Inflation expectations, learning and DGSE model estimation

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Abstract

Inflation expectations receive a lot of attention in the academy and industry because of their effect on actual inflation and on central banks' ability to achieve price stability. Surprisingly, few studies have investigated the capacity of state-of-art models for monetary policy analysis to match the data on inflation expectations.

I find that the benchmark New Keynesian dynamic stochastic general equilibrium (DSGE) model, Smets and Wouters (2007), under rational expectations does not reproduce neither the evolution of the series of survey expectations nor its cross correlation with respect to inflation and the interest rate. Estimates of this model under learning fit these problems but only when the forecasting models of inflation include just inflation for the period of high inflation (1968Q4-1983Q4) and the interest rate and hours worked for the period of low inflation (1989Q1-2008Q2). Additionally, these estimates imply that agents discard past information pretty fast, give support to the existence of *inflation scares* as pointed by Orphanides and Williams (2005) and show that learning *per se* does not generate higher levels of persistence than under rational expectations estimation. Finally, I find evidence of a significant increase in the response of interest rate to inflation expectations by the central bank at least since the late 80s.

This study employs quarterly US macroeconomic data for the period 1968Q4 – 2008Q2.

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1. Motivation

Inflation expectations receive a lot of attention from central banks for several reasons. There is a consensus that expectations greatly influence actual inflation and thus the ability of central banks to achieve price stability[1]. Additionally, given the lagged effect of monetary policy actions on output and inflation (Jevons, 1863, and Friedman, 1961), it is fundamental for central banks to keep a close eye on expected future conditions, and to behave preemptively. Finally, expectations of the private sector future inflation contain a wide variety of information about past and anticipated economic developments (Batini and Haldane, 1999).

Despite the central role of inflation expectations in practice, few studies have investigated the ability of state-of-art models for monetary policy analysis to match the data on inflation expectations. The scarcity of such studies can be explained by prevailing skepticism about the quality of the expectations collected in surveys (Roberts, 1998) and by the dominance of rational expectations (RE) paradigm[2].

However, in the last few years, promising results obtained from the direct use of surveys on inflation expectations had led researchers to reconsider the quality of the information contained in surveys[3]. Additionally, the development of alternatives to RE allows us to test how different ways to model expectations can affect the analysis that is being implemented. It follows that the joint use of information from surveys and alternative ways to model expectations could result in advances in our understanding of the dynamics of inflation and its relationship with other macroeconomic indicators.

In the light of this, the objective of the present study is threefold. First, I propose to determine the extent to which the current benchmark New Keynesian dynamic stochastic general

[1] It has long been recognized that monetary policy can be more successful when inflation expectations are well-anchored (Bernanke 2007, Mishkin 2007).

[2] In recent years, most of the developments in macroeconomic modeling where directed toward increasing the complexity of the microfoundations under the firmly-held assumption that expectations are compatible with the RE hypothesis. Specifically, models employed to explain the dynamics of inflation and its relationships with other macroeconomic aggregates have adopted features of stickiness *a la* Calvo and indexations in prices and wages, habits in consumption and investment adjustment cost among others. The most representative of these models are Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005).

[3] Early references are Roberts (1998) and Rudebusch (2002). Orphanides (2004) uses survey expectations of inflation and other real-time measures to estimate a Taylor rule, while Adam and Padula (2003), Nunes (2005) and Paloviita (2004) evaluate the characteristics of the Phillips curve when surveys are added as proxy of expected inflation. Using a TVC-VAR, Canova and Gambetti (2008) find that shocks related with survey expectations play a significant and important role in explaining the volatility and persistence of the inflation and output during the whole sample. Leduc et al (2007) use series of inflation surveys expectations in a VAR estimation and find that "shocks to expectations", jointly with an accommodating monetary policy explain the persistent high inflation in the 1970's.

equilibrium (DSGE) model, Smets and Wouters (2007), solved and estimated under the assumption of RE, is compatible with the surveys on inflation expectations. Second, I use surveys of inflation expectations to model how expectations of future values of this variable are generated. In order to do so, it is necessary to depart from the RE hypothesis. I choose *learning* as an alternative way to model expectations due to its increasing prominence in the evaluation of the persistence of macroeconomic indicators in general, and inflation in particular. After estimating the SW model under learning, I evaluate its performance to match dimensions of the data. Finally, as the third goal of this study, I analyze the implications of the estimated model with respect to the persistence of the inflation generated by learning, the reaction of expected inflation to the structural shocks and changes in the monetary policy rule.

The analysis that I implemented in this study employs quarterly US macroeconomic data for the period 1968Q4 – 2008Q2. The results of my estimation and simulation reveal that the SW model under RE is not compatible with the information of survey expectations on inflation. Assuming RE, the model of SW can neither reproduce the evolution of the series of survey expectations nor its cross correlation with inflation and interest rate. Although these statistics improve slightly when adding survey expectations of inflation to the estimation, the collateral effect is still negative. For instance, the model yields implausible values for some of the parameter estimates and correlations of inflation with respect to the interest rate and output which are not in line with the stylized facts.

The comparison of the series of survey expectations about inflation and the series of forecasts generated by small forecasting models indicates that for the period of high inflation (1968Q4-1983Q4) models that include inflation describe better the survey data, while for the period of low inflation (1989Q1-2008Q2), the interest rate and hours worked are among the most important regressors. Additionally, in contrast to previous studies, I find that agents discard past information pretty fast. 75 per cent of the information that the people employ to generate their inflation expectations is contained in the 9.8 most recent data observations (equivalent to approximately 2.5 years). For the period of low inflation, this number increases to 17 (equivalent to approximately 4.75 years). These results are significantly different from what other studies, such as Orphanides and Williams (2005), Milani (2008) and Slobodyan and Wouters (2007), without using the information of surveys, have found: according to these papers, 75 per cent of the information used to generate inflation expectations is contained in the most recent 68 data observations (or 17 years) – a considerably longer horizon[4].

As I show, the structural analysis implemented using the SW model estimated under learning when the representative agent uses the best small forecasting model indicates that learning cannot always generate high persistence per se, as pointed by Milani (2007). This result is conditional on the type of structural shock that affects the economy and the period under consideration. “Supply side” shocks (such as price and wage mark-up shocks and productivity shocks) generate less persistence response of inflation under learning than under the RE during the low inflation period. At the same time, I find evidence that support the analysis of Orphanides and Williams (2005) about the existence of *inflation scares*: under learning, inflation (and inflation expectations) react strongly and more persistently to non-expected shocks than under RE. These results are found for both the high and low inflation periods, although in the latter just for the “demand side” shocks. Finally, I find evidence of a structural break in the rule followed by the central bank: in the period of low inflation the response of the interest rate to expectations becomes significant. However, I uncover little support for the hypothesis that this change caused the reduction in inflation that occurred in the US during the 80s.

To the best of my knowledge, this study, together with parallel work by Marco del Negro and Stephano Eusepi (2009), is the first to use surveys formally in a DSGE estimation. My study shares some features with that of Del Negro and Eusepi such as the use of the SW framework and the focus on inflation expectations. However, the type of imperfect information they consider relates to the knowledge of time-varying policy-makers’ inflation targets as in Erceg and Levin (2003). In my case, the representative agent does not know the actual law of motion of inflation and I use survey expectations to determine the most likely model agents used to form their expectations.

Two of the closest references to my work are the studies of Slobodyan and Wouters (2007 and 2008). These are among the first applications of learning to medium-size DSGE models, although they do not use information about expectations. They shows that the gains of using this feature is very limited, in terms of general fit with the data or parameter estimates differences with respect to the RE case, when the forecasting model employed by the economic agent is close to the one implied by the RE. However, the gains are more significant

[4] The number of most recent observations used for the generation of expectations is originally expressed as the geometrically rate at which agents discard pass information or the “gain” parameter in the context of (adaptive) learning.

when small forecasting models are considered[5]. Another close reference is the study of Orphanides and Williams (2005). These authors use information from surveys to calibrate the speed at which agents discard information and implement counterfactual experiments in a reduced-form New Keynesian model in order to analyze what was the driving factor behind the high-inflation period of the 70s.

There are a number of important issues that are not addressed in the current version of this study. One of them is the way in which expectations are aggregated. The SW model is a model with a representative agent, while surveys report the expectations across a pool of forecasters. I assume that the representative agent's expectations are reflected by the median value across the pool of forecasters at each point in time. With respect to the equilibrium resulting from the use of misspecified models to forecast inflation, this is in line with the *restricted perceptions equilibrium* (RPE)[6]. The jointly determination of expectations and the dynamics of the model as well as stability conditions are considered at the time of solving and estimating the model. At the same time, optimality (in the mean-squared-error sense) of the forecast model's parameterization is given by the use of ordinary least squares. However, this feature is conditional on the selection of the forecasting model[7]. Finally, the robustness of the results presented in this document should ultimately be tested by using real-time data instead of revised data.

The remainder of the paper is organized in the following way. The next section describes the methodology used to incorporate survey expectations in the estimation of a DSGE model. Section 3 gives a brief of the SW model and introduces the data and priors assumed for the implementation of the Bayesian estimation. Section 4 presents the results of the estimations of the SW model under RE and learning, their compatibility with the data on surveys and other macroeconomic indicators and some structural analysis. Finally, Section 5 concludes.

[5] Other studies that estimate a DSGE models under learning are Milani (2007, 2008) and Jaaskela and McKibbin (2009)

[6] The name of Restricted Perceptions Equilibrium (RPE) was given by Evans and Honkapohja (2001). Branch (2004) discusses the generality of RPE as it encompasses my forms of misspecified equilibriums such as the Self-Confirming Equilibrium in Sargent (1999) and the Consistent Expectations Equilibrium in Hommes and Sorger (1998)

[7] See Adam (2005) for an illustration of how to incorporate microfoundations of the selection of competing forecasting models and to define a RPE.

2. Methodology

In this study I solve and estimate the SW model under RE and learning. In both cases the estimations are implemented by Bayesian techniques using and not using the information from survey expectations. The aim of this section is to clarify issues related to the estimation of learning, given that RE solution and estimation is very well-known. Additionally, I define how I select the forecasting models for inflation and other variables.

A. General learning setup

I follow the approach of adaptive learning developed by Evans and Honkapojha (2001). In order to translate this procedure in a matrix notational way, I consider the following general representation of a DSGE model:

$$A_0 E_t Y_{t+1} + A_1 Y_t + A_2 Y_{t-1} + B e_t = 0 \tag{1}$$

$$e_t = C e_{t-1} + D \varepsilon_t \tag{2}$$

Y and *e* are vectors that contain all the variables and structural shocks, respectively, that are included in the model. Matrices A_0, A_1, A_2 and *B* represent the way how these variables are interrelated. Given that not all the variables of the model appear in lags or leads, the previous matrices also contain zero elements. The shocks contained in the vector *e* are allowed to follows an AR(1) process where ε represents a vector of i.i.d. innovations.

As a referential point for the rest of the exposition, the solution of the previous model under RE is represented in the following way:

$$Y_t = PP^{re} Y^s_{t-1} + QQ^{re} e_t \tag{3}$$

where Y^s is a subsample of *Y* that contains all the state variables.

Under learning the agent of the model does not possess perfect knowledge about the structure of the economy (equations 1 and 2). Therefore, in the same fashion as applied economist or econometricians, she takes advantage of new data/information and attempts to improve her knowledge about the economy by reformulating and re-estimating her model, also called perceived law of motion (PLM).

The PLM can be represented in the following way:

$$Y^f_t = \beta'[G_1 Y_{t-1} + G_2 e_t] \equiv \beta' Z_{t-1} \tag{4}$$

Y_t^f is a subset of Y_t that contains those variables that appear with a lead in the model. Matrices G_1 and G_2 are filled with zeros and ones and indicate which of the variables of the model and the structural shocks form part of the PLM. If the PLM includes the same set of variables that characterize the solution under RE, then $G_1 Y_{t-1} \equiv Y_{t-1}^s$ and $G_2 = I$ (where *I* represent the identity matrix). β, usually referred as *beliefs*, is a matrix of reduced form coefficients and does not necessarily coincide with the matrices PP^{re} and QQ^{re}.

In adaptive learning it is commonly assumed that agents use least squares with finite memory to determine the value of β. More precisely, agents implement weighted least squares where the weight decline geometrically with the distance in time between the observation being weighted and the most recent observation. This procedure is also called *constant gain least squares*, where the "gain" is relative weight of the most recent observation (in ordinary least squares, the gain is equal to *1/t*, where *t* is the position of the observation since the beginning of the sample). Theoretically, the use of CG-LS implies that the agent is concerned for changes in the structural parameters of the economy. However, in this type of models, the introduction of the actual uncertainty in the structure of the economy is abstracted.

The recursive expression of the estimation of β under CG-LS is:

$$\hat{\beta}_t = \hat{\beta}_{t-1} + gR_t^{-1} Z_{t-1}(Y_t^f - \hat{\beta}_{t-1}{}' Z_{t-1})' \quad (5)$$

$$R_t = R_{t-1} + g(Z_{t-1} Z_{t-1}^{'} - R_{t-1}) \quad (6)$$

g represent the *constant gain parameter* and R_t is the variance-covariance matrix of the regressors included in the PLM.

With the estimates of β it is possible to generate the forecast (or expectations) for the variables Y_t^f:

$$\hat{E}_t Y_{t+1}^f = \hat{\beta}_{t-1}{}'[G_1 Y_t + G_2 C e_t] \quad (7)$$

$\hat{\beta}_{t-1}$ is employed instead of $\hat{\beta}_t$ in order to avoid an endogeneity problem in the estimation of the complete model[8].

