Disciplining Expectations: Survey Data on Inflation Expectations and the Sources of Inflation Persistence

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Abstract

Do survey data on inflation expectations play a role in identifying the sources of inflation persistence? In this study we address this question by using data from the Survey of Professional Forecasters to test the mechanisms of expectation formation derived under the assumptions of rational expectations (RE) and learning, and their implications for the sources of inflation persistence. We find, on the one hand, that when ignoring survey data, inflation expectations estimated under the assumption of learning capture some features of the data not related with actual expectations such as changes in the trend of inflation. On the other hand, estimates under RE require implausible high levels of nominal rigidities to match the data from surveys. Consistent results are obtained by learning models that incorporate survey data. Under this specification, learning plays a key role, explaining 30-40% of inflation persistence, while exogenous shocks are of minor importance. All the estimations are implemented using a dynamic stochastic general equilibrium model.

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1. Introduction

Explaining inflation persistence is one of the most challenging issues in modern macroeconomics. The precise nature of its sources has important implications for the conduct of monetary policy. The appropriate design of monetary policy depends on whether inflation persistence arises, for instance, from features of goods and labor markets (such as indexation to past inflation), from persistent shocks that affect the pricing decision of firms, or from people's way of forming their expectation about future inflation. As previous studies show, the assumption about how expectations are formed has important implications for the determinants of inflation persistence. If people are assumed to have a perfect understanding of how the economy works exogenous shocks explain an important part of inflation persistence. Yet if it is assumed that people have to learn about how inflation behaves and uses forecasting models to form their expectations, then this learning process is a key factor behind inflation persistence. In this study we use survey data on inflation expectations to test both assumptions of expectation formation and, therefore, to provide a more accurate picture of the sources of inflation persistence.

The notion that people have perfect knowledge about the functioning of the economy, referred to as the rational expectations hypothesis (henceforth RE), is a core assumption of many macroeconomic models currently used. However, given that such a high level of cognitive abilities and computational skills seem to be implausible in practice, researchers have started to develop models of imperfect knowledge and associated learning processes. One of the most popular learning mechanisms used in macroeconomics is adaptive learning. Under this approach agents are assumed to use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data becomes available (see Evans and Honkapohja 2001).

Learning is particularly attractive, in comparison with RE, as it does not require persistent exogenous shocks to the model to explain inflation persistence. However, learning is also subject to criticism as it relies heavily on the researcher's (arbitrary) assumption about the forecasting model used by the people to generate their expectations. This behavioral modeling choice is key for the results obtained under learning. Moreover, empirical studies based on learning models generally do not evaluate if the resulting series of expectations captures other features not necessarily related with actual expectations.

To address this criticism, the main objective of this study is to identify the sources of inflation persistence when forcing the model-implied inflation expectations to be compatible with existing data on inflation expectations provided by surveys. In this way, RE and learning assumptions about the formation of inflation expectations are tested. The survey data used in this study come from the Survey of Professional Forecasts (SPF) and are also employed to determine the forecasting model used by the agents under learning.

The results of our study provide evidence that inflation expectations generated by a model under learning, when survey data are ignored, capture only low frequency movements of inflation. This becomes evident as soon as the estimation is implemented using a period of stable inflation levels. In that case, learning seems to be irrelevant for the generation of inflation persistence while exogenous shocks play the most prominent role. However, the incorporation of survey data allows the model to capture other properties of the data, for instance inflation persistence. As a result, even in a period of stable inflation, learning plays a key role in explaining inflation persistence while exogenous shocks lose their predominant importance. Moreover, we find that the estimated interest rate reaction to changes in inflation is less pronounced, price indexation is higher and wage indexation is lower in comparison with estimated parameters under RE or learning without the use of survey data.

In order to match the survey data under the assumption of RE, the model requires high levels of nominal frictions over both prices and wages. This is especially clear for the period of low inflation, when exogenous shocks lose their predominant importance in explaining inflation persistence, just as under learning. Given the implausible values of the parameters that capture nominal rigidities, we conclude that the RE assumption is not consistent with the survey data.

The estimations implemented in this study are derived from the medium-size New Keynesian DSGE model developed by Smets and Wouters (2007), one of the benchmark models for empirical analysis. In this sense, this study is related with the work of Slobodyan and Wouters (2007 and 2009), who are the first to estimate learning in the context of a medium-size DSGE model. Working with this type of model reduces the risk that some omitted variable could distort the contribution of learning to the dynamics of the model.

To the best of my knowledge, only Del Negro and Eusepi (2009) have incorporated survey data into the estimation of a DSGE model so far. As in the present study, the work of Del Negro and Eusepi is based on a New Keynesian model. However, the type of imperfect information considered by these authors is related to time-varying inflation targets of policy-makers, as in Erceg and Levin (2003). Here, by contrast, the representative agent is not aware of the law of motion of inflation, and survey expectations are employed to determine the model which agents most likely use.

The remainder of the paper is organized as follows. In the next section we describe the model used for the estimation. Section 3 discusses the setup of the learning process, while Section 4 presents the series of macroeconomic indicators used in the estimation as well as their relationship with the variables of the model, and the forecasting model for inflation used in the estimation under learning. Section 5 contains the results of estimating the model under RE and learning with and without the use of survey data on inflation expectations. There, we show the sources of inflation persistence for each of the cases and how survey data improve their identification. Finally, Section 6 concludes and outlines possible avenues for future research.

2. The Model

Our estimation is based on a New Keynesian model, more precisely a variant of the one proposed by Smets and Wouters (2007), who represent one of the key references in the literature on the estimation of medium-size dynamic stochastic general equilibrium (DGSE) models.

This model incorporates several frictions affecting both nominal as well as real decisions of households and firms. Households, on the one side, maximize their utility over an infinite life-time horizon. Their utility function depends first on the consumption of goods, which is considered relative to a time-varying external habit variable, and second on the labor effort invested in production. They also own the stock of capital in the economy, which they can either rent to firms or accumulate, subject to an adjustment cost. Households' labor is assumed to be differentiated by a union, which therefore, has monopolistic power over wages. Firms, on the other hand, produce differentiated goods, decide on the amount of labor and capital services and finally set their prices. Both, prices and wages are affected by nominal rigidities *à la* Calvo and additionally incorporate partial indexation with respect to past inflation. Finally, the model features a deterministic growth rate driven by labor-augmenting technological progress.

The version used in this study departs from the original specification by Smets and Wouters (2007) in only two respects. First, monetary policy rule does not adjust to the output gap (i.e. the difference between the output obtained under nominal rigidities and under flexible prices). Instead, monetary policy reacts to changes in the level of output (produced by the economy with rigidities) from one period to the next. This modification allows us to avoid the estimation of a parallel economy under flexible prices, which reduces the number of forward looking variables contained in the model considerably (as in Slobodyan and Wouters, 2009)[1]. Second, the stochastic shocks that affect wages and prices directly, namely price and wage mark-up shocks, are assumed to be autoregressive processes that do not incorporate past perturbations (in other words they are AR(1) processes, not ARMA(1,1)).

The model contains the following thirteen endogenous variables: output, y ; consumption, c ; investment, i ; the value of the capital stock , Q^k ; the installed stock of capital, $\overline{k}$; stock of capital, k ; the inflation, π ; the capital utilization rate, u ; the real rental rate on capital, r^k ; the real marginal cost, mc ; real wages, w ; hours worked, L ; and the interest rate, R . In addition, the stochastic part of the model is characterized by seven exogenous autoregressive processes, each of them including an *iid*-normally distributed error term. After detrending the model with respect to the deterministic growth rate of the labor-augmenting technological progress and linearizing it around the steady-state of the detrended variables, the model can be summarized in the following set of equations (where ^ represents detrended variables and * their steady state values)[2, 3]:

(1) $$\hat{y}_t = \hat{g}_t + \frac{c_*}{y_*}\hat{c}_t + \frac{i_*}{y_*}\hat{i}_t + \frac{r_*^k k_*}{y_*}\hat{u}_t$$

(2) $$\hat{c}_t = \frac{1}{(1+h)}\left(E_t\hat{c}_{t+1} + h\hat{c}_{t-1}\right) - c_1(E_t\hat{L}_{t+1} - \hat{L}_t) - c_2(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \hat{b}_t$$

(3) $$\hat{i}_t = \frac{1}{1+\overline{\beta}\gamma}(\hat{i}_{t-1} + \overline{\beta}\gamma E_t\hat{i}_{t+1} + \frac{1}{\gamma^2 S''}\hat{Q}_t^k) + \hat{q}_t$$

[1] This modification, however, does not affect the results obtained by Smets and Wouters (2007).

[2] The optimization problem of the households, firms and government as well as the equilibrium conditions are shown in Appendix 1.

[3] The exact representation of the exogenous stochastic process is presented in Appendix 2.

