Optimal Fiscal Policy over the Business Cycle Revisited*

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Abstract

This paper studies optimal fiscal policy in a standard business cycle model with two departures: (i) government infrastructures, which constitute a fiscal policy instrument, are an input into production which firms do not control; (ii) we assume that investment in new capital becomes productive in the same period in which it is produced. Under a widely used class of utility functions, we show that in bad times the government should: (i) lower the tax rate on labor income, (ii) lower the tax rate on capital income; and (iii) increase spending (or investment) in infrastructures. Quantitatively, following a one-standard deviation negative productivity shock, the tax break amounts to 0.45% of GDP and government spending increases by 0.62% of GDP. When the fiscal authority is restricted to trade non state contingent bonds, the expansionary fiscal policy that follows a recession is financed by a permanent increase in government debt.

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1 Introduction

This paper revisits the age-old question of whether governments should respond to changes in economic activity. The actions recently undertaken by governments the world over clearly demonstrate that policy makers believe that the answer to the above question is a resounding yes. The idea behind the practice of a counter-cyclical fiscal policy typically revolves around the notion that idle resources (capital and labor) can be used during periods of low economic activity to produce some kind of public good, and that tax breaks help dampen the effects of bad shocks. The goal of this paper is to investigate whether this kind of policy response can be reconciled with standard neoclassical growth theory.

To that end, we investigate this question within the context of a stochastic neoclassical growth (or real business cycle) model in which government infrastructures are an input into production which firms do not control. Under a widely used class of utility functions, we show that in bad times the government should: (i) lower the tax rate on labor income, (ii) lower the tax rate on capital income; and (iii) increase spending (or investment) in infrastructures. Quantitatively, following a one-standard deviation negative productivity shock, the tax break amounts to 0.45% of GDP (most of which is accounted for by the labor income tax) and government spending increases by 0.62% of GDP.

While the pro-cyclicality of the labor income tax is not surprising, the result that capital income tax rates are pro-cyclical may seem not only surprising, but at odds with the work of Chari et al. (1994).[1] The difference comes from a different timing assumption in this paper relative to theirs. Chari et al. (1994) assume a conventional timing whereby investment made during the period becomes productive next period. As a result, the fiscal authority can promise a wide range of tax rates on capital income tomorrow while at the same time re-assuring investors that on average they will not be taxed. In other words, the government can induce an essentially undistorted investment decision while at the same time absorbing shocks *ex post* with a highly volatile tax on capital income. For example, the government taxes capital income heavily to absorb the negative effect of a bad shock on tax revenues. Indeed, under

[1] We do, however, establish properties of labor income taxes that not only apply to their environment, but also shed some light of their numerical findings.

standard specifications, in which shocks are persistent, the tax rate on capital income is so high that the government runs a surplus in bad times. This is optimal since it minimizes the need to resort to distortionary taxation, not only today but also in the future. While the intuition for the results in Chari et al. (1994) is clear, the results rely heavily on the notion that taxing capital tomorrow is a free lunch as the investment decision was made in the past.

A simple way to avoid the presence of an *ex post* free lunch is to assume that investment becomes productive in the same period in which it is undertaken. Under this timing assumption, the return on capital is realized during the same period in which the investment is made. Clearly, this timing assumption is not more realistic than the conventional one. However, in the context of analyzing optimal fiscal policy, it captures the idea that the tax authority does not hold an important timing advantage over investors. This implies that the tax rate is distortionary on a state by state basis.

One advantage of this approach is that the model remains sufficiently tractable to derive some analytical results. Theoretically, our environment produces results that are very much in the spirit of Chamley (1986) in the context of a deterministic neoclassical growth model. In particular, under a per-period utility function which is separable in consumption and leisure and feature constant elasticity of substitution in consumption, not only are *ex ante* capital income taxes zero, but realized tax rates as well (there is no distinction in our framework between *ex ante* and *ex post* tax rates). As mentioned above, under a widely used class of non-separable per-period utility functions, the tax rate on capital is pro-cyclical as long as labor is pro-cyclical.

We also show that if infrastructures depreciate at a faster pace than private capital, which is supported by the data, then the ratio of infrastructures to private capital is counter-cyclical. This result simply follows from the fact that because private capital depreciates faster than infrastructures, the marginal product of capital needs to be higher than the marginal product of infrastructures for their net returns to be equalized. This result is not new, in the sense that Jones et al. (2005) show in a model with two accumulable inputs (human and physical capital in their case) that the input with the higher depreciation rate responds more to a positive shock than the input with the lower depreciation rate. The reason is simple: shocks have a larger impact on the marginal product of the input which depreciates faster because the marginal product is higher (around the steady state or, in their case, the balanced growth path). As such, it should be clear that this result is entirely independent of our timing assumption, as it also holds (in expectation) under the more conventional timing used in Jones et al. (2005).

Despite pro-cyclical government spending and counter-cyclical taxes (on both capital and labor), nothing can be said about the value of government debt issued in the period of a bad shock. This is because in equilibrium, with state-contingent debt, the government tends to issue relatively few bonds that pay in the event of a bad shock tomorrow. This opens up the possibility that the debt to be repaid in the period of a bad shock is small enough to outweigh the extra spending and lower tax revenues raised during the period, thereby making it possible for the value of new debt issued to go down. Preliminary simulation results confirm that indeed the value of debt issued in bad times can either increase or decrease.

