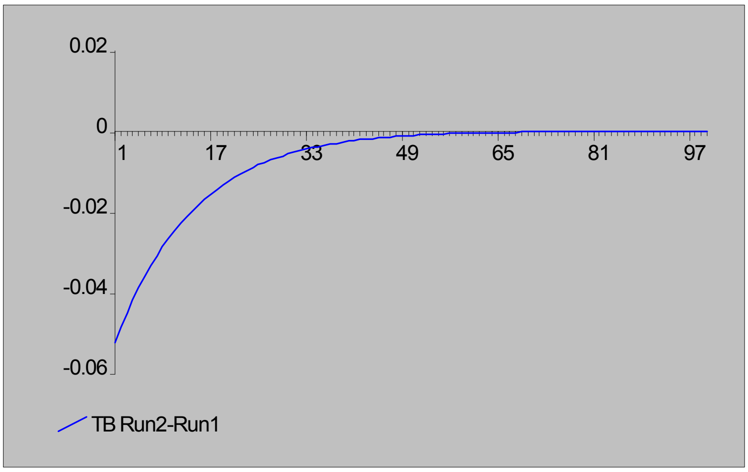

Figure 16: Shock response: trade deficit relative to fixed-price GDP

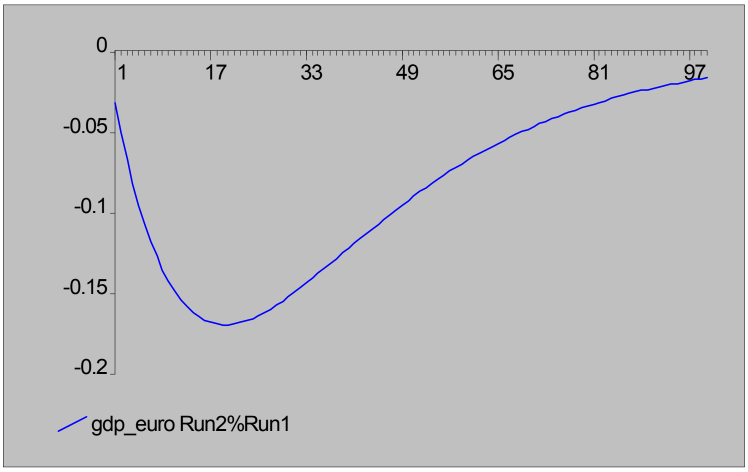

Figure 17: Shock response: evolution of fixed price GDP

5.2.2 Faster capital adjustment

This case corresponds to $c = 5000$ (instead of 3000). An initial 10% fall in capital has a half-time of 1.5 years (15 periods), somewhat still slower than the speed of the nominal adjustment.

The fall in capital is larger, but its level also returns faster to normal. All other impulse responses are very similar to the baseline. The cumulative loss of GDP is 1076 basis points, somewhat larger than the baseline number.

5.2.3 Initial capital stock

Instead of 100%, I also explored the choice of 90%. The loss of capital is somewhat larger, just like the initial increase of the nontraded-traded relative price. The cumulative loss of GDP, on the other hand, is only 854 basis points (in absolute terms, there is a smaller loss of capital). In fact, there is even an increase at the beginning.

5.2.4 Different speeds of nominal adjustment

Instead of $V = 0.1$, I have explored a faster ($V = 0.2$, a half life of an H-shock is 5 periods) and a slower ($V = 0.05$, a half life of 18 periods) adjustment scenario. The results (not reported) are easy to interpret: the impact effect on the relative price is largely independent from V. Under fast nominal adjustment, the price level quickly returns to equilibrium, so there is little response by capital accumulation. Cumulative GDP loss is also smaller, with the particular choice of V, the reduction is substantial (287 basis points of per period GDP). Under slow adjustment, all statements are reversed, capital accumulation is seriously affected, which translates into a cumulative GDP loss of 2745 basis points of per period GDP.

5.2.5 Initial money stock

In the baseline scenario, both capital and money started from its steady state value. I have also checked the implications of varying H_0 by $\pm 10\%$. In other words, it means that I check the implications of initial excess demand (+10%, overvaluation) and excess supply (-10%, undervaluation) on the responsiveness of the economy to nominal shocks. Under excess demand, all the impulse responses are bigger, but not substantially. An overall indicator could be the cumulative GDP loss number, which becomes 1063 basis points, somewhat larger than for the baseline case. With undervaluation, all reactions are smaller, and the loss reduces to 802 basis points.