(4) $\hat{Q}_t^k = -(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{r_*^k}{r_*^k + (1-\delta)} E_t\hat{r}_{t+1}^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} E_t\hat{Q}_{t+1}^k + \tilde{b}_t$

(5) $\hat{y}_t = \Phi(\alpha\hat{k}_t + (1-\alpha)\hat{L}_t + \hat{A}_t)$

(6) $\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1}$

(7) $\hat{u}_t = \frac{1-\psi}{\psi}\hat{r}_t^k$

(8) $\hat{\bar{k}}_t = (1 - i_* / \bar{k}_*)\hat{\bar{k}}_{t-1} + \frac{i_*}{\bar{k}_*}\hat{i}_t + \frac{i_*}{\bar{k}_*}\left(1+\bar{\beta}\gamma\right)\gamma^2 S''\hat{q}_t$

(9) $\hat{\pi}_t = \frac{1}{\left(1+\bar{\beta}\gamma l_p\right)}(l_p\hat{\pi}_{t-1} + \bar{\beta}\gamma E_t\hat{\pi}_{t+1} + c_{mc}\widehat{mc}_t) + \hat{\lambda}_{p,t}$

(10) $\widehat{mc}_t = \left(1-\alpha\right)\hat{w}_t + \alpha\hat{r}_t^k - \hat{A}_t$

(11) $\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t$

(12) $\hat{w}_t = \frac{1}{(1+\bar{\beta}\gamma)}(\hat{w}_{t-1} + \bar{\beta}\gamma(E_t\hat{w}_{t+1} + E_t\hat{\pi}_{t+1}) - (1+\bar{\beta}\gamma l_w)\hat{\pi}_t + l_w\hat{\pi}_{t-1} +$

$+c_w(\frac{1}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) + \sigma_l\hat{L}_t - \hat{w}_t)) + \hat{\lambda}_{w,t}$

(13) $\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(r_\pi\hat{\pi}_t + r_{\Delta y}(\hat{y}_t - \hat{y}_{t-1})) + \hat{r}_t$

Equations (1) to (4) represent the demand side of the economy. Equation (1) is the aggregate resource constraint of the economy and indicates that output is spent on consumption, investment or absorbed via capital-utilization costs that are a function of the capital utilization rate, and exogenous spending $\hat{g}_t$. Equation (2) represents the Euler equation for consumption where $h = \eta / \gamma$, $c_1 = \frac{(\sigma_c - 1)w_*^h L_* / c_*}{\sigma_c(1+h)}$, $c_2 = \frac{1-h}{\sigma_c(1+h)}$. Note that η captures the external habit formation, γ is the growth rate of the labor-augmenting technological process, σ_c is the inverse of the elasticity of intertemporal substitution and w_*^h is the nominal wage received by households in steady state. This equation implies that current consumption depends on a weighted average of past and expected future consumption, on the expected growth in hours worked, the ex-ante real interest rate $(\hat{R}_t - E_t\hat{\pi}_{t+1})$, and a disturbance term $(\hat{b}_t)$.The Euler equation for investment is represented by Equation (3), where $\bar{\beta} = \beta / \gamma^{\sigma_c}$. β represents the discount factor applied by households and S'' stands for the steady-state elasticity of the capital adjustment cost function. The impact of the real value of existing capital stock ($\hat{Q}^k$) on investment depends on this elasticity. $\hat{q}_t$ is a disturbance to the investment-specific technology process. Equation (4) represents the arbitrage equation for the value of capital. It states that the current value of the capital stock depends positively on its expected future value and the expected real rental rate on capital, but negatively on the ex ante real interest rate and the risk premium disturbance, $\tilde{b}_t = c_2 \hat{b}_t$. δ represents the depreciation rate.

The supply side of the economy is characterized by Equations (5) to (12). The aggregate production function, Equation (5), indicates that output is produced using capital and labor services as inputs and is affected by the total factor productivity $\hat{A}_t$. α captures the share of capital in production and Φ equals one plus the share of fixed costs in production. Current capital used in production is assumed to be a linear function of installed capital in the previous period, $\hat{\bar{k}}_{t-1}$. This reflects the assumption that new capital becomes effective only with a one-quarter lag, and the degree of capital utilization $\hat{u}_t$, Equation (6). The positive relationship between the degree of capital utilization and the rental rate of capital is represented by Equation (7). In this equation ψ is a positive function of the elasticity of the capital utilization adjustment cost function and is normalized to a value between zero and one. Equation (8) represents the accumulation of installed capital as a function of the flow of investment and the relative efficiency of the investment expenditures captured by the investment-specific technology disturbance. The New-Keynesian Phillips curve is represented by Equation (9) and incorporates partial indexation of lagged inflation, where l_p represents the degree of indexation to past inflation, ε_p the curvature of the Kimball goods market aggregator, $\phi_p - 1$ the share of the fixed cost in production, ξ_p the degree of price stickiness and c_{mc} the slope related to marginal cost, where

$$c_{mc} = \frac{(1-\xi_p \bar{\beta}\gamma)(1-\xi_p)}{\xi_p((\phi_p - 1)\varepsilon_p + 1)}.$$

Finally, $\hat{\lambda}_{p,t}$ stands for the price mark-up disturbance and follows an AR(1) process. The marginal cost $\widehat{mc}_t$ is defined by Equation (10). Equation (11) signifies that the rental rate of capital is positively related with the capital-labor ratio, but negatively with the real wage. In the same way that nominal rigidities affect the price level determination, real wages can only adjust gradually to their optimal level. Equation (12) shows how the real wage is determined, where l_w represents wage indexation, ε_w the curvature of the Kimball aggregator of labor, $\phi_w - 1$ the steady state labor market mark-up; and c_w represents

$$c_w = \frac{\left(1-\xi_w \overline{\beta}\gamma\right)\left(1-\xi_w\right)}{\xi_w\left(\left(\phi_w - 1\right)\varepsilon_w + 1\right)}.$$

Analogously to above, $\hat{\lambda}_{w,t}$ stands for the wage mark-up disturbance and it follows an AR(1) process.

Finally, Equation (13) represents the monetary policy rule where ρ_R captures the degree of smoothing over the policy instrument and r_π and $r_{\Delta y}$ represent the responses of this instrument to deviations of inflation and output growth from their targets. $\hat{r}_t$ represents the non-systematic component of the interest rate and is assumed to follow an AR(1) process.

3. Learning mechanism of expectations formation

The model of the previous section incorporates expectations of several future variables. When dealing with expectations, researchers have traditionally adopted the rational expectations (RE) assumption. This assumption implies that agents have perfect knowledge of the true stochastic process of the economy. Given that such high level of cognitive abilities and computational skills seem to be implausible in practice, researchers have developed models of imperfect knowledge and associated learning processes. One of the most popular learning mechanisms used in macroeconomics is adaptive learning. Under this approach agents are assumed to use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data becomes available (see Evans and Honkapohja 2001).

It is common in the literature on learning to assume that agents update the coefficients of their forecasting models using constant gain least squares (CG-LS). Least squares is a very popular estimation method in econometrics and the tool that probably most of real forecasters use in practice. Under CG-LS, the most recent observations receive higher weights in the least square estimation. More precisely, the weight that each observation receives decreases geometrically depending on the distance in time between the most recent observation and the one under consideration. This estimation procedure implies that agents are concerned about changes in the structural parameters of the economy.

In the remainder of this section we provide details of the algorithm followed by the representative agent to update her expectations and characterize the resulting dynamics of the economy. We also specify the forecasting models used in this study and the initial conditions of the recursive CG-LS.

3.1 Ordinary Least Squared with constant gain

Considering that the forecasting model used to generate one-period-ahead expectations of the set of variable Y^f can be represented as:

(14) $$Y_t^f = \beta \; 'X_{t-1},$$

the recursive expression for the estimate of β under CG-LS is:

(15a) $$\hat{\beta}_t = \hat{\beta}_{t-1} + g \; \left(R_t\right)^{-1} X_t \left(Y_t^f - \hat{\beta}_{t-1}\, 'X_t\right)'$$

(15b) $$R_t = R_{t-1} + g \; \left(X_t X_t \, ' - R_{t-1}\right)$$

where g represents the *constant gain parameter* and R_t the variance-covariance matrix of the regressors included in the forecasting model. The "gain" refers to the relative weight of the most recent observation and 1 – gain is the discount factor over less recent observations (in ordinary least squares, the gain is equal to *1/t*, where *t* is the position of the observation since the beginning of the sample).

Using β, we can generate the forecast of the variables included in Y_t^f :

(16) $$\hat{E}_t Y_{t+1}^f = \hat{\beta}_{t-1} \, '[X_t]$$

Employing $\hat{\beta}_{t-1}$ instead of $\hat{\beta}_t$ in equation (3.3) is a standard procedure in learning estimation in order to avoid the simultaneous determination of $\hat{\beta}_t$ and the variables included in the solution of the model.

3.2 Model expectations augmented by learning

Using equations (1) to (13), described in Section 2, we can derive the equilibrium conditions describing the dynamics and the interactions of all endogenous variables under RE. For ease of representation, it is useful to use a generic form of the solution under RE:

(17) $$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA^{re} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB^{re} \left[\vartheta_t \right]$$

where Y_t contains all the endogenous variables of the model, Z_t contains all the exogenous variables and ϑ_t contains all their iid-normal perturbations. AA^{re} and BB^{re} are matrices of very complex nonlinear functions of the structural parameters of the DSGE model.

When estimated under learning, the set of equations (1) – (13) is augmented by equation (16), by equations (15a) and (15b) which describe the estimation procedure for the β, and by some initial conditions for the CG-LS algorithm (which we describe later). The only parameter that is added to the set of structural parameters is the gain parameter.

Under learning, we can rewrite the system containing all endogenous variables of the model and replace those in expectations in the following compact way:

(18) $$\begin{bmatrix} Y_t \\ Z_t \end{bmatrix} = AA_{t-1}^{learning} \begin{bmatrix} Y_{t-1} \\ Z_{t-1} \end{bmatrix} + BB_{t-1}^{learning} \left[\vartheta_t \right]$$

Matrices $AA_{t-1}^{learning}$ and $BB_{t-1}^{learning}$ vary over time, as they contain not only the parameters of the structural model, but also the time-varying coefficients of the forecasting models ($\hat{\beta}_{t-1}$). The time variations of these coefficients depend on the value of the gain parameters *g*. If these parameters are equal to zero, the matrices $AA^{learning}$ and $BB^{learning}$ are constant. However, even in this situation they might not be equal to the corresponding matrices under RE. This depends on the selection of the forecasting model and the initial conditions of the CG-LS.

3.3 Learning setting used in this study

We use survey data on inflation expectations to determine the forecasting model for inflation used by agents. Due to lack of availability for the complete sample, surveys can unfortunately not be applied in the selection of the forecasting model for all other variables that appear in expectations in the model. For these cases, we choose not to depart indiscriminately far from the RE case and therefore use as forecasting models specifications with the same set of regressors that are included in the RE case. Slobodyan and Wouters (2007) find that when the forecasting model uses the same set of regressors than the RE case the differences between RE and learning are minimal. Thus, the discrepancies between the estimation under RE and learning we implement in this study are basically related with the learning process of inflation.