The above results lead us to study optimal fiscal policy without state-contingent debt. However, few results can be derived analytically in this case: only the pro-cyclical nature of government spending can be shown to hold in general. We also establish that under a very special per-period utility function—quasi-linear—capital income should never be taxed. As such, our work here complements that of Farhi (2005), who uses the conventional timing but imposes that the government sets capital income tax rates one period ahead, also to mitigate the free lunch of *ex post* volatile capital income tax rates.[2] To the same end, Scott (2007) and Marcet and Scott (2009) rule out capital income taxes altogether and show that the implications of their model without state contingent debt is more consistent with the data than models with state contingent debt. In particular, in models without state contingent bonds, government debt and labor tax rates inherit a unit root component which, as emphasized by Aiyagari et al. (2002) in a model without capital, lends some support to Barro (1990)'s conjecture. Qualitatively, our simulations confirm that these results hold even when the government sets capital tax rates optimally.

In related work, Lansing (1998) introduces government infrastructures in a model similar to that of Chari et al. (1994). Our work differs from his along several dimen-

[2]As shown by Chari et al. (1994), merely ruling out state contingent debt in their framework only serves to pin down the tax code and does not impose any restriction on the Ramsey problem.

sions. First, he also uses the conventional timing, and therefore obtains results similar to Chari et al. (1994) as far as capital income taxes are concerned. Second, we assume constant returns to scale technologies only in inputs chosen by firms, as opposed to the three inputs in the model (capital, labor, and government infrastructures). As such, we need not make assumptions about profit taxation. Our assumption is also consistent with the work of Aschauer (1989), who not only finds support for a production technology which is constant returns to scale in private inputs, but also argues that non-military structures should be part of the production function as these structures have a direct impact on productivity. Finally, we allow for the possibility that the government sector faces shocks that are different from the private sector. In particular, we pursue a specification in which the government sector is subject to shocks that are perfectly correlated with but less volatile than shocks to the private sector. Evidently, this shock structure increases the magnitude of the increase in government infrastructures in recessions. This is the type of mechanism we eluded to above, whereby recessions represent an opportunity for the government to use relatively cheap resources to produce government infrastructures. Note, however, that this last point is only important for quantitative purposes, as none of our analytical results rely on that shock structure.

The rest of the paper is organized as follows. The economic environment is presented in the next section. In Sections 3 and 4 we set up and analyze Ramsey problem with and without state-contingent debt, respectively. All our analytical results are contained in these two sections. The model is calibrated in Section 5, where we discuss our quantitative findings. A brief conclusion is offered in Section 6.

2 Economic Environment

The economic environment we consider is similar to that of Lansing (1998) who in turn builds on Chari et al. (1994). Our benchmark model consists of a one-sector stochastic neoclassical growth model modified so that government infrastructures are an input into production which firms do not control. In contrast to Lansing (1998), we assume firms have access to a technology which features constant returns to scale in the two inputs they control, capital and labor, as opposed to all three inputs. It follows that in our environment the scale of firms is irrelevant and, since firms make zero profits in equilibrium, there is no need to make an assumption regarding the taxation of profits.

Output produced during the period can be used either for consumption, investment, or government spending. We distinguish between two types of government spending: new infrastructures, which add to the un-depreciated stock of infrastructures inherited from the previous period, and non-government infrastructures. Similarly, investment in new capital adds to the un-depreciated stock of capital inherited from the previous period. As emphasized in the introduction, current investment (in capital or infrastructures) becomes productive immediately.[3]

Although our benchmark model, is a one-sector model, for the purpose of simulations we introduce the possibility that goods purchased by the government be produced by a technology that is less volatile than the technology that produces other goods. Accordingly, the model presented in this section allows for that generality.

Each period the economy experiences one of finitely many events $s_t \in S$. We denote histories of events by $s^t = (s_0, s_1, \ldots, s_t)$, where s_0 is taken as given. As of date 0, the probability that a particular history s^t will be realized is denoted $\pi(s^t)$.

Production The (private goods) production technology is represented by a neoclassical production function with constant returns to scale in capital (k^p) and labor (l^p)

$$y^p(s^t) = f^p\big(g(s^t), k^p(s^t), l^p(s^t), s_t\big) = A^p(s^t) g(s^t)^\gamma k^p(s^t)^\alpha l^p(s^t)^{1-\alpha}, \tag{1}$$

where $A^p(s_t)$ represents the state of technology in the private sector, $y^p(s^t)$ denotes the aggregate (or per capita) level of output in the private sector, and $k^p(s^t)$ and $l^p(s^t)$ denote capital and labor used in that sector. The distinguishing feature of this technology is that government infrastructures, $g(s^t)$, enter the production function. However, since firms take government structures as given, this technology retains all the properties of the standard neoclassical production function. In particular, the capital to labor ratio is independent of scale, firms make zero profits in equilibrium,

[3] The assumption that government infrastructures become productive within the period is made to maintain symmetry with private capital and therefore make the paper more readable: the results are robust to the adoption of the conventional timing whereby government infrastructures put in place today only become productive tomorrow.

and factors are paid their marginal products:[4]

$$\hat{r}(s^t) = f_k^p\big(g(s^t), k^p(s^t), l^p(s^t), s_t\big) - \delta \qquad = f_k^p(s^t) - \delta; \tag{2}$$

$$\hat{w}(s^t) = f_l^p\big(g(s^t), k^p(s^t), l^p(s^t), s_t\big) \qquad = f_l^p(s^t). \tag{3}$$