5.3 Government spending

This has similar implications than a nominal appreciation, though the evolution of government spending can have various dynamics, each producing different impulse responses. For example, the government can distribute 10% of GNP to consumers immediately, or during a certain number of periods. Later on, it might take it back or not (notice that such a future event is not correctly handled by my approximately forward-looking consumers). Qualitatively, we get the same answers as with a revaluation.

6 Discussion

The signs of the previous results are relatively easy to interpret. Both a fiscal expansion and a revaluation increases the value of H in terms of traded goods. This leads to an increase in spending on nontraded goods, which increases nontraded production. Given the short-run nonlinearity of the transformation curve, nontraded prices must increase. This is the dominant shock to the economy, all of the other results can be traced to this through the Stolper-Samuelson theorem: if the price of a sector increases, it leads to a more than proportional increase in the price of the factor which is used more intensively by the windfall sector. The price of the other factor of production decreases.

In our case, the price of the nontraded sector has increased, and it is more labor-intensive. This leads to a rise in wages and a fall in rental rates. Production becomes more capital-intensive, and the fall in rental rates decreases capital inflows (q falls).

What makes this situation relatively persistent? The explanation is closely related to the phenomenon of "Dutch disease": a country receiving a transfer also sees its terms of trade improving. The extra consumption enabled by the transfer falls partly on nontradables, which pushes domestic wages up. In our two factor model, we need some extra conditions for the transfer effect: if the only source of income of domestic consumers is their labor earnings, then the nontraded sector must be more labor intensive than the traded sector. The price of capital falls, but that does not influence domestic spending.

This is the underlying propagation mechanism: the initial shock to consumption increases domestic income, so the excess money stock will flow out only slowly. If some of the capital is domestic, and its income is used for consumption expenditures, then excess spending still creates some of its excess income, but to a smaller degree. In this case, we can get persistence even without the labor intensity assumption.

One can give a similar interpretation to the nominal convergence path starting from the medium-term (real) equilibrium position: since the money accumulation process governed by consumer optimization is not the same as the medium-term equilibrium path, period one money stocks differ. This changes all prices in equilibrium, through the "demand effect". For a smaller than equilibrium H, wages also become smaller, which then reinforces the initial undervaluation and makes it persistent. Undervaluation thus even increases initially. This is balanced by the effect on capital accumulation: due to a higher rental rate, there is more capital inflow, which leads to an increase of wages eventually. Undervaluation starts to disappear. With my choice of parameters (and approximations), the system converges to the equilibrium path (medium- and long-term) through undervaluation. In general, it is possible to have a shift to overvaluation during this process.

It is clear that the parameter V plays an important role in determining the speed of adjustment through the trade balance: excess spending is proportional to VH, so a small V leads to a slow outflow of the extra money. Looking one step behind, V is proportional to $(\delta + \pi) \frac{\gamma}{1-\gamma}$, the sum of the discount factor and inflation times the substitutability between consumption and money holdings. Consequently, V represents the degree of intertemporal consumption smoothing – how fast consumers deplete their excess money stocks. Another important determinant of persistence is the weight of nontradables in consumption expenditures, since the larger it is, the more valid the Keynesian thesis that "excess demand creates its supply".

It is important to note that the sectorial labor intensity assumption is not relevant for the increase of the price of nontradables. Its role is to make the price of capital fall and wages increase (through the Stolper-Samuelson theorem). The "wealth effect" of a revaluation hurts or benefits capital (investment), depending on relative factor intensities. I have explored a scenario with the traded sector being more labor intensive. Nontraded prices increased, wages fell, the rental rate increased, and capital accumulation accelerated.

The degree of substitutability between the two goods (by consumers) and the factors of production (by producers) also influences the quantitative behavior of the economy. Starting with the preference side, let us assume that consumption utility is

$$\left((1-\lambda)^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{NT}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} .$$

The choice of $\theta = 1$ corresponds to my Cobb-Douglas specification. Suppose that $\theta > 1$. An increase in p then implies a larger substitution towards traded goods, so an increase in consumption expenditure must lead to a smaller increase in C_{NT} and p. Keeping the same transformation curve between traded and nontraded goods, a smaller price increase leads to a smaller wage increase and a smaller decrease of the rental rate. This muted impact effect also weakens the endogenous persistence of the shock, since a smaller wage increase leads to a faster outflow of excess money. In summary, a higher degree of substitutability between traded and nontraded goods increases both the impact effect and the persistence of nominal shocks on the real economy. Conversely, $\theta < 1$ increases both the impact effect and the persistence. One can even find parameters such that a nominal appreciation initially *improves* the trade balance (wages increase more than one in one relative to the nominal exchange rate). Later on, the corresponding decline in r and K leads to a fall in w, and excess money flows out in the long-run.