In our estimations, Equation (14) can be rewritten in the following way:

(19a) $Y_t^{\pi} = \beta^{\pi}\,' X_{t-1}^{\pi}$

(19b) $Y_t^{non-\pi} = \beta^{non-\pi}\,' \left[X_{t-1}^{non-\pi} \right]$

where $Y_t^{\pi} = \left[dlP\right]_t$, $Y_t^{non-\pi} = [\hat{c}_t \quad \hat{i}_t \quad \hat{q}_t \quad \hat{r}_t^k \quad \hat{w}_t \quad \hat{L}_t]$, $X_{t-1}^{\pi} = \left[1 \quad dlP_{t-1}\right]$, and $X_{t-1}^{non-\pi} = [1 \quad Y_{t-1} \quad Z_{t-1}]$, where *dlP* refers to the actual series of inflation used in the estimation of the DSGE model (see the next section). $\beta^{non-\pi}$ contains rows of matrices AA^{re} and BB^{re} corresponding to the variables in $Y_t^{non-\pi}$. X_{t-1}^{π} and $X_{t-1}^{non-\pi}$ includes an intercept to allows for temporal deviations from the estimated steady state of the model with respect to the pure RE specification.

The previous equations indicate that we are dealing with two learning blocks: one related to the inflation process and the other to the processes of the remaining variables which appear with expectations in the model. Therefore, we have not one but two gain parameters and we need to define two sets of initial conditions for the CG-LS.

With respect to the forecasting model of inflation, given that actual series are employed to estimate β^{π}, the initial conditions for β_0^{π} and R_0^{π} are obtained using actual data of inflation for a pre-sample. The initial value for $\beta_0^{non-\pi}$ and $R_0^{non-\pi}$ for the forecasting model of the remaining variables are taken from the solution under rational expectations. The value of $\beta_0^{non-\pi}$ comes from the expression (17), while the value of $R_0^{non-\pi}$ can be derived from the expression of the unconditional variance matrix of Y_t obtained from the solution of RE. As pointed by Slobodyan and Wouters (2007), the only differences in the dynamics of this set of variables ("non-inflation" variables) in comparison to RE is related to the temporary deviations of beliefs from their RE values caused by in-sample data fluctuations and the related stochasticity of the constant gain. These deviations are, for instance, zero when the gain is zero.

Finally, it is important to mention that when a small forecasting model is used to generate expectations the equilibrium is not longer compatible with the equilibrium achieved under RE.

The relevant equilibrium concept in this case is the *restricted perceptions equilibrium* (RPE)[4] and is motivated by agents that, because they are unaware of the "true" structure of the economy, find it optimal to use small (misspecified) forecasting models. Two conditions are required for this equilibrium to exist: first, the selected forecasting model should generate a lower mean-squared error than the one produced by other potential models and second, the equilibrium achieved has to be expectationally stable. Both conditions hold in this study.

4. Data, measurement equations and priors

The model is estimated using the same quarterly macroeconomic indicators for the US as in Smets and Wouters (2007), but additionally employing the series of survey data on inflation expectations provided by the Survey of Professional Forecasters (SPF). In particular, for each quarter, we calculate the median value of the reported one-period-ahead forecast of the percentage increase of the GDP deflator. The resulting series is henceforth referred to as "*exp_dlP*". As this data is available only from 1968Q4 onward, this date marks the starting point for our sample. The sample covers all quarters until 2008Q2. The other macroeconomic indicators are the log difference of real GDP ("*dlGDP*"), of real consumption ("*dlCons*"), of real investment ("*dlInv*") and of the real wage ("*dlWage*"), as well as the log of hours worked ("*lHours*"), the log difference of the GDP deflator ("*dlP*") and the federal funds rate ("*FedFunds*"). Appendix 3 contains a description of the data.

The way in which these macroeconomic indicators are related with the variables of the model under RE and learning when survey data on inflation expectations are not included is summarized by the following measurement equations:

[4] The name of Restricted Perceptions Equilibrium (RPE) was given by Evans and Honkapohja (2001). Branch (2004) discusses the generality of RPE as it encompasses many forms of misspecified equilibria such as the Self-Confirming Equilibrium in Sargent (1999) and the Consistent Expectations Equilibrium in Hommes and Sorger (1998).

$$
\begin{bmatrix} dlGDP_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFunds_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}
$$

where $\overline{\gamma} = 100(\gamma - 1)$ represents the common quarterly trend growth rate, $\overline{l}$ the steady state hours worked, $\overline{\pi} = 100(\Pi_* - 1)$ the quarterly steady state inflation rate and $\overline{r} = 100(\gamma^{\sigma_c}\Pi_* / \beta - 1)$ the quarterly steady state nominal interest rate.

When survey data is incorporated in the estimation under RE and under learning, the measurement equations are:

$$
\begin{bmatrix} dlGDP_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFunds_t \\ \exp\_dlP_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{l} \\ \overline{\pi} \\ \overline{r} \\ \overline{\pi} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \\ E_t\hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \zeta_t \end{bmatrix}
$$

where ζ_t represents the measurement error (*iid*) related to the surveys on inflation expectations. Hence, survey data is taken as a noisy measurement of actual expectations. The new shock allows us to circumvent the "stochastic singularity problem" that would otherwise result from having more series included in the estimation than stochastic shocks.

Besides being used for estimating the DSGE model, the macroeconomic series are used to select the forecasting model for inflation employed in the learning estimation. In particular, we compare the models that can be obtained using as regressors, besides an intercept, all possible combinations of the lagged series of dlGDP, dlCons, dlInv, dlWage, dlP, FedFunds and lHours. Then, we rank these models (127 in total) according to their forecasting performance with respect to the survey data on inflation expectations, measured by the Mean Squared Error (MSE). Table 1 shows the five best-performing forecasting models for the periods 1968Q4 –

2008Q2 and 1984Q1 – 2008Q2 (the latter is used for robustness check of some of the results presented in the next section)[5]. In both cases, the best model is the one that considers as regressors only lagged inflation and the intercept.

Table 1
Ranking of forecasting models by MSE

	Period: 1968Q4 - 2008Q2			Period: 1984Q1 - 2008Q2		
Rank	Model	Gain	MSE	Model	Gain	MSE
1	dlP	0.125	0.0294	dlP	0.225	0.0238
2	dlP lHours	0.113	0.0300	dlP dlCons	0.238	0.0250
3	dlP dlCons	0.100	0.0302	dlP dlCons FedFunds	0.150	0.0264
4	dlP dlCons lHours	0.125	0.0303	dlP FedFunds	0.150	0.0266
5	dlP dlGDP	0.125	0.0315	dlP dlCons dlInv FedFunds	0.138	0.0277

Note: the models are estimated by recursive CG-LS. The initial conditions for the periods 1968Q4-2008Q2 and 1984Q1-2008Q2 are obtained from the periods 1950Q1-1968Q3 and 1974Q1-1983Q4, respectively. Regression: dlP_t = intercept + $model_{t-1}$

The model contains 38 structural parameters, and 33 are estimated. The learning estimation adds another two parameters (the gains for inflation and for the other variables that appear in expectations). When estimating the model with survey data, we consider one extra parameter, namely the standard deviation of the measurement error of the surveys (ζ_t). The prior distributions of the structural parameters are as in Smets and Wouters; for the gains, uniform distributions over the [0,0.30] domain; and for the standard error of ζ_t, an inverse gamma distribution with zero mean and standard deviation of 2. The prior distributions for all the parameters are presented in Appendix 4.

The estimation of the DSGE model is performed using Bayesian estimation methods. Employing the random walk Metropolis-Hastings algorithm I obtain 500 000 draws from each model's posterior distribution. The first half of these draws is discarded and 1 out of every 10 is selected in order to estimate the moments of the posterior distributions.

[5] Let us outline the manner in which this ranking is constructed. First, each model is estimated using a recursive CG-LS, which allows for compatibility with the algorithm behind the inflation expectations formation under learning. Second, recursive CG-LS requires the definition of the initial values for the coefficients to be estimated and of the variance-covariance matrix of the regressors. These values are obtained using Ordinary Least Squares (OLS) over a pre-sample. Third, different values of the constant gain are employed to produce forecasts for each of the models (these values are taken from a grid of points that goes from 0 to 0.30, with steps of 0.0125). The ranking is then established taking into account for each of the models the value of the constant gain that results in the best MSE. Finally, given that the ordering could vary depending on the choice of the pre-sample, different pre-samples are considered and we select the one with the lowest MSE among the top models.

5. Results

The main objective of this study is to identify the sources of inflation persistence when forcing the model-implied inflation expectations to be compatible with the data on inflation expectations provided by surveys. RE and learning assumptions about inflation expectations formation are tested. The analysis proceeds in the following three steps. First, we compare the predictions for inflation persistence yielded by the basic model under RE and the model under learning. In this step survey data on inflation expectations are only used to determine the type of forecasting model for inflation under learning. In a second step, we include data on inflation expectations in the set of information used to estimate the DSGE model under RE and learning and assess how the inclusion of this information affects the previous results. Finally, for robustness we also conduct a subsample analysis and discuss the robustness of our findings when different forecasting models for inflation are used under learning.

5.1 The importance of the assumption of expectations formation over the estimated sources of inflation persistence

As first step in our analysis we estimate the DSGE model under RE and learning for the complete sample (1968Q4-2008Q2) and discuss the results obtained in terms of the posterior statistics of the parameters which are closely related to the dynamics of inflation (Table 2). The posterior statistics of the complete set of parameters can be found in Appendix 5.

Table 2
Posterior distribution statistics: RE and learning estimations

		(1) RE		(2) Learning	
	Symbol	Median	Std	Median	Std
Wage stickiness	ξ_w	0,552	0,042	0,565	0,066
Price stickiness	ξ_p	0,648	0,042	0,489	0,038
Wage indexation	ι_w	0,491	0,137	0,335	0,101
Price indexation	ι_p	0,281	0,149	0,506	0,111
TR: inflation	r_π	1,660	0,120	1,390	0,120
TR: lag interest rate	ρ_R	0,756	0,028	0,778	0,025
TR: change in output	$r_{\Delta y}$	0,205	0,045	0,210	0,045
aut. Price Mk up shock	ρ_p	0,518	0,200	0,138	0,072
std. Price mkup shock	σ_p	0,135	0,026	0,211	0,013
gain - inflation	g^{π}			0,187	0,012
gain - others	$g^{non\pi}$			0,105	0,043
Log. Mg. Likelihood		-146,4		-144,2	

In line with the findings by Slobodyan and Wouters (2009), the differences between both estimations are mainly concentrated in the estimated values of the parameters that capture the nominal frictions in prices and wages, and in the stochastic shocks related with inflation, namely price mark-up shocks. In particular, the inclusion of learning in a DSGE model leads to a significant decrease in price stickiness and, though not statistically significant, in wage indexation but to an increase in price indexation. Moreover, the autoregressive component of the price mark-up shocks shrinks significantly and the response of the interest rate to changes in inflation decreases.