After replacing expression (7) into equation (1) and rearranging the terms we get the following expression:

[8]This procedure is standard in the learning literature. See for instance Carceles-Poveda and Giannitsarou (2007).

$$Y_t = PP_{t-1}^l Y_{t-1}^s + QQ_{t-1}^l e_t \qquad (8)$$

This expression represents the actual low of motion (ALM) of this economy. Notice that the matrices that describes the dynamics of the economy (PP_{t-1}^l and QQ_{t-1}^l) are time-varying. The time-variability is originated by the inclusion of the beliefs ($\hat{\beta}_{t-1}$).

The equilibrium in this economy is summarized by equations (2), (5), (6) and (8) for given initial conditions (β_0 and R_0). In order to estimate this model, we relate the variables of the models with series of actual data in the denominated measurement equation. It can be represented in the following simplistic way[9]:

$$W_t = \Phi Y_t \qquad (9)$$

When information from survey expectations are added in the estimation of the learning model, equation (7) is included in the set of relationships to evaluate. The set of observables now includes the series of surveys ($W_{t,t+1}^{survey}$) and the measurement equation changes to:

$$\begin{bmatrix} W_t \\ W_{t,t+1}^{survey} \end{bmatrix} = \Psi \begin{bmatrix} Y_t \\ \hat{E}_t Y_{t+1}^f \end{bmatrix} + \begin{bmatrix} 0 \\ \varsigma_t \end{bmatrix} \qquad (10)$$

ς_t represents measurement errors (1 per each series of surveys employed in the estimation). In this way, I am considering that survey expectations are a noisy measurements of actual expectations.

B. Selection of the PLMs and initial beliefs for “non-inflation” variables

The selection of the PLM and initial beliefs differs for inflation and for the rest of the variables. For the “non-inflation” variables, the set of regressors incorporated in the PLM is the same as the one resulting of the RE case (Y_{t-1}^s and e_t). The initial beliefs, β_0, correspond to the solution of the model under RE, while R_0 can be obtained from the expression of the unconditional variance matrix of Y_t obtained from the solution of RE (equation 3). As it is pointed by Slobodyan and Wouters (2007), the only source of differences in the dynamics of this set of variables (“non-inflation” variables) with respect to RE is related to the temporary deviations of beliefs from their RE values caused by in-sample data fluctuations and the related stochasticity of the constant gain. For instance, these deviations are zero when the gain is zero

[9] In SW model, the relationship between the set of observable (W_t) and the variables of the model (Y_t) is not only contemporaneous. For instance, the presence of data on real growth of consumption implies that not only the contemporaneous value of consumption in the model should appear in the measurement equation but also its lag. Additionally, constant terms (as in SW) and measurement errors can be added to this equation.

and therefore the initial beliefs remains constant during the whole estimation period (see equation 5).

Using PLM and initial beliefs compatible with the RE solution for the non-inflation variables makes more clear the gains (or losses) generated from different ways of modeling inflation expectations in the estimation of the DSGE model with respect to the RE estimation.

C. Selection of the PLM and initial belief for inflation

I use information from the surveys to determine the PLM of inflation that best describes the evolution of inflation expectations. The selection is implemented over a set of potential PLM specifications delimited by three criteria. First, I consider that the agent of the economy generates forecasts of inflation using the same information that the econometrician employs to estimate the model (W_t).[10] Second, the PLMs are linear and the regressors are only included with one lag as in the RE solution. Finally, I assume that the selected PLM remains unchanged during the estimation period. Although it is unrealistic to believe that agents use a single model over a long period of time to generate their expectations the aim of this paper is to try to keep some parallelism with the RE case. In order to deal with structural changes in the economy, which could have changed the way agents form their expectations, I also implement subsample estimations.

Considering these three criteria and the fact that the number of series used in the estimation is 7, the total number of combinations among all of them implies that the set of potential PLMs includes 127 specifications. Estimating the SW model for each of these specifications (for the complete sample and for each of the two subsamples) not only requires lot of time and computer resources but also it is not worth considering the many of them generate very badly approximations to the series of surveys. Therefore, a "partial equilibrium" exercise is implemented[11] and consists in estimating all the potential PLMs for inflation using contain-gain recursive least squares. Given that the gain parameter is estimated jointly with the rest of structural parameters of the DSGE model, for this exercise I use a discrete grid of values for this parameter (which goes from 0.001 to 0.25). At each point of the recursive estimation one-period-ahead forecast is generated and compared with the corresponding value of the survey. The Mean Square Errors (MSE) for each series of forecast errors is calculated and it is used to rank the models.

[10] Under RE, the agent includes as well "non-observable" data (such as e_t in equation 3).

[11] It is a "partial equilibrium" exercise because it does not consider the causality relationship from expectations to the set of variables used as regressors in the PLM. Additionally, in each of the estimations the constant gain parameter is taken as exogenous.

This procedure is implemented for different initial values of the beliefs which are generated by standard least squares regression using presample data. At the end, the ranking that presents the lowest MSE among its top 20 models is selected.

3. Model, data and priors

I use the same New Keynesian model presented in Smets and Wouters (2007) but with minor changes. First, as in Slobodyan and Wouters (2008), I do not include the modeling of the flexible economy. Adding this feature increases considerable the number of forward variables appearing in the model. As a result, the monetary policy does not react to the natural output level but to changes in the output. Second, price mark-up and wage mark-up shocks are not modeled as ARMA(1,1) processes as originally appear in SW but as AR(1). However, any of these two modifications prevent me to replicate their results.

In this section, I do not rewrite all the equations that define this model but recommend the reader to directly consult this reference. As a summary, this model incorporates external habit in consumption, capital adjustment cost, monopolistic competition on good and labor markets, Calvo stickiness and indexation for prices and wages. The model determines the evolution of thirteen endogenous variables (consumption, investment, wages, inflation, capital stock value, output, interest rate, capital services uses in production, hours worked, rental rate on capital, capital installed, marginal cost and the degree of capital utilization) and its stochastic behavior is driven by seven exogenous AR(1) processes: total factor productivity, investment-specific technology, risk premium, exogenous government spending, price mark-up and wage mark-up and monetary policy shocks.

The structure of the model is defined by 38 parameters. Table 1 shows prior distributions over the 33 parameters that are estimated. These distributions are exactly the same as the ones used originally by Smets and Wouters (2008).

Learning estimation implies the incorporation of two more parameters: the gain parameters of the PLMs of inflation and of the rest of variables. For these two parameters I use uniform distributions for the range from 0 to 0.30. One more parameter appears, the standard deviation of the measurement error related to the surveys, when this information is employed in the DSGE model estimation. Its prior distribution is the same as the one for the disturbance of the structural shocks. Finally, the parameter of the response of the interest rate to inflation expectations has a uniform prior distribution for the range from 0 to 3.

Table 1: Prior distributions of structural parameters

	Distribution	Mean	Std.
Share of capital in production	Normal	0.30	0.05
Inv. Elasticity of Intertemporal substitution	Normal	1.50	0.38
Fix cost in production	Normal	1.25	0.13
Adjust cost of investment	Normal	4.00	1.50
Habits in consumption	Beta	0.70	0.10
Wage stickiness	Beta	0.50	0.10
Elast. labor supply	Normal	2.00	0.75
Price stickiness	Beta	0.50	0.10
Wage indexation	Beta	0.50	0.15
Price indexation	Beta	0.50	0.15
Capital utilization elasticity	Beta	0.50	0.15
Taylor rule: response to inflation	Normal	1.50	0.25
Taylor rule: response to lagged interest rate	Beta	0.75	0.10
Taylor rule: response to changes in output	Normal	0.13	0.05
Trend growth rate	Normal	0.40	0.10
Steady state of inflation	Gamma	0.63	0.10
Steady state of hours worked	Normal	0.00	2.00
Steady state of nominal int rate	Gamma	1.15	0.30
Autocorrelation coef. Price Mk up shock	Beta	0.50	0.20
Autocorrelation coef. Wage Mk up shock	Beta	0.50	0.20
Autocorrelation coef. Product. Shock	Beta	0.50	0.20
Autocorrelation coef. Risk premium shock	Beta	0.50	0.20
Autocorrelation coef. Government shock	Beta	0.50	0.20
Autocorrelation coef. Investment-Specific shock	Beta	0.50	0.20
Autocorrelation coef. Monet policy shock	Beta	0.50	0.20
Correlation Government and productivity shocks	Normal	0.50	0.25
Std Price Mk up innovation	Inv. Gamma	0.10	2.00
Std. Wage Mk up innovation	Inv. Gamma	0.10	2.00
Std. Product. Innovation	Inv. Gamma	0.10	2.00
Std. Risk premium innovation	Inv. Gamma	0.10	2.00
Std. Government innovation	Inv. Gamma	0.10	2.00
Std. Inv. Specific innovation	Inv. Gamma	0.10	2.00
Std. Monet policy innovation	Inv. Gamma	0.10	2.00
Gain - no inflation	Uniform	0.00	0.30
Gain - inflation	Uniform	0.00	0.30
Std. measurement error on expectations	Inv. Gamma	0.10	2.00
Taylor rule: response to inflation expectations	Uniform	0.00	3.00

Note: for uniform distributions the values assigned as mean and standard deviations correspond to the range of the domain.

The model is estimated using the same number of quarterly macroeconomic indicators for the US as in SW. They are the log difference of real GDP (*dlGDP*), real consumption (*dlCONS*), real investment (*dlINV*) and the real wage (*dlWAG*), log hours worked (*lHOURS*), the log difference of the GDP deflator (*dlP*) and the federal funds rate (FEDFUNDS). Additionally, I use the data on survey of expected inflation (*exp_dlP*) collected by the Survey of Professional Forecasters (SFP). Per each point in time, I took the median of the one-period-ahead forecast of percentage increase of the GDP deflator across what the pool of forecasters have presented.

The measurement equation is:

$$\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUND_t \\ \exp dlP_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \\ \pi^e_{t/t+1} \end{bmatrix}$$

where $\bar{\gamma}$, $\bar{l}$, $\bar{\pi}$ and $\bar{r}$ represent the trend growth rate and the steady state values of the hours worked (l_t), the inflation (π_t) and the interest rate (r_t), respectively. y_t, c_t, i_t, w_t and $\pi^e_{t/t+1}$ represent output, consumption, investment, wages and one-period-ahead expectation of inflation.

In this study, the full sample period starts in 1968Q4 and ends in 2008Q2. The starting period of the sample is defined by the availability of the data on survey expectations. Additionally, subsample estimations are implemented for the periods 1968Q4-1983Q4 (or *high inflation* period) and 1989Q1-2008Q2 (or *low inflation* period). The beginning of the low inflation period is delayed under almost the end of the 80s to make possible to consider presamples that lay completely in the years where the inflation was under control.

The estimations are executed using Bayesian estimation methods. I use the random walk Metropolis-Hastings algorithm to obtain 200 000 draws from each model's posterior distribution. I use the draws to estimate the moments of the posterior distributions.

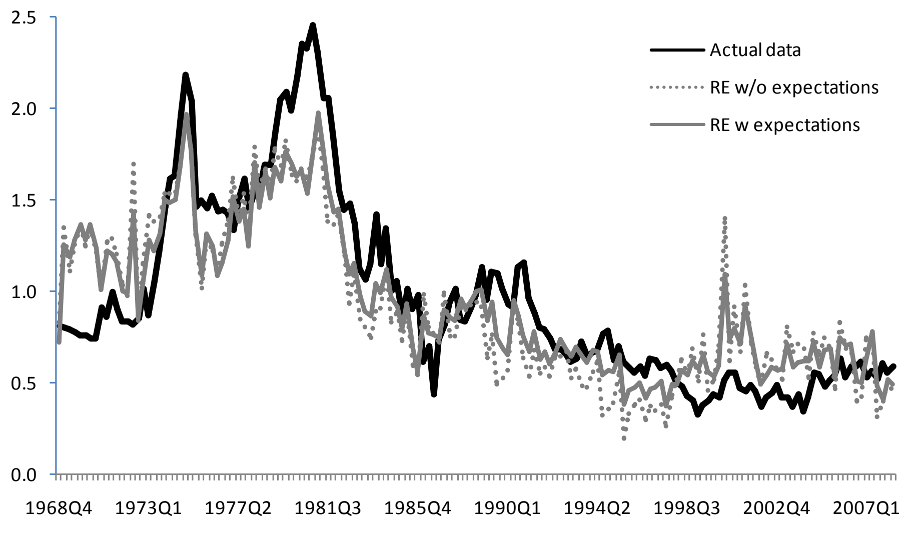
4. Results

The presentation of the results is divided in four subsections. At the beginning I evaluate the performance of the SW model under RE in order to describe the evolution of inflation expectation and its relationships with other variables such as inflation and the interest rate. I implement this evaluation under two different circumstances, including the data on surveys in the information set when estimating the model and then not including survey data. The results remain unchanged when a version of learning, one with RE-compatible PLMs and initial beliefs, is considered. I do not report these results, but they are compatible with the findings of Slobodyan and Wouters (2007). In the second subsection I employ survey expectations which allow me to determine which specifications of PLMs and initial beliefs provide series of one-period-ahead inflation forecast similar to the surveys. This analysis is implemented in a "partial equilibrium" setup. In the third subsection, I estimate the SW model using the best PLMs and initial beliefs for inflation. For this purpose, I use different measurements to determine the relative performance of this estimation. The parameter estimates, the IRFs analysis and a discussion about the existence of a structural change in the monetary policy rule are left for the last subsection.