The technology to produce government consumption goods ($c^g(s^t)$) and new government infrastructures ($i^g(s^t)$) is identical to the private production technology,

$$c^g(s^t) + i^g(s^t) = f^g\big(g(s^t), k^g(s^t), l^g(s^t), s_t\big) = A^g(s^t)g(s^t)^\gamma k^g(s^t)^\alpha l^g(s^t)^{1-\alpha}, \tag{4}$$

except for the stochastic process governing the technology shock $A^g(s_t)$.[5] We assume that while the shock to this technology is perfectly correlated with the shock affecting private production, its variance is a fraction θ of the variance of the private shock. Two special cases are of particular interest: $\theta = 0$ is a situation where the technology to produce government infrastructures is not subject to any shock, and $\theta = 1$ represents a situation in which both shocks have identical properties. In the latter case, there is no need to keep track of where factors are employed, and the model reduces to a one-sector model. This is our benchmark economy. Also note that since both production functions satisfy Inada conditions, both technologies will operate every period. As discussed below, it follows that the government sector pays the same price on factors of production as the private sector: $\hat{r}(s^t) = p^g(s^t)f_k^g(s^t) - \delta$ and $\hat{w}(s^t) = p^g(s^t)f_l^g(s^t)$, where $p^g(s^t)$ is the 'implicit relative price' of government produced goods—in term of private goods.[6]

Households The economy is populated by a large number of identical individuals who live for an infinite number of periods and are endowed with one unit of time every period. Individuals' preferences are ordered according to the following utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\big(c(s^t), l(s^t)\big), \tag{5}$$

[4] This is in contrast to Lansing (1998), who assumes constant returns to scale the three inputs: government capital; private capital; and labor.

[5] Whether government consumption goods are produced using the private or the public technology is immaterial. For calibration purposes, it is more attractive to have them produced by the public sector to better match the share of labor employed in the public sector.

[6] Formally, the government minimizes the cost of producing a given amount of goods, i.e. $\min\{wl^g + (r+\delta)k^g\}$ subject to $f^g \geq c^g + i^g$. Then p^g is the shadow value of an extra unit of investment in government infrastructures, i.e. the Lagrange multiplier on the production constraint.

where $c(s^t)$ and $l(s^t)$ represent consumption and hours worked at history s^t. We assume that the felicity function is increasing in consumption and leisure $(1 - l^{s^t})$, strictly concave, twice continuously differentiable, and satisfies the Inada conditions for both consumption and leisure.

Each period individuals face the budget constraint

$$c(s^t)+k(s^t)+\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) = w(s^t)l(s^t)+r(s^t)k(s^t)+k(s^{t-1})+b(s_t|s^{t-1}) \quad (6)$$

where $w(s^t) = [1 - \tau^w(s^t)]\hat{w}(s^t)$ and $r(s^t) = [1 - \tau^k(s^t)]\hat{r}(s^t)$, and where 'hats' denote pre-tax prices. The fiscal policy instruments τ^w and τ^k, as well as government debt $b(s_{t+1}|s^t)$ will be discussed in detail below.

Letting $p(s^t)$ denote the Lagrange multiplier on the budget constraint at history s^t, the first order necessary (and sufficient) conditions for a solution to the consumer's problem are given by (6) and

$$\beta^t \pi(s^t) U_c(s^t) = p(s^t), \quad (7)$$

$$\beta^t \pi(s^t) U_l(s^t) = -w(s^t)p(s^t), \quad (8)$$

at all dates t and histories s^t for consumption and labor,

$$-p(s^t)\left(1 - r(s^t)\right) + \sum_{s_{t+1}} p(s^{t+1}) = 0, \quad (9)$$

at all dates t and histories s^t for capital,

$$-p(s^t)(q(s_{t+1}|s^t)) + p(s^{t+1}) = 0, \quad (10)$$

at all dates t, histories s^t, and all states s_{t+1} tomorrow for bond holdings, as well as the transversality conditions

$$\lim p(s^t)a(s^t) = 0, \quad (11)$$

$$\lim \textstyle\sum_{s_{t+1}} p(s^{t+1})b(s_{t+1}|s^t) = 0. \quad (12)$$

The conditions above assume that individuals are indifferent between supplying factors to private or public production, and that they supply strictly positive factors to both sectors. Accordingly, it must be the case that after-tax wage rates are the same in both sectors, and that returns on capital be equalized in both sectors, as mentioned above.

Proposition 1 *An allocation solves the consumer's problem if and only if it satisfies equations (6)–(12), or, equivalently, if and only if it satisfies the implementability constraint*[7]

$$\sum_{t,s^t} \beta^t \pi(s^t) \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t) \right] = A_0, \tag{13}$$

where $A_0 = U_c(s_0)[k_{-1} + b{-1}]$*, and* a_{-1} *and* $b{-1}$ *are initial amounts of capital and government debt held by individuals.*

Proof. The proof is standard. [See for example Chari et al. (1994).] ■

The Government The government in this economy has full control over the entire fiscal policy in each period, except for the amount of government consumption $c^g(s^t)$, which it takes as given. The fiscal policy instruments available to the government consist of a proportional labor income tax $\tau^w(s^t)$; a proportional capital income tax $\tau^k(s^t)$; issuance of new government debt $b(s_{t+1}|s^t)$; and investment in new infrastructures $i_g(s^t)$ which requires employing factors of production k^g and n^g. As discussed before, investment in infrastructures becomes productive immediately: the law of motion for government infrastructures is

$$g(s^t) = g(s^{t-1}) + i^g(s^t) - \delta g(s^t), \tag{14}$$

where δ^g is the depreciation rate of government structures.[8]

3 The Ramsey Problem

To study optimal policy in this environment, we set up a standard Ramsey problem. As is well known, there is an equivalence between choosing fiscal policy instruments directly and choosing allocations among an appropriately restricted set of allocations.[9]

[7]To obtain the implementability constraint, multiply the budget constraint (6) by $p(s^t)$, add them up, and use the first order conditions (7)–(10) to replace prices.