All these changes in the parameter estimates and the adoption of learning have important effects on the estimated sources of the persistence of inflation. Thus, when estimating the model under RE, the autoregressive components of the structural shocks can explain 30% of inflation persistence (see Table 3[6], Column 1), while when estimating the model under learning (see Column 2), the relative importance of the autoregressive components is reduced to 10%. Moreover, the dynamics associated with the process of learning about future inflation rates explains 30% of total inflation persistence while learning associated with all other forecasted variables explains only 8%. Notice also that the part of the persistence explained by other features of the model not included in Table 3 represent roughly 70% under RE but only 52% under learning.

[6] The marginal contribution of the different sources of inflation persistence is calculated as follows. First, we simulate the macroeconomic indicators for a given set of innovations, taking into consideration (i) the complete model, (ii) the model without any autoregressive coefficients in the structural shock, (iii) the model with the previous features and, additionally, fixed beliefs about inflation only (fixed to their initial values), and (iv) the model with all previous features but, additionally, with fixed beliefs about all other variables that appear in expectations (fixed to their initial values). Second, we calculate for each of these cases the autocorrelation coefficient of inflation. Thus, the marginal contribution of the autoregressive coefficients over the total inflation persistence corresponds to the ratio of the difference between the coefficients obtained under the fist (i) and the second (ii) specification and the coefficient obtained under (i); the marginal contribution of learning associated with inflation can be measured as the ratio between the difference between the coefficients obtained under the third (iii) and the second (ii) specification and the coefficient under (i); etc.. Last, we repeat the previous steps for 10 sets of innovations per each draw of parameter sets taken from the posterior distributions. As a total, we use 500 draws of parameters sets to build the distributions of the marginal contributions per sources of the inflation persistence.

Table 3
Simulated inflation persistence

	(1) RE			(2) Learning		
Actual inflation persistence: **0.8693**	16%	Median	84%	16%	Median	84%
Complete model	0.770	0.815	0.852	0.794	0.845	0.882
% explained by autocorr shocks	21%	30%	41%	6%	10%	16%
% explained by inflation beliefs	--	--	--	22%	30%	39%
% explained by non infl. beliefs	--	--	--	2%	8%	17%

Note: This table shows the median and the 16th and 84th percentiles of the inflation persistence coefficient obtained from simulated series. The marginal contributions associated with the autoregressive part of the shocks and the learning component are measured in percentages.

These results illustrate the implications of adopting different assumptions about how expectations are formed for the identification of the sources of inflation persistence. Under RE most of inflation persistence is explained by the persistence of the price mark-up shocks, while under learning the process of expectation formation is itself one of the main driving forces behind inflation persistence. In the first case there is no obvious policy design that would reduce inflation persistence, while in the second case a more transparent conduct of monetary policy (or other policies designed to improve the learning process) could be effective.

Before continuing with the main line of our analysis, we discuss three important aspects of the results presented above that are related with the findings of previous research. First, the learning mechanism takes over the role of exogenous shocks in explaining inflation persistence to a significant extent, while basically leaving the total level of the simulated inflation persistence unchanged. By contrast, Orphanides and Williams (2004, 2005) show that adding learning, while leaving all other features of the model constant, increases the persistence of inflation. One has to bear in mind, however, that their calibration exercises do not allow for the joint determination of the roles of learning and the other features of the model in explaining inflation persistence. Thus, the omission of the substitution effect between learning and the exogenous part of the model in generating persistence could overestimate the duration and amplitude of the response of inflation in their exercise. Second, learning captures part of the persistence explained by some of the structural parameters. This finding is not obvious at first sight as the variations in the posterior distributions of the parameters resulting from the model with learning affect the inflation persistence in several opposing ways: for instance, inflation persistence is diminished by the reduction in price stickiness but raised by the increase in price indexation[7]. Finally, we observe only relatively modest gains in terms of

[7] Using a particular version of a small New Keynesian model, Milani (2007) finds that when adding learning price indexation drops to zero, as does the degree of habits in consumption, another parameter

the log marginal likelihoods between the estimation under RE and the one under learning (see Table 2)[8,9].

So far, the analysis has been implemented over the period 1968Q4-2008Q2. Within this period there are some years during which inflation fluctuated significantly and even changed its trend (for instance between years 1973 and 1982). In contrast to a model under RE, under learning there is no model-consistency condition imposed on expectations. Hence, generated inflation expectations could be capturing some features of inflation not necessarily related with actual expectations. A first warning bell should ring when tracing the evolution of the intercept and coefficient of lagged inflation from the forecasting model used in the estimation under learning. Figure 1 reveals that both coefficients change significantly over time. In particular, the intercept seems to reflect precisely the magnitude and the duration of the significant increase of inflation which occurred between the last quarter of 1977 and the beginning of 1982. The sharp and relatively short increase in inflation which resulted from the 1973 oil crisis is mainly captured by an increase in the coefficient of lagged inflation, and by a decrease in the intercept of the forecasting model. For the remaining sample, even though both coefficients keep varying, the intercept remains in the vicinity of 0.4 while the coefficient of the lagged inflation index fluctuates around 0.15.

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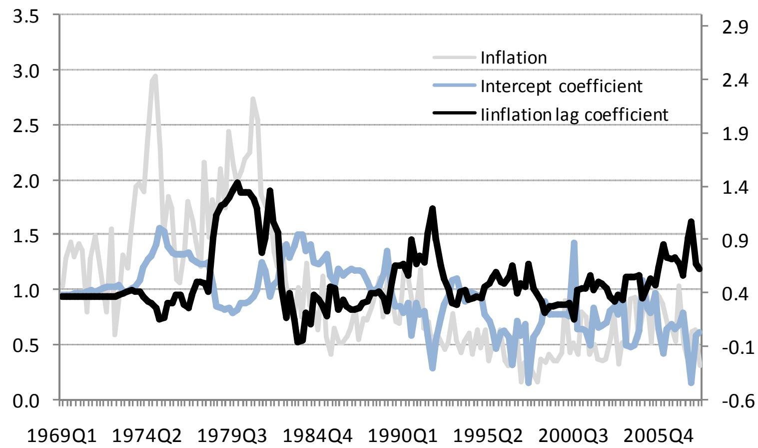
that affects the persistence of the model. He therefore suggests that the persistence arises in the model mainly from expectations and learning. However, this statement is not completely unambiguous given that his results also exhibit a sharp increase in the magnitude of the autocorrelation coefficient of the supply shocks under learning (the mean value of the posterior distribution increases from 0.02 in the RE case to 0.854 under learning). Therefore, it could be the case that learning only produces a switch from persistence generated by the price indexation or habits to persistence generated by the supply shocks.

[8] In order to favor the estimation under RE over the one under learning, a prior probability of the former is required which exceeds the prior probability of the latter by 8.9 times (which corresponds to exp(-144.2+146.4)). This value is relatively small in comparison to the findings of other studies (see Rabanal and Rubio-Ramirez 2005).

[9] Slobodyan and Wouters (2007) find more significant gains in terms of log marginal likelihoods from using learning. They find values for this statistic of -926 and -917 under RE and learning, respectively. They use small forecasting models for all the variables that appear with expectations. Therefore, probably, the gains in terms of log marginal likelihood do not come from learning process related with inflation but from the other variables.

Figure 1
Evolution of the coefficients of the forecasting model for inflation



For this reason, we repeat the previous analysis using a subsample during which inflation was more stable. According to the literature which analyses the changes in the volatility of macroeconomic indicators in the US, the year 1984 can be seen as the starting point of a more stable macroeconomic period (see Gali and Gambetti, 2009). Therefore, we choose the period between 1984Q1 and 2008Q2 for a subsample analysis.

By estimating the DSGE model under RE and learning for the period of stable inflation we find some important differences in terms of posterior distribution statistics with respect to those obtained for the complete sample. First, the median values of both gain parameters are now close to zero (see Table 4, Column 4). This result implies that the coefficients of the forecasting model associated with this parameter remain constant through the whole estimation period. As we show bellow, this result has important consequences in terms of inflation persistence. Second, learning estimates imply high degree of nominal frictions in both prices and wages with respect to the estimates under RE (under learning both medians of the posterior distributions for wage and price stickiness are higher than under RE). In addition, there is an increase in the difference of the degree of price indexation between learning and RE, which is higher under learning. There is also an increase in the differences of the degree of autocorrelation between learning and RE, which is higher under RE.

Table 4
Posterior distribution statistics: RE and learning estimations

		(1)		(2)		(3)		(4)	
		Complete sample*				Post-1984 sample*			
		RE		Learning		RE		Learning	
	Symbol	Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	ξ_w	0,552	0,042	0,565	0,066	0,248	0,057	0,464	0,070
Price stickiness	ξ_p	0,648	0,042	0,489	0,038	0,457	0,049	0,619	0,025
Wage indexation	ι_w	0,491	0,137	0,335	0,101	0,484	0,144	0,392	0,135
Price indexation	ι_p	0,281	0,149	0,506	0,111	0,170	0,070	0,463	0,118
TR: inflation	r_π	1,660	0,120	1,390	0,120	2,074	0,171	1,650	0,170
TR: lag interest rate	ρ_R	0,756	0,028	0,778	0,025	0,776	0,026	0,806	0,024
TR: change in output	$r_{\Delta y}$	0,205	0,045	0,210	0,045	0,169	0,044	0,183	0,048
aut. Price Mk up shock	ρ_p	0,518	0,200	0,138	0,072	0,978	0,011	0,075	0,053
std. Price mkup shock	σ_p	0,135	0,026	0,211	0,013	0,127	0,020	0,111	0,012
gain - inflation	g^{π}			0,187	0,012			0,006	0,004
gain - others	$g^{non\pi}$			0,105	0,043			0,047	0,037
Log. Mg. Likelihood		-146,4		-144,2		74,6		62,5	

* "Complete sample" and "Post-1984 sample" refer to the periods 1968Q4-2008Q2 and 1984Q1-2008Q2, respectively. Note: Columns 1 and 2 reproduce Table 2 results for easing the comparison

All these changes in the parameter estimates have first-order effects on the composition of the estimated sources of inflation persistence. The most significant of these changes is that learning is not longer a source of inflation persistence (Table 5, Column 2). The lack of any significant fluctuation in the trend of inflation seems to be connected with this result. Additionally, even though the autoregressive coefficient of the price mark-up shocks is slightly lower in this case, the importance of the structural shocks explaining inflation persistence has increased. This result is explained by the fact that the autocorrelation coefficients of other shocks such as wage price mark-up shocks, total factor productivity, investment specific technology shocks and monetary policy shocks have increased (see Appendix 5). Finally, under the case of RE, the importance of the structural shocks has also increased. Now, they explain 63 percent of inflation persistence.

