Self-fulfilling Liquidity Dry-ups

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Abstract

Secondary markets for long-term assets might be illiquid due to adverse selection. In a model in which moral hazard is confined to project initiation, I find that: (1) when agents expect a liquidity dry-up on such markets, they optimally choose to self-insure through the hoarding of non-productive but liquid assets; (2) such a response has negative externalities as it reduces ex-post market participation, which worsens adverse selection and dries up market liquidity; (3) liquidity dry-ups are Pareto inefficient equilibria; (4) the Government can rule them out. Additionally, when agents face idiosyncratic, privately known, illiquidity shocks, I show that: (5) it increases market liquidity; (6) illiquid agents are better-off when they can credibly disclose their liquidity position, but transparency has an ambiguous effect on risk-sharing possibilities.

1 Introduction

Liquidity can be understood as the ability to transform long-term assets into current consumption goods. In that sense, secondary markets play a crucial role in liquidity provision. However, we know from Akerlof (1970) that adverse selection may prevent such markets to work properly. What I present here is a model in which the fear of a market breakdown due to adverse selection might induce agents to adopt behaviors that would actually cause such a breakdown. In that case, it becomes extremely costly to transform long-term assets into consumption goods, which is the rationale for calling such an episode a self-fulfilling liquidity dry-up.

I build a three-date model in which ex-ante identical agents invest in long-term risky projects. They also have access to a riskless one-period storage technology. If successful, long-term projects yield a better return than storage, and conversely in they fail. Agents privately observe the quality (success or failure) of their projects at the interim date. At this

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point, they might want to liquidate a share of their long-term projects due either to private information about future payoffs or to provide for current consumption needs. Because of adverse selection, the price they can get on the secondary market is determined by the average seller's motive for trading. There is therefore a return-liquidity trade-off: long-term investment is on average more productive but liquidation on the secondary market might be endogenously costly because of adverse selection.[1]

Firstly, I explain how adverse selection may lead to self-fulfilling liquidity dry-ups. This is the main contribution of the paper. Then, as a secondary contribution, I introduce heterogeneity in preferences and discuss the interactions between idiosyncratic illiquidity shocks, market liquidity, and risk sharing.

In my model, adverse-selection-driven illiquidity leads to multiple equilibria for a wide range of parameter values. In this case, when agents expect the market to be illiquid, they optimally choose to self-insure through the hoarding of non-productive but liquid assets. Such a response has negative externalities as it reduces ex-post market participation which worsens adverse selection and dries up market liquidity. I derive the condition under which such an outcome is Pareto inefficient and I show how the Government can rule it out. A public liquidity insurance scheme implements the second-best allocation because the prospect of a market bailout suppresses the return-liquidity trade-off. This prevents wasteful self-insurance, boosts long-term investment and ex-post market participation, which then has a positive feedback effect on liquidity.

When agents anticipate that market liquidity will render storage wasteful, they are fully invested in the long-run technology. Consequently, any resource an agent consumes at date 1 should have been planned to come from liquidation, which is true irrespective of the agent's project quality. This implies full participation to the secondary market and a relatively high proportion of claims to high-return projects; liquidity is thus indeed high. However, if agents believe the market will become illiquid -that liquidation will hurt- they choose to self-insure. Therefore, they optimally store part of their initial resources; they hoard liquidity. Optimal self-insurance should naturally avoid liquidation in states of nature where the opportunity cost is high. There is thus limited participation and the agents with high-return projects are the first to exit the market. As liquidity decreases with average quality, this can account for another equilibrium where illiquidity is a self-fulfilling prophecy.

Whereas the decision to self-insure is individually optimal in a low-liquidity world, it is socially costly in two respects. First because it wastes resources on the storage technology (long-run investment is on average more productive) and second because self-insurance hinders risk sharing since it ex-post prevents agents from providing the positive externalities associated with the issuance of claims to high-return projects. The fact that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998), and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment is present in Eisfeldt (2004). However, that a liquidity dry-up can endogenously arise for the very reason that investors self-insure against it is a new result.

As the market may fail to allocate resources efficiently, expectations about market liquidity have a crucial impact on welfare. Because the Government can prevent the underlying coordination failure, the model has policy implications. Potential welfare losses may indeed arise if the law-maker overlooks the "liquidity expectation channel" when considering public intervention in the case of financial crisis. Caballero and Krishnamurthy (2008) find similar results in a model with Knightian uncertainty.

[1]This concept of adverse-selection-driven endogenous liquidity in asset markets is introduced by Eisfeldt (2004). In her model, current needs for resources depend on past decisions and on information about future income. Thus, there can be reasons (e.g. consumption smoothing in the case of a negative income shock) to sell high quality claims, even at a discount. The higher the discount one concedes to sell a good asset, the lower its liquidity. Similarly, the higher the proportion of agents trading for consumption-smoothing or risk-sharing purposes (instead of private information about payoffs), the lower the adverse selection and the higher the liquidity of the market.

When one thinks of financial institutions, including their off-balance structured investment vehicles, like the investors in my model, the kind of endogenous market breakdown I describe fits to the current situation of the securitization business quite well. Furthermore, it can simultaneously explain several stylized facts that are hardly mutually compatible within the traditional framework used to explain liquidity dry-ups: the cash-in-the-market pricing theory[2]. Indeed, whereas the latter relies on the inelasticity of short-term funding, my model allows for the presence of deep-pocket agents. My claim is that the availability of outside resources seems more reasonable when one looks at the recent low yields on T-Bills and at the tremendously high excess reserves of US depository institutions.

To expose the secondary contribution of the paper, I consider heterogeneous preferences: as in the banking literature, agents face idiosyncratic illiquidity shocks.

When the realization of such shocks is private information, I find that market liquidity improves. Because illiquid (those that are hit by the shocks) agents issue claims irrespective of the underlying project return, it indeed reduces adverse selection. As market liquidity improves risk sharing, this suggests that idiosyncratic liquidity shocks need not be socially a bad thing. Also, when there exists a technology that enables them to disclose their liquidity position, ex-post illiquid agents are better-off under disclosure. It isolates them from the negative externalities exerted by liquid lemon owners. However, ex-post liquid agents incur a bigger liquidity discount because the probability that they try to sell a lemon, conditionally to have a good liquidity position, increases. Therefore the effect on ex-ante risk sharing is ambiguous. This last result illustrates the ambiguity of transparency on liquidity suggested by Holmström (2008).

The relationship I show between liquidity, self-insurance and risk sharing complements the literature on the competing role of banks and markets for the provision of liquidity. In Diamond and Dybvig (1983), there is no market and banks can generally provide liquidity and improve on the autarky allocation. Jacklin (1987) shows that this is not the case if there exists a secondary market because the demand deposit contract is no longer incentive compatible. Diamond (1987) generalizes these results with a model of exogenous limited market participation. He finds that the lower the participation in the market, the greater the role of the banking sector. In that respect, the key differences of my model are that limited market participation is endogenous and investors are needed to run the initial phase of the project. Indeed, I assume that projects could not be run mutually in the first period and that there is no means by which agents could credibly commit to invest. Otherwise, agents could form a coalition in order to pool resources and diversify idiosyncratic risk away. This coalition would correspond to the bank[3] in Diamond and Dybvig (1983) and, as there is no aggregate shock to the fundamentals in my model, it could implement the first-best allocation. These assumptions are strong, as real banks do pool individual resources. However, there are frictions preventing them to ex-ante pool resources among themselves. That securitization thrived until 2007 is, in itself, evidence of the limits to ex-ante pooling of resources.

Section 2 presents the model, section 3 studies liquidity dry-ups, section 4 applies the model to the 2007-ongoing financial crisis, section 5 considers the impact of idiosyncratic liquidity shocks and section 6 concludes and draws the line for future research.

2 The Model

Technology

There are three dates ($t = 0, 1, 2$) with a unique consumption good that is also the unit of account. At dates 0 and 1, investor have access to a risk-free one-period storage technology

[2]See Allen and Gale (1994) for a theory of cash-in-the-market pricing and Brunnermeier and Pedersen (2009), Morris and Shin (2004) and Bolton, Santos and Scheinkman (2009) for applications to liquidity dry-ups.

[3]To avoid confusion, such a bank would correspond to a coalition of investors (or of small banks, seen as investors) in my model.

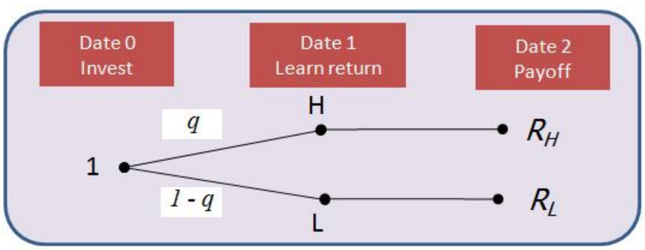
that yields an exogenous rate of return r. At date 0, they have also access to a risky long-run technology. Such projects are undertaken at date 0 and only pay off at date 2. They succeed with probability $q > 0$, in which case they yield a return R_H per unit invested. In case of failure, which occurs with probability $1-q$, they yield R_L, with $0 \leq R_L < R_H$. Investors are needed to initiate their own projects and have no means to credibly commit to properly invest. Moral hazard concerns consequently restrict ex-ante risk-sharing[4].

Projects cannot be physically liquidated at date 1[5]. However, at that date, investors may issue claims to their projects in a competitive and anonymous[6] secondary market. For simplicity, the output of the underlying project will be verifiable at date 2. Therefore, I abstract from moral hazard problems once the project has been properly initiated[7] . I do not allow for short-selling and there are thus no other way to borrow against future income than to issue claims to ongoing projects.

To make the analysis interesting, I assume $qR_H + (1-q)R_L > 1+r$ and $R_L < 1+r$: on average, long-term projects are more productive than storage, but they yield less than storage in case of failure.

At the beginning of date 1, investors observe their project's quality. This is private information, and quality is common to all the projects of a given investor. We can thus think of investors owning only one project of variable size. However, quality is independent across agents. Average quality is thus deterministic.

Figure 1: The long term technology



Investors

There is a measure one of ex-ante (at $t = 0$) identical *investors*[8] maximizing expected utility, which they may derive from consumption at date 1 and 2. Their period utility function $u(\cdot)$ is increasing, concave, and twice continuously differentiable. At date 0, they are endowed with one unit of the consumption good which they may allocate between the long-term risky investment and short-term storage.

These agents *may* face idiosyncratic illiquidity shocks: at date 1, they learn whether they are *normal* or *early* consumers[9]. These two kinds of agents differ by the subjective

[4]There is a consequent literature on the role of banks for such purpose when contracts are available to pool resources ex ante. See for instance Diamond and Dybvig (1983). What I study here is an economy in which such pooling is not possible.

[5]Equally, I could consider that the physical liquidation costs outweigh the residual value.

[6]The anonymity assumption simplifies the analysis. It might might be seen as a strong one, but it is not required to derive the main result.

[7]Moral hazard does of course play a crucial role in the funding of risky projects (see for instance Stiglitz and Weiss (1981), Kiyotaki and Moore (1997), Holmström and Tirole (1997 and 1998), and Bernanke, Gertler and Gilchrist (1989)). However, this paper focuses on adverse selection. It makes thus sense to contain the moral hazard channel.

[8]One can for instance think of these investors as entrepreneurs undertaking real projects or as banks issuing loans. I use the word "investors" because I model their problem as a portfolio choice.

[9]Most authors that use such preference shocks use the terminology *early* and *late* to distinguish between types

factor β they use to discount date-2 utility: $\beta \in \{0,1\}$ with $Prob(\beta = 1) = p$ and $0 < p \leq 1$. In the spirit of Diamond and Dybvig (1983), and in line with Eisfeldt (2004), *early* agents can also be viewed as agents incurring a need for liquidity due to either a current income shock or a very good investment opportunity[10]. Patience is independent across agents and is also independent of project returns. Ex-ante probabilities are common knowledge and aggregate investment is observable once it has been committed. There is thus no aggregate uncertainty in the fundamentals of this model. Ex-interim (at date 1) agents *may* thus differ in two dimensions: project quality and patience, which make four potential types of agents from that point in time. From an ex-ante point of view, these are four individual states of nature.

The presence or not of *early* consumers (that is whether $p < 1$ or $p = 1$) does not mater for the main contribution of the paper[11]. For this reason, and for the sake of simplicity, I will assume them away until section (5) where I present the secondary results.