[8]This law for motion is equivalent to a specification where

$$g(s^t) = (1 - \tilde{\delta}^g) \left(g(s^{t-1}) + i^g(s^t) \right),$$

with $1 + \delta^g = 1/(1 - \tilde{\delta}^g)$.

[9]See Chari and Kehoe (1999) or Erosa and Gervais (2001).

The government's problem consists of maximizing the utility of the representative individual (5) subject to the implementability constraint (13) and feasibility. If we denote λ the Lagrange multiplier on the implementability constraint, we can then define the pseudo-welfare function W by

$$W\big(c(s^t), l(s^t)\big) = U\big(c(s^t), 1 - l(s^t)\big) + \lambda \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)\right].$$

The Ramsey problem is thus as follows:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) W\big(c(s^t), l(s^t)\big) - \lambda A_0 \tag{15}$$

subject to the feasibility constraints

$$\begin{aligned} c(s^t) + k(s^t) &= A^p(s^t)g(s^t)^\gamma k^p(s^t)^\alpha l^p(s^t)^{1-\alpha} - \delta k(s^t) + k(s^{t-1}); \\ c^g(s^t) + g(s^t) &= A^g(s^t)g(s^t)^\gamma k^g(s^t)^\alpha l^g(s^t)^{1-\alpha} - \delta^g g(s^t) + g(s^{t-1}), \end{aligned} \tag{16}$$

where $l(s^t) = l^p(s^t) + l^g(s^t)$ and $k(s^t) = k^p(s^t) + k^g(s^t)$. It will prove useful to define $\kappa(s^t)$ as the fraction of the capital stock allocated to the private sector, that is, $k^p(s^t) = \kappa(s^t)k(s^t)$ and $k^g(s^t) = (1 - \kappa(s^t))k(s^t)$. When the government sector is subject to the same shock as the goods producing sector, the feasibility constraint simply becomes

$$\begin{aligned} &c(s^t) + c^g(s^t) + k(s^t) + g(s^t) \\ &\quad = A(s^t)g(s^t)^\gamma k(s^t)^\alpha l(s^t)^{1-\alpha} - \delta k(s^t) - \delta^g g(s^t) + k(s^{t-1}) + g(s^{t-1}). \end{aligned} \tag{17}$$

Proposition 2 *If an allocation satisfies the constraints of the Ramsey problem, then the allocation also satisfies the government budget constraint.*

Proof. First multiply the second feasibility constraints in (16) by $p^g(s^t)$ and add it to the first feasibility constraint

$$\begin{aligned} &c(s^t) + k(s^t) + p^g(s^t)c^g(s^t) + p^g(s^t)g(s^t) \\ &\quad = f^p(s^t) + p^g(s^t)f^g(s^t) + k(s^{t-1}) - \delta k(s^t) + p^g(s^t)[g(s^{t-1}) - \delta^g g(s^t)]. \end{aligned}$$

Since both production functions exhibit constant returns to scale in capital and labor, we have

$$\begin{aligned}
&c(s^t) + k(s^t) + p^g(s^t)c^g(s^t) + p^g(s^t)g(s^t) \\
&\qquad = f_k^p(s^t)k^p(s^t) + f_l^p(s^t)l^p(s^t) + f_k^g(s^t)p^g(s^t)k^g(s^t) + f_l^g(s^t)p^g(s^t)l^g(s^t) \\
&\qquad\qquad + k(s^{t-1}) - \delta k(s^t) + p^g(s^t)[g(s^{t-1}) - \delta^g g(s^t)].
\end{aligned}$$

Since both sectors must pay the same price on factors (2)–(3) and $k(s^t) = k^p(s^t) + k^g(s^t)$ and $l(s^t) = l^p(s^t) + l^p(s^t)$, and using the law of motion for government infrastructures (14) we have

$$c(s^t) + k(s^t) + p^g(s^t)[c^g(s^t) + i^g(s^t)] = (\hat{r}(s^t) + \delta)k(s^t) + \hat{w}(s^t)l(s^t) + k(s^{t-1}) - \delta k(s^t).$$

Now if an allocation satisfies the implementability constraint, then it must satisfy the budget constraint of individuals at all dates and histories. So we can use (6) to replace $c(s^t) + k(s^t)$ in the previous expression to obtain

$$\begin{aligned}
&w(s^t)l(s^t) + k(s^{t-1}) + b(s_t|s^{t-1}) + r(s^t))k(s^t) \\
&\qquad - \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) + p^g(s^t)[c^g(s^t) + i^g(s^t)] \\
&\qquad\qquad = \hat{r}(s^t)k(s^t) + \hat{w}(s^t)l(s^t) + k(s^{t-1}).
\end{aligned}$$

Using the fact that $\hat{w}(s^t) - w(s^t) = \tau^w(s^t)\hat{w}(s^t)$ and similarly $\hat{r}(s^t) - r(s^t) = \tau^k(s^t)\hat{r}(s^t)$, rearranging the last expression we get

$$\begin{aligned}
&p^g(s^t)[c^g(s^t) + i^g(s^t)] + b(s_t|s^{t-1}) \\
&\qquad = \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) + \tau^w(s^t)\hat{w}(s^t)l(s^t) + \tau_k(s^t)\hat{r}(s^t)k(s^t),
\end{aligned}$$

which is the budget constraint faced by the government. ■

The government typically has more instruments than it needs, in the sense that many tax systems can decentralize any given allocation (e.g. see Zhu (1992) or Chari et al. (1994)). Such is not the case in our environment: our tax code is unique. Essentially, this comes from the fact that the tax rate on capital is pined down by the marginal product of capital as well as the optimality conditions (7) and (9).