As illustrated by the previous results, we find that the relative importance of the estimated sources of inflation persistence obtained under learning is not robust to changes in the sample used. For periods of stable inflation, the mechanisms of expectations formation do not play any role explaining inflation persistence while when the sample includes significant fluctuations in inflation, learning seems to have predominant role. Thus, learning seems to be capturing these low frequency movements in inflation. This result could be explained by the fact that the learning estimation lacks model-consistency conditions over the model-implied expectations, in contrast with RE, which allow these expectations to capture other features of the data.

Table 5
Simulated inflation persistence, Post-84 sample

	(1) RE			(2) Learning		
Actual inflation persistence: **0.4829**	16%	Median	84%	16%	Median	84%
Complete model	0,596	0,675	0,737	0,660	0,762	0,846
% explained by autocorr shocks	51%	63%	78%	33%	46%	60%
% explained by inflation beliefs	--	--	--	0%	0%	0%
% explained by non infl. beliefs	--	--	--	0%	1%	2%

Note: This table shows the median and the 16^{th} and 84^{th} percentiles of the inflation persistence coefficient obtained from simulated series. The marginal contributions associated with the autoregressive part of the shocks and the learning component are measured in percentages.

Up to this point the discrepancy concerning the sources of inflation persistence seems to have disappeared: structural shocks are the predominant source, no matter what is the assumption about expectations formation.

5.2 Survey data on inflation expectations and their role in identifying the source of inflation persistence

So far, survey data on inflation expectations have been used only for the selection of the forecasting model for inflation under learning. However, this information can also be used directly in the estimation of the DSGE as a proxy of inflation expectations, under both RE and learning assumptions. By doing this, we are forcing the model-implied inflation expectations to be compatible with the data on inflation expectations provided by surveys. As Table 6 shows, adding surveys in the estimation of the DSGE model has important implications in terms of the posterior distribution statistics[10].

In the case of RE, adding survey data to the estimation affects the posterior distribution statistics of some of the structural parameters which are key for the dynamics of inflation. In particular, the posterior medians of wage and price stickiness increase significantly. A posterior median of 0.892 for the wage stickiness implies that wages are re-optimized, on average, once every 9.2 quarters. Given existing microeconomic evidence (see Heckel, et al, 2008), this frequency seems implausible. The same figure appears for the price stickiness: a posterior median value of 0.878 implies that prices are re-optimized, on average, once every 8.2 quarters. Additionally, the response of the interest rate to changes in inflation decreases from

[10] As before, the posterior distribution statistics for all the parameters estimated are shown in Appendix 6.

a posterior median value of 2.1 to 1.3, a low value considering that during this subsample the common consensus is that the US central bank reacted aggressively to inflation.

Table 6
Posterior distribution statistics: RE and learning estimations, Post-84 sample

		(1)		(2)		(3)		(4)	
		WITHOUT survey data				WITH survey data			
		RE		Learning		RE		Learning	
	Symbol	Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	ξ_w	0,248	0,057	0,464	0,070	0,892	0,032	0,461	0,067
Price stickiness	ξ_p	0,457	0,049	0,619	0,025	0,878	0,019	0,681	0,034
Wage indexation	ι_w	0,484	0,144	0,392	0,135	0,349	0,130	0,368	0,125
Price indexation	ι_p	0,170	0,070	0,463	0,118	0,222	0,105	0,665	0,102
TR: inflation	r_π	2,074	0,171	1,650	0,170	1,289	0,084	1,526	0,169
TR: lag interest rate	ρ_R	0,776	0,026	0,806	0,024	0,802	0,022	0,793	0,034
TR: change in output	$r_{\Delta y}$	0,169	0,044	0,183	0,048	0,183	0,050	0,181	0,044
aut. Price Mk up shock	ρ_p	0,978	0,011	0,075	0,053	0,158	0,089	0,113	0,067
std. Price mkup shock	σ_p	0,127	0,020	0,111	0,012	0,154	0,014	0,178	0,016
gain - inflation	g^{π}			0,006	0,004			0,201	0,006
gain - others	$g^{non\pi}$			0,047	0,037			0,005	0,009
Measurement exp error	σ_{exp}					0,157	0,013	0,151	0,010
Log. Mg. Likelihood		74,6		62,5		180,9		184,2	

Note: Columns 3 and 4 reproduce partially Table 4 results for easing the comparison

By contrast, when survey data is added to the estimation under learning, most of the parameters are barely altered. The only exception is the gain parameter with respect to the learning process of inflation. The median posterior value increases from zero to 0.20. This change has an important impact on the relative importance of different sources of inflation persistence.

Table 7
Simulated inflation persistence, Post-84 sample:
estimation using survey data

	(1)			(2)		
	RE			Learning		
Actual inflation persistence: **0.4829**	16%	Median	84%	16%	Median	84%
Complete model	0,875	0,923	0,958	0,691	0,795	0,872
% explained by autocorr shocks	63%	77%	90%	3%	9%	17%
% explained by inflation beliefs	--	--	--	29%	36%	41%
% explained by non infl. beliefs	--	--	--	0%	1%	2%

Note: This table shows the median and the 16^{th} and 84^{th} percentiles of the inflation persistence coefficient obtained from simulated series. The marginal contributions associated with the autoregressive part of the shocks and the learning component are measured in percentages.

As Table 7 shows, under RE the impact of structural shocks now explains 77% of inflation persistence. However, under learning the picture is completely different: the learning process of expectation formation can explain 36% of inflation persistence while the structural shocks do not play an important role.

Given the implausible estimates for some of the important parameters underpinning the dynamics of inflation, we conclude that the assumption of rational expectation, given the DSGE model used in this estimation, is not compatible with attempts to match survey data on inflation expectations. Learning, on the other hand, provides more plausible parameter estimates and can better explain the evolution of the series of inflation, inflation expectations, the interest rate, investment and real wage growth, in term of the root mean squared errors (RMSE) (see Table 8), and a slightly higher log marginal likelihood compared with RE (as shown in Table 6). Therefore, this assumption on expectation formation seems considerably more compatible with survey data.

Table 8
In sample RMSE, Post-84 sample: estimation using survey data

Variable	RE	Learning
Consumption growth	0,504	0,521
Investment growth	1,583	1,497
Wages growth	0,709	0,697
Inflation	0,233	0,228
Output growth	0,527	0,552
Interest rate	0,105	0,101
Hours worked	0,374	0,382
Inflation expectations	0,154	0,139

Why does adding survey data have such a significant effect under learning? Two factors explain this result. First, the gain parameter related with the learning process for inflation determines the degree of correlation between inflation and expected inflation. This relationship can be derived directly from the forecasting model used to predict inflation. Second, as shown in the previous section, the gain parameter is also related with inflation persistence. Because of this, low frequency movements of inflation can be captured by inflation expectations. Therefore, once survey data are incorporated in the estimation of the DSGE model under learning, the information about the correlation between inflation and inflation expectations is used to determine the value of the gain parameter, and the resulting estimate has a direct impact on the persistence of inflation. When survey data is not included, the role of learning in generating inflation persistence can be underestimated (as is the case Post-1984 sample) or overestimated (as it is the case for the complete sample).

5.3 Robustness check: adding different forecasting models

In order to examine the robustness of the results presented above, we use different forecasting models for inflation in the estimation of the DSGE model under learning. Table 9 reports the posterior distribution statistics of the gain parameter of the learning process with respect to inflation, the log marginal likelihood and the in-sample root of the mean of squared inflation innovations (R*MSE inflation*) for different forecasting models[11].

Table 9
Comparison across forecasting models, learning estimation Post-84 sample[12]

	WITHOUT survey data				WITH survey data			
	Gain		Log Mg.	RMSE	Gain		Log Mg.	RMSE
Forecasting model	Median	Std	likelihood	inflation	Median	Std	likelihood	inflation
dlP	0.006	0.004	62.5	0.227	0.201	0.006	184.2	0.228
dlP dlCons	0.206	0.047	49.5	0.235	0.199	0.007	185.4	0.234
dlP dlCons FedFunds	0.005	0.003	62.1	0.228	0.150	0.008	170.0	0.245
dlP FedFunds	0.165	0.012	53.9	0.246	0.160	0.009	171.8	0.236
dlP dlCons dlInv FedFunds	0.005	0.005	58.0	0.231	0.150	0.015	167.9	0.238

Without considering survey data, the median of the posterior distributions of the gain parameter varies depending on the selected forecasting model. However, the cases where this parameter is zero also display the highest log marginal likelihoods (measurement of global fit of the model) as well as the lowest RMSEs for inflation (individual measurement fit). In other words, models with the best fits are the ones that report zero gain parameter. As indicated previously, a gain parameter close to zero indicates that learning does not have any role in explaining inflation persistence.

When survey data is included in the estimation, the posterior median of the gain parameters lies between 0.15 and 0.20. These magnitudes imply an important time-variability in the structure of the economy and therefore a potential role for learning in explaining persistence. In fact, in terms of persistence the results are very similar to those reported in the previous subsections. Hence, survey data seem to be essential for clarifying the role of learning when explaining inflation persistence. Ignoring information from survey data can result in false conclusions being drawn; in particular without survey data the models that fit the data best are those where learning does not play any role at all in the dynamics of inflation (gain parameter equal to zero).

[11] The in-sample root of the mean of squared innovations represents a measurement for the individual fit of each of the series included in the estimation.