4.1 Is the SW model under RE compatible with survey expectations on inflation?

Figure 1 shows the evolution of inflation expectations generated by the model SW under RE when the estimation is implemented with ("w") and without ("w/o") the data on inflation expectations. The series of inflation expectations implicit in the estimation underestimates the surveys not only during the period of high inflation but also during almost all the 80s and 90s. It also highly overestimates actual expectations at the very beginning of the sample and during most of the 2000s.

Figure 1: Evolution of inflation expectations



The poor performance in replicating the evolution of inflation expectations is also reflected in the cross correlation of this series with inflation and the interest rate. Figure 2 shows these cross correlations not only for the complete sample but also for the subsamples 1968Q4-

1983Q4 (*low inflation* period) and 1989Q1-2008Q2 (*high inflation* period). In general, simulated cross correlations perform significantly poorly. For instance, the model predicts a high contemporaneous correlation between inflation expectations and inflation and sharp reductions in the correlation between inflation expectations and lagged and forward inflation. This pattern, however, does not coincide with the actual data: for the complete sample the non contemporaneous correlations do not descent so sharply, while expected inflation and past inflation exhibit a higher level of correlation for the period of low inflation and a lower level when the correlation is between expected inflation and forward inflation. With respect to the cross correlations between expected inflation and the interest rates, the simulated data underestimate the correlation between expected inflation and lagged interest rate.

Do the fit of the SW model under RE with respect to inflation expectations improve once surveys are used for estimating the model? The answer is negative. Although some of the cross correlations improve, the good fit of the simulated cross correlation between output and inflation is lost (see figures in Appendix 1).

Even worse, some parameter estimate values resulting from adding information from survey expectations are at odds with the findings of other studies. For instance, the median values of the posterior distributions of the long-run reaction on inflation in the Taylor rule are very close to 1 for both subsamples (high and low inflation periods). For the subsample of low inflation the median of the posterior distributions of the Calvo probability in prices and wages are high (0.904 and 0.844, respectively). For the complete sample estimation (1968Q4 – 2008Q2) the most striking anomaly is the implausible value of 0.922 of the posterior median of the Calvo probability for wages, which implies an average length of wage contract of 13 quarters. The posterior distribution statistics (median and standard deviation) for the complete set of parameters are reported in Appendix 2.

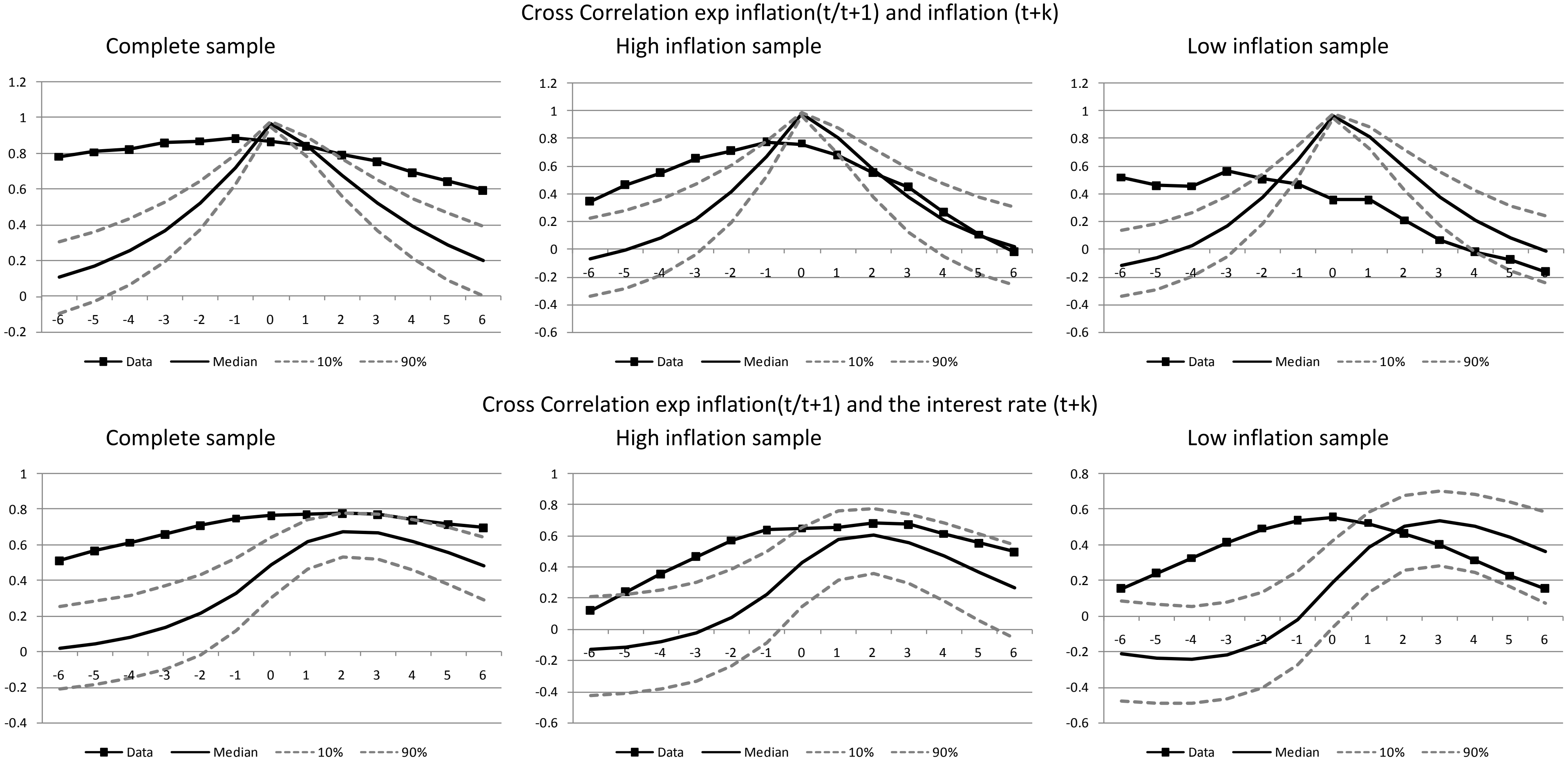
Finally, the performance of the out-of-sample forecast for inflation deteriorates severely for horizons of more than two quarters ahead. Forecast of consumption also deteriorates but less sharply. Mixed results are obtained for investment, wages, output and interest rate although there are clear improvements in the very short horizons. The clearest gains are obtained in the forecast of wages.

Table 2: Out-of-sample prediction performance

	dlCons	dlInv	dlWage	dlP	dlGDP	FedFunds	lHours
RE "without" Surveys							
RMSE-statistic for different forecast horizons							
1q	0.35	1.08	0.57	0.24	0.58	0.15	0.37
2q	0.42	1.32	0.66	0.25	0.49	0.30	0.56
4q	0.51	1.71	0.64	0.19	0.50	0.46	1.06
8q	0.59	1.63	0.54	0.29	0.48	0.44	1.62
12q	0.68	1.73	0.50	0.34	0.55	0.43	1.82
RE "with" Surveys							
Percentage gains(+) or losses(-) relative to RE "without" Surveys							
1q	2.35	9.85	17.59	11.83	5.81	11.97	12.10
2q	-0.53	9.08	11.17	18.46	5.82	13.59	5.39
4q	-2.20	2.51	-6.40	-42.21	0.51	11.45	16.42
8q	-8.99	-1.64	-4.82	-30.50	-11.01	-2.51	21.60
12q	-3.52	-4.44	1.21	-15.92	-12.42	-14.54	21.74

Note: For both cases, the estimations start in 1968Q4. The forecast period is 1993Q3 to 2008:Q2. Each year the models are reestimated.

Figure 2: Cross correlation between expected inflation and inflation and interest rate



Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.

4.2 Survey expectations and the determination of the PLM and initial beliefs for inflation

As explained in the methodological section, a simple partial equilibrium exercise is implemented in order to determine the specification for PLMs and initial beliefs that best capture the evolution of the expectations. Table 3 shows which variables have to be included in a regression against inflation to generate one-period-ahead forecast series that are close to the series of survey of expectations. The ranking indicator is the mean square error (MSE) and is calculated for both, the complete period as well as the subsamples of high and low inflation (including the information of the presample used to generate the initial beliefs).

Table 3: Ranking of PLM's specifications by minimum MSE

Complete sample Period: 1968Q4 - 2008Q2 Presample: 1950Q1 - 1968Q3				"High inflation" sample Period: 1968Q4 - 1983Q4 Presample: 1950Q1 - 1968Q3				"Low inflation" sample Period: 1989Q1 - 2008Q2 Presample: 1984Q1 - 1988Q4			
Rank	Model	Gain	MSE	Rank	Model	Gain	MSE	Rank	Model	Gain	MSE
1	PIE	0.125	0.0294	1	PIE	0.125	0.0330	1	L R	0.075	0.0148
2	PIE L	0.125	0.0300	2	PIE C	0.125	0.0333	2	L	0.100	0.0159
3	PIE C	0.125	0.0302	3	PIE Y	0.125	0.0343	3	PIE L R	0.075	0.0160
4	PIE C L	0.125	0.0303	4	PIE I	0.125	0.0355	4	R	0.088	0.0162
5	PIE Y	0.125	0.0315	5	PIE C L	0.113	0.0386	5	C L R	0.075	0.0164

Note: MSE with respect to one-period-ahead survey of inflation (SPF). Data is in % and nonannualized. PIE = inflation; L = labor; C = consumption; Y = output; I = investment; R = interest rate.

Table 3 shows that for the complete sample and for the period of high inflation, the most relevant variable explaining inflation expectations is its own lag. Inflation is the variable that appears the most among the best five PLMs. Of course, this is not surprising given that under high inflation its inertia is also high. Hence, today’s inflation is a good predictor for future inflation and the agent takes this into consideration when generating her expectations. For the low inflation period, as it is the case in the second subsample, the most important variables when explaining survey expectations are the interest rate and the number of hours worked. The presence of the interest rate among the most relevant determinants of survey expectations is compatible with a monetary policy that is clearly defined against high inflation (remember that in this study the interest rate represents the data on the Federal Fund interest rate, the most important indicator of the monetary policy stance). Labor (number of hours worked) is an indicator of real activity which is released with higher frequency than other indicators such as GDP or consumption, and it is less affected by posterior revisions.

Notice, models with relatively few variables match better the evolution of expectations on inflation. Even for the subsample estimations, the mean of the number of regressors included in the PLM is 3.3 for the top 20 models in the period of high inflation and 2.6 for the period of low inflation, in comparison with the 15 regressors implied by the RE solution (Appendix 3 contains the ranking for the best 20 PLM specifications for every sample estimates).

Taking together, these estimates indicate that the rate at which people discard past information to generate their expectations is higher than what it has been found in studies with learning models without using the information from surveys. For instance, Orphanides and Williams (2005) consider a baseline value of the gain parameter of 0.02 and Milani (2008) and Slobodyan and Wouters (2007) find posterior mean estimates that range between 0.01

and 0.025, and between 0.002 and 0.018, respectively. A value of 0.02 of the gain coefficient equal to 0.02 implies that the 75 per cent of the information that the agent uses to generate her expectation (on inflation) is contained in the most recent 68.6 (quarterly) observations (17.2 years). Gain parameters obtained in the partial equilibrium exercise (which do not differ significantly from the estimates values obtained when the complete model is estimated) lie at 0.125 for the period of low inflation and between 0.075 and 0.10 for the period of low inflation (see Table 3). Gain values of 0.132 and 0.078, posterior distribution medians obtained from DGSE model estimation using the best specifications of PLM for the period of high and low inflation, respectively, imply that 75 per cent of the information used to generate the inflation expectation is contained in the most recent 10.4 and 17.8 observations (2.6 and 4.4 years), respectively (see Table 4).

Table 4: Number of most recent observations incorporated in the formulation of inflation expectations

	Learning posterior medians		Learning
	High inflation	Low inflation	previous studies
Accum. Weight	gain = 0.132	gain = 0.078	g = 0.02
0.95	21.2	36.9	148.3
0.75	9.8	17.1	68.6
0.50	4.9	8.5	34.3

In the following subsection I implement the estimation of the complete model using the best PLM for each of the samples. In this estimation, the gain parameter is estimated jointly with the rest of the structural parameters of the economy.

4.3 DSGE estimations: gains (and losses) of using survey expectations

Table 5 shows the log marginal likelihood of the SW model solved under RE and learning. For the complete sample and for the high inflation sample, the PLM for inflation only includes the lagged inflation while for the low inflation sample the PLM includes the (lagged) interest rate and the hours worked. The estimation under RE and Learning is implemented with (“w”) and without (“w/o”) the inclusion of the survey of expected inflation[12].

Table 5: Model comparison (Log Marginal likelihood)

Sample	Complete sample		"High inflation"		"Low inflation"	
	w/o	w	w/o	w	w/o	w
RE	-146.4	-10.6	-136.0	-103.2	73.8	184.2
Learning	-144.2	45.7	-133.6	-85.4	86.6	204.9

Note: "w" = surveys are used in the estimation; "w/o" = surveys are not used in the estimation.