3.1 Optimality Conditions

Let $\beta^t \phi^p(s^t)$ and $\beta^t p^g(s^t)\phi^g(s^t)$ be the Lagrange multipliers on the first and second feasibility constraints in (16), respectively. The first order conditions with respect to consumption and labor used to produce private goods and government infrastructures are, respectively,

$$
\begin{aligned}
\pi(s^t)W_c(s^t) &= \phi^p(s^t), & (18)\\
\pi(s^t)W_l(s^t) &= -f_l^p(s^t)\phi^p(s^t), & (19)\\
\pi(s^t)W_l(s^t) &= -p^g(s^t)f_l^g(s^t)\phi^g(s^t). & (20)
\end{aligned}
$$

Since $f_l^p(s^t) = p^g(s^t)f_l^g(s^t)$, the last two equations imply that $\phi^p(s^t) = \phi^g(s^t) = \phi(s^t)$ for all t and s^t. The first order condition with respect to $\kappa(s^t)$ is

$$\beta^t\phi^p(s^t)k(s^t)f_k^p(s^t) = \beta^t\phi^g(s^t)p^g(s^t)k(s^t)f_k^g(s^t),$$

which implies that

$$f_k^p(s^t) = p^g(s^t)f_k^g(s^t), \tag{21}$$

as one should expect since the composition of the capital stock is chosen during the period. The first order condition with respect to capital is given by

$$\phi^p(s^t)\left[-1+\kappa(s^t)f_k^p(s^t)-\delta\right]+\beta\sum_{s_{t+1}}\phi^p(s^{t+1})+\phi^g(s^t)p^g(s^t)(1-\kappa(s^t))f_k^g(s^t)=0,$$

which, using (21), can be written as

$$\phi(s^t)\big[1-(f_k^p(s^t)-\delta)\big] = \beta\sum_{s_{t+1}}\phi(s^{t+1}). \tag{22}$$

Finally, the first order condition with respect to government infrastructures is given by

$$\phi^p(s^t)f_g^p(s^t)+p^g(s^t)\phi^g(s^t)\left[-1+f_g^g(s^t)-\delta^g\right]+\beta\sum_{s_{t+1}}p^g(s^{t+1})\phi^g(s^{t+1})=0,$$

or, simplifying,

$$\phi(s^t)\left[p^g(s^t)\big(1-(f_g^g(s^t)-\delta^g)\big)-f_g^p(s^t)\right] = \beta\sum_{s_{t+1}}p^g(s^{t+1})\phi(s^{t+1}). \tag{23}$$

3.2 Optimal Fiscal Policy

The rest of this section is devoted to characterize optimal fiscal policy. Our characterization, which requires making assumptions about the form of the utility function, involves in turn the labor income tax, the capital income tax, and finally government infrastructure spending.

Our first two Propositions show that while the labor tax does not depend of the state of the economy if the per-period utility is separable between consumption and labor and both part exhibit constant elasticity of substitution (CES), it becomes pro-cyclical when individual care about leisure, even if the utility function is CES in leisure.

Proposition 3 *Assume that the felicity function is separable with $u(c)$ and $v(l)$ both exhibiting constant elasticity of substitution. Then the tax rate on labor income is invariant to the productivity shock.*

Proof. Combining the first order conditions with respect to consumption (18) and labor (19) from the Ramsey problem and using (3), we get

$$-\frac{W_l(s^t)}{W_c(s^t)} = \hat{w}(s^t). \tag{24}$$

The derivatives W_c and W_l are given by

$$\begin{aligned} W_c(s^t) &= (1+\lambda)U_c(s^t) + \lambda U_c(s^t)H_c(s^t), \\ W_l(s^t) &= (1+\lambda)U_l(s^t) + \lambda U_l(s^t)H_l(s^t), \end{aligned}$$

where

$$\begin{aligned} H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}, \\ H_l(s^t) &= \frac{U_{l,c}(s^t)c(s^t) + U_{l,l}(s^t)l(s^t)}{U_l(s^t)}. \end{aligned}$$

Now pick two histories as of date t, s^t and $\tilde{s}^t$. From (24), it must be that

$$\frac{W_l(s^t)}{W_c(s^t)\hat{w}(s^t)} = \frac{W_l(\tilde{s}^t)}{W_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)},$$

or, equivalently,

$$\frac{\left[1+\lambda+\lambda H_l(s^t)\right]U_l(s^t)}{\left[1+\lambda+\lambda H_c(s^t)\right]U_c(s^t)\hat{w}(s^t)}=\frac{\left[1+\lambda+\lambda H_l(\tilde{s}^t)\right]U_l(\tilde{s}^t)}{\left[1+\lambda+\lambda H_c(\tilde{s}^t)\right]U_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)}.$$

Since the felicity function is separable, the functions H_c and H_l become

$$\begin{aligned} H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t)}{U_c(s^t)}, \\ H_l(s^t) &= \frac{U_{l,l}(s^t)l(s^t)}{U_l(s^t)}. \end{aligned}$$

And since the sub-utilities for consumption and labor are both from the constant elasticity of substitution class of utility, that means both H_c and H_l are constant. Accordingly, the last expression reduces to

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)}=\frac{\hat{w}(s^t)}{\hat{w}(\tilde{s}^t)}.$$

But the first order conditions for consumption and labor from the household's problem (equations (7) and (8)) under histories s^t and $\tilde{s}^t$ imply

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)}=\frac{w(s^t)}{w(\tilde{s}^t)}=\frac{(1-\tau^w(s^t))\hat{w}(s^t)}{(1-\tau^w(\tilde{s}^t))\hat{w}(\tilde{s}^t)}.$$

For the last two equations to hold it must be the case that $\tau^w(s^t)=\tau^w(\tilde{s}^t)$. ■

The intuition for this result is that because the elasticity of the labor supply does not vary with the shock, there is no reason for the government to tax labor at rates that vary with the shock. Note that the utility function as specified in (5) will not generally satisfy the assumption of the above proposition, as individuals care about leisure, as opposed to disliking labor. The following proposition shows that indeed labor income taxes will in general not be constant when individuals care about leisure.