Intuitively, one would expect the opposite impact of substitutability between factors of production: if it is easy to substitute labor with capital, then the same price increase leads to a smaller wage increase. Consequently, the same increase in nontraded expenditure (pC_{NT}) leads to a smaller increase in p and w, thus a smaller impact effect and smaller persistence. The combination of nonunit substitutability both in preferences and technology has very complicated general equilibrium cross-effects, which can be addressed only numerically.

Recent developments in the Hungarian economy (rising wages, stagnating investment, a marked asymmetry between traded and nontraded investment behavior) suggest an overall decline in $\frac{r}{w}$. If we do not attribute this entirely to changes in minimum wages and public sector wages, then the monetary restriction ("revaluation") and the overall fiscal expansion offers an explanation. Note that this requires the sectorial factor intensity assumption, which seems to be somewhat problematic with Hungarian data. One resolution would be to assume similar factor intensities but then relax the equalization of rental rates across sectors. The reverse factor intensity scenario should have counteracted the exogenous rise in wages, but there are no signs for such an effect.

One comment is in line here, about the large sectorial reallocations showed by the results. These are the consequences of the assumption that only the cross-border adjustment of capital is slow. The price of domestic labor and installed domestic capital is equalized between the two sectors, so there is free sectorial mobility. Regarding capital, this does not necessarily imply a large reallocation: remember that all the results are in effective terms, so the capital stock is constantly adjusted to TFP growth. A decline in effective capital stocks corresponds to a much smaller decline or even an increase in regular capital. It seems plausible that current investment is always channeled in a way that sectorial differences in rental rates are minimal. As long as regular investment is positive in both sectors, the equalization of rental rates can be resolved only through the appropriate allocation of new capital between sectors.

For labor, such an argument does not work, and one has good reasons to believe that labor also adjusts slowly. This leads to sectorial wage differences, which is likely to increase the impact effect of the nominal shocks (nontraded prices must increase even more, since near fixed capital and labor imply a large increase in the cost of production), but its persistence should decrease – by the time labor starts to switch sectors, the price and wage differentials have been nearly eliminated. My results still issue a warning about the direction of asymmetries between sectors and factors, and indicate the direction of reallocation. For example, with gradual labor flows, nontraded prices should increase even more, nontraded sector wages should increase heavily, while traded sector wages may even fall temporarily.

An important extension of the model is the following (this is also the direction of future developments). Maintain the assumption of labor mobility, but allow for temporary firm profits, and gradual capital flows between sectors and countries. One would then have monopolistic competitors in the domestic nontraded and traded sectors, and also in the world traded market. One can assume free entry (or even perfect competition) abroad, and a gradual entry and exit of domestic firms in response to profits. In a q theory fashion, entry and exit could be a function of current and expected future profits (discounted by world interest rates). This would allow for a gradual passthrough of the nominal exchange rate into domestic tradable prices, and also a more complete and realistic treatment of investment, profits and the reallocation of capital between sectors.

One can reinterpret the "money effect" as a "wealth effect", or even as a portfolio resizing and rebalancing effect. The common is that a nominal shock will influence the value or the returns of nominal wealth/assets, leading to a change in the intertemporal behavior of consumers. Such a change will translate into a change in consumption expenditures, which is the necessary starting point of my model.

In the wealth effect interpretation, it is implausible to assume that expenditures are determined entirely by wealth. In terms of a consumption function

$$E = \mu V H + (1 - \mu) Y,$$

I have assumed so far that μ is one (here H is total wealth, and not just money, though it does not yet earn any returns). This implies that V controls both the steady state wealth-income ratio and the persistence of the nominal shock response. The model can be extended to $\mu \neq 1$, which yields

$$\begin{aligned}
\hat{K}_t &= \hat{k}_T(t)\left(1+\lambda(1-\mu)\frac{\beta-\alpha}{1-\beta}\right)+\lambda\mu V\frac{\hat{H}}{e}\hat{k}_T(t)^{\beta}\left(\frac{1-\alpha}{1-\beta}-\frac{\alpha}{\beta}\right) \qquad (16)\\
r_t &= (1-\beta)\,e\hat{k}_T(t)^{-\beta}\\
\hat{K}_{t+1} &= \frac{\hat{K}_{t+2}}{2+r^*}+\frac{\hat{K}_t(1+r^*)}{2+r^*}+\frac{c}{2+r^*}\left(r_t\left(\hat{K}_t\right)/e-r^*\right)\\
\hat{H}_{t+1}-\hat{H}_t &= \frac{e}{1+g}\left(\beta+\lambda(1-\mu)(2\alpha-\beta)\right)\hat{k}_T\left(\hat{K}_t\right)^{1-\beta}-\frac{V\hat{H}_t}{1+g}\frac{1-\beta+\lambda(1-\mu)(\beta-\alpha)\frac{\beta-2\alpha}{\beta}}{1-\beta+\lambda(1-\mu)(\beta-\alpha)}.
\end{aligned}$$