The time line

At date 0, investors:

- Form anticipations about date-1 secondary market price.
- Choose λ the share of endowment they invest in the long-term technology, the remaining being stored.

At date 1, they:

- Learn their true type: project return and patience.
- Choose how much claims to issue out of their long-term investment and how much to consume at date 1 and store until date 2.
- Take P, the price at which they may issue claims on their projects (liquidate them) as given.

At date 2:

- Projects pay off and output is distributed to claimants.
- Agents consume their remaining resources and die.

Demand for claims and market price

There is also a measure 1 of "deep-pocket" agents which have available resources but do not have access to the long-term technology. They only have access to storage and to the market for claims to ongoing projects. For simplicity, they are risk-neutral, and hence they are ready to buy any asset at the expected discounted value of the underlying payoffs. I assume thus that they have, on aggregate, enough resources to clear the competitive secondary market at that price.[12]

When project quality is private information, a key variable to determine asset prices on secondary market is average quality (Akerlof 1970). As the deep-pocket agents have access

of consumer. I do not stick to that terminology because whereas the typical *late* consumers do only care about date 2 consumption, the *normal* agents of my model are standard consumption smoother.

[10] In Eisfeldt (2004), liquidity shocks are endogenous as they take the form of current income shocks and information about future income shocks that both depend on past investment decisions.

[11] This is formally proved in the appendix.

[12] Eisfeldt (2004) proposes an alternative formalization: she assumes that such agents are risk-averse and that their endowment streams and utility function are such that they want to save, for instance for precautionary saving motive. Then, she assumes perfect divisibility of claims and costless diversification. This generates the same perfectly elastic demand function.

to the storage technology, a simple no-arbitrage argument suffices to establish the market price P of an asset, given average quality on that market:

$$P(\eta) = \frac{R_L + \eta(R_H - R_L)}{1+r} \tag{1}$$

Where η denotes the proportion of good quality claims in the secondary market. Note that I do not need η to be directly observable since it could easily be inferred in equilibrium.

Endogenous market liquidity

When there is at least one low-return claim on the market, claims to high-return projects are sold (if any) at a discount with respect to $R_H/(1+r)$, the price that would prevail absent asymmetry of information. It is in that sense that adverse selection makes high-quality claims "illiquid".

For it is a direct measure of the proportion of agents trading for other reasons than private information about future payoffs and because it determines the illiquidity discount, η embodies market liquidity in this model.

Equilibrium definition

A triple $\gamma \equiv (P^*, \lambda^*, \eta^*)$ is an equilibrium for this economy if and only if:

$$\begin{cases} P^* = P(\eta^*) \\ \lambda^* \in \lambda(P^*) \\ \eta^* = \eta(P^*, \lambda^*) \end{cases} \tag{2}$$

That is, P^* is the price buyers are ready to pay for the average quality implied by η^*; λ^* is an optimal investment decision given P^*; and η^* is the proportion of high-return claims in the market implied by optimal liquidation behavior at the level of investment λ^* and at price P^*.

There always exists at least one such equilibrium[13]. Uniqueness, however, depends on parameter values. Roughly speaking, it at least requires that average project return $E[R]$ is low enough or sufficiently high. For all the intermediate cases, there are multiple equilibria.

3 Liquidity dry-ups

In this Section, I solve the model backward and I show how the negative externalities linked to self-insurance may lead to multiple equilibria that can be Pareto ranked according to their respecting level of market liquidity. To illustrate the crucial role of self-insurance on liquidity and risk sharing, I also give a numerical example and compare equilibria with a benchmark first-best allocation. Then, I show how the Government can implement the second-best by the mean of a public liquidity insurance scheme and I discuss policy implications.

Throughout this section, I make the following assumption:

Assumption 1

- Return to storage is normalized: $r = 0$.
- All agents are *normal*: $p = 1$
- Projects succeed and fail with equal probabilities: $q = 0.5$

[13] Kakutani's theorem ensures that there exists a price P' such that: $P' \in \eta\left(P', \lambda(P')\right) R_H/(1+r) + \left(1 - \eta\left(P', \lambda(P')\right)\right) R_L/(1+r)$. Such a price pins down an equilibrium.

- Period utility is logarithmic: $u(C_t) = \ln C_t$

I only use *assumption 1* for simplicity. The main results are extended to the general model in the appendix.

3.1 Equilibria

A first implication of *assumption 1* is that there are only two types of date-1 agents and, by the law of large number, they are exactly one half of each type. I therefore consider two date-1 representative agents named after their type $j \in (H, L)$. Where H stands for agents with high-return projects and L for low-return.

The problem of the agent

Long term investment is risky: on the one hand, it pays well in the case of success but, on the other hand, it might yield a relatively low return in case of failure or early liquidation. Conversely, storage yields the same amount in each state of nature and is liquid; it can be consumed 1 to 1 at each period. There is thus a *return-liquidity* trade-off and risk-averse agents might use storage to self-insure against the risk they face.

Formally, at date 0, agents solve:

$$\max_{\lambda, L_j, S_j} U_0 = E_0\left[\ln(C_1) + \ln(C_2)\right] \tag{3}$$

Subject to:

$$s.t. \begin{cases} C_{1j} + S_j = 1 - \lambda + L_j P \\ C_{2j} = (\lambda - L_j) R_j + S_j \\ 0 \leq L_j \leq \lambda \leq 1 \\ prob(j = H) = 0.5 \end{cases}$$

Where $E_t[.]$ is the conditional (upon information available at date t) expectation operator[14], λ is the share of endowment invested in the long-term technology, and the following variables are contingent on being in state of nature j: C_{tj} is consumption at date t , $S_j \geq 0$ is storage between dates 1 and 2 , L_j is the number of claims (to unit projects) issued at date 1.

The budget constraints state the following: date-1 resources consist of storage $(1 - \lambda)$ from date 0 plus the revenue from claim issuance (L_j) at the market price (P). These resources can be consumed (C_{1j}) or transferred to date 2 through storage (S_j). At date 2, resources available for consumption consist of the output from long-term investment that has not been liquidated $(\lambda - L_j)R_j$ plus storage from date 1.

To determine optimal behavior with respect to this trade-off, I solve the problem backward.

Date-1 optimal liquidation policy

Let $L_j(P, \lambda)$ denote the optimal correspondence[15] that solves the date-1 problem for agent j for each couple (P, λ). For simplicity, I restrict my analysis to prices that are consistent with (1): $P \in [R_L, R_H]$.

Agent L knows he owns lemons and he sells off any project he holds as soon as $P > R_L$. In the case $P = R_L$, optimal liquidation is undetermined[16]. I assume for simplicity that he sells off any project he holds too. Accordingly:

[14] Expectation is thus taken with respect to type, that is projects return. In the general model, expectation is also taken with respect to patience (β).

[15] $L_j(P, \lambda)$ might not be a function. Indeed, an agent j may reach the same utility level for several values of L.

[16] At an optimum, the agent could choose to increase (decrease) liquidation a bit and increase (decrease) storage to date 2 by the same amount, which would not affect utility.

$$L_L(P,\lambda) = \lambda \tag{4}$$

The problem of H, the agent with high-return projects, is given by:

$$\max_{L_H,S_H} U_1 = \ln(C_1) + \ln(C_2)$$

Subject to:

$$s.t. \begin{cases} C_{1H} + S_H = 1 - \lambda + L_H P \\ C_{2H} = (\lambda - L_H) R_H + S_H \\ 0 \leq L_H \leq \lambda \end{cases}$$

From the first order conditions, I have:

$$L_H(P,\lambda) = \max\left\{0; \frac{P\lambda - 1 + \lambda}{2P}\right\} \tag{5}$$

The intuition is the following. The optimal liquidation of agent H is weakly increasing in P and in λ. If both are high enough, $\frac{P\lambda-1+\lambda}{2P}$ is positive because the resources available at date 1 (before liquidation) are smaller than the share of wealth he wants to dedicate to consumption at that period. Conversely, if $\frac{P\lambda-1+\lambda}{2P}$ is negative, the agent would like to "create" ongoing projects. This is of course ruled out by the definition of the long-term technology. In that case, agent H does not participate in the market and $L_H(P,\lambda) = 0$. Note that if λ is really small, he might roll part of its storage over to date 2.

Date-0 optimal investment policy

Agents choose investment according to expected utility maximization.

*Proposition 1 (**self-insurance**)*

Let $\lambda(P) \equiv \arg\max_{\lambda} U_0(\lambda, P)$ be the set of solutions for a given P to the date 0 problem (3), then:

$$\lambda(P) = \begin{cases} \{1\} & ;P > 1 \\ \left\{\left[\frac{1}{2}, 1\right]\right\} & ;P = 1 \\ \left\{\tilde{\lambda}\right\} & ;P < 1 \end{cases} \tag{6}$$

With $0 < \tilde{\lambda} < \frac{1}{2}$.

This proposition states that when P is anticipated to be smaller than the gross return to storage, investment is low because it hurts to liquidate. Self-insurance is thus crowding out productive investment. Conversely, if P is anticipated to be high, investment is high too because it dominates storage -even in the case of early liquidation- and makes thus the agent better-off in all states of nature. If $P = 1$, investment is rather high, though undetermined over the range $\left[\frac{1}{2}, 1\right]$: whereas agent L is indifferent over the whole range of admissible values $[0,1]$, agent H is indifferent over this specific range and strictly prefers it to any lower value. This is the reason why the optimal investment policy has not a functional form at $P = 1$.

Proof: see appendix

Supply for claims and average quality

I can now evaluate the optimal liquidation functions (4) and (5) at the optimal investment level given price P (*proposition 1*):

$$L_L(P,\lambda(P)) = \lambda$$

$$L_H(P,\lambda(P)) = \begin{cases} 0 & ;P<1 \\ \frac{1}{2} & ;P>1 \\ \in \left[0,\frac{1}{2}\right] & ;P=1 \end{cases}$$

And I can define $\eta(P)$, the proportion of claims to high-return projects for a given P. I have:

$$\eta(P) \equiv \frac{L_H(P,\lambda(P))}{L_L(P,\lambda(P))+L_H(P,\lambda(P))} = \begin{cases} \eta_{illiq}=0 & ;P<1 \\ \eta_{liq}=\frac{1}{3} & ;P>1 \\ \eta_1 \in \left[\eta_{illiq},\eta_{liq}\right] & ;P=1 \end{cases} \tag{7}$$

If the price is anticipated to be low, relative to return on storage, market participation is anticipated to be limited: there will only be lemons in the market. However, in the case of a high price, there is full participation and therefore there is a higher proportion of claims to high-return projects in the market. The latter case implies a smaller discount and thus greater market liquidity (recall that η is a direct measure of liquidity in this model).

Equilibria and liquidity dry-ups

In this economy, the same fundamentals ($R_H, R_L, r=0, p=1, q=0{,}5$) might lead to multiple equilibria that primarily differ by their level of liquidity. Accordingly, I interpret equilibria with the lowest level of liquidity as a liquidity dry-ups.

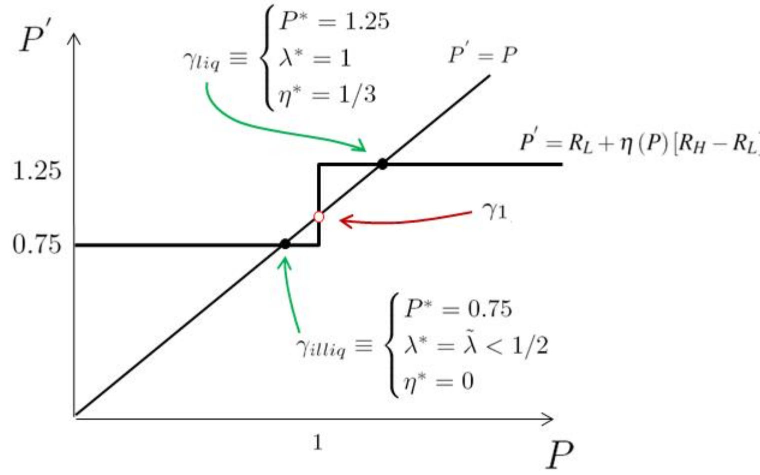
To find the equilibria of this economy, I define the implied price correspondence:

$$P^{'} = R_L + \eta(P)\left[R_H - R_L\right]$$

$P^{'}$ is the market price corresponding to a proportion of high-return projects $\eta(P)$. Therefore, a fixed point $P^{'}=P$ pins down an equilibrium price (call it P^{*}) for the economy. The corresponding values of λ^{*} and η^{*} are given by (6) and (7) respectively.