When the information from survey expectations is not included in the estimation, the log marginal likelihood is similar between RE and Learning for the complete sample and the high inflation sample. Nevertheless this is not the case for the low inflation sample: learning

[12] The information set used to calculate the log marginal likelihood for RE and Learning is not exactly the same. Although the set of observable variables is the same in both estimations, learning includes extra information in the form of the initial beliefs for the PLM of inflation. However, a sensitivity analysis with respect to the initial beliefs of the selected PLM indicates that the fit of the model does not change importantly.

estimation reports a 12.8 points higher log marginal likelihood. This important improvement on the fit of the model results just from the selection of the PLM and initial for inflation. Information about fit per variable contained in Appendix 4 shows that the improvements come from a better fit of investment growth, inflation, hours worked and interest rate.

Table 5 shows moreover the log marginal likelihood when survey expectations on inflation are incorporated in the information set to estimate the SW model (columns label with "w"). Under this case, learning estimates outperform RE in all samples. The improvement of the log likelihood is of 56.3, 17.8 and 20.7 points for the complete sample, the high inflation and the low inflation samples, respectively. Appendix 4 contains information about the "winners" and "losers" of adopting learning with respect to the RE case. For the high inflation subsample, inflation, output and wages growths show higher root mean squared errors (RMSEs) under learning while for the rest of the variables the fit improves. The improvements are more general for the low inflation sample estimate. For this case, the fit of all the variables increase except for consumption growth.

The gains in terms of cross correlations between inflation expectations and other variables such as the interest rate, the inflation and the hours worked are presented in Appendix 5. Most of the gains are concentrated in the low inflation sample while in the high inflation sample the results barely alter except for the good match of the cross correlation between inflation expectations and hours worked (remember that hours worked is not incorporated as a regressor in the PLM for this sample).

More importantly, improvements on the fit of inflation expectations under learning are not accompanied by deteriorations in the dynamics of other variables. Appendix 6 shows the cross correlations of inflation with respect to the output and the interest rate, for the RE case (with and without expectations) and the learning estimation (with expectations). Looking at the subsample analyses it becomes clear that the results from the learning estimations are generally in line with the results by the RE case estimated without using survey expectation (except for the case of the cross correlation between inflation and the interest rate in the low inflation sample where learning performs better), but are significantly better when comparing them with the results yielded by the RE case estimated using survey expectations.

Finally, I implement an out-of-sample forecasting exercise such as the one implemented by Smets and Wouters (2007). Table 6 reports the out-of-sample root mean squared errors (RMSEs) for different forecast horizons over the period 1993Q3 to 2008:2. For this purpose I first estimate the SW model under RE and under learning (which includes only the lagged inflation in the PLM of inflation) over the sample 1968Q4 – 1993Q2. Then I employ these models to forecast the series used in the estimation from 1993Q3 to 2008:2[13], reestimating them every year.

The out-of-sample statistics provide mixed results. The clearest gains are related with the forecast of hours worked while the clearest losses are found for the forecast of consumption growth. Most of the remaining gains are concentrated in the short horizons (except for the

[13] Expectations on inflation were not forecaster because this series was not include in the estimation under the RE case.

case of inflation). This result can be explained by the fact that the structure of the model under learning is time-varying and therefore more flexible. The longer the forecast horizon the more misspecified the model becomes.

Table 6: Out-of-sample prediction performance, a comparison wrt learning

	dlCons	dlInv	dlWage	dlP	dlGDP	FedFunds	lHours
RE "without" Surveys							
RMSE-statistic for different forecast horizons							
1q	0.35	1.08	0.57	0.24	0.58	0.15	0.37
2q	0.42	1.32	0.66	0.25	0.49	0.30	0.56
4q	0.51	1.71	0.64	0.19	0.50	0.46	1.06
8q	0.59	1.63	0.54	0.29	0.48	0.44	1.62
12q	0.68	1.73	0.50	0.34	0.55	0.43	1.82
Learning "with" Surveys*							
Percentage gains(+) or losses(-) relative to RE "without" Surveys							
1q	-7.15	11.03	1.57	0.68	8.48	14.85	22.89
2q	-4.83	12.33	-2.82	-6.42	12.52	9.55	27.97
4q	-5.08	7.19	-3.63	-9.10	2.75	-4.25	33.62
8q	-5.34	-3.28	-2.63	24.07	-9.04	-21.21	23.23
12q	-1.50	-2.72	0.88	30.49	-5.59	-12.26	11.39

Note: For both cases, the estimations start in 1968Q4. The forecast period is 1993Q3 to 2008:Q2. Each year the models are reestimated.
*The PLM of inflation includes only lagged inflation.

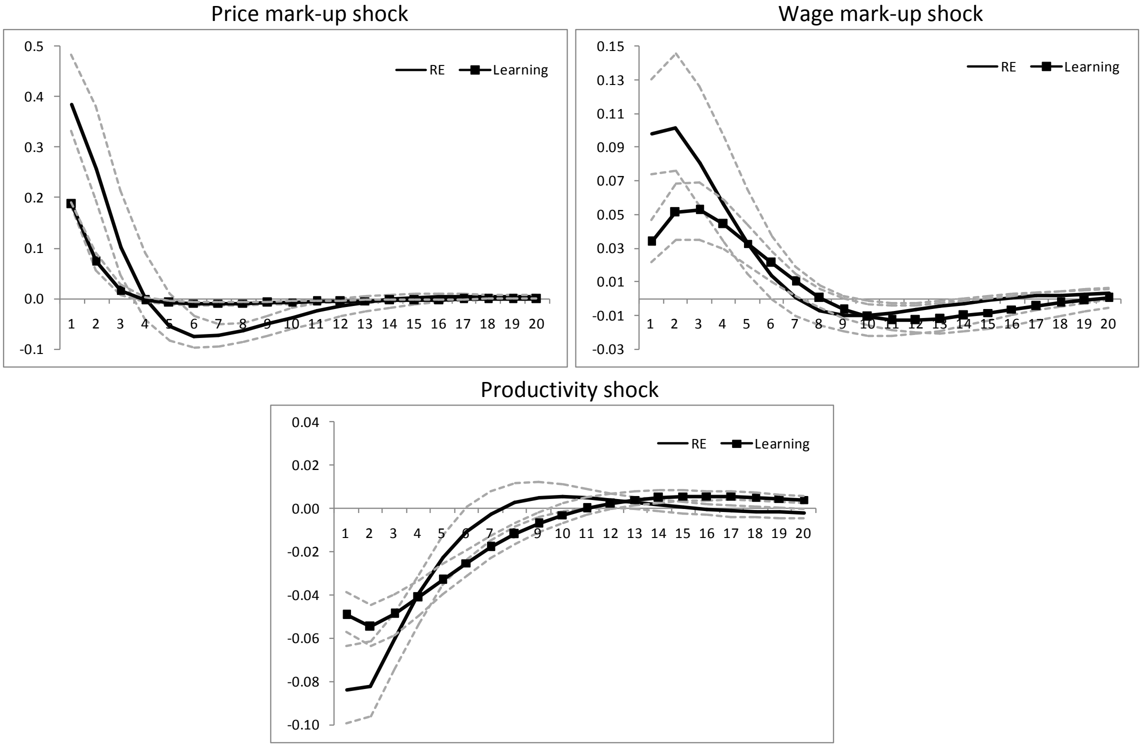
4.4 Structural comparison and a Taylor rule evaluation

In this section I compare the posterior parameter estimates for the RE case (without using the survey) and the learning case (using survey information) for the complete sample and high and low inflation samples[14].

Posterior distribution statistics (median and standard deviations) for all parameters are reported in Appendix 7[15]. The clearest difference between RE and learning estimations is the reduction in the persistence of the price mark-up shock in the latter one. For the complete sample the posterior median of the autoregressive coefficient of this structural shock is 0.518 (with a posterior standard deviation of 0.200) for the RE case while these statistics are 0.168 (0.089) for the learning case. For the subsample estimates we observe even higher reductions. Moreover, I find that the indexation of prices is higher (although statistically different only in the high inflation sample) and the stickiness of prices is lower (significantly different in the low inflation sample). These results coincide with Slobodyan and Wouters (2008).

[14] I show in subsection 5.1 that the posterior parameter estimates of RE case that uses information from surveys have implausible values. For this reason, this case is not including in the results presented in this subsection.
[15] Estimates of the trend growth rate and steady-state nominal interest rate, inflation and hours worked are difficult to be estimated correctly for the period of high inflation. For this reason these values are fixed to the values obtained from the actual data. The steady-state of hours worked in general shows trouble to be estimated but fixing it or not do not affect neither the posterior estimations nor the simulated data.

Figure 3: Responses of inflation to supply-side shocks for low inflation sample

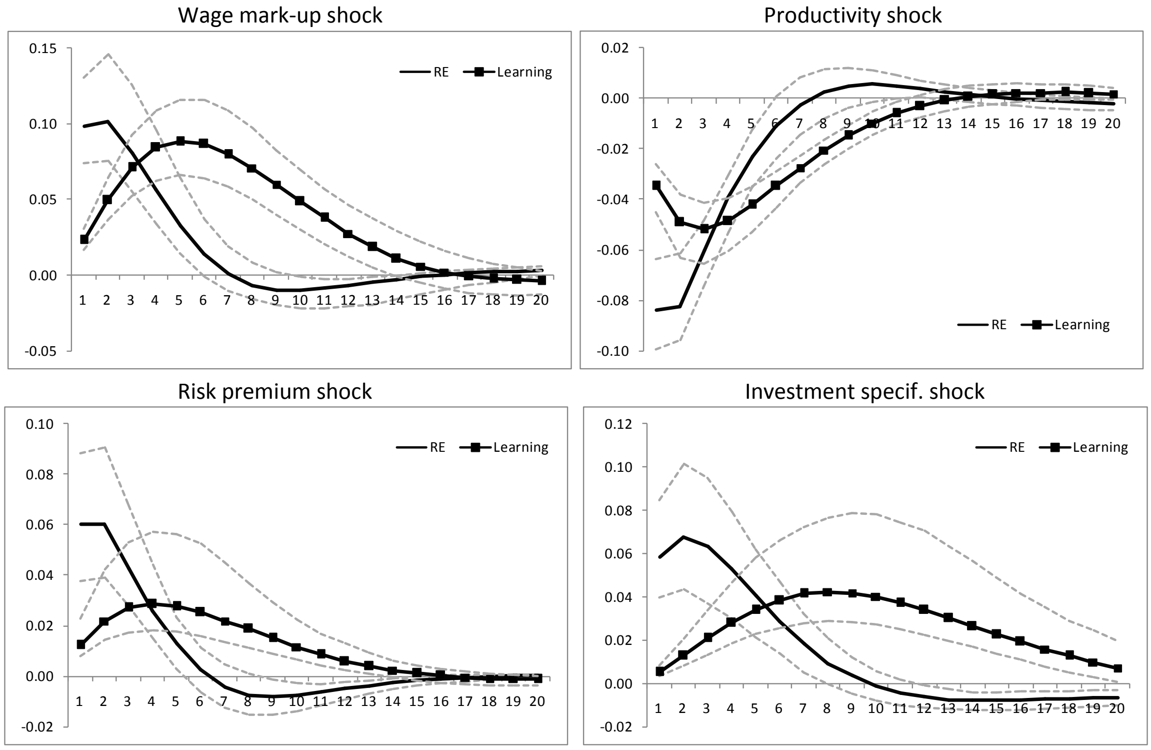


Note: 10 000 draws from the posterior distributions of the model parameters are used to generate IRFs and the median, 16 and 84 percentiles are derived. Under learning, this operation is repeated using the parameter values at each point in time.

Some studies, such as Milani (2007), interpret learning as a way to explain observed persistence in inflation without using features such as inflation indexation. According to the results of this study, this is not necessarily true. As Figure 3 shows, the response of inflation to "supply side" shocks under learning have a lower impact and are less persistent than under RE for the low inflation sample. This result is directly related with the choice of the PLM for inflation. Considering the interest rate and the hours worked as the only regressors in the PLM implies that the mark-up shock has a reduced role in the dynamic of expectations and therefore its impact to inflation via expectations is weaker under learning.

The results obtained from the IRFs can also be related with the episodes of *inflation scares*, defined as the "significant and persistent deviations of inflation expectations from those implied by rational expectations" (Orphanides and Williams 2005). Although I do not evaluate the impact of a permanent shock over inflation and expected inflation, the results of temporary shocks indicate that the persistence of both variables have importantly increased (see Figure 4). This happens for most of the shocks in the period of high inflation and just for the "demand side" shocks in the period of low inflation (by the reason previously exposed).

Figure 4: Responses of inflation to different structural shocks for high inflation sample



Note: 10 000 draws from the posterior distributions of the model parameters are used to generate IRFs and the median, 16 and 84 percentiles are derived. Under learning, this operation is repeated using the parameter values at each point in time.

As a final step, I estimate the SW model under learning for both high and low inflation samples but this time adding a response of the interest rate to inflation expectations. During the period of high inflation, inflation-expectation augmented Taylor rule estimates under RE and Learning point at very small improvements on the fit of the data. However, discrepancies arise for the period of low inflation. RE estimations indicates that adding this feature in the model does not result in a clear improvement on the fit of the model (the log marginal likelihood increases from 73.8 to 75.7). Under learning, however, the increase of the log marginal is more notorious (from 204.9 to 211.3). These discrepancies can be explained by the inability of the RE setup to detect the important reduction of correlation between inflation and expected inflation during this period (see Figure 2). Under RE both variables are "by construction" highly correlated and therefore, reacting to one or the other does not make a big different in the dynamic of the model. Under learning both variables are not necessarily highly correlated and therefore reacting to one or the other can make a significant difference in the stability of the model. Therefore, this setup allowed us to detect differences in the reaction of the interest rate to inflation and its expectations in the low inflation period.