Proposition 4 *Assume that the felicity function is given by $u(c)v(l)$, with $u(c)=(1-\sigma)^{-1}c^{1-\sigma}$ and $v(l)=(1-l)^{\nu(1-\sigma)}=(1-l)^{\eta}$, with $\sigma>1$ and $\nu>0$, and $\ln(c)+\eta\ln(1-l)$ for $\sigma=1$. Pick two states s^t and $\tilde{s^t}$ such that $l(s^t)>l(\tilde{s}^t)$. Then $\tau^w(s^t)>\tau^w(\tilde{s}^t)$ if and only if*

$$\lambda<\frac{-1}{(1-\sigma)(1+\nu)}. \tag{25}$$

Proof. From equations (7)–(8) and (24), the tax rate on labor income is given by

$$\tau^w(s^t) = \frac{\lambda\big(H_l(s^t) - H_c(s^t)\big)}{1 + \lambda + \lambda H_l(s^t)}. \tag{26}$$

Under the stated utility function, H_c and H_l are such that

$$\begin{aligned} H_l(s^t) - H_c(s^t) &= \frac{-1}{1 - l(s^t)}, \\ H_l(s^t) &= -\sigma + \frac{1 - \eta l(s^t)}{1 - l(s^t)}. \end{aligned}$$

Using these expression in equation (26) we have

$$\tau^w(s^t) = \frac{\lambda}{1 - \lambda(\sigma - 2) - l(s^t)\big(1 + \lambda(1 - \sigma)(1 + \nu)\big)}.$$

It follows that the tax rate is higher under state s^t than $\tilde{s}^t$ if the term in from of labor is positive, which is the condition given above. ■

Notice that under logarithmic utility, i.e. when $\sigma = 1$, the condition is always satisfied.[10] In this case, as long as labor is pro-cyclical, so will the tax on labor income. This Proposition is useful to interpret the finding in Chari et al. (1994) that the correlation between the shock and labor taxes changes sign as they change the risk aversion parameter. Notice as well that what is key here is whether the utility function exhibits constant elasticity of substitution in labor or in leisure. When it is CES in leisure, the labor supply elasticity varies with the level of the labor supply, becoming more inelastic as the labor supply increases.

Our next results pertain to the tax on capital or interest income. We first show that the interest income should not be taxed if the utility function is separable and exhibits constant elasticity of substitution in consumption. We then argue that under non-separable preferences, the tax rate on interest income is likely to be pro-cyclical.

Proposition 5 *Assume that the felicity function is separable and $u(c)$ exhibits constant elasticity of substitution. Then the capital income tax is zero at all dates and histories (other than the first period).*

[10] Of course $\tau^w(s^t) = 0$ if $\lambda = 0$.

Proof. Recall that the first order conditions (7) and (9) from the households' problem imply that

$$(1 - r(s^t)) = \sum_{s_{t+1}} \frac{\beta\pi(s^{t+1})U_c(s^{t+1})}{\pi(s^t)U_c(s^t)}. \tag{27}$$

Similarly, combining first order conditions (18) and (22) from the Ramsey problem we have

$$\left[1 - (f_k^p(s^t) - \delta)\right] = (1 - \hat{r}(s^t)) = \sum_{s_{t+1}} \frac{\beta\pi(s^{t+1})W_c(s^{t+1})}{\pi(s^t)W_c(s^t)}. \tag{28}$$

But with separable utility and constant elasticity of substitution,

$$W_c(s^t) = (1 + \lambda + \lambda H^c(s^t))U_c(s^t) = (1 + \lambda - \lambda\sigma)U_c(s^t),$$

where σ is the inverse of the intertemporal elasticity of substitution. Hence we can replace W_c with U_c in equation (28). But then the only way for both equation (28) and equation (27) to hold is if we have $\tau^k(s^t) = 0$. ■

This Proposition is in sharp contrast to the results in Chari et al. (1994), where the *ex post* tax rate on capital income is extremely volatile. The intuition is that in their set up, the return on investment made today is taxed tomorrow. Since the investment decision has already been made when the tax authority sets the tax rate on capital income, this instrument is extremely useful to absorb shocks to the budget of the government. For example, if the economy experiences a bad shock today, then the government will tax capital income at a high rate to absorb the loss in revenue. The more persistent the shock is, the higher the tax rate. In fact, under standard parameter specifications, the increase in capital income taxes is so large that the government runs a surplus in the period of the bad shock, thereby absorbing the future path of low government revenue with very little change to the tax rate on labor income. Of course, the tax authority always promises individuals that *on average* capital income will not be taxed. This is what Chari et al. (1994) refer to as the *ex ante* tax rate on capital income, which, under the assumptions of proposition 5, is zero.

In our setup, the return on capital is known at the time individuals make their investment decision, thereby eliminating the distinction between *ex ante* and *ex post* taxes on capital. In particular, the tax authority no longer has the ability to absorb shocks in an essentially non-distortionary fashion.