Figure (2) gives an example with three equilibria for a given set of parameters.

Figure 2: Multiple equilibria ($R_H = 2.25$ and $R_L = 0.75$)



In a first equilibrium, which I denote γ_{illiq}, agents anticipate a liquidity dry-up. They take as given that the price will be low ($P < 1$) and, according to *proposition 1,* they choose $\lambda(P) = \tilde{\lambda} < 1/2$. This in turn imply that agents with high-return projects will not enter the secondary market at date 1 and that the resulting liquidity will be low ($\eta^* = 0$) which make the anticipation of a low price a self-fulfilling prophecy: the liquidity has dried up.

Similar argument apply to γ_{liq}, the self-fulfilling high-liquidity equilibrium. Expecting high liquidity, which means that liquidation does not hurt ($P \geq 1$), agents invest only in the long term technology. Given that investment is high ($\lambda^* = 1$), agent H enters the market, and its participation increases the proportion of trade for other reasons than private information about future payoff. Hence, equilibrium liquidity and price are indeed high ($\eta^* > 1/3$ and $P^* = 1.2$). Both equilibria are locally stable in the sense that agents best-response to any small perturbation to the equilibrium price would bring the price back to equilibrium. There is also an equilibrium (call it γ_1) which is unstable. As they are of low economic relevance and they are not fundamental to my argument, I do not discuss such unstable equilibria further in the text. However, an interpretation is proposed in the appendix.

One might wonder if the example depicted in figure (2) is an exception. It is actually much more part of the rule than an exception in such a setup. I formalize this statement in *proposition 2*:

*Proposition 2 (**dry-ups**)*

Under assumption 1. $\forall R_H > 3 - 2R_L$*, problem (3) has at least two distinct solutions with different level of liquidity.*

This simply means that for any admissible low return ($0 \leq R_L < 1$) there is a threshold for the high return (R_H) from which these fundamentals lead to multiple equilibria.

Proof: Let $\Gamma(R_L, R_H)$ denote the set of equilibria for these parameters. From (1), (7) and *proposition 1*, I directly get that, if $R_H > 3 - 2R_L$, then the following two elements belong to $\Gamma(R_L, R_H)$:

$$\gamma_{illiq}(R_L, R_H) \equiv \begin{cases} P^*_{illiq} = R_L \\ \lambda^*_{illiq} = \tilde{\lambda} < 1/2 \\ \eta^*_{illiq} = 0 \end{cases}$$

$$\gamma_{liq}(R_L, R_H) \equiv \begin{cases} P^*_{liq} = R_L + \eta^*_{liq}(R_H - R_L) \\ \lambda^*_{liq} = 1 \\ \eta^*_{liq} = \frac{1}{3} \end{cases}$$

The existence of a high-return threshold for multiple equilibria depends neither on the assumption that utility is logarithmic nor on the choice of parameter values. Instead, it relies on the non-convexities due to positive externalities. As the key ingredient to the existence of a high-liquidity equilibrium is agent H's participation to the secondary market, the only needed restriction in the general model is $u'(0)$ not being too low[17].

In the appendix, I propose a generalization of *proposition 2* with respect to p, q, r, and $u(.)$, and I show that this result extend to the general model.

3.2 Externalities, self-insurance and welfare

This subsection studies the mechanisms by which endogenous liquidity dry-ups affects welfare. The key starting point of this exercise is that, in this economy, the market may fail to allocate resources efficiently. I first state it formally, then give the main intuition and propose a numerical illustration based on the same parameter values as those used for figure 2.

*Proposition 3 (**market failure**)*

A liquidity dry-up is a Pareto-dominated equilibrium, both from an ex-ante and an ex-post point of view.

Ex-post inefficiency means that conditionally on his type, no agent ends up better-off[18] in a liquidity dry-up than in the corresponding high-liquidity equilibrium. Ex-post inefficiency of course implies ex-ante inefficiency: expected utility is lower in the case of a dry-up.

When both exists, how can every agents be worse-off in $\gamma_{illiq}(R_L, R_H)$ than in $\gamma_{liq}(R_L, R_H)$?

- First, because resources are wasted in the storage technology (long-run investment is on average more productive).
- Second, because self-insurance has negative externalities (it ex-post decreases market participation which hinders risk sharing).

When an agent optimally chose to issue a claim on a high-return project, it increases average quality and all claims can be sold for a better price. The social benefit is thus higher than the individual cost. However, in a liquidity dry-up, liquidation is anticipated to be so painful that agents optimally choose to self-insure. The fact is that being self-insured prevents them ex post to issue claims on high-return projects. The social cost of self-insurance is therefore higher than the private benefit.

In order to illustrate this, it is useful to identify the first best allocation of resources as a benchmark.

[17] For instance, Inada conditions are sufficient but not necessary.

[18] In fact, in this example, all types of agents are strictly worse-off in a dry-up.

The first-best allocation

To help compare risk sharing across allocations, I define ex-interim and ex-post wealth as follows:

$$\begin{cases} W_j \equiv 1-\lambda+\lambda R_j \\ W_j^* \equiv 1-\lambda+L_jP+(\lambda-L_j)R_j \end{cases} \tag{8}$$

They are, respectively, the date-2 wealth of agent j computed at the *beginning* of date 1 (that is, before liquidation decision) and the date-2 wealth of agent j computed at the *end* of date 1 (that is, after liquidation decision).

At the first-best allocation, aggregate output is maximized and agents are perfectly insured. Output maximization requires that all agents invest their full endowment in the risky technology ($\lambda = 1$). As all ex-post agents have the same preference, full insurance implies that they receive the same share of the pie.

The maximum per capita resources as of date 1 in the economy considered in figure 2 is given by $0.5\,(W_H+W_L) = E\,[R] = 1.5$, which allow for 0.75 unit of consumption good available for consumption at each date for each agent. Of course, with this allocation, marginal utility is equal across periods and states of nature, and ex-post wealth is hence equal across agents ($W_H^* = W_L^*$). The corresponding date-0 expected utility is $U_0^{FB} = -1.15$.

Table 1 displays such allocation[19].

Table 1: The first-best allocation ($R_H = 2.25$ and $R_L = 0.75$)

C_{tj}	Date 1	Date 2
State H	0.75	0.75
State L	0.75	0.75

W_j	W_j^*
$R_H = 2.25$	1.5
$R_L = 0.75$	1.5

In this allocation, per capita resources are maximal: $0.5\,(W_H+W_L) = E\,[R]$, consumption is equal across states and date: $C_{tj} = E[R]/2, \forall t, j$ and the reached level of expected utility is $U_0^{FB} = -1.15$.

This allocation maximizes date-0 expected utility but is not ex-interim incentive compatible under the assumption of private information on project return. Indeed, if a planner were to bundle these assets and sell them in the secondary market[20], the corresponding unit price would be $P_{FB} = E\,[R]$. However, as $R_L < P_{FB} < R_H$ incentives to sell are distorted: agents with high-return projects have an incentive to retain part of their projects as it is a more efficient way to provide for date-2 consumption needs. Consequently P_{FB} cannot be a competitive equilibrium price.

The competitive allocations

As depicted in figure 2, there are two stable competitive allocations when $R_H = 2.25$ and $R_L = 0.75$.

In γ_{liq}, all resources are invested in the long run. From a date-1 perspective, and because investment has been determined at date 0, it is as if agent H had a date-2 endowment of R_H. Of course, this agent wants to smooth consumption and wishes to transfer resources across period. Because of adverse selection, he cannot do it 1 to 1: the rate of transformation is 1 to $\frac{P^*}{R_H} = 0.56$. This means that his investment is relatively illiquid and that issuing claims on it is costly. However, such issuance has a positive externality on agent L, the lemon owner: he does an arbitrage and transfers resources at the rate 1 to $\frac{P^*}{R_L} = 1.67$. Still, it is *ex-ante* and *ex-post* incentive-compatible for agent H to issue claims on his projects. *Ex ante* because liquidation still dominates storage and *ex post* because it is, in this case, the only way to

[19] It is easy to check that this allocation maximizes date-0 expected utility subject to the following aggregate resources constraint only ($0.5\,(C_{1L}+C_{1H}+C_{2L}+C_{2H}) \leq E\,[R]$).

[20] A way to implement this allocation would be to do so and to share the proceeds equally.

obtain consumption goods at date 1. The latter justification is crucial and illustrate the fact that long-term investment can be seen has a secondary-market-participation commitment technology. So, in γ_{liq}, thanks to their decision to *ex ante* tie their hands, agents are *ex post* pretty well insured: they still face the risks of project failure, but in that case they take advantage of adverse selection and get compensated by a relatively high price on the market. Furthermore, they do not face the risk to liquidate good assets in an illiquid market[21]. Table 2 displays this allocation.

Table 2: The high-liquidity competitive allocation (γ_{liq})

C_{tj}	Date 1	Date 2
State H	0.625	1.125
State L	0.625	0.625

W_j	W_j^*
$R_H = 2.25$	1.75
$R_L = 0.75$	1.25

In this allocation, the size of the pie is still maximum ($0.5\,(W_H + W_L) = 1.5$) but the market fails to share it equally ($W_H^* > W_L^*$). As a consequence, expected utility is lower: $U_0^{\gamma_{liq}} = -1.29$. However, the market still provides some insurance as there is a positive transfer of resources from the "lucky" state H to the "unlucky" state L ($W_H - W_H^* = W_L^* - W_L = 0.5$).

On the other hand, in γ_{illiq}, agents anticipate that the secondary market will be illiquid and will provide poor insurance. Accordingly, they optimally choose to compensate with self-insurance: they use storage to hoard liquidity. This is a coordination failure: as no agent ties his hands, there is individually no incentive to do so, and no one provides externalities in this low-liquidity equilibrium. Thus, paradoxically, when agents choose to self-insure, they end-up rather poorly insured. They are not able anymore to transfer resources from state H to state L ($W_j = W_j^*$). Even worse, as long-term investment dominates storage, self-insurance is wasteful and the size of the pie decreases. ($0.5\,(W_H + W_L) = 1.22 < 1.5$). As a result of these two combined effects, expected utility drops further $U_0^{\gamma_{illiq}} = -2.21$. The low-liquidity competitive allocation is shown in table 3.

Table 3: The low-liquidity competitive allocation (γ_{illiq})

C_{tj}	Date 1	Date 2
State H	0.57	0.97
State L	0.45	0.45

W_j	W_j^*
1.53	1.53
0.9	0.9

In this allocation, there is no transfer across states ($W_j = W_j^*$) and the size of the pie is not maximized ($0.5\,(W_H + W_L) = 1.22 < 1.5$). As a result of these two combined effects, expected utility drops further $U_0^{\gamma_{illiq}} = -2.21$.

The fact that liquidity hoarding might be wasteful is not new (see for instance Diamond (1997), Holmström and Tirole (1998) and Caballero and Krishnamurthy (2008)) and the feedback effect between liquidity and investment is present in Eisfeldt (2004). However, that liquidity dry-ups can endogenously arise for the very reason that investors self-insure against it is a new result[22].

Market failure opens the door to government intervention. This is the subject of the next subsection.

[21] There are empirical evidences that such a risk is priced on financial markets. See Acharya and Pedersen (2005).

[22] The traditional tool used to generate liquidity dry-ups is cash-in-the-market pricing. See the following section for intuition and references.

3.3 The Government

Implementing the second-best allocation in the presence of positive externalities is a standard and rather simple problem[23]. I propose in this subsection a public liquidity insurance scheme that enables the government to achieve such a goal. For simplicity, I do this under *assumption 1*, but it can readily be extended to the general case. Also, I assume that the fundamentals are such that a market failure is possible[24].

The public liquidity insurance

The idea for the insurance is extremely simple. The bad equilibrium is a coordination failure, which happen when investors fear to have to sell claims in an illiquid market. Therefore, if the government pledge to compensate them for the loss (with respect to storage) in such a case, the incentive to self-insure vanishes and the only possible outcome is the high-liquidity equilibrium. Of course, this result rely on the possibility for the government to levy a break-even lump-sum tax after observing aggregate agent behavior. This is the reason why a private agent could not do it (the reader might want to check that the tax scheme is not incentive compatible). This public liquidity insurance is actually very similar to Diamond and Dybvig's (1983) demand deposit insurance with the same assumptions about the fiscal ability of the Government

Proposition 4 ***(public insurance)***

The public liquidity insurance implements the second-best.