The posterior statistics of the Taylor rule parameters in Table 6 show a significant change in the responses of the interest rate between the high and low inflation subsamples (see Table 7; Appendix 8 contains the posterior statistics of all the parameters of the model). In the period of high inflation, the interest rate reacts to inflation but not to expectations while in the period of low inflation, interest rate reacts more to expectations than to actual inflation. However, the importance of this change in the policy rule to explain the end of the Great Inflation and the change in the PLMs of inflation of the private agent is still inconclusive. Some preliminary counterfactual estimates (which are not reported here) show that the impact over inflation is limited.

Table 7: Posterior distribution statistics for
the inflation-forecast augmented Taylor rule case

	High inflation sample*		Low inflation sample**	
	Median	Std.	Median	Std.
Inflation	1,372	0,202	1,070	0,290
Lag interest rate	0,725	0,055	0,888	0,026
Change in output	0,155	0,048	0,153	0,048
Inflation expectations	0,346	0,259	2,321	0,514
Log mg. Likelihood	-84,908		211,288	

Note: "High inflation" sample represents the period 1968Q4 - 1983Q4 and "Low inflation" sample represents the period 1989Q1 - 2008Q2.
*Learning estimation considers a PLM for inflation that only includes lagged inflation rate.
**Learning estimation considers a PLM for inflation that includes lagged interest rates and hours worked.

5. Summary and concluding remarks

In this paper I use information of survey of inflation expectations to evaluate the performance of the benchmark New Keynesian model of Smets and Wouters (2007) in matching this data and to improve the way how inflation expectations have been modeled.

The results of my estimation and simulation reveal that the SW model under RE is not compatible with the information of survey expectations on inflation when this model is estimated using US macroeconomic indicator over the period 1968-2008. The propose alternative, which uses survey information to model how agents generates forecast of inflation, gives a good fit not only over expectations (and its correlation with other variables) but also keeps the good properties of the estimation of Smets and Wouters in terms of plausible parameter estimates, cross correlation between output and inflation and out-of-sample forecast.

The analysis of the series of survey expectations on inflation indicates that this data is better described by simple models that include inflation for the high inflation period (1968-1983) and the interest rate and hours worked for the low inflation period (1989-2008). These results evidence the existence of an important change in the way how agents generate their expectations about future inflation. Additionally, I find that agents discard past information pretty fast. 75 per cent of the information that people employ to generate their expectations on inflation is contained in the 9.8 and 17 most recent data observations during the period of high and low inflation, respectively (equivalent to 2.5 and 4.75 years approx). These results are significantly different to what other studies, without using the information of surveys, have found: the most recent 68 data observations (or 17 years) contain 75 per cent of the information used to generate expectations.

The structural analysis implemented using the SW model shows evidence in favor of the existence of “inflation scares”, stronger reaction under learning to unexpected shock than under RE, as pointed by Orphanides and Williams (2005). On the other hand, learning models do not generate *per se* more persistence as Milani (2007) concludes. This statement is conditional on the type of shock, being the “supply side” shocks those that generate less persistence under learning than under RE during the period of low inflation. Finally, even though my estimates indicate the existence of a break in the monetary policy rule between the periods of high and low inflation, my preliminary results imply little support for the hypothesis that this change caused the reduction in inflation that occurred in the US during the 80s. However, more work still has to be done in order to determine the usefulness of survey expectations on inflation to address this issue.

In general, the use of the data on survey expectations has been relatively low even thought the existence of complete databases and the promising good results obtained in previous studies. One of the goals of this study, therefore, is to show that this information is valuable and that can and must be used for macroeconomic analysis.

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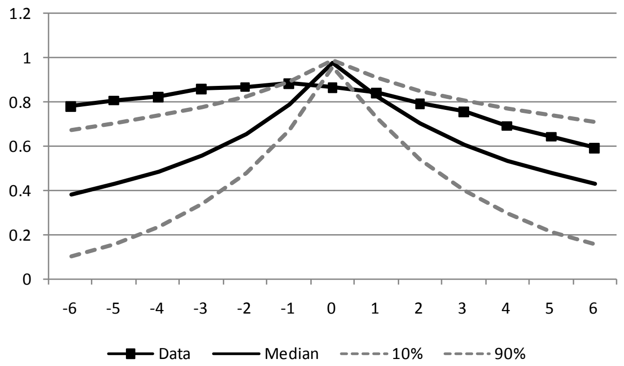
[TO BE COMPLETED]

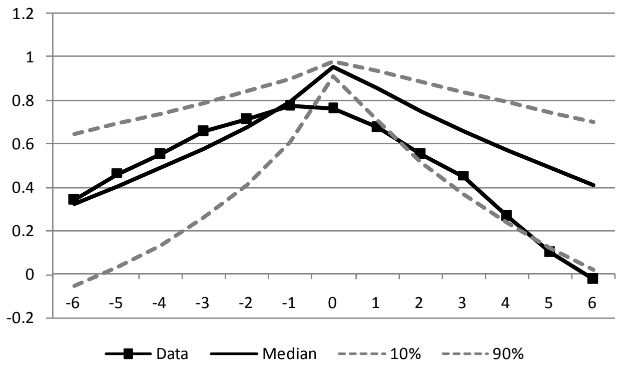
Appendix 1a: Cross correlation between expected inflation and inflation and interest rate (SW under RE using Surveys)

Cross Correlation exp inflation(t/t+1) and inflation (t+k)

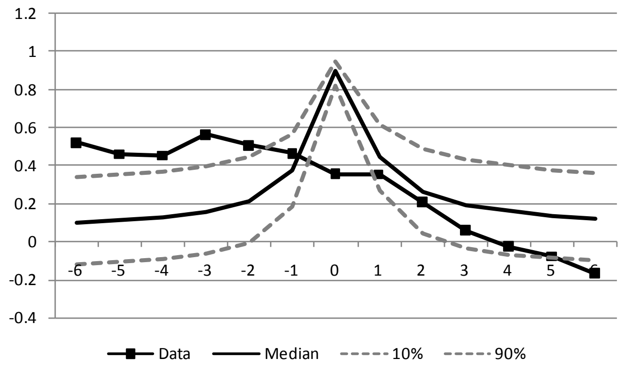
Complete sample



High inflation sample

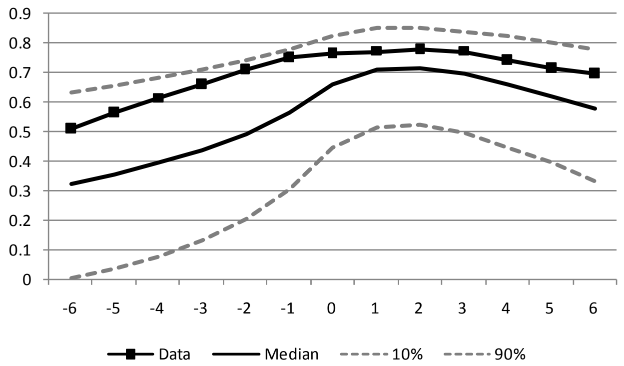


Low inflation sample

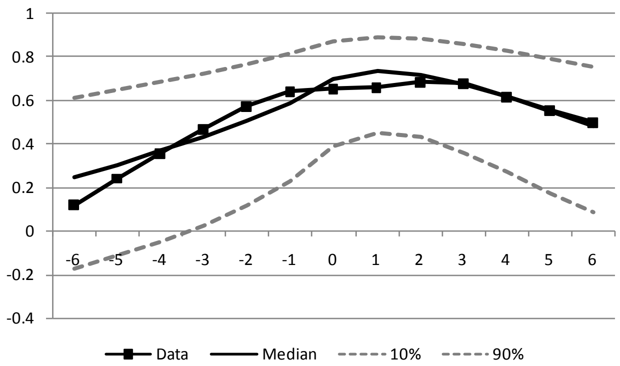


Cross Correlation exp inflation(t/t+1) and the interest rate (t+k)

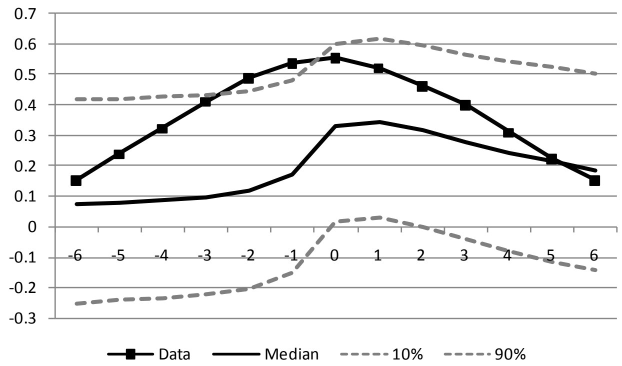
Complete sample



High inflation sample



Low inflation sample



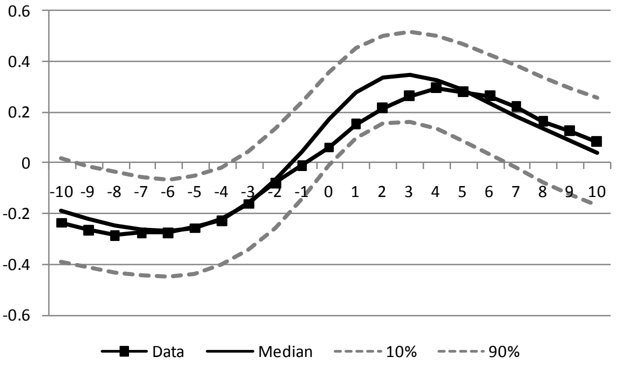
Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.

Appendix 1b: Cross correlation between output and inflation (SW under RE)

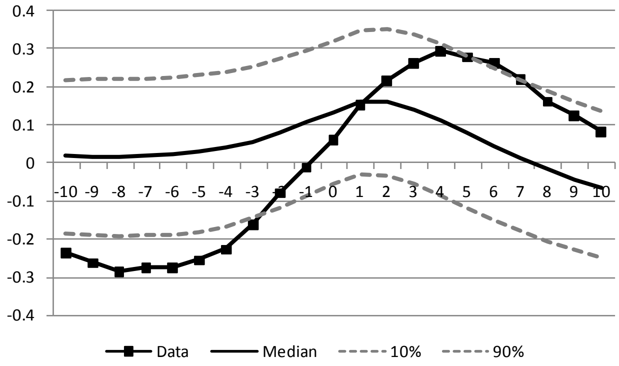
Cross Correlation output (t) and inflation (t+k)

Complete sample

“With out” surveys

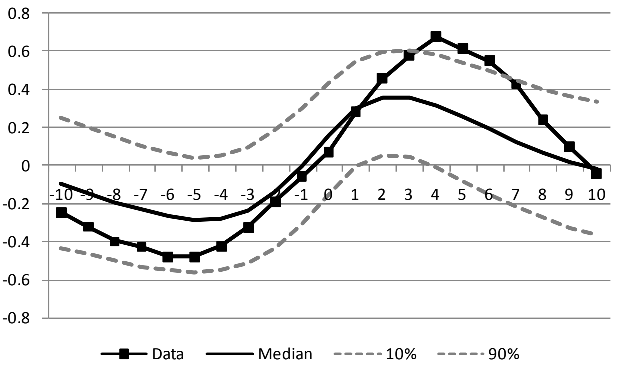


“with” surveys

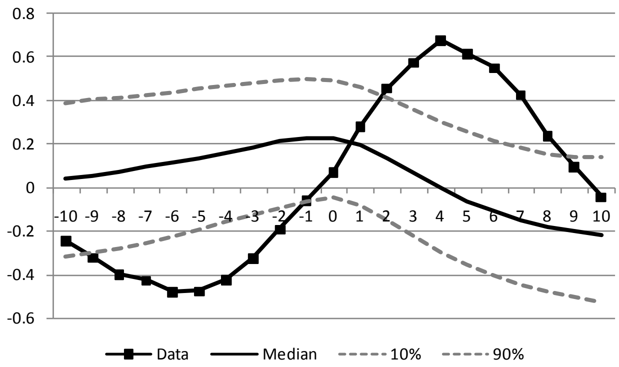


High inflation sample

“With out” surveys

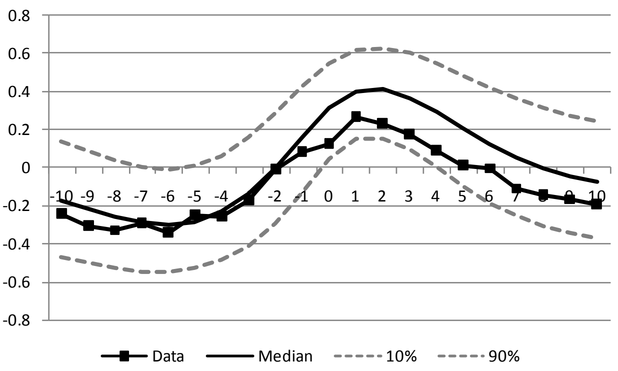


“with” surveys

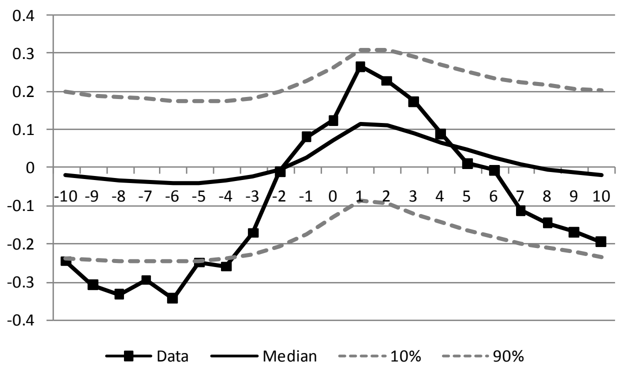


Low inflation sample

“With out” surveys



“with” surveys



Note: Simulated correlations obtained from estimations that do not include information of survey expectations are called “*with out*” *surveys*. *“with” surveys* refers to the case when this data is added to the information set.
Output is Hodrick-Prescott filtered real GDP.