Under more general preferences, the tax rate on capital will not in general be equal to zero. For instance, if $U(c,l) = u(c)v(l)$, with $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$ and $v(l) = (1-l)^{\nu(1-\sigma)} = (1-l)^{\eta}$, with $\sigma > 1$ and $\nu > 0$, then capital income will tend to be subsidized in bad times and taxed in good times. To see this, note that the function $H_c(s^t)$ under this utility function is given by

$$H_c(s^t) = -\sigma - \eta \frac{l(s^t)}{1-l(s^t)},$$

which, since $\eta < 0$, is increasing in l. Now from equations (27) and (28), we have

$$\frac{1-r(s^t)}{1-\hat{r}(s^t)} = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1+\lambda+\lambda H^c(s^t)\big)U_c(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1+\lambda+\lambda H^c(s^{t+1})\big)U_c(s^{t+1})}. \tag{29}$$

When this ratio is smaller than 1, capital income is subsidized, and capital income is taxed if the ratio is greater than 1. In particular, capital income is subsidized when $H_c(s^t)$ is relatively low, i.e. when the labor supply is relatively low. Much like the labor income tax, the capital income tax is thus likely to be pro-cyclical as long as labor is pro-cyclical.

We now move on to study the behavior of government infrastructures over the business cycle. The following Proposition states that in a one-sector growth model, if government infrastructures depreciate at the same rate as capital, then the ratio of the two stocks of capital will always be constant.

Proposition 6 *Assume that feasibility is given by (17) and that $\delta^g = \delta$. Then the ratio of government infrastructures to capital is constant at all dates and histories, and is given by $g(s^t)/k(s^t) = \gamma/\alpha$.*

Proof. With only one production function, the first order conditions with respect to capital and government infrastructures become

$$\phi(s^t)\big[1 - (f_k(s^t) - \delta)\big] = \beta \sum_{s_{t+1}} \phi(s^{t+1}),$$

$$\phi(s^t)\big[1 - (f_g(s^t) - \delta^g)\big] = \beta \sum_{s_{t+1}} \phi(s^{t+1}),$$

which, since $\delta^g = \delta$ implies that

$$\frac{\alpha y(s^t)}{k(s^t)} = \frac{\gamma y(s^t)}{g(s^t)}.$$

■

As we will see in the next section, however, government infrastructures and capital do not depreciate at the same rate. In particular, for the historical stocks of infrastructure and capital to be consistent with historical series on investment in capital and infrastructures, it must be the case that infrastructures depreciate at a much slower pace than capital. In other words, $f_k(s^t) > f_g(s^t)$. As our next Proposition shows, this is sufficient for the ratio of g/k to be counter-cyclical.

Proposition 7 *Assume that $\delta^g < \delta$. Then g/k is counter-cyclical.*

Proof. Pick two states s^t and $\tilde{s}^t$ such that productivity is higher in state s^t then in state $\tilde{s}^t$. Without loss of generality, assume that $A(\tilde{s}^t) = 1$, and that $f_k(\tilde{s}^t) - \delta = f_g(\tilde{s}^t) - \delta^g$ for some $\tilde{g}(\tilde{s}^t)$, $\tilde{k}(\tilde{s}^t)$ and $\tilde{l}(\tilde{s}^t)$. Clearly, with $A(s^t) = \omega < 1$,

$$\omega f_k(\tilde{s}^t) - \delta < \omega f_g(\tilde{s}^t) - \delta^g,$$

which implies that $g(s^t)/k(s(t)) > \tilde{g}(s^t)/\tilde{k}(s(t))$ for the equality of the marginal products to hold under state s^t. ■

This result is not new, in the sense that Jones et al. (2005) show in a model with two accumulable inputs (human and physical capital in their case) that the input with the higher depreciation rate responds more to a positive shock than the input with the lower depreciation rate. The reason is simple: shocks have a larger impact on the marginal product of the input which depreciates faster because the marginal product is higher (around the steady state or, in their case, the balanced growth path). of course, this Proposition only states that investment in infrastructures will be high *relative to* investment in capital, not that investment in infrastructures will be higher than otherwise. The direction of investment in infrastructures itself depends on parameter values. As it turns out, under our benchmark calibration, which only seeks to have the right long run ratios of government infrastructures to capital and as well as the right investment ratios, government spending (investment) in infrastructures goes up in bad times.

4 Ruling out State-Contingent Debt

Ruling out state-contingent debt in the standard neoclassical growth model has proven difficult. As shown in Chari and Kehoe (1999), ruling out state-contingent debt in this model amounts to imposing, at all dates, implementability constraints of the form[11]

$$\sum_{t=m}^{\infty}\sum_{s^t}\beta^{t-m}\pi(s^t|s^m)\left[Uc(s^t)c(s^t)+U_l(s^t)l(s^t)\right]=U_c(s^m)\big(k(s^{m-1})+b(s^{m-1})\big). \quad (30)$$

Notice that relative to Chari et al. (1994) or Scott (2007), the right-hand-side of (30) does not involve the after-tax interest rate. This is a direct consequence of our timing assumption, which greatly simplifies the problem as there is no need to impose that the interest rate need to be consistent with an Euler equation which depends on consumption today and tomorrow. In particular, we can directly apply the methodology developed by Marcet and Marimon (1994) to obtain the following Ramsey problem in Lagrangian form:

$$\max\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\Big\{U(c(s^t),l(s^t))+\mu(s^t)\left[U_c(s^t)c(s^t)+U_l(s^t)l(s^t)\right]$$
$$-\lambda(s^t)U_c(s^t)\left[k(s^{t-1})+b(s^{t-1})\right]\Big\}, \quad (31)$$

where $\mu(s^t)=\mu(s^{t-1})+\lambda(s^t)$ and $\mu_{-1}=0$, subject to feasibility (17) at all dates and history, given $k(-1)$ and $b(-1)$.