Formal proof: see appendix.

Here is the intuition. Under this public liquidity insurance, the ex-ante trade-off between return and liquidity (with respect to storage) disappears and the date-0 first order condition for $\lambda = 1$ always holds:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

Whatever the anticipated date-1 market price, $\lambda^* = 1$ maximizes expected utility. Hence, it is a dominant strategy to be fully invested in the long-term technology. Under this scheme, the only one equilibrium corresponds to $\gamma_{liq}(R_L, R_H)$ and the insurance is never claimed. As in Dybvig and Spatt (1983), such an insurance is thus free.

The Pareto improvement through public liquidity provision *absent aggregate shocks* departs from the literature. In Diamond and Dybvig (1983) and Holmström and Tirole (1998), there is a role for the government only in the case of aggregate uncertainty about the fundamentals. This discrepancy might be understood when related to Allen and Gale (2003). They show that under complete markets for aggregate risk, if intermediaries can offer complete contracts, the equilibrium is incentive-efficient (corresponds to the first-best allocation). When contracts are incomplete (they focus on demand-deposit contracts) the equilibrium is constrained efficient (second-best). However, if markets are not complete, the outcome is generally inefficient. I my model, demand-deposit contracts are exogenously ruled-out and the market can fail, even under complete markets for aggregate risk.

Implementation

There are several ways to implement the public liquidity insurance. For instance, the government may pledge to buy any claim at a price of 1. Of course[25], sellers would only claim

[23] See Dybvig and Spatt (1983).

[24] That is: $R_H > 3 - 2R_L$, see *proposition 2*.

[25] The following subsidy to liquidation (and a break even lump-sum tax) would lead to an equivalent outcome: $Subs(L) = \min\{(1-P), 0)\}$

this insurance in the case of a dry-up. To break even, the government needs to levy the following lump-sum tax:

$$\tau(P) = \begin{cases} (1-P)\sum_j \frac{L_j}{2} & ;P < 1 \\ 0 & ;P \geq 1 \end{cases}$$

Where τ is the per capita lump-sum tax needed to fund the insurance.

The net effect of such a scheme is thus a transfer from agents that liquidate few to agents that liquidate more:

$$transfer_j = \begin{cases} (1-P)\left[L_j - \frac{\sum_j L_j}{2}\right] & ;P < 1 \\ 0 & ;P \geq 1 \end{cases}$$

Such a transfer is always feasible. As the incentive constraints are circumvented thanks to government regalian power to raise lump-sum taxes, the only remaining issue is the one of binding resource constraints. However, they will never be binding. First, from an aggregate point of view, because the value of aggregate resources cannot decrease over time. Second, from an individual point of view, because a highly negative transfer to agent H would force him to liquidate part of its portfolio. This would trigger positive externalities and increase the average value of traded claims which, in turn, would decrease the size of the needed transfer and relax the government budget constraint. Consequently, agent H could never run out of resources because of this scheme[26].

Policy implications

During financial crises, the fear of a credit crunch[27] might lead to various public interventions such as liquidity injection, bank recapitalization or even nationalization. To assess them in the light of self-fulfilling liquidity dry-up mechanisms, it is first important to note that the public liquidity insurance is only effective ex ante. Assume the Government did not commit to bail the market out and agents coordinated on the low-liquidity equilibrium. It is still feasible to implement the scheme, but it would not be a Pareto improvement anymore. As all agents would be self-insured, there would be mostly lemons in the market, it would not restore liquidity[28], and the taxpayer would pay the burden of the operation.

Since only the promise of a future market bailout could restore future liquidity in such a world, it raises the question of moral hazard. This is indeed well known that public intervention might induce investor to take on too much risk in the future[29] and sow then the seeds of the subsequent crisis. My results suggest the existence of an additional term to this usual trade-off. Indeed, the public liquidity insurance improves expectations about market liquidity, shifts investors horizon towards the long-run and actually avoids liquidity dry-ups. So, this model advocates for policies that insure investors against sudden generalized price drops but let them with the long-run risk. If the floor price is enough penalizing with respect to average long-run returns, the moral hazard problem might be contained. In a sense, the Government should design a policy that reduces enough adverse selection so that it restores self-fulfilling market liquidity but not too much. Otherwise, agents will find it profitable to game the system and invest in -or simply roll over- bad projects. Anyway,

[26] Where agent H to liquidate its whole portfolio, the equilibrium price would be $E[R] > 1$ which is not consistent with agents selling claims to the Government.

[27] See Bernanke, Gertler and Gilchrist (1989) for an illustration of the financial accelerator effect.

[28] While public interventions such as liquidity injections (through TARP for instance), eased the short-term funding of financial institution in the fall 2008, one might easily argue that they did not restore liquidity in the securitized markets.

[29] See for instance Diamond (1984) and Freixas et al. (2004).

the underlying moral hazard problem might be the price to pay to exit a flight-to-liquidity trap and avoid a credit crunch.[30]

If the government does not want to provide such insurance because of moral hazard concerns, it might still try to forbid coordination mechanisms that would lead to a liquidity-dry-up. A temporary ban on short-selling might be viewed as such an attempt (see Brunnermeier and Pedersen (2005) for an illustration that short-selling might coordinate a switch toward a liquidity dry-up). Also, and more generally, lengthening investors horizon might reduce adverse selection on asset markets. This improves liquidity and increases the propensity to invest in long-term risky projects that are on average more productive than short-term investment. If investors horizon is short, such socially profitable opportunities are not concreted and there is a welfare loss. According to that mechanism, the law-maker should avoid policies that shorten investors horizon, as might for instance be the case with policies based on mark-to-market accounting[31]. Finally, transaction taxes, which have been considered as a mean to reduce speculation and promote long-term investment (see for instance Tobin (1974) and Stiglitz (1989)) might in fact prove counter-productive: it would decrease liquidity and could reduce the propensity to invest in long-term projects (through an increase in self-insurance) which would deter liquidity further.

4 Liquidity dry-ups and the *2007-ongoing* financial crisis

The literature on financial crises is abundant and is considerably expanding since Summer 2007 and the beginning of the current crisis[32]. In this section, I present an application of my model to that financial crisis and I show how it can contribute to a better joint explanation of some empirical observations.

Stylized facts

Consider the following stylized observations related to the crisis (supporting tables and graphs can be found in the appendix):

1. MARKET BREAKDOWN. The Asset Backed Securities (ABS) business collapsed and has not recovered yet: prices dramatically dropped and so did new issuance.

2. UNDERPRICING. Recent prices seem to be "too low" with respect to fundamentals. For instance, according to the computations of the Bank of England (2008), the Spring 2008 prices of the ABX.HE index[33] implied an expected loss of 38% on subprime mortgages, which could correspond to a 76% foreclosure rate with a 50% recovery value. Both not really credible. In June 2009, the ABX.HE prices are even lower than at that time and the foreclosure rate on subprime mortgages "only" hiked to around 20%. This "underpricing" view might be all wrong, but if it is shared by a number of key observers (central bankers and leading academics[34] for instance), it is at least worth to confront it to economic theory.

[30]These results are related to the policy recommendations of Ricardo Caballero about the 2007-2008 financial turmoil. He indeed advocated for not caring so much about moral hazard *during* financial crisis (Financial Times, August 08), and he for instance proposed, in order to launch a virtuous circle, that: "The government pledges to buy up to twice the number of shares currently available, at twice some recent average price, five years from now" (VOX, 22 February 2009)

[31]Allen and Carletti (2008b) shows that mark-to-market accounting is not desirable during cash-in-the-market pricing episode. My results suggest that it might also have an impact on the ex-ante propensity to invest in long-run productive projects.

[32]See Brunnermeier (2009) for a chronology of the crisis and Allen and Carletti (2008) for a focus on liquidity issues.

[33]http://www.markit.com/en/products/data/indices/structured-finance-indices/abx/documentation.page?#. See Gorton (2008) for a description.

[34]For instance, Allen and Carletti (2008) endorse the analysis of the BoE and Uhlig (2009) states that prices appear low compared to some benchmark fundamental value.

3. SURGE IN DEMAND FOR SAFE ASSETS. From September 2008, there has been a dramatic increase of aggregate excess reserves of the major deposit institutions in the US: it increased by a factor bigger than 18. Also, despite the huge increase in supply (both from the FED and the US Government), T-Bills prices have been and are still high. Therefore, demand for short term safe assets should be very high too.

The limits of the classic view

One of the classic view on fact 1 and 2 is that the market incurred an episode of cash-in-the-market pricing (Allen and Gale, 1994)[35]. The idea is the following: there is an opportunity cost to hoard cash if the alternative, a productive long-term asset, has a higher expected return. This cost should be compensated by gains in some states of the world. These gains are realized when there are "many" sellers of long-term assets. In this case, as demand is limited by the amount of cash hoarded, the price might drop below the fundamental value. Were some deep-pocket investors presents, they would do an arbitrage and bring the price back to the fundamental value. Cash-in-the-market pricing is an application of the limits of arbitrage put forward by Shleiffer and Vishny (1997). Clearly, there were episodes of the crisis that had the flavor of such a mechanism.

However, such mechanisms do not really help to explain fact 3. Indeed, limits of arbitrage do certainly exist, but are likely to fade away with time. I certainly do not want to discuss how to tell between what is the short run and what is the long run, but I point out that fact 3 resembles more a situation where deep-pocket agents exist and that we need another theory to explain why they require such a discount to buy those assets. To be sure, my point is not to dismiss the cash-in-the-market explanation, but rather to stress the need for a complementary mechanism that could explain why the ABS market seem not to recover once there potentially is a lot of cash-in-the-market.

A possible joint explanation: adverse selection and liquidity premium

Think of financial institutions, including their off-balance structured investment vehicles, as the investors of my model[36]. If they anticipate future market illiquidity, it is optimal for them to hoard liquidity. If they anticipate that the other financial institution will do the same, market illiquidity is self-fulfilling since self-insured investors would never sell good assets at fire-sale prices. This might help explain facts 1 and 2 *together* with fact 3. Indeed, the model can explain a drop in volume and price (fact 1), down to a level lower than *average* fundamental price[37] (fact 2), and together with a surge in demand for safe short-term assets (fact 3), despite the presence of deep-pocket agents. As the model does not rely on limits to arbitrage, it might be seen as better suited to capture long lasting effects.

[35] Cash-in-the-market and similar mechanisms have been used in the literature to "generate" liquidity dry-ups and related events. In Bolton, Santos, and Scheinkman (2009), adverse selection is combined with cash-in-the-market pricing to deliver multiple equilibria with prices below fundamentals. In Brunnermeier and Pedersen (2009), arbitrageur have limited resources and face margin calls from their funding institutions. When these margin calls increase with volatility, they create a liquidity spiral and assets might be traded below fundamental value. Morris and Shin (2004) and Genotte-Leland (1990) can also be understood in a similar way: there is a given downward sloping demand curve and arbitrageurs face resources constraints that are functions of price.

[36] Investors would correspond to financial institutions that have limited resources that they lend to private agents (mortgages or corporate loans for instance). Only these institutions have the needed skills to screen applicants. Proper screening is assumed costly so that a moral hazard problems prevent them to do it within a principal-agent relationship. However, monitoring of ongoing projects is assumed costless and the moral hazard problem does not apply after the investment is indeed committed. The secondary market can therefore be viewed as the securitization and sale of the initial loans. Buffer agents could be, for instance, US pension funds or sovereign funds from oil-exporting countries.

[37] The underlying assumption the BoE's computation is that traded assets are a representative sample of the whole set of existing ones. This kind of *Mark-to-Market* valuation procedure is of course biased in presence of adverse selection: retained assets are on average better than traded ones.

5 When illiquidity shocks improve... liquidity

In this section, I consider the case where investors face idiosyncratic preference shocks. At date 1, they indeed learn whether they are *normal* or *early* consumers. The latter deriving utility from consumption at date 2 only. While the presence of such *early* agents is not necessary to the existence of liquidity dry-ups, it still has an impact on market liquidity and hence risk sharing. In the classic banking literature, such preference shocks are interpreted as illiquidity shocks and incentive compatibility problems arise when patient agents pretend they are impatient (see for instance Diamond and Dybvig (1983) and Jacklin (1987)). This might generate bank runs and hinder the ability of deposit contracts to improve risk sharing.