[12] The forecasting models behind the results of this table are the top-five forecasting models obtained using the same steps presented in subsection 5.1 but for the simple 1984Q1-2008Q2. The pre-sample used to initialize the CG-LS is the period 1974Q1-1983Q4.

In conclusion, our findings confirm that survey data on inflation expectations is crucial for an adequate identification of the role of learning in explaining the persistence of inflation. Ignoring survey data, different results can be obtained depending on the exact forecasting model for inflation which is selected. As soon as survey data is taken into account the message is unambiguous: learning plays a key role in explaining inflation persistence.

6. Conclusions

In this paper we evaluate the role of learning in explaining the persistence of inflation when the model-implied inflation expectations are forced to be compatible with survey data on inflation expectations.

Employing surveys in the context of a DSGE model under learning is a novel strategy and we use it in the following two ways. First, we exploit the information provided by surveys to determine the forecasting model used by agents. Second, we add surveys to the estimation of a DSGE which allows us to test for the compatibility between the mechanism of expectation formation and data on expectations.

The results of our study provide evidence that inflation expectations generated by a model under learning, when ignoring survey data, capture only low frequency movements of inflation. This becomes evident as soon as the estimation is implemented using a period of stable inflation levels. In this case, learning seems to be irrelevant for the generation of inflation persistence while exogenous shocks play the most prominent role. However, adding survey data allows the model to capture other properties of the data, for instance inflation persistence. As a result, even in a period of stable inflation, learning plays a key role in explaining inflation persistence while exogenous shocks lose their predominant importance. Moreover, we find that the estimated interest rate reaction to changes in inflation is less pronounced, price indexation is higher and wage indexation is lower in comparison with estimated parameters under RE or learning without using survey data.

Finally, we use survey data to test the expectation mechanism under the RE assumption. The implausibility of some of the parameter estimates lead us to conclude that the assumption of rational expectation, given the DSGE model used in this estimation, is not compatible in order to match survey data on inflation expectations.

There are some important issues not yet addressed in this study. Three of them constitute our agenda for future research. First, in order to show that learning without surveys captures only low frequency movements of inflation, we use a sub-sample during which inflation was more stable. One alternative to this procedure is to model the low frequency movements of inflation explicitly, as in Sbordone (2007), and then to evaluate whether learning still adds persistence to inflation and whether survey data helps to identify this role correctly. In fact, considering all potential sources of inflation persistence (trend inflation, learning, exogenous shocks and frictions such as indexation), and using survey data to incorporate a compatibility condition over the expectation mechanism, constitutes a completely novel framework of analysis. Second, we only use the median value of inflation forecasting across the reports of all forecasters included in the SPF at each point in time. However, information about other moments such as the dispersion can be exploited to evaluate issues such as credibility of the central bank and how periods of high disagreement in expectations affect the conduct of monetary policy. Finally, survey data are also available for a variety of other macroeconomic indicators which could be used and potentially change the results about the identification of the sources of the business cycle, among other things. It is true that surveys about inflation expectations have been the subject of academic studies more often and have attracted less criticism concerning their quality than other variables. Nevertheless, this should not imply that survey data of other indicators do not contain any useful information at all.

To conclude, this study is one of the first to show how the appropriate use of information about expectations formation contained in survey data can have significant macroeconomic implications. So far, however, the information collected by surveys such as Survey of Professional Forecasters (SPF), but also Livingstone and Michigan surveys or the Greenbook have been largely neglected by empirical macroeconomic studies. The use of this information could improve our understanding of the workings of the economy and might change some pre-established ideas.

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Appendix 1: Optimization problem of the agents and equilibrium conditions

1. Final good producers

As in Kimball (1995), the final good Y_t is a composite made of a continuum of intermediate goods $Y_t(i)$. The final good producers buy intermediate goods, package Y_t, and sell it to consumers, investors and the government in a perfectly competitive market. Their maximization problem is as follows:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

$$s.t. \left[\int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \varepsilon_t^p\right) di \right] = 1$$

where P_t and $P_t(i)$ are the prices of the final and intermediate goods, respectively. G is a strictly concave and increasing function characterized by *G(1)=1 and* ε_t^p is a stochastic parameter that determines the time-varying markup in the goods market. Combining the first order conditions (FOCs) of the above outlined maximization process yields the following expression:

$$Y(i) = Y_t G'^{-1}\left[\frac{P_t(i)}{P_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}\right) \frac{Y_t(i)}{Y_t} di \right]$$

Hence, the assumptions on *G()*, as defined in Kimball (1995), imply a demand for intermediate goods that is decreasing in its relative price, while the elasticity demand is increasing in the relative price.

2. Intermediate goods producers

The technology used by the intermediate good producer *i* is defined like:

$$Y_t(i) = \varepsilon_t^a K_t(i)^\alpha \left[\gamma^t L_t(i) \right]^{1-\alpha} - \gamma^t \Phi$$

where $K_t(i)$ is the capital services used in production, $L_t(i)$ is a composite labor input and Φ is a fixed cost. γ^t represents the labor-augmenting deterministic growth in the economy and ε_t^a is the total factor productivity.

Considering that profits are defined as:

$$P_t(i) Y_t(i) - W_t L_t(i) - R_t^k K_t(i)$$

where W_t is the aggregate nominal wage rate and R_t^k is the rental rate of capital. The resulting cost minimization conditions are:

$$(\partial L_t(i)) : \Theta_t(i)\gamma^{(1-\alpha)t}(1-\alpha)\varepsilon_t^{\alpha} K_t(i)^{\alpha} L_t(i)^{-\alpha} = W_t$$

$$(\partial K_t(i)) : \Theta_t(i)\gamma^{(1-\alpha)t}\alpha\varepsilon_t^{\alpha} K_t(i)^{\alpha-1} L_t(i)^{1-\alpha} = R_t^k$$

where $\Theta_t(i)$ is the Lagrange multiplier associated with the production function and equals the marginal cost MC_t.

Combining the previous optimization conditions and considering that the capital-labor ratio is equal across the firms implies that:

$$K_t = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^k}L_t$$

The marginal cost is the same across all the firms and equals to:

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \gamma^{-(1-\alpha)t}(\varepsilon_t^{\alpha})^{-1}$$

The optimal price set by the firm is determined considering a Calvo pricing setup with partial indexation in order to pass inflation. The optimization problem that the intermediate firm faces is as follows:

$$\max_{\widetilde{P}_t(i)} E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\widetilde{P}_t(i) \left(\prod_{l=1}^{s} \pi_{t+l-1}^{l_p} \pi_*^{1-l_p} \right) - MC_{t+s} \right] Y_{t+s}(i)$$
$$s.t. Y_{t+s}(i) = Y_{t+s} G'^{-1}\left(\frac{P_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}\right)$$

where $\widetilde{P}_t(i)$ is the newly set price, 1-ξ^p is the Calvo probability of being allowed to optimize, π_t is the gross inflation, where $\pi_t = \frac{P_t}{P_{t-1}}$, $\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}}$ is the nominal discount factor for firms, which equals the discount factor of the households who are the final owner of the firms. Lastly, $\tau_t = \int_0^1 G'(\frac{Y_t(i)}{Y_t})\frac{Y_t(i)}{Y_t} di$ and $X_{t,s} = \begin{Bmatrix} 1 \; for s=0 \\ \left(\prod_{l=1}^{s} \pi_{t+l-1}^{l_p} \pi_*^{1-l_p}\right) for s=1,...,\infty \end{Bmatrix}$.

The first order condition is given by:

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{t+s}(i) \left[X_{t,s} \widetilde{P}_t(i) + \left(\widetilde{P}_t(i) X_{t,s} - MC_{t+s} \right) \frac{1}{G'^{-1}\left(z_{t+s}\right)} \frac{G'\left(x_{t+s}\right)}{G''\left(x_{t+s}\right)} \right]$$

where $x_t = G'^{-1}(z_t)$ and $z_t = \dfrac{P_t(i)}{P_t}\tau_t$.

The aggregate price index is in this case given by:

$$P_t = (1-\xi_p) P_t(i) G'^{-1}\left[\frac{P_t(i)\tau_t}{P_t}\right] + \xi_p \pi_{t-1}^{l_p} \pi_*^{1-l_p} P_{t-1} G'^{-1}\left[\frac{\pi_{t-1}^{l_p} \pi_*^{1-l_p} P_{t-1}\tau_t}{P_t}\right]$$

3. Households

Household *j* chooses consumption $C_t(j)$, hours worked $L_t(j)$, bonds $B_t(j)$, investment $I_t(j)$ and capital utilization $Z_t(j)$ in order to maximize the following objective function:

$$E_t \sum_{s=0}^{\infty} \left[\frac{1}{1-\sigma_c} (C_{t+s}(j) - \eta C_{t+s-1})^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+s}(j)^{1+\sigma_l}\right)$$

subject to the budget constraint:

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_t^b R_{t+s} P_{t+s}} - T_{t+s}$$

$$\leq \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{t+s}^h(j) L_{t+s}(j)}{P_{t+s}} + \frac{R_{t+s}^k Z_{t+s}(j) \bar{K}_{t+s-1}(j)}{P_{t+s}} - a(Z_{t+s}(j)) \bar{K}_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}}$$

and the capital accumulation equation:

$$\bar{K}_t(j) = (1-\delta)\bar{K}_{t-1}(j) + \varepsilon_t^q \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j)$$

The degree of external habit formation is captured by η while σ_c and σ_l denote the inverse of the elasticity of intertemporal substitution (for constant labor) and the inverse of the elasticity of labor supply, respectively. The one-period bond is expressed on a discount basis. The term ε_t^b represents an exogenous premium on the return to bonds and should be interpreted as a reflection of inefficiencies in the financial sector that generates a premium on the deposit rates with respect to the risk free rate set by the central bank or a risk premium that households require in order to hold one-period bond. T_{t+s} are lump sum taxes or subsidies. In nominal terms, the income from labor effort is $W_{t+s}^{h}(j)L_{t+s}(j)$ and from renting capital services $R_{t+s}^{k}Z_{t+s}(j)\bar{K}_{t+s-1}(j)$, while the cost of changing the capital utilization is $P_{t+s}a(Z_{t+s}(j))\bar{K}_{t+s-1}(j)$. In period t, the amount of effective capital that households can rent to the firms is denoted as:

$$K_t(j) = Z_t(j)\bar{K}_{t-1}(j)$$

With respect to the capital accumulation equation, δ represents the depreciation rate, $S(\cdot)$ the adjustment cost, with $S(\gamma)=0$, $S'(\gamma)=0$ and $S''(\cdot)>0$. ε_t^q is a stochastic shock to the price of investment relative to consumption goods.