The equations are somewhat more complicated, but they remained quite similar. Linearization is also similar here than before. It is visible that both μ and V influence the persistence of shock responses. So far, I have not experimented with solving (16).

7 Some concluding empirical considerations

There is vast literature on exchange rate based disinflations (for example, Rebelo and Végh, NBER Macroeconomics Annual 1995), their stylized facts, the sources of success or failure. Darvas (2003) identifies two main groups of countries who experienced large (nominal and real) appreciations at the start of disinflations: fixed exchange rate, mostly Latin American countries with a failure in their stabilization programs, and floating countries, mostly industrial, with a success. The question is then whether these differences in country experiences can be related to differences in the strength of the mechanisms of my model. Naturally, there are many additional issues and considerations determining the overall success or failure (credibility, fiscal side, just to name a few), but this "transfer effect" might also be an important contributor.

According to my results, a nominal appreciation has a negative side effect, through nontraded prices: even if there is immediate and full passthrough of the nominal appreciation into tradable prices, the "equilibrium" (flexible price, but gradual adjustment of money balances/wealth) relative price of nontradables increases. Under certain factor intensity assumptions (the nontraded sector is more labor intensive than the traded sector), it is also true that wages increase (relative to tradable prices, which is the same as relative to wages in the rest of the world). Assume that these *qualitative* effects are equally present in successful and failed disinflation episodes.

What can determine the difference in success? If nontraded inflation or wage inflation remains relatively high, then expected inflation may adjust only little. Somewhat more formally: if we manage to introduce inflation persistence (expectations) into the model, then a slow adjustment of wages and nontraded prices should increase inflation persistence (expectations), thus making the success of the disinflation less probable. This interpretation would trace the difference in success or failure into differences in the impact and persistence of the wealth (money) effect of an appreciation.

These differences can come from the role of money/wealth directly (like parameters V and μ), or from different production functions, factor intensities, preferences, intertemporal and intratemporal substitutability of goods. The distance from steady state (industrial country or emerging market), or the role of capital income within GNP may also matter. Thinking in terms of an external asset position, it may also matter whether the country is a net lender or borrower, since a revaluation decreases the value of external assets, so it decreases the value of outstanding foreign debt. Besides this direct wealth effect, there can be a portfolio rebalancing between foreign and domestic assets as well.

These are all differences which can be measured for both groups of countries. For most of them, one would expect that the failure countries are subject to larger wealth (nominal) effects. It is central to have data on wages and nontraded-traded relative prices. Burstein et al (2002) document a systematic behavior of prices after large *devaluations*: once one accounts for various conceptual and measurement issues in tradable prices, the only remaining mystery is the surprisingly low increase in nontradable prices. That effect can also be related to the negative wealth or money effect of a devaluation, cutting domestic spending, thus decreasing the relative price of nontradables.

Apart from data availability issues, a major problem of implementing this empirical cross-country comparison is that success and failure countries also differed in their fiscal policies. Overall, successful countries did not tend to "match" the appreciation with a fiscal expansion, which is much less true for failure countries. As argued earlier, a fiscal expansion has qualitatively similar implications for relative prices and factor rewards as an appreciation. In order to relate success or failure to a different money or wealth effect, one needs to filter out the effect of fiscal policy. It is in fact possible that fiscal policy gets all the blame of a failure, but it may also happen that disinflation would have failed even without a fiscal expansion. Unfortunately, this filtering might require extensive data on government behavior, and its result might be sensitive to parameters and model specification.

The last piece of speculation concerns the reason for certain countries switching to a fiscal expansion around the appreciation. Is there a potential mechanism that leads to an automatic fiscal expansion, depending on the equilibrium implications and price incidence of the nominal appreciation? For example, a government closely allied with exporters or domestic capital owners might be tempted to compensate domestic firms for the strong exchange rate. A government concerned with slowing investment may also adopt a fiscal expansion. The common theme is that an appreciation hurts exporters and capital, and a fiscal policy aimed at offsetting these effects ends up giving substantial extra income to domestic consumers as a byproduct.

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