When ex-ante pooling of resources is ruled out and when private information about future payoffs is already a source of adverse selection, I find that private information about idiosyncratic liquidity shocks enhances market liquidity. Adverse selection is indeed reduced because illiquid agents issue claims irrespective of the underlying project project return. As market liquidity improves risk sharing, this suggests that idiosyncratic liquidity shocks need not be socially a bad thing[38].

Introducing illiquidity shocks also permits to study the question of transparency[39]. I show in subsection 5.2 that transparency about liquidity position has two effects. First, it decreases the liquidity discount for "illiquid" investors[40]. This can be counter-intuitive, but it explains why sellers on a secondary market often pretend they sell for reasons exogenous to the asset's quality[41]. Second, transparency increases the liquidity discount for the others investors: as they did not incur a liquidity shock, it is more likely that they sell because of adverse selection. As a consequence, the effect of transparency on ex-ante risk sharing is ambiguous, as suggested by Holmström (2008).

Formally, in this extension, agents differ by the subjective factor β they use to discount date-2 utility: $\beta \in \{0,1\}$ with $Prob(\beta = 1) = p$ and $0 < p < 1$. Also, for the sake of simplicity, I keep the assumption that period utility is logarithmic and that projects succeed or fail with equal probabilities.

The machinery of the model is basically the same, the major difference is that there are now *four* types of agents as of date 1. Agents indeed differ on two dimensions: project return R_j and patience β_i. From here onward, I will name agents after their type ij where j still reflects projects return and $i = e,n$ accounts for *early* ($\beta_e = 0$) and *normal* ($\beta_n = 1$) agents respectively. Indexes on date-1 decision variables are modified accordingly.

5.1 Equilibrium with illiquidity shocks

Investors are still ex-ante identical. They solve:

$$\max_{\lambda, L_{ij}, S_{ij}} U_0 = E_0\left[\ln(C_1) + \beta_i \ln(C_2)\right] \tag{9}$$

$$s.t. \begin{cases} C_{1ij} + S_{ij} = 1 - \lambda + L_{ij}P \\ C_{2ij} = (\lambda - L_{ij})R_j + S_{ij} \\ 0 \le L_{ij} \le \lambda \le 1 \end{cases}$$

Where $i \in \{e,n\}$with $Prob(i = n) = p > 0$, $\beta_e = 0$ and $\beta_n = 1$, and $j \in \{L,H\}$with $Prob(j = H) = 0.5$.

[38]In the June 2009 release of its Financial Stability Report, the Bank of England calls for calling for a better self-insurance against such shocks. As we have seen, such a policy might have serious drawbacks with respect to future market liquidity.

[39]This is relevant because the lack of transparency as been widely blamed for the current crisis.

[40] I thank Jean Tirole for pointing out this relationship between liquidity position disclosure and market liquidity discount, as well as for the example below.

[41]For instance, in their ads for second-hand cars, student often mention that they want to sell because they are graduating and moving out of town.

While the liquidation decision of *normal* agents (nL and nH) are still given by (4) and (5), those of *early* agents (eL and eH) are now determined by their first order condition for $L_{ej} = \lambda$, which always holds:

$$\frac{P}{C_{1ej}} \geq 0 \quad ; \quad j = H, L$$

As they only care about utility of consumption at date 1, they sell off any project they hold, whatever the quality[42]:

$$L_{eH}(P,\lambda) = L_{eL}(P,\lambda) = \lambda$$

Accordingly, the proportion of claim to high-return projects is:

$$\eta(P) = \begin{cases} \eta_{illiq} = \frac{1-p}{2-p} & ;P < 1 \\ \eta_{liq} = \frac{2-p}{4-p} & ;P > 1 \\ \eta_1 \in \left[\eta_{illiq}, \eta_{liq}\right] & ;P = 1 \end{cases}$$

Which implies that equilibrium market liquidity depends on p. Equilibrium prices are still given by the fixed point $P^{'} = P$, with:

$$P^{'} = R_L + \eta(P)(R_H - R_L)$$

It follows that for any admissible value of p and R_L there is a range for R_H that implies multiple equilibria (see appendix for the proof). And the two possible levels of liquidity in stable equilibria depend on p:

$$\begin{cases} \eta^*_{illiq} = \frac{1-p}{2-p} & ;P < 1 \\ \eta^*_{liq} = \frac{2-p}{4-p} & ;P > 1 \end{cases}$$

Effect on equilibrium market liquidity

I present here a short comparative statics exercise.

*Proposition 5 (**market liquidity**)*

For any admissible value of R_L, as *the probability $(1-p)$ of being hit by an illiquidity shock increases:*

1. *Equilibrium market liquidity increases:* $\frac{\partial \eta^*}{\partial(1-p)} > 0$
2. *The range for a high-liquidity equilibrium increases*
3. *The range for a low equilibrium decreases*

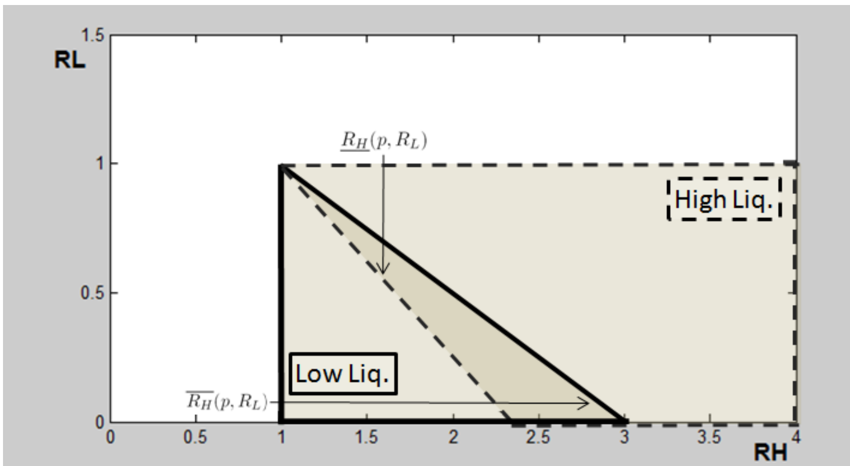
Proof: straightforward.

First, it might appear counter-intuitive but illiquidity shocks enhance market liquidity. In fact, investors hit by such shocks liquidate assets irrespective of their quality. As a consequence, average quality increases with $1-p$, which is also the proportion of *early* agents. Second, when this proportion increases, a lower R_H is needed for a high-liquidity equilibrium to exist. As *early* agent's liquidation behavior imply more units of claims to high-return projects, a lower return par unit is needed to bring the price up to 1. Third, as soon as $p < 1$, there are always claims to high-return projects that are traded. So, if R_H is sufficiently high for the price at the low level of market liquidity to be higher than 1, it cannot be an equilibrium. Obviously, the more the *early* agents, the lower that upper bound.

[42] Again, if $P = R_L$ they are indifferent, in which case I assume for simplicity that they liquidate their whole position.

The lower bound for a high-liquidity equilibrium is $\underline{R_H}(p,R_L) \equiv R_L + \frac{4-p}{2-p}(1-R_L)$. The upper bound for a low-liquidity equilibrium is $\overline{R_H}(p,R_L) \equiv R_L + \frac{2-p}{1-p}(1-R_L)$. For a given p, they define three regions in the space of admissible parameter values for R_H and R_L. Figure 3 illustrates this for $p = 0.5$.

Figure 3: 3 regions ($p = 0.5$)



This figure presents the three regions that define multiplicity of equilibrium. As $(1-p)$ increases, the low-liquidity-equilibrium region (delimited by the solid lines) shrinks and the high-liquidity-equilibrium one (dashed lines) widens. The overlap is the region with multiple equilibria.

Impact on risk sharing

To interpret the implications of the presence of illiquidity shock on the risk sharing, I first generalize (8), to the four-agent case:

$$\begin{cases} W_{ij} \equiv 1-\lambda+\lambda R_j \\ W_{ij}^* \equiv 1-\lambda+L_{ij}P+(\lambda-L_{ij})R_j \end{cases}$$

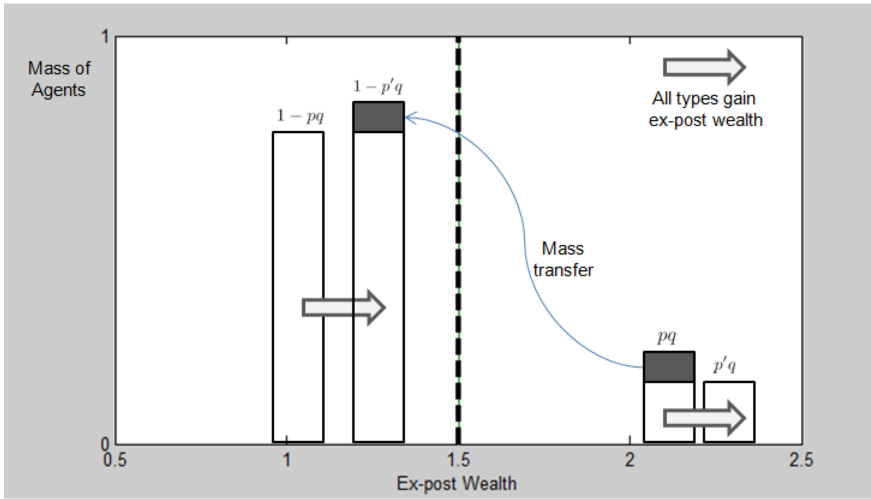
And I define the wealth gains from trade as the difference between the two: $\phi_{ij} \equiv W_{ij}^* - W_{ij} = L_{ij}(P-R_j)$.

Corollary 1

For any given ex-post agent, the wealth gains from trade increases with the proportion of early agents: $\frac{\partial \phi_{ij}}{\partial (1-p)} \geq 0$, $\forall ij$

Proof: see appendix. Conditionally of being of any given type, ex-post wealth increases with liquidity. How can it be possible? Here is the intuition: The mass of agents in state eH increases, so they provide more externalities on aggregate (to other type of agents *and* among themselves). This increase in positive externalities suggest a better risk sharing. Indeed, the unlucky lemon owners (eL and nL) benefit from a bigger transfer. However, in the high-liquidity equilibrium, agents nH, which were already the best-off, also take an advantage thereof. This is depicted, for a high-liquidity equilibrium, in figure4.

Figure 4: The effect of an decrease in p on ex-post wealth



This figure illustrates, in a high liquidity equilibrium, the effect on ex-post wealth of an increase in the probability of being an *early consumer* $(1-p)$. As p decreases to p', the mass of agents nH decreases, but liquidity increases and ex-post wealth increases for all type of agents.

In other words, an increase in $1-p$ (the probability of being hit by a liquidity shock), enhance risk sharing across types, on the project return dimension (from iH to iL thus) but deteriorates risk sharing across types, on the patience dimension[43]. Note that in a low-liquidity equilibrium, as there is limited participation, agent nH does not take advantage of the increase in liquidity.

This comparative statics exercise does not directly apply to ex-ante welfare as a change in p as an impact on the expected utility function itself. Nevertheless, it is still worth trying to disentangle the different forces at stake:

- When there are illiquidity shocks, liquidity increases and it becomes less costly to transfer resources intertemporally which is likely to be welfare improving[44].
- There is a cross-subsidy from "lucky" high-return project owners to "unlucky" lemon owners. As long as moral hazard is not concerned, this is also likely to improve welfare.
- There might be a cross subsidy from agents incurring the liquidity shock to agents the best-off agents. This is generally ex-ante undesirable.

The conclusion of this subsection is that the existence of idiosyncratic illiquidity shocks need not be socially a "bad thing" once market liquidity considerations are taken into account.

5.2 Transparency and liquidity

Now, still in the model with illiquidity shocks, I assume that there exists a technology that enables agents to credibly disclose their patience parameter β_i. What I have in mind is for instance a bank that could credibly disclose its liquidity position. In that case, buyers are able to classify sellers in two categories and they update their priors about average quality accordingly. There are thus two separated markets.

[43]Table 4, in the appendix, displays the high-liquidity competitive allocation as a function of p.

[44]The best way to illustrate it is to let p go to 0. In that case, the market price is $E[R]$ and all agents have the same ex-post wealth. Note that this first best allocation does not exactly correspond to the one of section 3 as agents decide to consume at date 1 only.