In equilibrium the decisions about consumption, hours worked, bonds, investment and capital utilization are the same across all the households. The first order conditions with respect to each of the previously mentioned variables are as follows:

$$(\partial C_t): \Xi_t = \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_t^{1+\sigma_l}\right)(C_t - \eta C_{t-1})^{-\sigma_c}$$

$$(\partial L_t): \left[\frac{1}{1-\sigma_c}(C_t - \eta C_{t-1})^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_t^{1+\sigma_l}\right)(\sigma_c - 1)L_t^{\sigma_l} = -\Xi_t \frac{W_t^h}{P_t}$$

$$(\partial B_t): \Xi_t = \beta \varepsilon_t^b R_t E_t\left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right]$$

$$(\partial I_t): \Xi_t = \Xi_t^k \varepsilon_t^q \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) + \beta E_t\left[\Xi_{t+1}^k \varepsilon_{t+1}^q S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

$$(\partial \bar{K}_t): \Xi_t^k = \beta E_t\left[\Xi_{t+1}\left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1})\right) + \Xi_{t+1}^k(1-\delta)\right]$$

$$(\partial u_t): \frac{R_t^k}{P_t} = a'(Z_t)$$

where Ξ_t and Ξ_t^k are the Lagrange multipliers associated with the budget and capital accumulation constraint respectively. Tobin's q is $Q_t = \frac{\Xi_t^k}{\Xi_t}$ and equals one in the absence of adjustment costs.

4. Intermediate labor union sector

As mentioned above there is a labor union in the economy that differentiates the labor services provided by the households and sets wages subject to a Calvo probability scheme with the labor packers. Labor packers take labor services from the union $L_t(l)$, package L_t and resell it to the intermediate goods producers. L_t is a composite "product" that aggregates $L_t(l)$ using the aggregator proposed by Kimball (1995).

Labor packers maximize profit in a perfectly competitive environment:

$$\max_{L_t, L_t(I)} W_t L_t - \int_0^1 W_t(I) L_t(I) dI$$

$$s.t. \left[\int_0^1 H\left(\frac{L_t(I)}{L_t}; \varepsilon_t^w\right) dI \right] = 1$$

where W_t and $W_t(I)$ represent the prices of the composite and intermediate labor services respectively, and H is a strictly concave and increasing function characterized by $H(1) = 1$. ε_t^w is an exogenous process that reflects shocks to the aggregator function that result in changes in the elasticity of demand and therefore in the mark up. We will constrain $\varepsilon_t^w \in (0, \infty)$. Combining FOCs yields the following result:

$$L_t(I) = L_t H'^{-1} \left[\frac{W_t(I)}{W_t} \int_0^1 H'\left(\frac{L_t(I)}{L_t}\right) \frac{L_t(I)}{L_t} dI \right]$$

The labor unions represent the intermediates between households and the labor packers. In their negotiations with the labor packers they consider the marginal rate of substitution between consumption and labor of the households. Given that unions possess of market power they can generate some markups that are distributed among the households. However, the choice of the wage level is subject to nominal rigidities *à la* Calvo. Unions can adjust wages with a probability $1-\xi_w$ in each period. For those unions that cannot re-optimize within one period, $W_t(l)$ will increase at the deterministic growth rate of γ and weighted average rate of the steady-state inflation π_* and of the last period's inflation (π_{t-1}). The maximization problem faced by the unions allows for the possibility of getting stuck with the determined wage level for the following infinite periods having the previously mentioned indexation mechanism as the only way to adjust nominal wages. Thus, optimal wage level $\widetilde{W}_t(l)$ maximize the value of the following object:

$$\max_{\widetilde{W}_t(I)} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \left[\widetilde{W}_t(I) \left(\prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_w} \pi_*^{1-l_w} - W_{t+s}^h \right) \right] L_{t+s}(I)$$

$$s.t. L_{t+s}(I) = L_{t+s} H'^{-1} \left(\frac{W_t(I) X_{t,s}^w}{W_{t+s}} \tau_{t+s}^w \right)$$

where $\tau_t^w = \int_0^1 H' \left(\frac{L_t(I)}{L_t} \right) \frac{L_t(I)}{L_t} dI$ and $X_{t,s} = \left\{ \begin{matrix} 1 \; for s=0 \\ \left(\prod_{l=1}^{s} \gamma \pi_{t+l-1}^{l_w} \pi_*^{1-l_w} \right) for s=1,\ldots,\infty \end{matrix} \right\}$.

The FOC is given by

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} L_{t+s}(I) \left[X_{t,s}^w \widetilde{W}_t(I) + \left(\widetilde{W}_t(I) X_{t,s}^w - W_{t+s}^h \right) \right] \frac{1}{H'^{-1}(z_{t,s}^w)} \frac{H'(x_{t,s}^w)}{H''(z_{t,s}^w)} = 0$$

where $x_{t,s}^w = H'^{-1}(z_t^w)$ and $z_t^w = \frac{W_t(I)}{W_t} \tau_t^w$

After some algebraic manipulation one achieves the following expression for aggregate wages:

$$W_t = (1-\xi_w) \widetilde{W}_t H'^{-1} \left[\frac{\widetilde{W}_t \tau_t^w}{W_t} \right] + \xi_w \gamma \pi_{t-1}^{l_w} \pi_*^{1-l_w} W_{t-1} H'^{-1} \left[\frac{\gamma \pi_{t-1}^{l_w} \pi_*^{1-l_w} W_{t-1} \tau_t^w}{W_t} \right]$$

5. Government policies

The central bank adjusts the nominal interest rate in response to deviations of the inflation and the output growth to their respective target levels:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho} \left[\left(\frac{\pi_t}{\pi^*} \right)^{r_\pi} \left(\frac{Y_t}{Y_{t-1}} \Big/ \frac{Y_t^*}{Y_{t-1}^*} \right)^{r_{\Delta y}} \right]^{1-\rho} \varepsilon_t^r$$

where R^* is the steady state nominal gross interest rate and Y_t^* is defined as the potential output taking into account only the exogenous process for total factor productivity and the trend growth of the economy:

$$Y_t^* = \varepsilon_t^a \overline{K}^{\alpha} \left[\gamma^t \overline{L} \right]^{1-\alpha} - \gamma^t \Phi$$

The government budget constraint has the following form:

$$P_tG_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

Government spending (G_t) is exogenous and expressed relative to the steady-state output path as $\varepsilon_t^g = \frac{G_t}{Y\gamma^t}$.

6. Resource constraints

The market clearing condition for the final goods market can be obtained by integrate the households' budget constraint across all of them and combine with the government budget constraint. The resulting resource constraint is:

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t$$

Appendix 2: Stochastic part of the model

The stochastic part of the model is characterized by seven exogenous processes: two which affect the intertemporal margin, such as the risk premium shocks, $\widehat{b_t}$, and the investment-specific technology shocks, $\widehat{q_t}$; further two that affect the intratemporal margin, such as the wage mark-up shocks, $\widehat{\lambda_{w,t}}$, and the price mark-up shocks, $\widehat{\lambda_{p,t}}$; another two policy shocks, the exogenous spending, $\widehat{g_t}$, and the monetary policy shocks, $\widehat{r_t}$; and last the total factor productivity shocks, $\widehat{A_t}$.

Each of these processes are characterized as first-order autoregressive (AR(1)) process with and *iid*-normal distributed error term. Their representations are the following:

- $\hat{g}_t = \rho_g \hat{g}_{t-1} + \rho_{ga} \varepsilon_t^a + \varepsilon_t^g$
- $\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_t^b$
- $\hat{q}_t = \rho_q \hat{q}_{t-1} + \varepsilon_t^\mu$
- $\hat{A}_t = \rho_a \hat{A}_{t-1} + \varepsilon_t^a$
- $\widehat{\lambda_{p,t}} = \rho_p \widehat{\lambda_{p,t-1}} + \varepsilon_t^p$
- $\widehat{\lambda_{w,t}} = \rho_w \widehat{\lambda_{w,t-1}} + \varepsilon_t^w$
- $\hat{r}_t = \rho_R \widehat{r_{t-1}} + \varepsilon_t^r$

Appendix 3: Definition of the dataset[13]

Definition of data variables

- consumption = LN((PCEC / GDPDEF) / LNSindex) * 100
- investment = LN((FPI / GDPDEF) / LNSindex) * 100
- output = LN(GDPC96 / LNSindex) * 100
- hours = LN((PRS85006023 * CE16OV / 100) / LNSindex) * 100
- inflation = LN(GDPDEF / GDPDEF(-1)) * 100
- real wage = LN(PRS85006103 / GDPDEF) * 100
- interest rate = Federal Funds Rate / 4

Source of the original data:

GDPC96 : Real Gross Domestic Product - Billions of Chained 1996 Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

GDPDEF : Gross Domestic Product - Implicit Price Deflator - 1996=100, Seasonally Adjusted Source: U.S. Department of Commerce, Bureau of Economic Analysis

PCEC : Personal Consumption Expenditures - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

FPI : Fixed Private Investment - Billions of Dollars, Seasonally Adjusted Annual Rate. Source: U.S. Department of Commerce, Bureau of Economic Analysis

CE16OV : Civilian Employment: Sixteen Years & Over, Thousands, Seasonally Adjusted. Source: U.S. Department of Labor: Bureau of Labor Statistics

CE16OV index : CE16OV (1992:3)=1

Federal Funds Rate : Averages of Daily Figures – Percent. Source: Board of Governors of the Federal Reserve System (Before 1954: 3-Month Treasury Bill Rate, Secondary Market Averages of Business Days, Discount Basis)

LFU800000000 : Population level - 16 Years and Older - Not Seasonally Adjusted. Source: U.S. Bureau of Labor Statistics

LNS10000000 : Labor Force Status : Civilian noninstitutional population - Age : 16 years and over . Seasonally Adjusted - Number in thousands. Source: U.S. Bureau of Labor Statistics (before 1976: LFU800000000 : Population level - 16 Years and Older)