Appendix 2

Posterior distribution statistics of RE estimations

	"With out" surveys						"With surveys"					
	Complete		High inflation		Low inflation		Complete		High inflation		Low inflation	
	Median	Std.	Median	Std.	Median	Std.	Median	Std.	Median	Std.	Median	Std.
Share of K in production	0.182	0.018	0.198	0.025	0.159	0.021	0.153	0.017	0.205	0.023	0.145	0.015
Inv. Elast. Intertp. Sust.	1.159	0.084	1.099	0.175	1.528	0.141	1.245	0.073	1.106	0.105	1.252	0.114
Fix cost product.	1.574	0.074	1.476	0.089	1.517	0.089	1.411	0.072	1.502	0.085	1.617	0.122
Adj.cost inv.	5.841	0.948	4.999	1.032	6.803	1.228	5.941	0.843	4.005	1.064	7.191	1.069
Habits	0.814	0.028	0.813	0.062	0.741	0.069	0.799	0.028	0.772	0.055	0.802	0.043
Wage stickiness	0.552	0.042	0.735	0.058	0.519	0.124	0.922	0.012	0.681	0.055	0.844	0.038
Elast. labor supply	2.261	0.579	2.155	0.578	1.906	0.655	2.046	0.593	0.972	0.621	1.247	0.682
Price stickiness	0.648	0.042	0.484	0.073	0.651	0.054	0.765	0.027	0.724	0.034	0.904	0.024
Wage indexation	0.491	0.137	0.620	0.116	0.506	0.150	0.678	0.089	0.551	0.121	0.381	0.143
Price indexation	0.281	0.149	0.185	0.076	0.239	0.114	0.062	0.030	0.182	0.087	0.217	0.087
Cap. Utiliz. Elast.	0.638	0.098	0.541	0.134	0.643	0.128	0.630	0.139	0.589	0.136	0.438	0.135
TR: inflation	1.660	0.120	1.544	0.189	1.692	0.215	1.343	0.161	1.079	0.035	1.093	0.066
TR: lag interest rate	0.756	0.028	0.699	0.047	0.840	0.025	0.749	0.024	0.584	0.052	0.793	0.029
TR: change in output	0.205	0.045	0.159	0.049	0.162	0.047	0.212	0.044	0.152	0.047	0.151	0.046
Trend growth rate	0.363	0.010			0.428	0.015	0.383	0.011			0.478	0.028
St-st inflation	0.834	0.070			0.642	0.075	0.726	0.080			0.713	0.054
St-st hours worked	-0.872	0.510			-0.728	0.608	-1.748	0.623			1.079	0.627
St-st nominal int rate	1.351	0.112			1.137	0.100	1.260	0.126			1.152	0.090
aut. Price Mk up shock	0.518	0.200	0.922	0.050	0.673	0.146	0.602	0.068	0.271	0.122	0.134	0.074
aut. Wage Mk up shock	0.960	0.013	0.620	0.147	0.866	0.106	0.288	0.070	0.793	0.073	0.433	0.102
aut. Product. Shock	0.960	0.012	0.973	0.020	0.922	0.025	0.975	0.006	0.988	0.004	0.926	0.037
aut. Risk premium	0.124	0.060	0.280	0.111	0.188	0.114	0.147	0.067	0.279	0.106	0.238	0.121
aut. Government shock	0.991	0.004	0.952	0.014	0.985	0.007	0.996	0.002	0.995	0.016	0.985	0.006
aut. Inv. Specific shock	0.841	0.034	0.771	0.091	0.683	0.072	0.949	0.019	0.898	0.026	0.711	0.113
aut. Monet policy shock	0.217	0.067	0.233	0.102	0.519	0.063	0.192	0.059	0.289	0.098	0.442	0.067
Corr. Gov & product sks	0.588	0.086	0.692	0.133	0.494	0.094	0.594	0.070	0.629	0.127	0.494	0.117
std. Price Mk up shock	0.135	0.026	0.124	0.027	0.093	0.019	0.115	0.018	0.219	0.035	0.164	0.017
std. Wage Mk up shock	0.185	0.030	0.144	0.029	0.231	0.060	0.223	0.023	0.119	0.020	0.229	0.033
std. Product. Shock	0.458	0.029	0.595	0.065	0.397	0.035	0.498	0.033	0.608	0.056	0.392	0.032
std. Risk premium	0.248	0.021	0.303	0.049	0.179	0.025	0.254	0.025	0.292	0.046	0.176	0.026
std. Government shock	0.481	0.028	0.619	0.061	0.360	0.030	0.474	0.031	0.626	0.055	0.356	0.023
std. Inv. Specific shock	0.342	0.030	0.466	0.091	0.360	0.058	0.250	0.037	0.371	0.055	0.295	0.071
std. Monet policy shock	0.264	0.016	0.392	0.042	0.108	0.011	0.262	0.016	0.397	0.039	0.110	0.012
Measurement exp error							0.258	0.018	0.271	0.038	0.131	0.014
Log mg. Likelihood	-146.4		-136.0		73.8		-10.6		-103.2		184.2	

Note: Complete sample represents the period 1968Q4 - 2008Q2; "High inflation" sample represents the period 1968Q4 - 1983Q4; "Low inflation" sample represents the period 1989Q1 - 2008Q2.

Estimations that do not include information of survey expectations are called "*with out*" *surveys*. *"with" surveys* refers to the case when this data is added to the information set.

Appendix 3

Ranking of PLM's specifications by minimum MSE

Complete sample Period: 1968Q4 - 2008Q2 Presample: 1950Q1 - 1968Q3				"High inflation" sample Period: 1968Q4 - 1983Q4 Presample: 1950Q1 - 1968Q3				"Low inflation" sample Period: 1989Q1 - 2008Q2 Presample: 1984Q1 - 1988Q4			
Rank	Model	Gain	MSE	Rank	Model	Gain	MSE	Rank	Model	Gain	MSE
1	PIE	0.125	0.0294	1	PIE	0.125	0.0330	1	L R	0.075	0.0148
2	PIE L	0.125	0.0300	2	PIE C	0.125	0.0333	2	L	0.100	0.0159
3	PIE C	0.125	0.0302	3	PIE Y	0.125	0.0343	3	PIE L R	0.075	0.0160
4	PIE C L	0.125	0.0303	4	PIE I	0.125	0.0355	4	R	0.088	0.0162
5	PIE Y	0.125	0.0315	5	PIE C L	0.113	0.0386	5	C L R	0.075	0.0164
6	PIE I	0.125	0.0323	6	PIE L	0.113	0.0396	6	PIE L W R	0.075	0.0167
7	PIE R	0.063	0.0330	7	PIE Y C	0.125	0.0411	7	L W R	0.063	0.0168
8	PIE I L	0.125	0.0333	8	PIE Y I	0.125	0.0418	8	C R	0.088	0.0170
9	PIE C R	0.063	0.0335	9	PIE I L	0.125	0.0424	9	C L	0.100	0.0171
10	PIE Y L	0.125	0.0336	10	PIE Y L	0.113	0.0441	10	PIE L	0.100	0.0173
11	PIE Y C	0.125	0.0352	11	PIE L W R	0.050	0.0458	11	W R	0.063	0.0177
12	C	0.238	0.0358	12	PIE I L W F	0.050	0.0459	12	PIE C L R	0.075	0.0177
13	PIE I R	0.050	0.0360	13	PIE Y L W I	0.050	0.0461	13	C	0.113	0.0178
14	PIE Y C L	0.113	0.0364	14	PIE Y I L W	0.050	0.0471	14	PIE L W	0.100	0.0180
15	PIE Y I	0.125	0.0366	15	PIE R	0.063	0.0473	15	L W	0.088	0.0185
16	PIE W	0.113	0.0370	16	PIE Y C L	0.113	0.0477	16	C L W R	0.075	0.0187
17	PIE C I L	0.125	0.0372	17	PIE L W	0.088	0.0478	17	PIE C L W	0.075	0.0188
18	PIE Y I R	0.050	0.0373	18	PIE C L W	0.050	0.0479	18	PIE C L	0.100	0.0188
19	PIE L W	0.100	0.0376	19	PIE I L W	0.088	0.0481	19	PIE R	0.088	0.0190
20	PIE Y I L	0.113	0.0382	20	PIE C R	0.063	0.0481	20	C W R	0.063	0.0192

Appendix 4

In-sample a priori RMSEs: Estimation WITHOUT survey expectations as observable variable

Variable	Whole period			"High inflation" period			"Low inflation" period		
	(1)	(2)	(1)-(2)	(1)	(2)	(1)-(2)	(1)	(2)	(1)-(2)
Consumption growth	0.566	0.581	-0.015	0.743	0.701	0.041	0.441	0.461	-0.020
Investment growth	1.885	1.889	-0.004	2.277	2.366	-0.089	1.358	1.336	0.022
Wages growth	0.575	0.602	-0.028	0.504	0.470	0.035	0.674	0.677	-0.003
Inflation	0.304	0.305	-0.002	0.386	0.347	0.039	0.245	0.202	0.043
Output growth	0.714	0.715	0.000	1.044	0.997	0.046	0.483	0.491	-0.008
Interest rate	0.259	0.264	-0.005	0.391	0.374	0.017	0.089	0.090	0.000
Hours worked	0.523	0.519	0.005	0.714	0.687	0.027	0.397	0.375	0.022

(1) Estimation under RE

(2) Estimation under learning with PLM survey obtained

In-sample a priori RMSEs: Estimation WITH survey expectations as observable variable

Variable	Whole period			"High inflation" period			"Low inflation" period		
	(1)	(2)	(1)-(2)	(1)	(2)	(1)-(2)	(1)	(2)	(1)-(2)
Consumption growth	0.576	0.578	-0.002	0.727	0.703	0.025	0.428	0.444	-0.016
Investment growth	1.958	1.886	0.071	2.547	2.363	0.184	1.354	1.344	0.011
Wages growth	0.559	0.597	-0.039	0.516	0.526	-0.009	0.692	0.674	0.018
Inflation	0.294	0.302	-0.008	0.393	0.421	-0.028	0.240	0.201	0.039
Output growth	0.721	0.716	0.005	0.991	0.994	-0.002	0.495	0.481	0.014
Interest rate	0.253	0.256	-0.003	0.394	0.380	0.014	0.098	0.090	0.007
Hours worked	0.514	0.517	-0.003	0.686	0.678	0.007	0.380	0.379	0.000
Exp. Inflation	0.253	0.188	0.065	0.320	0.244	0.076	0.125	0.124	0.001

(1) Estimation under RE

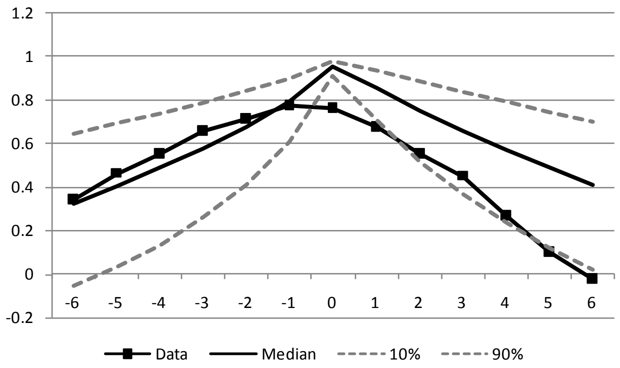
(2) Estimation under learning with PLM survey obtained

Note. These tables show the Root Mean Squared forecast Errors obtained from the Kalman filter at the posterior distribution median value of the parameters.