*Proposition 6 (**transparency**)*

Early agents are better-off disclosing their liquidity position

Proof (for the high-liquidity equilibrium case): Assume disclosure and let η_e^* and P_e^* denote equilibrium liquidity and price, conditionally on the seller to be illiquid (to be an *early* agent). Such agents liquidate any project they hold: $L_{eH} = L_{eL} = \lambda = 1$. Thus $\eta_e^* = \frac{1}{2}$ and $P_e^* = E[R] > P_{liq}^*$ with $P_{liq}^* = R_L + \eta_{liq}^*(R_H - R_L)$ being the equilibrium price without disclosure. Therefore, as these agents get a better price, their ex-post wealth increase, they are better-off and they have no incentive to deviate (that is, to issue claims without liquidity position disclosure).

However, the price *normal* agents can get is back to the level without illiquidity shocks ($P_n^* = P_{liq}^*(p = 1) < P_{liq}^*(p < 1)$) and they are thus worse-off under disclosure. In that sense, transparency modify ex-ante risk-sharing opportunities and might deteriorate liquidity. The view that it is not the lack of transparency but rather the asymmetry of information that can cause illiquidity is expressed by Gorton (2008) and Holmström (2008). An extreme way to show this in my model is the following: imagine for a moment that agents only learn the quality of their projects between date 1 and date 2. Under such an assumption, agents are less informed but adverse selection is not an issue anymore and the first best allocation is the only equilibrium of the model.

6 Conclusion

In this paper, I have shown the major role adverse selection plays in self-fulfilling liquidity dry-ups. In normal times, the feedback effect between liquidity and investment magnifies the externalities linked to the issuance of good quality claims for consumption reallocation motives. However, the anticipation of a dry-up suffices to reverse the process: the feedback effect magnifies the negative externalities linked to self-insurance. This sheds new light on financial crises. The result that the promise of a market bailout implements the second-best might be important as well. Indeed, most of the debates on public intervention during financial crises focus on moral hazard, and the implications of adverse selection have been rather overlooked.

This mechanism seems powerful but the model does not help to assess the magnitude of the potential welfare gain in comparison with welfare losses usually attributed to moral hazard. Still, I suspect that the introduction of moral hazard would not negate the results and that it is worth extending the model in that direction. This view is based on the fact that, even in the high-liquidity equilibrium, agents are only partially insured. There is thus still room for incentive to exert effort or to prevent risk-shifting. On the other hand, aggregate shocks to preferences or productivity might also have a negative impact on the government budget constraint and the public liquidity insurance would probably need to be more sophisticated to ensure that this constraint is satisfied in all states of nature.

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A Appendix: Stylized Facts

Here is the first line of the *2008 Global Securitisation Annual* Edited by Barclays Capital Research:

> "2008 may well become known in the global securitisation markets as the annus horribilis."

Figure 5: Collapse of the securitization business

F.126 Issuers of Asset-Backed Securities (ABS)
Billions of dollars; quarterly figures are seasonally adjusted annual rates

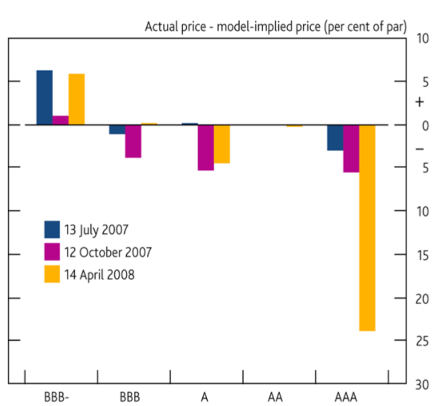
		2003	2004	2005	2006	2007	2008	2008 Q1	2008 Q2	2008 Q3	2008 Q4	2009 Q1	
1	**Gross saving**	**1.1**	**1.0**	**0.8**	**0.7**	**0.6**	**0.6**	**0.6**	**0.6**	**0.5**	**0.5**	**0.5**	1
2	**Fixed nonresidential investment**	**0.9**	**0.2**	**0.2**	**0.2**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	2
3	**Net acquisition of financial assets**	**249.0**	**439.3**	**725.0**	**807.7**	**333.2**	**-422.0**	**-254.8**	**-455.8**	**-374.8**	**-602.8**	**-623.4**	3
4	Treasury securities	1.9	5.2	19.7	28.7	21.0	-12.3	-2.8	-11.4	-15.7	-19.4	-20.7	4
5	Agency- and GSE-backed securities (1)	82.3	-5.4	-32.4	23.9	19.3	-19.0	9.6	-19.5	-22.4	-43.8	-136.0	5
6	Other loans and advances	5.1	20.2	21.0	55.8	89.1	20.4	19.4	9.1	51.7	1.5	-17.3	6
7	Mortgages (2)	173.3	434.3	687.1	630.5	180.9	-365.2	-302.6	-379.5	-347.7	-430.8	-349.5	7
8	Home	122.9	382.2	572.4	513.7	33.9	-320.6	-269.3	-346.5	-313.1	-353.5	-332.0	8
9	Multifamily residential	7.9	6.4	16.5	13.9	22.0	-11.0	-10.6	-11.5	-9.8	-12.1	-4.9	9
10	Commercial	42.6	45.7	98.1	102.9	125.0	-33.6	-22.7	-21.5	-24.9	-65.2	-12.6	10
11	Consumer credit	-22.5	-25.3	32.5	60.2	19.5	-29.8	-5.2	-2.2	1.7	-113.7	-62.3	11
12	Trade credit	8.8	10.3	-2.8	8.5	3.4	-16.1	26.8	-52.5	-42.4	3.5	-37.6	12
13	**Net increase in liabilities**	**248.8**	**438.6**	**724.3**	**807.2**	**332.7**	**-422.5**	**-255.2**	**-456.3**	**-375.2**	**-603.2**	**-623.8**	13
14	Commercial paper	-36.0	-3.9	148.3	162.9	-194.2	-83.9	-134.7	-145.8	-123.1	68.1	-211.8	14
15	Corporate bonds (net) (5)	284.8	442.5	576.0	644.2	527.0	-338.6	-120.6	-310.5	-252.1	-671.3	-412.0	15
16	Discrepancy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	16
	Memo:												
	Securitized assets not included above												
17	Consumer leases (3)	-0.2	-0.7	-0.6	-0.5	-0.5	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	17
18	REIT assets (4)	15.6	59.6	17.3	2.8	-17.9	-39.7	-79.7	-19.4	-26.2	-33.4	-12.4	18

(1) Agency- and GSE-backed mortgage pool securities backing privately issued CMOs.
(2) Mortgages backing privately issued pool securities and privately issued CMOs.
(3) Receivables from operating leases, such as consumer automobile leases, are booked as current income when payments are received and are not included in financial assets (or household liabilities).
The leased automobile is a tangible asset; depreciation flows are included in line 1, and fixed investment flows are included in line 2.
(4) Included in table F.128.
(5) Net issuance less net acquisition of corporate bonds held as assets.

This figure displays the shrinkage of the balance sheet of ABS issuers.

Source: www.federalreserve.org

Chart 5 Anomalies in the prices of the ABX sub-prime index (2007 H1 vintage)(a)(b)



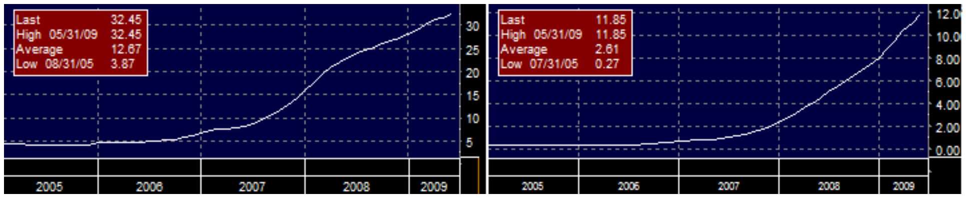
Source: Bank calculations using data from JPMorgan Chase & Co.

(a) The pricing model is an adaptation of that used in 'A simple CDO valuation model', Bank of England Financial Stability Review, Box 1, December 2005, pages 105–06.
(b) The loss given default rate on the underlying collateral is uncertain, but is assumed for the purposes of this chart to be 50%.

Underpricing of ABX Subprime index. Source: BoF Financial Stability Review October 2008

Figure 6: Fundamentals

(a) Delinquency rates

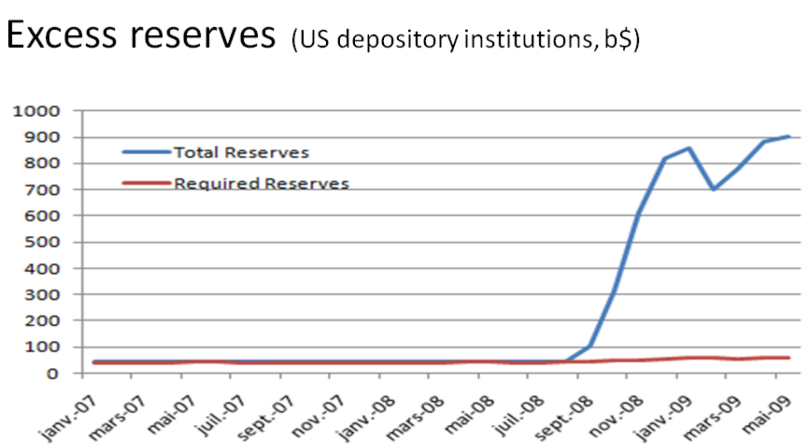


Delinquency rates on subprime (left panel) and prime (right panel) residential mortgages in the US. Source: www.bloomberg.com

(b) Foreclosure rates

Foreclosure rates on subprime (left panel) and prime (right panel) residential mortgages in the US. Source: www.bloomberg.com

Figure 7



While excess reserve never exceeded 10% of required reserves between January 2007 and August 2008, then it skyrocketed to more than 1000%. Source: http://www.federalreserve.org.

Figure 8

(a) Holdings of Treasury Securities

L.209 Treasury Securities
Billions of dollars; amounts outstanding end of period, not seasonally adjusted