4.1 Analysis

We first establish that the evolution of the multiplier μ, which reflects the distortionary nature of taxation over time, contains a permanent component—a result first discussed in Aiyagari et al. (2002) in a model without capital, and more recently by Scott (2007) in a model with capital in which capital income taxation is ruled out. To establish this result, notice that the first-order condition for government debt states that

$$\sum_{s^{t+1}|s^t}\beta^{t+1}\pi(s^{t+1})\lambda(s^{t+1})U_c(s^{t+1})=0. \quad (32)$$

[11] Imposing this constraint at all nodes is equivalent to imposing the consumer's budget constraint period by period.

Now since $\lambda(s^{t+1}) = \mu(s^{t+1}) - \mu(s^t)$, multiplying $\lambda(s^{t+1})$ by $U_c(s^{t+1})$ and using (32) establishes that

$$\mu(s^t) = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1}) \mu(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t) U_c(s^{t+1})}, \tag{33}$$

so that the multiplier μ follows a risk-adjusted Martingale. An interesting special case, which we study in more details below, is one where the felicity function is quasi-linear, i.e. $U(c, l) = c + v(l)$. In this case, the marginal utility of consumption is constant at unity, and so the stochastic process for the multiplier μ becomes a martingale. Indeed, Farhi (2005) shows that if the government faces natural debt limits and the stochastic process governing the state s_t converges to a unique (non-degenerate) stationary distribution, then μ_t converges to zero, which implies that the Ramsey allocation converges to a first-best allocation (i.e. all taxes are zero in the long run). This result holds in our economy as well.

In general not much can be said analytically about the behavior of optimal taxes in this environment. In particular, nothing can be said about the labor income taxes, at least as far as we can tell. For the capital income tax, we will now establish one special case where it is always zero. If we let $\beta^t \pi(s^t) \phi(s^t)$ be the multipliers on the feasibility constraint, the first order condition with respect to capital reads

$$- \sum_{s^{t+1}|s^t} \beta^{t+1} \pi(s^{t+1}) \lambda(s^{t+1}) U_c(s^{t+1}) \\ - \beta^t \pi(s^t) \phi(s^t) \Big(1 - \big(f_k(s^t) - \delta\big)\Big) + \sum_{s^{t+1}|s^t} \beta^{t+1} \pi(s^{t+1}) \phi(s^{t+1}) = 0,$$

which, given (32), implies that

$$1 - \big(f_k(s^t) - \delta\big) = 1 - \hat{r}(s^t) = \frac{\sum_{s^{t+1}|s^t} \beta \pi(s^{t+1}) \phi(s^{t+1})}{\pi(s^t) \phi(s^t)}. \tag{34}$$

As usual, recalling equation (27)—which holds here as well—interest income should not be taxes if the shadow value of resources is equal to marginal utility at all dates and states, i.e. if $\phi(s^t) = U_c(s^t)$. This will in general not be the case, even under a per-period utility function separable between consumption and leisure. In this case, the value of the multiplier ϕ, from the first order condition for consumption, is given by

$$\phi(s^t) = U_c(s^t)\left[1 + \mu(s^t)\left(\frac{U_{cc}(s^t)c(s^t)}{U_{c(s^t)}} + 1\right) - \lambda(s^t)\frac{U_{cc}(s^t)}{U_c(s^t)}\Big(k(s^{t-1}) + b(s^{t-1})\Big)\right]. \tag{35}$$

Clearly, the term inside the square brackets will not in general be equal to one. There is, however, one special case under which we can establish that capital income should not be taxed, as we state in the following proposition.

Proposition 8 *If the per-period utility function is quasi-linear in consumption, i.e.* $U(c, l) = c + v(l)$, *then the tax rate on capital income is zero.*

Proof. First note that under this utility function, because the marginal utility of consumption is fixed at unity, (27) implies that $1 - r(s^t) = \beta$. From (35), the value of the multiplier on the feasibility constraint is given by $\phi(s^t) = 1 + \mu(s^t)$. Furthermore, (33) implies that $\mu(s^t) = \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\mu(s^{t+1})$. Using these facts in equation (34) imply that $\hat{r} = \beta$. ■

Before moving to simulations, we note that Propositions 6 and 7 continue to hold in this environment. This follows simply by comparing the first order condition for government infrastructures, given by

$$\pi(s^t)\phi(s^t)\Big(1 - \big(f_g(s^t) - \delta^g\big)\Big) = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1})\phi(s^{t+1}),$$

to the optimality condition for capital (34).

5 Simulations

To be completed.

6 Conclusion

This paper revisits the question as to how fiscal policy should be conducted over the business cycle. We do so in a standard neoclassical growth model modified along two important dimensions: (i) we introduce government infrastructures as an (government chosen) input into production which firms take as given, and (ii) we assume that investment becomes productive immediately.

Under a widely used class of utility functions, we show that in bad times the government should: (i) lower the tax rate on labor income, (ii) lower the tax rate on capital income; and (iii) increase spending (or investment) in infrastructures. While the increase in spending only relies on the fact that infrastructures depreciate at a slower pace than business capital, the fact that the capital income tax is pro-cyclical comes from our timing assumption. Quantitatively, following a one-standard deviation negative productivity shock, the tax break amounts to 0.45% of GDP (most of which is accounted for by the labor income tax) and government spending increases by 0.62% of GDP. Despite the counter-cyclical nature of fiscal policy, the behavior of (state-contingent) government debt is not pinned down by the model. Accordingly, we study a situation in which the government is precluded from issuing state-contingent bonds. Aside from the pro-cyclical nature of government spending, very little can be said about optimal tax rates in this case.

Overall our findings support the adoption of a counter-cyclical fiscal policy whereby taxes should be cut and productive government expenditures increased. However, it should be emphasized that infrastructures only constitute a small fraction of overall government spending, and that our theory is silent as to how the rest of government spending should be set.

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