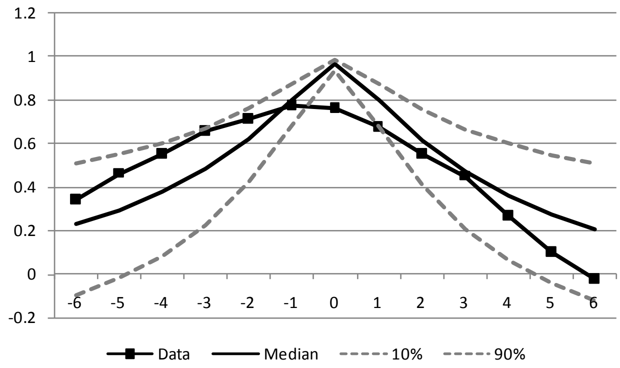
Appendix 5a: Cross correlations, SW model using Surveys, period 1968Q4-1983Q4

Cross Correlation exp inflation(t/t+1) and inflation (t+k)

RE

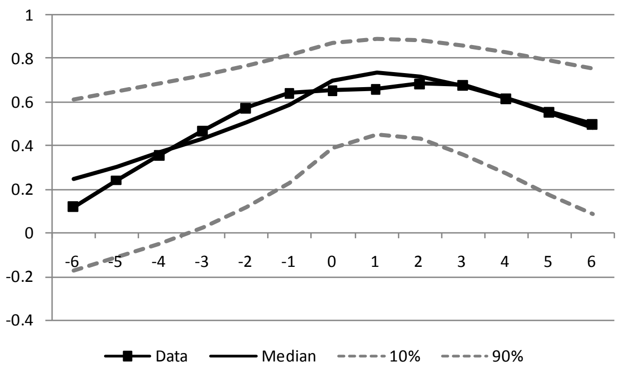


Learning

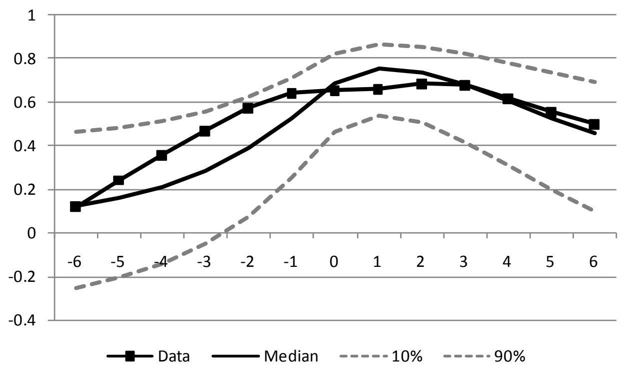


Cross Correlation exp inflation(t/t+1) and the interest rate (t+k)

RE

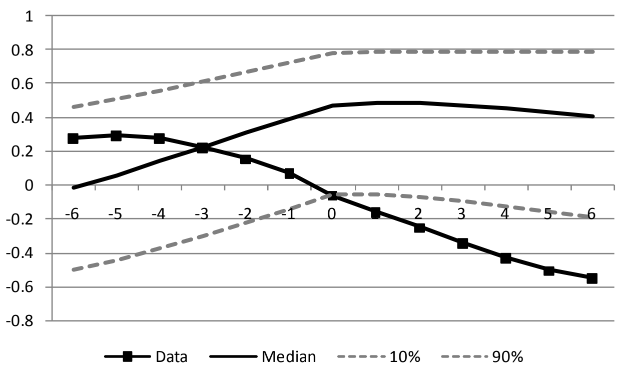


Learning



Cross Correlation exp inflation(t/t+1) and hours worked (t+k)

RE



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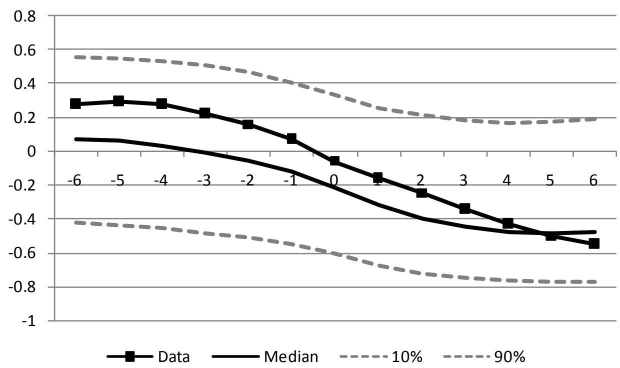
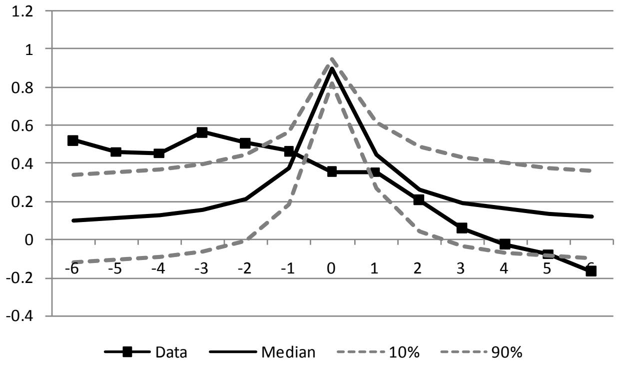


Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.

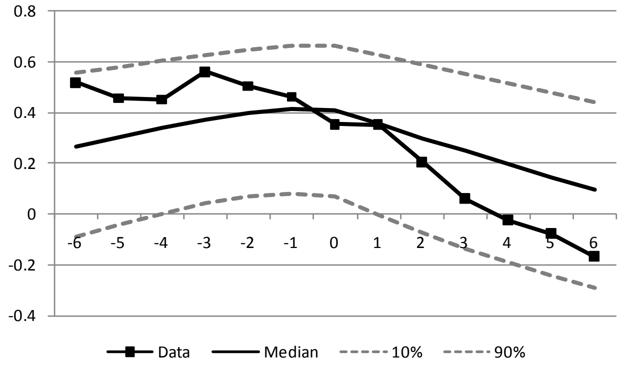
Appendix 5b: Cross correlations, SW model using Surveys, period 1989Q1-2008Q2

Cross Correlation exp inflation(t/t+1) and inflation (t+k)

RE

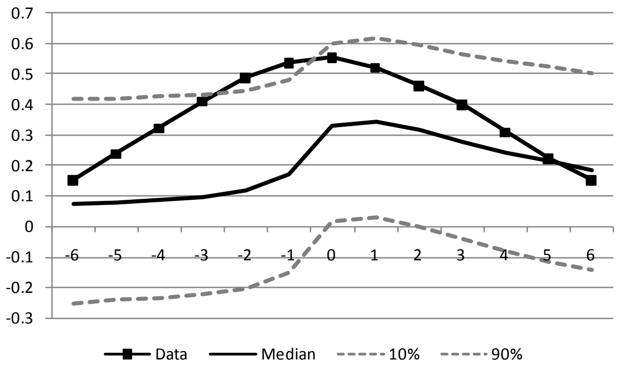


Learning

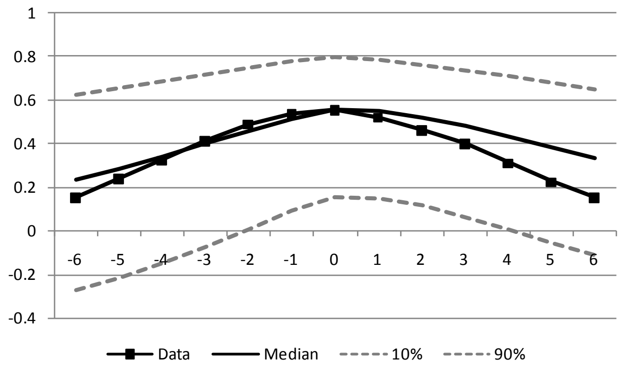


Cross Correlation exp inflation(t/t+1) and the interest rate (t+k)

RE

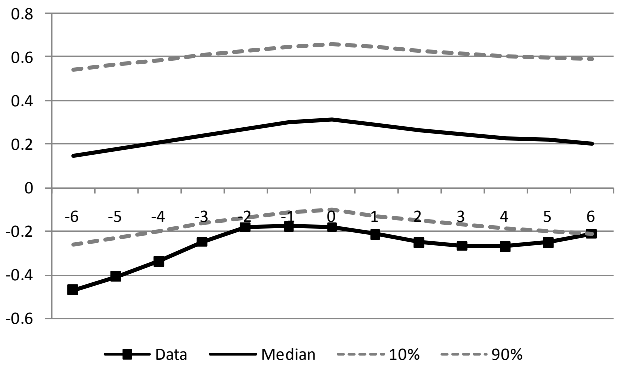


Learning

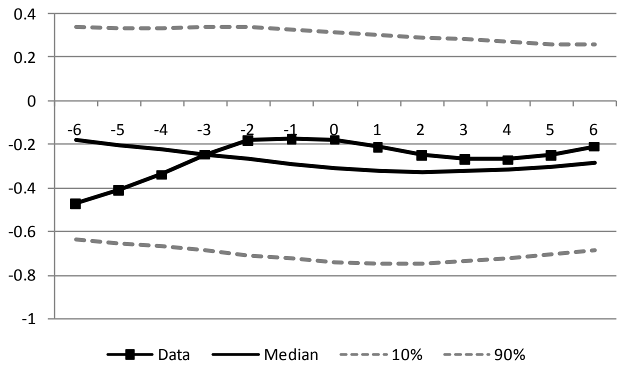


Cross Correlation exp inflation(t/t+1) and hours worked (t+k)

RE



Learning

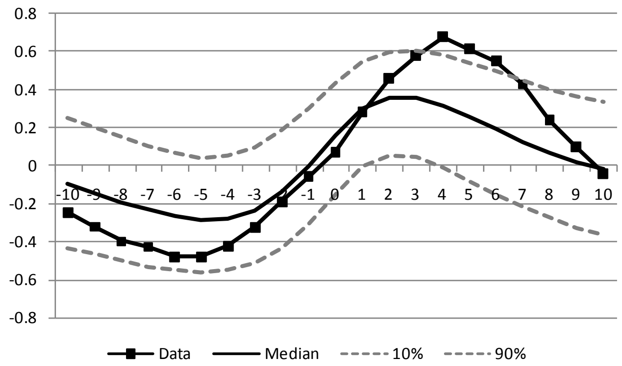


Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.

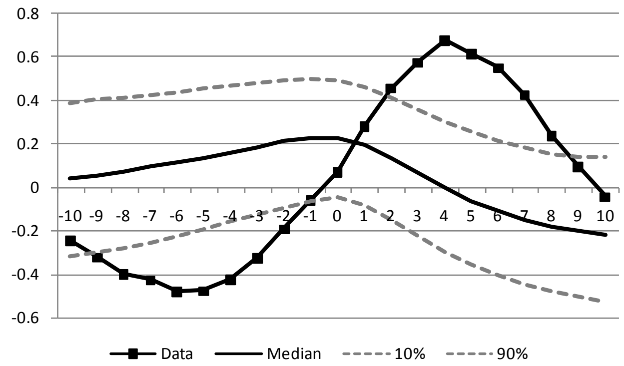
Appendix 6a: Cross correlation among inflation, output and interest rate, period 1968Q4-1983Q4

Cross Correlation output (t) and inflation (t+k)

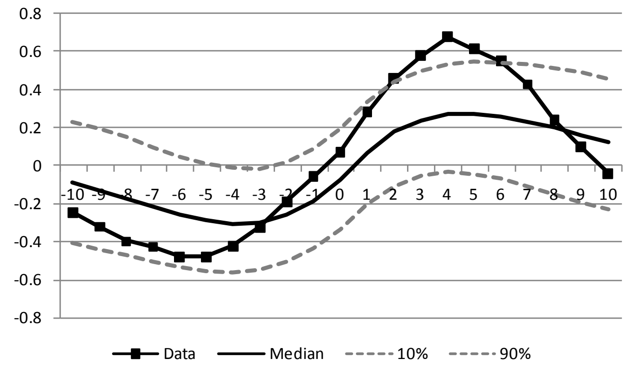
RE without Surveys



RE with Surveys

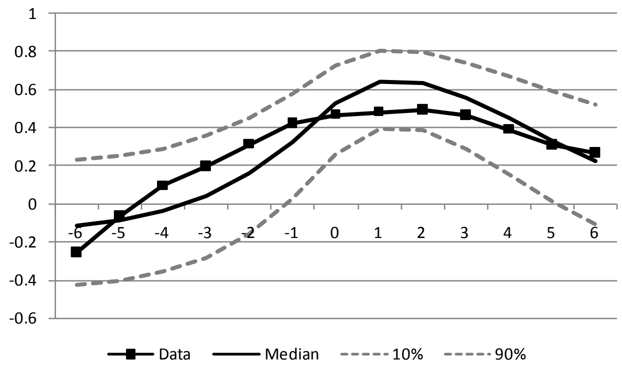


Learning with Surveys

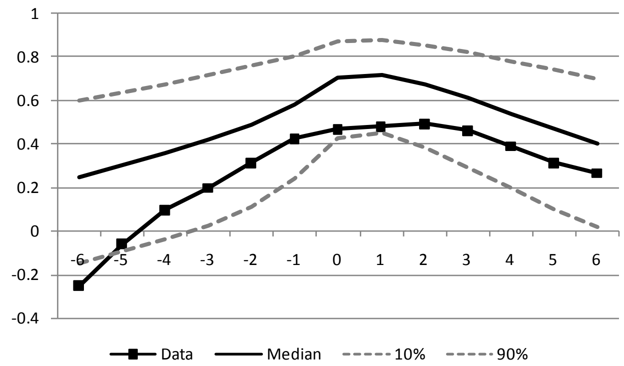


Cross Correlation output (t) and the interest rate (t+k)

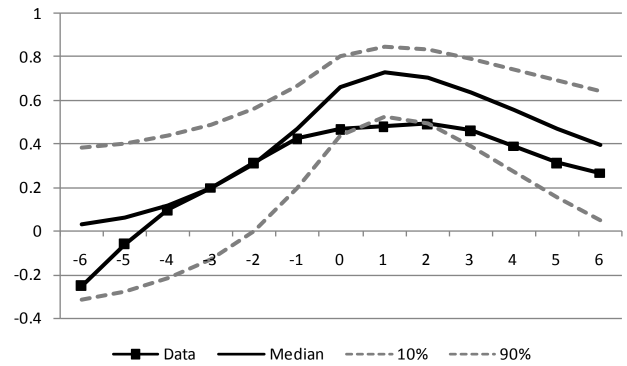
RE without Surveys



RE with Surveys



Learning with Surveys

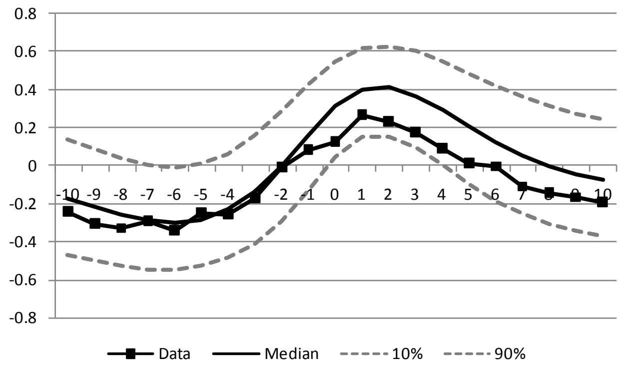


Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.
Output is Hodrick-Prescott filtered real GDP.