		2002	2003	2004	2005	2006	2007	2008 Q1	2008 Q2	2008 Q3	2008 Q4	2009 Q1	
1	**Total liabilities**	**3609.8**	**4008.2**	**4370.7**	**4678.0**	**4861.7**	**5099.2**	**5299.1**	**5250.6**	**5777.5**	**6338.2**	**6804.4**	1
2	Savings bonds	194.9	203.8	204.4	205.1	202.4	196.4	195.3	194.9	194.2	194.0	193.9	2
3	Other Treasury issues	3414.9	3804.4	4166.3	4472.9	4659.4	4902.8	5103.8	5055.7	5583.4	6144.2	6610.5	3
4	**Total assets**	**3609.8**	**4008.2**	**4370.7**	**4678.0**	**4861.7**	**5099.2**	**5299.1**	**5250.6**	**5777.5**	**6338.2**	**6804.4**	4
5	Household sector	285.3	438.6	532.2	507.5	433.0	196.5	234.6	251.8	262.3	266.6	643.9	5
6	Savings bonds	194.9	203.8	204.4	205.1	202.4	196.4	195.3	194.9	194.2	194.0	193.9	6
7	Other Treasury issues	90.4	234.8	327.8	302.4	230.7	0.1	39.3	56.9	68.1	72.6	450.0	7
8	Nonfinancial corporate business	31.4	32.8	33.1	50.7	44.3	37.3	28.0	30.3	28.0	32.8	35.7	8
9	Nonfarm noncorporate business	42.8	44.9	50.2	56.2	56.3	65.8	67.4	68.4	69.3	69.3	68.3	9
10	State and local governments	354.7	364.2	389.1	481.4	516.9	531.5	523.6	522.2	535.7	522.7	526.3	10
11	Rest of the world	1285.5	1513.5	1813.6	1984.4	2126.2	2432.1	2584.1	2707.9	2913.5	3182.0	3341.0	11
12	Monetary authority	629.4	666.7	717.8	744.2	778.9	740.6	591.2	478.8	476.6	475.9	492.3	12
13	Commercial banking	205.8	132.3	107.8	97.1	95.2	112.5	111.9	99.7	117.6	95.8	116.2	13
14	U.S.-chartered commercial banks	86.4	93.5	74.2	64.2	61.9	73.0	64.1	49.5	71.5	53.4	69.5	14
15	Foreign banking offices in U.S.	116.7	33.7	27.8	27.9	27.1	30.6	40.1	37.2	31.3	38.1	45.3	15
16	Bank holding companies	1.3	2.7	2.0	1.1	2.5	7.1	6.3	11.8	13.7	3.1	0.6	16
17	Banks in U.S.-affiliated areas	1.5	2.5	3.8	3.9	3.7	1.8	1.4	1.2	1.2	1.2	0.8	17
18	Savings institutions	9.2	12.2	8.4	12.3	12.4	7.0	6.0	4.9	2.9	2.9	3.6	18
19	Credit unions	7.5	8.8	8.9	7.7	7.4	10.4	9.6	10.2	9.9	8.8	7.6	19
20	Property-casualty insurance companies	61.2	64.7	71.3	69.2	75.8	55.1	54.8	54.7	54.2	53.8	56.1	20
21	Life insurance companies	78.5	71.8	78.5	91.2	83.2	68.3	73.8	79.2	84.1	87.5	93.5	21
22	Private pension funds	103.6	116.9	113.0	116.5	130.8	169.5	173.7	177.9	185.8	184.9	185.9	22
23	State and local govt. retirement funds	158.9	148.6	151.0	153.8	157.1	168.8	169.4	169.1	171.6	174.6	177.3	23
24	Federal government retirement funds	50.2	55.3	60.7	68.4	76.7	88.0	96.6	98.9	105.9	112.3	116.7	24
25	Money market mutual funds	142.2	132.1	98.2	88.6	82.7	178.1	296.3	283.7	460.0	577.7	519.7	25
26	Mutual funds	134.8	144.2	149.0	155.7	160.7	179.2	182.7	188.3	191.3	187.9	194.2	26
27	Closed-end funds	3.9	4.6	6.8	7.1	7.4	5.6	5.4	5.2	4.7	3.2	3.3	27
28	Exchange-traded funds	2.0	2.1	4.9	9.7	13.1	19.7	21.0	22.6	26.6	27.0	31.6	28
29	Government-sponsored enterprises	25.7	13.5	12.9	13.1	14.2	15.5	15.9	16.2	16.6	16.7	17.6	29
30	ABS issuers	0.9	2.8	8.0	27.7	56.4	77.5	76.8	73.9	70.0	65.1	60.0	30
31	Brokers and dealers	-3.9	37.8	-44.6	-64.6	-67.0	-60.0	-23.7	-93.3	-9.2	190.6	113.8	31
	Memo:												
32	Federal government debt (1)	3637.0	4033.1	4395.0	4701.9	4885.3	5122.3	5322.6	5274.1	5800.6	6361.5	6826.9	32

(1) Total Treasury securities (table L.209, line 1) plus budget agency securities (table L.210, line 2) and federal mortgage debt (table L.217, line 12).

Figure 8a shows that Treasury securities issuance has been rather high as from 2008Q3. It also shows that the Household sector (which includes domestic hedge funds) and the Rest of the World are still big players on the demand side. Source: http://www.federalreserve.org.

(b) Yield on short-term (4w) Treasury Bills.

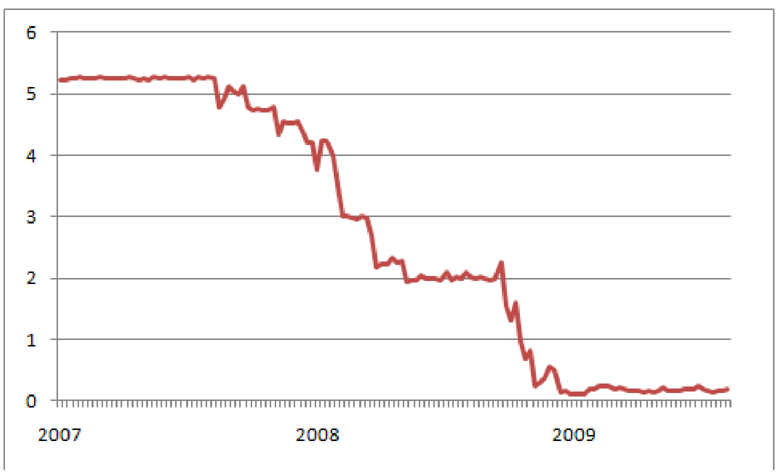


Figure 8b shows that despite the surge in supply there are still enough buyers of such assets for the yield to stay low, i.e. close to the Fed Funds target rate, without requiring that the Monetary Authorities increases substantially its position. Source: www.federalreserve.org.

B Appendix: proofs

B.1 Proof of *proposition 1* (self-insurance)

*Proposition 1 (**self-insurance**)*

Let $\lambda(P) \equiv \arg\max_{\lambda} U_0(\lambda, P)$ *be the set of solutions for a given* P *to the date* 0 *problem (3), then:*

$$\begin{cases} \lambda(P>1) = \{1\} \\ \lambda(P=1) = \left\{\left[\frac{1}{2},1\right]\right\} \\ \lambda(P<1) = \left\{\tilde{\lambda}\right\} \end{cases}$$

With $0 < \tilde{\lambda} < \frac{1}{2}$.

Proof

Let $\lambda_j^*(P) \equiv \arg\max_\lambda \ln((1-\lambda)P) + \ln(\lambda R_j)$ the optimal investment level conditionally on being of type j. I have,

$$\lambda_L^* = \begin{cases} 1 & ;P>1 \\ \{[0,1]\} & ;P=1 \\ 0 & ;P<1 \end{cases}$$

$$\lambda_H^* = \begin{cases} 1 & ;P>1 \\ \left\{\left[\frac{1}{2},1\right]\right\} & ;P=1 \\ 0.5 & ;P<1 \end{cases}$$

The cases for $P>1$ and $P=1$ are straightforward.

Consider now $P<1$ and let $U_j' \equiv \left[\frac{\partial U_0}{\partial \lambda} \,|\, j\right]$ be the marginal utility of λ conditionally of being in the state j. It is easy to check that:

$$\begin{cases} U_H'\Big|_{\lambda \in \left\{\left[0,\frac{1}{2}\right]\right\}} > 0 \\ U_L'\Big|_{\lambda \in \left\{\left[0,\frac{1}{2}\right]\right\}} < 0 \end{cases}$$

Expected utility maximization requires thus an interior solution in that case: $\tilde{\lambda} \equiv \left\{\lambda; U_H' + U_L' = 0\right\}$

B.2 Proof of *proposition 2* (Dry-ups)

Derive $P^* = R_L + \eta(R_H - R_L)$ with respect to p. It gives:

$$\begin{cases} \frac{\partial P^*_{illiq}}{\partial p} = \frac{-(R_H-R_L)}{(2-p)^2} < 0 \\ \frac{\partial P^*_{liq}}{\partial p} = \frac{-2(R_H-R_L)}{(2-p)^2} < 0 \end{cases}$$

First show that the lower bound on R_H for the existence of a high-liquidity equilibrium is decreasing in $1-p$:

Define $\underline{R}(p) \equiv R_L + \frac{1-R_L}{\eta_{liq}(p)} = R_L + \frac{4-p}{2-p}(1-R_L)$. Hence $\frac{\partial P^*_{illiq}}{\partial p} < 0$ and $\forall R_H \leq \underline{R}(p)$, $R_L + \eta_{liq}(p)(R_H - R_L) \geq 1$.

Then show that there is an upper bound for the existence of a low-liquidity equilibrium and that it is decreasing in $1-p$:

Define $\overline{R}(p) \equiv R_L + \frac{1-R_L}{\eta_{illiq}} = R_L + \frac{2-p}{1-p}(1-R_L)$. Hence $\frac{\partial P^*_{illiq}}{\partial p} < 0$ and $\forall R_H \geq \underline{R}(p)$, $R_L + \eta_{illiq}(p)(R_H - R_L) \geq 1$.

These do the proof.

B.3 Dry-ups: conditions for multiple equilibria in the general model

Consider the general problem:

$$\max_{\lambda, L_{ij}, S_{ij}} U_0 = E_0\left[u(C_1) + \beta_i u(C_2)\right]$$

$$s.t. \begin{cases} C_{1ij} + S_{ij} = 1 - \lambda + L_{ij}P \\ C_{2ij} = (\lambda - L)R_j + S_{ij}(1+r) \\ 0 \leq L_{ij} \leq \lambda \leq 1 \end{cases}$$

Where $i \in \{e, n\}$with $\beta_e = 0$, $\beta_n = 1$ and $Prob(i = n) = p > 0$, and $j \in \{L, H\}$with $Prob(j = H) = q$ and $0 < q < 1$.

*Proposition 2bis (**dry-ups generalized**)*

Let $\bar{\Omega}$ be the range of admissible values[45] *for parameters$\{R_L, p, q, r\}$. Let $\Gamma(\Omega, R_H) = \{(P^*, \lambda^*, \eta^*)\}$ denote the set of stable equilibria defined by (2) for a vector $(\Omega \in \bar{\Omega}, R_H)$ of parameters. Assume $u'(0) > u'(R_H)\frac{R_H}{R_L}$.*

$\forall \Omega \in \bar{\Omega}$, $\exists \underline{R_H} < \overline{R_H}$ such that $\forall R_H \in]\underline{R_H}; \overline{R_H}[$, $\Gamma(\Omega, R_H)$ has at least two distinct elements corresponding to equilibria with different level of liquidity.

Let define the generalized implied price correspondence $P'(P, R_H) : \left\{\left[\frac{R_L}{1+r}, \frac{R_H}{(1+r)}\right], R_H\right\} \rightarrow \left[\frac{R_L}{1+r}, \frac{R_H}{(1+r)}\right]$:

$$P'(P, R_H) = \begin{cases} P'_{illiq}(P, R_H) = \frac{R_L}{1+r} + \eta_{illiq}(P, R_H)\frac{R_H}{(1+r)} & ; P \leq 1+r \\ P'_{liq}(P, R_H) = \frac{R_L}{1+r} + \eta_{liq}(P, R_H)\frac{R_H}{(1+r)} & ; P \geq 1+r \\ P'(1+r, R_H) \in \left[P'_L(P, R_H), P'_H(P, R_H)\right] & ; P = 1+r \end{cases}$$

With $\eta_{illiq}(P, R_H) = \frac{q-pq}{1-pq} \equiv \eta_{illiq}$, $\eta_{liq}(P, R_H) = \frac{q-pq\lambda^*_{nH}(P,R_H)}{1-pq\lambda^*_{nH}(P,R_H)}$ and $\lambda^*_{nH}(P, R_H) \equiv \arg\max_\lambda u((1-\lambda)P) + u(\lambda R_H)$ being the optimal endowment share invested in the long-run technology, for a given P and conditionally on being of type nH.

Before turning to the core of the proof, I establish the continuity of $P'_{liq}(P, R_H)$ and the fact that liquidity is always higher when agent H participates the market.

Condition 1: $u'(0) > u'(R_H)\frac{R_H}{1+r}$

This (mild) condition will ensure $C^*_{1,nH} > 0$, that is I rule out the case where agent nH optimally chooses not to consume at date 1.

Lemma 1 (liquidity dominance)

Under condition 1,

$$\eta_{illiq} < \eta_{liq}(P, R_H) < q\,,$$

$\forall P \geq 1+r$.

[45] $0 < p \leq 1, 0 < q < 1, 0 \leq R_L < 1+r, r > -1$

Proof: First, from $\lambda_{nH}^* \in [0,1]$, I have:

$$\eta_{illiq} \leq \eta_{liq}(P,R_H) < q$$

Then, *Condition 1* implies that: $u^{'}(0) > u^{'}(R_H)\frac{R_H}{P_{liq}(P,R_H)}$, $\forall P \geq 1+r$. This, in turn, implies that $\lambda_{nH}^*(P,R_H) < 1, \forall P \geq 1+r,$ which implies *Lemma* 1.

Lemma 2 (continuity)

$P_{liq}^{'}(P,R_H)$ is continuous in P and R_H.

Proof: As $\lambda_{nH}^* \in [0,1]$, $P_{liq}^{'}$ is a continuous function of $\lambda_{nH}^*(P,R_H)$. The implicit functions theorem applied to the first order condition for an interior λ_{nH}^* ensures that $\lambda_{ns}^*(P,R_H)$ is continuous in both its arguments which implies *Lemma 2*.