[13] Taken from the data documentation of Smets and Wouters (2007)

LNSindex : LNS10000000(1992:3)=1

PRS85006023 - Nonfarm Business, All Persons, Average Weekly Hours Duration : index, 1992 = 100, Seasonally Adjusted. Source : U.S. Department of Labor

PRS85006103 - Nonfarm Business, All Persons, Hourly Compensation Duration : index, 1992 = 100, Seasonally Adjusted. Source : U.S. Department of Labor

Appendix 4: Prior distributions of structural parameters

	Symbol	Distribution	Mean	Std.
Share of capital in production	α	Normal	0.30	0.05
Inv. Elasticity of Intertemporal substitution	σ_c	Normal	1.50	0.38
Fix cost in production	Φ	Normal	1.25	0.13
Adjust cost of investment	S''	Normal	4.00	1.50
Habits in consumption	η	Beta	0.70	0.10
Wage stickiness	ξ_w	Beta	0.50	0.10
inv. Elast. labor supply	σ_l	Normal	2.00	0.75
Price stickiness	ξ_p	Beta	0.50	0.10
Wage indexation	ι_w	Beta	0.50	0.15
Price indexation	ι_p	Beta	0.50	0.15
Capital utilization elasticity	ψ	Beta	0.50	0.15
Taylor rule: response to inflation	r_π	Normal	1.50	0.25
Taylor rule: response to lagged interest rate	ρ_R	Beta	0.75	0.10
Taylor rule: response to changes in output	$r_{\Delta y}$	Normal	0.13	0.05
Trend growth rate	γ	Normal	0.40	0.10
Steady state of inflation	π _bar	Gamma	0.63	0.10
Steady state of hours worked	l _bar	Normal	0.00	2.00
Steady state of nominal int rate	r _bar	Gamma	1.15	0.30
Autocorrelation coef. Price Mk up shock	ρ_p	Beta	0.50	0.20
Autocorrelation coef. Wage Mk up shock	ρ_w	Beta	0.50	0.20
Autocorrelation coef. Product. Shock	ρ_a	Beta	0.50	0.20
Autocorrelation coef. Risk premium shock	ρ_b	Beta	0.50	0.20
Autocorrelation coef. Government shock	ρ_g	Beta	0.50	0.20
Autocorrelation coef. Investment-Specific shock	ρ_q	Beta	0.50	0.20
Autocorrelation coef. Monet policy shock	ρ_r	Beta	0.50	0.20
Correlation Government and productivity shocks	ρ_{ga}	Normal	0.50	0.25
Std Price Mk up innovation	σ_p	Inv. Gamma	0.10	2.00
Std. Wage Mk up innovation	σ_w	Inv. Gamma	0.10	2.00
Std. Product. Innovation	σ_a	Inv. Gamma	0.10	2.00
Std. Risk premium innovation	σ_b	Inv. Gamma	0.10	2.00
Std. Government innovation	σ_g	Inv. Gamma	0.10	2.00
Std. Inv. Specific innovation	σ_q	Inv. Gamma	0.10	2.00
Std. Monet policy innovation	σ_r	Inv. Gamma	0.10	2.00
Gain - no inflation	g^{π}	Uniform	0.00	0.30
Gain - inflation	$g^{non\pi}$	Uniform	0.00	0.30
Std. measurement error on expectations	σ_{exp}	Inv. Gamma	0.10	2.00

Note: for uniform distributions the values assigned as mean and standard deviation correspond to the range of the domain.

Appendix 5: Posterior distribution statistics: RE and learning estimations

		RE		Learning	
	Symbol	Median	Std.	Median	Std.
Share of K in production	α	0,182	0,018	0,184	0,018
Inv. Elast. Intertp. Sust.	σ_c	1,159	0,084	1,242	0,105
Fix cost product.	Φ	1,574	0,074	1,640	0,080
Adj.cost inv.	S''	5,841	0,948	7,115	1,242
Habits	η	0,814	0,028	0,822	0,029
Wage stickiness	ξ_w	0,552	0,042	0,565	0,066
Elast. labor supply	σ_l	2,261	0,579	2,426	0,612
Price stickiness	ξ_p	0,648	0,042	0,489	0,038
Wage indexation	ι_w	0,491	0,137	0,335	0,101
Price indexation	ι_p	0,281	0,149	0,506	0,111
Cap. Utiliz. Elast.	ψ	0,638	0,098	0,640	0,105
TR: inflation	r_π	1,660	0,120	1,390	0,120
TR: lag interest rate	ρ_R	0,756	0,028	0,778	0,025
TR: change in output	$r_{\Delta y}$	0,205	0,045	0,210	0,045
aut. Price Mk up shock	ρ_p	0,518	0,200	0,138	0,072
aut. Wage Mk up shock	ρ_w	0,960	0,013	0,946	0,029
aut. Product. Shock	ρ_a	0,960	0,012	0,962	0,012
aut. Risk premium	ρ_b	0,124	0,060	0,155	0,071
aut. Government shock	ρ_g	0,991	0,004	0,993	0,004
aut. Inv. Specific shock	ρ_q	0,841	0,034	0,842	0,035
aut. Monet policy shock	ρ_r	0,217	0,067	0,195	0,063
Corr. Gov & product sks	ρ_{ga}	0,588	0,086	0,572	0,098
std. Price Mk up shock	σ_p	0,135	0,026	0,211	0,013
std. Wage Mk up shock	σ_w	0,185	0,030	0,211	0,032
std. Product. Shock	σ_a	0,458	0,029	0,440	0,025
std. Risk premium	σ_b	0,248	0,021	0,247	0,022
std. Government shock	σ_g	0,481	0,028	0,497	0,027
std. Inv. Specific shock	σ_q	0,342	0,030	0,327	0,028
std. Monet policy shock	σ_r	0,264	0,016	0,257	0,014
Gain - others	$g^{non\pi}$			0,105	0,043
Gain - inflation	g^{π}			0,187	0,012
Measurement exp error	σ_{exp}				
Log Mg. Likelihood		-146,4		-144,2	

Total number of draws is 500 thousands. After discarding the first half, 1 out of every 10 is selected to estimate the moments of the posterior distribution.

Appendix 6: Posterior distribution statistics: RE and learning estimations, Post-84 sample

		WITHOUT survey data				WITH survey data			
		RE		Learning		RE		Learning	
	Symbol	Median	Std.	Median	Std.	Median	Std.	Median	Std.
Share of K in production	α	0,159	0,021	0,173	0,023	0,167	0,023	0,185	0,023
Inv. Elast. Intertp. Sust.	σ_c	0,839	0,125	1,531	0,222	1,430	0,128	1,477	0,207
Fix cost product.	Φ	1,447	0,097	1,581	0,085	1,697	0,087	1,632	0,090
Adj.cost inv.	S''	4,847	1,208	7,121	1,231	8,615	1,004	7,147	1,170
Habits	η	0,654	0,060	0,741	0,076	0,839	0,024	0,751	0,051
Wage stickiness	ξ_w	0,248	0,057	0,464	0,070	0,892	0,032	0,461	0,067
Elast. labor supply	σ_l	2,678	0,687	2,603	0,666	1,758	0,730	2,382	0,571
Price stickiness	ξ_p	0,457	0,049	0,619	0,025	0,878	0,019	0,681	0,034
Wage indexation	ι_w	0,484	0,144	0,392	0,135	0,349	0,130	0,368	0,125
Price indexation	ι_p	0,170	0,070	0,463	0,118	0,222	0,105	0,665	0,102
Cap. Utiliz. Elast.	ψ	0,762	0,100	0,578	0,116	0,556	0,122	0,632	0,118
TR: inflation	r_π	2,074	0,171	1,650	0,170	1,289	0,084	1,526	0,169
TR: lag interest rate	ρ_R	0,776	0,026	0,806	0,024	0,802	0,022	0,793	0,034
TR: change in output	$r_{\Delta y}$	0,169	0,044	0,183	0,048	0,183	0,050	0,181	0,044
aut. Price Mk up shock	ρ_p	0,978	0,011	0,075	0,053	0,158	0,089	0,113	0,067
aut. Wage Mk up shock	ρ_w	0,991	0,006	0,977	0,010	0,603	0,089	0,969	0,018
aut. Product. Shock	ρ_a	0,988	0,007	0,995	0,005	0,998	0,001	0,996	0,003
aut. Risk premium	ρ_b	0,818	0,049	0,332	0,199	0,099	0,062	0,264	0,194
aut. Government shock	ρ_g	0,983	0,011	0,975	0,011	0,987	0,007	0,972	0,012
aut. Inv. Specific shock	ρ_q	0,809	0,063	0,909	0,048	0,890	0,048	0,929	0,030
aut. Monet policy shock	ρ_r	0,405	0,069	0,392	0,067	0,435	0,059	0,438	0,063
Corr. Gov & product sks	ρ_{ga}	0,420	0,099	0,461	0,102	0,420	0,105	0,435	0,094
std. Price Mk up shock	σ_p	0,127	0,020	0,111	0,012	0,154	0,014	0,178	0,016
std. Wage Mk up shock	σ_w	0,564	0,151	0,259	0,052	0,173	0,033	0,260	0,058
std. Product. Shock	σ_a	0,384	0,032	0,363	0,029	0,366	0,026	0,360	0,029
std. Risk premium	σ_b	0,061	0,011	0,161	0,040	0,207	0,018	0,175	0,038
std. Government shock	σ_g	0,396	0,029	0,391	0,027	0,386	0,027	0,387	0,032
std. Inv. Specific shock	σ_q	0,289	0,038	0,276	0,036	0,200	0,032	0,262	0,032
std. Monet policy shock	σ_r	0,139	0,011	0,125	0,012	0,120	0,008	0,125	0,011
Gain - others	$g^{non\pi}$			0,0465	0,0366			0,0045	0,009
Gain - inflation	g^{π}			0,0055	0,0042			0,2008	0,0059
Measurement exp error	σ_{exp}					0,157	0,013	0,151	0,010
Log Mg. Likelihood		74,6		62,5		180,917		184,2	

Total number of draws is 500 thousands. After discarding the first half, 1 out of every 10 is selected to estimate the moments of the posterior distribution.