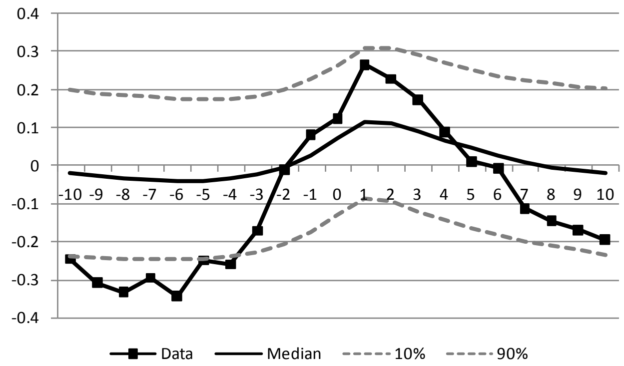
Appendix 6b: Cross correlation among inflation, output and interest rate, period 1989Q1-2008Q2

Cross Correlation output (t) and inflation (t+k)

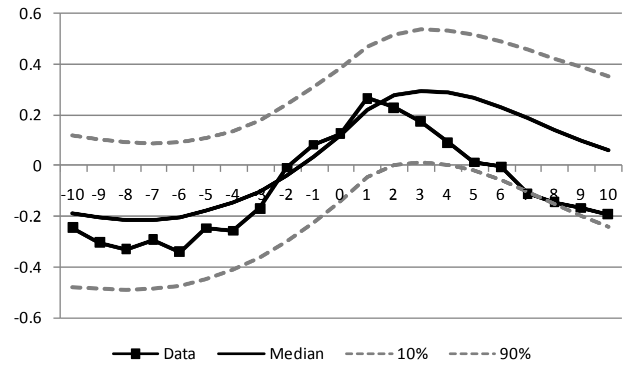
RE without Surveys



RE with Surveys

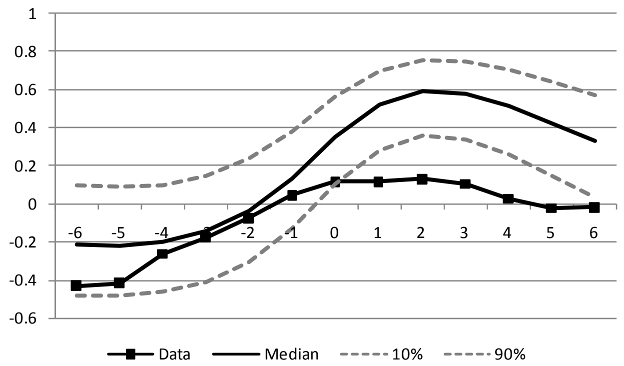


Learning with Surveys

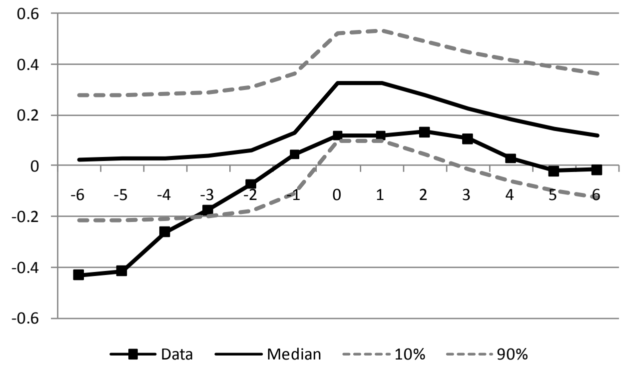


Cross Correlation output (t) and the interest rate (t+k)

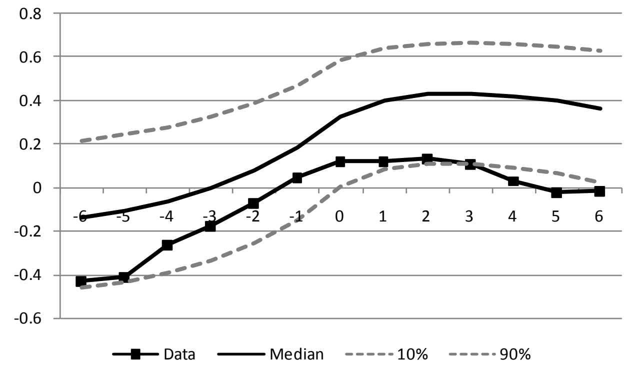
RE without Surveys



RE with Surveys



Learning with Surveys



Note: in order to generate simulated distribution of the cross correlation, 10 000 draws from the posterior distributions of the model parameters are used to generate artificial samples of the same sample size as the actual dataset. For each of those 10 000 artificial samples, the autocorrelation function is calculated and the median, 10 and 90 percentiles are derived.
Output is Hodrick-Prescott filtered real GDP.

Appendix 7

Posterior distribution statistics: RE v.s. Learning

	Complete sample				High inflation sample*				Low inflation sample**			
	RE		Learning		RE		Learning		RE		Learning	
	Median	Std.	Median	Std.	Median	Std.	Median	Std.	Median	Std.	Median	Std.
Share of K in production	0,182	0,018	0,183	0,019	0,198	0,025	0,197	0,024	0,159	0,021	0,159	0,019
Inv. Elast. Intertp. Sust.	1,159	0,084	1,309	0,140	1,099	0,175	1,092	0,124	1,528	0,141	1,530	0,135
Fix cost product.	1,574	0,074	1,653	0,079	1,476	0,089	1,511	0,092	1,517	0,089	1,457	0,087
Adj.cost inv.	5,841	0,948	7,059	1,017	4,999	1,032	5,333	1,089	6,803	1,228	7,604	1,117
Habits	0,814	0,028	0,805	0,031	0,813	0,062	0,812	0,043	0,741	0,069	0,790	0,039
Wage stickiness	0,552	0,042	0,552	0,046	0,735	0,058	0,647	0,053	0,519	0,124	0,601	0,095
Elast. labor supply	2,261	0,579	2,491	0,627	2,155	0,578	1,803	0,640	1,906	0,655	2,099	0,713
Price stickiness	0,648	0,042	0,470	0,032	0,484	0,073	0,553	0,052	0,651	0,054	0,475	0,041
Wage indexation	0,491	0,137	0,334	0,115	0,620	0,116	0,381	0,119	0,506	0,150	0,438	0,145
Price indexation	0,281	0,149	0,518	0,115	0,185	0,076	0,570	0,117	0,239	0,114	0,345	0,134
Cap. Utiliz. Elast.	0,638	0,098	0,649	0,108	0,541	0,134	0,487	0,132	0,643	0,128	0,670	0,118
TR: inflation	1,660	0,120	1,409	0,113	1,544	0,189	1,308	0,213	1,692	0,215	1,408	0,188
TR: lag interest rate	0,756	0,028	0,773	0,029	0,699	0,047	0,692	0,053	0,840	0,025	0,863	0,025
TR: change in output	0,205	0,045	0,204	0,045	0,159	0,049	0,150	0,048	0,162	0,047	0,167	0,045
Trend growth rate	0,363	0,010	0,368	0,010					0,428	0,015	0,419	0,014
St-st inflation	0,834	0,070	0,713	0,092					0,642	0,075	0,637	0,094
St-st hours worked	-0,872	0,510	-0,719	0,429					-0,728	0,608	-1,148	0,349
St-st nominal int rate	1,351	0,112	1,279	0,134					1,137	0,100	1,157	0,123
aut. Price Mk up shock	0,518	0,200	0,168	0,089	0,922	0,050	0,200	0,085	0,673	0,146	0,160	0,088
aut. Wage Mk up shock	0,960	0,013	0,948	0,018	0,620	0,147	0,890	0,052	0,866	0,106	0,769	0,128
aut. Product. Shock	0,960	0,012	0,971	0,008	0,973	0,020	0,942	0,020	0,922	0,025	0,945	0,021
aut. Risk premium	0,124	0,060	0,156	0,073	0,280	0,111	0,276	0,123	0,188	0,114	0,205	0,098
aut. Government shock	0,991	0,004	0,993	0,004	0,952	0,014	0,953	0,018	0,985	0,007	0,988	0,006
aut. Inv. Specific shock	0,841	0,034	0,837	0,033	0,771	0,091	0,830	0,064	0,683	0,072	0,746	0,074
aut. Monet policy shock	0,217	0,067	0,195	0,064	0,233	0,102	0,222	0,090	0,519	0,063	0,514	0,068
Corr. Gov & product sks	0,588	0,086	0,592	0,093	0,692	0,133	0,686	0,116	0,494	0,094	0,481	0,101
std. Price Mk up shock	0,135	0,026	0,203	0,014	0,124	0,027	0,266	0,033	0,093	0,019	0,205	0,020
std. Wage Mk up shock	0,185	0,030	0,218	0,030	0,144	0,029	0,152	0,025	0,231	0,060	0,286	0,040
std. Product. Shock	0,458	0,029	0,448	0,032	0,595	0,065	0,604	0,058	0,397	0,035	0,402	0,038
std. Risk premium	0,248	0,021	0,239	0,023	0,303	0,049	0,285	0,045	0,179	0,025	0,170	0,024
std. Government shock	0,481	0,028	0,495	0,029	0,619	0,061	0,594	0,052	0,360	0,030	0,356	0,032
std. Inv. Specific shock	0,342	0,030	0,340	0,031	0,466	0,091	0,397	0,077	0,360	0,058	0,328	0,060
std. Monet policy shock	0,264	0,016	0,259	0,016	0,392	0,042	0,392	0,037	0,108	0,011	0,098	0,008
Gain - no inflation			0,017	0,024			0,041	0,040			0,064	0,046
Gain - inflation			0,140	0,008			0,132	0,011			0,078	0,006
Measurement exp error			0,176	0,011			0,207	0,026			0,126	0,010

Note: Complete sample represents the period 1968Q4 - 2008Q2; "High inflation" sample represents the period 1968Q4 - 1983Q4; "Low inflation" sample represents the period 1989Q1 - 2008Q2.
*Learning estimation considers a PLM for inflation that only includes lagged inflation rate.
**Learning estimation considers a PLM for inflation that includes lagged interest rates and hours worked.

Appendix 8

Posterior distribution statistics for the inflation-forecast augmented Taylor rule case

	High inflation sample*		Low inflation sample**	
	Median	Std.	Median	Std.
Share of K in production	0.208	0.024	0.165	0.025
Inv. Elast. Intertp. Sust.	1.219	0.197	1.642	0.164
Fix cost product.	1.549	0.085	1.451	0.089
Adj.cost inv.	5.451	1.115	6.965	0.993
Habits	0.785	0.058	0.728	0.058
Wage stickiness	0.615	0.050	0.491	0.101
Elast. labor supply	1.700	0.602	2.439	0.642
Price stickiness	0.539	0.046	0.474	0.039
Wage indexation	0.360	0.119	0.426	0.138
Price indexation	0.607	0.114	0.379	0.143
Cap. Utiliz. Elast.	0.464	0.131	0.673	0.113
TR: inflation	1.372	0.202	1.070	0.290
TR: lag interest rate	0.725	0.055	0.888	0.026
TR: change in output	0.155	0.048	0.153	0.048
TR: inflation expectations	0.346	0.259	2.321	0.514
Trend growth rate			0.414	0.017
St-st inflation			0.655	0.066
St-st hours worked			-0.982	0.337
St-st nominal int rate			1.242	0.168
aut. Price Mk up shock	0.193	0.088	0.146	0.086
aut. Wage Mk up shock	0.885	0.055	0.855	0.128
aut. Product. Shock	0.957	0.017	0.939	0.023
aut. Risk premium	0.281	0.125	0.348	0.161
aut. Government shock	0.951	0.015	0.988	0.007
aut. Inv. Specific shock	0.802	0.061	0.725	0.064
aut. Monet policy shock	0.207	0.078	0.605	0.078
Corr. Gov & product sks	0.696	0.119	0.495	0.106
std. Price Mk up shock	0.269	0.030	0.206	0.018
std. Wage Mk up shock	0.163	0.027	0.326	0.046
std. Product. Shock	0.601	0.061	0.391	0.037
std. Risk premium	0.288	0.050	0.151	0.036
std. Government shock	0.588	0.059	0.366	0.028
std. Inv. Specific shock	0.449	0.062	0.370	0.052
std. Monet policy shock	0.388	0.037	0.088	0.007
Gain - no inflation	0.067	0.046	0.015	0.019
Gain - inflation	0.137	0.011	0.075	0.005
Measurement exp error	0.202	0.017	0.125	0.010

Note: Complete sample represents the period 1968Q4 - 2008Q2; "High inflation" sample represents the period 1968Q4 - 1983Q4; "Low inflation" sample represents the period 1989Q1 - 2008Q2.
*Learning estimation considers a PLM for inflation that only includes lagged inflation rate.
**Learning estimation considers a PLM for inflation that includes lagged interest rates and hours worked.