Proof (of proposition 2bis): As I only consider stable equilibria, I am not interested in the vertical locus for $P'(P,R_H)$ which corresponds to $P = 1+r$. I will consider separately the two functions $P_{illiq}^{'}(P,R_H)$ and $P_{liq}^{'}(P,R_H)$ defined respectively on the sets $\left\{\left[\frac{R_L}{(1+r)}, 1+r\right], R_H\right\}$ and $\left\{\left[1+r, \frac{R_H}{(1+r)}\right], R_H\right\}$ and show that there exists a range of R_H that generates at least an equilibrium for both functions:

The upper bound for a low-liquidity equilibrium Brouwer's fixed point theorem gives the necessary and sufficient condition on R_H for a unique fixed point $P_L^{'}(P,R_H) = P$:

$\exists P \in \left[\frac{R_L}{1+r}, 1+r\right]$ such that $P_L^{'}(P,R_H) = P \Longleftrightarrow R_H \leq R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$.

There exists thus a unique low-liquidity equilibrium if and only if R_H is low enough, and I can thus set:

$$\overline{R_H} \equiv R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$$

The lower bound for a high-liquidity equilibrium: In order to derive a sufficient condition for the existence of a fixed point $P_{liq}^{'}(P,R_H) = P$, I construct a function $G(P,R_H) \equiv P - P_H^{'}(P,R_H)$, defined on the interval for P: $\left[1+r, \frac{R_H}{(1+r)}\right]$. Clearly, given the continuity of $P_{liq}^{'}(P,R_H)$ (*Lemma 2*), if $G(P,R_H)$ changes sign on its domain, there is a fixed point for $P_H^{'}(P,R_H)$.

Since $P_H^{'}(P,R_H) = \frac{R_L}{1+r} + \eta_{liq}(P,R_H)\frac{R_H}{(1+r)}$ and $\eta_{liq}(P,R_H)$ is bounded above by $q < 1$, I have: $P_{liq}^{'}(P,R_H) < \frac{R_H}{(1+r)}$ and thus $P_{liq}^{'}\left(\frac{R_H}{(1+r)}, R_H\right) < \frac{R_H}{(1+r)}$. It implies:

$$G\left(\frac{R_H}{(1+r)}, R_H\right) > 0 \tag{10}$$

For any R_H, given (10), a sufficient condition for the existence of a high-liquidity equilibrium is thus:

$$G((1+r), R_H) \leq 0$$

Which is equivalent to:

$$P_{liq}^{'}(1+r, R_H) \geq (1+r)$$

And thus to:

$$R_H \geq R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r, R_H)} \tag{11}$$

As $\eta_{liq}(P,R_H)$ is bounded, there will always exist a R_H high enough such that condition (11) is satisfied. Yet, I am interested in a lower bound on R_H for this condition to hold. That is, a $\underline{R_H}$ such that:

$$\forall R_H \geq \underline{R_H}, R_H \geq R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r, R_H)}$$

In order to find $\underline{R_H}$, I construct the function $R_H^{'}(R_H) \equiv R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Given *lemma 1*, it is bounded below by $R_L + \frac{(1+r)^2-R_L}{q}$ and above by $R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}$. Also, given *Lemma* 2, it is continuous over the range corresponding to these bounds. It admits thus at least a fixed point: $R_H^{'} = R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H^{'})}$.

For a wide class of utility functions[46], $\eta_{liq}(P,R_H)$ is monotonic in R_H. It implies that there is a unique fixed point; call it $R_H^{'*}$. *Lemma 1* ($\eta_{illiq} < \eta_{liq}(P,R_H)$) implies that $R_H^{'}\left(R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}\right) < R_L + \frac{(1+r)^2-R_L}{\eta_{illiq}}$ and thus $\forall R_H \geq R_H^{'*}$, $R_H > R_L + \frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Hence, this unique fixed point is a lower bound on R_H for the existence of a high-liquidity equilibrium. I can thus choose $\underline{R_H} \equiv R_H^{'*}$. If there are multiple fixed points, the correct lower bound is the highest valued fixed point:

$$\underline{R_H} \equiv \max\left\{R_H^{'} : R_H^{'} = R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r, R_H^{'})}\right\}$$

The range for multiple equilibria *Lemma 1* implies $\underline{R_H} < \overline{R_H}$ which concludes the proof: $\forall R_H \in]\underline{R_H}; \overline{R_H}[$ there exists at least two equilibria.

B.4 Proof of *proposition 3* (market failure)

A liquidity dry-up is a <u>*Pareto-dominated*</u> *equilibrium, both from an* <u>*ex-ante*</u> *and an* <u>*ex-post*</u> *point of view.*

Let $R_H > 3 - 2R_L$. By *proposition 2*, I have:

$$\left\{\gamma_{illiq}(R_L,R_H) = \left(R_L, \tilde{\lambda}_L, 0\right); \gamma_{liq}(R_L,R_H) = \left(R_L + \frac{(R_H - R_L)}{3}, 1, \frac{1}{3}\right)\right\} \in \Gamma(R_L,R_H)$$

Denote C_{tj}^{γ} the optimal consumption of agent j at date t in equilibrium γ.
If both agents are better-off in γ_{liq}, it ex-post Pareto dominates γ_{illiq}.

i) Agent L is better-off

Obvious since $C_{1L}^{\gamma_{illiq}} = C_{2L}^{\gamma_{illiq}} = \frac{1-\lambda(R_L-1)}{2} < \frac{R_L + \frac{(R_H-R_L)}{3}}{2} = C_{1L}^{\gamma_{liq}} = C_{2L}^{\gamma_{liq}}$

[46] Including CARA, CRRA and quadratic utility functions.

ii) Agent H is better-off

If S_1^*denotes the optimal level of savings of this agent at date 1, I have:

$$\begin{cases} C_{1H}^{\gamma_{illiq}} = 1-\tilde{\lambda} - S_1^* \\ C_{2H}^{\gamma_{illiq}} = \tilde{\lambda} R_H + S_1^* \end{cases}$$

In γ_{liq}, the budget constraints of this agent are:

$$\begin{cases} C_{1H}^{\gamma_{liq}} \leq P_{liq} L \\ C_{2H}^{\gamma_{liq}} \leq (1-L) R_H \end{cases}$$

Assume he sets: $L = \frac{C_{1H}^{\gamma_L}}{P_{liq}}$, then:

$$\begin{cases} C_{1ns}^{\gamma_{liq}} = C_{1ns}^{\gamma_{illiq}} \\ C_{2ns}^{\gamma_{liq}} = \left(1 - C_{1ns}^{\gamma_{illiq}}\right) R_H \end{cases}$$

and $C_{2H}^{\gamma_{liq}} > C_{2H}^{\gamma_{illiq}}$ as $\tilde{\lambda} < 1$ and $S_1^* \geq 0$. So, the optimal choice of this agent in γ_{illiq} is still feasible and lets some spare resources. He can thus do strictly better than in γ_{illiq}.

If utility is strictly higher in all states of the world (L and H), γ_{liq} Pareto dominates γ_{illiq} , both ex-ante and ex-post.

B.5 Proof of *proposition 4* (public liquidity insurance)

Under this liquidity insurance, date-1 budget constraints are then contingent to P:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + L \max(P,1) - \tau(P) \\ C_2 = (\lambda - L) R_j + S_1 \end{cases}$$

Where:

$$\tau(P) = \begin{cases} (1-P) \sum_j \frac{L_j}{2} & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

It simply states that if the market liquidation price is low, agents will have to pay $\tau(P)$ but they will also be compensated for the loss of value with respect to the opportunity cost - the return on storage.

Of course, I still have: $L_L^*(P) = \lambda^*(P)$. Such a subsidy will not decrease the willing to liquidate of these agents.

Thus:

$$\begin{cases} C_1 + S_1 = 1 - \lambda + \lambda \max(P,1) - \tau(P) \\ C_2 = (\lambda - L) R_j + S_1 \end{cases}$$

Conditionally on being L, the date-0 first order condition of problem (9) for ($\lambda = 1$) always holds:

$$E_0 \left[\max(P-1,0) \ln'(C_1) + \ln'(C_2) R_j \,|L \right] > 0 \tag{12}$$

The return-liquidity trade-off has well disappeared. As it is true irrespective to the competitive market price, $\lambda = 1$ is a dominant strategy.

Conditionally on H, there are two cases depending on P:

If $P \geq 1$, the budget constraints are:

$$\begin{cases} C_1 = 1 - \lambda + \max(\lambda - 1/2; 0) P \\ C_2 = (\lambda - \max(\lambda - 1/2; 0)) R_H \end{cases}$$

From this, I can consider the first order condition of problem (9) with respect to λ:

- if $\lambda > \frac{1}{1+P}$ => $L_H = \frac{P\lambda - 1 + \lambda}{2P}$ => $\frac{(P-1)}{2} \ln^{'}(C_1) > 0$
- if $\lambda \leq \frac{1}{1+P}$ => $L_H = 0$ => $-\ln^{'}(1-\lambda) + \ln^{'}(\lambda R_H) R_H \geq 0$

If $P < 1$, the budget constraints are:

$$\begin{cases} C_1 = 1 - \lambda + L - \tau \\ C_2 = (\lambda - L) R_H \end{cases}$$

Which implies the following on the first order condition of problem (9) with respect to λ:

- if $\lambda > \frac{1-\tau}{2}$ => $L_{ns} = \lambda - \frac{1-\tau}{2}$ => $-\ln^{'}(1-\lambda-\tau) + \ln^{'}(\lambda R_H) R_H = 0$
- if $\lambda < \frac{1-\tau}{2}$ => $L_{ns} = 0$ => $-\ln^{'}(1-\lambda-\tau) + \ln^{'}(\lambda R_H) R_H > 0$

Hence:

$$\frac{\partial U_H(\lambda, P)}{\partial \lambda} \geq 0 \tag{13}$$

and (12) plus (13) gives:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

Which concludes the proof.

B.6 Proof of Corollary 1

The fourth column of table 4 gives the ϕ_{ij} as a function p. To obtain corollary 1, just derive with respect to $(1-p)$.

Table 4: Resources across states with private info over β_i

Type	Mass	W_{ij}	ϕ_{ij}	$W^*_{ij}/E[R]$
nH	$\frac{p}{2}$	R_H	$-\left(\frac{1}{4-p}\right)(R_H - R_L)$	$\left(1 + \frac{2-p}{4-p}\right)$
nL	$\frac{p}{2}$	R_L	$\left(\frac{2-p}{4-p}\right)(R_H - R_L)$	$\left(1 - \frac{p}{4-p}\right)$
eH	$\frac{1-p}{2}$	R_H	$-\left(\frac{2}{4-p}\right)(R_H - R_L)$	$\left(1 - \frac{p}{4-p}\right)$
eL	$\frac{1-p}{2}$	R_L	$\left(\frac{2-p}{4-p}\right)(R_H - R_L)$	$\left(1 - \frac{p}{4-p}\right)$

C Appendix to section 3.1: Unstable equilibria

There are several ways to interpret this equilibrium: for instance, it can be seen as a Nash equilibrium in pure or mixed strategy. In this equilibrium, agents expect $P = 1$. They are therefore indifferent with respect to investment choice over a wide range of value: $\lambda(P = 1) = \left[\frac{1}{2}, 1\right]$. At date 1, I have $L_L(1,\lambda) = \lambda$ and $L_H(1,\lambda) = \lambda - 0.5$. To indeed have $P = 1$ as an outcome, the actual investment density function $f(x)$ defined over the interval $\left[\frac{1}{2}, 1\right]$ should be such that: $R_L + \eta_{f(x)}(R_H - R_L) = 1$. With $\eta_{f(x)}$ being the level of liquidity implied by distribution $f(x)$, that is:

$$\eta_{f(x)} = \int \frac{x - 0.5}{2x - 0.5} f(x)dx$$

In the example of figure 2, one could for instance consider the degenerate distribution:

$$f(x) = \begin{cases} 1 & ; x = 5/8 \\ 0 & ; x \neq 5/8 \end{cases}$$

Which gives $\eta_{f(x)} = 1/6$ and indeed $P = 1$.

Obviously, in such cases, any small perturbation (to the expected price for instance) would switch the best response to either $\lambda^*(P > 1) = 1$ or $\lambda^*(P < 1) < \bar{\lambda}_1^*$ and best response iteration would never bring the economy back to γ_1.